Multi-gluon correlations in the CGC

(An analytical solution to JIMWLK)

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Hard Probes, Cagliari, May 2012

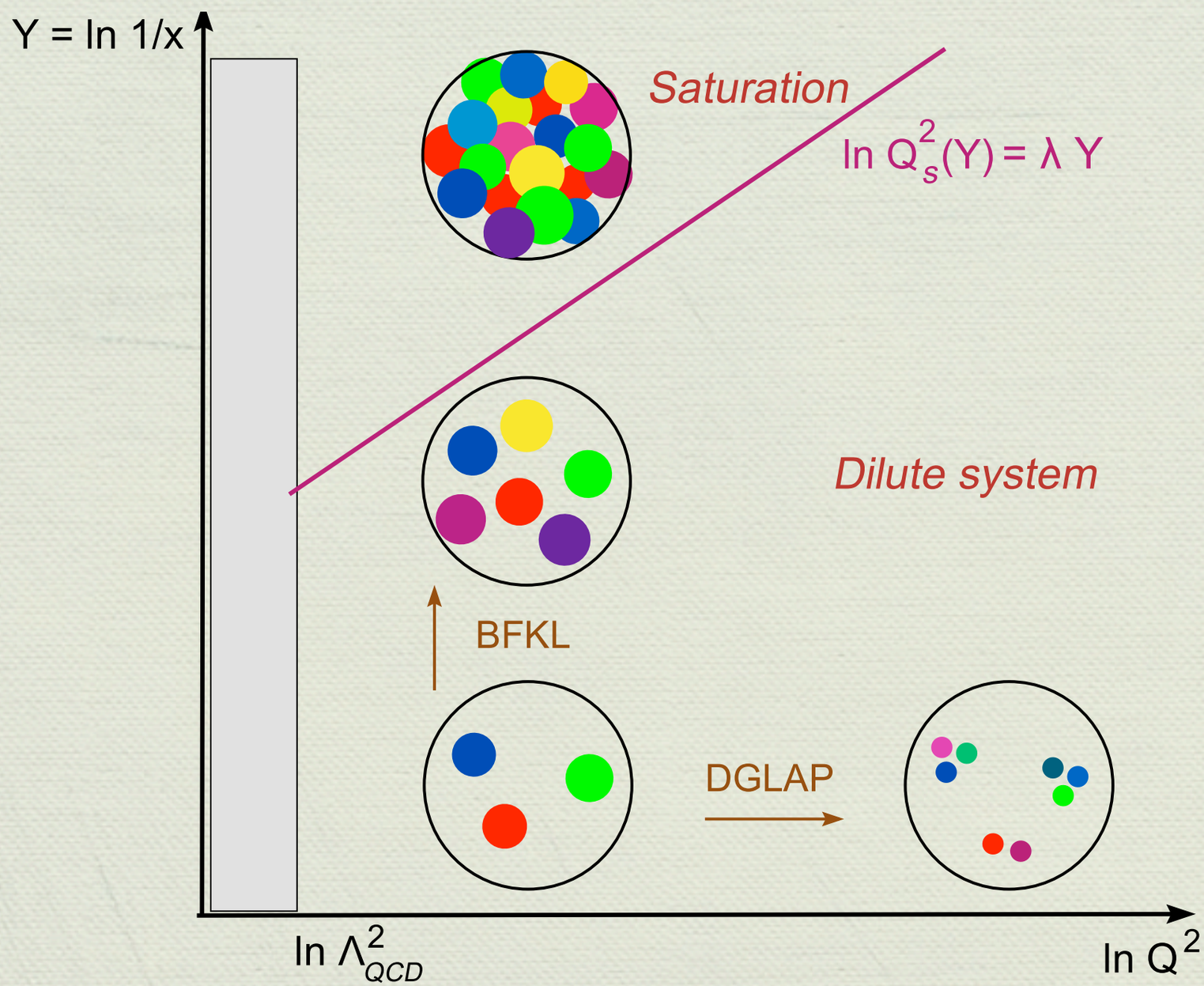
E. Iancu, DNT: JHEP 11 (2011) 105 [1109:0302]
E. Iancu, DNT: JHEP 04 (2012) 025 [1112.1104]

Iancu, Itakura, McLerran: NPA 724 (2003) 181 [hep-ph/0212123]
Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan: PLB 706 (2011) 219 [1108.4764]

Outline

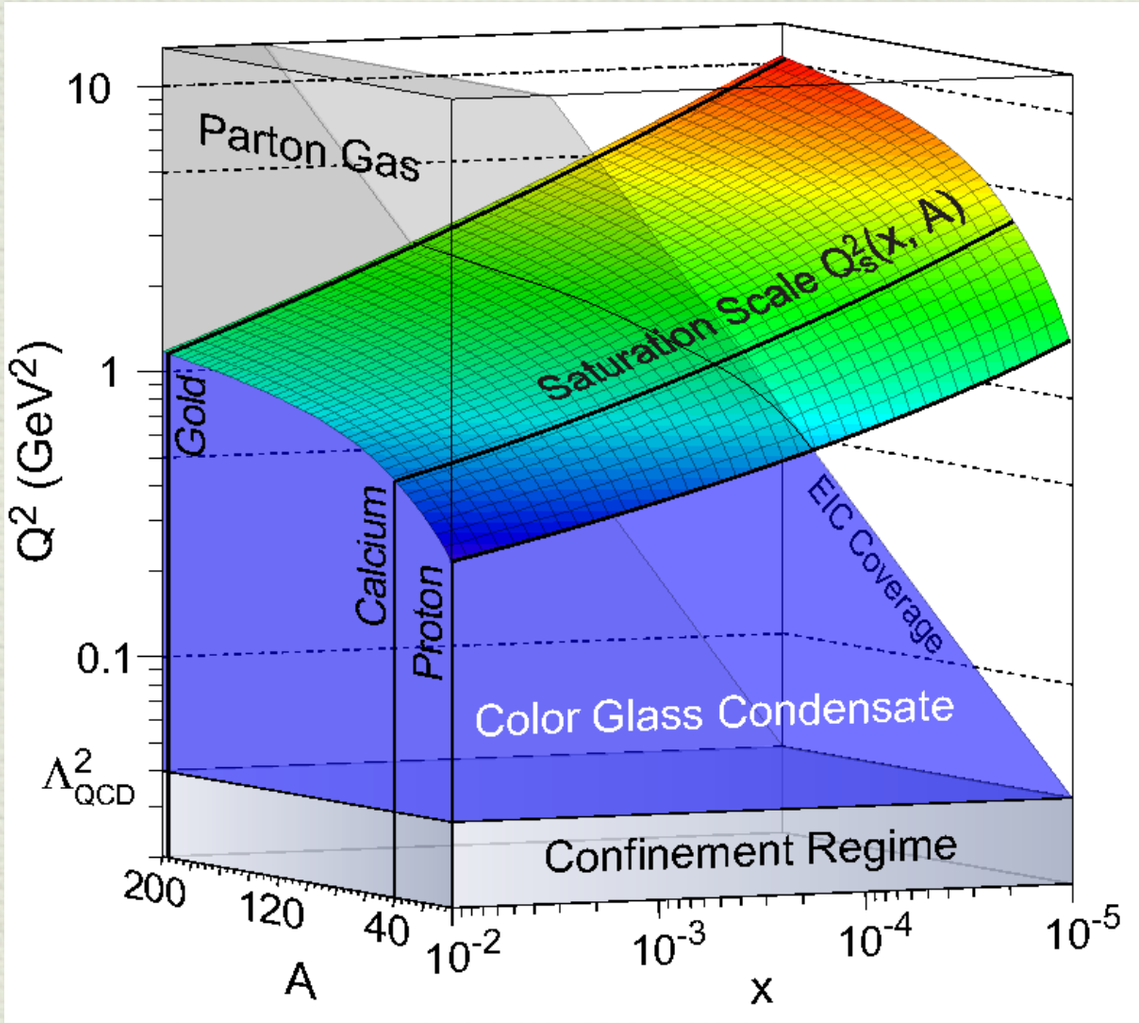
- The partonic “phase diagram” and saturation
- High energy scattering and Wilson lines
- Dihadron production in forward region in pA collisions
- JIMWLK evolution of dipoles, quadrupoles, ...
- The approximate excellent solution
- Conclusion

Partonic "phase diagram"



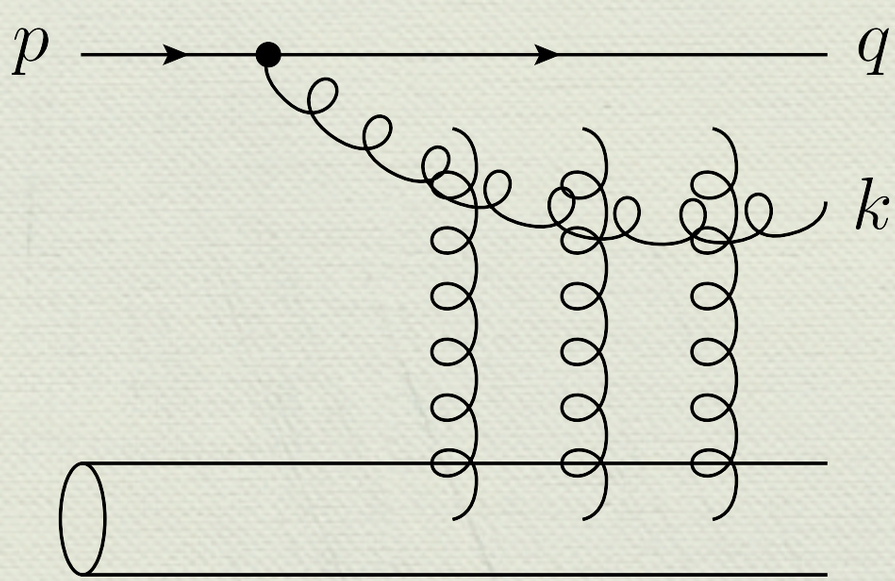
Saturation momentum

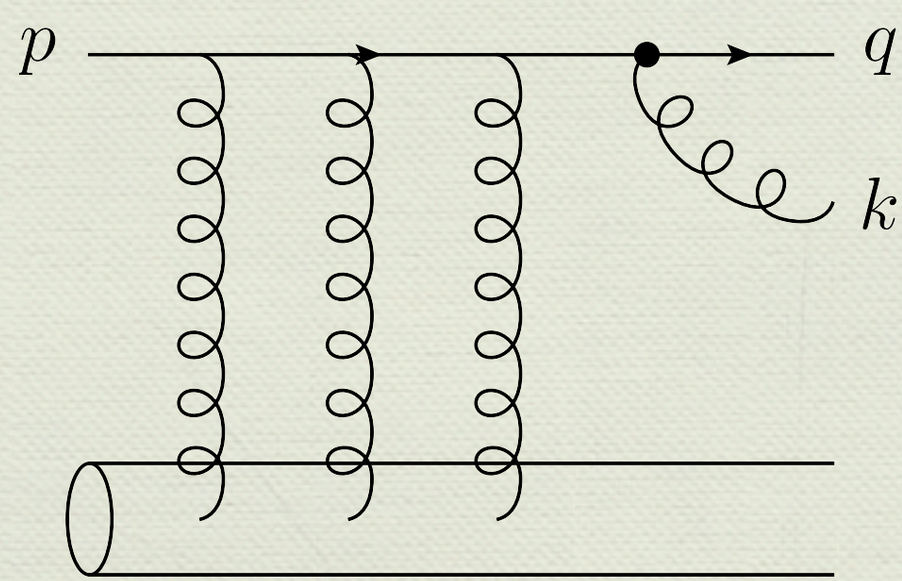
- Saturation when $\dfrac{xg(x,Q_s^2)}{Q_s^2 R^2} \sim \dfrac{1}{\alpha_s}$
- $Q_s^2(x,A) \sim Q_0^2 A^{1/3} \left(\dfrac{x_0}{x}\right)^{\lambda}$ with $\lambda = 0.2 \div 0.3$



The process

- Large-x quark from proton splits into quark-gluon pair
- Interacts with soft components of nucleus
- Quark-gluon pair “measured” in forward region





The outgoing state

- Mixed representation: transverse momenta → coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction → Wilson lines: $V^\dagger_{\boldsymbol{x}} = P \exp\left[\mathrm{i}\,\mathrm{g} \int \mathrm{d}x^- t^a \mathcal{A}^{+a}_{\boldsymbol{x}}(x^-)\right]$

$$|\Psi_{\mathrm{out}}\rangle = \int D\mathcal{A}^+ \, \Phi_{\mathrm{Y}}[\mathcal{A}^+] \int_{\boldsymbol{x},\boldsymbol{b}} \mathrm{d}z \, p^- g \, \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{b}} \sum_{j\beta c\lambda} \phi^\lambda_{\alpha\beta}(p, zp^-, \boldsymbol{x}-\boldsymbol{b})$$
$$[T^d V(\boldsymbol{b}) \tilde{V}^{dc}(\boldsymbol{x}) - V(\boldsymbol{b} + z(\boldsymbol{x}-\boldsymbol{b}))T^c]_{ij}$$
$$|(1-z)p^-, \boldsymbol{b}, j, \beta; zp^-, \boldsymbol{x}, c, \lambda\rangle \otimes |\mathcal{A}^+\rangle$$

Marquet '07

The cross section

- From $\langle \Psi_{\rm out} | N_q(q) N_g(k) | \Psi_{\rm out} \rangle$ calculate cross section

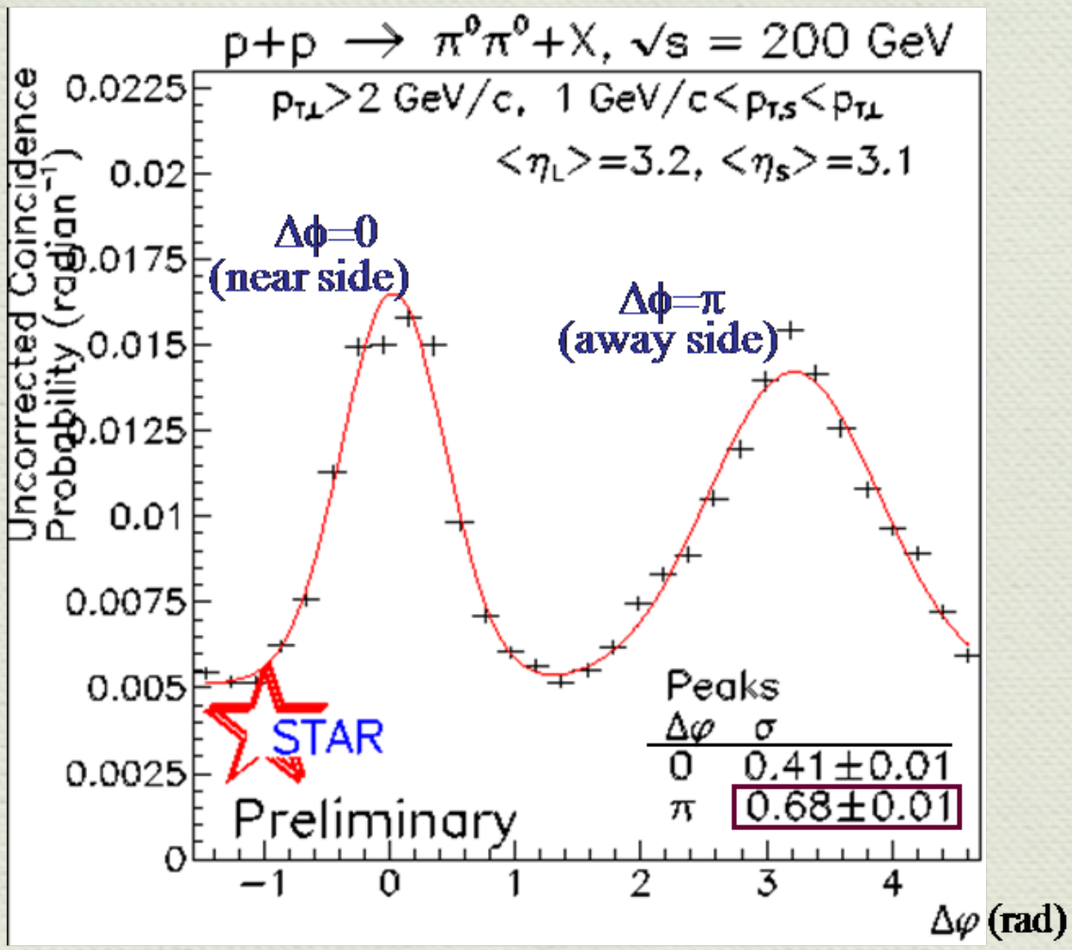
$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k\mathrm{d}^3q} = \frac{\alpha_s N_c}{2} \int_{\boldsymbol{x}\acute{\boldsymbol{x}}\boldsymbol{b}\acute{\boldsymbol{b}}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\acute{\boldsymbol{x}})+\mathrm{i}(\boldsymbol{q}-\boldsymbol{p})\cdot(\acute{\boldsymbol{b}}-\boldsymbol{b})}$$

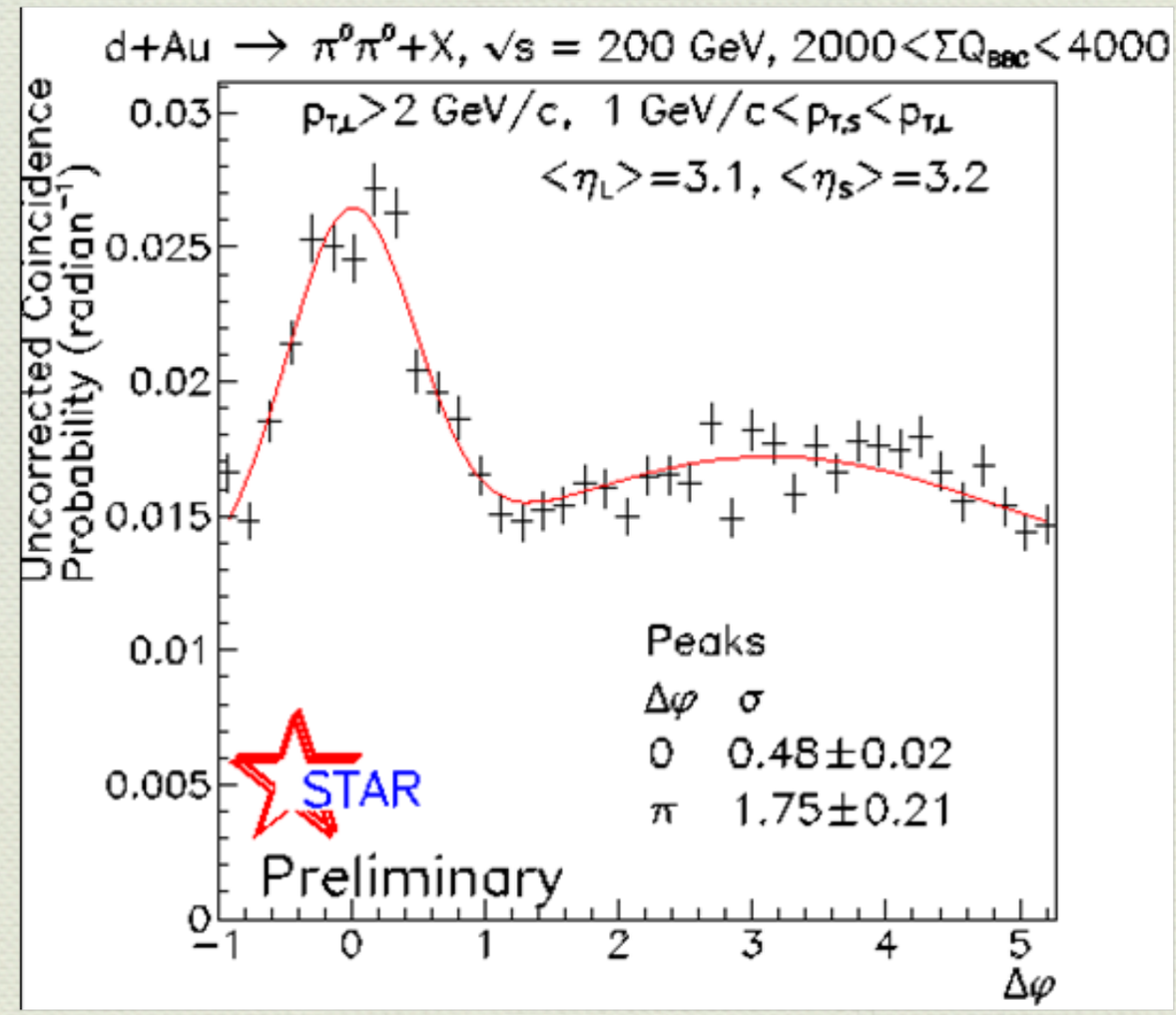
$$\sum_{\lambda\alpha\beta} \phi^{\lambda *}_{\alpha\beta}(p, zp^-, \acute{\boldsymbol{x}} - \acute{\boldsymbol{b}}) \phi^{\lambda}_{\alpha\beta}(p, zp^-, \boldsymbol{x} - \boldsymbol{b})$$

$$\left\langle \frac{1}{N_c} \mathrm{tr}[V^\dagger(\boldsymbol{x}) V(\boldsymbol{b}) V^\dagger(\acute{\boldsymbol{b}}) V(\acute{\boldsymbol{x}})] \frac{1}{N_c} \mathrm{tr}[V^\dagger(\acute{\boldsymbol{x}}) V(\boldsymbol{x})] + \ldots \right\rangle_Y$$

- QCD dynamics in $\langle \ldots \rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+] \ldots$
- $\mathrm{e}^{-Y} = x = \dfrac{|\boldsymbol{k}|\mathrm{e}^{-y_k} + |\boldsymbol{q}|\mathrm{e}^{-y_q}}{\sqrt{s}} \ll 1$ in forward region

Di-hadron azimuthal correlations





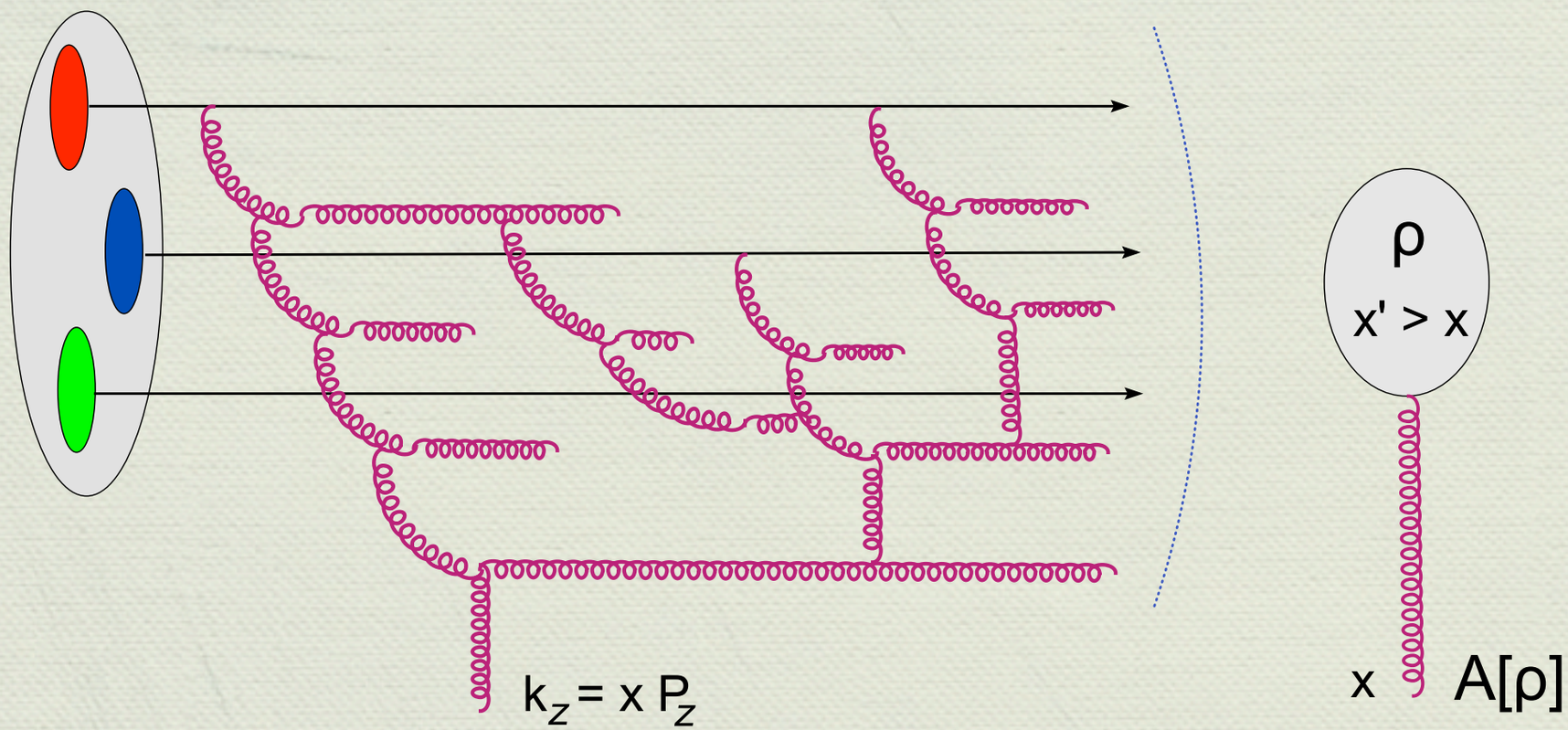
Albacete, Marquet '10

Wilson line correlators

- Dipole operator: $\hat{S}_{12} = \frac{1}{N_c}\mathrm{tr}(V_1^\dagger V_2)$
- Quadrupole operator: $\hat{Q}_{1234} = \frac{1}{N_c}\mathrm{tr}(V_1^\dagger V_2 V_3^\dagger V_4)$
- 2n-point operator: $\hat{S}^{(2n)}_{12\ldots(2n-1)2n} = \frac{1}{N_c}\mathrm{tr}(V_1^\dagger V_2 \ldots V_{2n-1}^\dagger V_{2n})$
- Finite Nc, given wave-function, calculate each correlator
- Large Nc, factorization: $\langle \hat{S}^{2n_1} \ldots \hat{S}^{2n_k} \rangle_Y \to \langle \hat{S}^{2n_1} \rangle_Y \ldots \langle \hat{S}^{2n_k} \rangle_Y$
- Still infinite number of correlators, e.g. $\langle \hat{Q}_{1234} \rangle_Y = ?$

Color Glass Condensate

- QCD, frozen sources, occupation numbers of order $1/\alpha_s$
- All orders in $\alpha_s \ln 1/x$ and classical field $A_a^\mu \sim \mathcal{O}(1/g)$



Evolution of correlators

- QCD dynamics encoded in JIMWLK Hamiltonian

$$H = -\frac{1}{16\pi^3}\int_{\boldsymbol{uvz}} \mathcal{M}_{\boldsymbol{uvz}}[1 + \widetilde{V}^\dagger_{\boldsymbol{u}}\widetilde{V}_{\boldsymbol{v}} - \widetilde{V}^\dagger_{\boldsymbol{u}}\widetilde{V}_{\boldsymbol{z}} - \widetilde{V}^\dagger_{\boldsymbol{z}}\widetilde{V}_{\boldsymbol{v}}]^{ab}\frac{\delta}{\delta\alpha^a_{\boldsymbol{u}}}\frac{\delta}{\delta\alpha^b_{\boldsymbol{v}}}$$

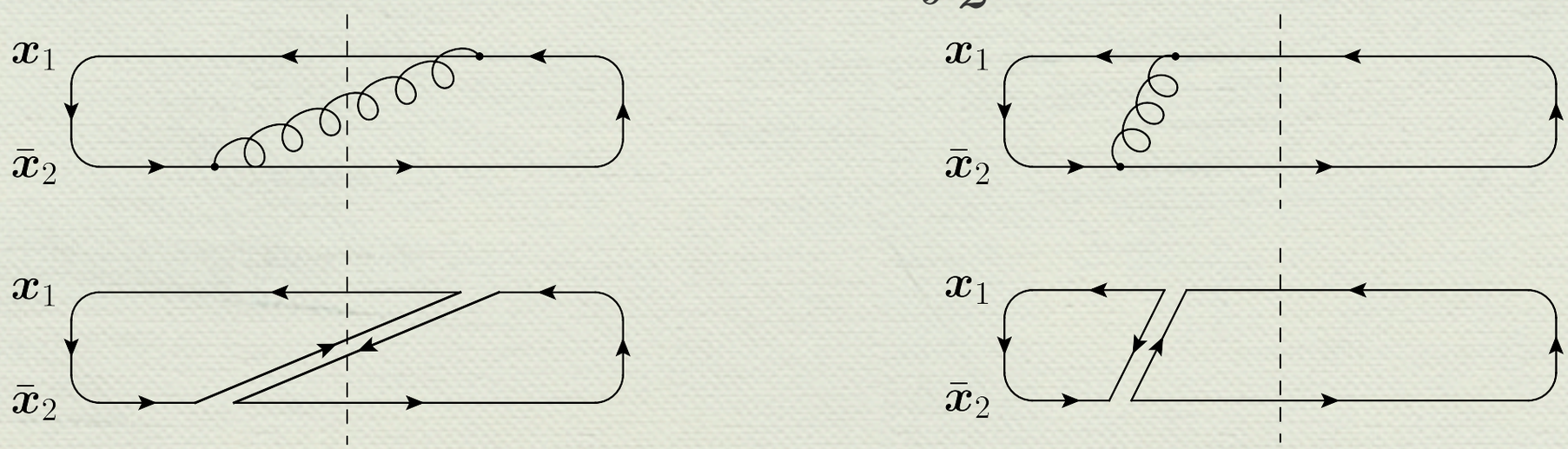
- Evolution of expectation value of arbitrary correlator

$$\frac{\partial W_Y[\alpha]}{\partial Y} = HW_Y[\alpha] \quad \Longrightarrow \quad \frac{\partial\langle\hat{\mathcal{O}}\rangle_Y}{\partial Y} = \langle H\hat{\mathcal{O}}\rangle_Y$$

- Easy to work out: act on end-point, use Fierz identities.

The Dipole

- Well-known eqn: $$\frac{\partial\langle\hat{S}_{12}\rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int_{\boldsymbol{z}} \mathcal{M}_{12\boldsymbol{z}}\langle\hat{S}_{1\boldsymbol{z}}\hat{S}_{\boldsymbol{z}2} - \hat{S}_{12}\rangle_Y$$



- Weak scattering T = 1 - S small: linear, BFKL, easy to solve

- Strong scattering, assume large Nc: linear in S

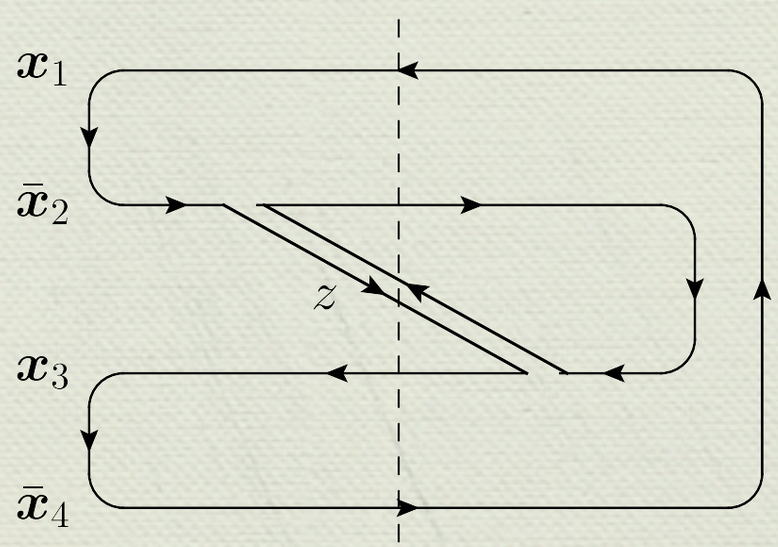
$$\frac{\partial\langle\hat{S}_{12}\rangle_Y}{\partial Y} = -\bar{\alpha}_s\int_{1/Q_s^2}^{r_{12}^2}\frac{\mathrm{d}z^2}{z^2}\langle\hat{S}_{12}\rangle_Y = -\bar{\alpha}_s\ln(r_{12}^2Q_s^2)\langle\hat{S}_{12}\rangle_Y$$

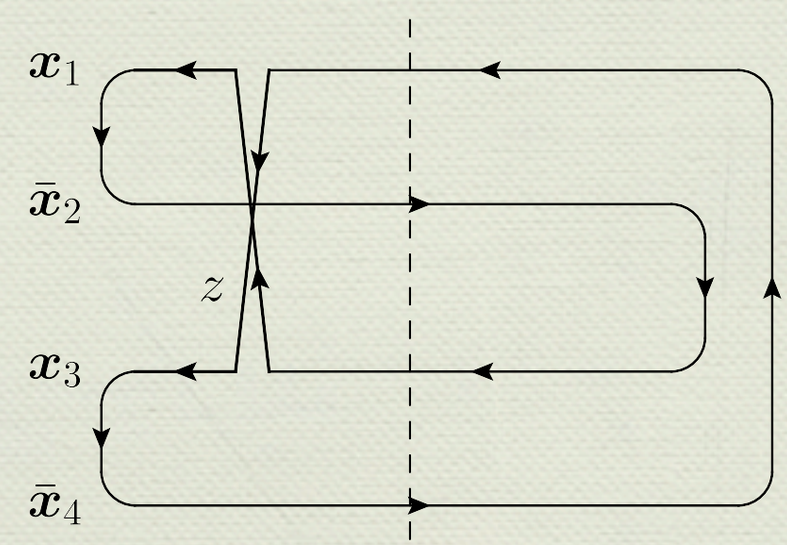
- Local in S, trivially solved if we know Qs(Y)

The Quadrupole

- The evolution equation...

$$
\begin{aligned}
\frac{\partial\langle\hat{Q}_{1234}\rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{4\pi}\int_{\boldsymbol{z}} & (\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}})\langle\hat{S}_{1\boldsymbol{z}}\hat{Q}_{\boldsymbol{z}234}\rangle_Y \\
& +(\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}})\langle\hat{S}_{\boldsymbol{z}2}\hat{Q}_{1\boldsymbol{z}34}\rangle_Y \\
& +(\mathcal{M}_{23\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}})\langle\hat{S}_{3\boldsymbol{z}}\hat{Q}_{12\boldsymbol{z}4}\rangle_Y \\
& +(\mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}})\langle\hat{S}_{\boldsymbol{z}4}\hat{Q}_{123\boldsymbol{z}}\rangle_Y \\
& -(\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}})\langle\hat{Q}_{1234}\rangle_Y \\
& -(\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}})\langle\hat{S}_{12}\hat{S}_{34}\rangle_Y \\
& -(\mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}})\langle\hat{S}_{14}\hat{S}_{23}\rangle_Y
\end{aligned}
$$





Quadrupole in limiting cases

- Weak scattering, expand Wilson lines, 2-gluon exchange

$$\hat{Q}_{1234} \simeq 1 - \hat{T}_{12} + \hat{T}_{13} - \hat{T}_{14} - \hat{T}_{23} + \hat{T}_{24} - \hat{T}_{34}$$

- Evolving like "six BFKL's"

- Strong scattering, assume large Nc, keep quadratic terms, local

$$\begin{aligned}\frac{\partial\langle\hat{Q}_{1234}\rangle_Y}{\partial Y} \simeq & -\frac{\bar{\alpha}_s}{2}\left[\ln(r_{12}^2Q_s^2)+\ln(r_{34}^2Q_s^2)+\ln(r_{14}^2Q_s^2)+\ln(r_{23}^2Q_s^2)\right]\langle\hat{Q}_{1234}\rangle_Y \\ & -\frac{\bar{\alpha}_s}{2}\left[\ln(r_{12}^2Q_s^2)+\ln(r_{34}^2Q_s^2)-\ln(r_{13}^2Q_s^2)-\ln(r_{24}^2Q_s^2)\right]\langle\hat{S}_{12}\rangle_Y\langle\hat{S}_{34}\rangle_Y \\ & -\frac{\bar{\alpha}_s}{2}\left[\ln(r_{14}^2Q_s^2)+\ln(r_{23}^2Q_s^2)-\ln(r_{13}^2Q_s^2)-\ln(r_{24}^2Q_s^2)\right]\langle\hat{S}_{32}\rangle_Y\langle\hat{S}_{14}\rangle_Y,\end{aligned}$$

- Given Qs(Y) and dipole, can solve for quadrupole, but better ...

Look for functional form

- Write logs in terms of log-derivative of dipole
  - Leads to functional form: Quadrupole in terms of dipole
  - Better than log-accuracy
  - Extends to running coupling
  - An, *a priori*, unexpected result
- Ordinary 1st order inhomogeneous differential equation

Solution to the quadrupole

$$\langle \hat{Q}_{1234} \rangle_Y = \sqrt{\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{32} \rangle_Y \langle \hat{S}_{34} \rangle_Y \langle \hat{S}_{14} \rangle_Y} \left[\frac{\langle \hat{Q}_{1234} \rangle_{Y_0}}{\sqrt{\langle \hat{S}_{12} \rangle_{Y_0} \langle \hat{S}_{32} \rangle_{Y_0} \langle \hat{S}_{34} \rangle_{Y_0} \langle \hat{S}_{14} \rangle_{Y_0}}} \right.$$

$$\left. + \frac{1}{2} \int_{Y_0}^{Y} \mathrm{d}y \, \frac{\langle \hat{S}_{13} \rangle_y \langle \hat{S}_{24} \rangle_y}{\sqrt{\langle \hat{S}_{12} \rangle_y \langle \hat{S}_{32} \rangle_y \langle \hat{S}_{34} \rangle_y \langle \hat{S}_{14} \rangle_y}} \frac{\partial}{\partial y} \frac{\langle \hat{S}_{12} \rangle_y \langle \hat{S}_{34} \rangle_y + \langle \hat{S}_{14} \rangle_y \langle \hat{S}_{32} \rangle_y}{\langle \hat{S}_{13} \rangle_y \langle \hat{S}_{24} \rangle_y} \right]$$

- Expanding solution for small T: correct result !
  Linear Hamiltonian, Q linear in T for small T
- Valid in two limits, not exact at transition but cannot be bad
- Can integrate over y for simple configurations

An even simpler expression

- Deep at saturation solve for dipole (fixed coupling)

$$\langle \hat{S}_{ij} \rangle_Y \simeq \exp\Big[-\frac{1}{2\omega} \ln^2(r_{ij}^2 Q_s^2) \Big]$$

- Possible to integrate over y

$$\begin{aligned}\langle \hat{Q}_{1234} \rangle_Y =& \frac{\ln[\langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{34}\rangle_Y / \langle \hat{S}_{13}\rangle_Y \langle \hat{S}_{24}\rangle_Y]}{\ln[\langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{34}\rangle_Y / \langle \hat{S}_{14}\rangle_Y \langle \hat{S}_{23}\rangle_Y]} \langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{34}\rangle_Y \\ &+ \frac{\ln[\langle \hat{S}_{14}\rangle_Y \langle \hat{S}_{23}\rangle_Y / \langle \hat{S}_{13}\rangle_Y \langle \hat{S}_{24}\rangle_Y]}{\ln[\langle \hat{S}_{14}\rangle_Y \langle \hat{S}_{23}\rangle_Y / \langle \hat{S}_{12}\rangle_Y \langle \hat{S}_{34}\rangle_Y]} \langle \hat{S}_{14}\rangle_Y \langle \hat{S}_{23}\rangle_Y\end{aligned}$$

Jalilian-Marian, Kovchegov '04
Dominguez, Marquet, Xiao, Yuan '11

- Still correct for small T, symmetric under exchange of 2 and 4

The Gaussian approximation

- At saturation, dropping real terms, cutoff z-integration

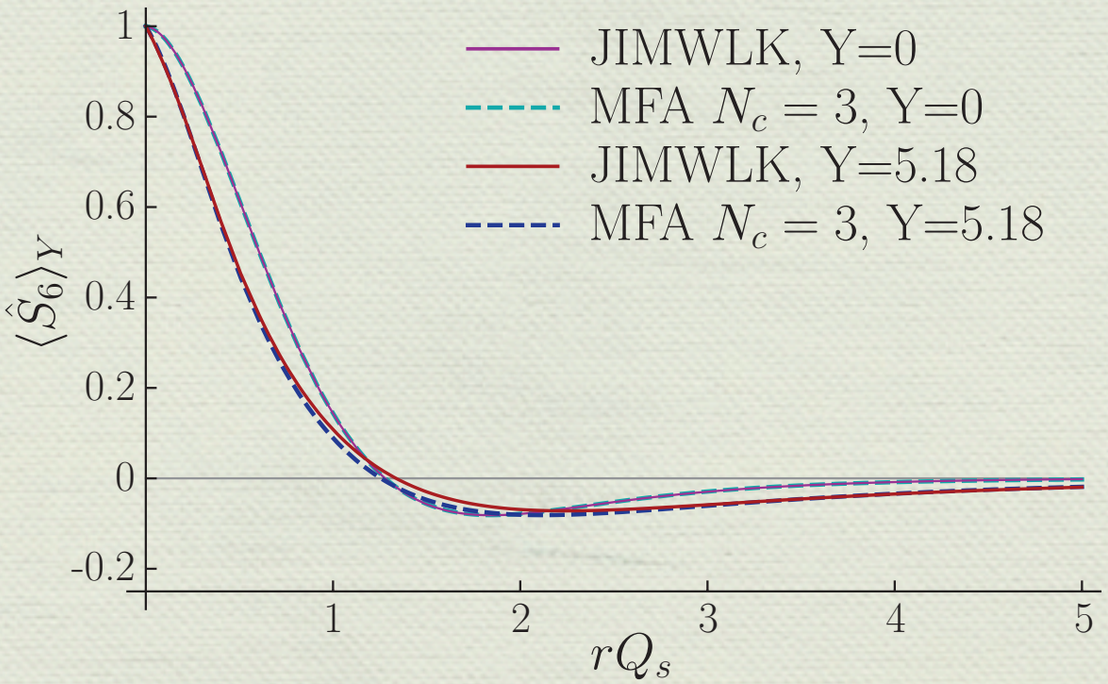
$$H_{\rm sat} \simeq -\frac{1}{8\pi^2}\int_{\boldsymbol{uv}} \ln\left[(\boldsymbol{u}-\boldsymbol{v})^2 Q_s^2(Y)\right]\left(1+\widetilde{V}_{\boldsymbol{u}}^\dagger \widetilde{V}_{\boldsymbol{v}}\right)^{ab}\frac{\delta}{\delta\alpha_{\boldsymbol{u}}^a}\frac{\delta}{\delta\alpha_{\boldsymbol{v}}^b}$$

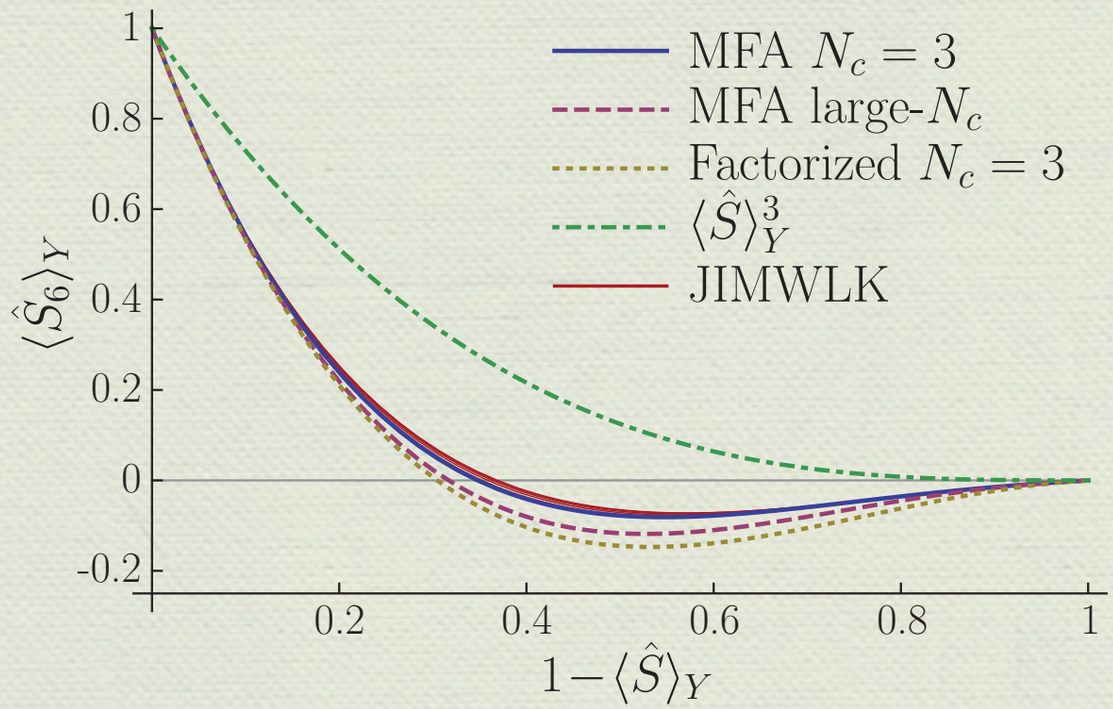
- Modify the "Sudakov" kernel to extend at low density

$$\frac{1}{4\pi^2}\ln\left[(\boldsymbol{u}-\boldsymbol{v})^2 Q_s^2(Y)\right] \to \gamma_Y(\boldsymbol{u},\boldsymbol{v}) = -\frac{1}{2g^2 C_F}\frac{\partial \ln\langle \hat{S}_{\boldsymbol{uv}}\rangle_Y}{\partial Y}$$

- Possible to express in terms of the BK solution

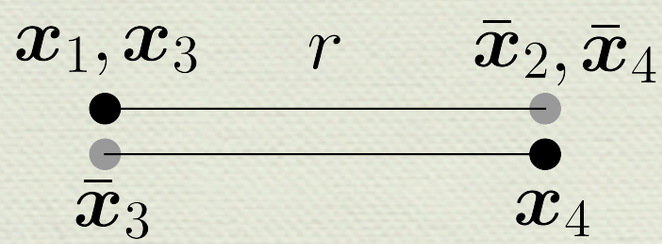
vs numerical solution



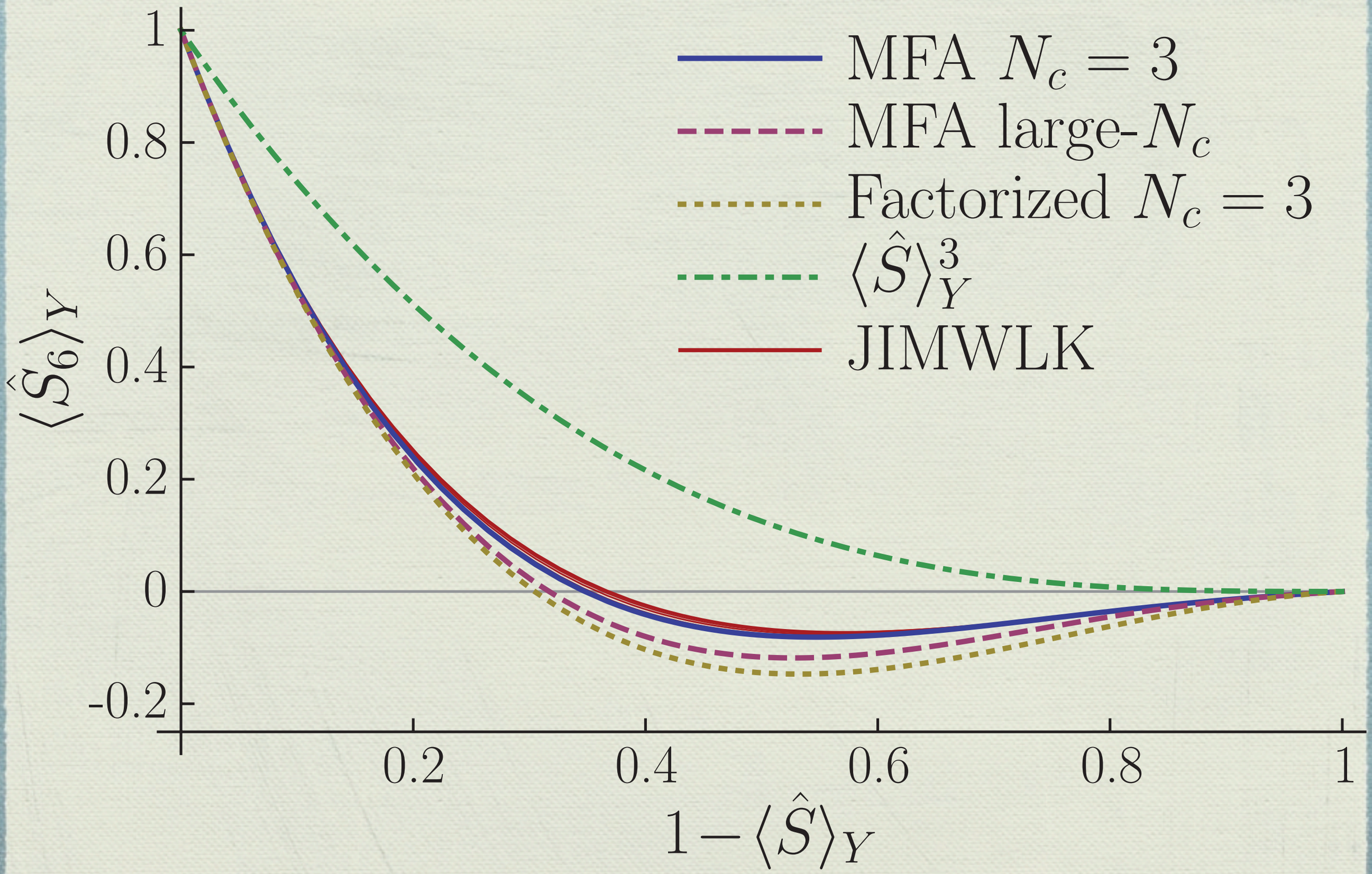


- 6-point function $\hat{S}_{6\,\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4} = \frac{N_c^2}{N_c^2-1}\hat{Q}_{1234}\hat{S}_{43} - \frac{1}{N_c^2-1}\hat{S}_{12}$

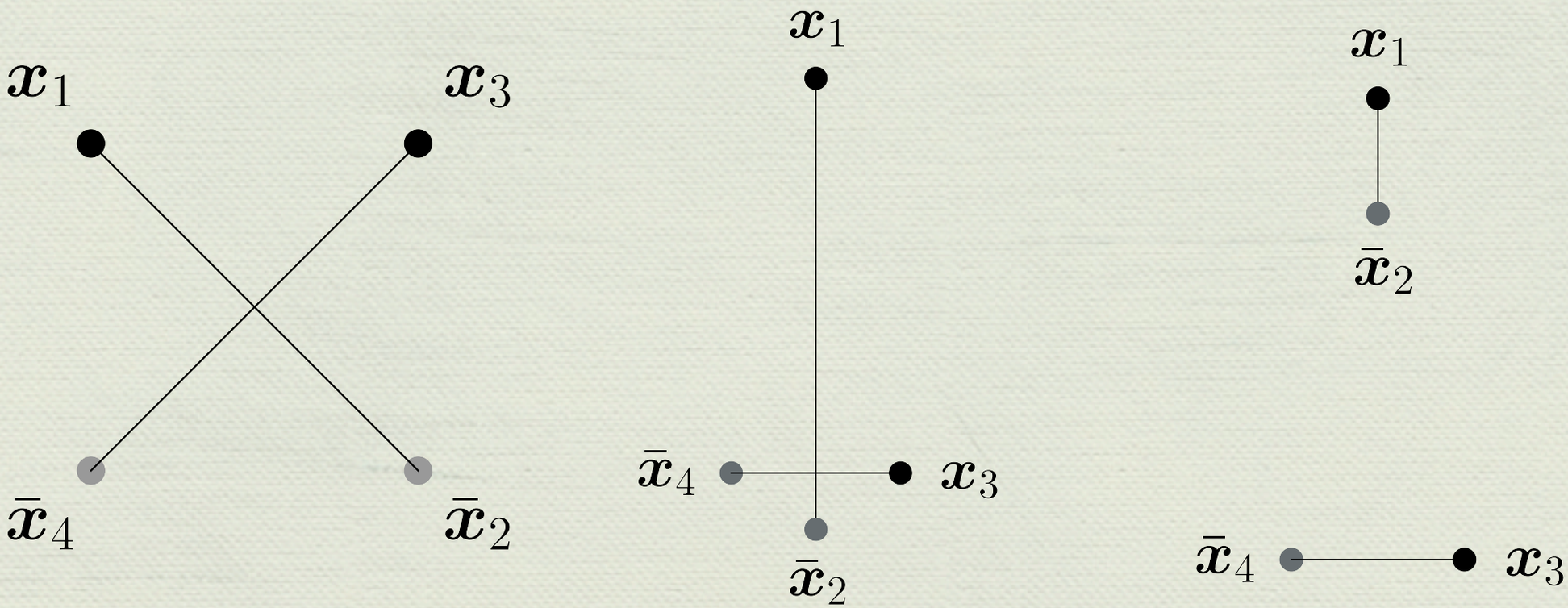
- Line configuration:



$$\langle \hat{Q}\hat{S} \rangle_Y = \frac{(N_c+2)(N_c-1)}{2N_c}\langle \hat{S} \rangle_Y^{\frac{3N_c-1}{N_c-1}} - \frac{(N_c+1)(N_c-2)}{2N_c}\langle \hat{S} \rangle_Y^{\frac{3N_c+1}{N_c+1}}$$


MFA $N_c = 3$
MFA large-N_c
Factorized $N_c = 3$
$\langle \hat{S} \rangle_Y^3$
JIMWLK
$\langle \hat{S}_6 \rangle_Y$
$1 - \langle \hat{S} \rangle_Y$
1
0.8
0.6
0.4
0.2
0
-0.2
0.2
0.4
0.6
0.8
1

Other special configurations



$$\langle \hat{Q}_{1234} \rangle_Y = \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y$$

- Factorization violations at finite Nc (at saturation)

$$\left\langle \hat{S}_{13}\hat{S}_{32} - \frac{1}{N_c^2}\,\hat{S}_{12} \right\rangle_Y = \frac{N_c^2-1}{N_c^2}\left[\frac{\langle \hat{S}_{13}\rangle_Y \langle \hat{S}_{32}\rangle_Y}{\langle \hat{S}_{12}\rangle_Y}\right]^{\frac{1}{(N_c^2-1)}} \langle \hat{S}_{13}\rangle_Y \langle \hat{S}_{32}\rangle_Y$$

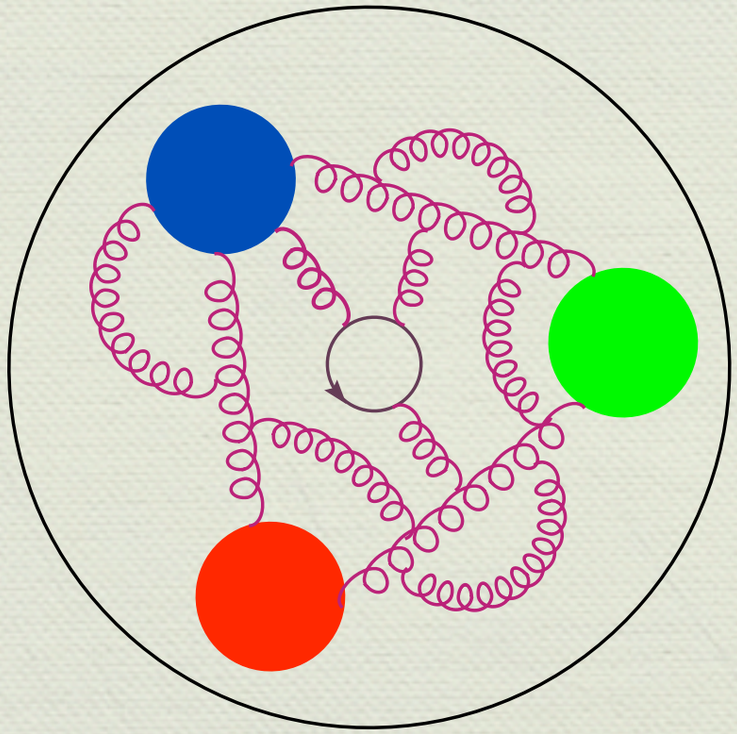
Conclusion

- Justify Gaussian approximation at finite Nc
- In practice: an analytical solution to JIMWLK
- Study more configurations, self-consistent checks
- Wilson lines expand symmetrically
- Access to more observables, more accurate phenomenology

BACKUP SLIDES

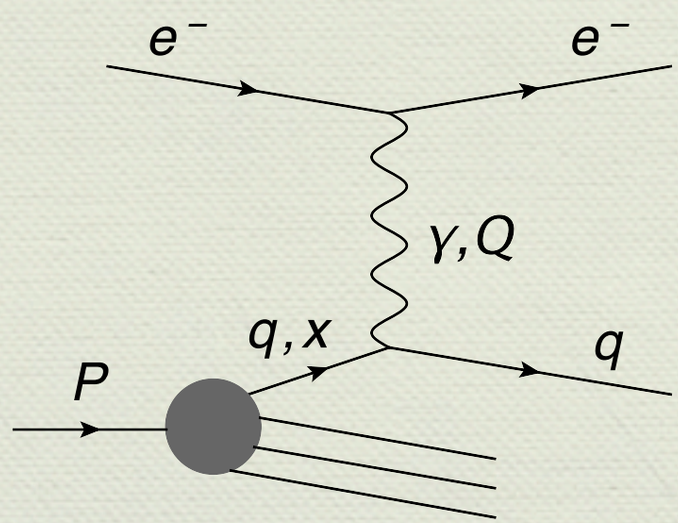
Constituents of a hadron

- Proton, or generic hadron, is complicated in rest frame
- Hadronic and vacuum fluctuations
- Non-perturbative with same lifetime $\Delta t_{\mathrm{RF}} \sim 1/\Lambda_{\mathrm{QCD}}$



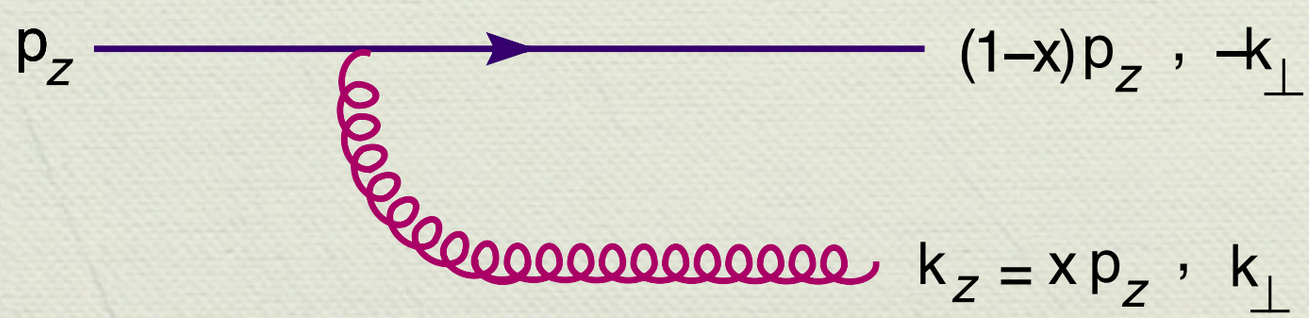
Infinite momentum frame and DIS

- IMF: Hadronic fluctuations live longer $\Delta t_{\rm IMF} \sim \gamma/\Lambda_{\rm QCD}$
- Longer than vacuum fluctuations
- Longer than collision time, e.g. in DIS $\Delta t_{\rm coll} \sim 2xP/Q^2$
- Quark with $\Delta t_{\rm fluct} \sim 2xP/k_\perp^2 \gtrsim \Delta t_{\rm coll}$ seen by photon



Soft and collinear gluons

- $\mathrm{d}P = C_R \frac{\alpha_s(k_\perp^2)}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$
- Emission of soft and collinear gluons is favored
- Large logs can overcome smallness of coupling
- Source lives longer than emitted parton: frozen
- Gluons dominate at small-x



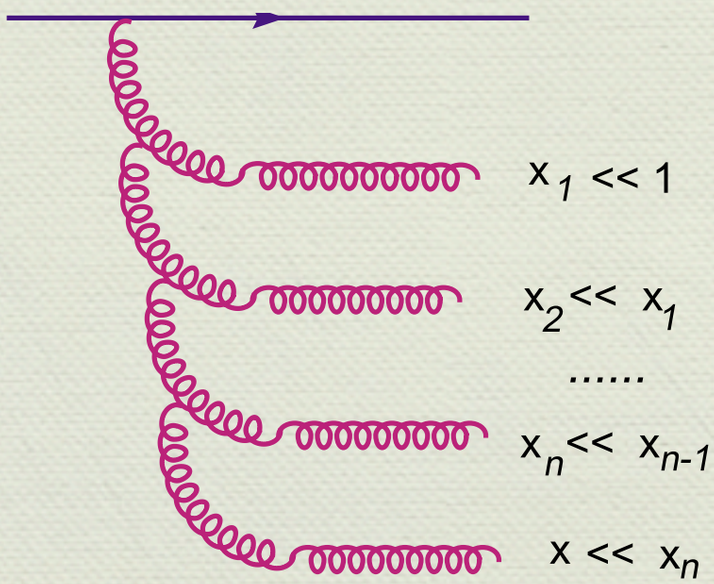
Cascades and evolution

- Successive emissions: DGLAP or BFKL cascade

$$\left(\frac{\alpha_s N_c}{\pi}\right)^n \int_x^1 \frac{\mathrm{d}x_n}{x_n} \int_{x_n}^1 \frac{\mathrm{d}x_{n-1}}{x_{n-1}} \cdots \int_{x_2}^1 \frac{\mathrm{d}x_1}{x_1} = \frac{1}{n!}\left(\bar{\alpha}_s \ln \frac{1}{x}\right)^n$$

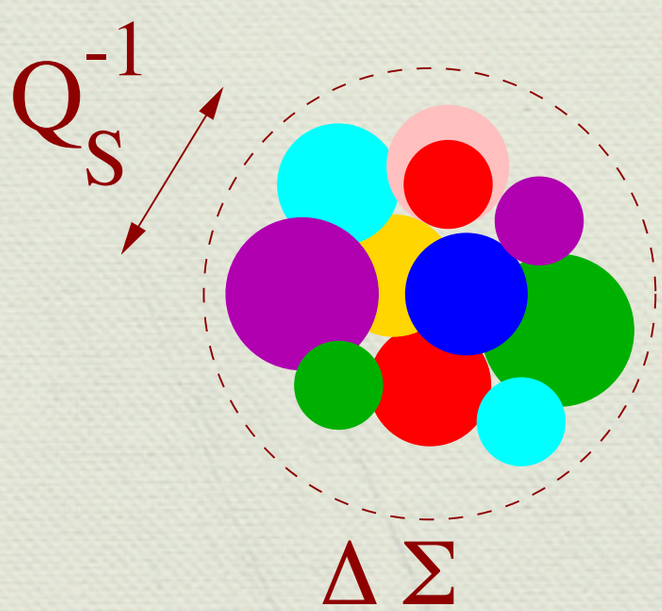
- Resum all diagrams $f_g = \frac{\mathrm{d}N_g}{\mathrm{d}Y\,\mathrm{d}k_\perp^2} \sim \frac{\alpha_s C_F}{\pi}\frac{1}{k_\perp^2}\exp(\omega\bar{\alpha}_s Y)$

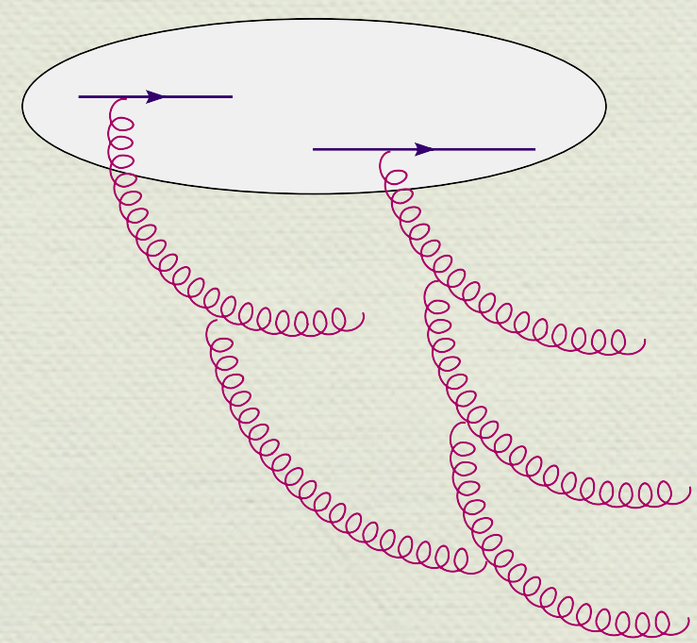
- Evolution equation $\frac{\mathrm{d}}{\mathrm{d}Y} f_g = \omega\bar{\alpha}_s f_g$



Kinematic regimes

- BFKL and DGLAP: linear, incoherent emissions
- DGLAP: smaller and smaller partons of size $1/Q^2$
- BFKL: typically same size partons
- Partons will "overlap", coherent, non-linear evolution





Single particle production

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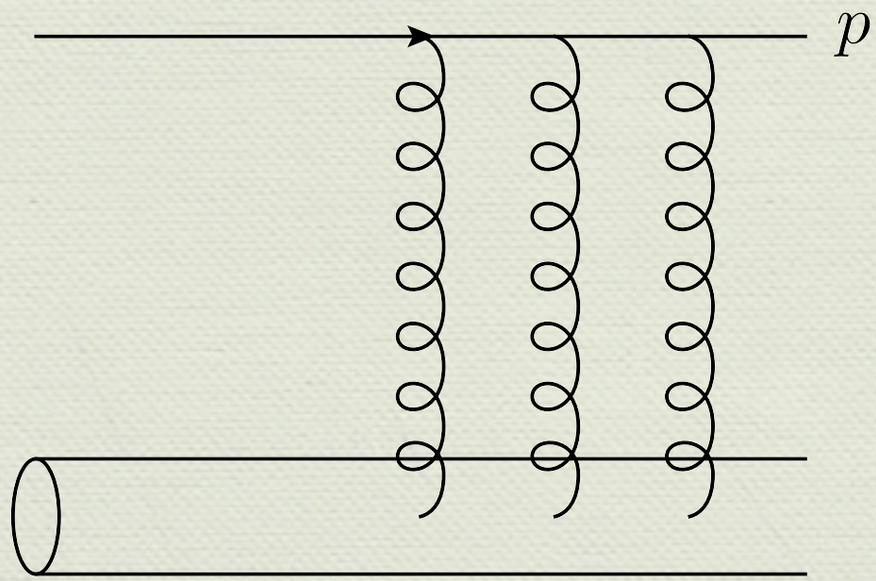
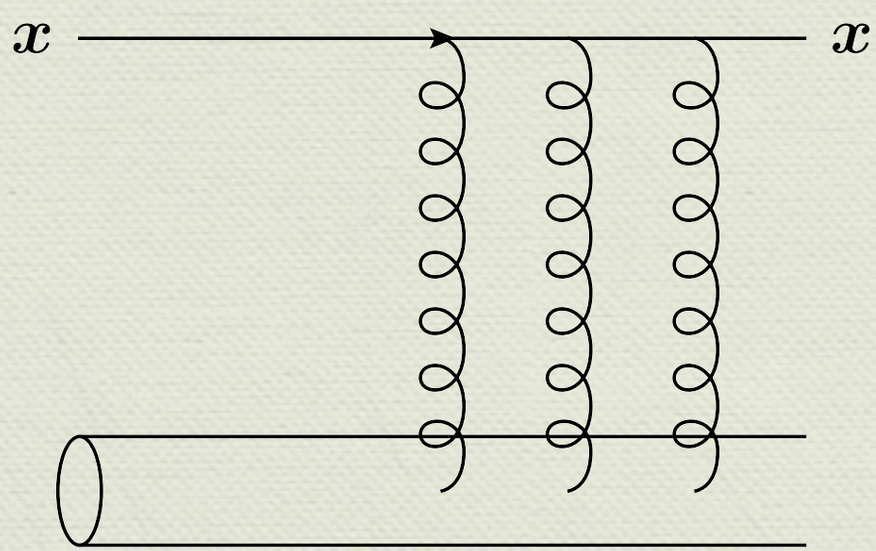


Wilson lines

- Mixed representation: transverse momenta → coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction → Wilson lines: $V^\dagger_{\boldsymbol{x}} = P \exp\left[\mathrm{i}\,\mathrm{g} \int \mathrm{d}x^- t^a \mathcal{A}^{+a}_{\boldsymbol{x}}(x^-)\right]$



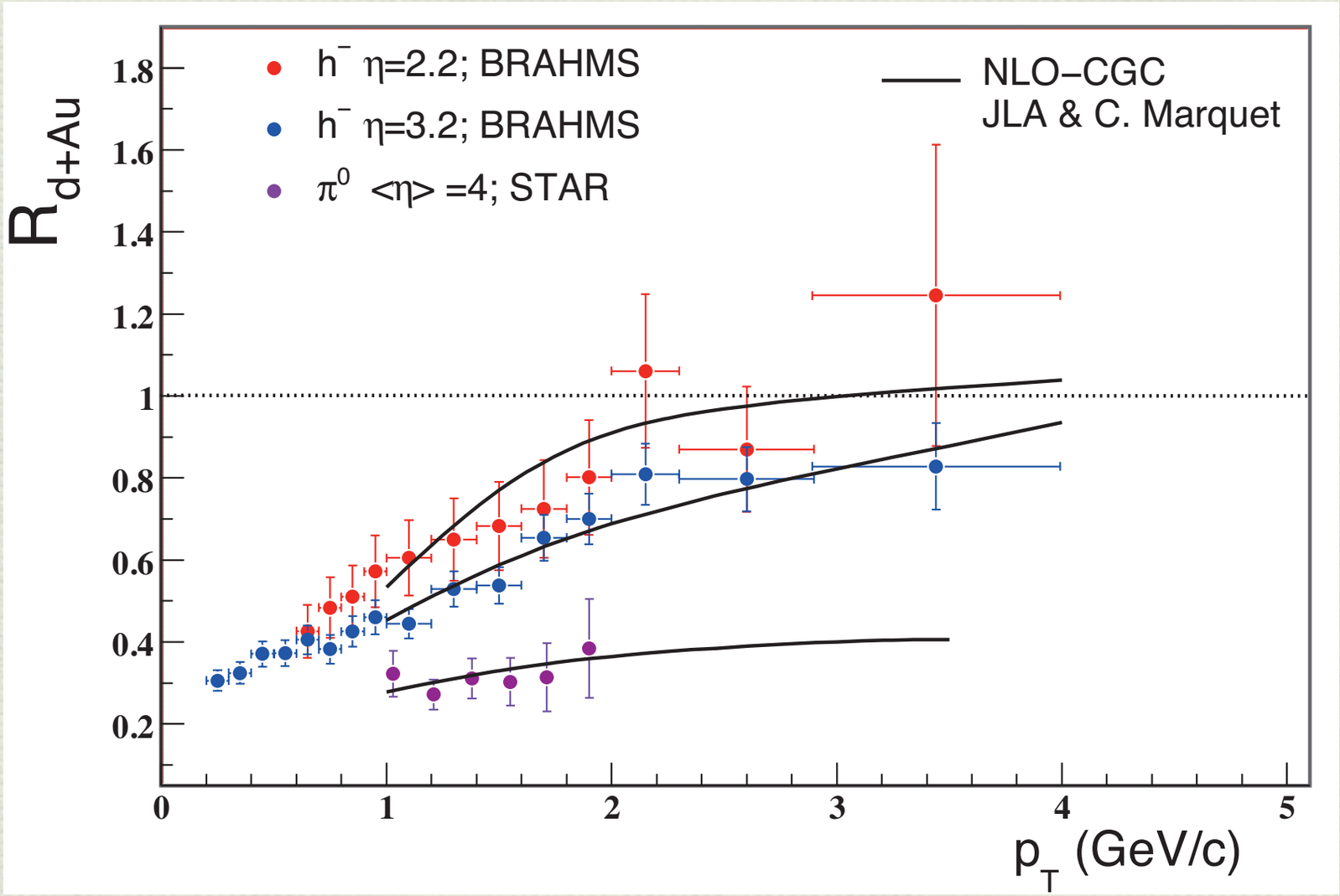
The cross section

- Multiply by c.c. and F.T. to calculate cross section

$$\frac{\mathrm{d}\sigma^{qA\to qX}}{\mathrm{d}y\mathrm{d}^2\boldsymbol{p}} = x_1 f_q(x_1,\boldsymbol{p}^2)\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{p}}\frac{1}{N_c}\Big\langle \mathrm{tr}[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})]+\ldots\Big\rangle_Y$$

- Summing over final color, average initial: 1/Nc tr ...
- QCD dynamics in $\langle\ldots\rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+]\ldots$
- $\mathrm{e}^{-Y} = x_2 = \frac{|\boldsymbol{p}|\mathrm{e}^{-y}}{\sqrt{s}} \ll 1$ in forward region, $x_1 = \frac{|\boldsymbol{p}|\mathrm{e}^{y}}{\sqrt{s}}$

Comparing with data



$$R_{pA}(\eta, \boldsymbol{p}) \equiv \frac{1}{A^{1/3}} \frac{\mathrm{d}N_h/\mathrm{d}\eta \mathrm{d}^2\boldsymbol{p}\big|_{pA}}{\mathrm{d}N_h/\mathrm{d}\eta \mathrm{d}^2\boldsymbol{p}\big|_{pp}}$$

JIMWLK evolution

- Evolution for functional of color sources

$$\frac{\partial W_Y[\rho]}{\partial Y} = H_{\mathrm{JIMWLK}} W_Y[\rho]$$

- Average over color sources

$$\langle \mathcal{O}[A] \rangle_Y = \int \mathcal{D}\rho W_Y[\rho] \mathcal{O}[A(\rho)]$$

- Solve classical Yang-Mills equations to get the field

$$D_\nu^{ab} F_b^{\nu\mu} = \delta^{\mu+} \rho^a(x^-, \boldsymbol{x}_\perp)$$

The right-derivatives

- Functional derivatives act on upper endpoint: L-derivative

$$\frac{\delta}{\delta\alpha^a_{\boldsymbol{u}}}\, V^\dagger_{\boldsymbol{x}} = \mathrm{i}g\delta_{\boldsymbol{xu}}\, t^a V^\dagger_{\boldsymbol{x}}$$

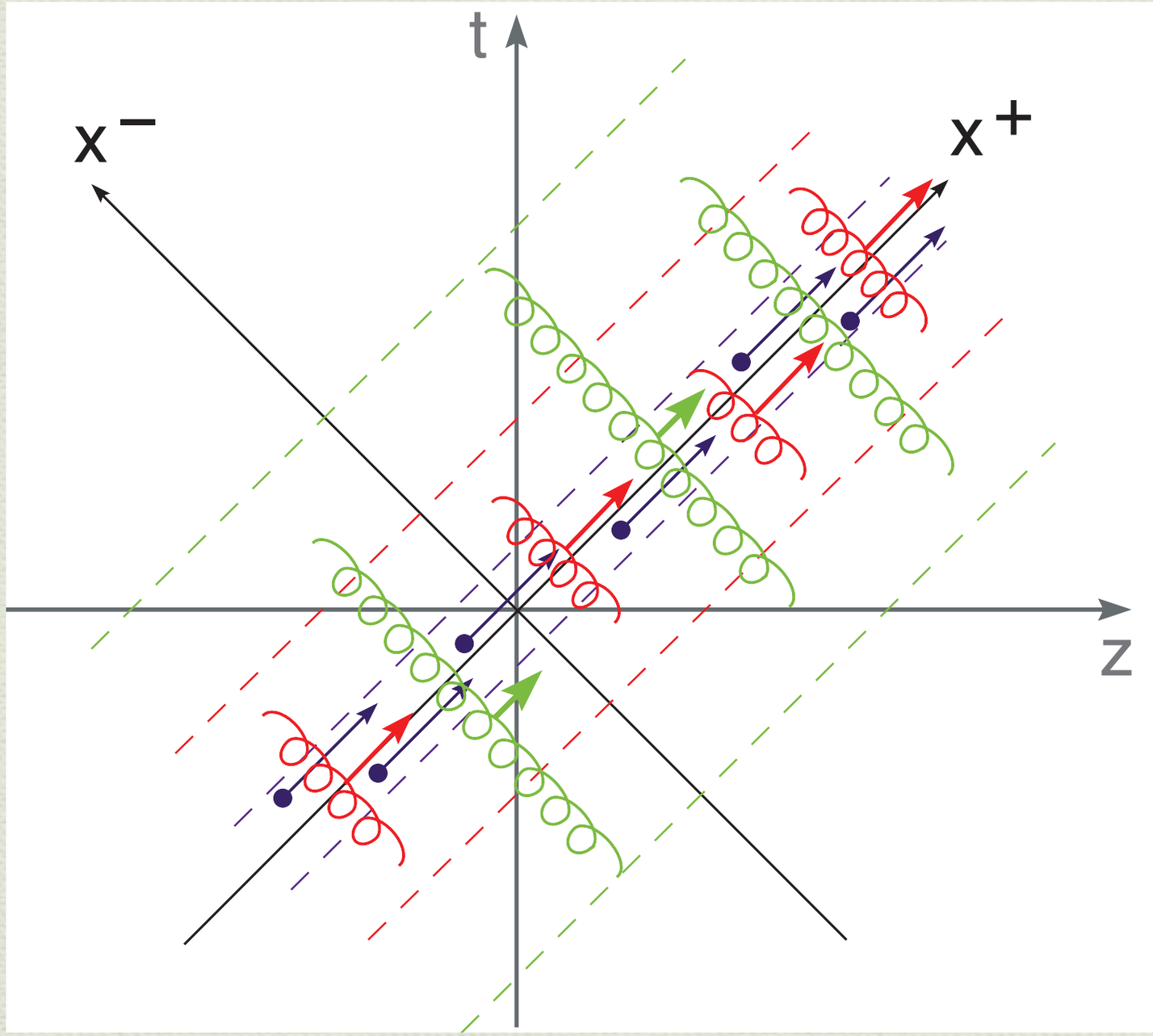
- Adjoint Wilson lines transform to R-derivatives: lower endpoint

$$(\widetilde{V}^\dagger)^{ac}\frac{\delta}{\delta\alpha^a_{\boldsymbol{u}}}\, V^\dagger_{\boldsymbol{x}} = \mathrm{i}g\delta_{\boldsymbol{xu}}\, V^\dagger_{\boldsymbol{x}} t^c$$

- Wilson lines expand symmetricaly in longitudinal direction

$$V^\dagger_{n+1}(\boldsymbol{x}) = \exp[\mathrm{i}g\epsilon\alpha_{n+1}(\boldsymbol{x})]\, V^\dagger_n(\boldsymbol{x})\, \exp[\mathrm{i}g\epsilon\alpha_{-(n+1)}(\boldsymbol{x})]$$

Symmetric longitudinal expansion



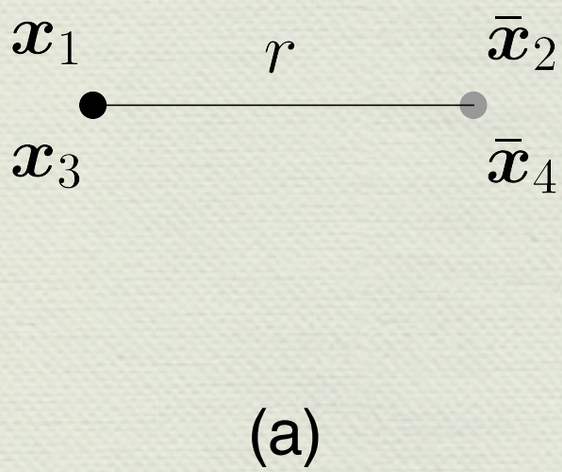
The Gaussian approximation

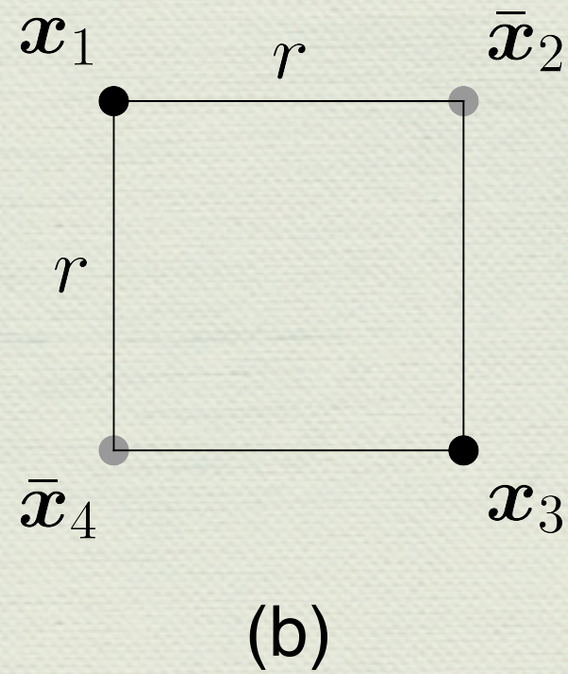
- Expression for Q first derived in MV model = special Gaussian

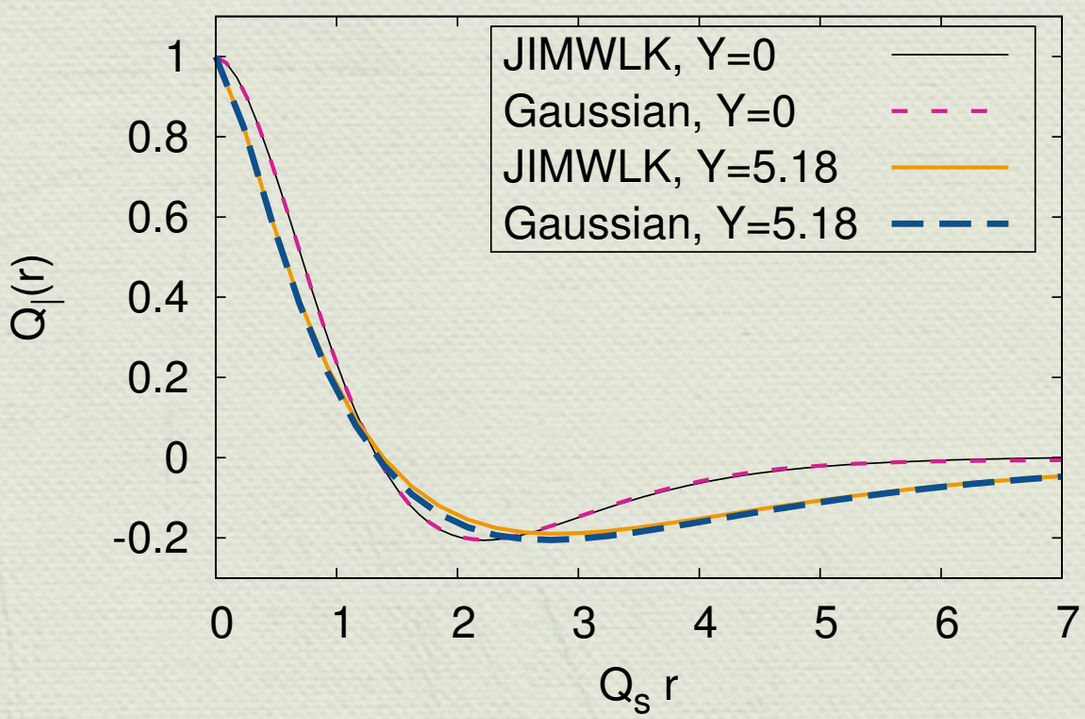
$$\langle \rho_a(x_1^-, \boldsymbol{x}_1)\rho_b(x_2^-, \boldsymbol{x}_2)\rangle = \delta_{ab}\Theta(x_0^- - |x_1^-|)\delta(x_1^- - x_2^-)\delta_{\boldsymbol{x}_1\boldsymbol{x}_2}\lambda(x_1^-, \boldsymbol{x}_1)$$

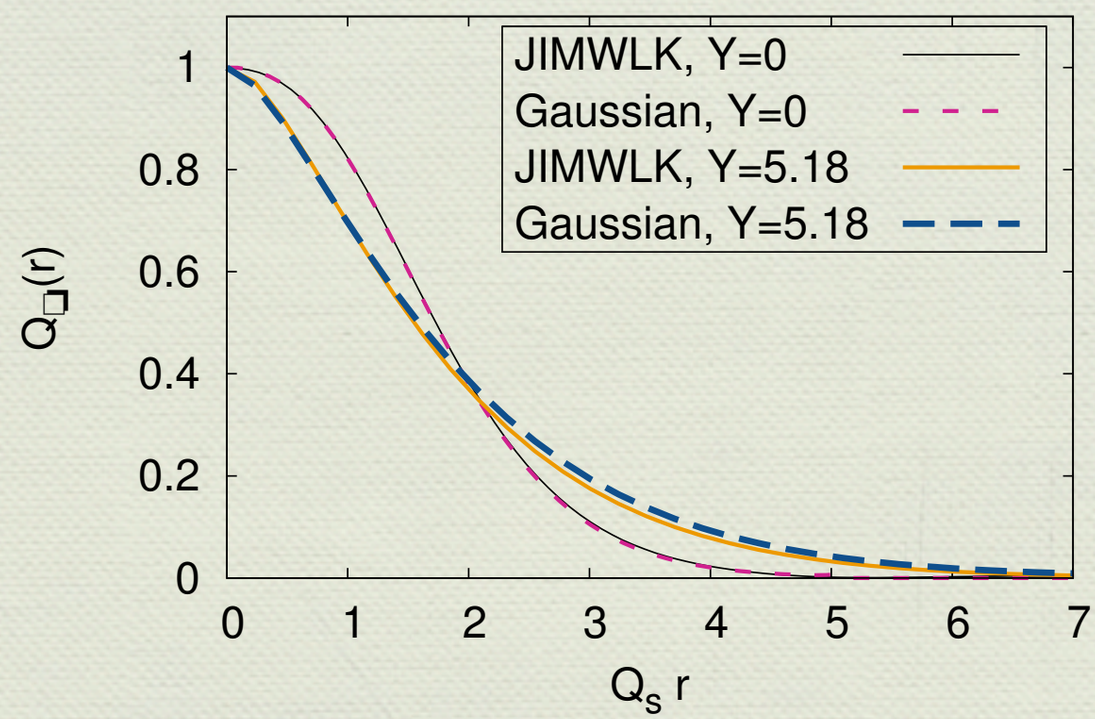
- Virtual terms arise from Gaussian part of H
- All correlators in terms of 2-point function → Gaussian kernel
- Valid at finite Nc
- At saturation drop last two terms of gluon emission
- At weak scattering only 2-point function and products

vs numerical solution



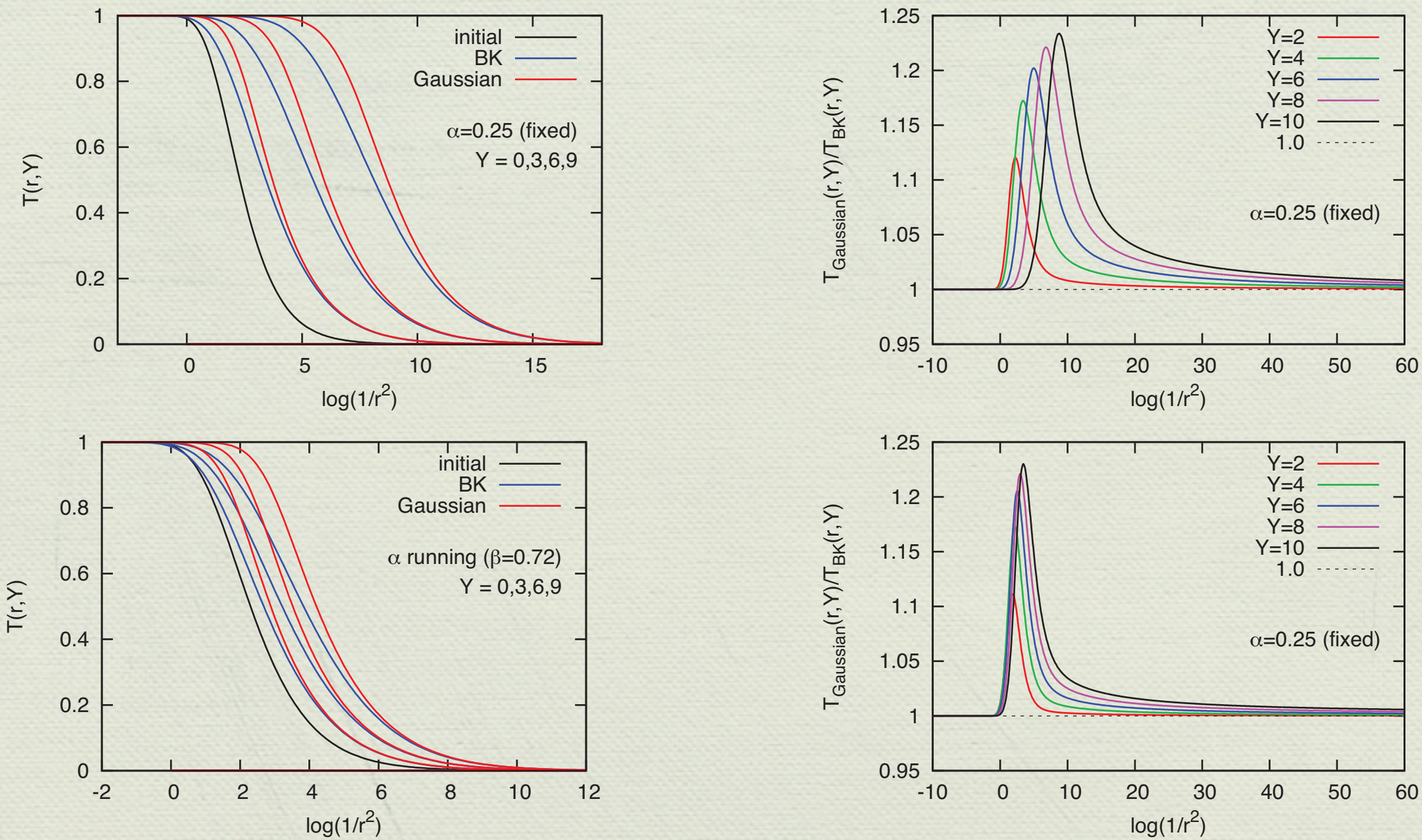






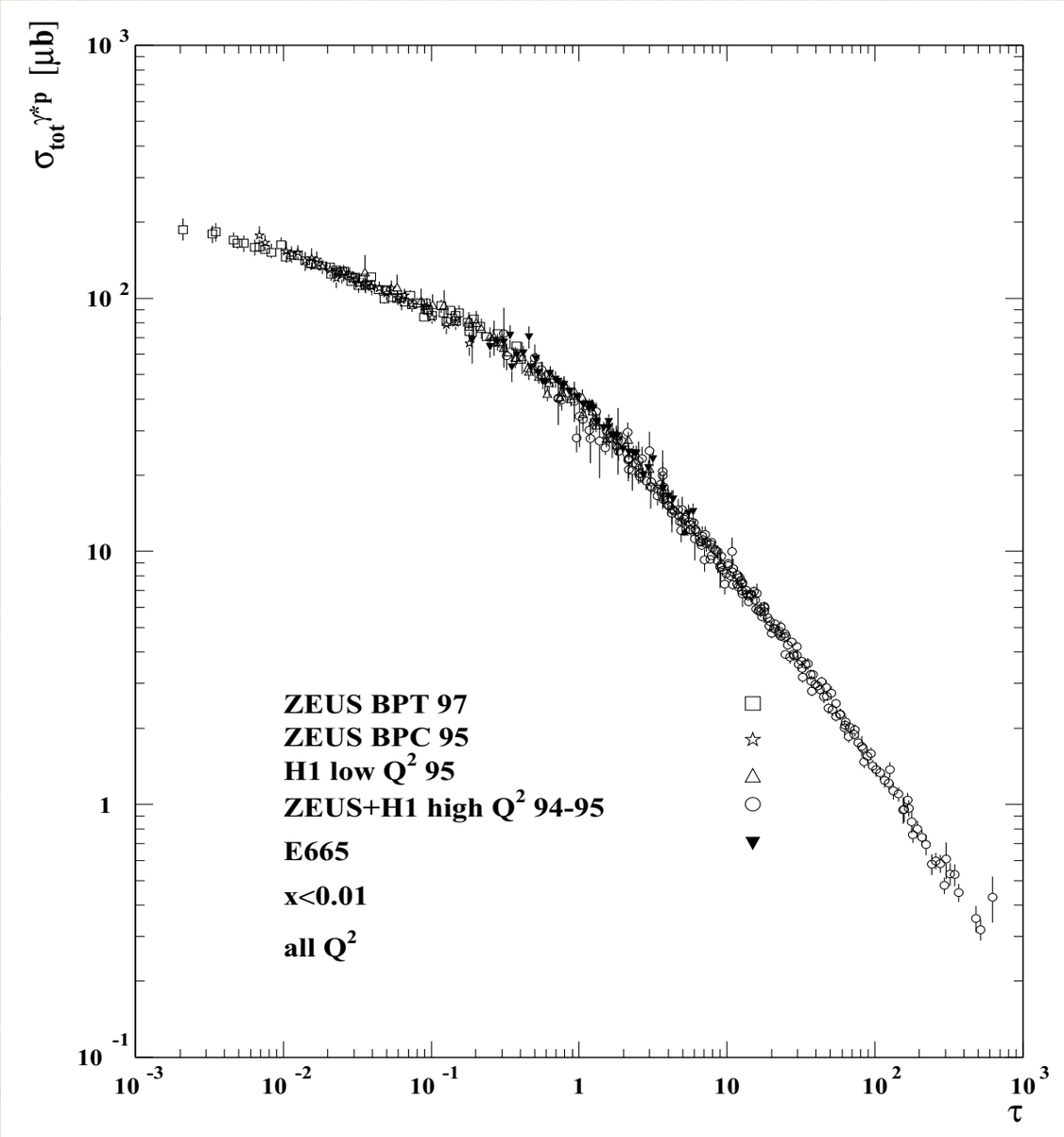
"Internal check"

- At large Nc for example:
  Solve BK and MFA from averaging JIMWLK kernel



Scaling

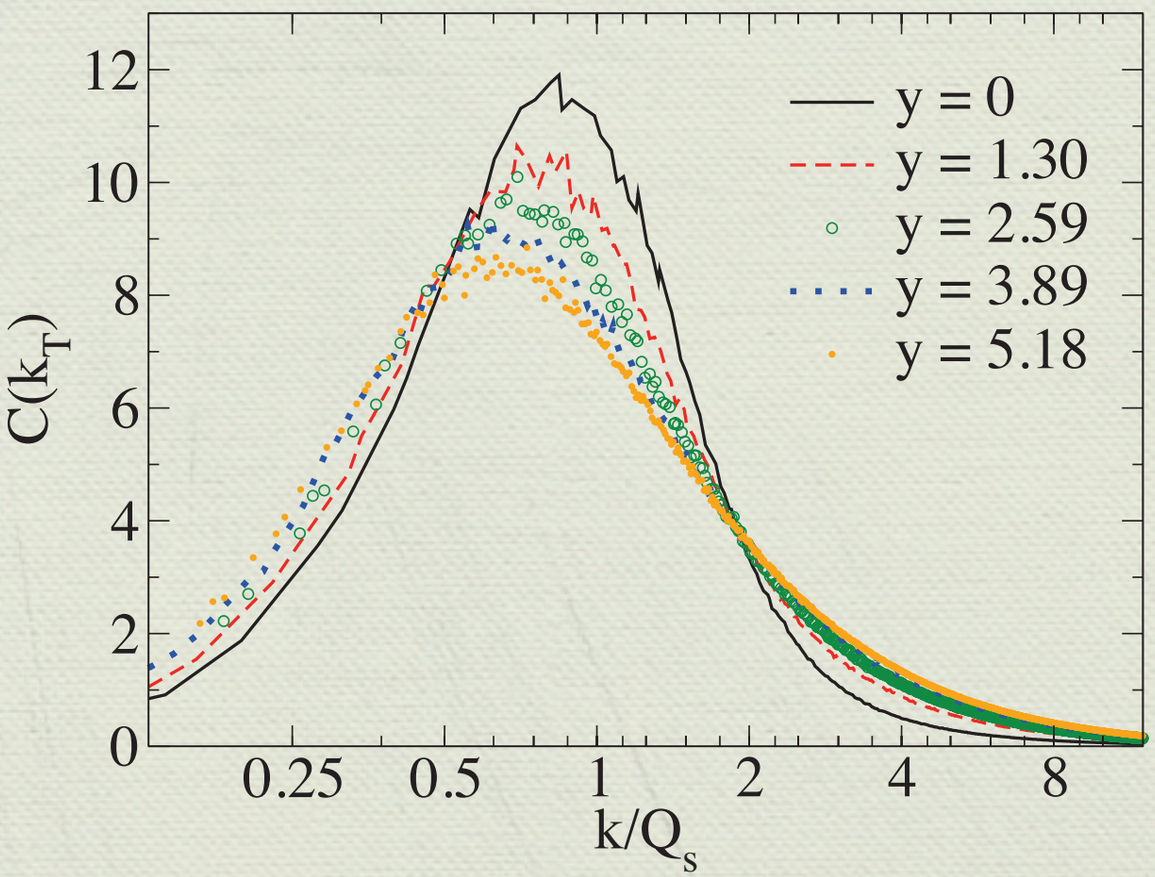
- Qs(Y) sets the scale: dependence only on rQs(Y) around Qs

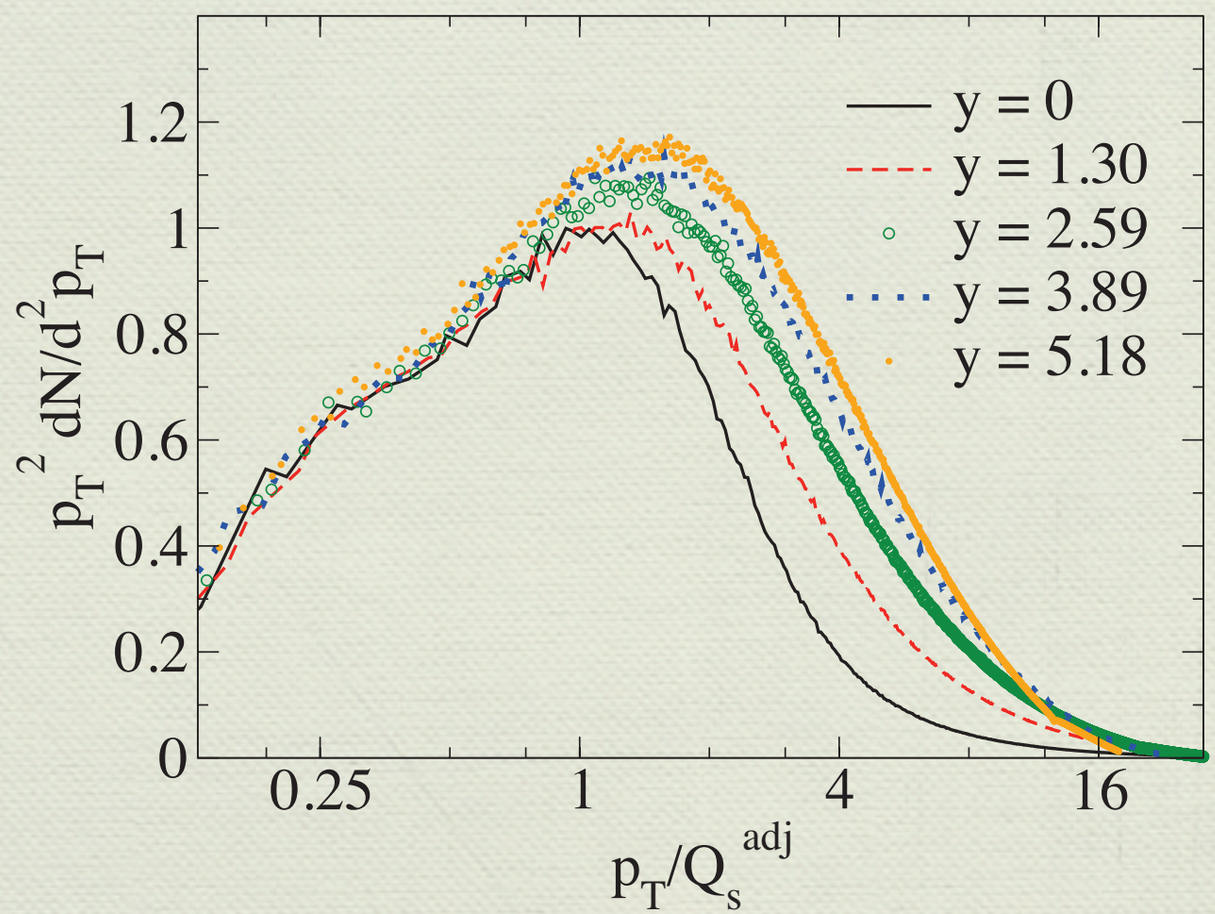


Gluon distribution and gluon production in AA

- Distribution $C(Y,\boldsymbol{p}) = \boldsymbol{p}^2 \int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{p}} \frac{1}{N_c} \left\langle \mathrm{tr}[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})] + \ldots \right\rangle_Y$

- Production: collide two "color glasses"





Multiplicities in AA

- Multiplicities should not be affected by final state