Calculating the jet quenching parameter $\hat{q}$ in lattice gauge theory

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Hard Probes 2012, Cagliari, Italy

Motivation !

Duke QCD Theory

Hot QCD matter properties

Which **properties of hot QCD matter** can we hope to determine with the help of hard probes ?

Easy for LQCD

$$m_D = -\lim_{|x|\to\infty} \frac{1}{|x|} \ln\left\langle E^a(x)E^a(0)\right\rangle$$

Color screening: Quarkonium states

Hard for LQCD

$$\Pi^{\mu\nu}_{\rm em}(k) = \int d^4x\, e^{ikx} \left\langle j^\mu(x) j^\nu(0)\right\rangle$$

QGP Radiance: Lepton pairs, photons

Very Hard for LQCD

$$\hat{q} = \frac{4\pi^2\alpha_s C_R}{N_c^2-1} \int dy^- \left\langle F^{a+i}(y^-)F_i^{a+}(0)\right\rangle$$

$$\hat{e} = \frac{4\pi^2\alpha_s C_R}{N_c^2-1} \int dy^- \left\langle i\partial^- A^{a+}(y^-)A^{a+}(0)\right\rangle$$

$$\hat{e}_2 = \frac{4\pi^2\alpha_s C_R}{N_c^2-1} \int dy^- \left\langle F^{a+-}(y^-)F^{a+-}(0)\right\rangle$$

Momentum diffusion:
parton energy loss, jet quenching

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Monday, May 28, 12

The Setup

Hard jets described by pQCD (analytic or MC)

Factorized from soft non-perturbative medium

Medium influences jet evolution via transport coeffs.

Medium evolves hydrodynamically (most of the time)

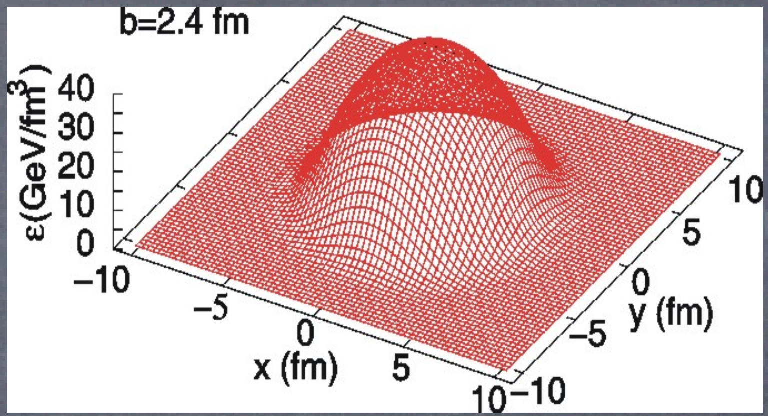
Transport coefficients parametrized not calculated

Momentum structure unknown !

See Talk by T. Renk for other approaches

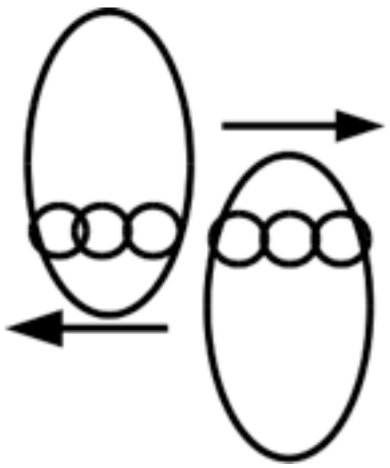
A status report on pQCD jet modification on a non-perturbative medium

Fit the $\hat{q}$ at initial T in the hydro in central coll.

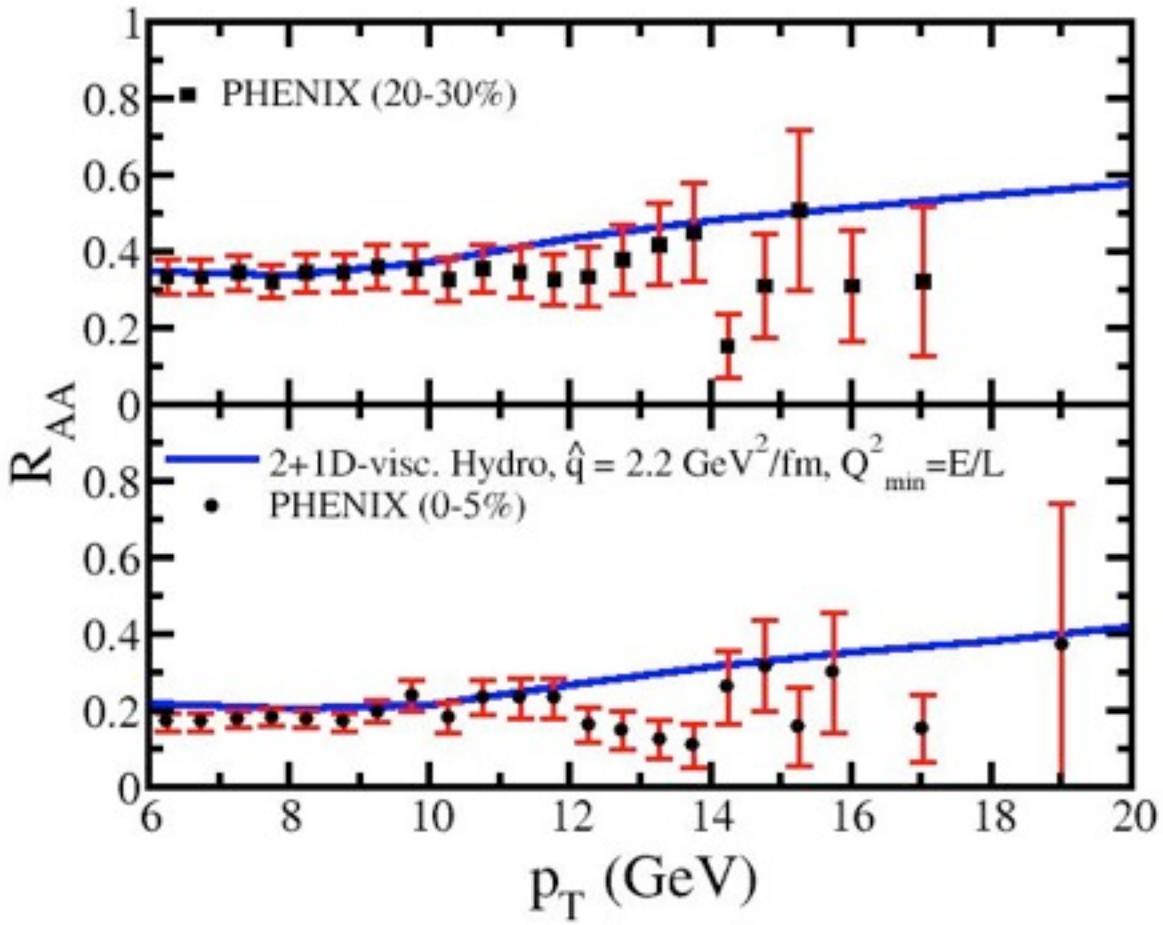


$$\hat{q}(\vec{r},t) = \hat{q}_0 \frac{s(\vec{r},t)}{s_0}$$

$$s_0 = s(T_0)$$

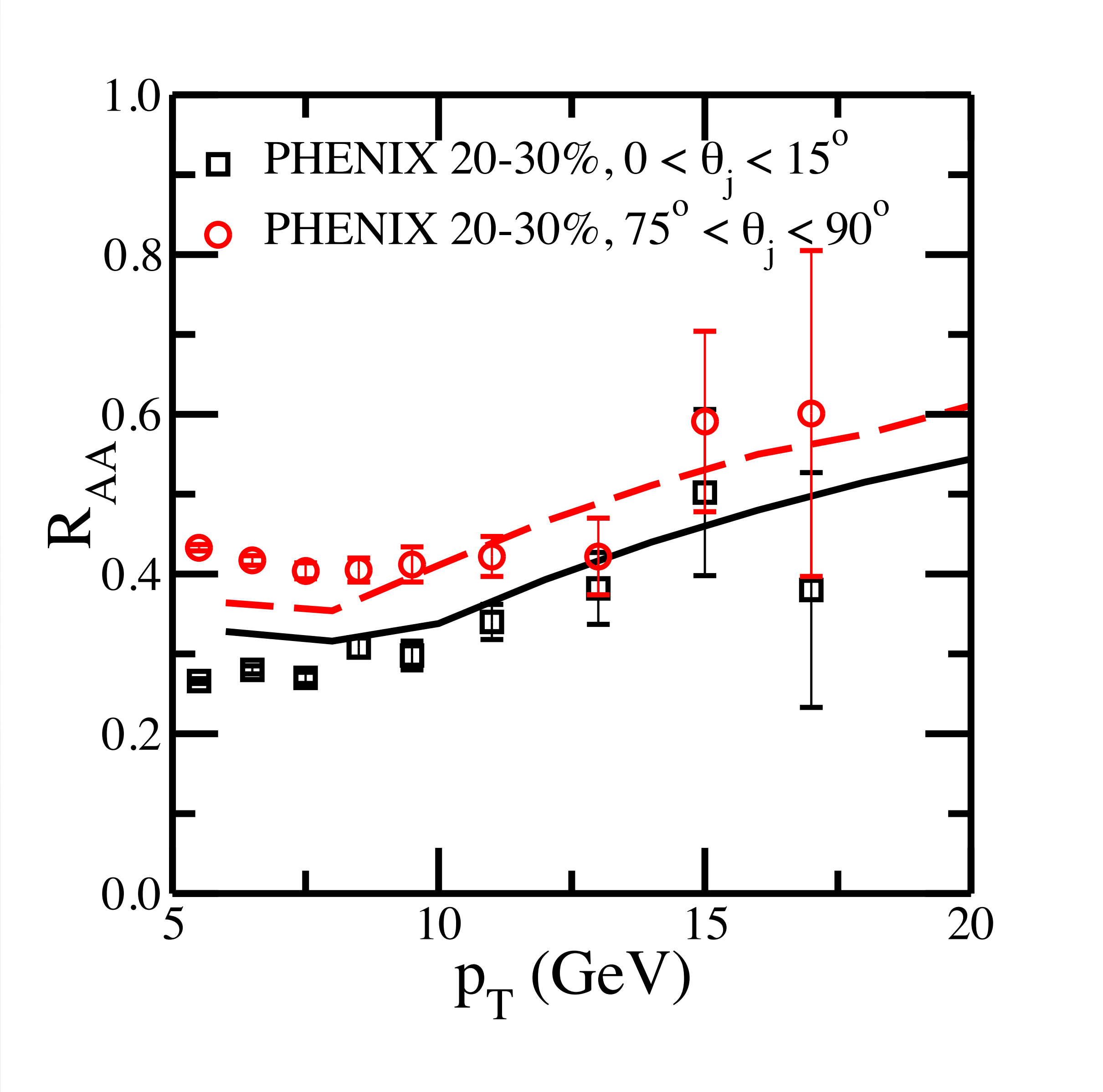


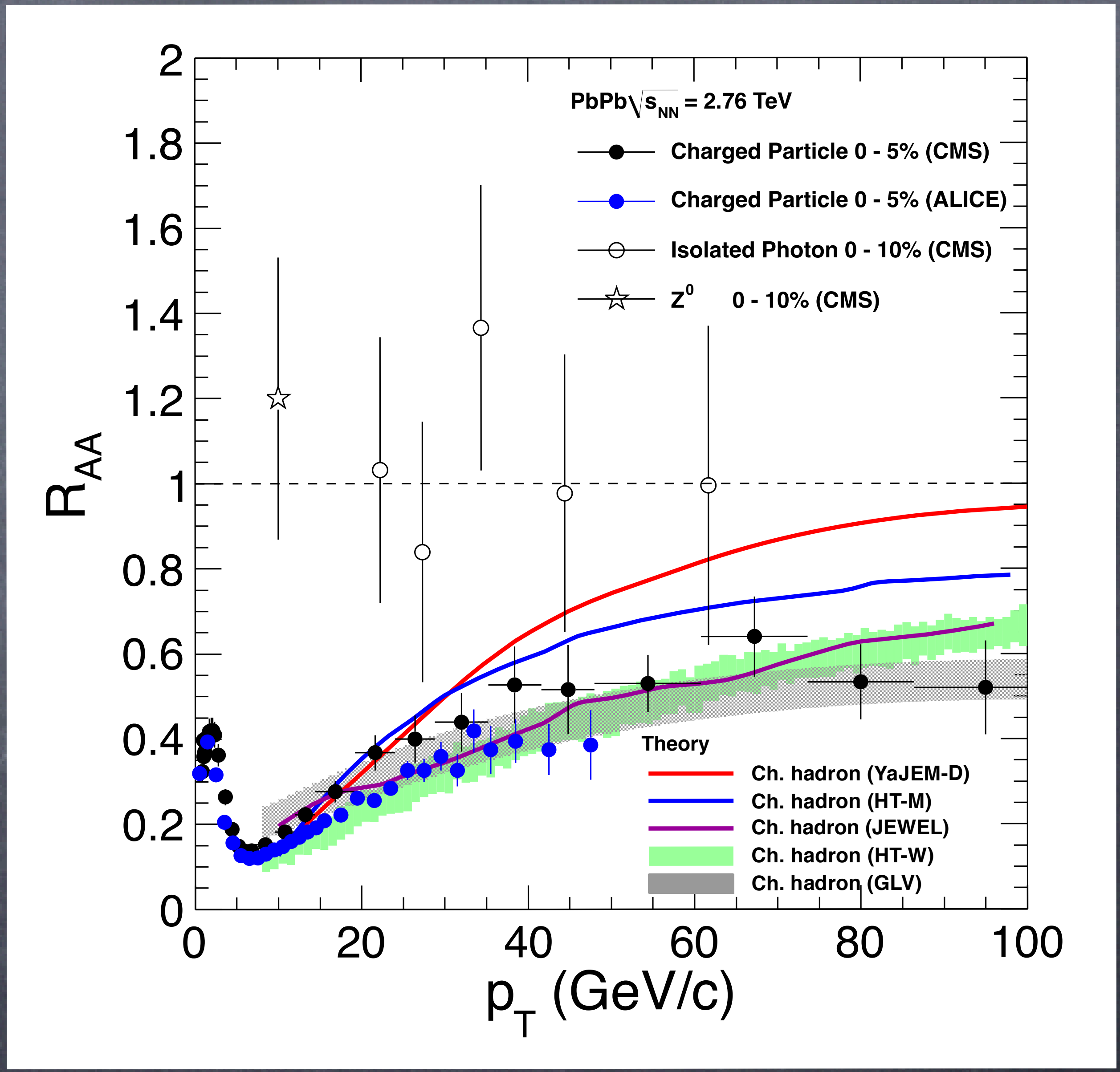
$$R_{AA} \sim \frac{\frac{dN_{AA}}{dp_T dy}}{N_{bin}\frac{dN_{pp}}{dp_T dy}}$$



Versus reaction plane, versus energy

Reasonable agreement with data





See talk by B. Mueller

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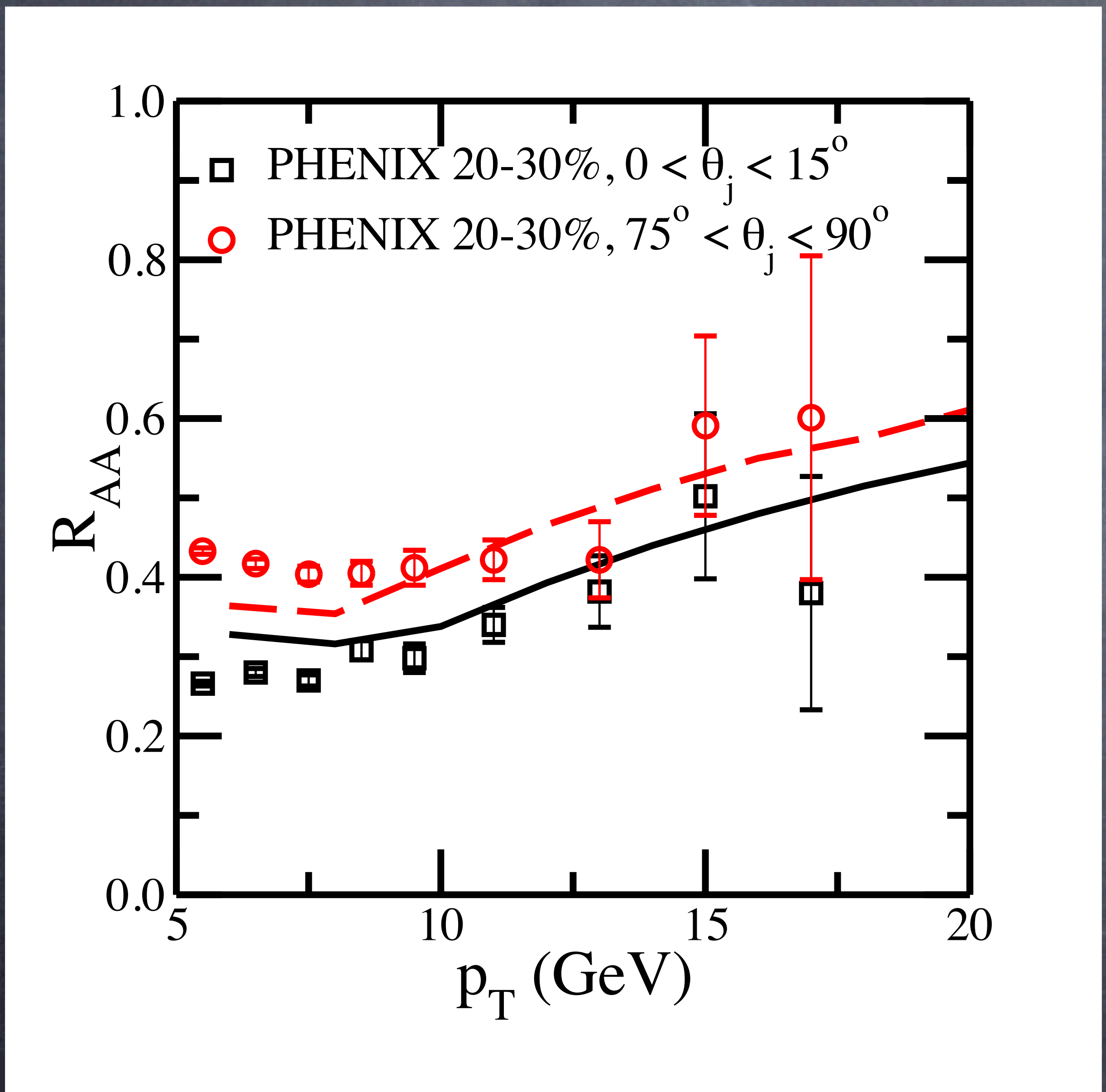

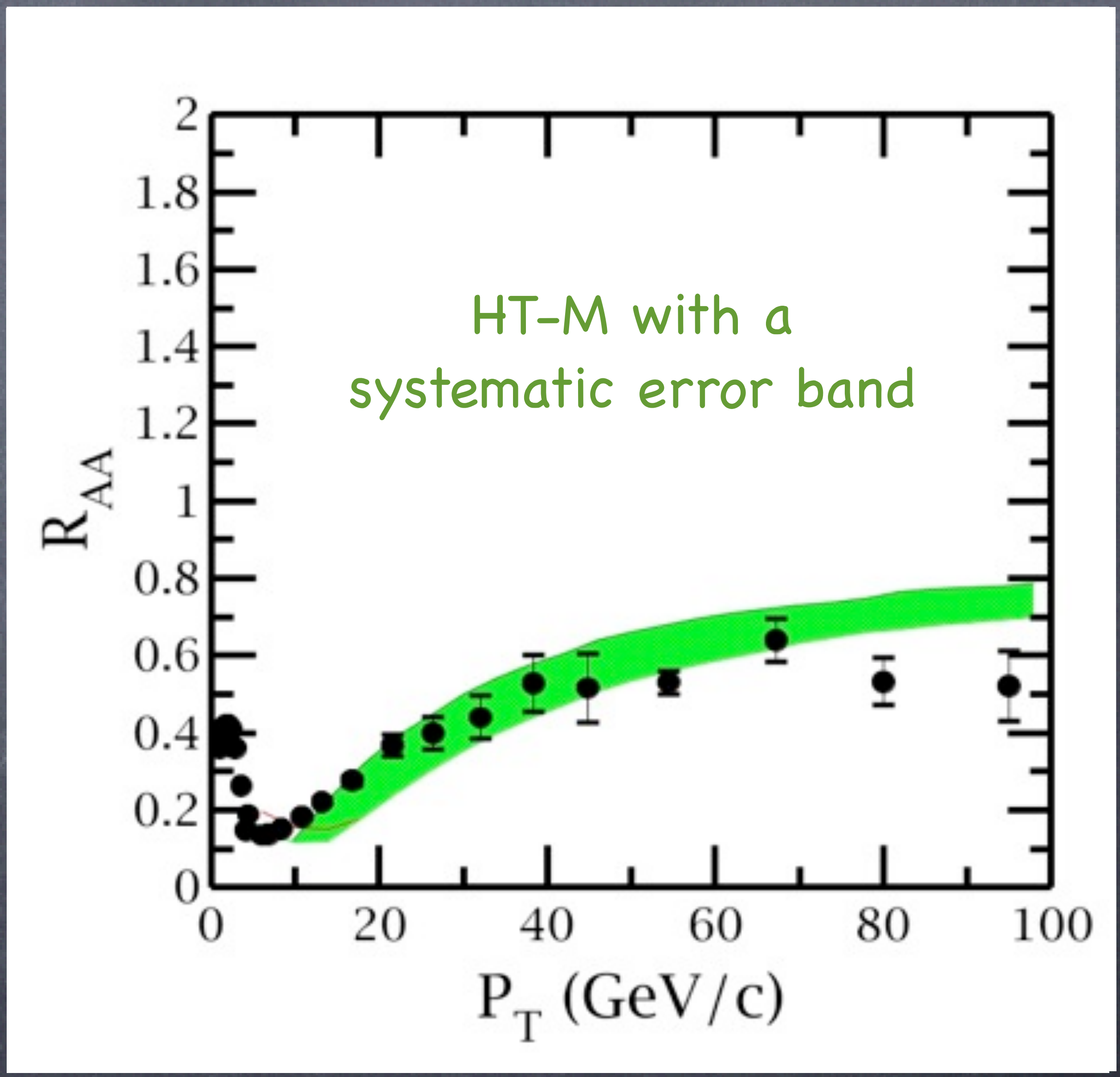

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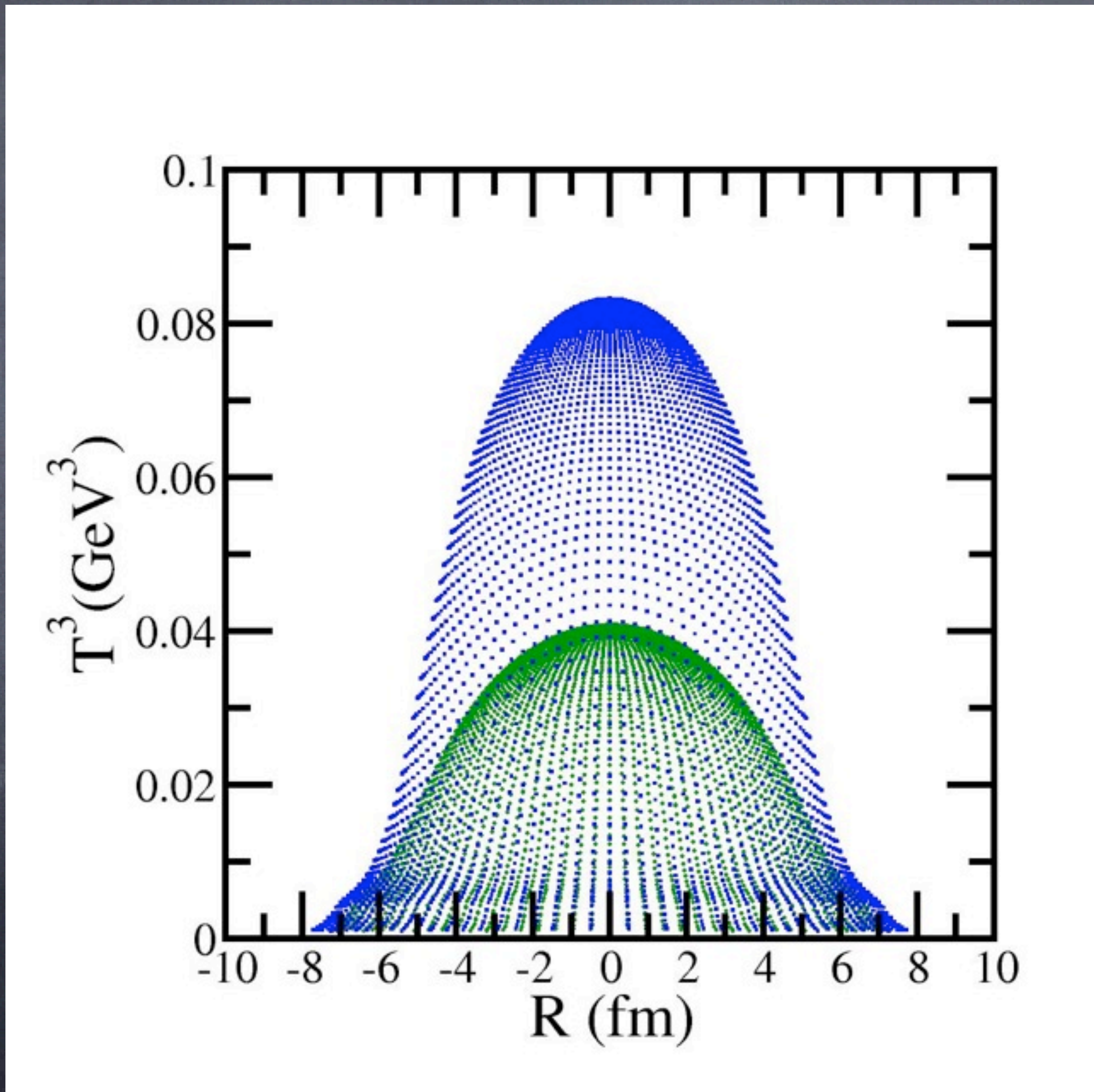
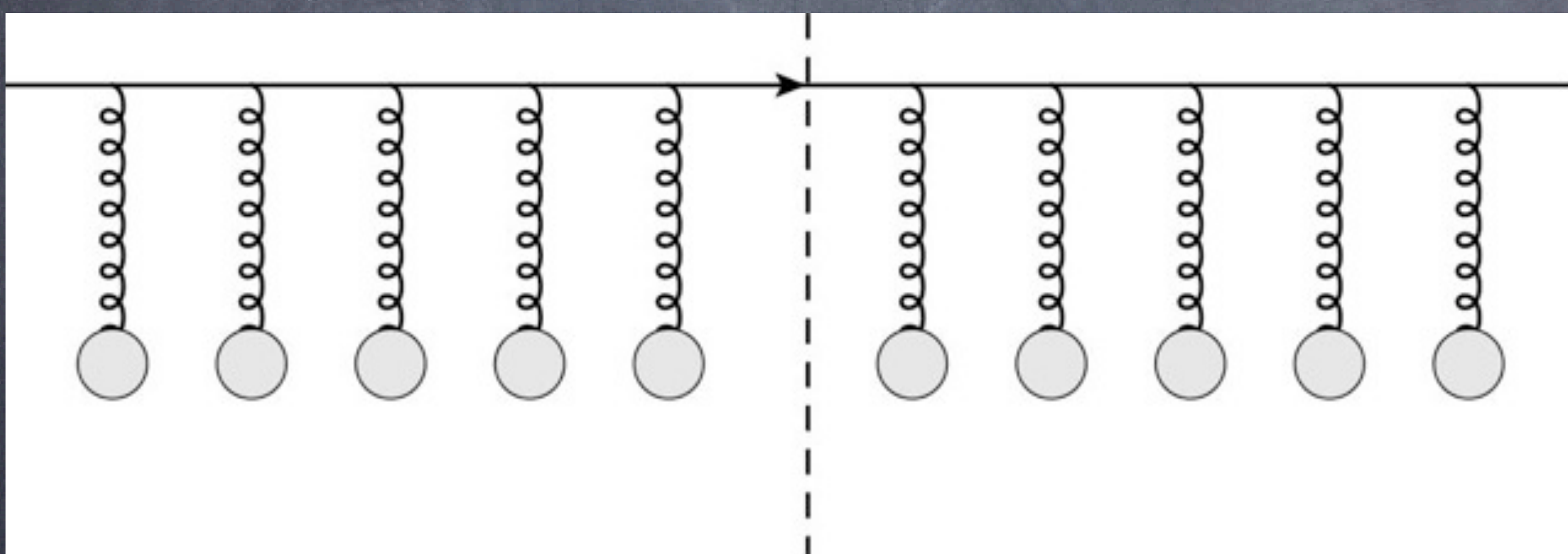
Note: no refitting between RHIC and LHC.



Back to the question of how the medium effects the parton.

A parton in a jet shower, has momentum components

$q = (q^-, q^+, q_T) = (1, \lambda^2, \lambda)Q$, Q: Hard scale, $\lambda \ll 1$, $\lambda Q \gg \Lambda_{QCD}$

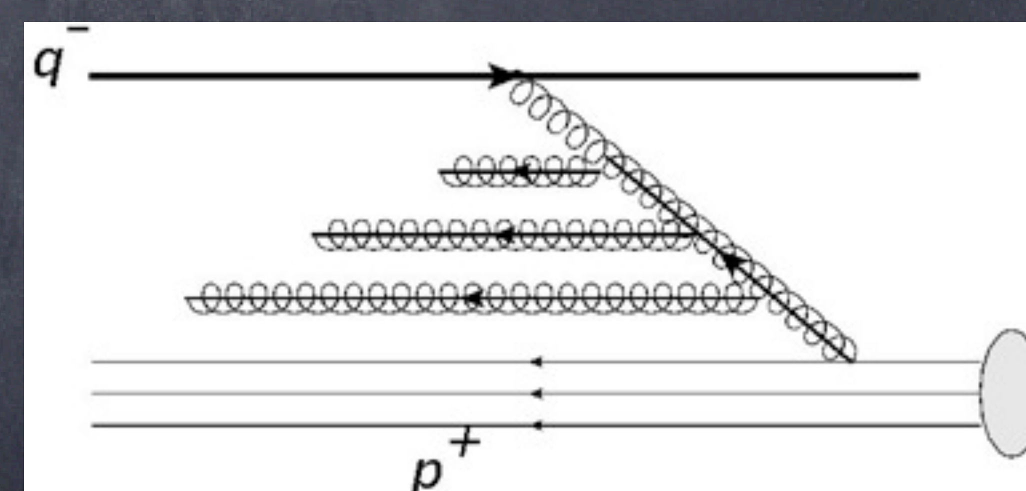


hence, gluons have

$$k_\perp \sim \lambda Q, \qquad k^+ \sim \lambda^2 Q$$

could also have $k^- \sim \lambda Q$

G.Y. Qin and A. M. , arXiv: 1205.5741 [hep-ph]



Assuming the medium has a large length.

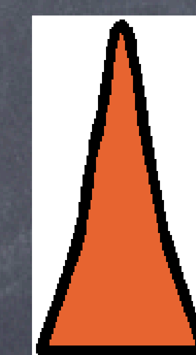
Or, the parton has a long life time, $1/(\lambda^2 Q)$

Multiple independent scattering dominates over multiple correlated scattering

Resumming gives a diffusion equation for the p_T distribution

$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$

$$\langle p_\perp^2 \rangle = 4Dt$$



$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_s C_R}{N_c^2 - 1} \int d\tilde{t} \langle F^{\mu\alpha}(\tilde{t}) v_\alpha F_\mu^\beta(0) v_\beta \rangle$$

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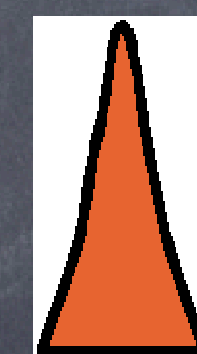


$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_S C_R}{N_c^2 - 1} \int dt \left\langle X \left| \mathrm{Tr} \left[\mathrm{U}^\dagger(\mathrm{t}, \mathrm{vt}; 0) \mathrm{t}^{\mathrm{a}} \mathrm{F}^{\mathrm{a}\mu\rho} \mathrm{v}_\rho \mathrm{U}(\mathrm{t}, \mathrm{vt}; 0) \mathrm{t}^{\mathrm{b}} \mathrm{F}^{\mathrm{b}\,\sigma}_{\ \ \mu}(0) \mathrm{v}_\sigma \right] \right| X \right\rangle$$

See talk by Michael Benzke

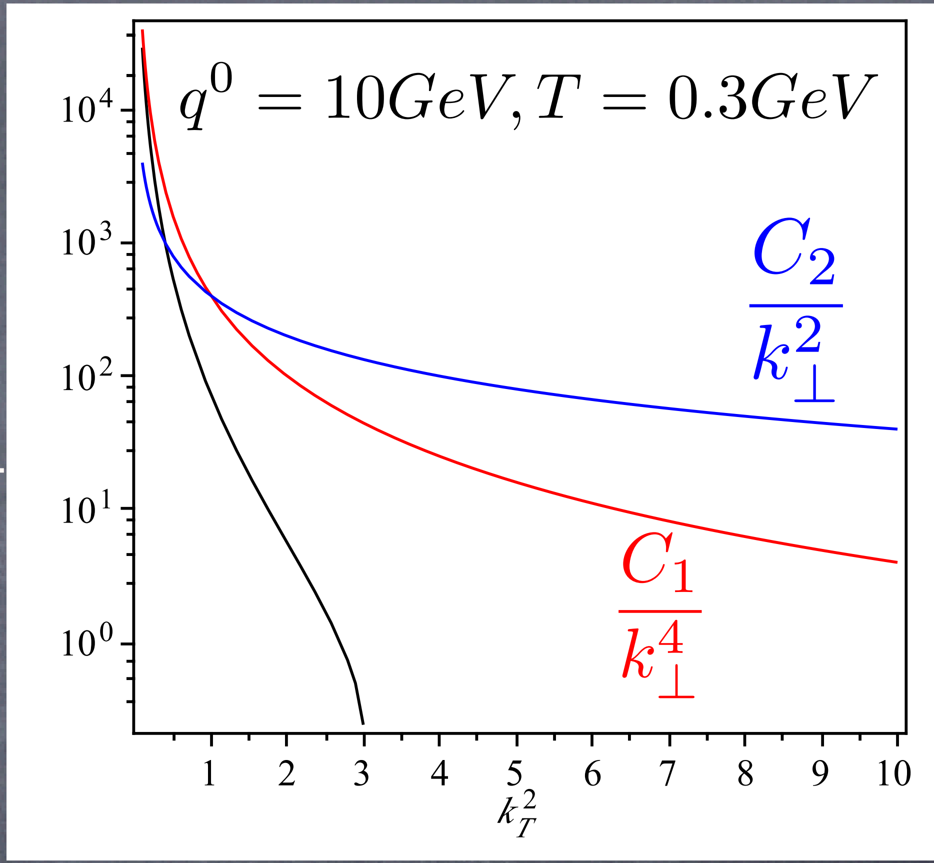
Gaussian distribution/temperature dependence/fit parameter !!!

Multiple scattering off any distribution samples a Gaussian

$$\hat{q} \sim T^3, s, \epsilon^{3/4}$$

is basically a model

$$\frac{d\sigma}{dk_\perp^2}$$

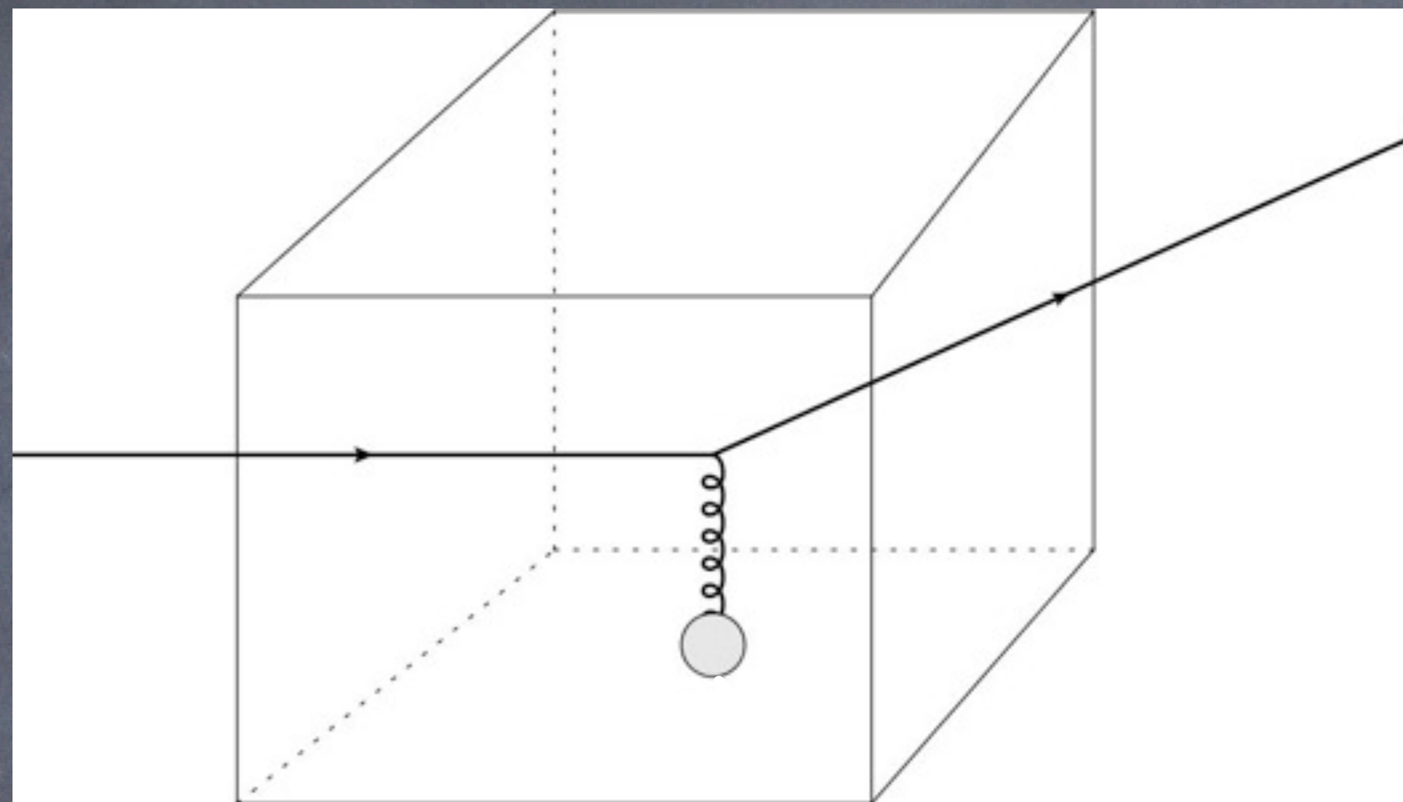


Ultimately you have to fit the normalization to 1 data point at one centrality, one value of p_T , one HIC energy

``So, its not really first principles!'', S.S. Gubser

A first principles method to calculate $\hat{q}$

(-)

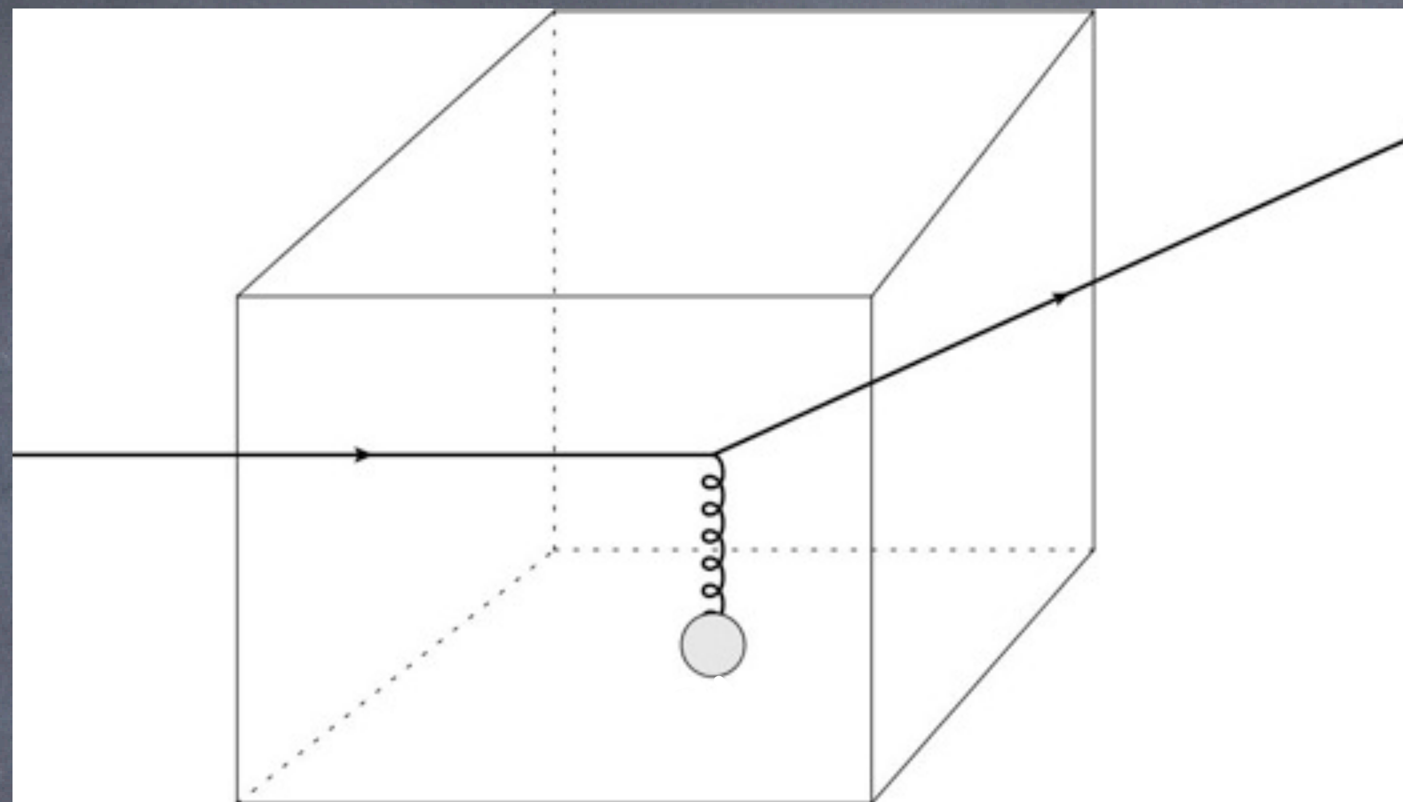


$$W(k) = \frac{g^2}{2N_c}\int d^4x d^4y \langle q^-; M|\bar{\psi}(y)\, \not{A}(y)\psi(y)|q^- + k_\perp; X\rangle$$
$$\times \quad \langle q^- + k_\perp; X|\bar{\psi}(x)\, \not{A}(x)\psi(x)|q^-; M\rangle$$

in terms of W, we get $\hat{q} = \sum_k k_\perp^2 \frac{W(k)}{t},$

Another first principles method to calculate $\hat{q}$

(-)



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Use a cut and put Final state "on-shell"

$$\delta[(q+k)^2] \simeq \frac{1}{2q^-}\delta\left(k^+ - \frac{k_\perp^2}{2q^-}\right).$$

Also we are calculating in a finite temperature heat bath

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}\cdot y^- + i\vec{k}_\perp\cdot\vec{y_\perp}}$$

$$\langle n|\frac{e^{-\beta E_n}}{Z}F^{+,}{}_\perp(y^-)F_\perp^+(0)|n\rangle$$

physical $\hat{q}(q^-,q^+)$ where $q^+ \sim \lambda^2 Q$

Consider a more general object

$$\hat{Q} \;=\; \frac{4\pi^2\alpha_s}{N_c}\int \frac{d^4y\,d^4k}{(2\pi)^4} e^{ik\cdot y}\frac{2(q^-)^2}{\sqrt{2}q^-}\frac{\langle M|F^{+\perp}(0)F^{+}_{\perp,}(y)|M\rangle}{(q+k)^2+i\epsilon}.$$

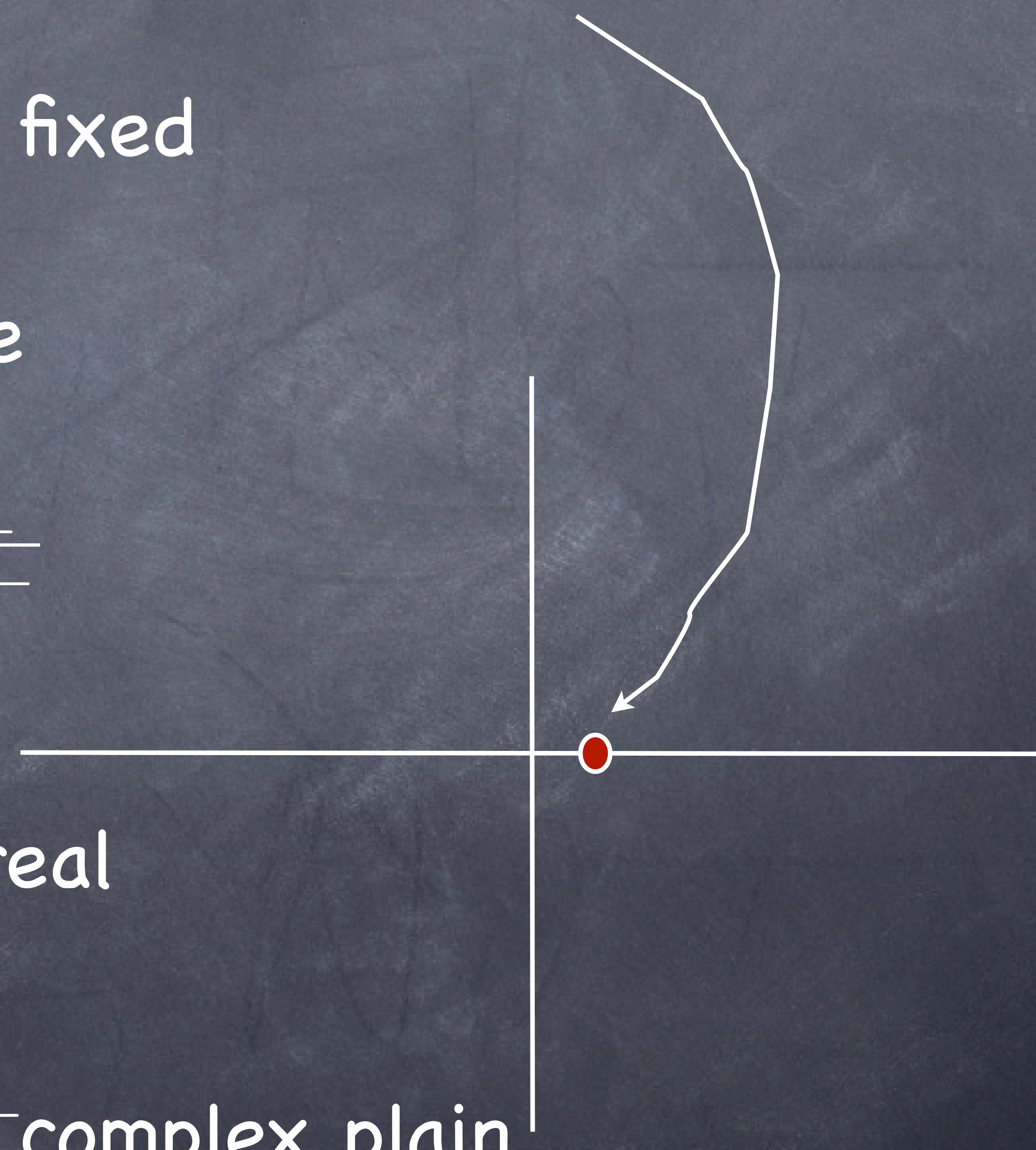
Consider q^- large (~Q) and fixed

Consider q^+ to be a variable

$\frac{d^2\hat{Q}}{dk_\perp^2}$ has a pole at $k^+ = \frac{k_\perp^2}{2q^-}$

$\hat{Q}$ has a branch cut on the real axis at $q^+ \sim \lambda^2 Q$

$\hat{q} = Im(\hat{Q})$



q^+complex plain

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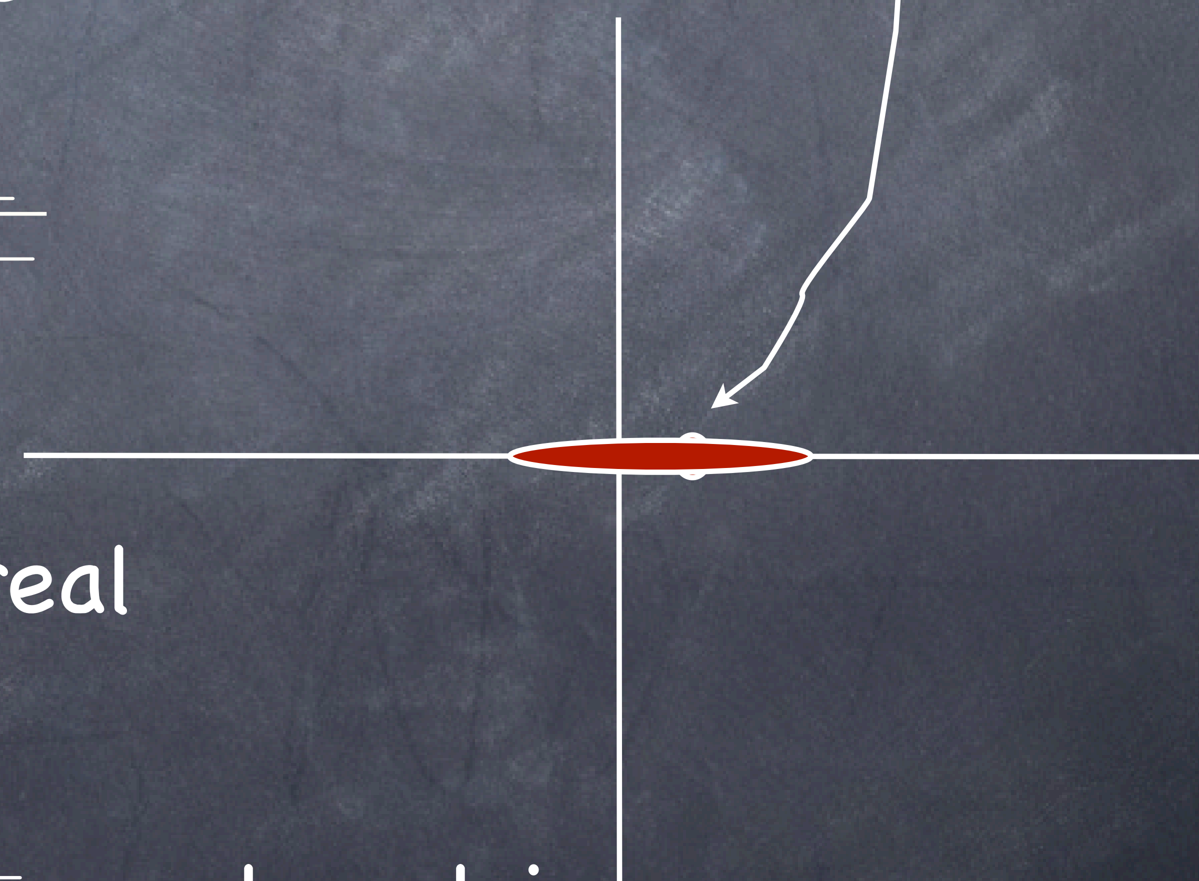

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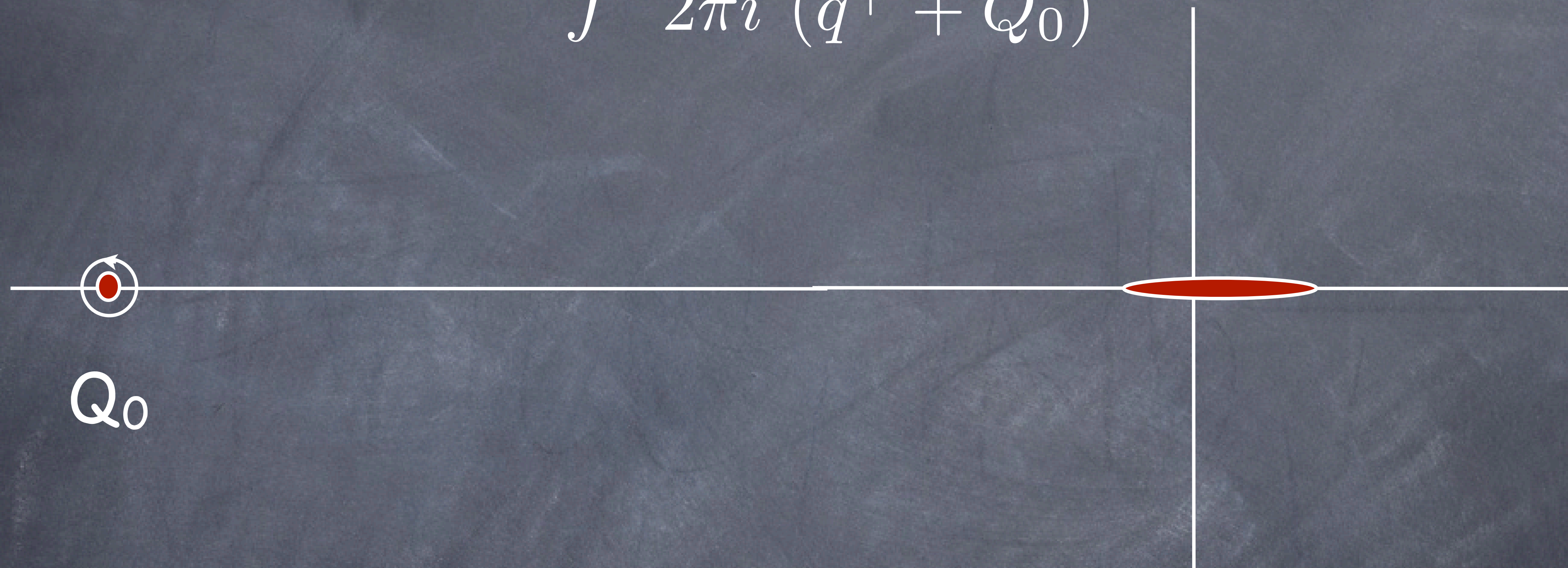
q^+complex plain

Consider the following integral

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + Q_0)}$$



For Q_0 ~ -Q, can Taylor expand $\hat{Q}$ in terms of local operators

$$I_1 = \frac{4\sqrt{2}\pi^2\alpha_s \langle M| F_\perp^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-q\cdot i\mathcal{D} - \mathcal{D}_\perp^2}{2q^- Q_0}\right)^n F^+_{\perp,\mu} |M\rangle}{N_c 2 Q_0}$$

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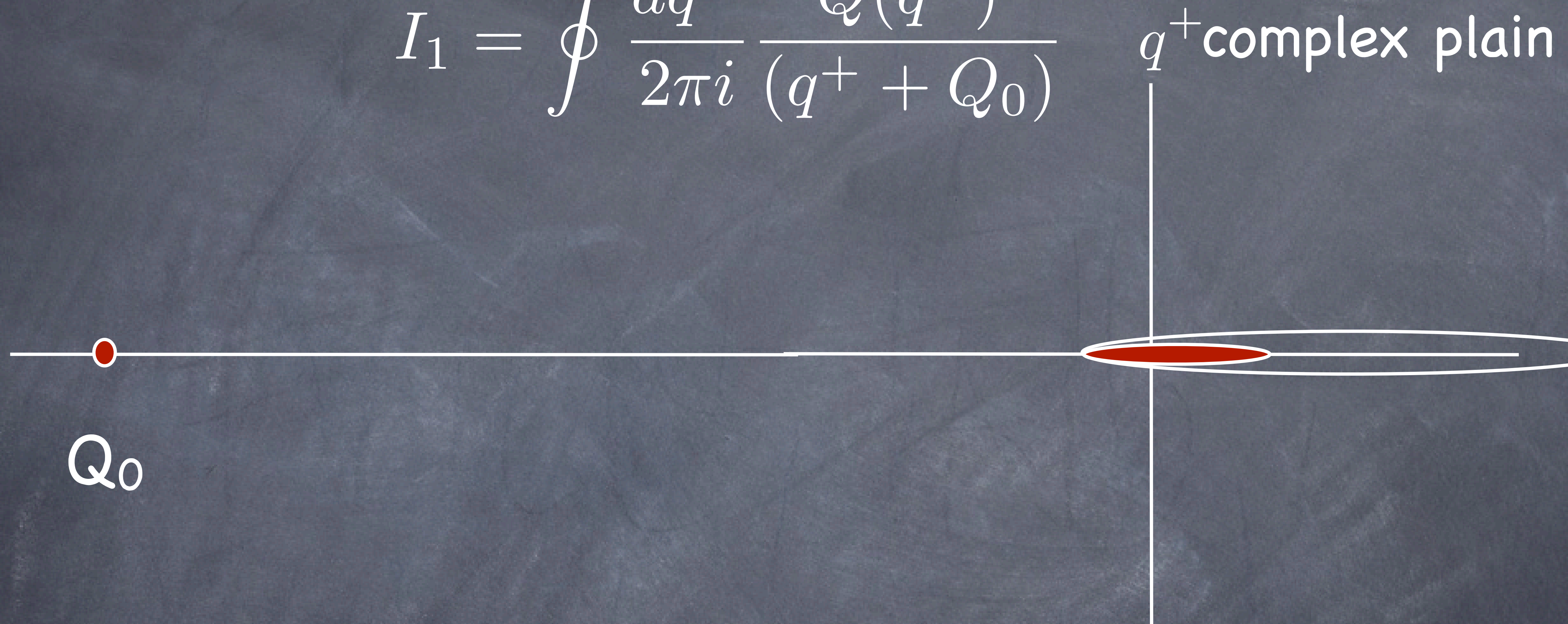

For $Q_0 \sim -Q$, can Taylor expand $\hat{Q}$ in terms of local operators

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Deforming the contour

$$I_1 = \int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \frac{\hat{q}(q^+)}{q^+ + Q_0} + \int_0^\infty dq^+ V(q^+)$$

set $Q_0 = q^-$

Taylor expand I_1 on the real side and do the integral

$$\int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \hat{q}(q^+) \simeq 2\hat{q}\lambda^2 Q + \frac{\hat{q}''(\lambda^2 Q)^3}{3}$$

Match powers of q^-

in the unphysical region with that in the physical region

Calculate local operators on the Lattice

Consider the unordered correlator

$$\mathcal{D}^{>}(t) = \sum_n \langle n| e^{-\beta H} \mathcal{O}_1(t)\mathcal{O}_2(0)|n\rangle$$

convert thermal weight to evolution in imaginary time

$$\mathcal{D}^{>}(-i\tau) = \Delta(\tau) = \mathrm{Tr}\left[e^{-\int_0^\beta d\tau H(\tau)} \mathcal{O}_i(\tau)\mathcal{O}_2(0)\right].$$

with time derivaives

$$\mathcal{D}^{>}(-i\tau) = i^{N_t}\Delta(\tau)$$

But local operators are easy

$$\mathcal{D}^{>}(t=0) = i^{N_t}\Delta(\tau=0)$$

Rotating everything to Euclidean space and calculating

$$x^0 \rightarrow -ix^4 \text{ and } A^0 \rightarrow iA^4$$
$$\rightarrow F^{0i} \rightarrow iF^{4i}$$

Calculate in quark less SU(2) gauge theory

Turn the box into a lattice of $n_t X n_s^3$ points
we use Wilson's gauge action
up to 5000 heat bath sweeps per point

Set the scale with Creutz formula
$$a_L = \frac{1}{\Lambda_L}\left(\frac{11g^2}{24\pi^2}\right)^{-\frac{51}{121}} \exp\left(-\frac{12\pi^2}{11g^2}\right),$$

Temperature $T = \dfrac{1}{n_t a_L}$

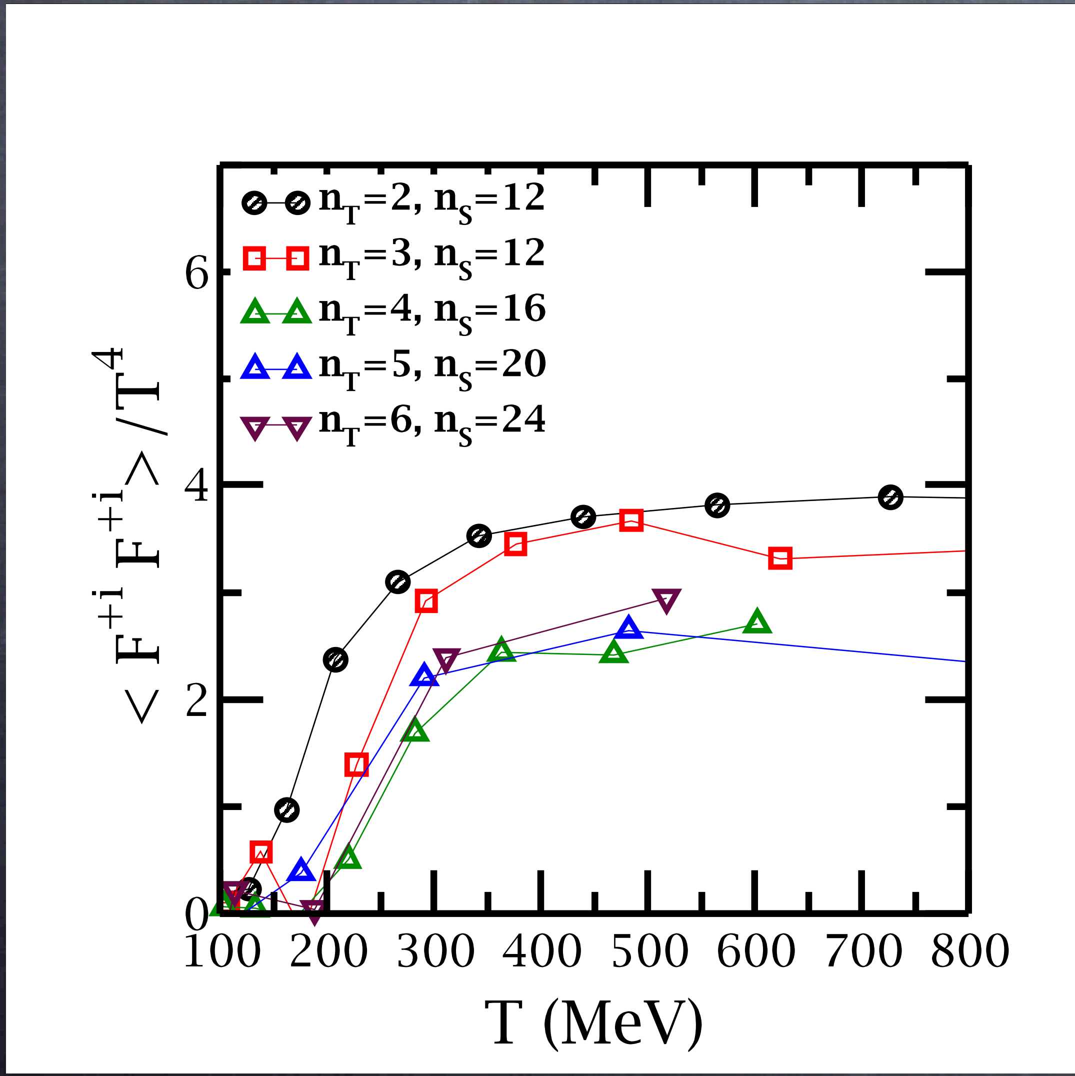
Λ_L = 5.3 MeV

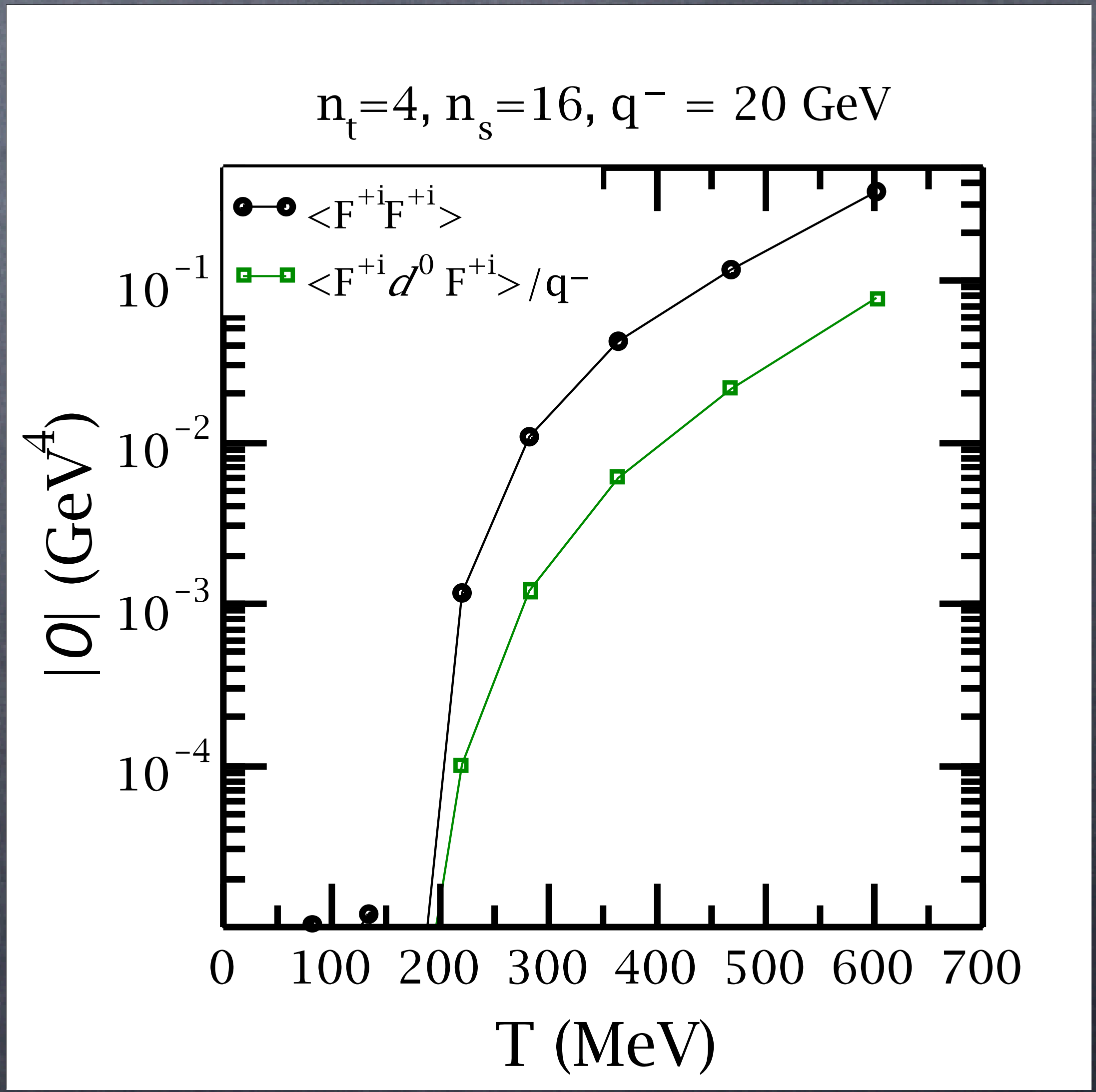
first operator to evaluate $F^{+i}F^{+i} \rightarrow F^{3i}F^{3i} - F^{4i}F^{4i}$

The measurements

Series to evaluate $\langle M | F_{\perp}^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-q \cdot i\mathcal{D} - \mathcal{D}_{\perp}^2}{2q^- Q_0} \right)^n F_{\perp,\mu}^{+} | M \rangle$

for large q⁻ 2nd operator ~ $F^{+\mu} \mathcal{D}^0 F^{+\mu}$





Concluding and Extrapolating !

Need to calculate in SU(3)
Better renormalization prescription
More complicated processes on the lattice
Need to do a higher order perturbative calculation
But lets estimate anyways

at T=363, FF = 0.04 GeV^4
Lattice size ~ 2fm, E = 20 GeV, μ^2 = 1.3 GeV^2

Gluon $\hat{q}$ is C_A/C_F of quark $\hat{q}$

SU(2) has 3 gluons, SU(3) has 8,
and 6 quarks + antiquarks

$$\hat{q}(T = 363\mathrm{MeV}) = 3.7\mathrm{GeV}^2/\mathrm{fm} - 6.5\mathrm{GeV}^2/\mathrm{fm}$$

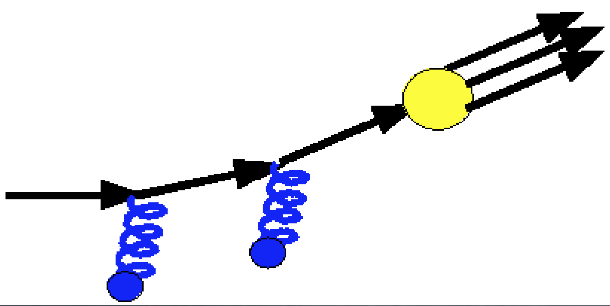
Back up!

What is our status

Assume factorization

Parametrized our lack of knowledge about the medium

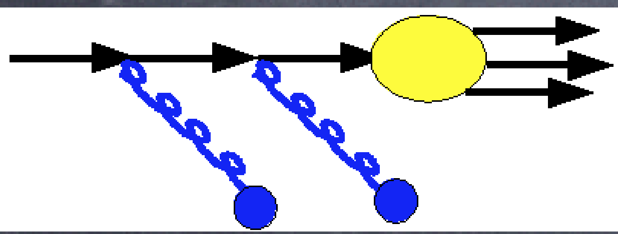
Effect of medium on parton in terms of jet transport coeffs



$$D\left(\frac{\vec{p}_h}{|\vec{p}+\vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_T^2 \rangle_L}{L}$$

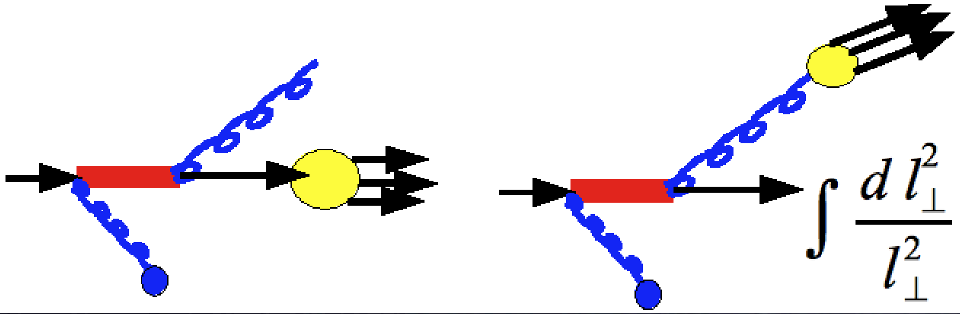
Transverse momemtum diffusion rate



$$D\left(\frac{p_h}{p-k}, m_J^2\right)$$

$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

Elastic energy loss rate also diffusion rate e_2



$$\int \frac{d\,l_\perp^2}{l_\perp^2} \int_{\frac{p_h}{p}}^{1} \frac{d\,y}{y} P(y)\, M(\vec{r}, y, l_\perp)\, D\left(\frac{p_h}{p\,y}\right)$$

Gluon radiation is sensitive to all these transport coefficients