Gauge invariant definition of the jet quenching parameter $\hat{q}$

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Outline

1. Introduction: The jet quenching parameter

2. Theoretical background
   - Soft-Collinear Effective Theory
   - The Glauber mode
   - Singular Gauges

3. Calculation

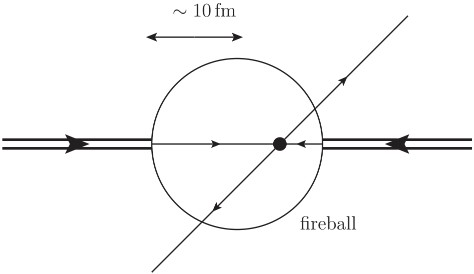
4. Results and Conclusions

The jet quenching parameter $\hat{q}$

- A jet moving through the medium (e.g. QGP) may lose energy and is subject to momentum broadening

  $\rightarrow$ **jet quenching**

- **Broadening** refers to a change of the momentum perpendicular to the original direction of motion

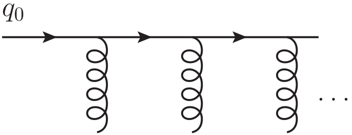


The jet quenching parameter $\hat{q}$

- Define the **jet quenching parameter**

$$\hat{q} = \int d^2k_\perp \, k_\perp^2 \, \frac{d\Gamma}{d^2k_\perp}$$

- Γ is rate of elastic collisions of a parton with the medium particles



Goal

Find field theoretic definition of $\hat{q}$

The jet quenching parameter $\hat{q}$

Intuitive interpretations

- $\hat{q} = \frac{1}{L}\langle k_\perp^2 \rangle = \frac{1}{L}\int d^2k_\perp\, k_\perp^2\, P(k_\perp)$
  $P(k_\perp)$ is probability to acquire a perpendicular momentum $k_\perp$ after travelling through a medium with length L
- When describing the broadening of the $k_\perp$-distribution while travelling a distance through the medium by a diffusion equation, $\hat{q}$ is related to the diffusion constant

Motivation

Basic ingredient in jet quenching calculations
e.g. when considering medium modified fragmentation functions or transverse momentum dependent parton distribution functions

The effective field theory approach

- Several scales appear in the process, most notably

  The energy of the jet Q

  The scale of the medium (temperature) T
- Small dimensionless ratio $\lambda = T/Q \ll 1$

Conclusion

Use an effective field theory that provides a systematic expansion in λ

- When dealing with jets and their interactions with soft particles **Soft-Collinear Effective Theory** (SCET) is the appropriate EFT

  Bauer et al. '01; Beneke at al. '02

Soft-Collinear Effective Theory

- Classify modes by the scaling of their momentum components in the different light-cone directions $(n, \bar{n})$

  $(p^+, p^-, p_\perp) = (Q, Q, Q) \sim (1, 1, 1)$ is called hard

  $(p^+, p^-, p_\perp) = (T, T, T) \sim (\lambda, \lambda, \lambda)$ is called soft

  $(p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$ is called collinear
- **Jets** have a collinear momentum, i.e., they have a large momentum component in one light cone direction, but only a small invariant mass
- Integrate out the hard modes and the off-cone components of the collinear modes to find the **SCET Lagrangian** for collinear fields

$$\mathcal{L} = \bar{\xi} i \bar{n} \cdot D \frac{\not{n}}{2} \xi + \bar{\xi} i \not{D}_\perp \frac{1}{in \cdot D} i \not{D}_\perp \frac{\not{n}}{2} \xi + \mathcal{L}_{\mathrm{Y.M.}}, \quad iD = i\partial + gA$$

The Glauber mode

- When considering momentum broadening a further mode becomes relevant

  $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda)$ is called **Glauber mode**

  Idilbi, Majumder '08

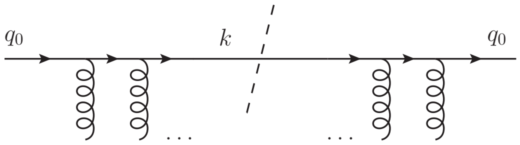
- Introduce it into the SCET Lagrangian as an effective classical field of the medium particles
- At leading order in the power counting the extended SCET Lagrangian for the interaction of collinear particles with Glauber gluons is just

$$\mathcal{L} = \bar{\xi} i \bar{n} \cdot D \frac{\not{n}}{2} \xi, \quad iD = i\partial + gA$$

The Glauber mode

- Use SCET_G to calculate $\hat{q}$

  D'Eramo, Liu, Rajagopal '10



- Initially on-shell quark scattering on an arbitrary number of medium particles via Glauber exchange
- Use **covariant gauge**

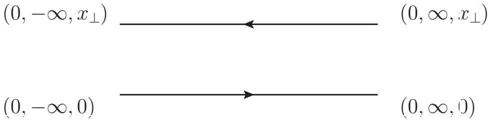
$\hat{q}$ in covariant gauge

- Result is the Fourier transform of the medium averaged expectation value of two Wilson lines

$$P(k_\perp) = \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr}\left[W^\dagger_{\mathcal{R}}[0, x_\perp] W_{\mathcal{R}}[0,0] \right] \right\rangle$$

$$W_{\mathcal{R}}[y^+, y_\perp] = \mathcal{P}\left\{ \exp\left[ig \int_{-\sqrt{2}L/2}^{\sqrt{2}L/2} dy^- A^+(y^+, y^-, y_\perp) \right] \right\}$$

- Agrees with Baier et al. '97; Zakharov '96; Casalderrey-Solana, Salgado '07 except time ordering



- Not gauge invariant ($W_{\mathcal{R}} = 1$ in light-cone gauge $A^+ = 0$)

Changes in arbitrary gauge

Goal

Want to show that SCET_G is complete and find a gauge invariant expression of $\hat{q}$ for applications

- In singular gauges, such as light-cone gauge the **scaling of the Glauber field itself** is different

  Idilbi, Majumder '08; Ovanesyan, Vitev '11

  $A_\perp^{\text{cov}} \ll A_\perp^{\text{lcg}} \sim A^{+,\text{cov}}$

- Additional leading power interaction term in the Lagrangian becomes relevant

$$\bar{\xi} i\not{D}_\perp \frac{1}{Q} i\not{D}_\perp \frac{\not{n}}{2} \xi$$

Changes in arbitrary gauge

- Additional vertices for collinear-Glauber interaction

Will show

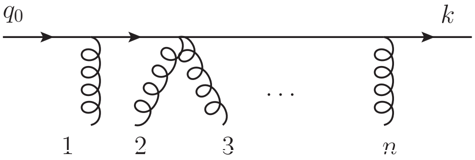
Sum over all possible interactions gives rise to a gauge invariant result

The gauge field at light-cone infinity

- The gluon field may be decomposed
  $A^i_\perp(x^+, x^-, x_\perp) =$
  $A^{\mathrm{cov},i}_\perp(x^+, x^-, x_\perp) + \theta(x^-) A^i_\perp(x^+, \infty, x_\perp) + \theta(-x^-) A^i_\perp(x^+, -\infty, x_\perp)$
  where the leading power comes from the terms at ∞
  Echevarria, Idilbi, Scimemi '11
- For $x^- \to \infty$ the field strength must vanish
  $\to A_\perp(x^+, \infty, x_\perp)$ is a pure gauge
  $A_\perp(x^+, \infty, x_\perp) = \nabla_\perp \phi(x^+, \infty, x_\perp)$
  $\phi(x^+, \infty, x_\perp) = \int_{-\infty}^0 ds\, l_\perp \cdot A_\perp(x^+, \infty, x_\perp + l_\perp s)$
  Belitsky, Ji, Yuan '02

Calculation

- Define the (amputated) diagram with n gluon interactions

$G_n(k) =$ 

- We can calculate this in a recursive fashion

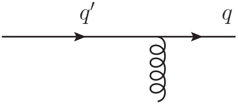
Calculation

- Decompose into fields at $\pm\infty$

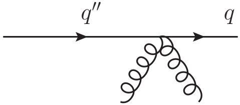
  $$G_n(k^-, k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4}\, G^+_{n-j}(k^-, k_\perp, q)\, \frac{iQ\,\not{\bar{n}}}{2Qq^+ - q_\perp^2 + i\epsilon}\, G^-_j(q)$$

  where $G^\pm$ contains only the gluon at $\pm\infty$
- The recursive definition of G^- is then

$$G^-_n(q) = \int \frac{d^4q'}{(2\pi)^4}\, G^-_{n-1}(q')$$



$$+ \int \frac{d^4q''}{(2\pi)^4}\, G^-_{n-2}(q'')$$



- and G^+_n correspondingly

Results

- Squaring the amplitude and summing over any number of gluon interactions, we find

$$
\begin{aligned}
P(k_\perp) &= \frac{1}{N_c} \int d^2x_\perp \, e^{ik_\perp \cdot x_\perp} \\
&\quad \left\langle \mathrm{Tr}\left[T^\dagger(0,-\infty,x_\perp)\, T(0,\infty,x_\perp)\; T^\dagger(0,\infty,0)\, T(0,-\infty,0) \right] \right\rangle
\end{aligned}
$$

- with
  $T(x_+, \pm\infty, x_\perp) = \mathcal{P}\, e^{ig \int_{-\infty}^{0} ds\; l_\perp \cdot A_\perp(x_+, \pm\infty, x_\perp + l_\perp s)}$
  the transverse Wilson line

Results

- Wilson lines in the perpendicular plane at $\pm\infty$ for light-cone gauge

Results

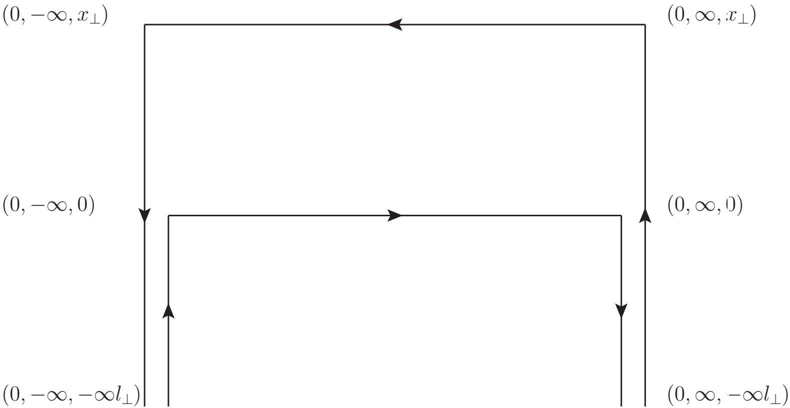
- Combining the results with the ones in covariant gauge we find

$$
\begin{aligned}
P(k_\perp) \;=\;& \frac{1}{N_c}\int d^2x_\perp\, e^{ik_\perp\cdot x_\perp} \\
& \big\langle \mathrm{Tr}\big[\, T^\dagger(0,-\infty,x_\perp)\, W_F^\dagger[0,x_\perp]\, T(0,\infty,x_\perp) \\
& T^\dagger(0,\infty,0)\, W_F[0,0]\, T(0,-\infty,0)\big]\big\rangle
\end{aligned}
$$

- The fields on the lower line are time ordered, the ones on the upper line anti-time ordered

  $\rightarrow$ Use Keldysh-Schwinger contour in path integral formalism

Results


$(0, -\infty, x_\perp)$
$(0, \infty, x_\perp)$
$(0, -\infty, 0)$
$(0, \infty, 0)$
$(0, -\infty, -\infty l_\perp)$
$(0, \infty, -\infty l_\perp)$

Conclusions

- SCET$_G$ is a suitable framework for the calculation of gauge invariant results in jet quenching
- The jet quenching parameter $\hat{q}$ can be expressed as the medium average of two longitudinal and four transverse Wilson lines
- The operators are ordered along a Keldysh-Schwinger contour
- The results may be used to determine $\hat{q}$ in different frameworks also using lattice computations (see Parallel IIIB tomorrow)

  work in progress...

Thank you for your attention!

Bonus Slides

Soft-Collinear Effective Theory

- A jet originates from the fragmentation of a parton with high energy E_j and a much smaller invariant mass $m_j = \sqrt{p_j^2}$
  $\rightarrow$ Almost light-like $\rightarrow$ use light-cone coordinates

$$p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp^\mu \quad \text{with } n,\ \bar{n} \text{ light-cone vectors}$$
$$n\cdot p \sim E_j \gg p_\perp,\ \bar{n}\cdot p$$

- Introduce a **scaling parameter** $\lambda \ll 1$
  $n\cdot p \sim E_j \quad m_j \sim \lambda E_j$
  $p^2 = n\cdot p\,\bar{n}\cdot p + p_\perp^2$
  $\rightarrow p_\perp \sim \lambda E_j,\ \bar{n}\cdot p \sim \lambda^2 E_j^2$
- Jet momentum p_j scales like $(n\cdot p, \bar{n}\cdot p, p_\perp) \sim (\lambda^2, 1, \lambda)$
  "hard-collinear"

Scaling of the Glauber field

- Consider the form of the effective Glauber field

$$A^\mu(x) = \int d^4y \, D_G^{\mu\nu}(x-y) \, f_\nu(y)$$

$$D^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \, \frac{-i}{k^2 + i\epsilon} \left(g^{\mu\nu} - \frac{k^\mu \bar{n}^\nu + k^\nu \bar{n}^\mu}{[k^+]} \right) e^{-ik(x-y)}$$

- Source f_ν only knows about the soft scale $\sim \lambda^3$