# The relation between cross-section, decay width and imaginary potential of heavy quarkonium in a quark-gluon plasma

Miguel A. Escobedo

Physik-Department T30f. Technische Universität München

28th of May

Nora Brambilla, MAE, Jacopo Ghiglieri, Antonio Vairo, JHEP12(2011)116
and in preparation.
Hard Probes 2012, Cagliari

# Outline

# Motivation

# What has been found until now using EFTs in Quarkonia?

- EFT provide a systematic way to extract information from the fact that $m_Q \gg \frac{1}{r} \gg E$ in Quarkonia. Computations are easier and it is more difficult to neglect a needed resummation.
- For $T \gg \frac{1}{r} \sim m_D$ we recover the perturbative potential with an imaginary part found by Laine, Philipsen, Romatschke and Tassler (2007).
- For $T \lesssim \frac{1}{r}$ we were able to compute thermal corrections to the binding energies and the decay width.
- For the decay width we found two different mechanism. The breaking of the singlet into an octet due to the absorption of a gluon from the medium and the Landau damping of the gluons that are exchanged between the heavy quarks.

# Other approach to quarkonia decay width

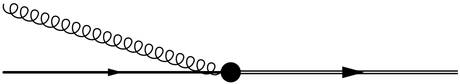
- Use a cross-section computed at $T = 0$, $\sigma(k)$.
- Convolute with the thermal distribution

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)\sigma(k)$$

- These cross-sections are computed in perturbation theory and later they are "adapted" to strong coupling by using $\alpha_{\rm s}$ as a free parameter, introducing thermal masses...
- This information is used as an input to predict the observed suppression in nowadays experiments. See for example Zhao and Rapp (2010).

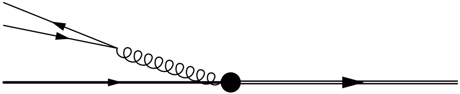
# Perturbative computations of cross-section for quarkonia in the literature

Gluo-dissociation



Bhanot and Peskin (1979)

Quasi-free dissociation



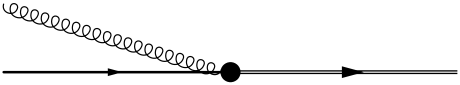
Combridge (1978), Park, Kim, Song, Lee and Wong (2007)

# Motivation

- Translate the EFT results that have been found to cross-sections convoluted with distribution function "language".
- Analyze the assumptions made by previous perturbative computations and check if they agree or disagree with the EFT framework.

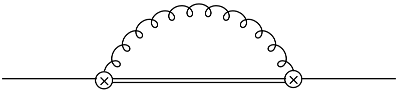
# Gluo-dissociation

# Gluo-dissociation in Bhanot and Peskin



- They use OPE. The interaction between the singlet, the octet and the gluon is a color dipole interaction.
- This approximation is convenient because the gluo-dissociation is the dominant dissociation mechanism only for $E \gg m_D$. It is very similar to what is done in pNRQCD.
- They use the large $N_c$ limit approximation. In this limit $V_o = 0$ and computations are simplified.
- We are going to see that the large $N_c$ limit is a good approximation for $T \gg E$ but not for $T \sim E$.

# Gluo-dissociation in pNRQCD



- Computed for $T \gg E$ in HQ. Brambilla, MAE, Ghiglieri, Soto and Vairo (2010)

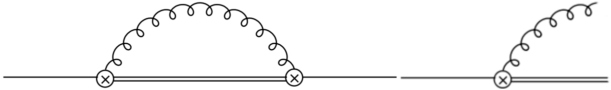
$$\delta\Gamma_n = \frac{1}{3}N_C^2 C_F \alpha_s^3 T - \frac{16}{3m} C_F \alpha_s T E_n + \frac{4}{3} N_C C_F \alpha_s^2 T \frac{2}{mn^2 a_0}$$

where $E_n$ is the binding energy and $a_0$ the Bohr radius.

- Computed for $T \sim E$ in the hydrogen atom. MAE and Soto (2008).

$$\delta\Gamma_n = \frac{4}{3}\alpha_s C_F T \langle n | r_i \frac{|E_n - h_o|^3}{e^{\beta |E_n - h_o|} - 1} r_i | n \rangle$$

# Cutting rules at finite temperature

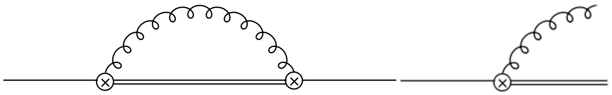


Similar to what is found at $T = 0$.

- Multiply by $n_B(k)$ $(n_F(k))$ for in-coming bosons (fermions).
- Multiply by $1 + n_B(k)$ $(1 - n_F(k))$ for out-going bosons (fermions).

Kobes and Semenoff (1986)

# Cutting rules at finite temperature



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- Multiply by $n_B(k)$ ($n_F(k)$) for in-coming bosons (fermions).
- Multiply by $1 + n_B(k)$ ($1 - n_F(k)$) for out-going bosons (fermions).

In this case we get a structure

$$\delta\Gamma_n = \int \frac{d^3k}{(2\pi)^3} n_B(k) \langle n | h_\sigma(r, p, k) | n \rangle$$

# A choice

$$\delta\Gamma_n = \int \frac{d^3k}{(2\pi)^3} n_B(k) \langle n | h_\sigma(r, p, k) | n \rangle$$

- If we integrate out $k$ first we recover the pNRQCD result
- If we choose for example $n = 1S$ and compute the matrix element.

$$\delta\Gamma_{1S} = \int \frac{d^3k}{(2\pi)^3} n_B(k) \sigma_{gd}(k)$$

# pNRQCD gluo-dissociation $\sigma_{gd}$ for $1S$

- If we do the same approximations as Bhanot and Peskin (large $N_c$ limit) we recover their result.
- Without doing this approximation we get

$$\sigma_{gd}(k) = \frac{8\pi^2 C_F \alpha_{\rm s} m a_0^2 k}{3} |\langle 1S|r_i|ma_0^2(k+E_1)\rangle_o|^2 \Theta(k+E_1)$$

$|\epsilon\rangle_o$ are the octet wave function taking into account the octet potential.

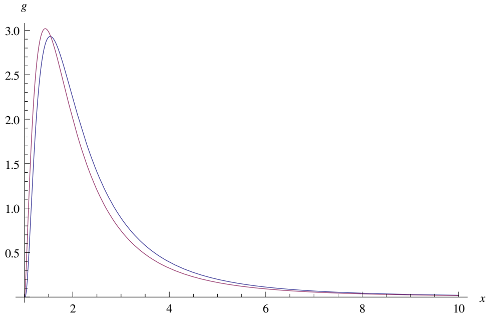
$$\int_0^\infty d\epsilon \langle\epsilon|\epsilon\rangle_o = 1$$

Agrees with Brezinski and Wolschin (2011)

# Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation

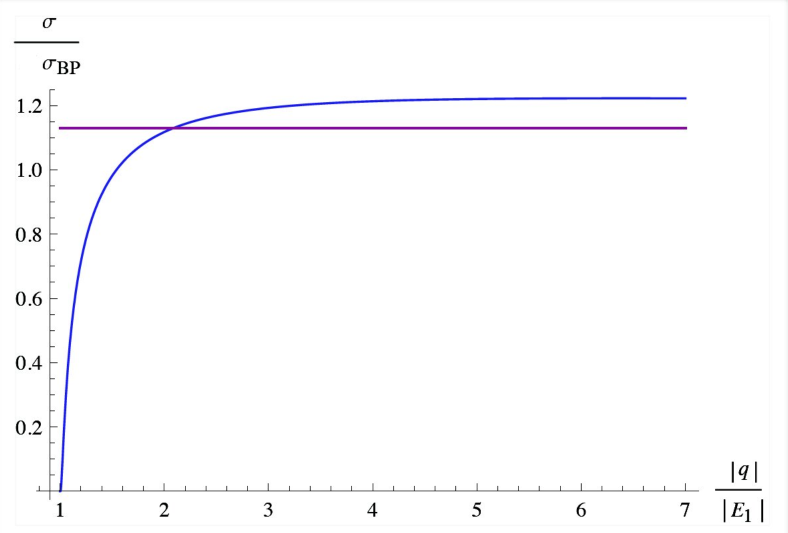
$$\sigma_{gd}(k) = \sigma_R g(x)$$

with $\sigma_R = \frac{32\pi C_F \alpha_s a_0^2}{3}$ and $x = \frac{k}{|E_1|}$

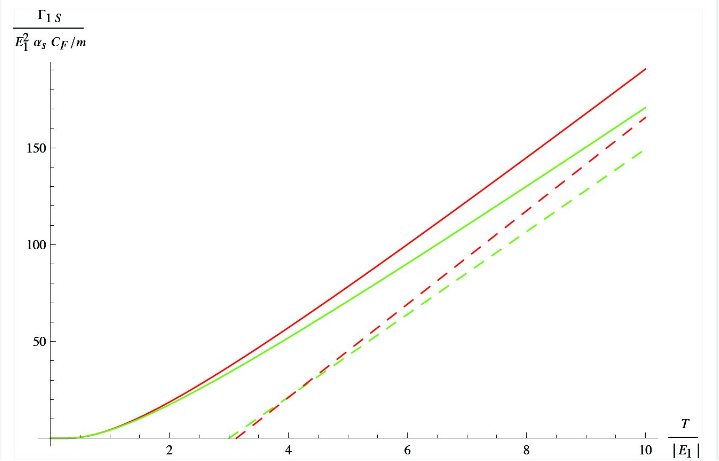



Bhanot and Peskin large $N_c$ limit
pNRQCD

# Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation

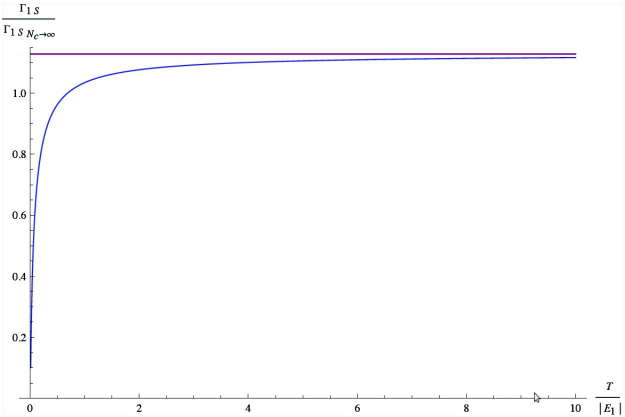


# Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation




Bhanot and Peskin, pNRQCD

# Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation

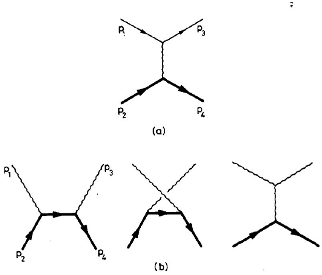


# Quasi-free dissociation

# (Or Landau damping)

# Quasi-free in Combridge

He computed the process $qc \to qc$ for a charm quark, no information of the bound state is included.



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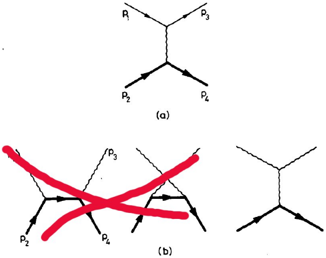
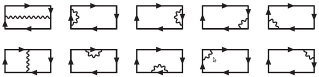



Note that in NRQCD (valid for $m_Q \gg T$) and using the Coulomb gauge the crossed diagrams are subleading.

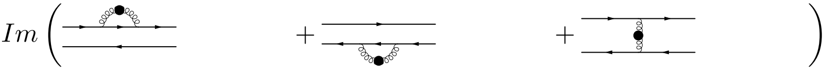
# HQ potential for $T \gg \frac{1}{r} \sim m_D$

Laine, Philipsen, Romatschke and Tassler

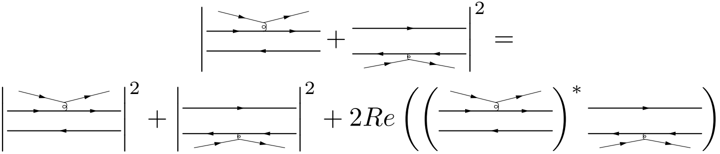


pNRQCD

# Imaginary part of the potential




By the optical theorem



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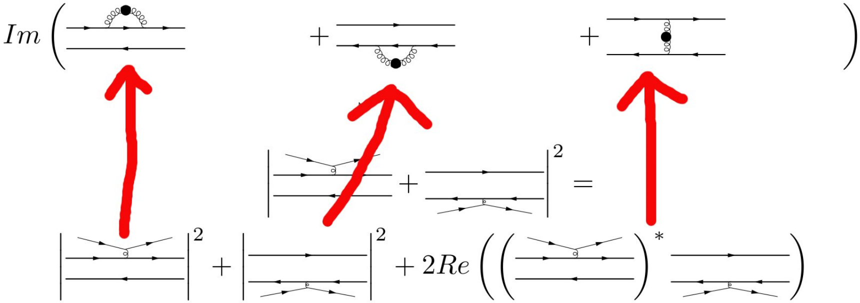
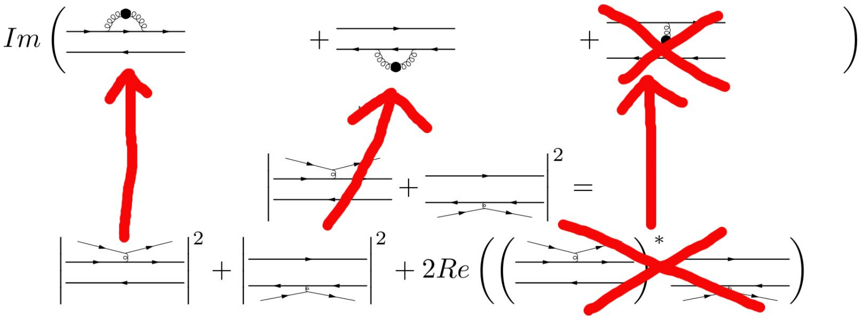
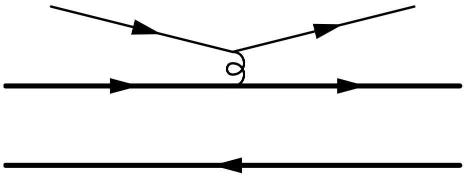


# Combridge approximation




Interference term is neglected. Good approximation for $T, m_D \gg \frac{1}{r}$.

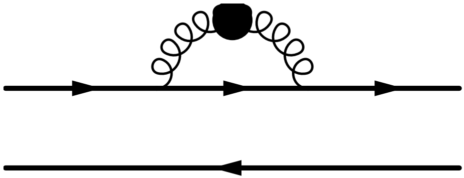
# From the cross-section to the decay width



- Apart from the heavy quarks that are not thermalized, there is an in-coming parton and an out-going parton.
- The decay width then has the structure

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)(1 + f(k))\sigma(k)$$

# From the cross-section to the decay width



The information of in-coming and out-going parton is included in EFT formulation in the symmetric self-energy that is included in the gluon propagator (often in the HTL approximation). For example, the part related with fermion loops is

$$\Pi^S_{00}(q \gg q_0) = \frac{4ig^2 T_F N_F}{\pi q} \int_{k > \frac{q}{2}} dk k^2 \left(1 - \frac{q^2}{4k^2}\right) n_F(k)(1 - n_F(k))$$

# From the cross-section to the decay width

In conclusion, thermal field theory does not justify in this case

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)\sigma(k)$$

but instead

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)(1 + f(k))\sigma(k)$$

# Cross-section for the $1S$ state

We proceed in a similar way to what is done for the gluo-dissociation. We start by our previous EFT computations and "translate" them

- In gluo-dissociation only a energy scale was relevant. This is not the case now.
- As we need information of the scale $m_D$ the HTL has to be performed at some part of the computation. $\sigma$ is going to depend also on the temperature due to this.

# Some notation

$$\sigma(k, m_D) = \sigma_R f(x, y)$$

where

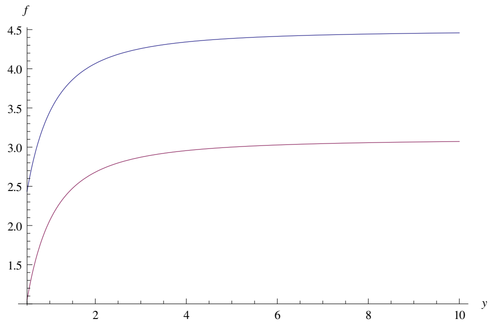
$$\sigma_R = 8\pi C_F \alpha_s^2 N_F a_0^2$$
$$x = m_D a_0$$
$$y = k a_0$$

I will only show the result for the fermion part, the boson part is quantitatively and qualitatively very similar.

# $T \sim \frac{1}{r} \gg m_D$ cross-section for $1S$

$$f(x,y) = -\frac{3}{2} + 2\log\left(\frac{2}{x}\right) + \log\left(\frac{y^2}{1+y^2}\right) - \frac{1}{y^2}\log(1+y^2)$$

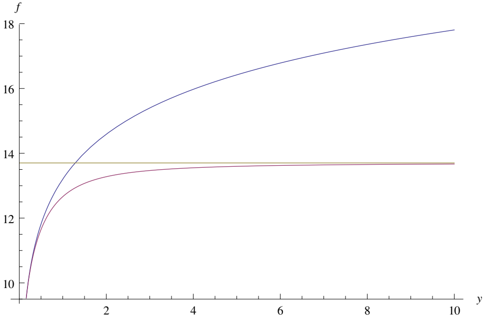
$x \ll 1$ and $y \sim 1$.




$m_D a_0 = 0.1$ and $m_D a_0 = 0.2$.

# Summary cross-section for $1S$

$m_D a_0 = 0.001$




$\frac{1}{r} \gg T \gg m_D$, $T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

# Conclusions

- The Bhanot and Peskin result that is normally used correspond to the large $N_c$ limit of pNRQCD result. This is a good approximation for $T \gg E$ but it is not so good for $T \sim E$.
- The imaginary part of the potential and the quasi-free dissociation describe the same physical process at different temperatures.
- The perturbative computations of the cross-section that existed before the use of EFT techniques ignored all bound state properties or used a cross-section equivalent to color dipole approximation.
- The quasi-free cross-section goes to an asymptotic value for large incoming momentum. This is the physics described by the imaginary part of Laine et al. potential.

# The End

# pNRQCD gluo-dissociation $\sigma_{gd}$ for $1S$

The Coulomb wave-function with a repulsive potential (as the one of the octet) were taken from Abramowitz and Stegun (1972)

$$\sigma_{gd}(k) = \frac{32\pi C_F \alpha_{\rm s} m a_0^3 k \left(C_1\left(\frac{1}{8\sqrt{\tau}}\right)\right)^2 \left(f\left(\frac{1}{\sqrt{\tau}}\right)\right)^2}{3\tau^{7/2}}\Theta(\tau)$$

where

$$\tau = m a_0^2 (k + E_1)$$

$$C_1(x) = \frac{\sqrt{1+x^2}}{3}\sqrt{\frac{2\pi x}{e^{2\pi x}-1}}$$

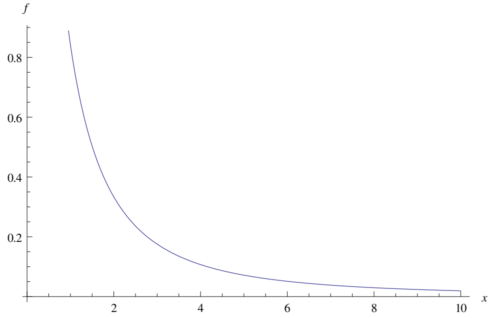
$$f(x) = \frac{51}{2}\frac{x e^{\frac{x}{4} arccot(x)}}{(x+1)^3}$$

Agrees with Brezinski and Wolschin (2011)

# $T \gg \frac{1}{r} \sim m_D$ cross-section for $1S$

$$f(x,y) = 2\left(1 - 4\frac{x^4 - 16 + 8x^2\log\left(\frac{4}{x^2}\right)}{(x^2-4)^3}\right)$$

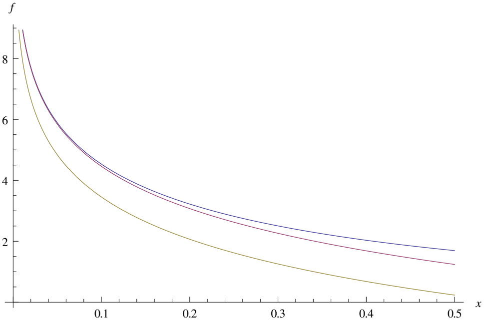
$x \sim 1$ and $y \gg 1$



# $T \sim \frac{1}{r} \gg m_D$ cross-section for $1S$

$$f(x,y) = -\frac{3}{2} + 2\log\left(\frac{2}{x}\right) + \log\left(\frac{y^2}{1+y^2}\right) - \frac{1}{y^2}\log(1+y^2)$$

$x \ll 1$ and $y \sim 1$.




$T \gg \frac{1}{r} \sim m_D$, $ka_0 = 10$ and $ka_0 = 1$. Discrepancy between the blue and red line signals the need for HTL resummation.

# $\frac{1}{r} \gg T \gg m_D \gg E$ cross-section for $1S$

$$f(x,y) = 2\left(\log\left(\frac{2y}{x}\right) - 1\right)$$

$1 \gg y \gg x.$