COHERENCE AND BROADENING EFFECTS IN MEDIUM INDUCED GLUON RADIATION

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Hard Probes 2012

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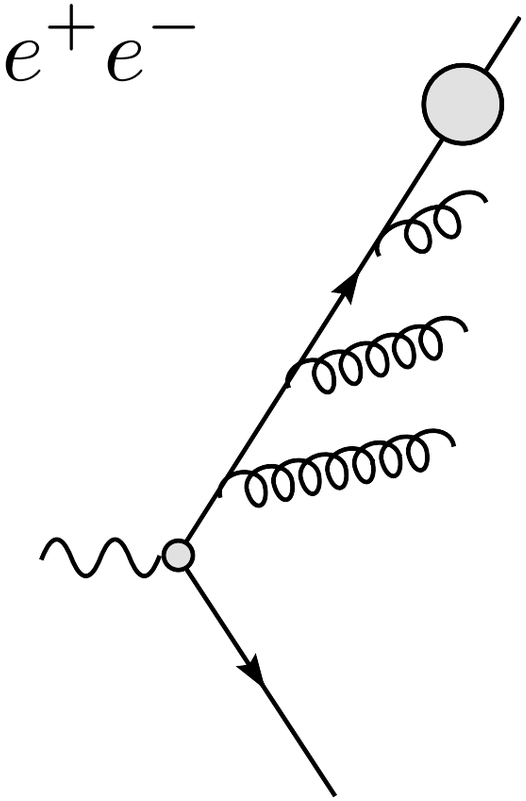
Work in progress

UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

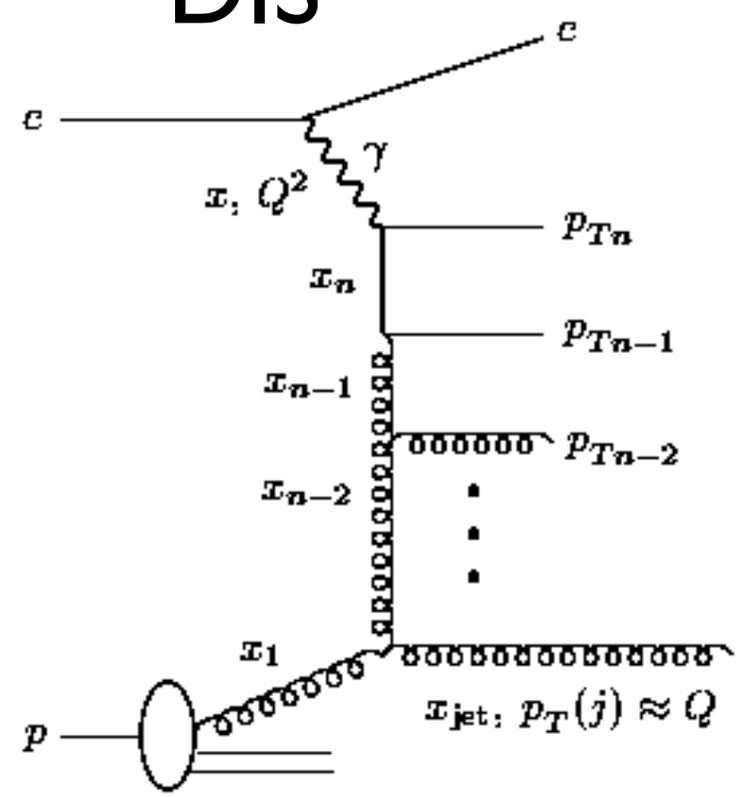
Color coherence in vacuum

e^+e^-



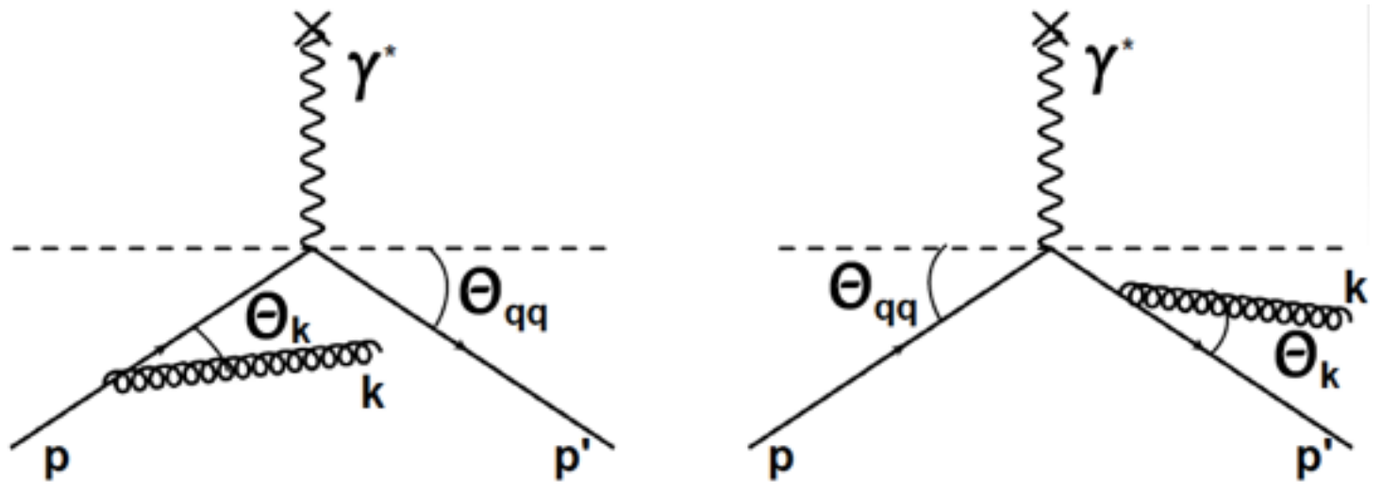
Time-like
Fragmentation functions

DIS



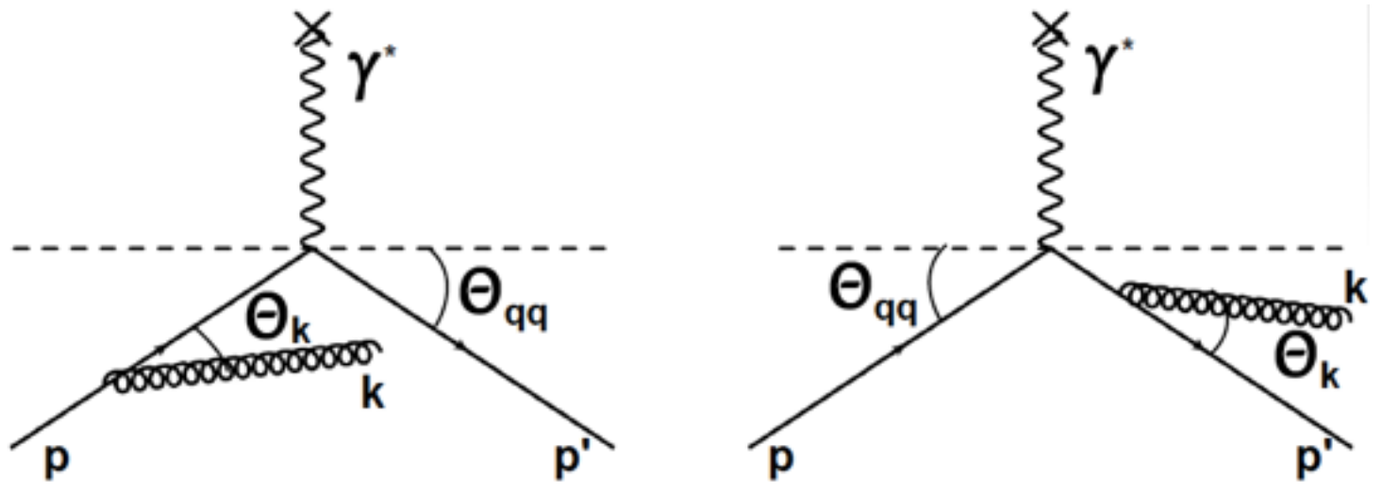
Space-like
PDF's

Angular ordering in Initial State Radiation



$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} = \frac{\alpha_s\, C_F}{(2\pi)^2\, \omega^2} \left(\mathcal{P}_{in} + \mathcal{P}_{out}\right)$$

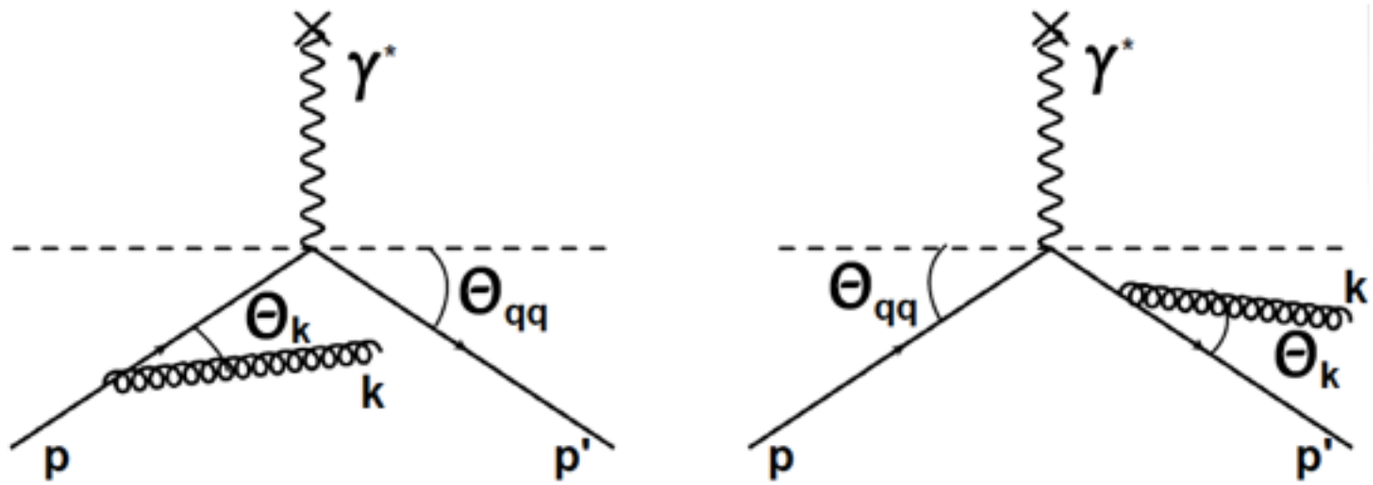
Angular ordering in Initial State Radiation



$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} = \frac{\alpha_s C_F}{(2\pi)^2\,\omega^2}\left(\mathcal{P}_{in} + \mathcal{P}_{out}\right)$$

Color charge

Angular ordering in Initial State Radiation



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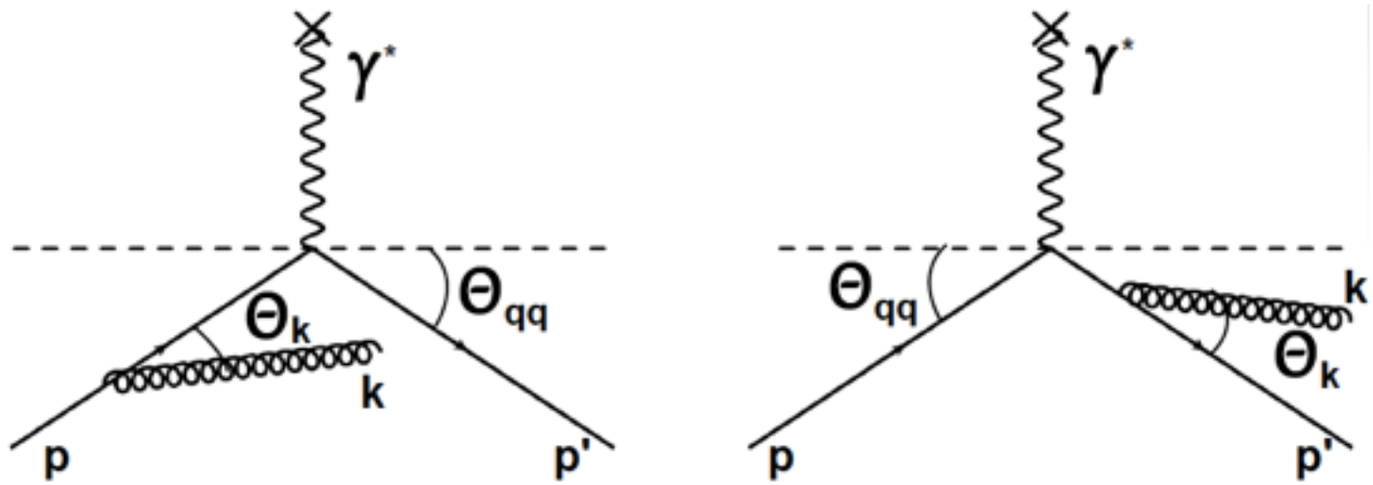
Color charge

Independent emissions

Angular ordering in Initial State Radiation

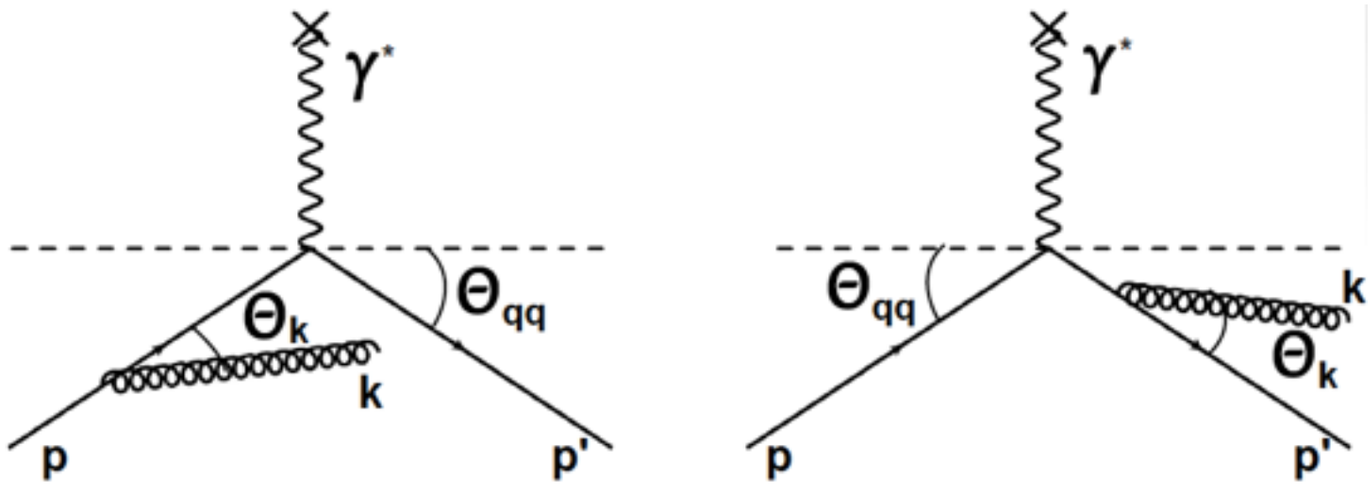


$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} = \frac{\alpha_s\, C_F}{(2\pi)^2\, \omega^2} \left(\mathcal{P}_{in} + \mathcal{P}_{out}\right)$$

$$\mathcal{P}_{in} = \mathcal{R}_{in} - \mathcal{J} \qquad \kappa = k_\perp - x P_\perp$$

Incoherent emission: $4\omega^2 \frac{1}{\kappa^2}$

Angular ordering in Initial State Radiation



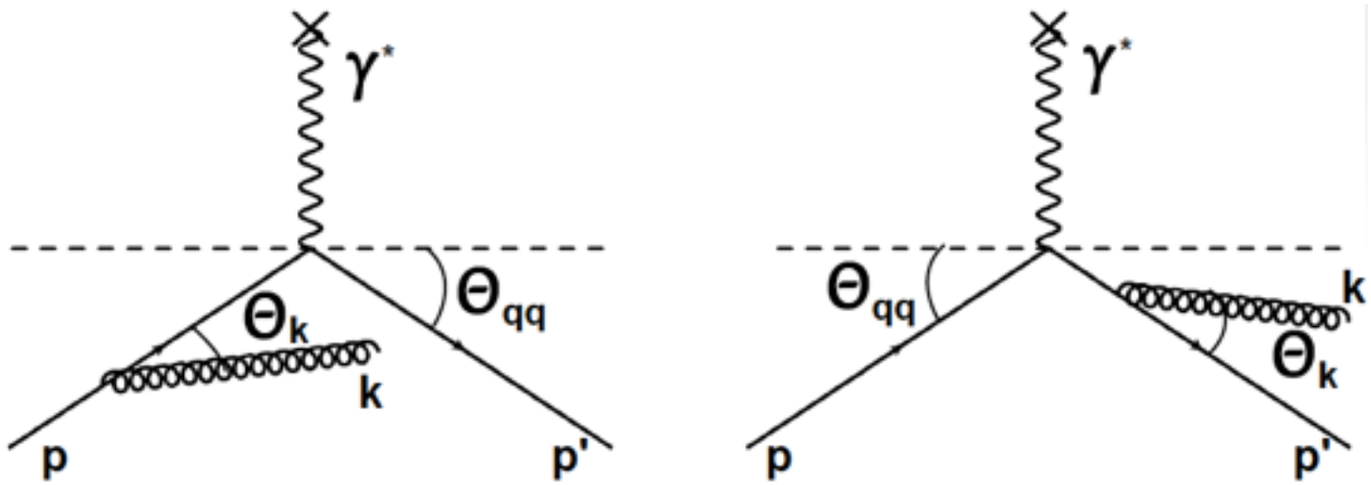
$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} = \frac{\alpha_s\, C_F}{(2\pi)^2\, \omega^2} \left(\mathcal{P}_{in} + \mathcal{P}_{out}\right)$$

$$\mathcal{P}_{in} = \mathcal{R}_{in} - \mathcal{J} \qquad \kappa = k_\perp - x P_\perp$$

Incoherent emission: $4\omega^2 \frac{1}{\kappa^2}$

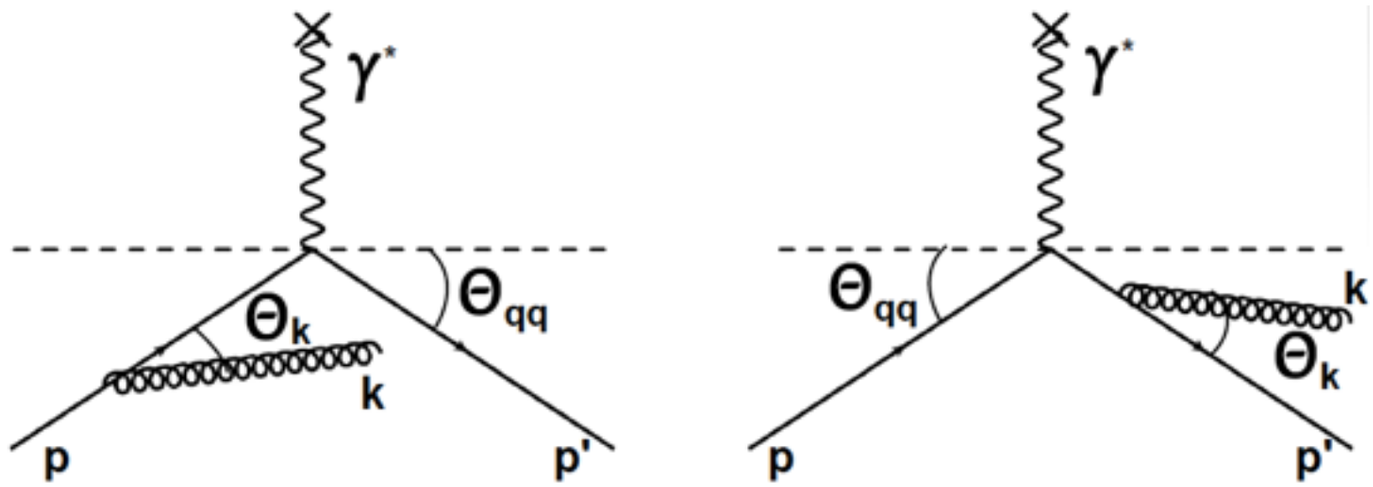
Interference: $4\omega^2 \frac{\kappa \cdot \bar{\kappa}}{\kappa^2 \bar{\kappa}^2}$

Angular ordering in Initial State Radiation



$$\langle dN_{in} \rangle \propto \frac{d\omega}{\omega} \frac{d\theta_{in}}{\theta_{in}} \Theta\left(\theta_{p\bar{p}} - \theta_{in}\right)$$

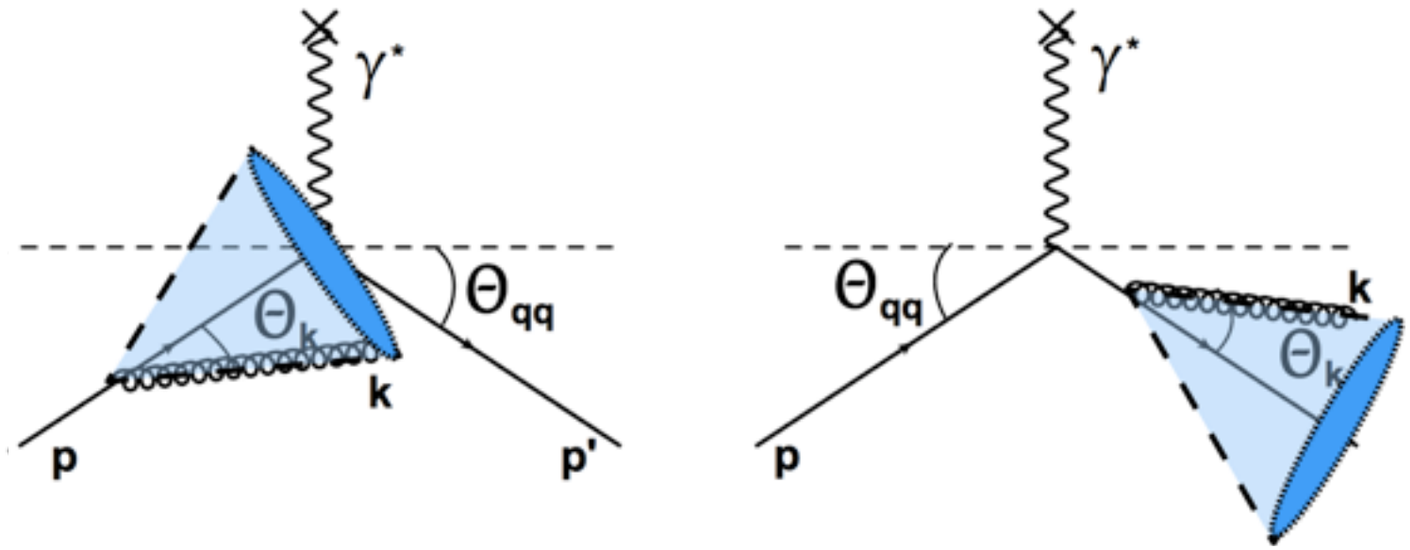
Angular ordering in Initial State Radiation



$$\langle dN_{in} \rangle \propto \boxed{\frac{d\omega}{\omega}\frac{d\theta_{in}}{\theta_{in}}}\Theta\left(\theta_{p\bar{p}} - \theta_{in}\right)$$

- Infrared and collinear divergences

Angular ordering in Initial State Radiation

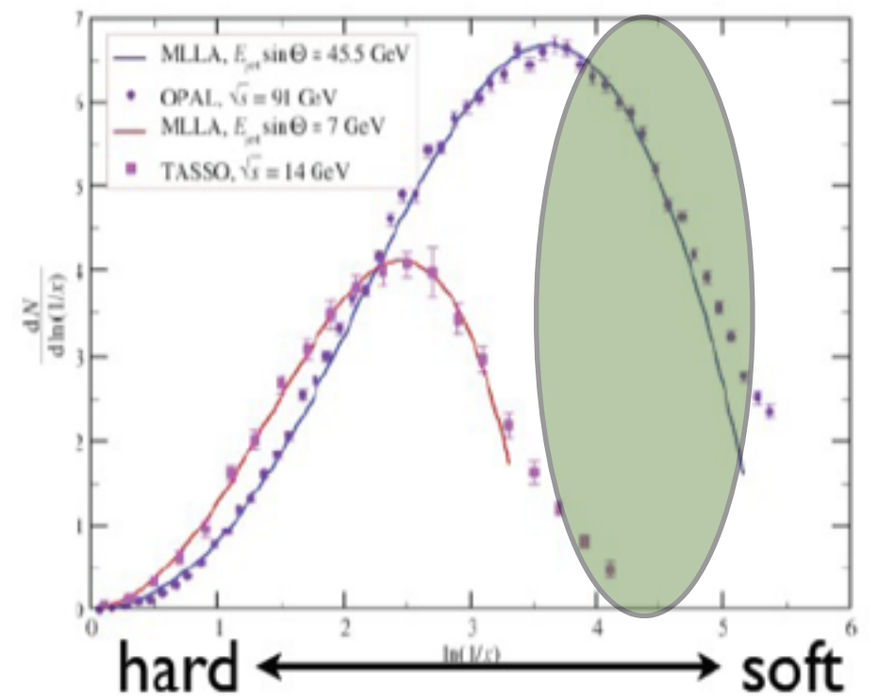


$$\langle dN_{in} \rangle \propto \frac{d\omega}{\omega}\frac{d\theta_{in}}{\theta_{in}}\Theta\left(\theta_{p\bar{p}} - \theta_{in}\right)$$

- Infrared and collinear divergences
- Similar coherence effects leading

to angular ordering in e^+e^-

$\Rightarrow$ reduction of soft gluon emissions



TASSO Collaboration, Z. Phys. C 47 (1990) 187

OPAL Collaboration, Phys. Lett. B 247 (1990) 617

First steps towards understanding coherence in a QCD medium

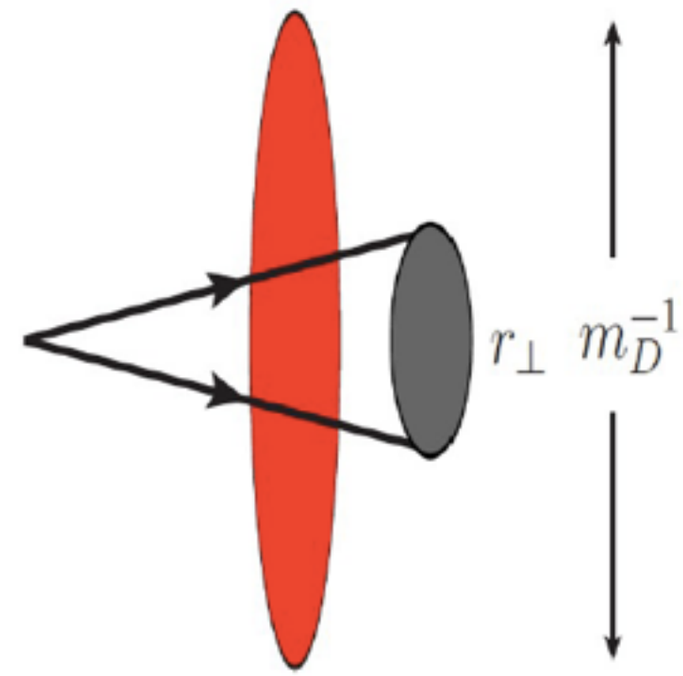
Dilute medium

- Massless antenna:

Y. Mehtar-Tani, C. Salgado and K. Tywoniuk, PRL 106 (2011) 122002, JHEP 1204 (2012) 064.

- Massive antenna:

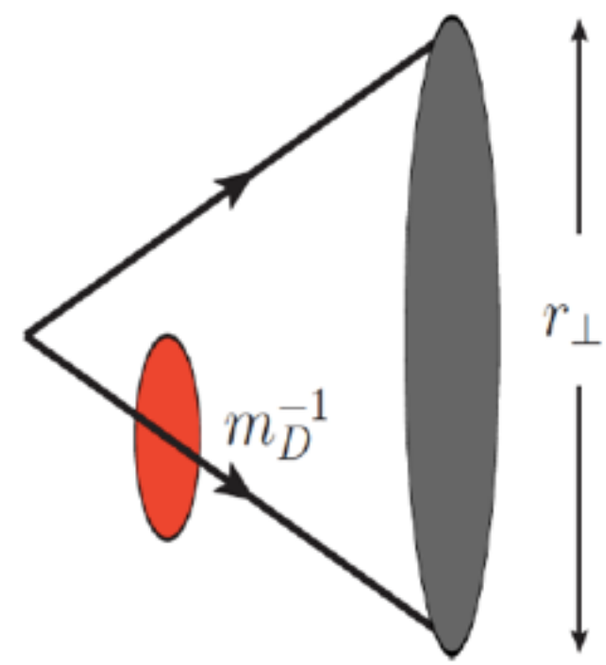
A. Armesto, H. Ma, Y. Mehtar-Tani, C. Salgado and K. Tywoniuk, JHEP 1201 (2012) 109.



Opaque dense medium

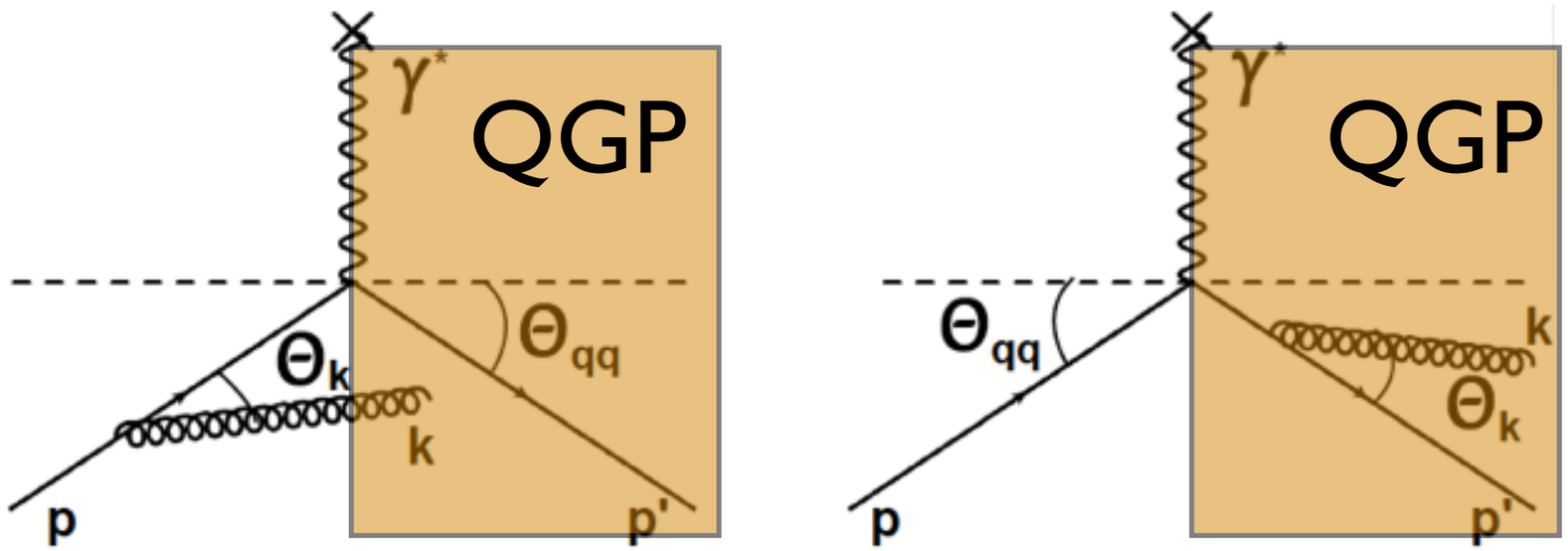
- Massless antenna:

Y. Mehtar-Tani, C. Salgado and K. Tywoniuk, PLB 707 (2012), 156.
Y. Mehtar-Tani and K. Tywoniuk, arXiV:1105.1346.
J. Casalderrey and E. Iancu, JHEP 1108 (2011) 015.
Y. Mehtar-Tani, C. Salgado and K. Tywoniuk, arXiv: 1205.5739

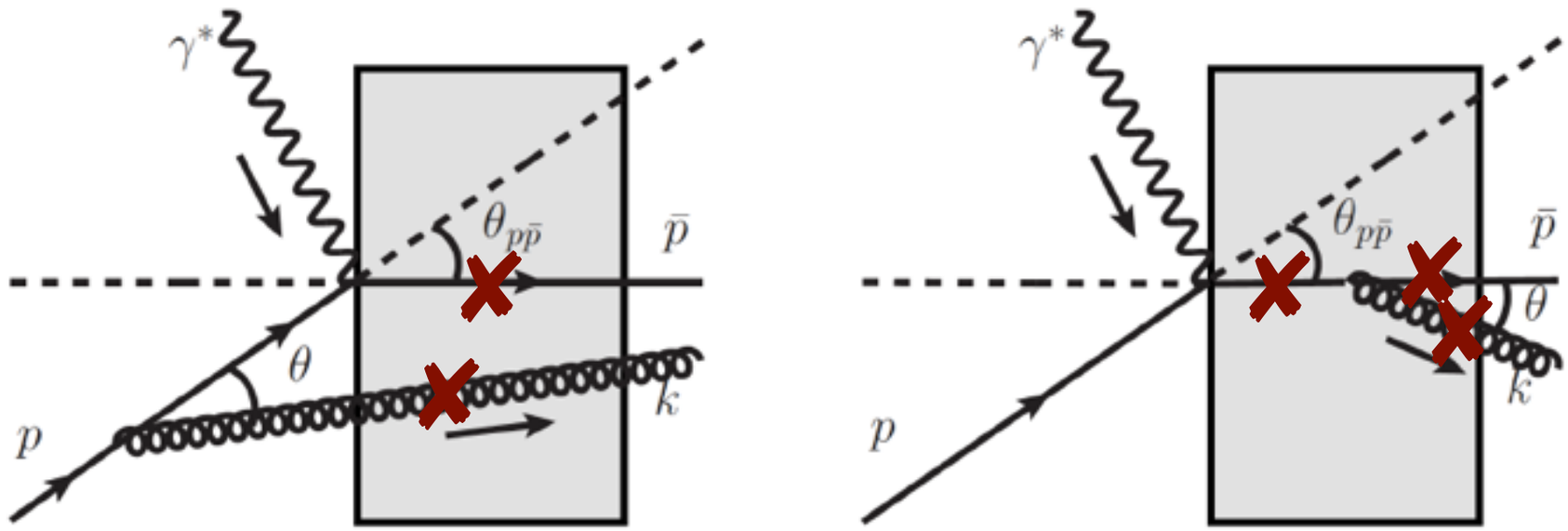


⇒ **K. Tywoniuk and Y. Mehtar-Tani's talks**

Coherence effects and medium modifications to the initial state radiation

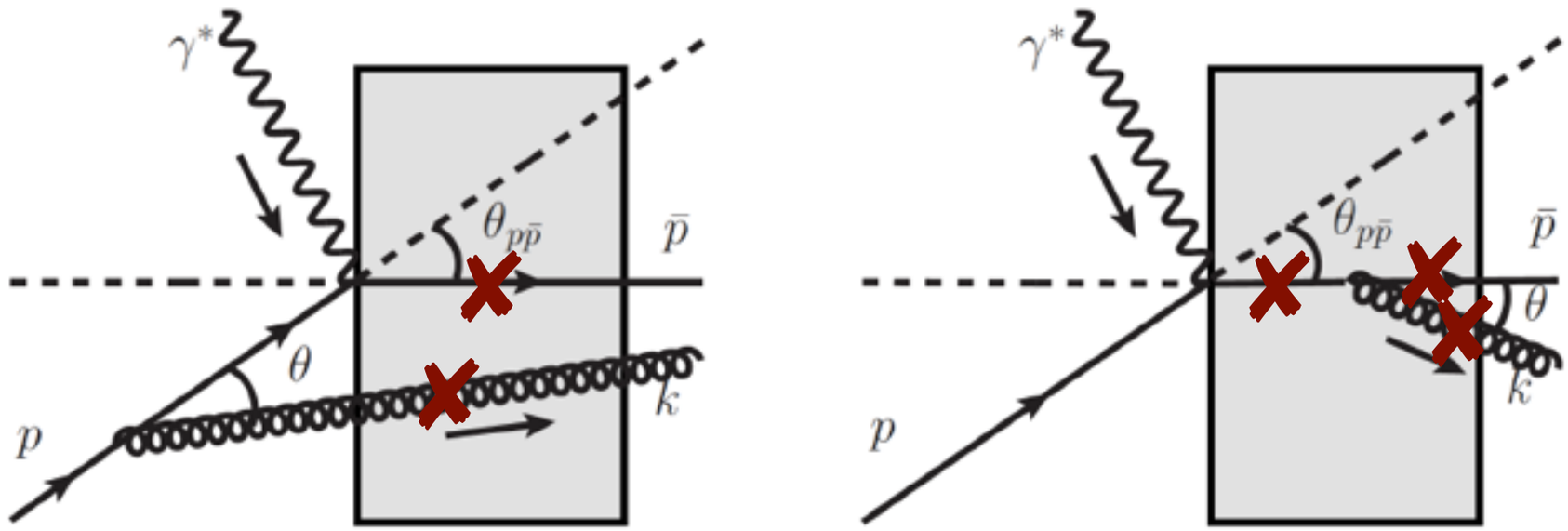


Setup



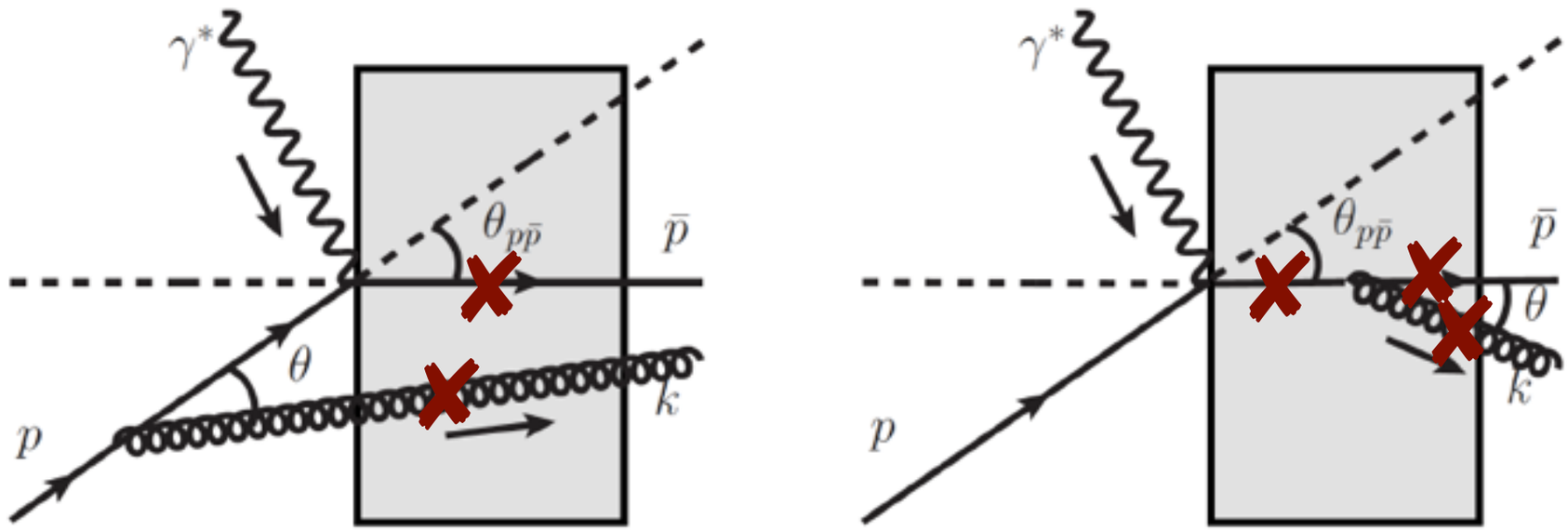
- The QCD medium starts after the hard scattering

Setup



- The QCD medium starts after the hard scattering
- Eikonal approximation: $E \gg \omega \gg k_\perp$

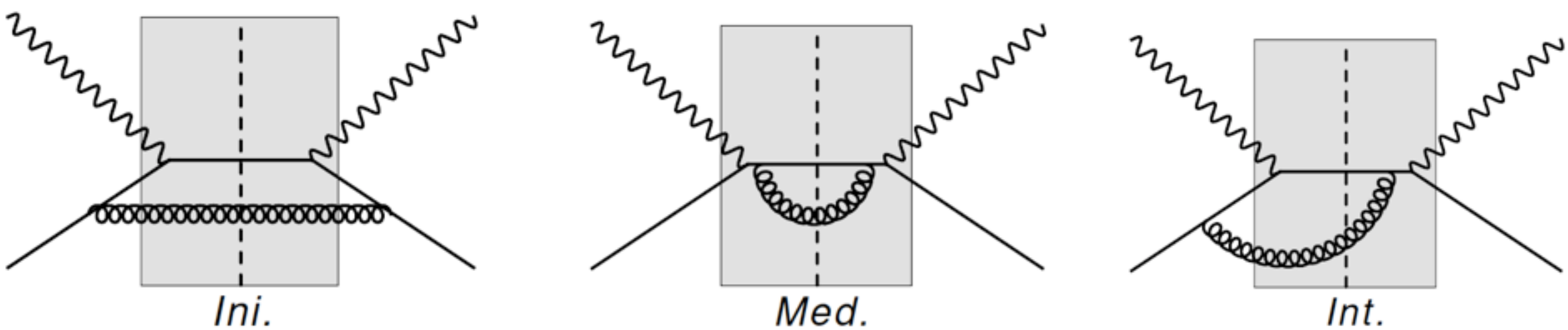
Setup



- The QCD medium starts after the hard scattering
- Eikonal approximation: $E \gg \omega \gg k_\perp$
- Dilute medium (N=1 opacity)

The medium induced gluon spectrum

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{4\,\alpha_s C_F\,\hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{L^+} dx^+ \left[\frac{1}{(\boldsymbol{\kappa}-\boldsymbol{q})^2} - \frac{1}{\boldsymbol{\kappa}^2} + 2\,\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{q}}{\bar{\boldsymbol{\kappa}}^2(\bar{\boldsymbol{\kappa}}-\mathbf{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}_\perp-\boldsymbol{q})^2}{2k^+}x^+\right]\right) - 2\left\{\boldsymbol{L}\cdot\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2} + \bar{\boldsymbol{L}}\cdot\frac{(\boldsymbol{\kappa}-\boldsymbol{q})}{(\boldsymbol{\kappa}-\boldsymbol{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{2k^+}x^+\right]\right)\right\}\right]$$



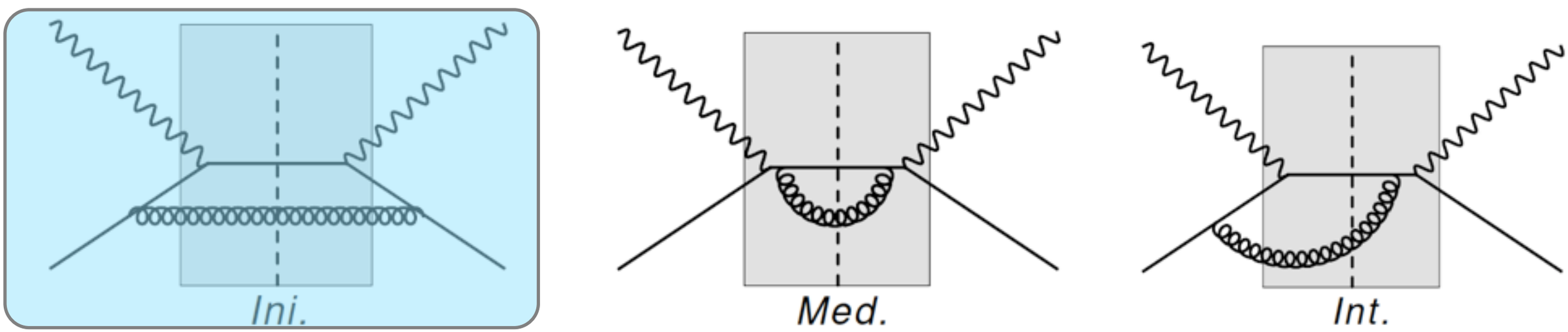
The medium induced gluon spectrum

Reshuffling of the off shell incoming parton

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{4\,\alpha_s C_F\,\hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{L^+} dx^+ \left[\frac{1}{(\boldsymbol{\kappa}-\boldsymbol{q})^2} - \frac{1}{\boldsymbol{\kappa}^2} \right.$$

$$+\, 2\,\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{q}}{\bar{\boldsymbol{\kappa}}^2(\bar{\boldsymbol{\kappa}}-\mathbf{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}_\perp-\boldsymbol{q})^2}{2k^+}x^+\right]\right)$$

$$\left. -\,2\left\{\boldsymbol{L}\cdot\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2} + \bar{\boldsymbol{L}}\cdot\frac{(\boldsymbol{\kappa}-\boldsymbol{q})}{(\boldsymbol{\kappa}-\boldsymbol{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{2k^+}x^+\right]\right)\right\}\right]$$



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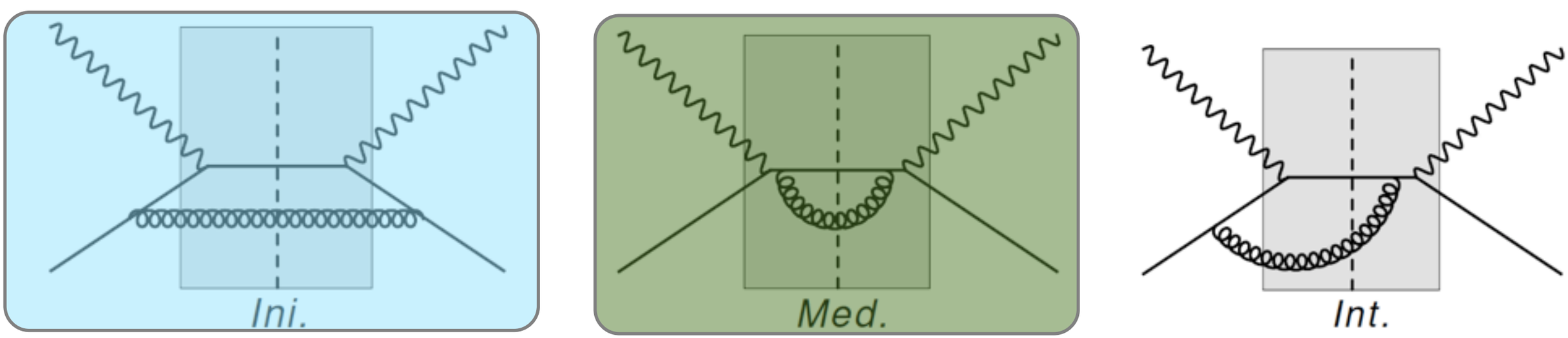
GLV



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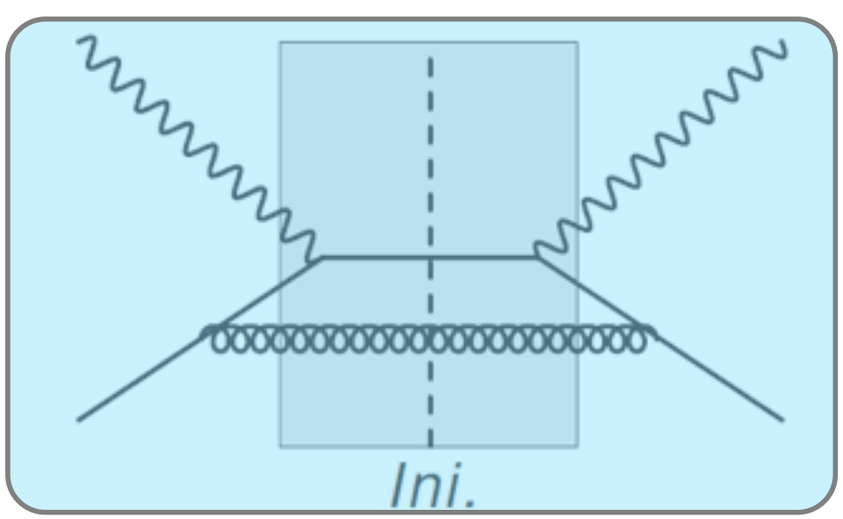
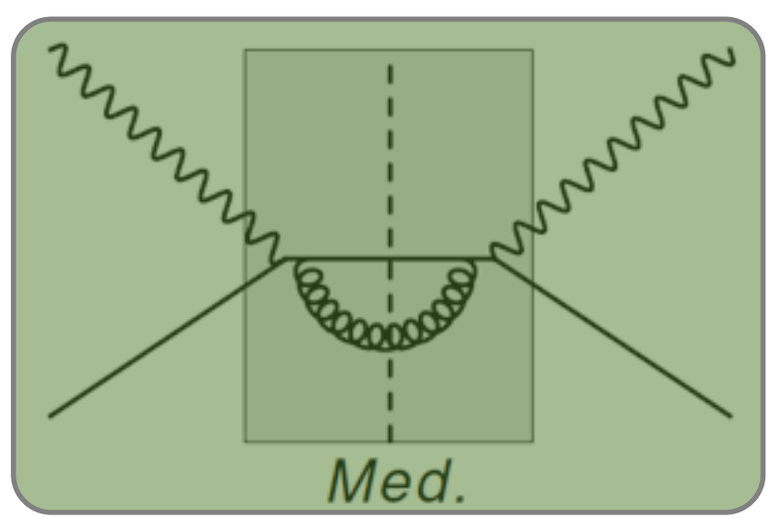
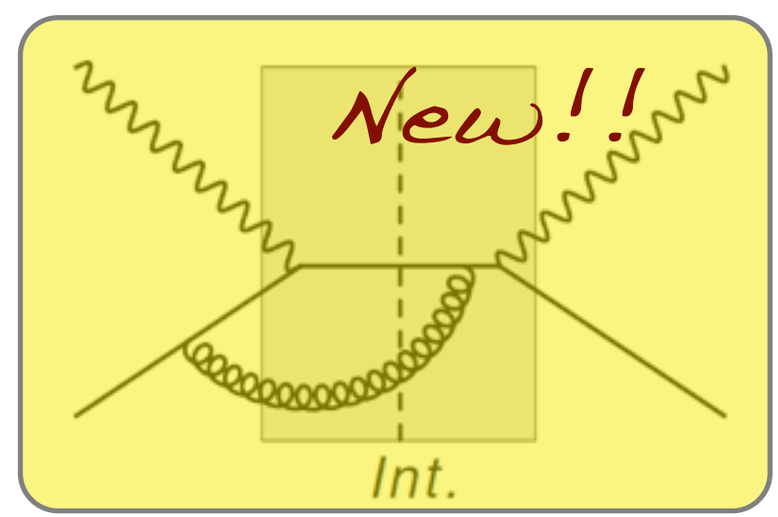
$$+\ 2\,\frac{\bar{\boldsymbol{\kappa}}\cdot\boldsymbol{q}}{\bar{\boldsymbol{\kappa}}^2(\bar{\boldsymbol{\kappa}}-\mathbf{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}_\perp-\boldsymbol{q})^2}{2k^+}x^+\right]\right)$$

Interferences $$\left. 2\left\{ \boldsymbol{L}\cdot\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2} + \bar{\boldsymbol{L}}\cdot\frac{(\boldsymbol{\kappa}-\boldsymbol{q})}{(\boldsymbol{\kappa}-\boldsymbol{q})^2}\left(1-\cos\left[\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{2k^+}x^+\right]\right)\right\}\right]$$







The medium induced gluon spectrum: Incoherent limit

$$\omega \left.\frac{dN^{med}}{d^3\vec{k}}\right|_{\tau_f<L} = \frac{4\alpha_s C_F \hat{q}}{\pi} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{V}(\mathbf{q}) \int_0^{L^+} dx^+ \{\bar{\mathbf{L}}^2 + \mathcal{C}^2(\kappa - q) - \mathcal{C}^2(\kappa)\}$$

$$\bar{L} = \frac{\bar{\kappa} - q}{(\bar{\kappa} - q)^2} - \frac{\bar{\kappa}}{\bar{\kappa}^2}$$

$$\mathcal{C}(\kappa) = \frac{\kappa}{\kappa^2} - \frac{\bar{\kappa}}{\bar{\kappa}^2}$$

$$\mathcal{C}(\kappa - q) = \frac{\kappa - q}{(\kappa - q)^2} - \frac{\bar{\kappa} - q}{(\bar{\kappa} - q)^2}$$

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$$\bar{L} = \frac{\bar{\kappa} - q}{(\bar{\kappa} - q)^2} - \frac{\bar{\kappa}}{\bar{\kappa}^2}$$

Lipatov vertex

$$\mathcal{C}(\kappa) = \frac{\kappa}{\kappa^2} - \frac{\bar{\kappa}}{\bar{\kappa}^2}$$

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Current associated to the bremsstrahlung

$$\mathcal{C}(\kappa - q) = \frac{\kappa - q}{(\kappa - q)^2} - \frac{\bar{\kappa} - q}{(\bar{\kappa} - q)^2}$$

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Lipatov vertex

$$\mathcal{C}(\kappa) = \frac{\kappa}{\kappa^2} - \frac{\bar{\kappa}}{\bar{\kappa}^2}$$

Current associated to the bremsstrahlung

$$\mathcal{C}(\kappa - q) = \frac{\kappa - q}{(\kappa - q)^2} - \frac{\bar{\kappa} - q}{(\bar{\kappa} - q)^2}$$

same + rescatt.

The medium induced gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \left. \frac{dN^{med}}{d^3\vec{k}} \right|_{\omega \to 0} = \frac{\alpha_s C_F}{(2\pi)^2} \Delta_{med} \left(2\mathcal{J} - \mathcal{R}_{in} \right)$$

The medium induced gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \left.\frac{dN^{med}}{d^3\vec{k}}\right|_{\omega\to 0} = \frac{\alpha_s\, C_F}{(2\pi)^2}\, \Delta_{med}\, (2\mathcal{J} - \mathcal{R}_{in})$$

$$\Delta_{med} = \frac{\hat{q}\, L^+}{m_D^2} \approx \frac{L}{\lambda}$$

The medium induced gluon spectrum: Soft limit and probabilistic interpretation

Interference

$$\omega \left.\frac{dN^{med}}{d^3\vec{k}}\right|_{\omega\to 0} = \frac{\alpha_s C_F}{(2\pi)^2}\Delta_{med}\left(2\mathcal{J} - \mathcal{R}_{in}\right)$$

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The medium induced gluon spectrum: Soft limit and probabilistic interpretation

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$$\Delta_{med} = \frac{\hat{q}\, L^+}{m_D^2} \approx \frac{L}{\lambda}$$

Emission

The full gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \omega \frac{dN^{\rm vac}}{d^3\vec{k}}\bigg|_{\omega\to 0} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\omega\to 0}$$

$$= \frac{\alpha_s\, C_F}{(2\pi)^2}(\mathcal{P}_{in} + \mathcal{P}_{out})$$

The full gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \omega \frac{dN^{\rm vac}}{d^3\vec{k}}\bigg|_{\omega\to 0} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\omega\to 0}$$

$$= \frac{\alpha_s\, C_F}{(2\pi)^2}(\mathcal{P}_{in} + \mathcal{P}_{out})$$

$$\mathcal{P}_{in} = \big(1 - \Delta_{med}\big)\big(\mathcal{R}_{in} - \mathcal{J}\big)$$

$$\mathcal{P}_{out} = \mathcal{R}_{out} - \big(1 - \Delta_{med}\big)\mathcal{J}$$

The full gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \omega \frac{dN^{\rm vac}}{d^3\vec{k}}\bigg|_{\omega\to 0} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\omega\to 0}$$

$$= \frac{\alpha_s\, C_F}{(2\pi)^2}(\mathcal{P}_{in} + \mathcal{P}_{out})$$

$$\mathcal{P}_{in} = \big(1 - \Delta_{med}\big)\big(\mathcal{R}_{in} - \mathcal{J}\big)$$

Reduction of coherent gluon emission of the initial state

$$\mathcal{P}_{out} = \mathcal{R}_{out} - \big(1 - \Delta_{med}\big)\mathcal{J}$$

The full gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \omega \frac{dN^{\rm vac}}{d^3\vec{k}}\bigg|_{\omega\to 0} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\omega\to 0}$$

$$= \frac{\alpha_s\, C_F}{(2\pi)^2}(\mathcal{P}_{in} + \mathcal{P}_{out})$$

$$\mathcal{P}_{in} = \left(1 - \Delta_{med}\right)\left(\mathcal{R}_{in} - \mathcal{J}\right)$$ ⇒ Reduction of coherent gluon emission of the initial state

$$\mathcal{P}_{out} = \mathcal{R}_{out} - \left(1 - \Delta_{med}\right)\mathcal{J}$$ ⇒ Partial decoherence of the final state

The full gluon spectrum: Soft limit and probabilistic interpretation

$$\omega \frac{dN^{\rm tot}}{d^3\vec{k}}\bigg|_{\omega\to 0} = \omega \frac{dN^{\rm vac}}{d^3\vec{k}}\bigg|_{\omega\to 0} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}\bigg|_{\omega\to 0}$$

$$= \frac{\alpha_s\, C_F}{(2\pi)^2}(\mathcal{P}_{in} + \mathcal{P}_{out})$$

$$\mathcal{P}_{in} = \big(1 - \Delta_{med}\big)\big(\mathcal{R}_{in} - \mathcal{J}\big)$$ ⇒ Reduction of coherent gluon emission of the initial state

$$\mathcal{P}_{out} = \mathcal{R}_{out} - \big(1 - \Delta_{med}\big)\mathcal{J}$$ ⇒ Partial decoherence of the final state

Valid as far as $\omega\theta_{qq}, k_\perp \ll m_D$ ⇒ Setting the scale !!

Conclusions and outlook

- We study interferences between initial and final state radiation in a QCD medium.
- A probabilistic interpretation is found in the incoherent and soft limit of the gluon spectrum.

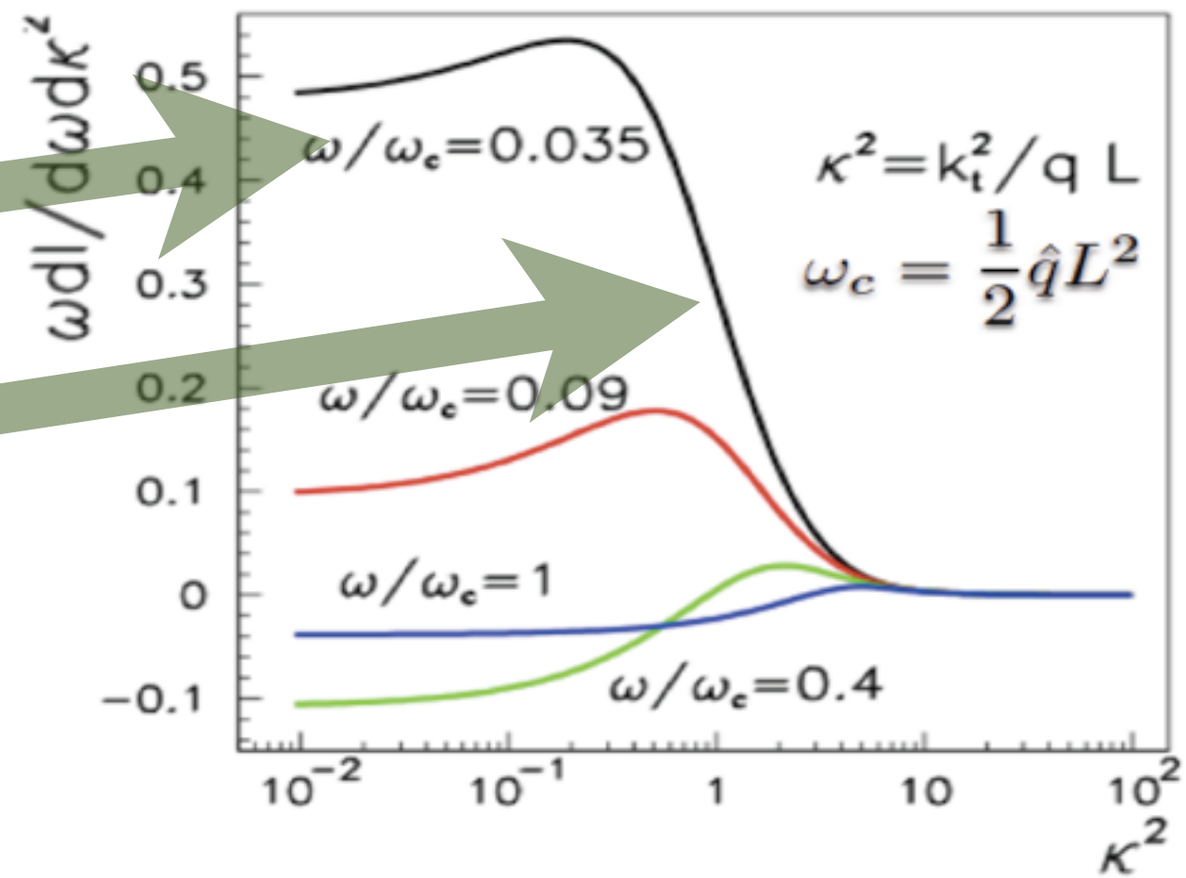
- **Future work (stay tuned):**
  - Numerical results for the dilute regime case
  - Analytical studies for an opaque medium (multiple scatterings).
  - Use these results for phenomenological studies...

Backup slides

GLV Spectrum

$$\omega \frac{dN_q^{\rm GLV}}{d\omega d^2k_\perp} = \frac{8\,\alpha_s C_F\,\hat{q}}{\pi}\int \frac{d^2q_\perp}{(2\pi)^2}\int_0^L dt\,\frac{1-\cos\frac{(k_\perp-q_\perp)^2}{2\omega}t}{(q_\perp^2+\mu_D^2)^2}\,\frac{k_\perp\cdot q_\perp}{k_\perp^2(k_\perp-q_\perp)^2}$$

- Gluon spectrum is infrared and collinearly safe
- LPM suppression: large formation times are suppressed



GLV Spectrum

$$\omega \frac{dN_q^{\rm GLV}}{d\omega d^2k_\perp} = \frac{8\,\alpha_s C_F\,\hat{q}}{\pi}\int \frac{d^2q_\perp}{(2\pi)^2}\int_0^L dt\,\frac{1-\cos\frac{(k_\perp-q_\perp)^2}{2\omega}t}{(q_\perp^2+\mu_D^2)^2}\,\frac{k_\perp\cdot q_\perp}{k_\perp^2(k_\perp-q_\perp)^2}$$

Incoherent limit: $\tau_f \ll L$

$$\omega\,\frac{dN_q^{\rm GLV}}{d\omega d^2k_\perp}\bigg|_{\tau_f\ll L} = \frac{4\,\alpha_s C_F\,\hat{q}L^+}{\pi}\int_{\mathcal{V}(\mathbf{q})}\left[\mathbf{L}^2+\frac{1}{(\mathbf{k}-\mathbf{q})^2}-\frac{1}{\mathbf{k}^2}\right]$$

- ☆ Induced radiation of an asymptotic color charge (Gunion- Bertsch)
- ☆ Bremstrahlung of an accelerated color charge

$$\mathbf{L}^2 = \frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$