

Probabilistic description of in-medium jet evolution

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In collaboration with

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(In preparation)

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Outline

- ✓ One-gluon emission in a dense QGP (BDMPS-Z)
- ✓ In-medium Color decorrelation (decoherence)
- ✓ Multiple-gluon emissions: Master Equation

1-gluon emission

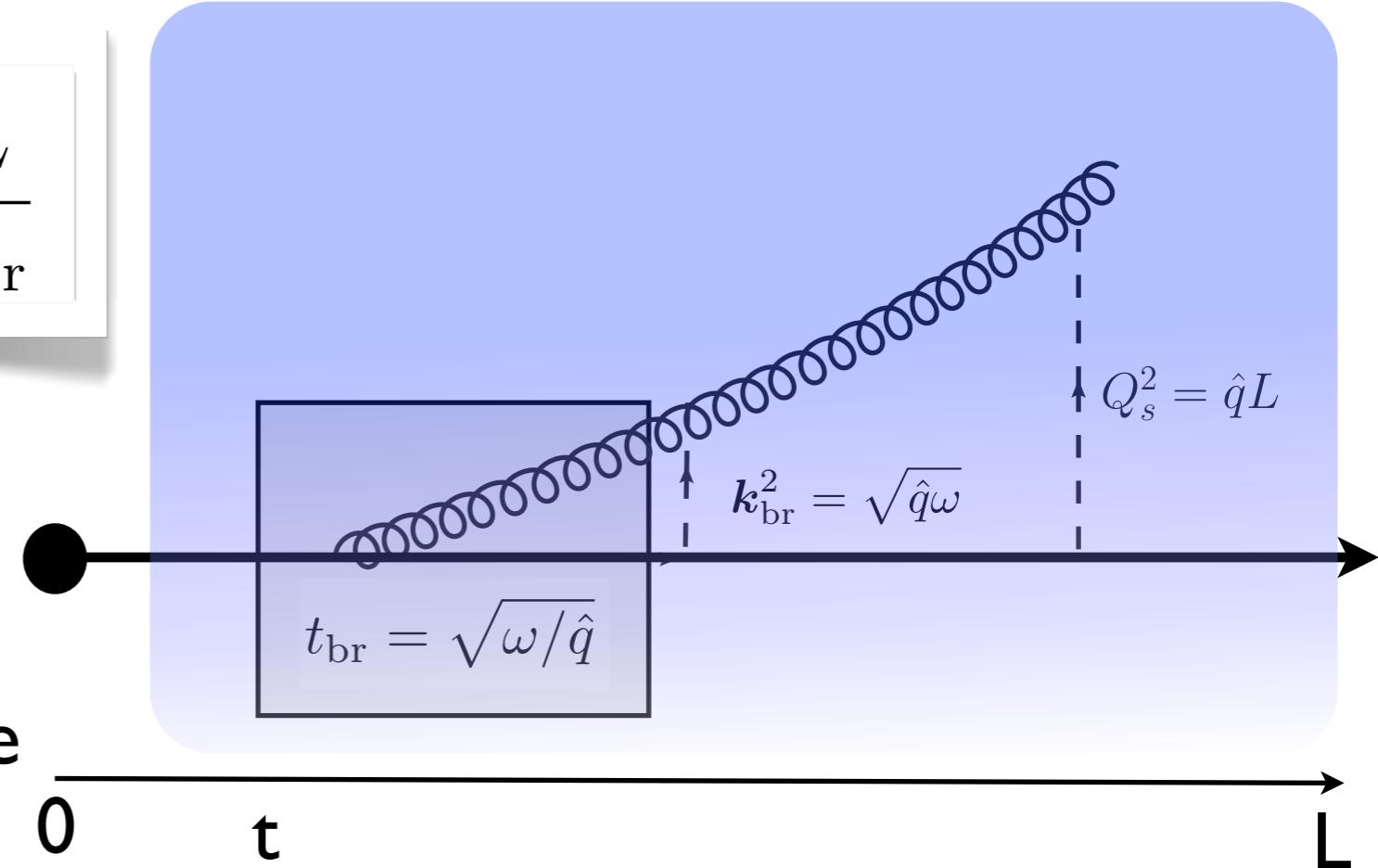
Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

- Medium-induced gluon radiation in the regime $\omega \ll \omega_c = \hat{q}L^2$

$$\omega \frac{dN}{d\omega} = \frac{C_F \alpha_s}{\pi} \sqrt{\frac{\hat{q}L^2}{\omega}} \propto \alpha_s \frac{L}{t_{\text{br}}}$$

- $t_{\text{br}} = \sqrt{\frac{\omega}{\hat{q}}}$ branching time (formation time)

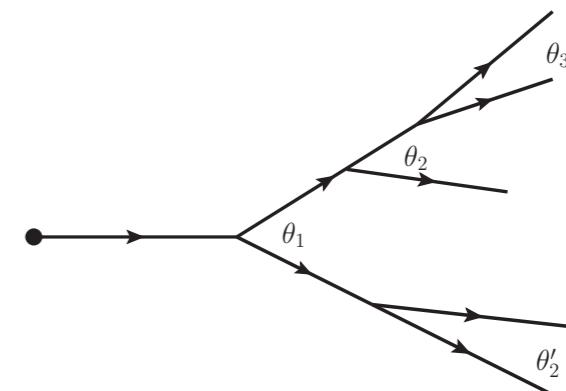
- Gluons can be emitted anywhere phase space: $t \sim L \gg t_{\text{br}}$



- 1-gluon \sim n-gluon when $\alpha_s \frac{L}{t_{\text{br}}} \sim 1$

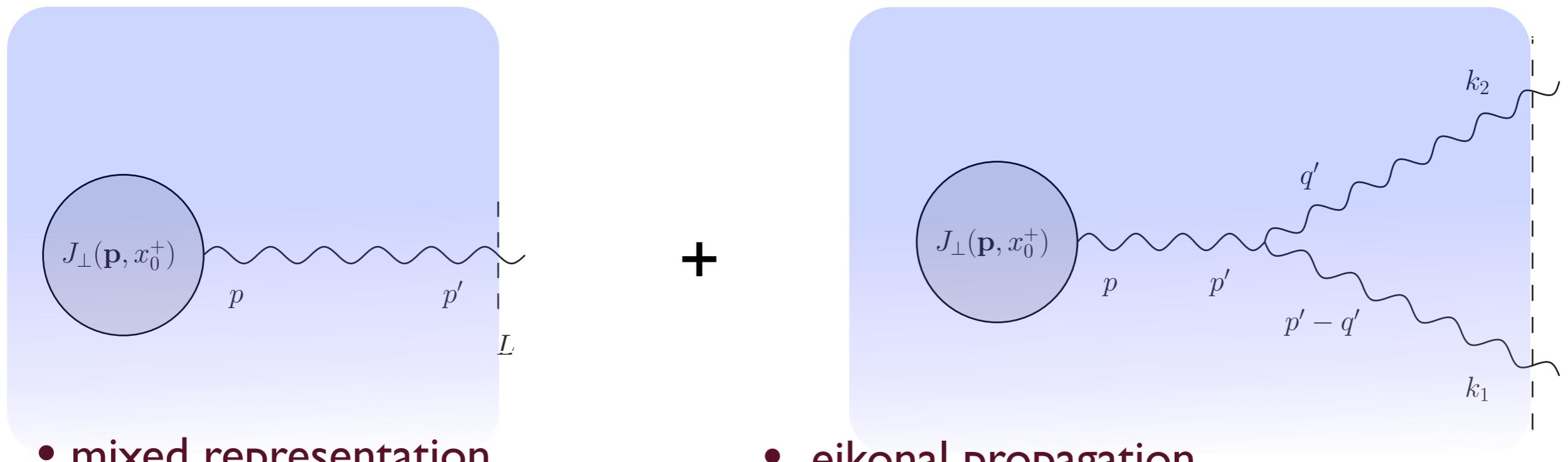
- Resummation?
Probabilistic Scheme?

$$\sigma = \sum_n a_n \left(\alpha_s \frac{L}{t_{\text{br}}} \right)^n$$



Medium-induced branching

0th order (no-splitting) and 1st order (I-branching)



- mixed representation
 $(p_{\perp}, p^+, x^+ \equiv t)$
- Brownian motion in transverse plan

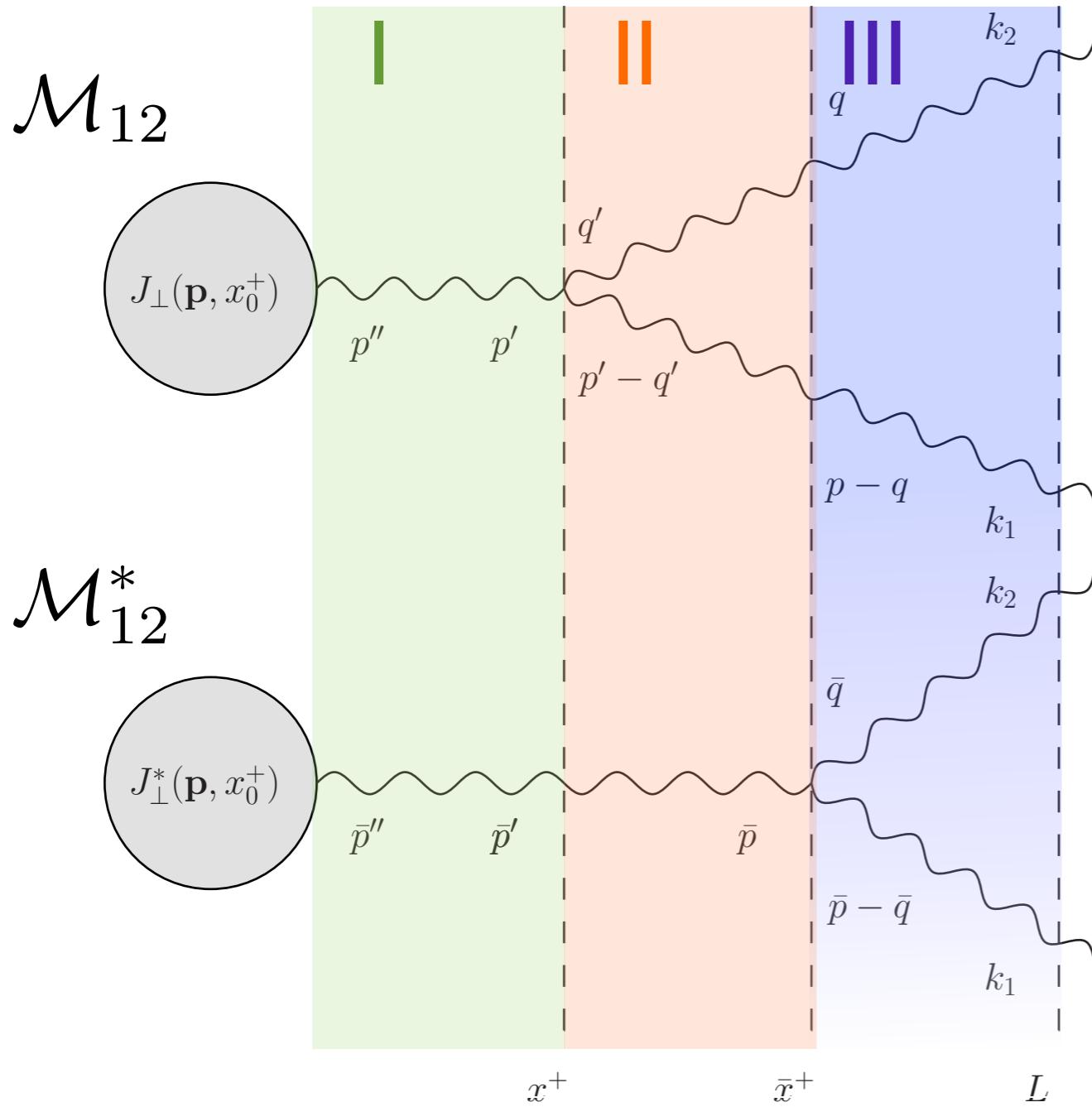
- eikonal propagation
 $p^+ \gg k_{\perp} \sim p_{\perp}$

$$\tilde{U} \equiv P \exp \left[ig \int d\xi A_{\text{med}}^-(\xi) \right]$$

$$\mathcal{G}_{ac}(X, Y; k^+) = \int \mathcal{D}\mathbf{r}_{\perp} e^{i \frac{k^+}{2} \int_{y^+}^{x^+} d\xi \dot{\mathbf{r}}_{\perp}^2(\xi)} \tilde{U}_{ac}(x^+, y^+; \mathbf{r}_{\perp})$$

Medium-induced branching

→ (amplitude) times (complex conjugate amplitude)



$$\mathcal{M}_{12}$$

$$\mathcal{M}_{12}^*$$

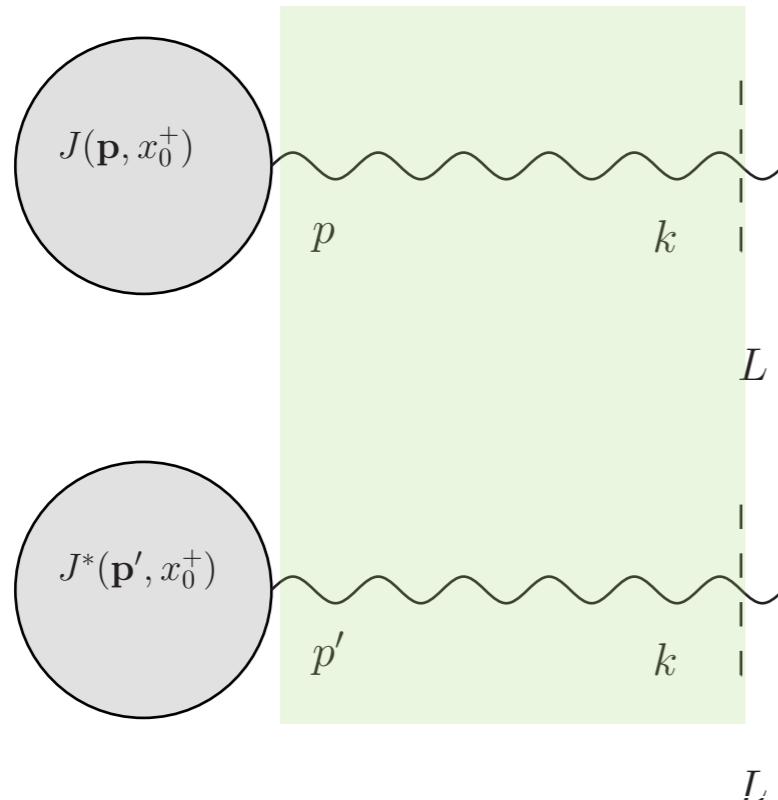
$$S^{(2)} \equiv \langle \mathcal{G}_0 \mathcal{G}_0^\dagger \rangle$$

$$S^{(3)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_0^\dagger \rangle$$

$$S^{(4)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_1^\dagger \mathcal{G}_2^\dagger \rangle$$

Transverse momentum broadening

Phase I (for illustration)

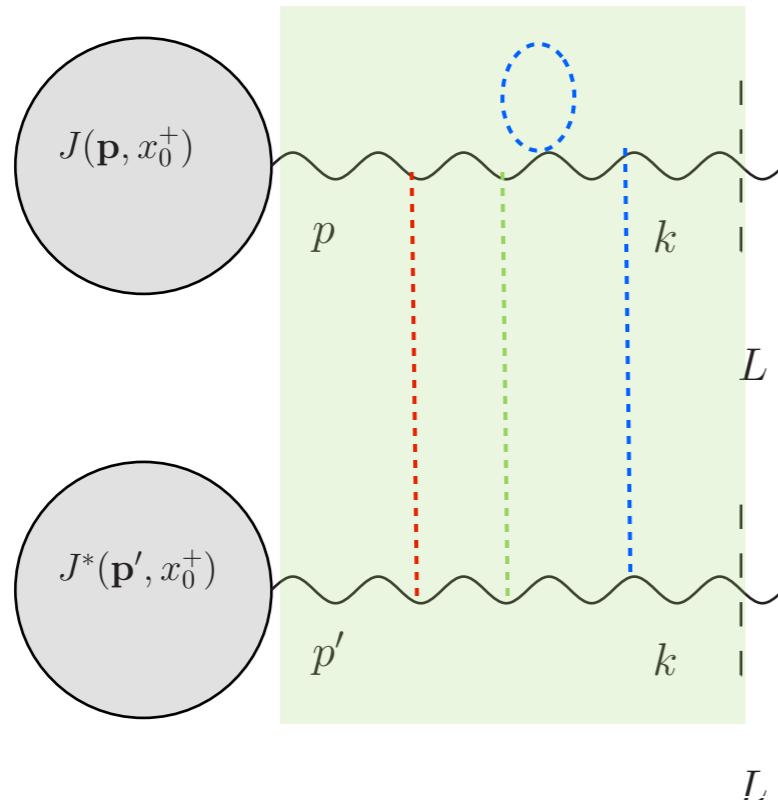


- First building block for in-medium jet branching
- 2-point function correlator

$$S^{(2)} \equiv \langle \mathcal{G}_0 \mathcal{G}_0^\dagger \rangle$$

Transverse momentum broadening

Phase I (for illustration)



- First building block for in-medium jet branching
- 2-point function correlator

$$S^{(2)} \equiv \langle \mathcal{G}_0 \mathcal{G}_0^\dagger \rangle$$

$$\delta^{ab} \langle \mathcal{G}^{aa'}(\mathbf{k}, L^+; \mathbf{p}, x_0^+) \mathcal{G}^{\dagger b'b}(\mathbf{p}', x_0^+; \mathbf{k}, L^+) \rangle = \delta^{a'b'} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{p}') \mathcal{P}(\mathbf{k} - \mathbf{p}, L^+ - x_0^+)$$

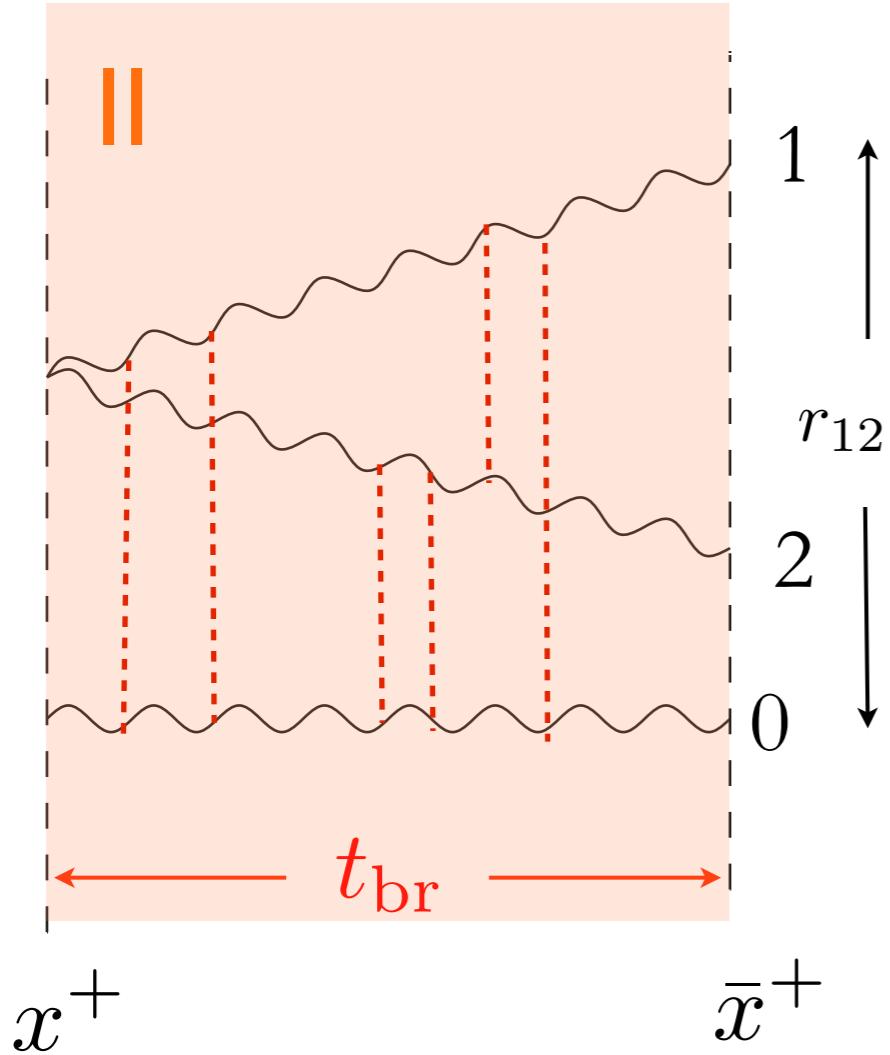
- Prob. for kt broadening

$$\mathcal{P}(\mathbf{k}, L) = \frac{4\pi}{\hat{q}L} e^{-\frac{\mathbf{k}^2}{\hat{q}L}}$$

The branching process

The 3-point function

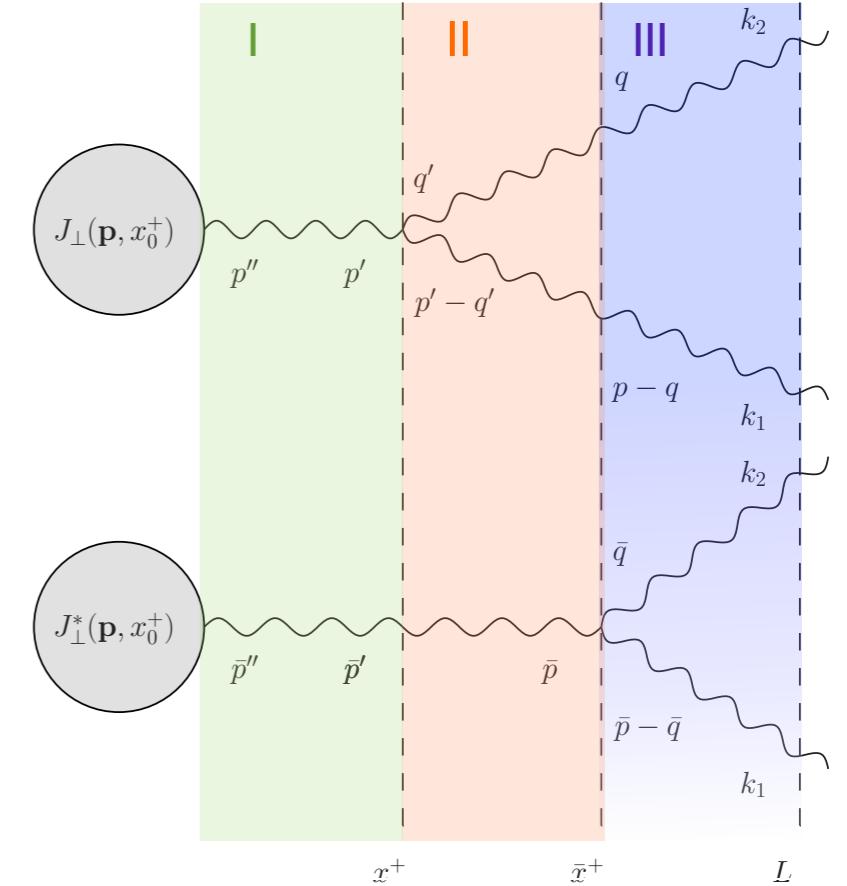
→ Multiple-scatterings destroy color coherence btw produced gluons and their parent



Color coherence
up to

$$r_{12}(t_{\text{br}}) \sim \frac{1}{\sqrt{\hat{q} t_{\text{br}}}}$$

$$t_{\text{br}} \sim \sqrt{\frac{z(1-z)p^+}{\hat{q}}}$$

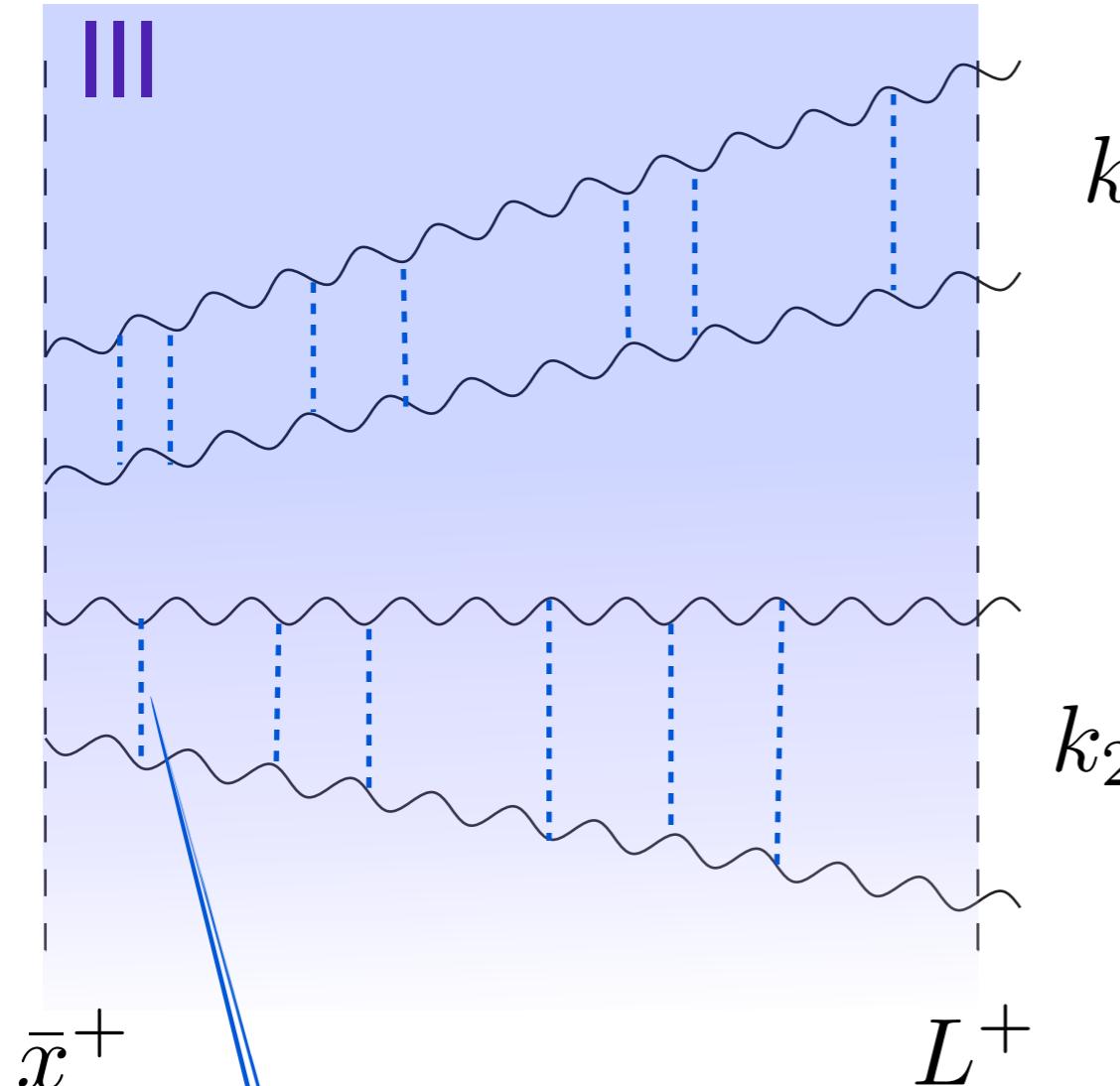


$$S^{(3)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_0^\dagger \rangle \propto e^{-\frac{\bar{x}^+ - x^+}{t_{\text{br}}}}$$

→ Full transverse dependence of the emission vertex computed

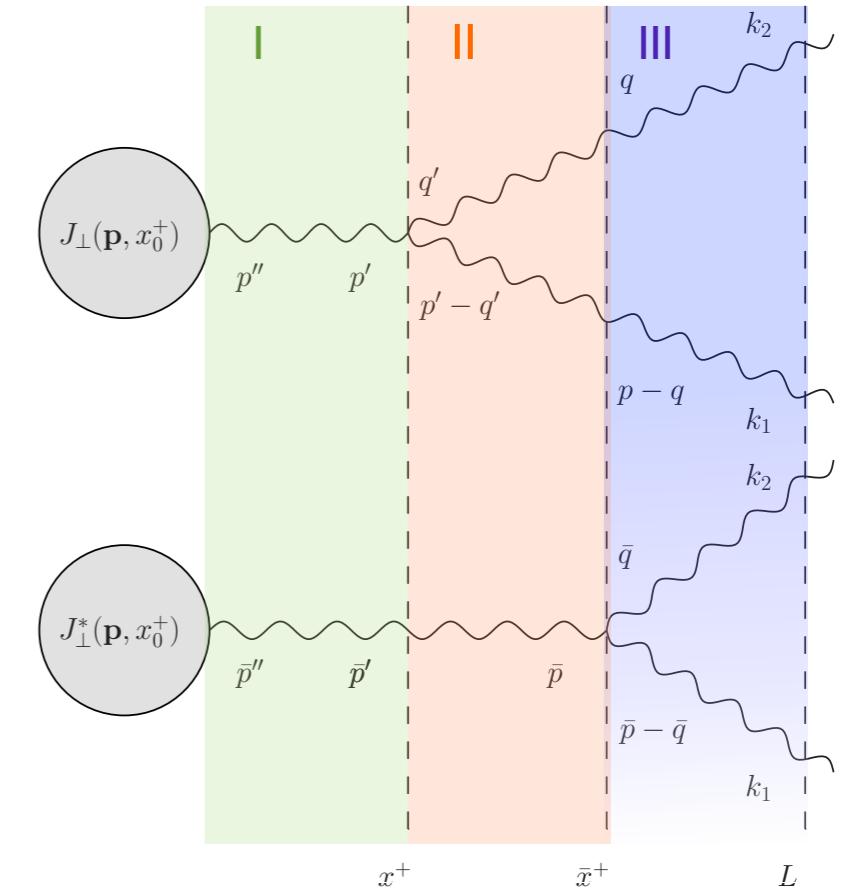
Color decoherence

Factorization of the 4-point function



$$S^{(4)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_1^\dagger \mathcal{G}_2^\dagger \rangle$$

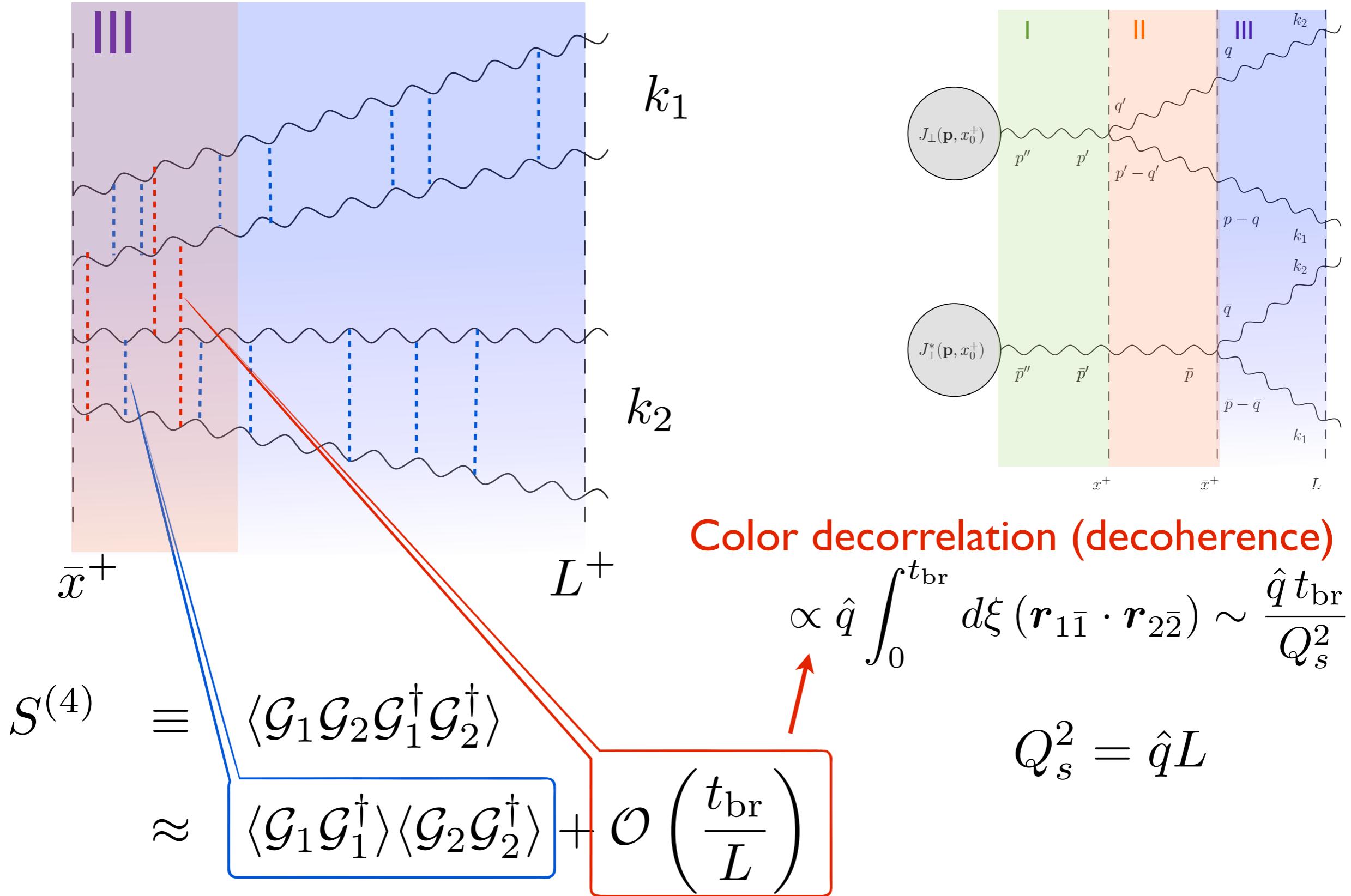
$$\approx \boxed{\langle \mathcal{G}_1 \mathcal{G}_1^\dagger \rangle \langle \mathcal{G}_2 \mathcal{G}_2^\dagger \rangle} + \mathcal{O}\left(\frac{t_{\text{br}}}{L}\right)$$



Color decorrelation (decoherence)

Color decoherence

Factorization of the 4-point function

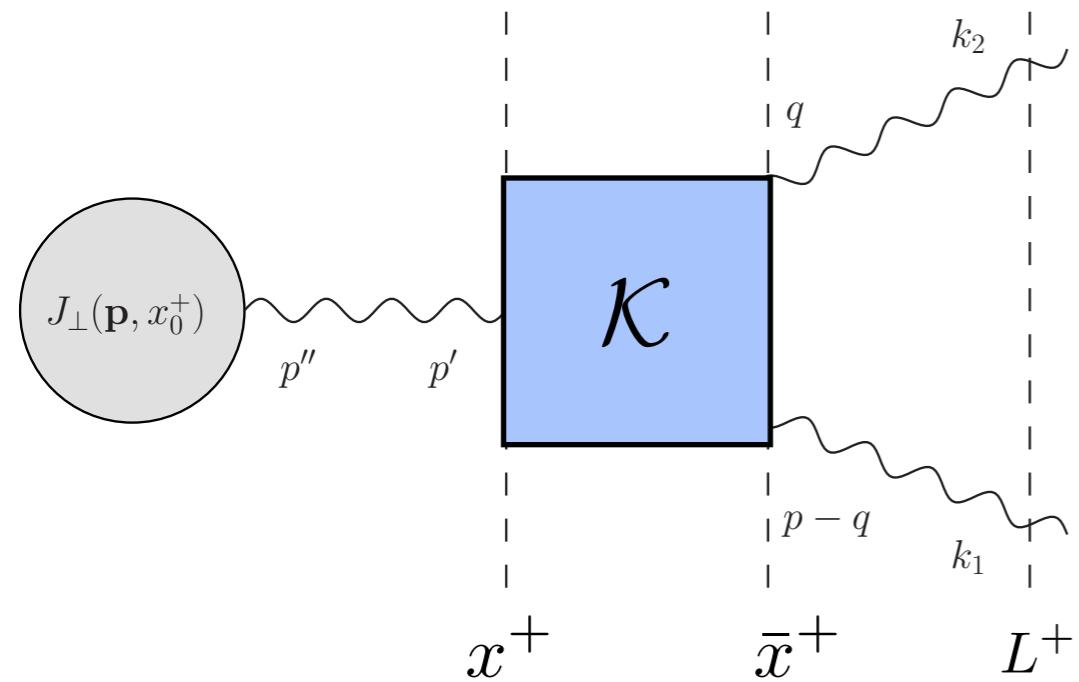


The branching process (result)

$$\frac{d^2\sigma_1}{d\Omega_{k_1} d\Omega_{k_2}} = g^2 \int_{x_0^+}^{L^+} d\bar{x}^+ \int_{x_0^+}^{\bar{x}^+} dx^+ \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{d\mathbf{p}}{(2\pi)^2} \frac{d\mathbf{p}'}{(2\pi)^2} \frac{d\mathbf{p}''}{(2\pi)^2}$$

$$\mathcal{P}(\mathbf{k}_2 - \mathbf{q}, L^+ - \bar{x}^+) \mathcal{P}(\mathbf{k}_1 - \mathbf{p} + \mathbf{q}, L^+ - \bar{x}^+)$$

$$\mathcal{K}_{gg}^g(\mathbf{q} - z\mathbf{p}, \mathbf{p} - \mathbf{p}', z; \bar{x}^+ - x^+) \mathcal{P}(\mathbf{p}' - \mathbf{p}'', x^+ - x_0^+) J^2(\bar{\mathbf{p}}'', x_0^+)$$

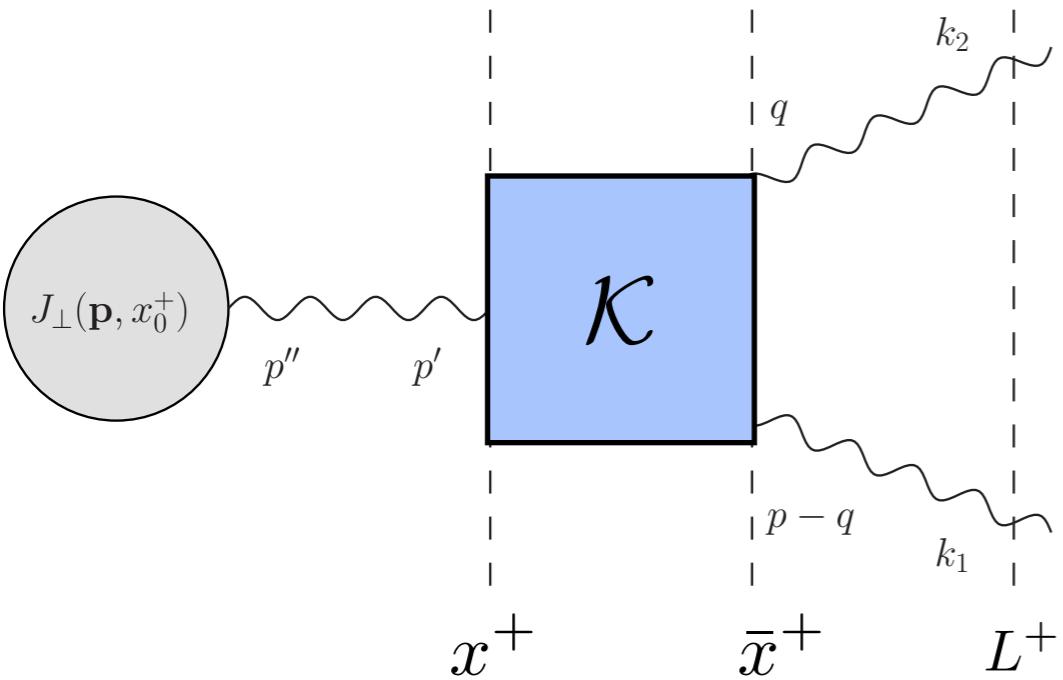


$$\mathcal{K}_{gg}^g \propto S^{(3)}$$

- Integrating over transverse momenta we recover the BDMPS-Z result
- letting $p^+ \rightarrow \infty$ we recover Wiedeman's result

The branching process (result)

$$\begin{aligned}
 \frac{d^2\sigma_1}{d\Omega_{k_1} d\Omega_{k_2}} = & g^2 \int_{x_0^+}^{L^+} d\bar{x}^+ \int_{x_0^+}^{\bar{x}^+} dx^+ \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{d\mathbf{p}}{(2\pi)^2} \frac{d\mathbf{p}'}{(2\pi)^2} \frac{d\mathbf{p}''}{(2\pi)^2} \\
 & \mathcal{P}(\mathbf{k}_2 - \mathbf{q}, L^+ - \bar{x}^+) \mathcal{P}(\mathbf{k}_1 - \mathbf{p} + \mathbf{q}, L^+ - \bar{x}^+) \\
 & \mathcal{K}_{gg}^g(\mathbf{q} - z\mathbf{p}, \mathbf{p} - \mathbf{p}', z; \bar{x}^+ - x^+) \mathcal{P}(\mathbf{p}' - \mathbf{p}'', x^+ - x_0^+) J^2(\bar{\mathbf{p}}'', x_0^+)
 \end{aligned}$$



- Large L approximation :

$$\bar{x}^+ - x^+ \sim t_{\text{br}} \ll x^+ \sim L^+$$

$$p_\perp - p'_\perp \sim \sqrt{\hat{q} t_{\text{br}}} \equiv k_{\text{br}}$$

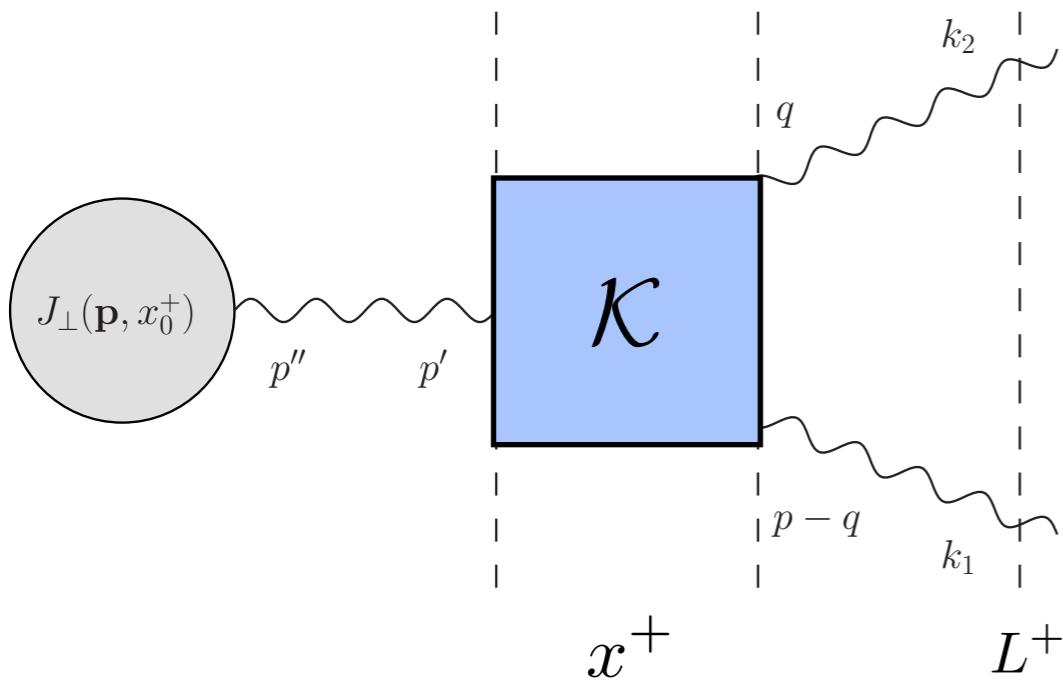
$$p_\perp \sim \sqrt{\hat{q} L^+} \equiv Q_s$$

$$p_\perp - p'_\perp \ll p_\perp$$

The branching process (factorization limit)

$$\frac{d^2\sigma_1}{d\Omega_{k_1} d\Omega_{k_2}} = g^2 \int_{x_0^+}^{L^+} dx^+ \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{d\mathbf{p}}{(2\pi)^2} \frac{d\mathbf{p}''}{(2\pi)^2}$$
$$\mathcal{P}(\mathbf{k}_2 - \mathbf{q}, L^+ - x^+) \mathcal{P}(\mathbf{k}_1 - \mathbf{p} + \mathbf{q}, L^+ - x^+)$$
$$\mathcal{K}_{gg}^g(\mathbf{q} - z\mathbf{p}, z) \mathcal{P}(\mathbf{p} - \mathbf{p}'', x^+ - x_0^+) J^2(\bar{\mathbf{p}}'', x_0^+).$$

Classical branching



- Large L approximation :

$$\bar{x}^+ - x^+ \sim t_{\text{br}} \ll x^+ \sim L^+$$

$$p_\perp - p'_\perp \sim \sqrt{\hat{q} t_{\text{br}}} \equiv k_{\text{br}}$$

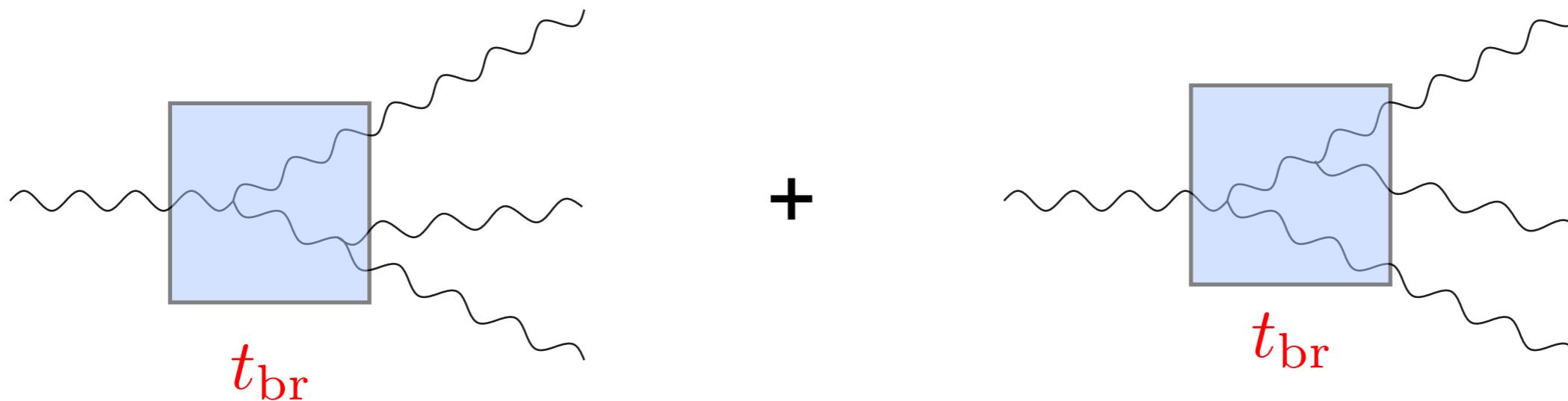
$$p_\perp \sim \sqrt{\hat{q} L^+} \equiv Q_s$$

$$p_\perp - p'_\perp \ll p_\perp$$

The branching process (factorization limit)

→ Interferences are limited to emissions within t_{br}

Interf. suppressed as $\sim \frac{t_{\text{br}}}{L}$



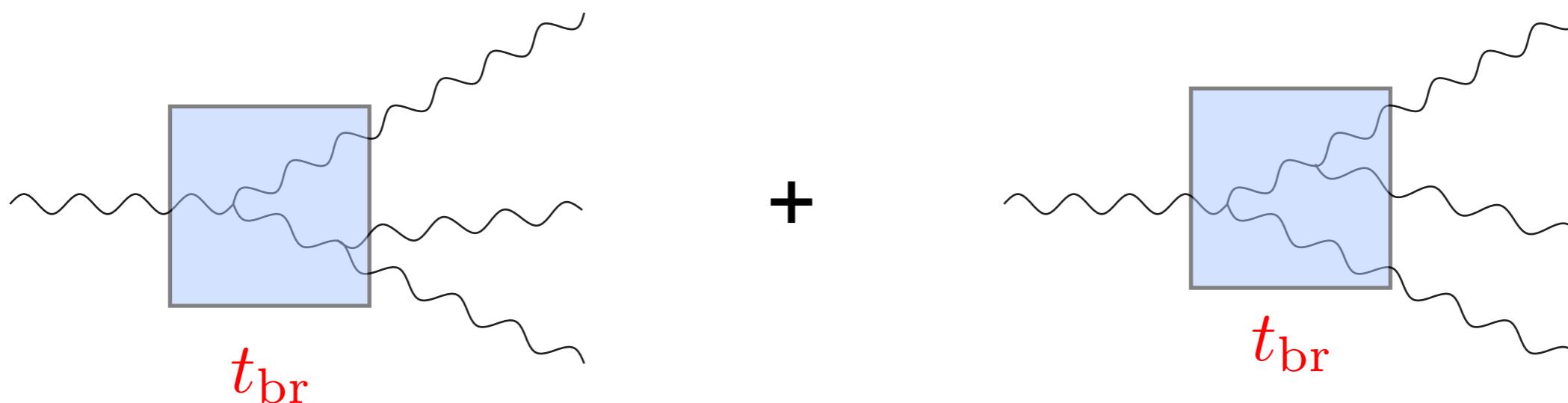
- Decoherence of successive splittings: ⇒ No Angular Ordering!

Y.M.-T., K.Tywoniuk, C.A. Salgado (2011)
E. Iancu, J. Casalderrey Solana (2011)

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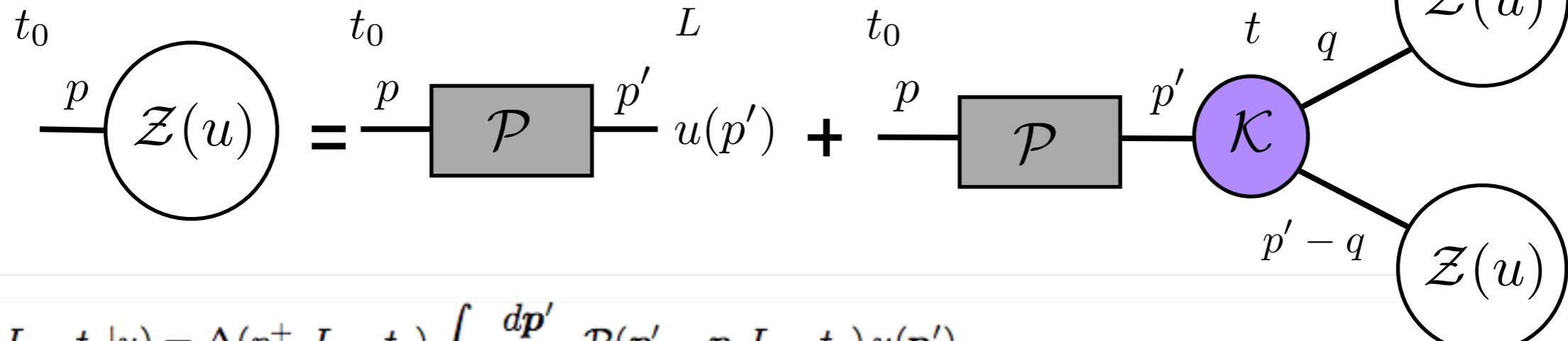
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⇒ Probabilistic Scheme

$$\sigma = \sum_n a_n \left(\alpha_s \frac{L}{t_{\text{br}}} \right)^n$$

Master Equation for Generating Functional



$$\mathcal{Z}(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}')$$

$$+ \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) \mathcal{Z}(\mathbf{q}, L - t | u) \mathcal{Z}(\mathbf{p}' - \mathbf{q}, L - t | u)$$

Master Equation for Generating Functional

$$Z(u) = \mathcal{P}^p_{t_0} u(p') + \mathcal{P}^p_{t_0} \mathcal{K}^{p'}_q Z(u) + \mathcal{P}^p_{t_0} \mathcal{K}^{p'-q}_{t_0} Z(u)$$

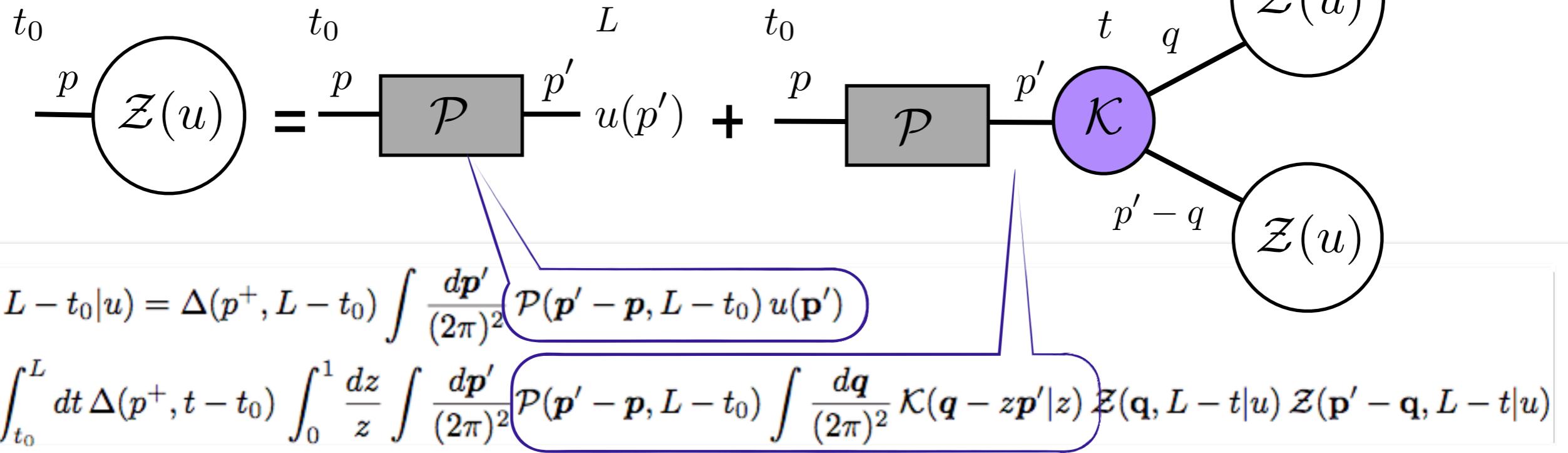
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Master Equation for Generating Functional

$$Z(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}') + \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) Z(\mathbf{q}, L - t | u) Z(\mathbf{p}' - \mathbf{q}, L - t | u)$$

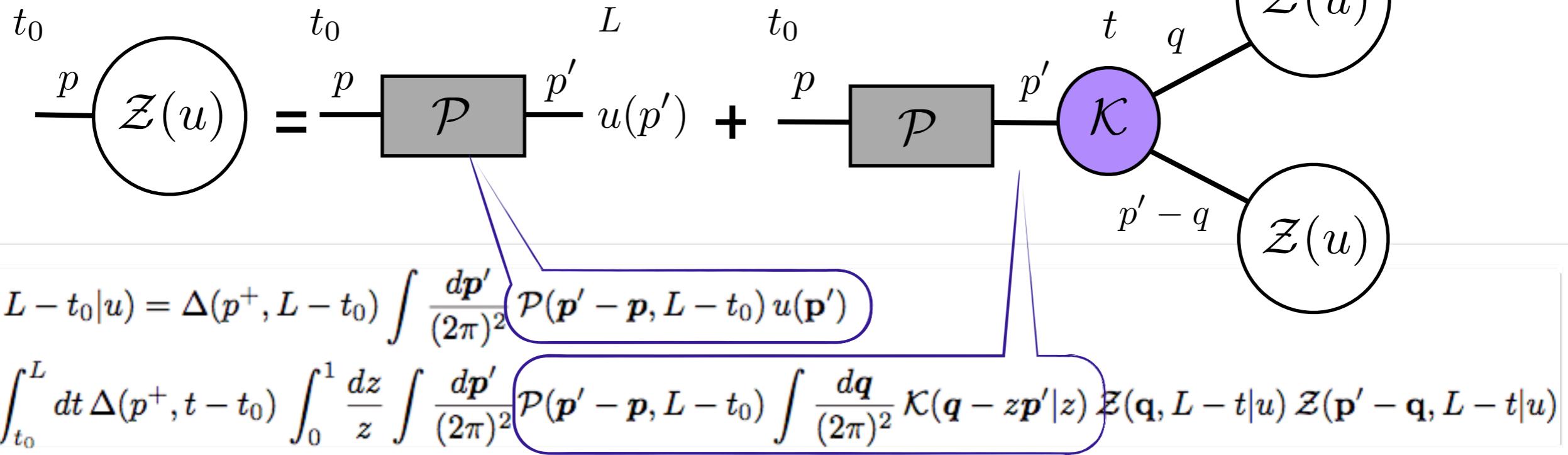
Master Equation for Generating Functional



- In-medium splitting function
- Relative pT at branching time

$$\mathcal{K}_{BC}^A(\mathbf{q} - z\mathbf{p}, z) = \frac{2}{p^+} P_{AB}(z) \sin \left[\frac{(\mathbf{q} - z\mathbf{p})^2}{2k_{\text{br}}^2} \right] \exp \left[-\frac{(\mathbf{q} - z\mathbf{p})^2}{2k_{\text{br}}^2} \right] \quad k_{\text{br}}^2 = \sqrt{z(1-z)p^+\hat{q}_{\text{eff}}}$$

Master Equation for Generating Functional



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- Sudakov form factor:
Prob. not to emit
(Unitarity)

$$\Delta(p^+, L-t_0) = \exp \left[-\alpha_s (L-t_0) \int_0^1 \frac{dz}{z} \mathcal{K}(z) \right]$$

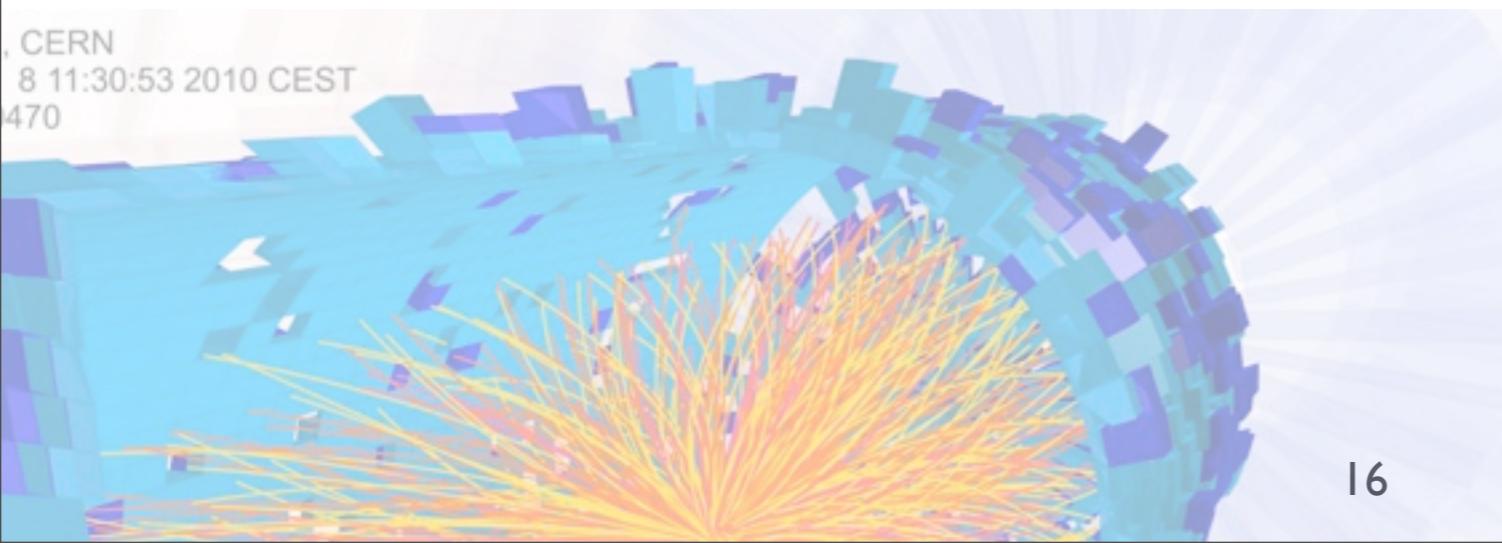
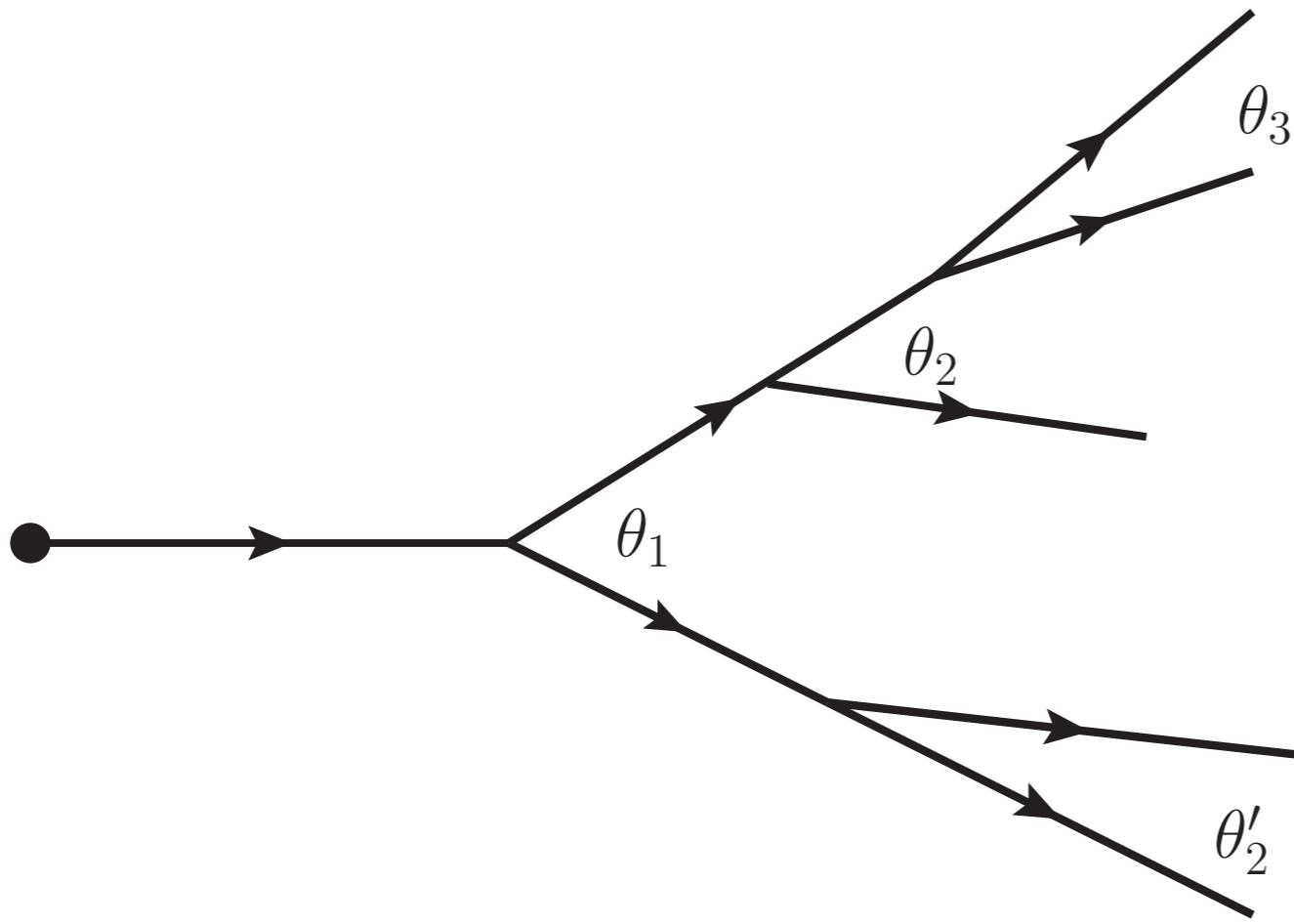
Summary

- ✓ In the limit of a dense medium, parton branchings are **incoherent** due to rapid color randomization.
- ✓ A probabilistic description of in-medium jet evolution is formulated in terms of a Master Eq. for Generating Functional.
- ✓ \Rightarrow Fully exclusive description of the jet including **momentum broadening**
- ✓ Possible implementation in a Monte Carlo generator

Back-up

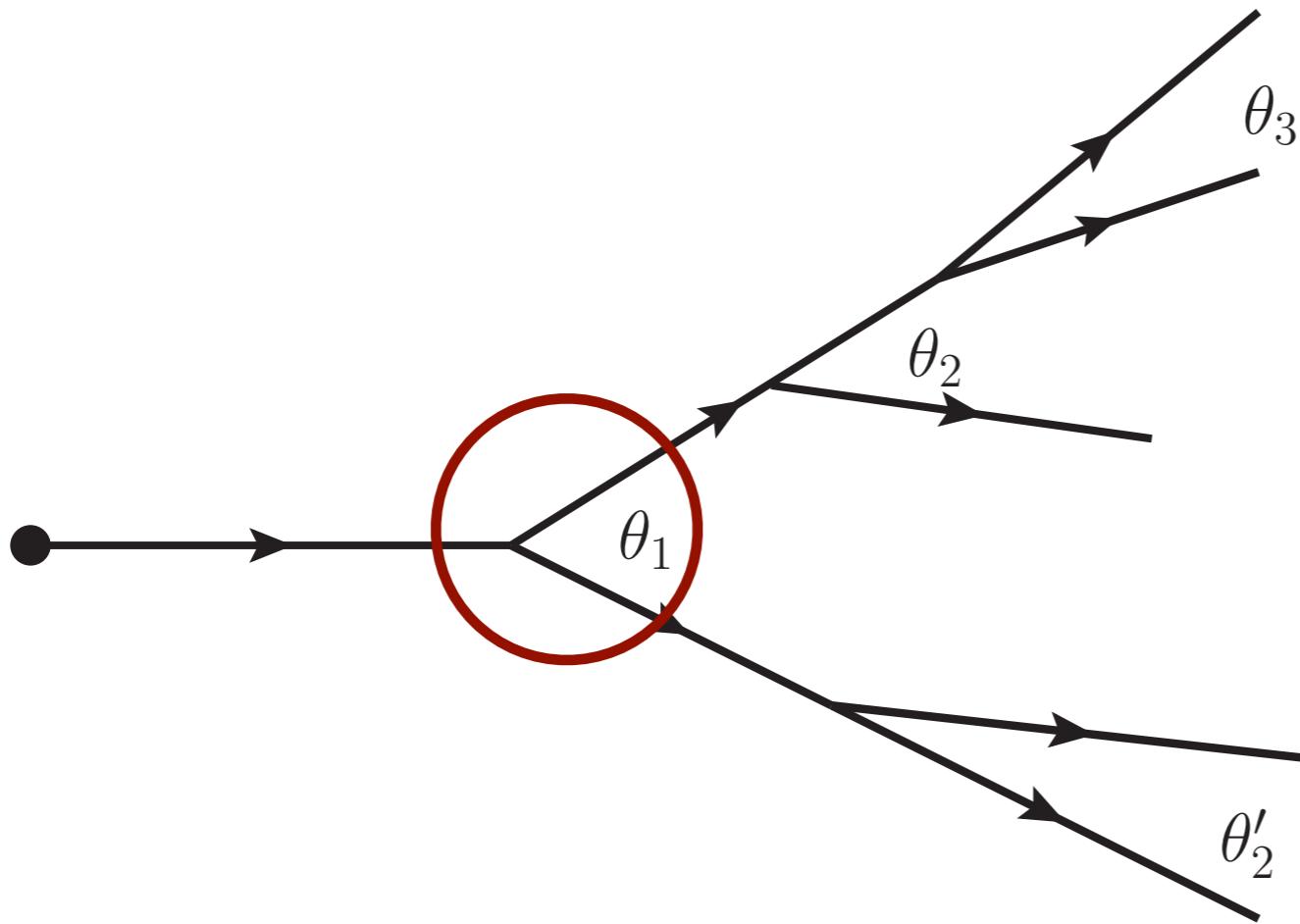
Jets in vacuum

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]



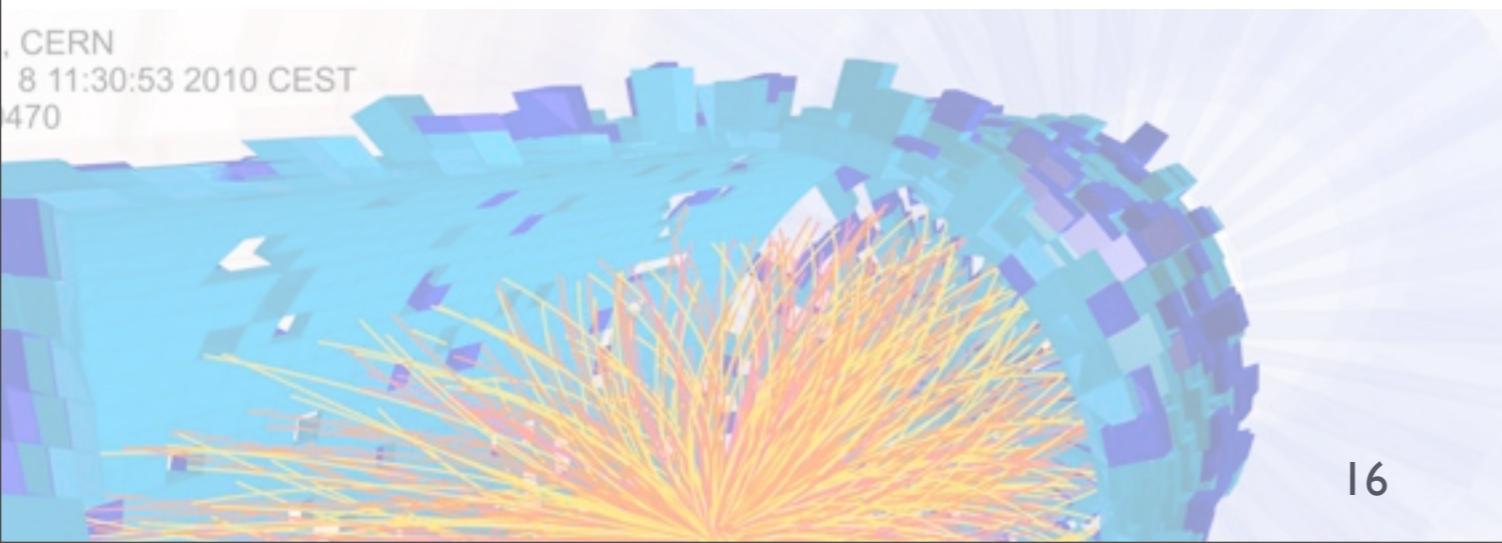
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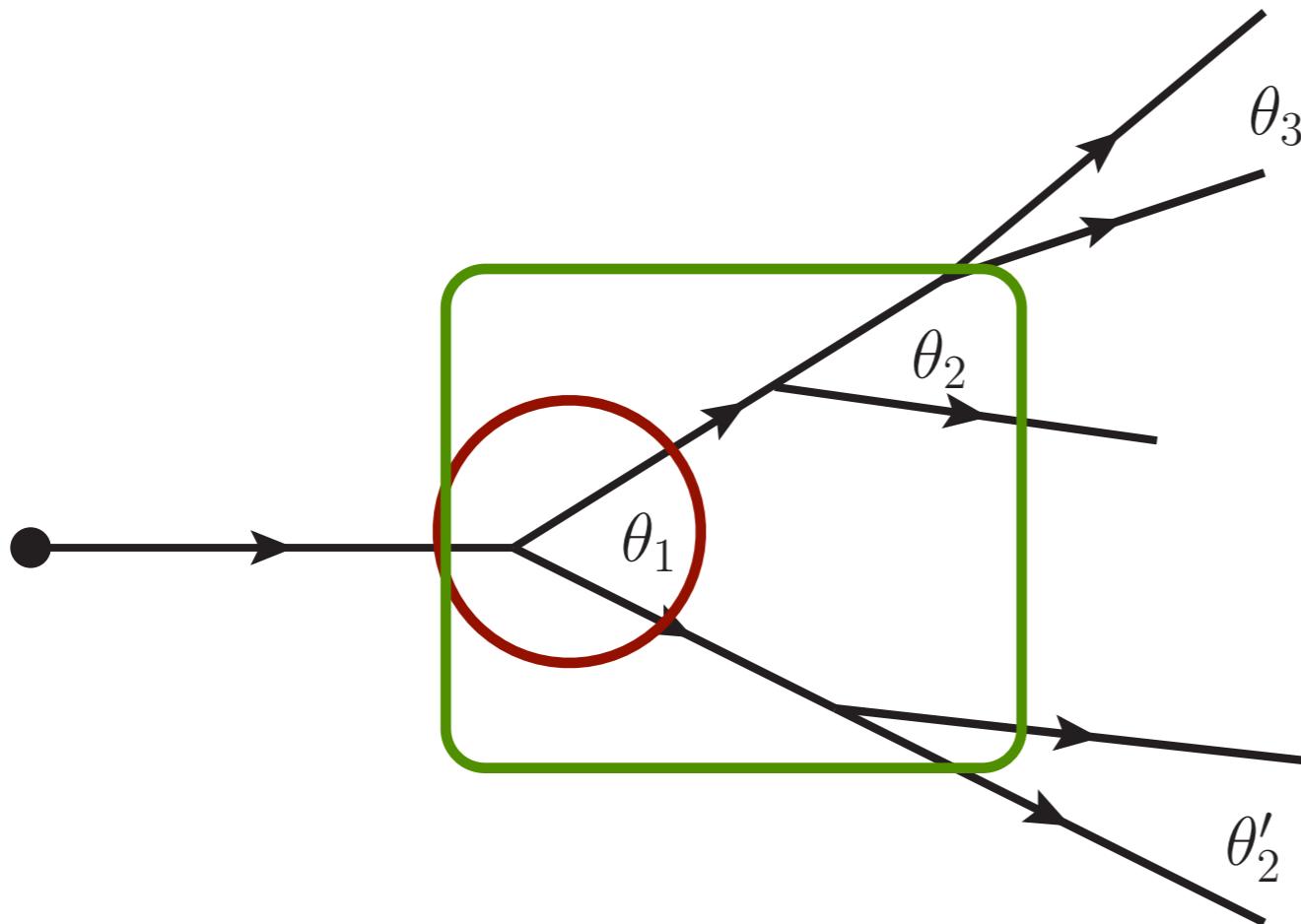
Leading Logarithms

$$dP \propto \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$



Jets in vacuum

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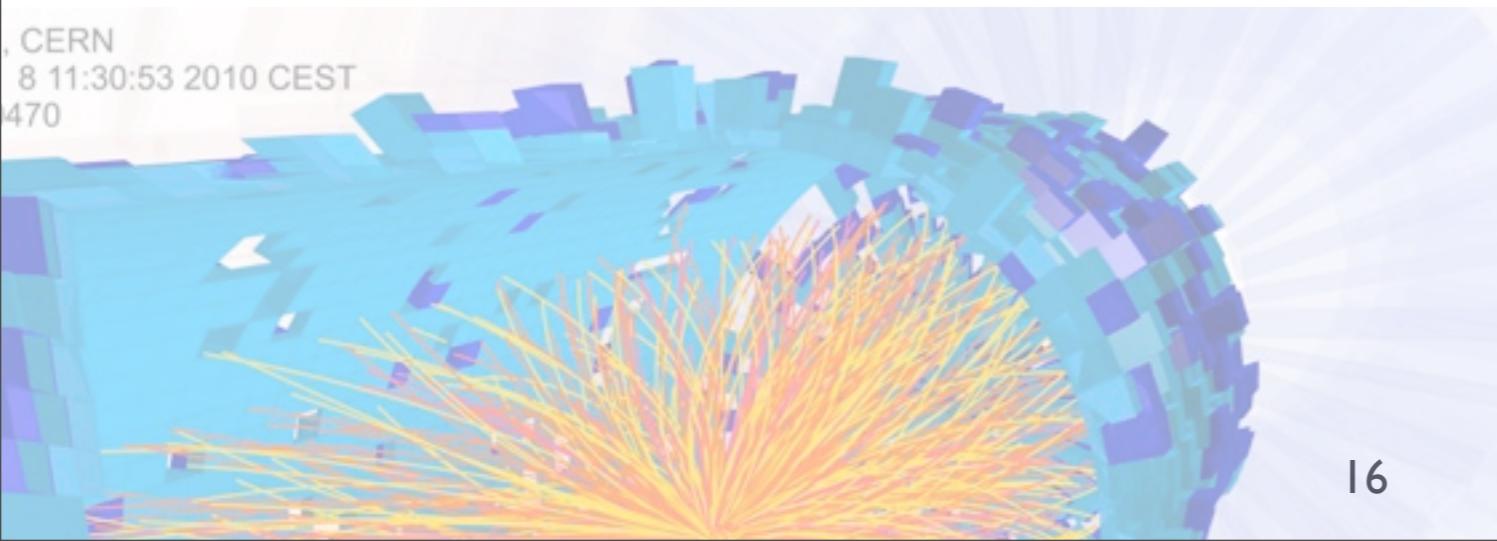


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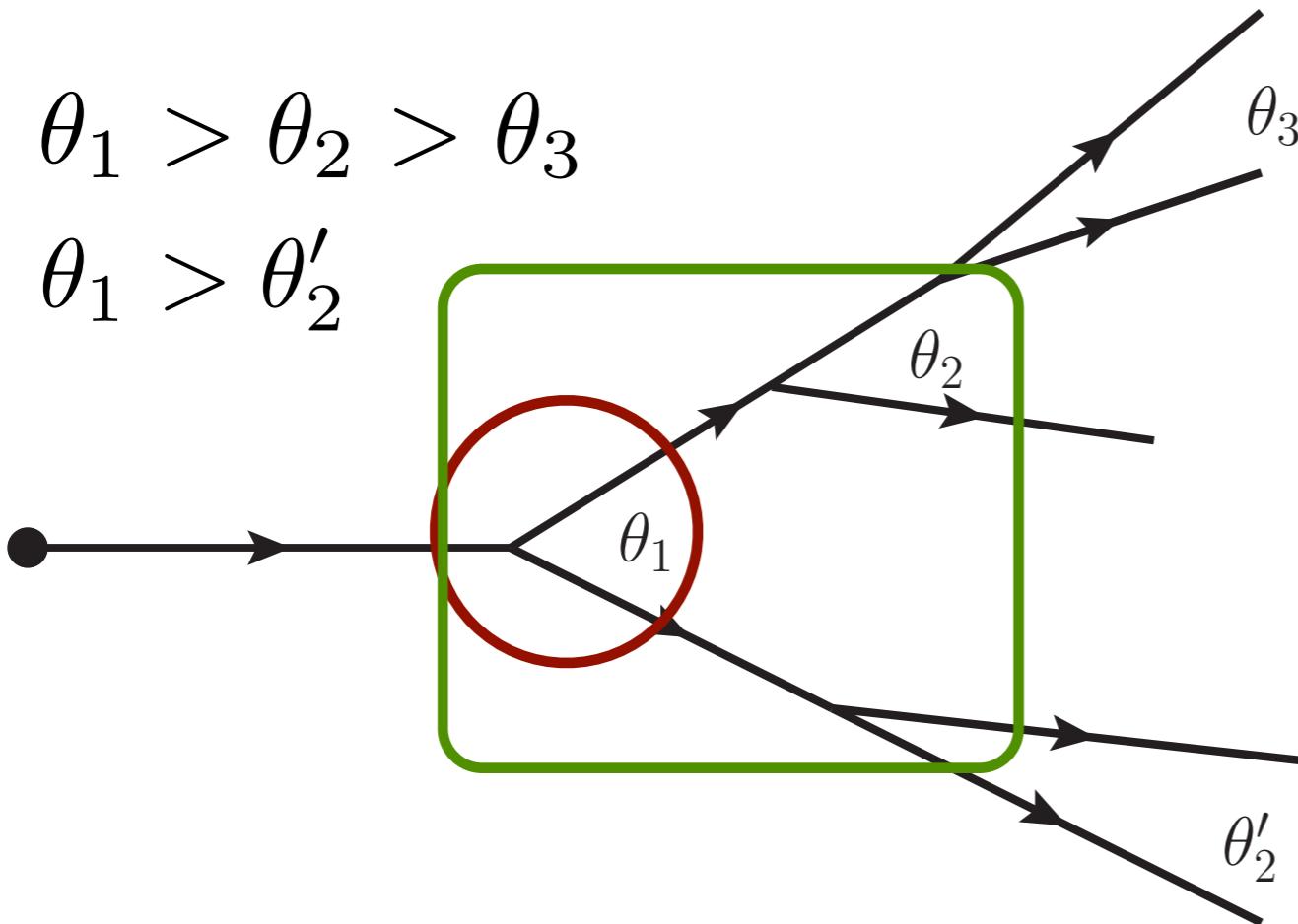
Angular ordering

$$d^2 P \propto \Theta(\theta_1 - \theta_2) dP_1 dP_2$$



Jets in vacuum

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]

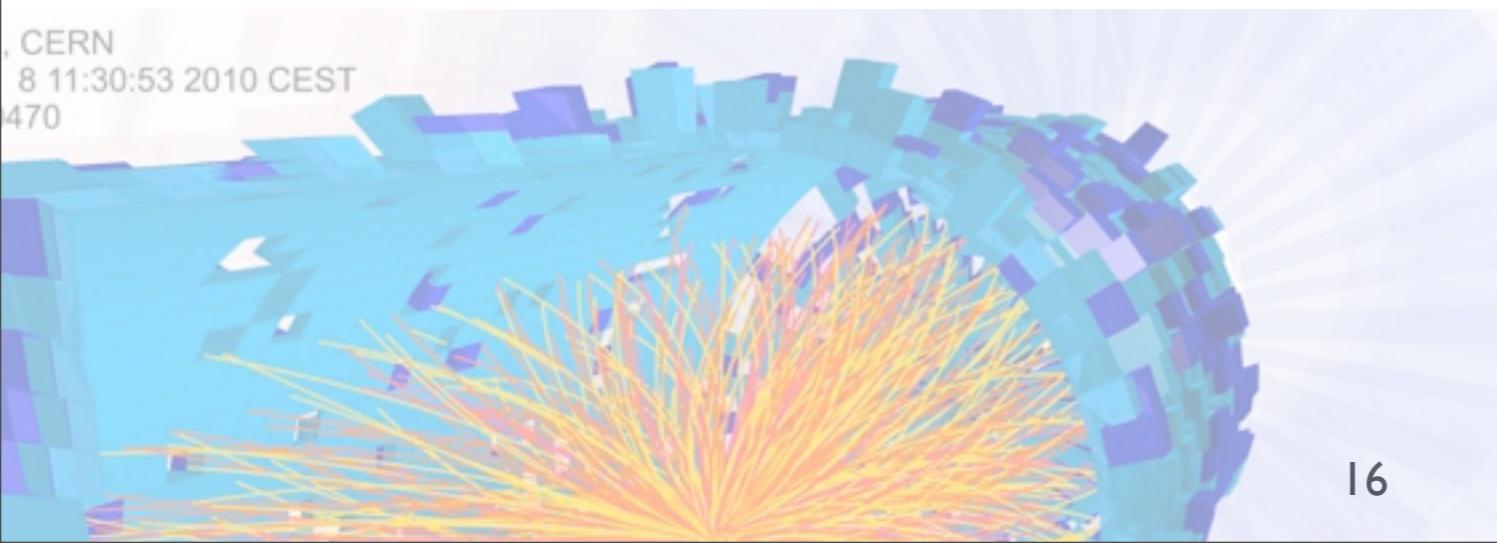


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Angular ordering

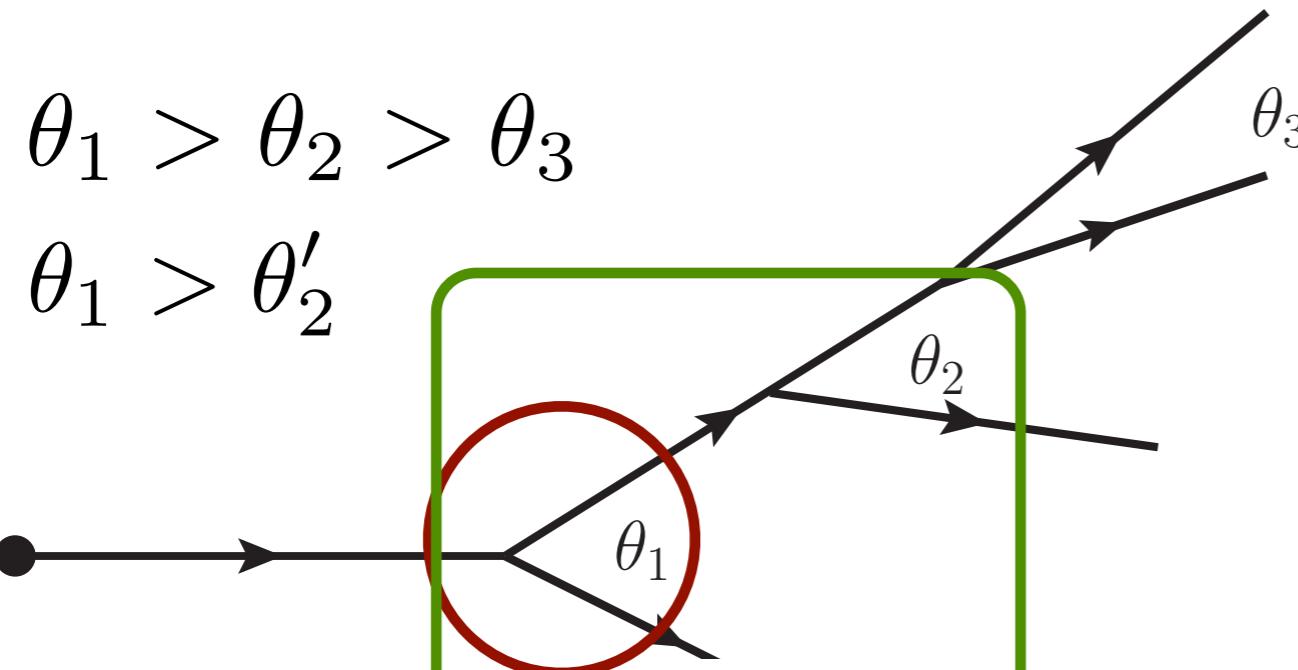
$$d^2 P \propto \Theta(\theta_1 - \theta_2) dP_1 dP_2$$



Markov process!

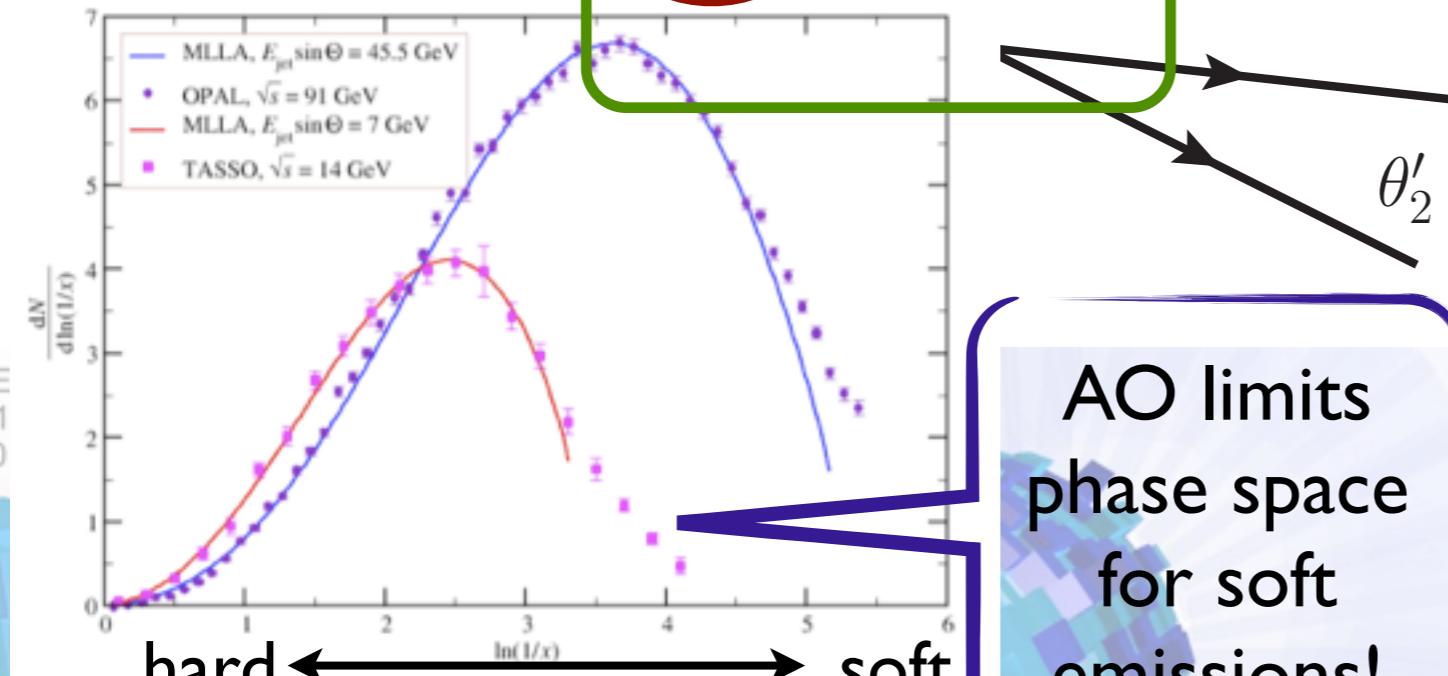
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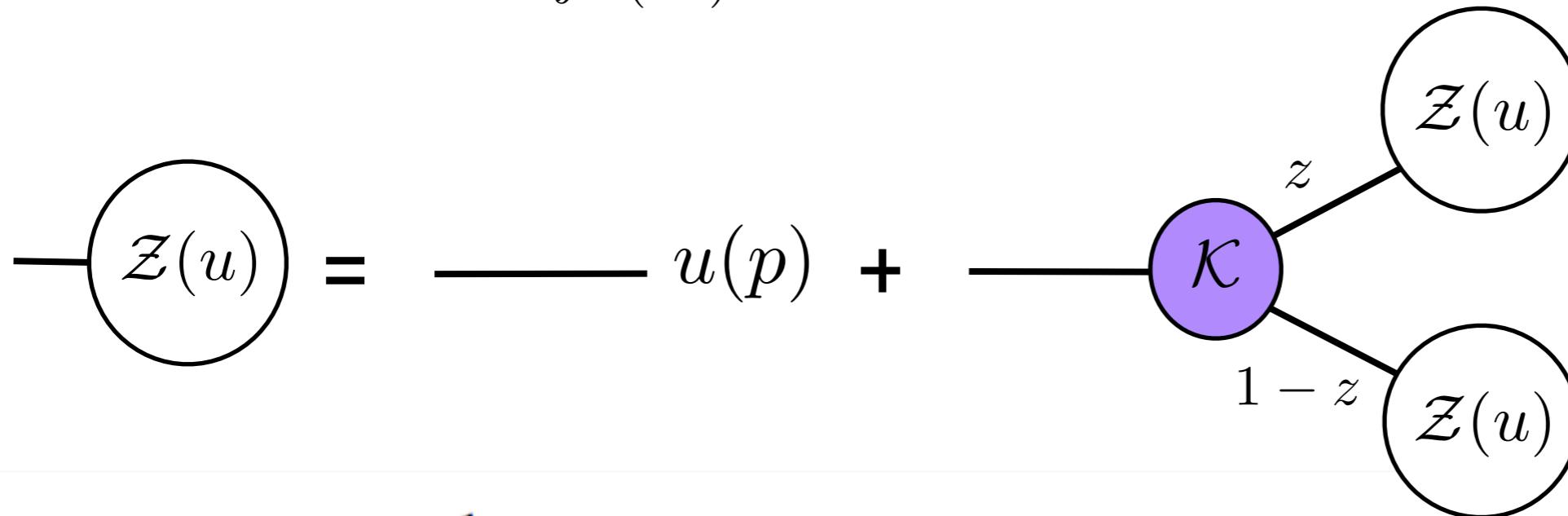
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Markov process!

Application I: gluon distribution

- Integrating over transverse momenta:

$$\int \frac{d^2 k}{(2\pi)^2} \mathcal{P}(k - q, L - t) = 1$$

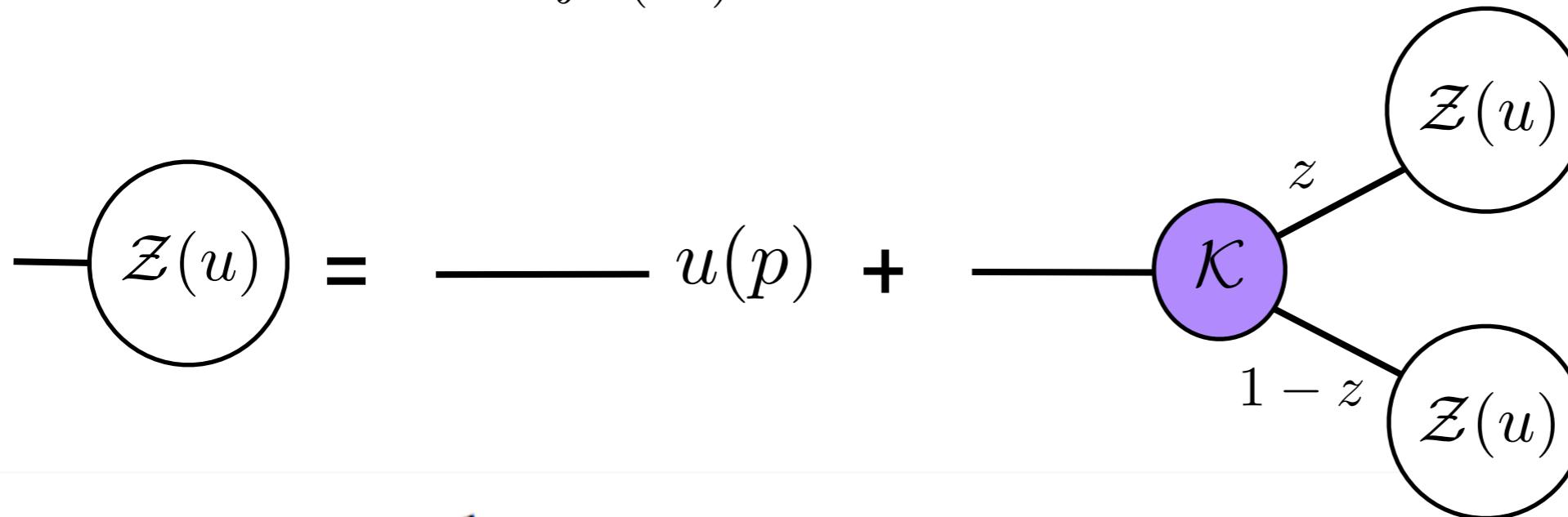


$$\frac{\partial}{\partial L} \mathcal{Z}(E, u) = \alpha_s \int_0^1 \frac{dz}{z} \mathcal{K}(z) [\mathcal{Z}(zE, u) \mathcal{Z}((1-z)E, u) - \mathcal{Z}(E, u)] ,$$

Application I: gluon distribution

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$$\frac{\partial}{\partial L} \mathcal{Z}(E, u) = \alpha_s \int_0^1 \frac{dz}{z} \mathcal{K}(z) [\mathcal{Z}(zE, u) \mathcal{Z}((1-z)E, u) - \mathcal{Z}(E, u)] ,$$

- Gluon distribution

$$D(x, E) \equiv x \frac{dN}{dx} = \omega \frac{\delta \mathcal{Z}(E, u)}{\delta u(\omega)} \Big|_{u=1}$$

$$x = \omega/E$$

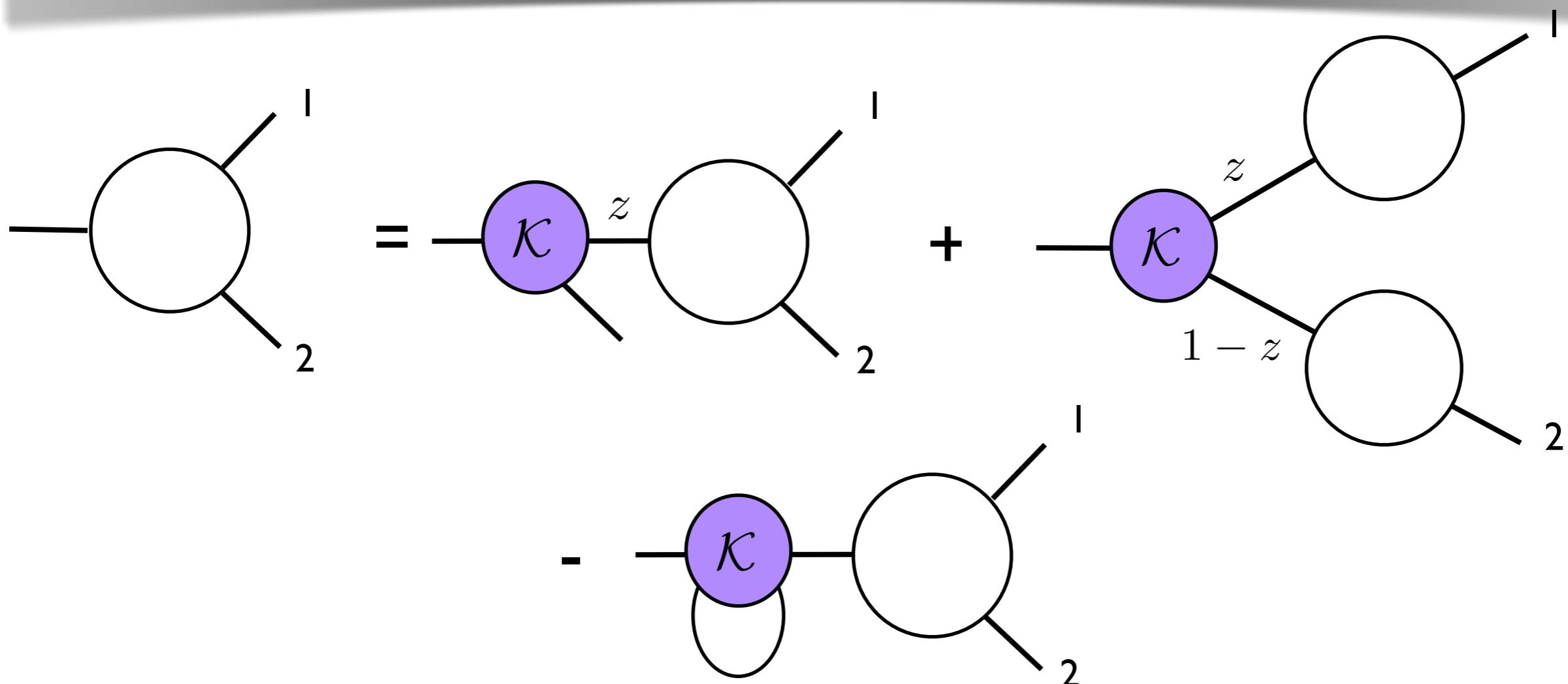
$$\frac{\partial}{\partial L} D(x) = \alpha_s \int_0^1 \frac{dz}{z} \mathcal{K}(z) \left[D\left(\frac{x}{z}\right) + D\left(\frac{x}{1-z}\right) - D(x) \right]$$

Application II: Correlations

- 2-particle correlations inside the jet

$$D(x_1, x_2) \equiv \omega_1 \omega_2 \frac{\delta^2 \mathcal{Z}(E, u)}{\delta u(\omega_1) \delta u(\omega_2)} \Big|_{u=1}$$

$$\frac{\partial}{\partial L} D(x_1, x_2) = \alpha_s \int_0^1 \frac{dz}{z} \mathcal{K}(z) \left[D\left(\frac{x_1}{z}, \frac{x_2}{z}\right) + D\left(\frac{x_1}{z}\right) D\left(\frac{x_2}{1-z}\right) + \text{sym} - D(x_1, x_2) \right]$$



Generating Functional Method

- n-gluon probability P_n

$$\mathcal{Z}(u) = \sum_{n=1}^{\infty} P_n u^n$$

- Probability conservation

$$\mathcal{Z}(u = 1) = 1$$

- Average gluon number

$$\langle n \rangle \equiv \frac{d}{du} \mathcal{Z}(u = 1)$$

- Higher moments

$$\langle n(n-1)\dots(n-m+1) \rangle = \left(\frac{d}{du} \right)^m \mathcal{Z}(u = 1)$$

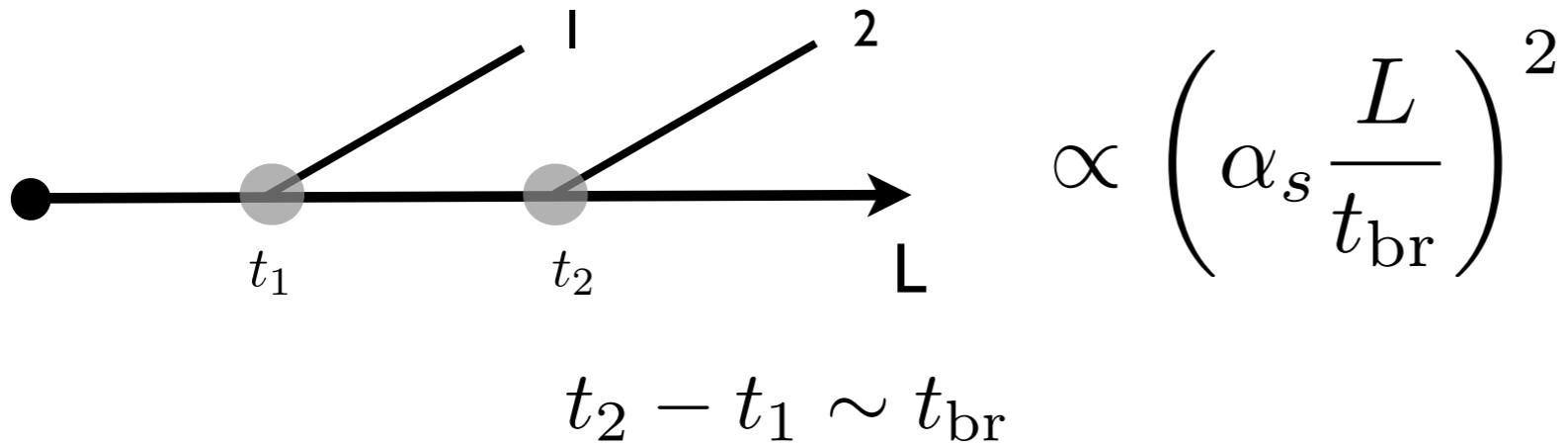
- To compute differential distributions in k

$$u \rightarrow u(k)$$

$$\frac{\delta u(k)}{\delta u(p)} = \delta^{(3)}(k - p)$$

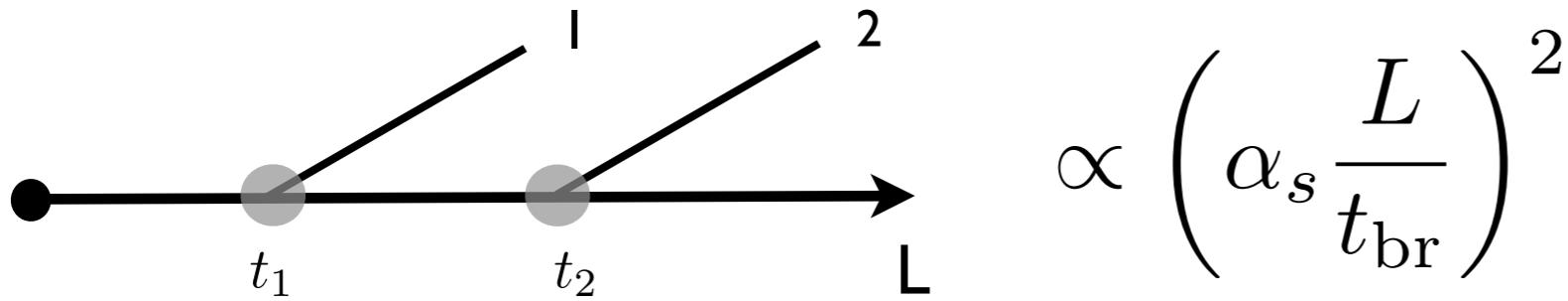
N-gluon emissions

- Factorized contribution $t_1 \sim t_2 \sim L \gg t_{\text{br}}$ or ($\omega_1 \sim \omega_2 \ll \omega_c = \hat{q}L^2$)

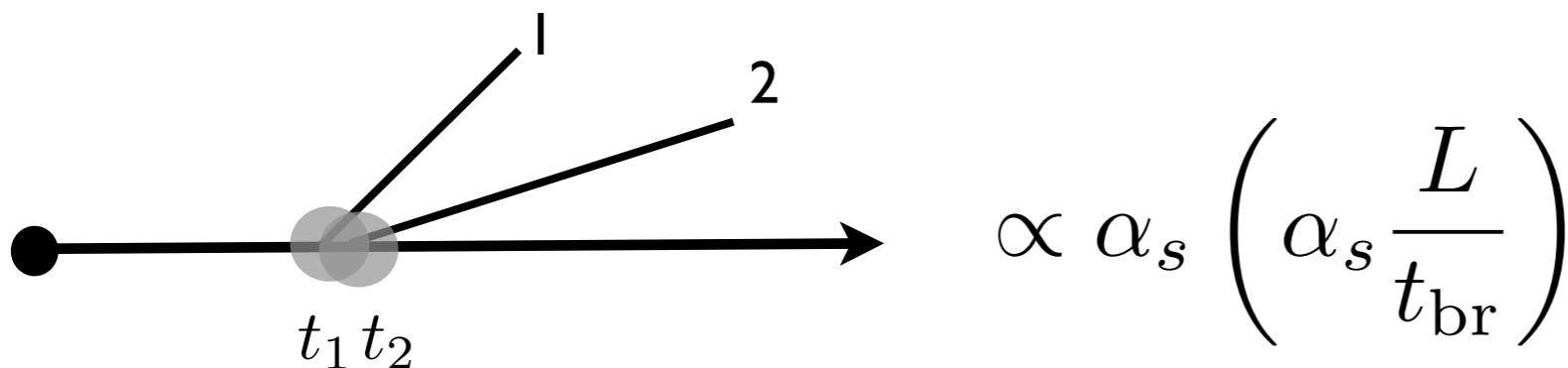


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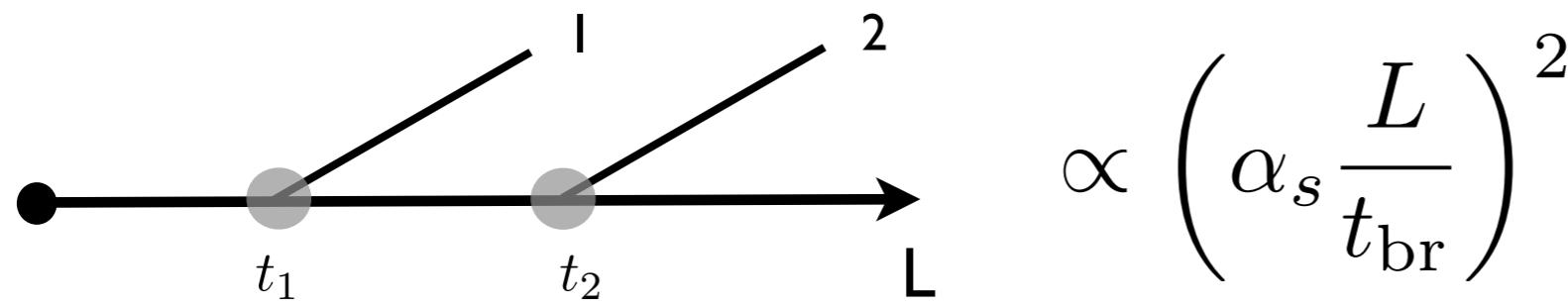


- Interferences $t_2 - t_1 \sim t_{\text{br}}$



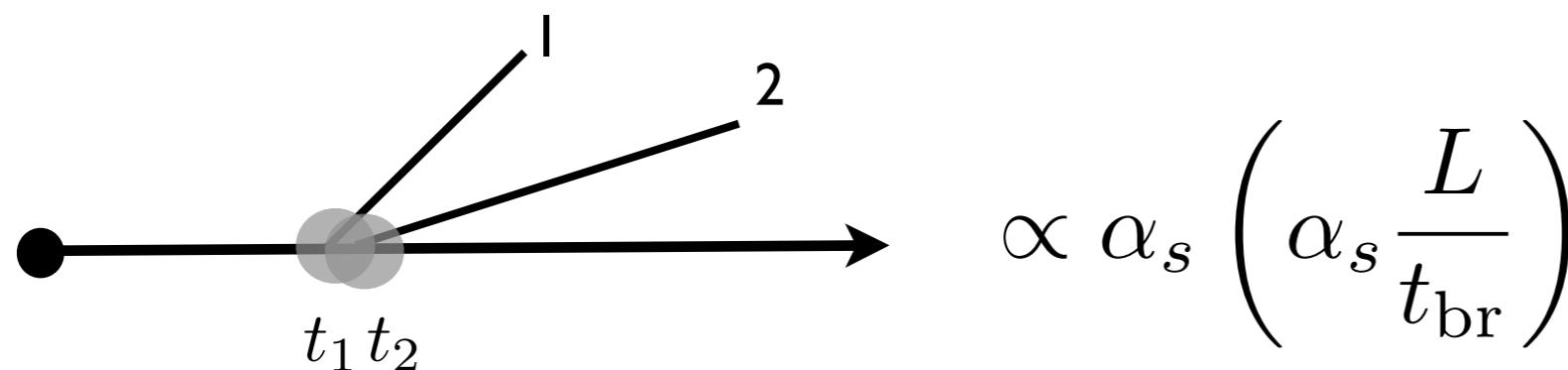
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- Decoherence of successive splittings:
Interferences are suppressed in a dense medium \Rightarrow No Angular Ordering!

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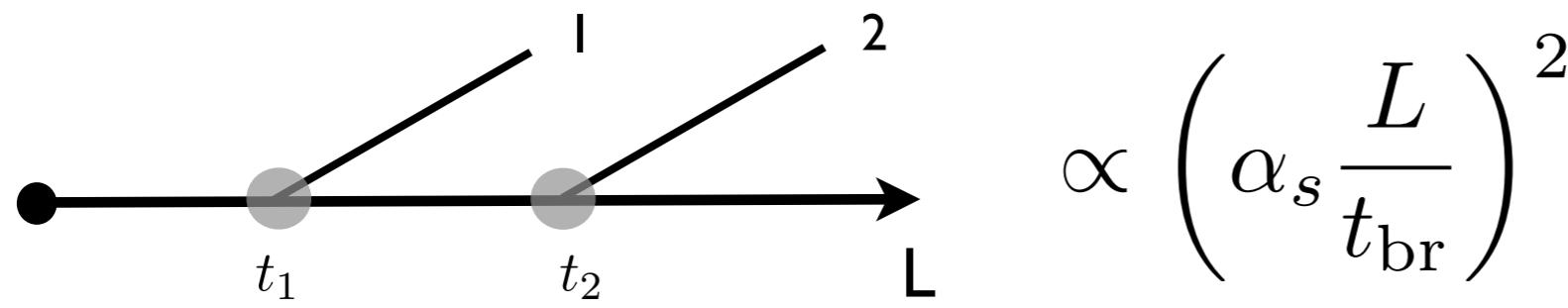


Y.M.-T., K.Tywoniuk, C.A. Salgado (2011)
E. Iancu, J. Casalderrey Solana (2011)

- Ordering variable:
emission time $t_1 < t_2$

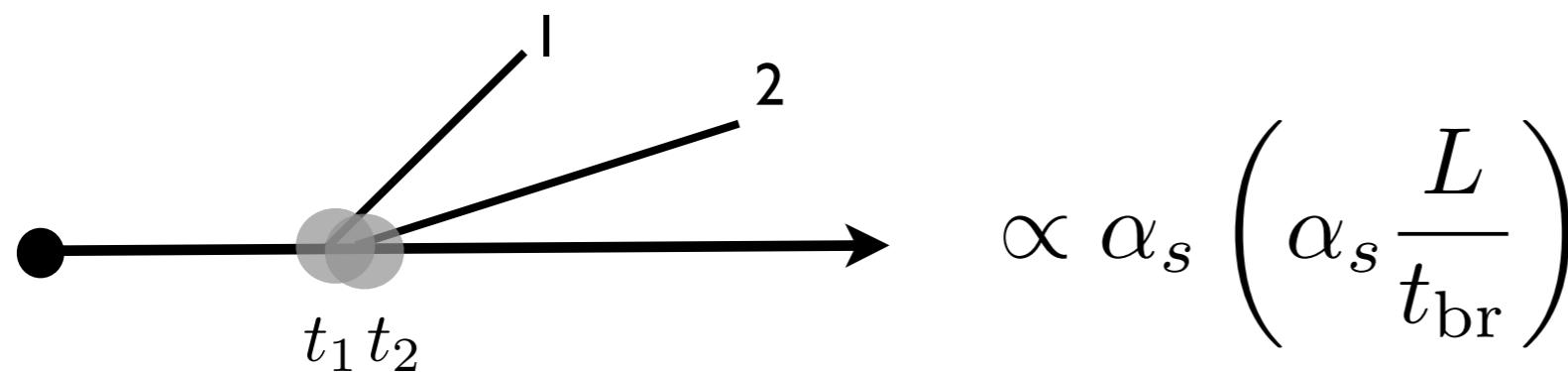
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\Rightarrow Probabilistic Scheme

- Resummation:

$$\sigma = \sum_n a_n \left(\alpha_s \frac{L}{t_{\text{br}}} \right)^n$$

- Ordering variable:
emission time $t_1 < t_2$