Di-hadron Angular Correlations as a Probe of Saturation Dynamics

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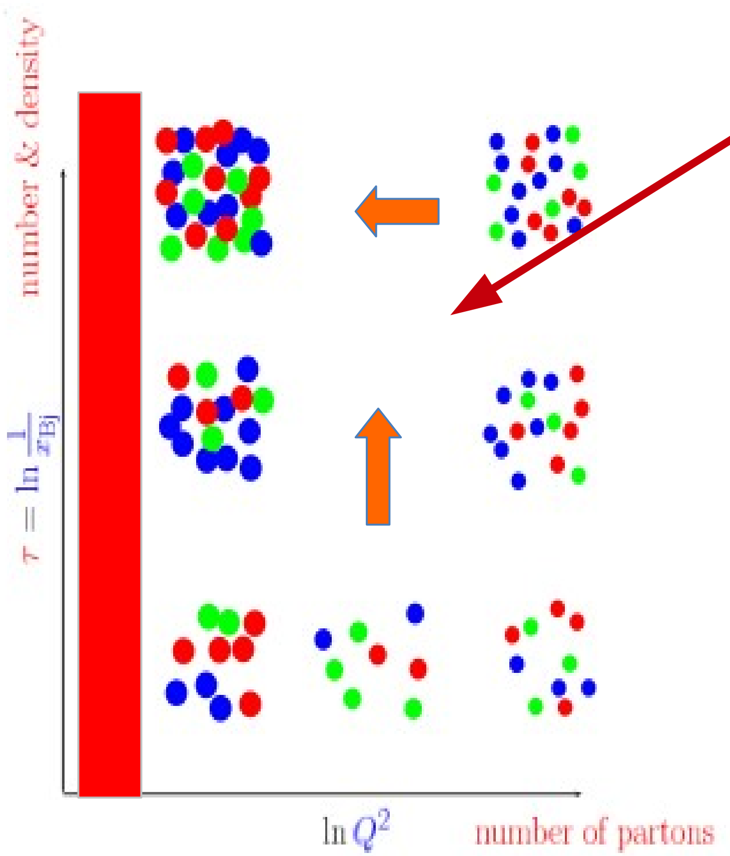
Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

How does the coupling run ?

How does saturation transition to chiral symmetry breaking and confinement

Color Glass Condensate

JIMWLK evolution equation

$$\frac{d}{d\, ln 1/x} \langle O \rangle = \frac{1}{2} \left\langle \int d^2x \, d^2y \, \frac{\delta}{\delta \alpha_x^b} \, \eta_{xy}^{bd} \, \frac{\delta}{\delta \alpha_y^d} \, O \right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi} \int \frac{d^2z}{(2\pi)^2} \frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2} \left[\underbrace{1 + U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}} \right]^{bd}$$

U is a Wilson line in adjoint representation

QCD at low x: CGC
(a high gluon density environment)

two main effects: *"multiple scatterings"*
evolution with ln (1/x)

CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^{\dagger} >$ with $\mathbf{V(x_t) = \hat{P} e^{ig \int dx^- A_a^+ t_a}}$

$$\mathbf{A}_{\mathbf{a}}^{\mu}(\mathbf{x_t}, \mathbf{x}^-) \sim \delta^{\mu +} \delta(\mathbf{x}^-)\, \alpha_{\mathbf{a}}(\mathbf{x_t}) \qquad \alpha^{\mathbf{a}}(\mathbf{k_t}) = \mathbf{g}\, \rho^{\mathbf{a}}(\mathbf{k_t})/\mathbf{k_t^2}$$

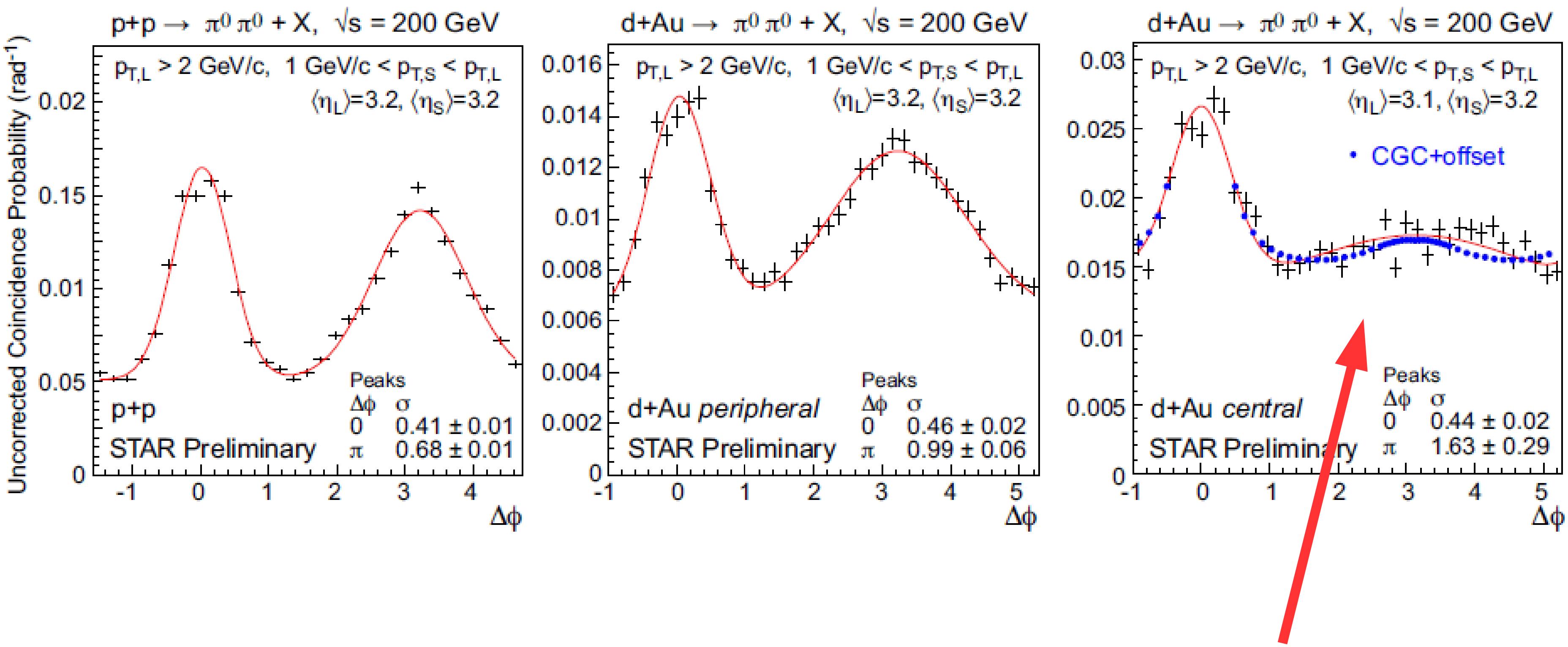
gluon distribution: $\mathrm{xG(x, Q^2)} \sim \int^{\mathrm{Q^2}} \frac{\mathrm{d^2k_t}}{\mathrm{k_t^2}}\, \phi(\mathrm{x, k_t})$ with $\phi(\mathbf{x}, \mathbf{k_t^2}) \sim < \rho_{\mathbf{a}}^{\star}(\mathbf{k_t})\, \rho_{\mathbf{a}}(\mathbf{k_t}) >$

pQCD with collinear factorization: *single scattering*
evolution with ln Q²

disappearance of back to back jets

PHENIX PRL107 (2011)

Recent STAR measurement (arXiv:1008.3989v1):



multiple scatterings de-correlate the hadrons

CGC fit from Albacete + Marquet, PRL (2010)
Also by Tuchin, NPA846 (2010) and
A. Stasto, B-W. Xiao, F. Yuan, arXiv:1109.1817

Di-jet production: pA

recall DIS, single inclusive production in pA probe ***dipoles***

$$< \mathbf{Tr\, V\, V^\dagger} >$$

di-jet production in pA (and DIS) probe ***quadrupoles***

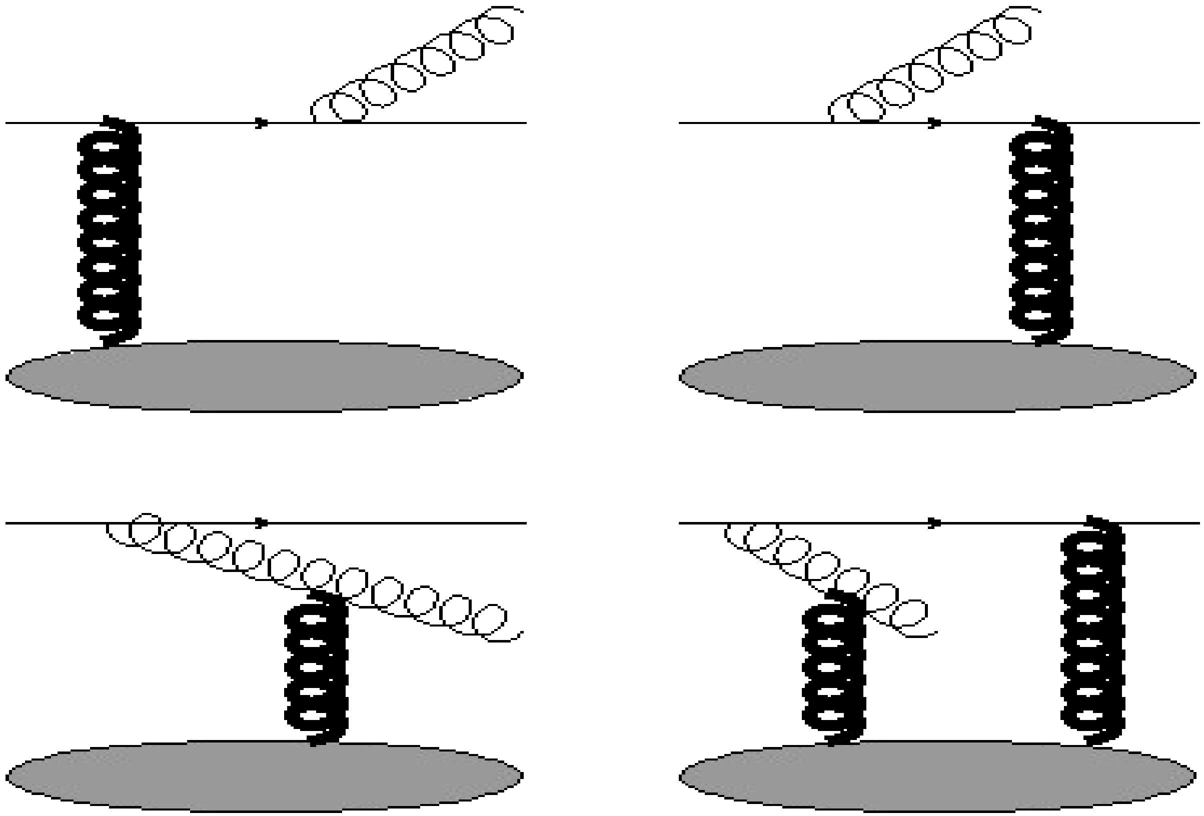
$$< \mathbf{Tr\, V\, V^\dagger\, V\, V^\dagger} >$$

momentum space: J. Jalilian-Marian, Y. Kovchegov, PRD70 (2004) 114017

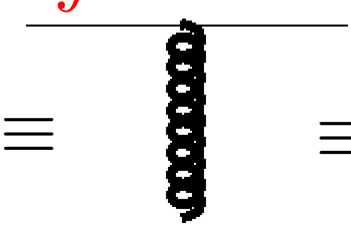
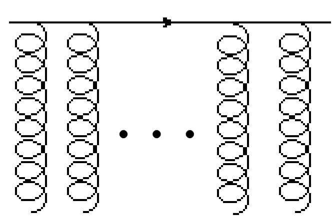
coordinate space: *C. Marquet, NPA796 (2007) 41*

including gluons in the projectile *F. Dominguez,C. Marquet, B-W. Xiao, F. Yuan, PRD83 (2011) 105005*

Di-jet production: pA $\quad \mathbf{q(p)\,T \to q(q)\,g(k)}$



$$\mathbf{d}\sigma \sim \int \mathbf{K} \otimes \left[< \mathbf{Tr V\, V^\dagger} > + < \mathbf{Tr V\, V^\dagger\, V\, V^\dagger} > + \cdots \right]$$

$\mathbf{V} \equiv$  $\equiv$  $\sim \mathbf{1} + \mathbf{O}(\mathbf{g}\,\rho) + \mathbf{O}(\mathbf{g^2}\,\rho^2)$

di-jet production in pA

$O_2(r,\bar{r}) \equiv Tr V_r\, V^\dagger_{\bar{r}}$ **dipole** $\longrightarrow$ **F2 in DIS, single hadron in pA**

$$O_4(r,\bar{r}:s) \equiv Tr V^\dagger_r\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\Big[Tr V^\dagger_r\, V_s\;\; Tr V_{\bar{r}}\, V^\dagger_s - \frac{1}{N_c} Tr V^\dagger_r\, V_{\bar{r}}\Big]$$

$$O_6(r,\bar{r}:s,\bar{s}) \equiv Tr V_r\, V^\dagger_{\bar{r}}\, t^a\, t^b\, [U_s\, U^\dagger_{\bar{s}}]^{ba} = \frac{1}{2}\Big[Tr V_r\, V^\dagger_{\bar{r}}\, V_{\bar{s}}\, V^\dagger_s\; Tr V_s\, V^\dagger_{\bar{s}} - \frac{1}{N_c} Tr V_r\, V^\dagger_{\bar{r}}\Big]$$

quadrupole

calculations: classical

how about quantum corrections (energy dependence) ?

energy (rapidity) dependence from JIMWLK evolution of O's

<u>evolution of a dipole is well known: BK eq.</u>

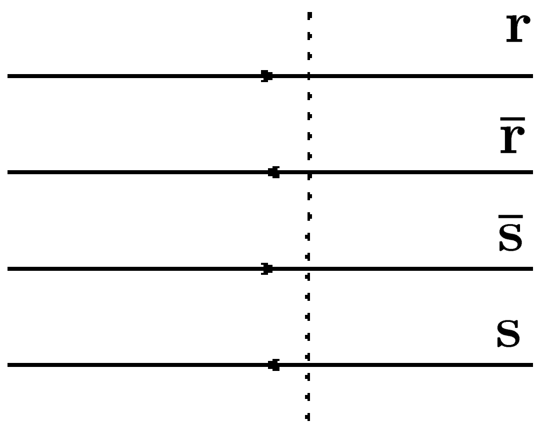
how does a quadrupole evolve?

Evolution of quadrupole from JIMWLK

$$\mathbf{Q}(\mathbf{r},\bar{\mathbf{r}},\bar{\mathbf{s}},\mathbf{s}) \equiv \frac{1}{\mathbf{N_c}} < \mathbf{Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s)} >$$

radiation kernels
as in dipole

$$\frac{\mathbf{d}}{\mathbf{dy}}$$



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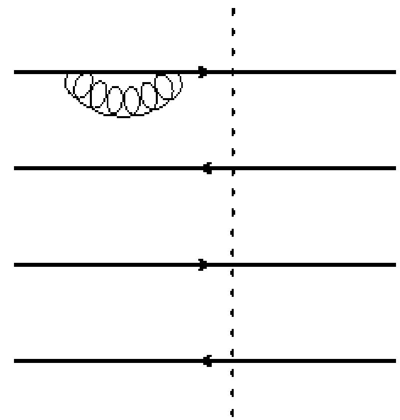
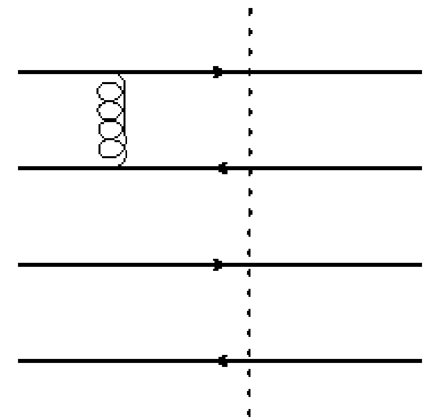
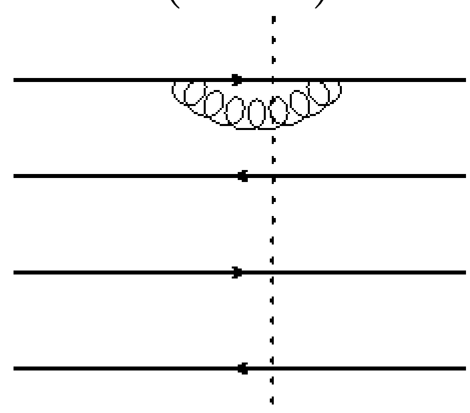
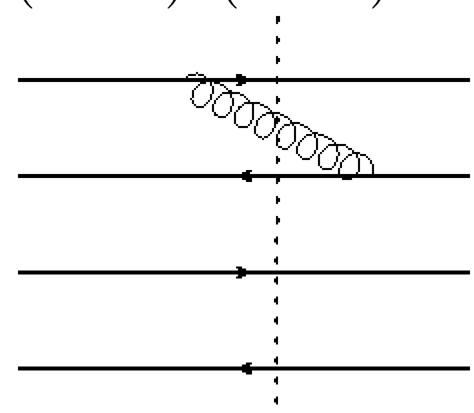


$$+$$



$$\frac{\mathbf{1}}{(\mathbf{r}-\mathbf{z})^2} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}$$





$$+\ \cdots$$

Evolution of quadrupole from JIMWLK

$$
\begin{aligned}
&\frac{d}{dy}\,\langle Q(r,\bar{r},\bar{s},s)\rangle\\
= \;& \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z \Bigg\{\Bigg\langle \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(r-s)^2}{(r-z)^2(s-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right] Q(z,\bar{r},\bar{s},s)\,S(r,z)\\
+\;& \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] Q(r,z,\bar{s},s)\,S(z,\bar{r})\\
+\;& \left[\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(s-z)^2(\bar{r}-z)^2}\right] Q(r,\bar{r},z,s)\,S(\bar{s},z)\\
+\;& \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] Q(r,\bar{r},\bar{s},z)\,S(z,s)\\
-\;& \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} + \frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}\right] Q(r,\bar{r},\bar{s},s)\\
-\;& \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\right] S(r,s)\,S(\bar{r},\bar{s})\\
-\;& \left[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2} + \frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2} - \frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2} - \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right] S(r,\bar{r})\,S(\bar{s},s)\Bigg\rangle\Bigg\}
\end{aligned}
$$

$$\frac{d}{d\,y}Q = \int P_1\,[Q\,S] - P_2\,[Q] + P_3[S\,S]$$ with $$P_1 - P_2 + P_3 = 0$$

J. Jalilian-Marian, Y. Kovchegov: PRD70 (2004) 114017
Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037

quadrupole evolution in the linear regime

define $\mathbf{T}(\mathbf{r},\bar{\mathbf{r}}) \equiv \mathbf{1} - \mathbf{S}(\mathbf{r},\bar{\mathbf{r}})$ $\quad \mathbf{T_Q}(\mathbf{r},\bar{\mathbf{r}},\bar{\mathbf{s}},\mathbf{s}) \equiv \mathbf{1} - \mathbf{Q}(\mathbf{r},\bar{\mathbf{r}},\bar{\mathbf{s}},\mathbf{s})$

re-write the evolution eq. for T_Q rather than Q

expand in powers of gauge fields (or color charges)

ignore contribution of non-linear terms: T T and T_Q T

$\mathbf{O}(\alpha^{\mathbf{2}})$ $\quad \mathbf{T_Q}(\mathbf{r},\bar{\mathbf{r}},\bar{\mathbf{s}},\mathbf{s}) \to \mathbf{T}(\mathbf{r},\bar{\mathbf{r}}) + \mathbf{T}(\mathbf{r},\mathbf{s}) + \cdots$

with $\mathbf{T}(\mathbf{r},\bar{\mathbf{r}}) \sim \alpha^{\mathbf{2}}(\mathbf{r},\bar{\mathbf{r}})$

quadrupole evolution reduces to a sum of BFKL evolution eqs

Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037
D. Triantafyllopoulos

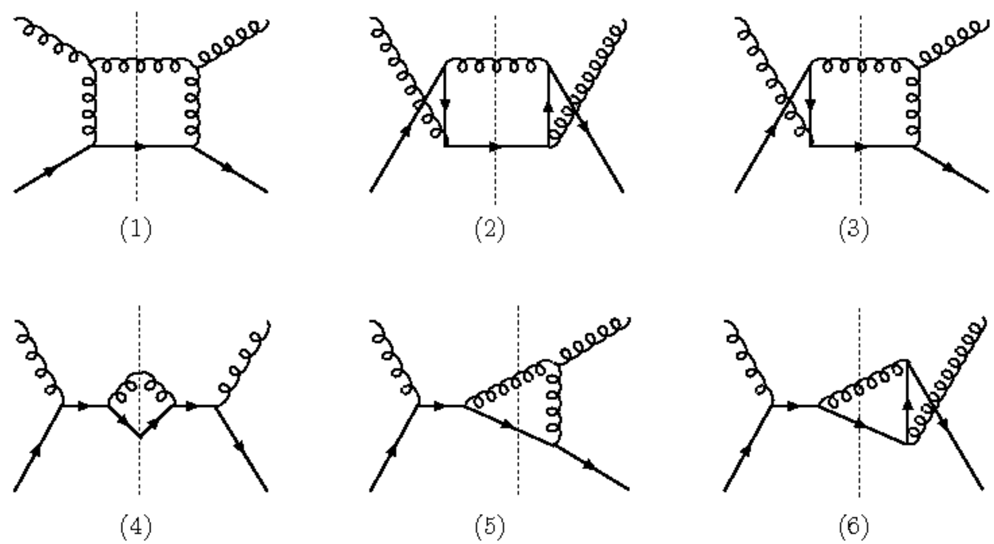
di-hadron correlations in the high p_t limit

$$\mathbf{O}(\alpha^{\mathbf{2}})$$

Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Xiao, Yuan (2011)

factorization of target distribution functions and hard scattering matrix element

$$\mathbf{d}\sigma \sim \mathbf{\Phi} \otimes \frac{\mathbf{d}\sigma}{\mathbf{dt}}^{\mathbf{2 \to 2}}$$



$$\frac{d\sigma}{dt}^{q\,g \to q\,g} \sim \frac{1}{s^2}\left[\frac{4}{9}\frac{s^2+u^2}{-s\,u} + \frac{s^2+u^2}{t^2}\right]$$

partons are back to back

quadrupole evolution in the linear regime

$\mathbf{O}(\alpha^{\mathbf{4}})$

momentum space

define $$\hat{\mathbf{T}}_{\mathbf{4}}(\mathbf{l_1}, \mathbf{l_2}, \mathbf{l_3}, \mathbf{l_4}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{Tr}\, \rho(\mathbf{l_1})\, \rho(\mathbf{l_2})\, \rho(\mathbf{l_3})\, \rho(\mathbf{l_4})$$

assume $\mathbf{l_1} \neq \mathbf{l_2} \neq \mathbf{l_3} \neq \mathbf{l_4}$ subject to an overall delta function

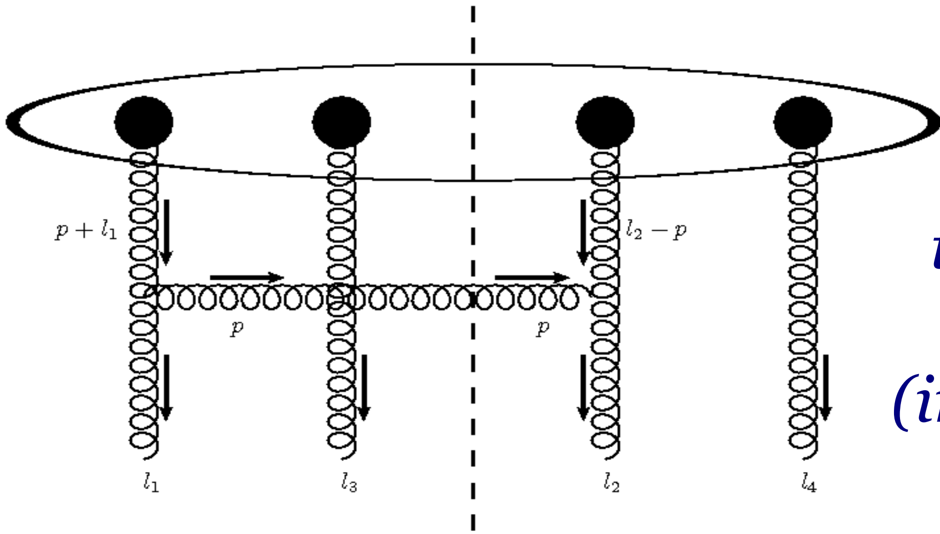
contribution only from linear term in expansion of Wilson lines (except for the z-dependent ones)

quadrupole evolution eq. reduces to Bartels-Jaroszewicz-Kwiecinski-Praszalowicz (BJKP) eq. for evolution of 4-Reggeized gluons in a singlet state

quadrupole evolution in the linear regime

BJKP equation

$\mathbf{O}(\alpha^4)$: 4-gluon exchange



J. Jalilian-Marian, PRD85 (2012) 014037

the color structure is identical on both sides of this eq. (independent of color averaging)

$$\frac{d}{dy}\hat{T}_4(l_1,l_2,l_3,l_4) = \frac{N_c\,\alpha_s}{\pi^2}\int d^2p_t\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_1^i)}{(p_t+l_1)^2}\right]\cdot\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_2^i)}{(p_t+l_2)^2}\right]$$
$$\hat{T}_4(p_t+l_1,l_2-p_t,l_3,l_4)+\cdots$$
$$-\;\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2p_t\left[\frac{l_1^2}{p_t^2(l_1-p_t)^2}+\{l_1\to l_2,l_3,l_4\}\right]\hat{T}_4(l_1,l_2,l_3,l_4)$$

this will <u>de-correlate</u> the produced partons at high $p_t > Q_s$

color structure

$$\mathbf{\hat{T}_4(l_1, l_2, l_3, l_4) \equiv \frac{1}{N_c} Tr\, \rho(l_1)\, \rho(l_2)\, \rho(l_3)\, \rho(l_4) = Tr\,(t^a\, t^b\, t^c\, t^d)\, \rho^a(l_1)\, \rho^b(l_2)\, \rho^c(l_3)\, \rho^d(l_4)}$$

$$\begin{aligned} Tr\,(t^a\, t^b\, t^c\, t^d) &= \frac{1}{4N_c}\left[\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right] \\ &+ \frac{1}{8}\left[d^{abr}\, d^{cdr} - d^{acr}\, d^{bdr} + d^{adr}\, d^{bcr}\right] \\ &+ \frac{i}{8}\left[d^{abr}\, f^{cdr} - d^{acr}\, f^{bdr} + d^{adr}\, f^{bcr}\right] \end{aligned}$$

overall state is a singlet, how about pairwise?

for $N_c = 3$

$$\left[\delta^{\mathbf{ab}}\delta^{\mathbf{cd}} + \delta^{\mathbf{ac}}\delta^{\mathbf{bd}} + \delta^{\mathbf{ad}}\delta^{\mathbf{bc}}\right] = \mathbf{3}\left[\mathbf{d^{abr}d^{cdr} + d^{acr}d^{bdr} + d^{adr}d^{bcr}}\right]$$

the linear regime

$\mathbf{O}(\alpha^{\mathbf{3}})$ **:** 3-gluon (odderon) exchange

Dipole odderon: *Kovchegov, Szymanowski, Wallon*

$\mathrm{Tr}\, V\, V^{\dagger}\, V$ *Hatta, Iancu, Itakura, McLerran* **BJKP equation**

BJKP equation describes evolution of n-Reggeized gluons in a singlet state

JIMWLK (linear) and BJKP eqs. agree for n=2,3,4

non-linear interactions:

1) MV action with JIMWLK evolution
2) Triple (and more?) pomeron vertices
Chirilli, Szymanowski, Wallon (2010)

QCD at high energy

Two distinct approaches:

1) CGC

McLerran-Venugopalan effective action
JIMWLK-BK evolution

2) Reggeized-gluon exchange

BJKP equation
triple,... pomeron vertex

Conjecture: CGC contains BJKP + multi-pomeron vertices

The role of initial conditions

McLerran-Venugopalan (93)

$$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S}_\perp}$$

$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)}^\dagger\, \mathbf{V(0)}] > \sim \mathbf{1 - e}^{-[\mathbf{r_t^2\, Q_s^2}]^\gamma \mathbf{\log(e} + \frac{\mathbf{1}}{\mathbf{r_t}\,\Lambda_{\mathbf{QCD}}})}$$

with $\gamma = \mathbf{1.119}$

how about higher order terms in ρ ?

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}} - \frac{\mathbf{d^{abc}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})}{\kappa_{\mathbf{3}}} + \frac{\mathbf{F^{abcd}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})\rho^{\mathbf{d}}(\mathbf{x_t})}{\kappa_{\mathbf{4}}}\right]}$$

these higher order terms make the single inclusive spectra steeper and give <u>leading N_c</u> correlations (ridge)

Dumitru-Jalilian-Marian-Petreska, PRD84 (2011) 014018
Dumitru-Petreska,, NPA879 (2012) 59

Two-hadron angular correlations

<u>A unique window to dynamics of high energy QCD</u>

We have just started to scratch the surface: there is much more to be understood