Linear vs non-linear QCD evolution: from HERA data to LHC phenomenology

Paloma Quiroga Arias
LPTHE, UPMC UNIV. Paris VI & CNRS

[Javier L Albacete, Guilherme Milhano and Juan Rojo]

arXiv:1203.1043[hep-ph]

Hard Probes 2012, Cagliari

Proton partonic structure - QCD evolution: linear vs non-linear

- Scale dependence of parton distribution functions - two different QCD approaches
  - Q^2 dependence: DGLAP evolution equations $\left(\sim \alpha_s \ln \frac{Q^2}{Q_0^2}\right)$
    applicable in collinear factorization
  - small x evolution: BFKL $\left(\sim \alpha_s \ln \frac{x_0}{x}\right) \xrightarrow[\text{non linear terms}]{}$ BK-JIMWLK equations - BK + running coupling

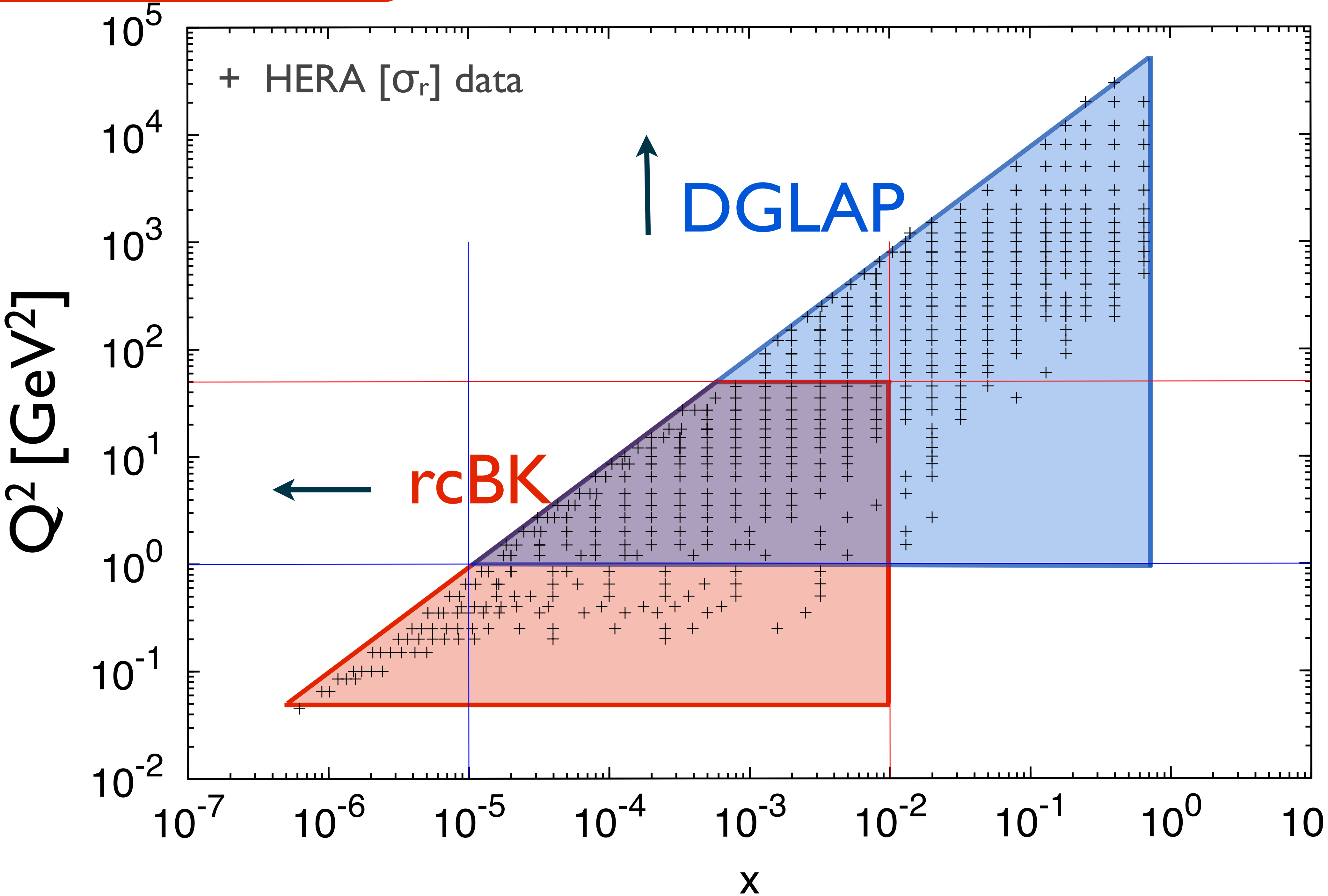
- Region of applicability of the two orthogonal approaches
  - DGLAP approach: $x > 10^{-5}$, $Q^2 > Q_0^2 \sim 1$ GeV^2
  - running coupling BK (rcBK) fits: $x < 10^{-2}$, $Q^2 < 50$ GeV^2

- DGLAP linear evolution eqs. provide accurate description of data [so does rcBK]
  - legitimate question: flexibility of i.c. hiding some interesting QCD dynamics [non-linear behavior]?
  - recent NNPDF [no i.c. bias] fits find deviations w.r.t. low x data excluded from fits

Kinematic range - data & theory

- DGLAP: $x>10^{-5}$, $Q^2 > Q_0 \sim 1\text{-}4$ GeV^2
- rcBK: $x<10^{-2}$, $Q^2 < 50$ GeV^2

both approaches **coexist** in a region



linear approach - DGLAP

- DGLAP evolution equation for vector PDFs f(x,Q^2):

$$\boxed{\frac{\partial f(x,Q^2)}{\partial \ln(Q^2/Q_0^2)} = \int_x^1 \frac{dy}{y} P\left(\alpha_s(Q^2), x/y\right) f(y,Q^2)\Big)}$$

linear equation

- Provides evolution to large Q^2 and has no predictive power in the orthogonal x-direction [values of $x \leq x_{min}$ DGLAP predictions become unreliable]

x_{min} = lowest value of x from experimental data

- Initial conditions: specify the PDFs at some low initial scale for all values of x

$$\boxed{xf(x, Q^2 = Q_0^2)}$$

NNPDF approach: initial conditions parametrized with artificial neutral networks

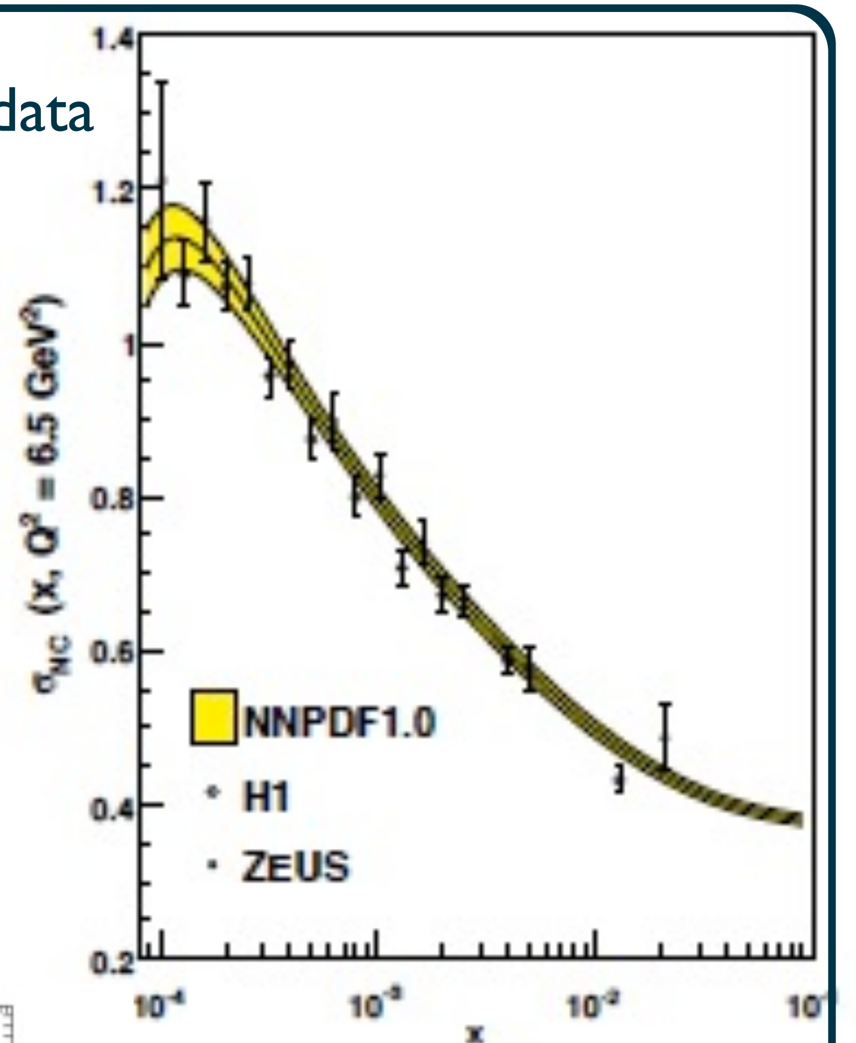
[avoid theoretical biases of choosing a particular functional form for the input PDF]

- Linear equation => expected to break for sufficiently small values of Q^2

[gluon densities are higher => higher twists important]

linear approach - DGLAP

- Historically: for many years has provided excellent description of data
- NNPDF implementation (MC based):
  - very sophisticated fitting technology [error propagation]

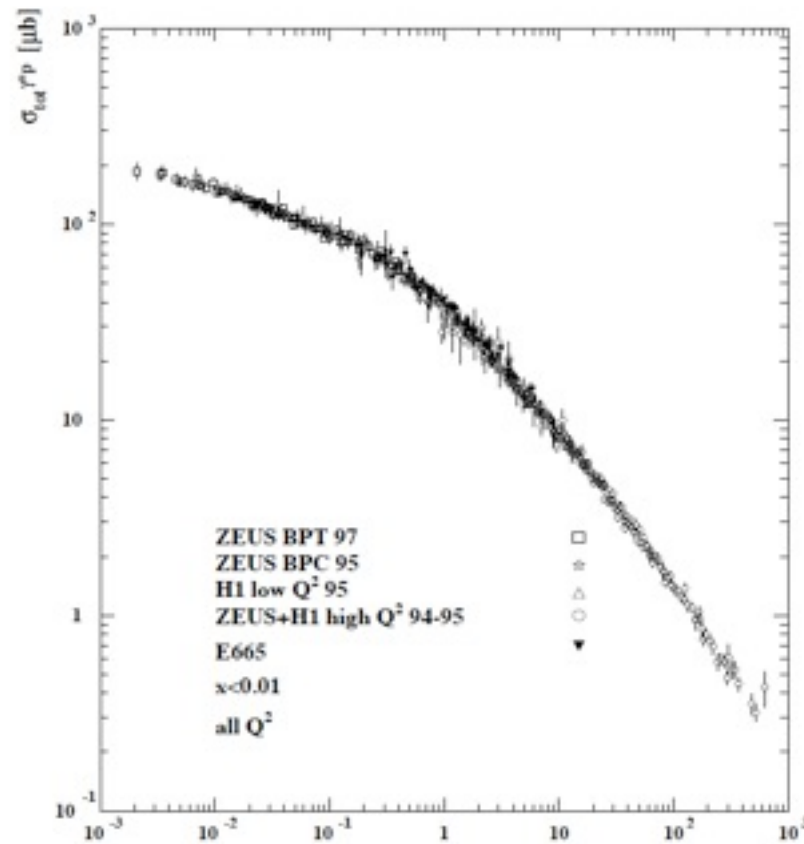


- Recently: studies show deviations [PLB:686,2010, F.Caola, S.Forte, J.Rojo]
- Difficulty accommodating some phenomena

e.g. geometric scaling

$$\sigma^{\gamma^* p}(x,Q) = \sigma^{\gamma^* p}(\tau), \;\; \tau = \log\left(\frac{Q^2}{Q_s^2(x)}\right)$$

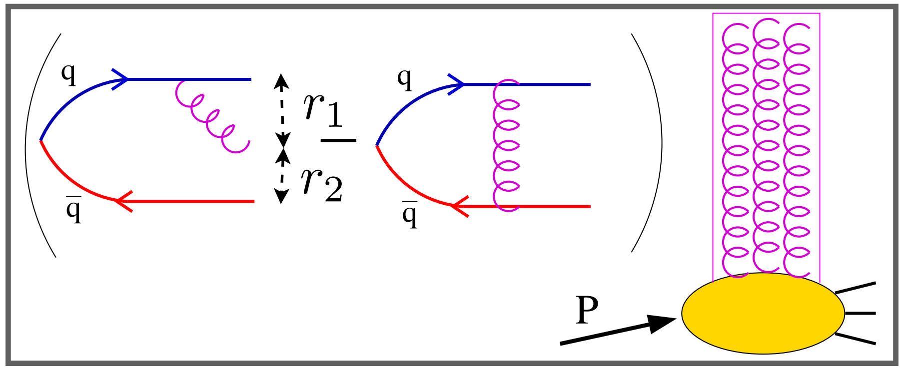
[can be accommodated but no strong theoretical argument]



non-linear approach - running coupling BK

- rcBK evolution equation for scattering amplitude of q-qbar color dipole with hadronic target:

$$\boxed{\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)} = \int d^2 r_1 \mathcal{K}^{run}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2)[\mathcal{N}(r_1,x) + \mathcal{N}(r_2,x) - \mathcal{N}(r,x) - \mathcal{N}(r1,x)\mathcal{N}(r_2,x)]}$$



non-linear equation

[change of hadron structure as smaller values of x are probed]

- Provides evolution in Bjorken-x. No predictive power in Q^2

- Onset of black-disk limit: $\boxed{\mathcal{N}(r_s = 1/Q_s(x), x) = \kappa \sim 1}$ [def. saturation scale $Q_s(x)$]

- Non-linear equation [non-linear terms required by unitarity preservation. Gluon recombination]

- Applicable for very small values of Q^2

Physical interpretation of dipole amplitude $\mathcal{N}$ $\xrightarrow{\text{F.T.}}$ $\boxed{\phi(x,k_t) = \int d^2 r e^{-i\vec{r}\vec{k}_t} \mathcal{N}(r,x)}$ $\xrightarrow{\text{LO}}$ $\boxed{xg(x,Q^2) = \int^{Q^2} d^2 k_t \phi(x,k_t)}$

UGD

integrated gluon distribution

non-linear approach - rcBK

- Similarly good fits to DGLAP + naturally accommodates geometric scaling

Stasto, Golec-Biernat, Kwiecinski arXiv:0007.192[hep-ph]

- AAMQS implementation: does a very good job describing HERA data

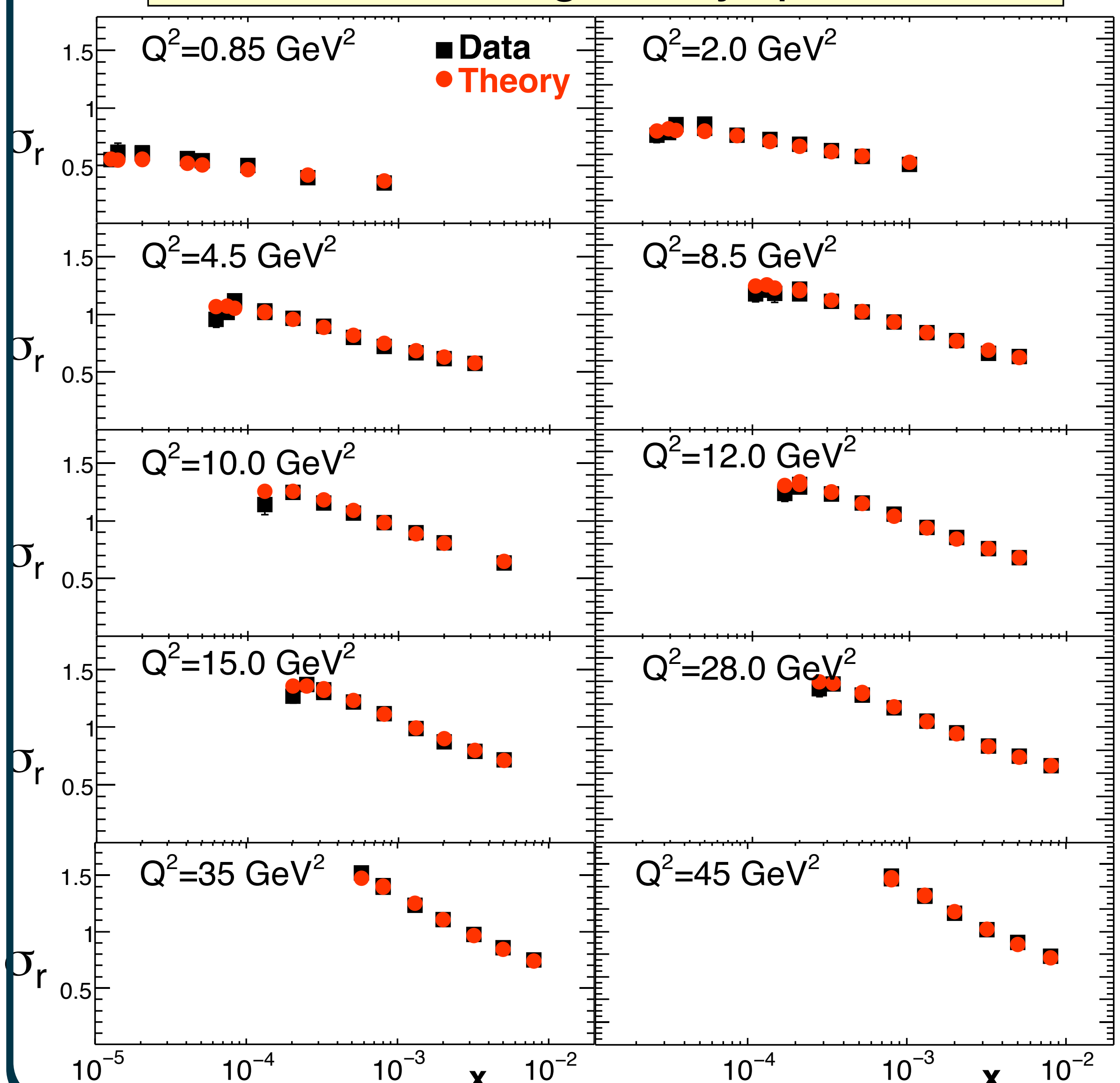
arXiv:1012.4408
arXiv:0902.1112

global fits to HERA e-p data (4 free parameters): calculate σ_r and F_2 according to the dipole model with small-x dependence described by rcBK equation. MV initial condition for the dipole amplitude

Albacete, Armesto, Milhano, Quiroga, Salgado

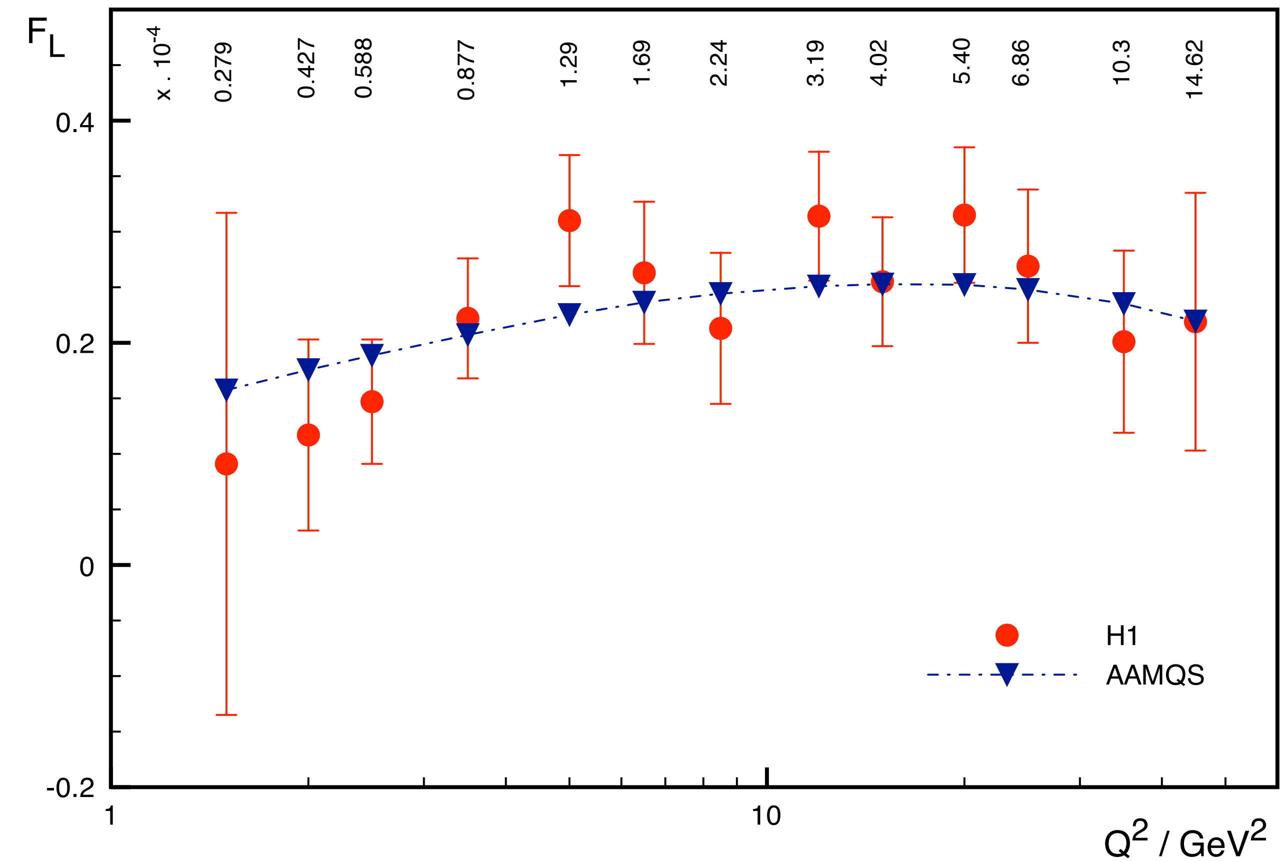
especially latest data (combined H1-ZEUS analysis) quite challenging!

Fit including heavy quarks



AAMQS calculation of F_L vs latest data

[independent test of the method]



there is some non-linear physics going on here

Interplay between the two approaches

- applicability of both theories based on purely theoretical arguments: asymptotic limits
    - DGLAP: large Q^2
    - rcBK: low x

  $\left.\begin{array}{l}\text{DGLAP: large } Q^2 \\ \text{rcBK: low } x\end{array}\right\}$ unclear in the intermediate kinematic region

- in the intermediate region agreement with data necessary but not sufficient

- **Pertinent question**: "are corrections to the limit in which both theories are well defined important in the intermediate region?"
    - is the flexibility of initial conditions in DGLAP masking the presence of some underlying physics (like saturation)?
    - is x_0=0.01 small enough for the dipole model of AAMQS (rcBK) to be applicable?

- need for systematic studies comparing both approaches
    - check stability of both approaches under changes of the boundary conditions

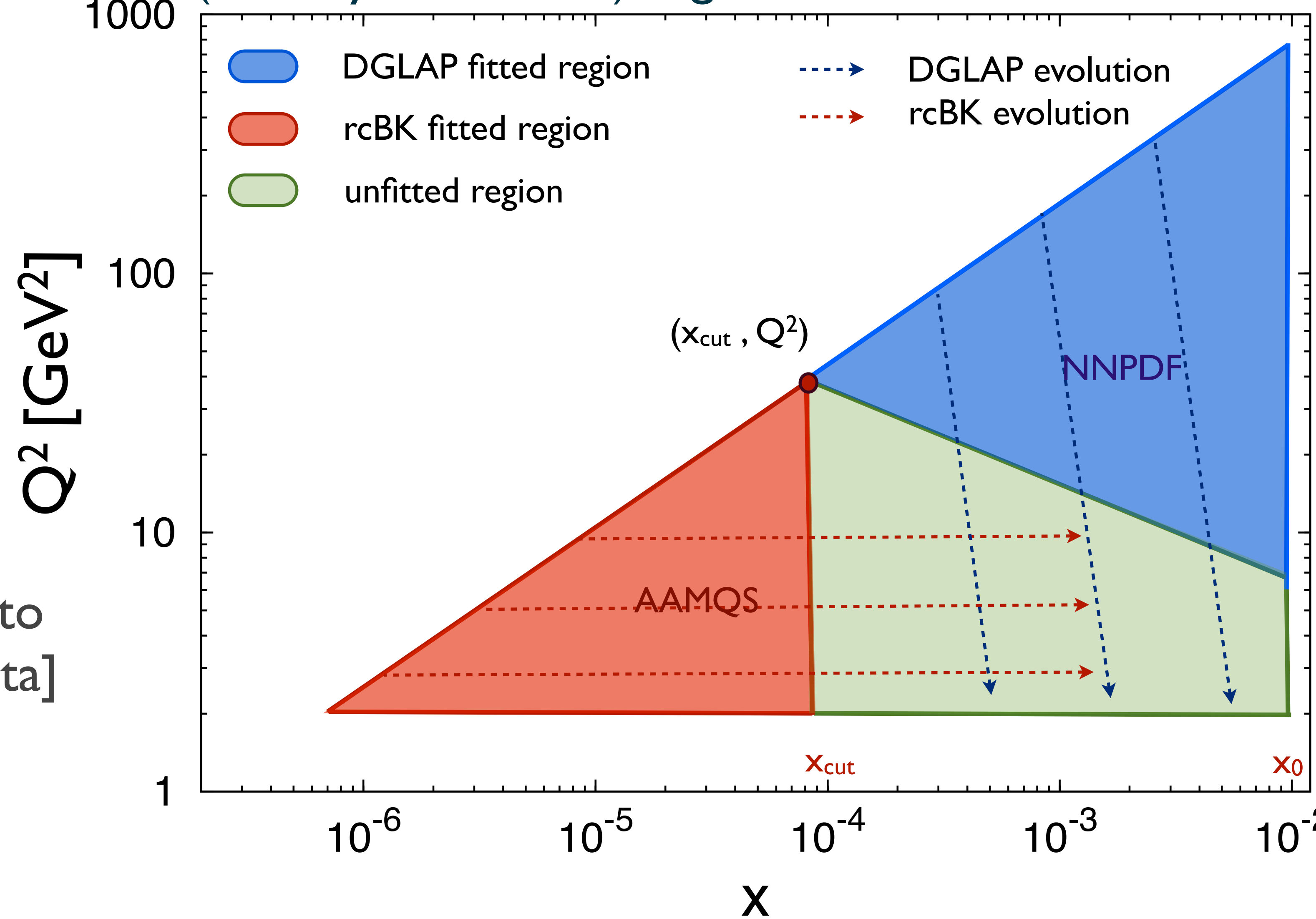
Strategy

[PLB:686,2010, F.Caola, S.Forte, J.Rojo]

- Fit to a subset of data in a reduced kinematic regime [specific to each approach]
- Then extrapolated to the common unfitted (causally connected) region

Test the evolution NOT the choice of initial conditions

[No assumptions on i.c.: only evolve to points where all i.c. info is given by data]



- NNPDF: fit large Q^2 region - backwards evolution towards smaller Q^2

saturation inspired cut $Q^2 > Q_{cut}^2 = A_{cut} x^{-\lambda}$

- AAMQS: fit small x region - use resulting dipole parametrization to predict at larger x

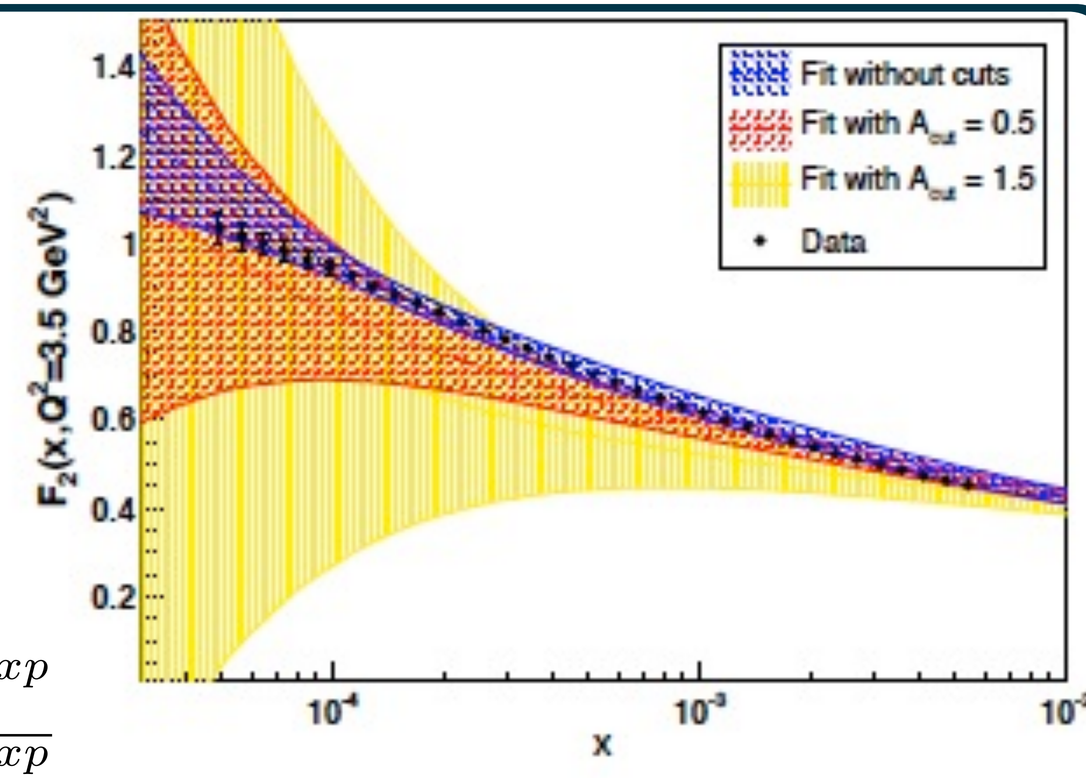
$x < x_{cut} < 0.01$

(Non-linear?) deviations from NLO DGLAP evolution

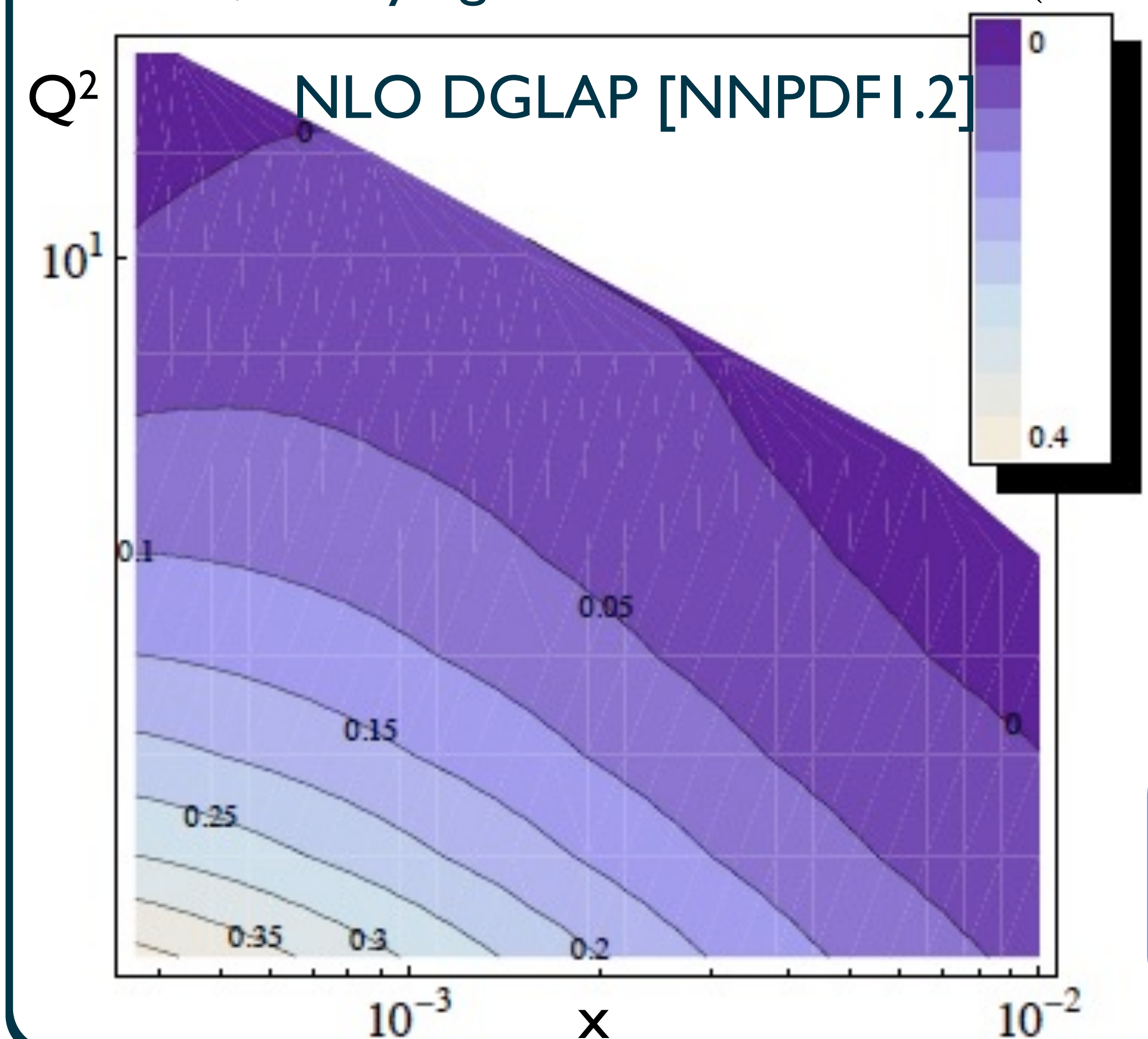
Caola, Forte, Rojo, PLB 686, 2010

- NNPDF: fits with cuts $Q^2 > Q_{cut}^2 = A_{cut}x^{-\lambda}$

fits tend to systematically underestimate the data



- Quantifying the deviations $$d_{rel}(x,Q^2) = \frac{F_2^{th} - F_2^{exp}}{F_2^{th} + F_2^{exp}}$$



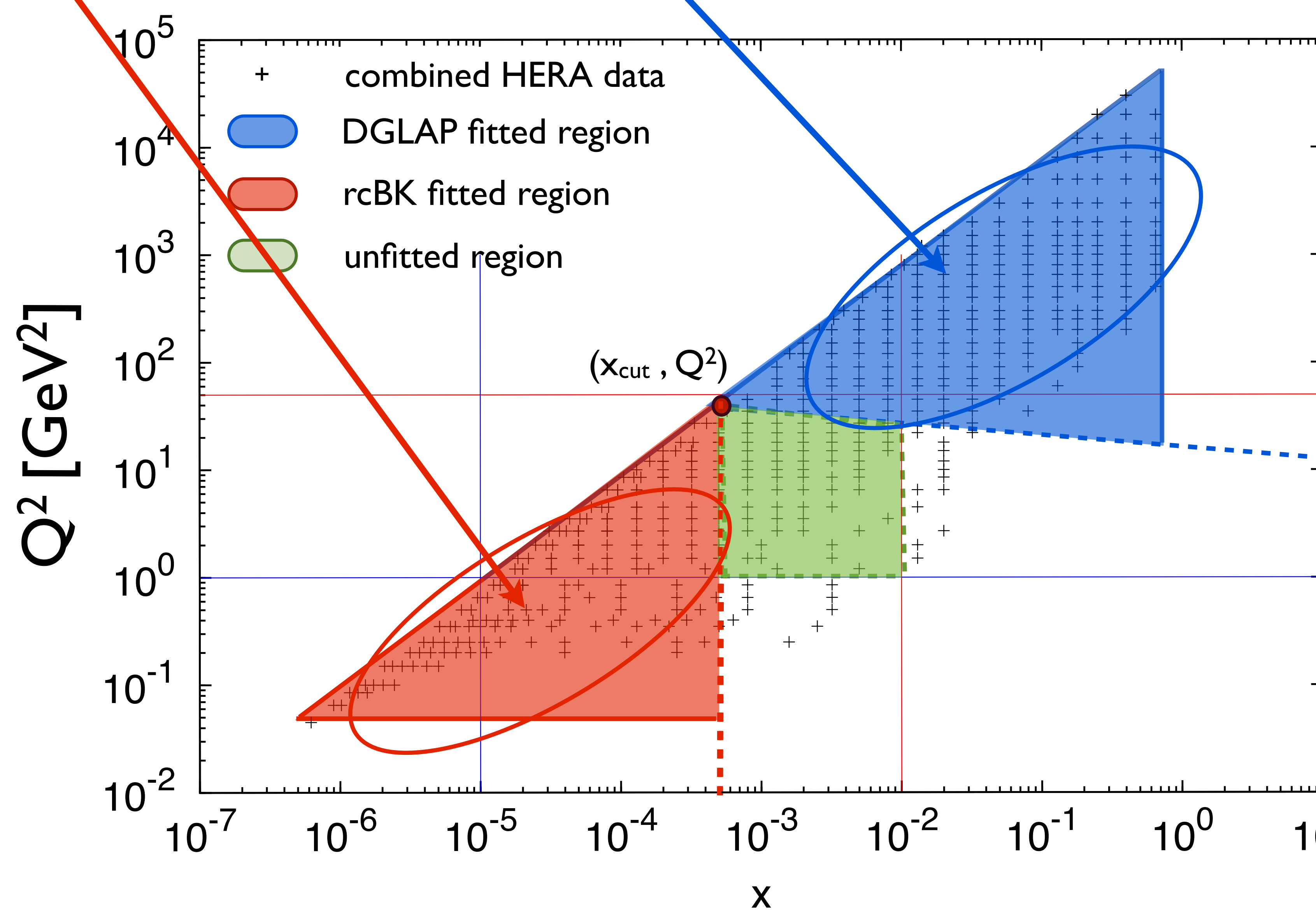
NLO DGLAP: deviations as large as 35% !!
[at low x and low Q^2]

- not corrected by
  - NNLO corrections
  - improved treatment of heavy quark effects

Hints of physics effects beyond the dynamical content of DGLAP evolution equation in the intermediate kinematical region (**non-linear effects?**)

Strategy - data cuts

- DGLAP-NNPDF cuts: $Q^2 > Q^2_{cut} = A_{cut}\, x^{-1/3}$: $A_{cut}=1.5$
- rcBK-AAMQS cuts: $x < x_{cut} = 3\times10^{-3}, 1\times10^{-3}, 3\times10^{-4}, 1\times10^{-4}$
- Comparison of extrapolation from both formalisms to **same** data in unfitted region

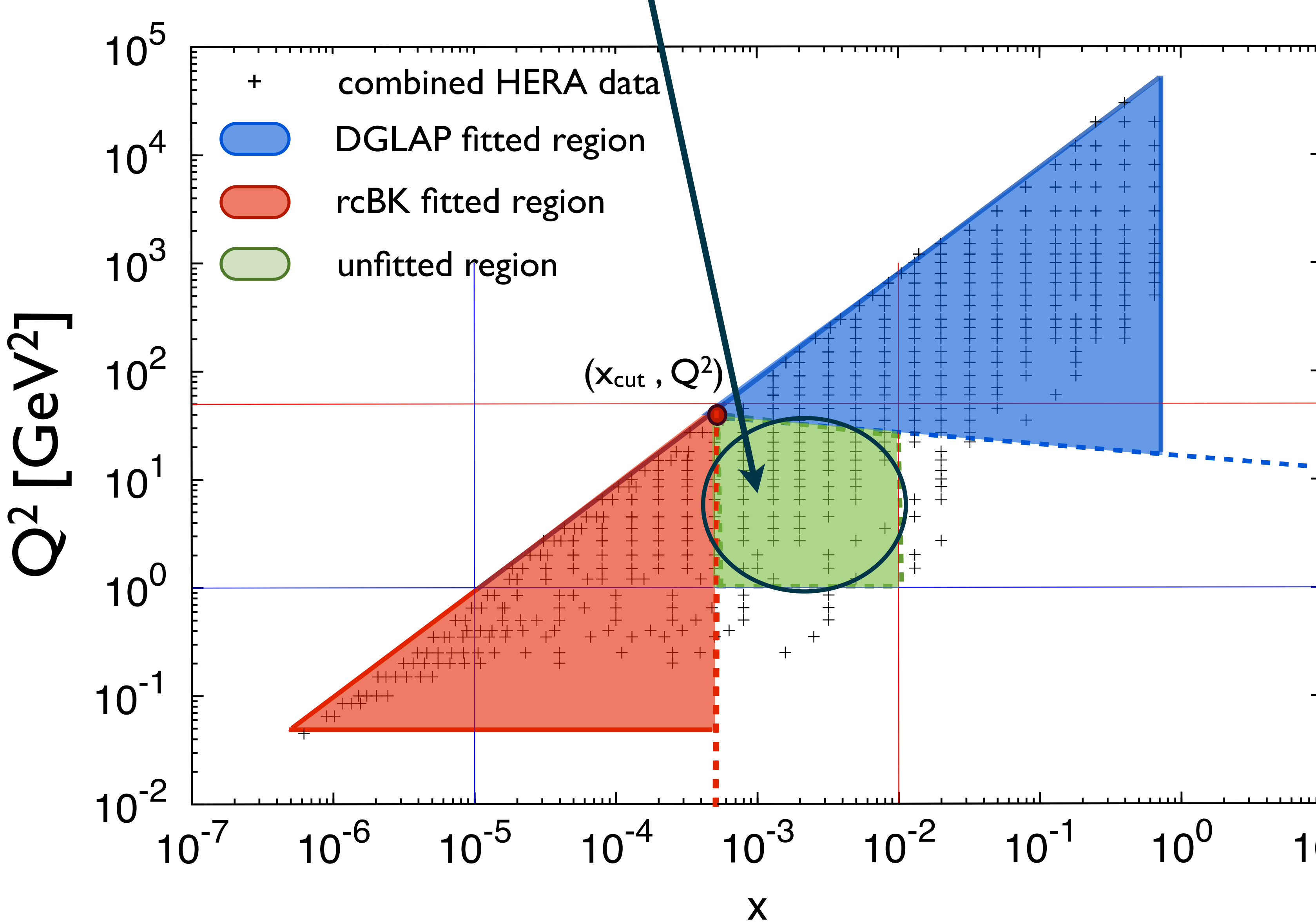


Strategy - data cuts

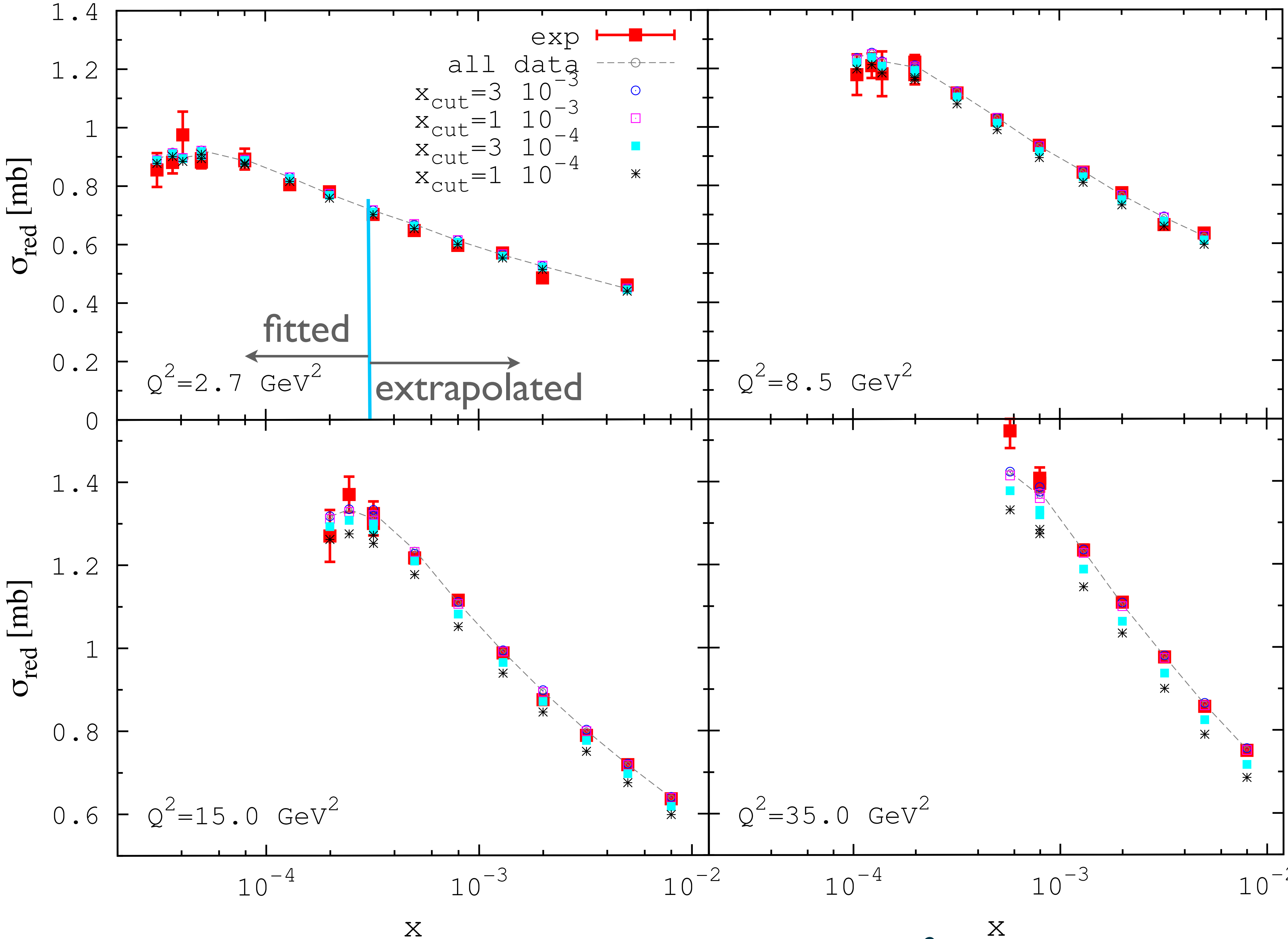
- DGLAP-NNPDF cuts: $Q^2 > Q^2_{cut} = A_{cut}\, x^{-1/3}$: $A_{cut}=1.5$: [59 HERA data points in unfitted region]

- rcBK-AAMQS cuts: $x < x_{cut} = 3\times10^{-3}, 1\times10^{-3}, 3\times10^{-4}, 1\times10^{-4}$

- Comparison of extrapolation from both formalisms to **same** data in unfitted region



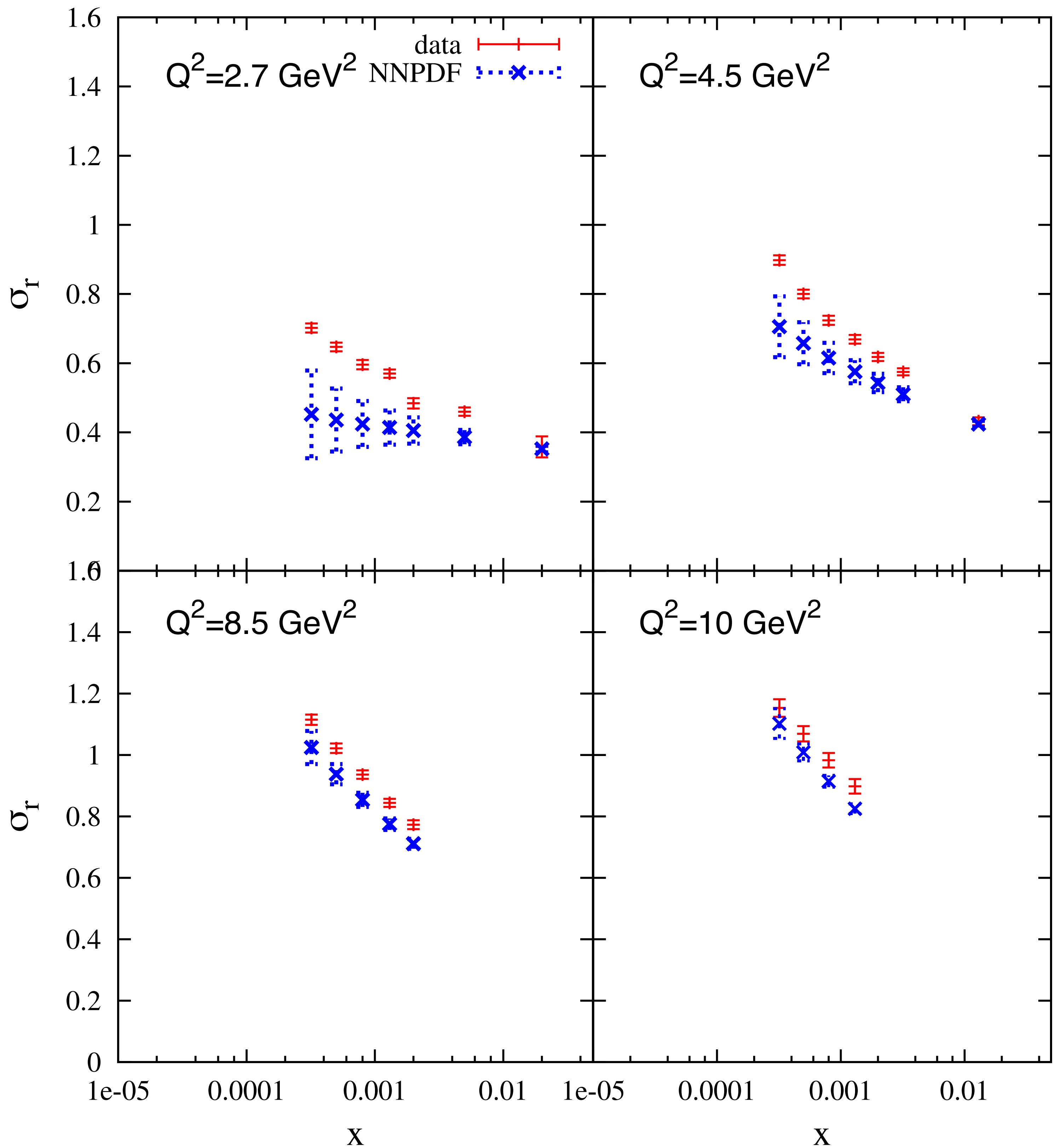
results - rcBK AAMQS different cuts



- Deviations increase with decreasing x_{cut} and increasing Q^2. MAKES PERFECT SENSE
- rcBK (AAMQS) fits: stable under changing boundary condition
  - non-linear small-x dynamics describes scale dependence of the proton structure in the intermediate (x,Q^2)

results NNPDF - NLO DGLAP

- NLO DGLAP - NNPDF extrapolation to the common unfitted region



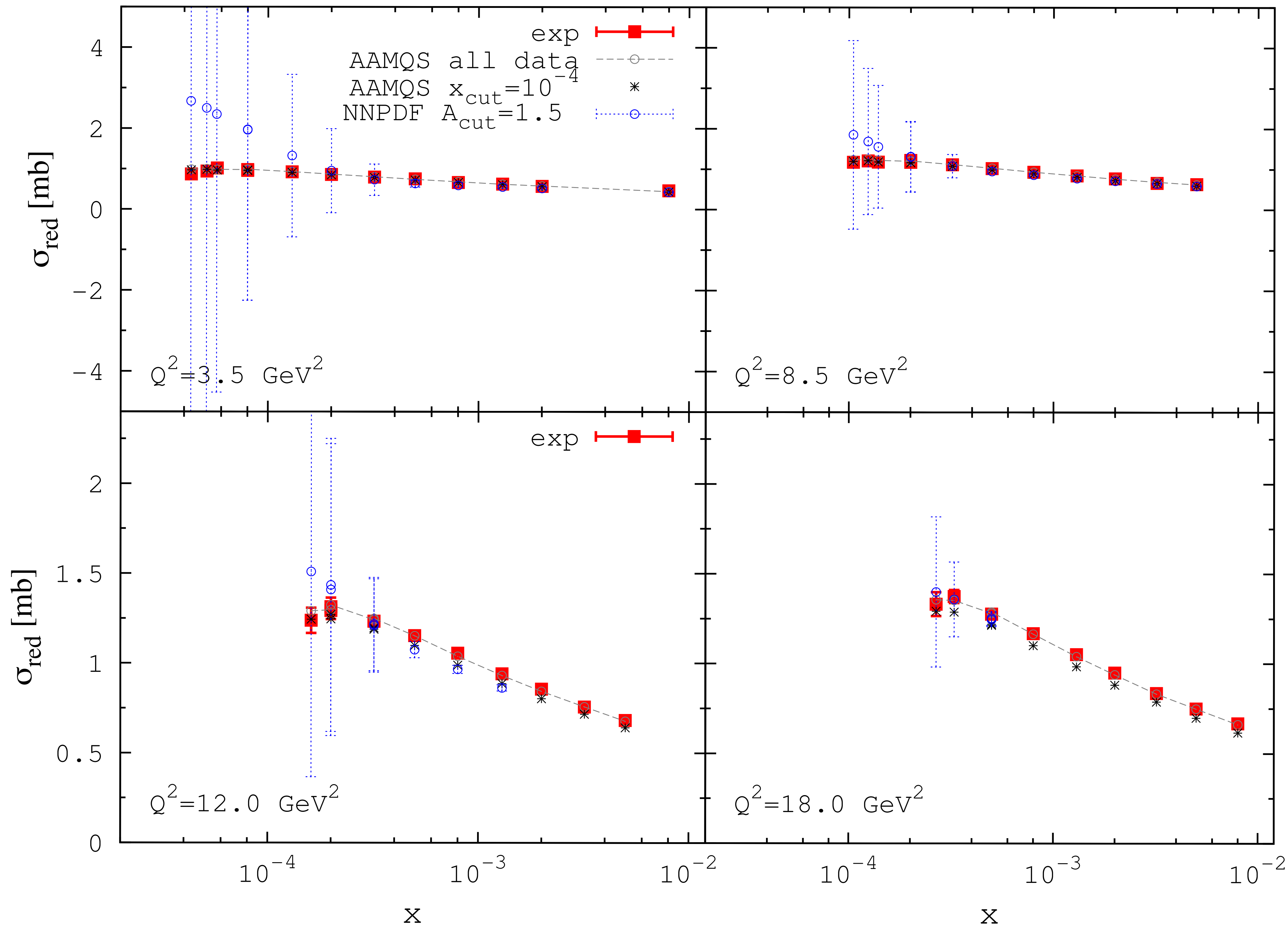
NLO DGLAP [NNPDF1.2]

A_{cut}=1.5

deviation from data at low x and low Q^2

results NNPDF - NNLO DGLAP with heavy quarks

NNLO DGLAP [NNPDF2.1] includes heavy quarks



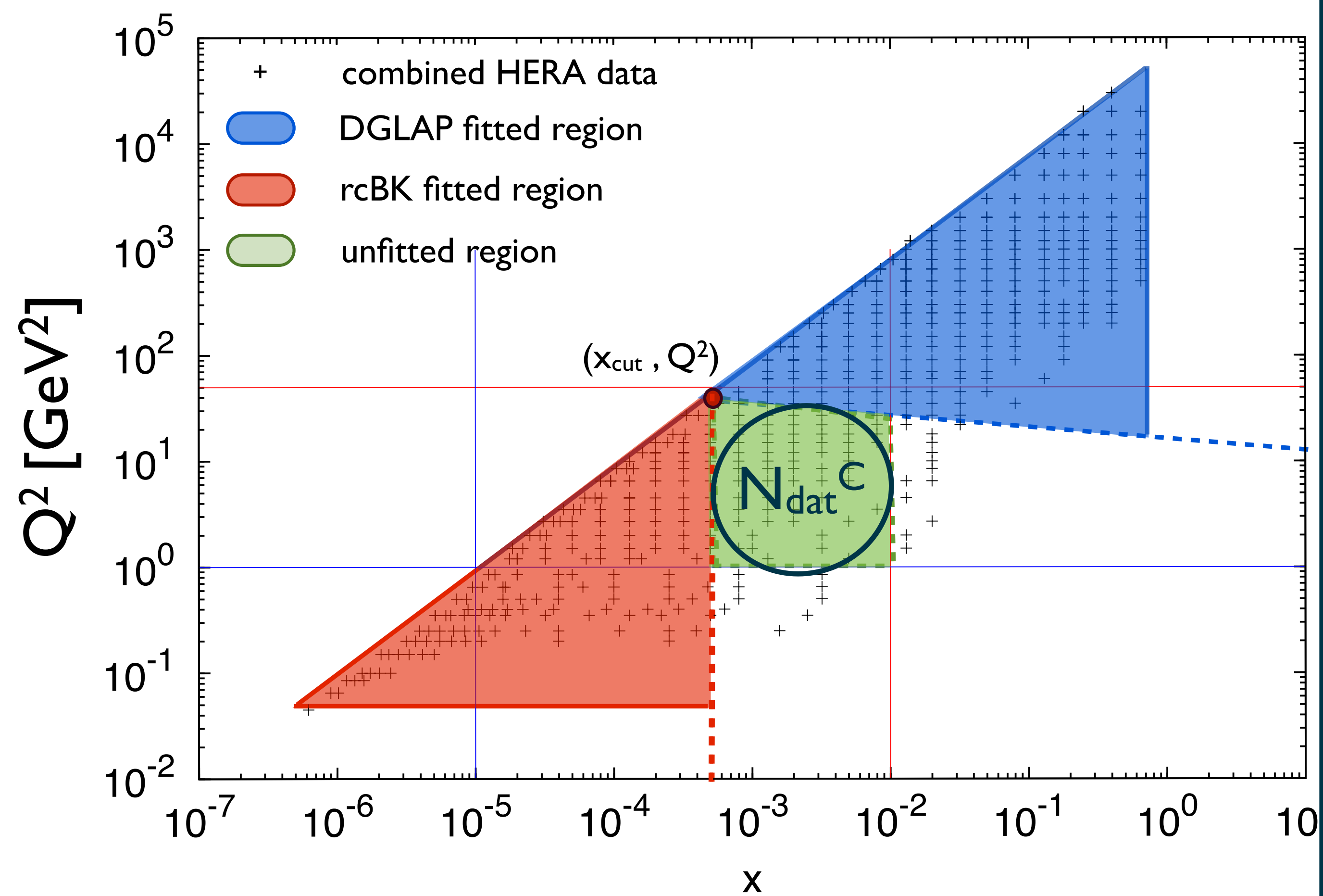
results - all fits

[rcBK] AAMQS

x_{cut}	N_{dat}	N^C_{dat}
$1 \cdot 10^{-2}$	271	0
$3 \cdot 10^{-3}$	237	34
$1 \cdot 10^{-3}$	205	66
$3 \cdot 10^{-4}$	148	123
$1 \cdot 10^{-4}$	105	166

no cut

$x < 10^{-2}, Q^2 < 50\mathrm{GeV}^2$



[DGLAP] NNPDF

A_{cut}	N_{dat}	N^C_{dat}	N^D_{dat}	$(x_{min}, Q^2\ [\mathrm{GeV}^2])$
no cuts	3372	0	0	$(4.1 \cdot 10^{-5}, 2.5)$
0.2	3363	4	5	$(8 \cdot 10^{-5}, 3.5)$
0.3	3350	14	8	$(10^{-4}, 6.5)$
0.5	3333	25	15	$(1.4 \cdot 10^{-4}, 8.5)$
0.7	3304	38	16	$(1.6 \cdot 10^{-4}, 12)$
1.0	3228	44	19	$(2.1 \cdot 10^{-4}, 15)$
1.2	3164	53	30	$(2.4 \cdot 10^{-4}, 15)$
1.5	3084	59	38	$(2.7 \cdot 10^{-4}, 20)$

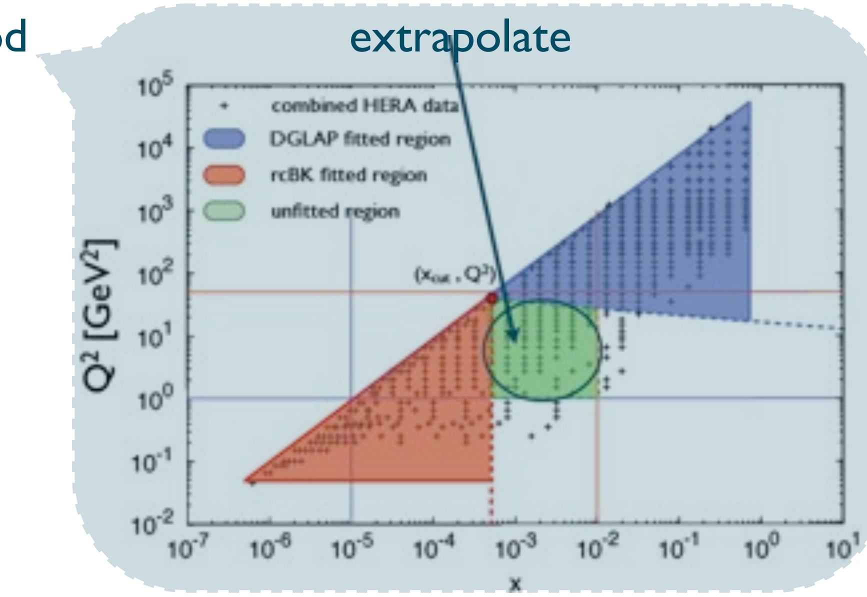
N_{dat}=data included in the fit

N^C_{dat}=data in the causally connected region

N^D_{dat}=data in the disconnected region

results - measuring the deviations

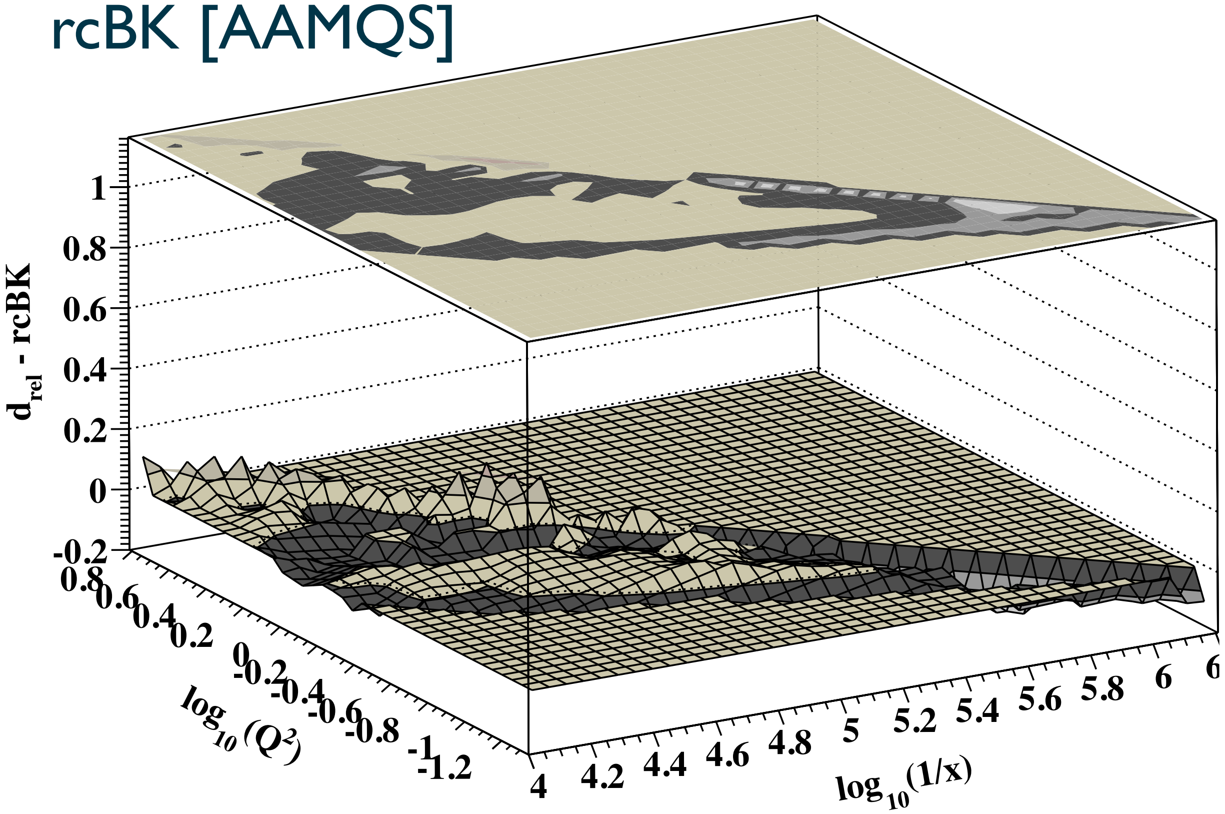
- Relative distance between theoretical and experimental results: measures the absolute size of deviations



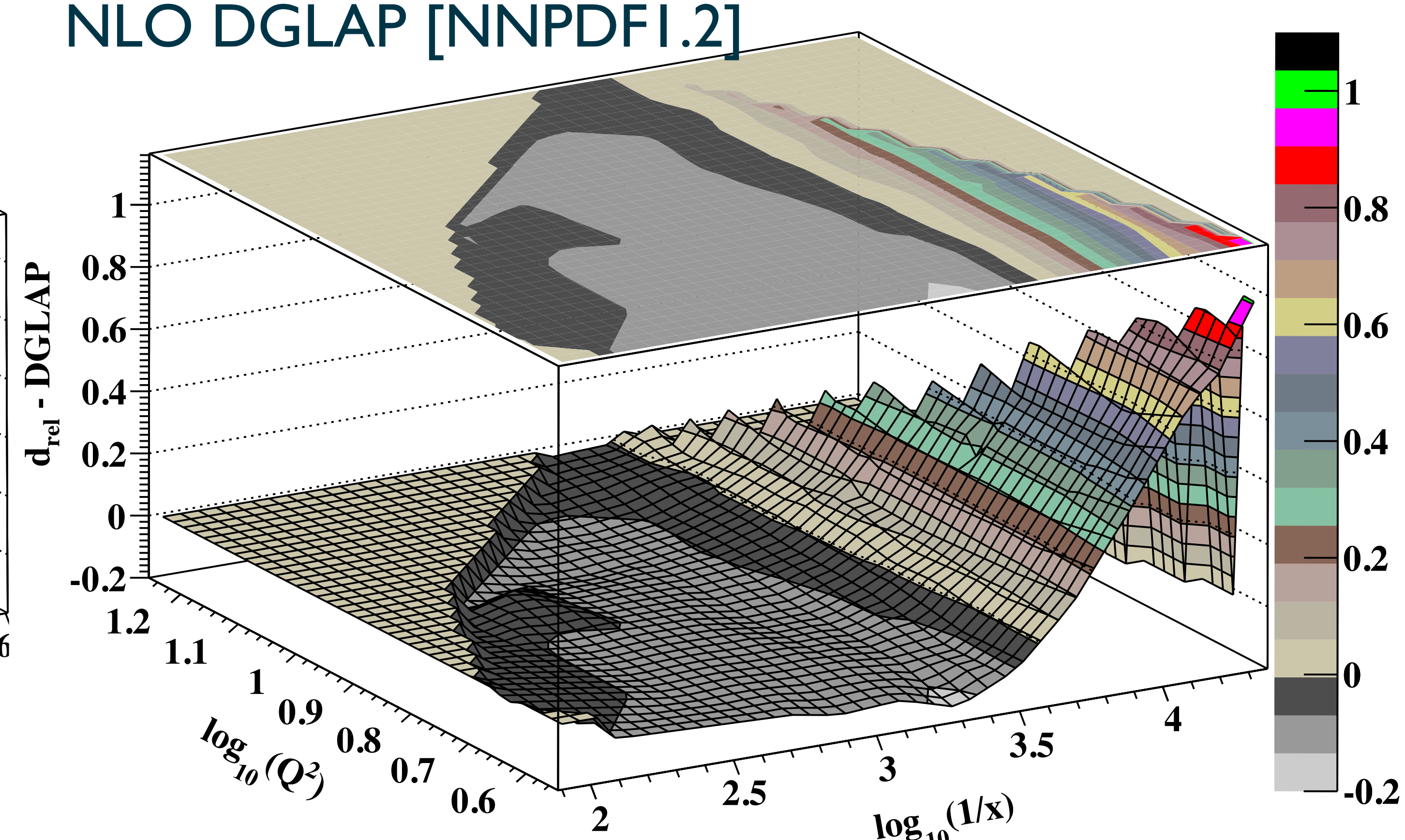
$$d_{rel}(x,Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\frac{1}{2}\left(\sigma_{r,th} + \sigma_{r,exp}\right)}$$

fit with $x_{cut} = 10^{-4}$, $A_{cut} = 1.5$

rcBK [AAMQS]



NLO DGLAP [NNPDF1.2]



small deviations & alternate in sign in all unfitted region

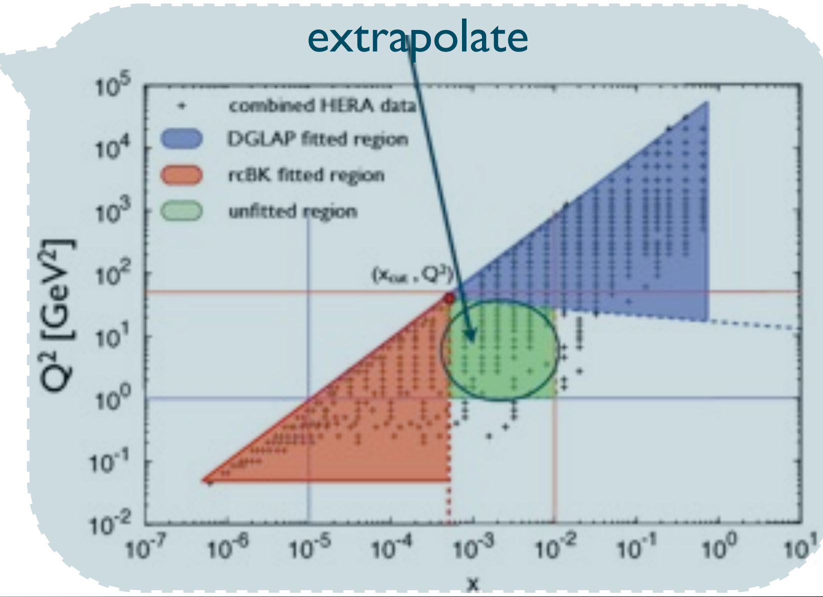
Systematic trend to underestimate small-x data and overshoot at larger-x

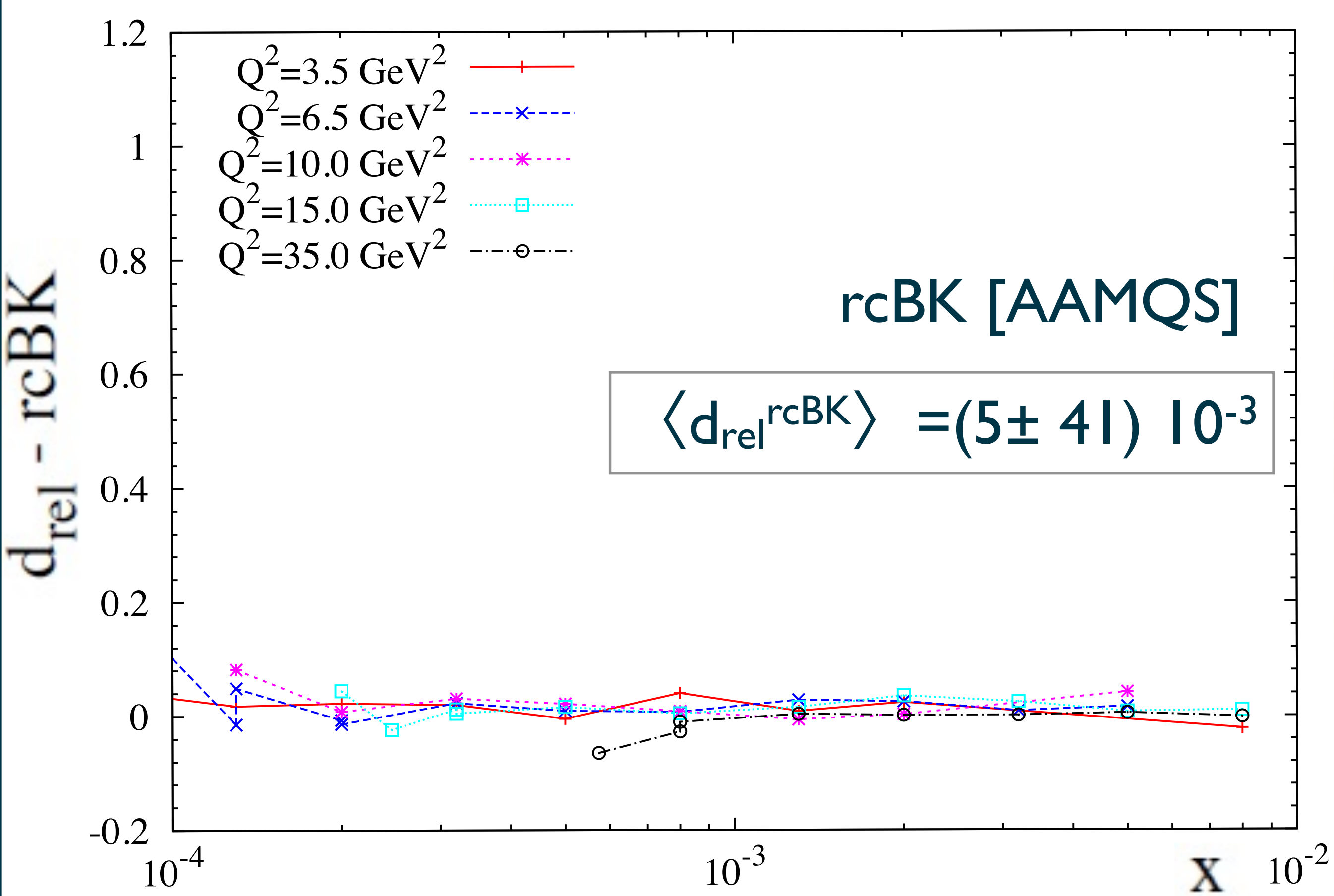
results - measuring the deviations

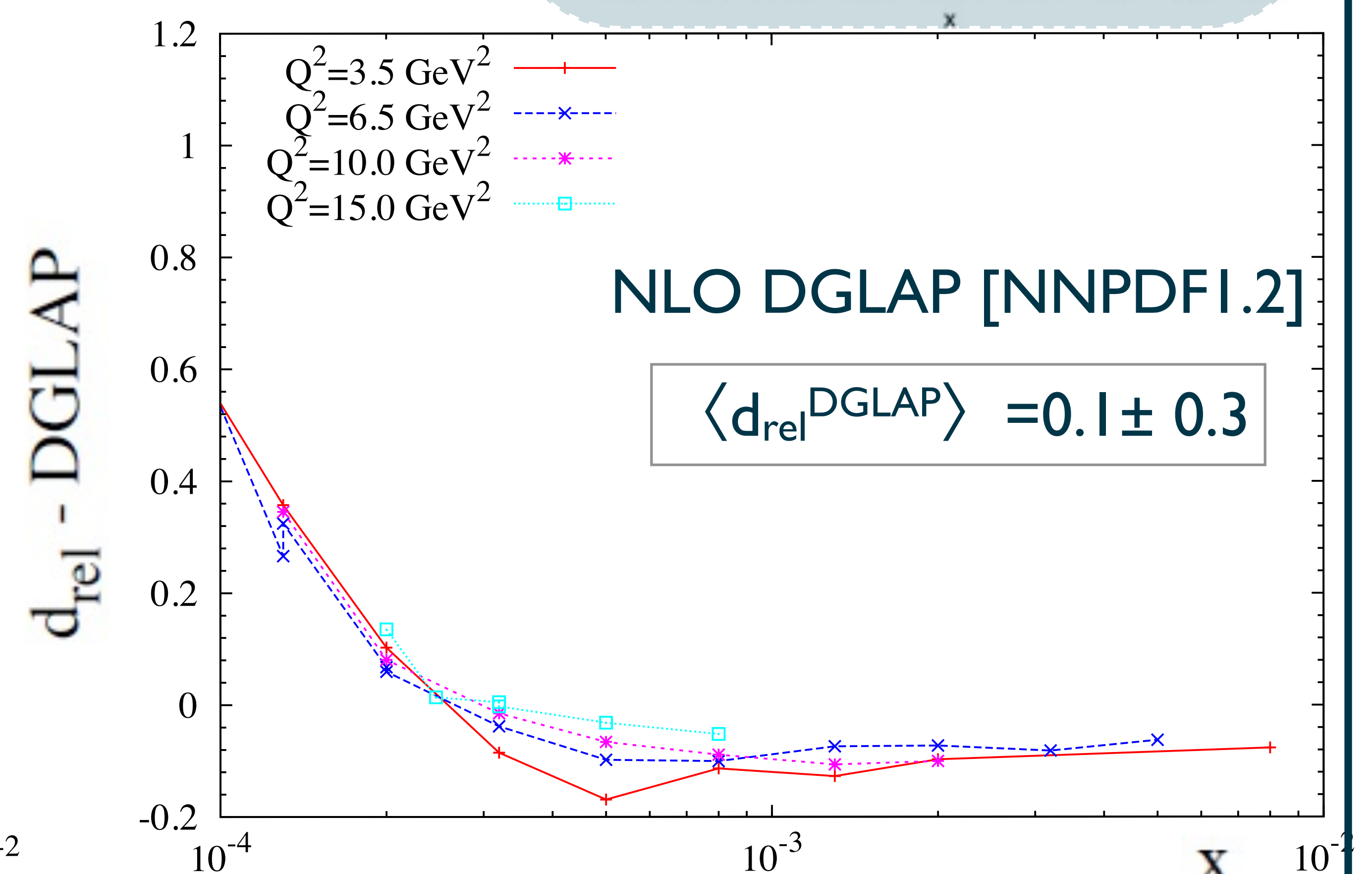
- Relative distance between theoretical and experimental results: measures the absolute size of deviations

$$d_{rel}(x,Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\frac{1}{2}\left(\sigma_{r,th} + \sigma_{r,exp}\right)}$$

method







small deviations & alternate in sign in all unfitted region

Systematic trend to underestimate small-x data and overshoot at larger-x

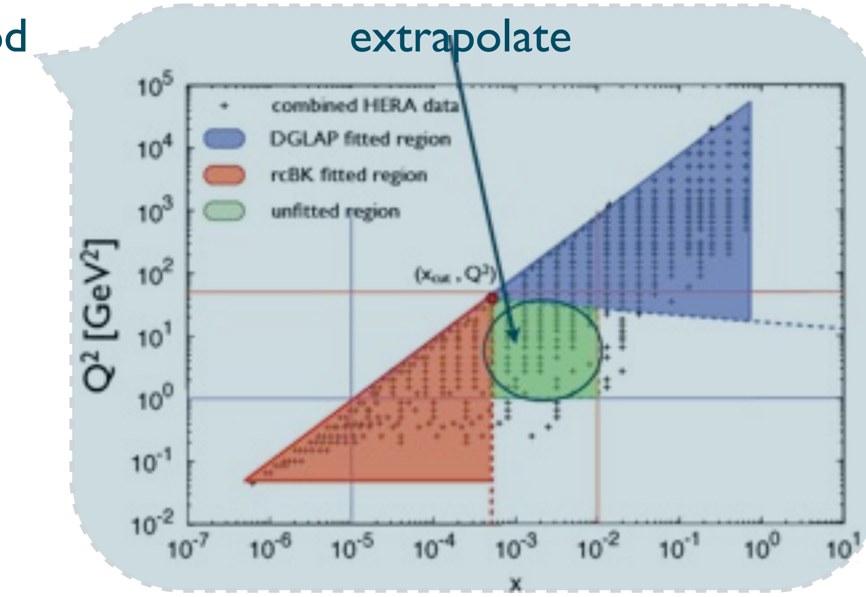
results - measuring the deviations

- Statistical distance between theoretical and experimental results: measures statistical significance of the deviation in units of standard deviation

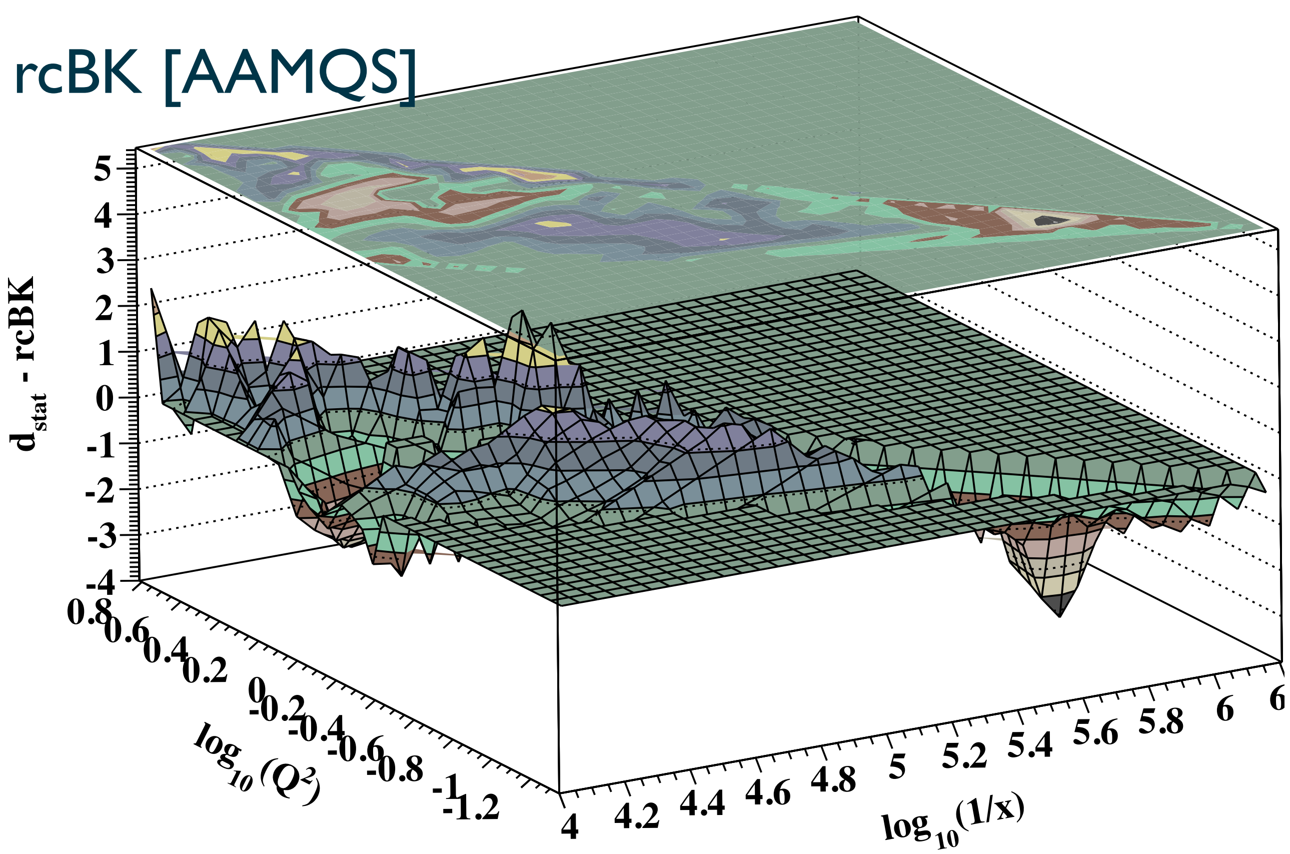
$$d_{stat}(x, Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\sqrt{\left(\Delta\sigma^2_{r,th} + \Delta\sigma^2_{r,exp}\right)}}$$

meaningless when large theory errors

method



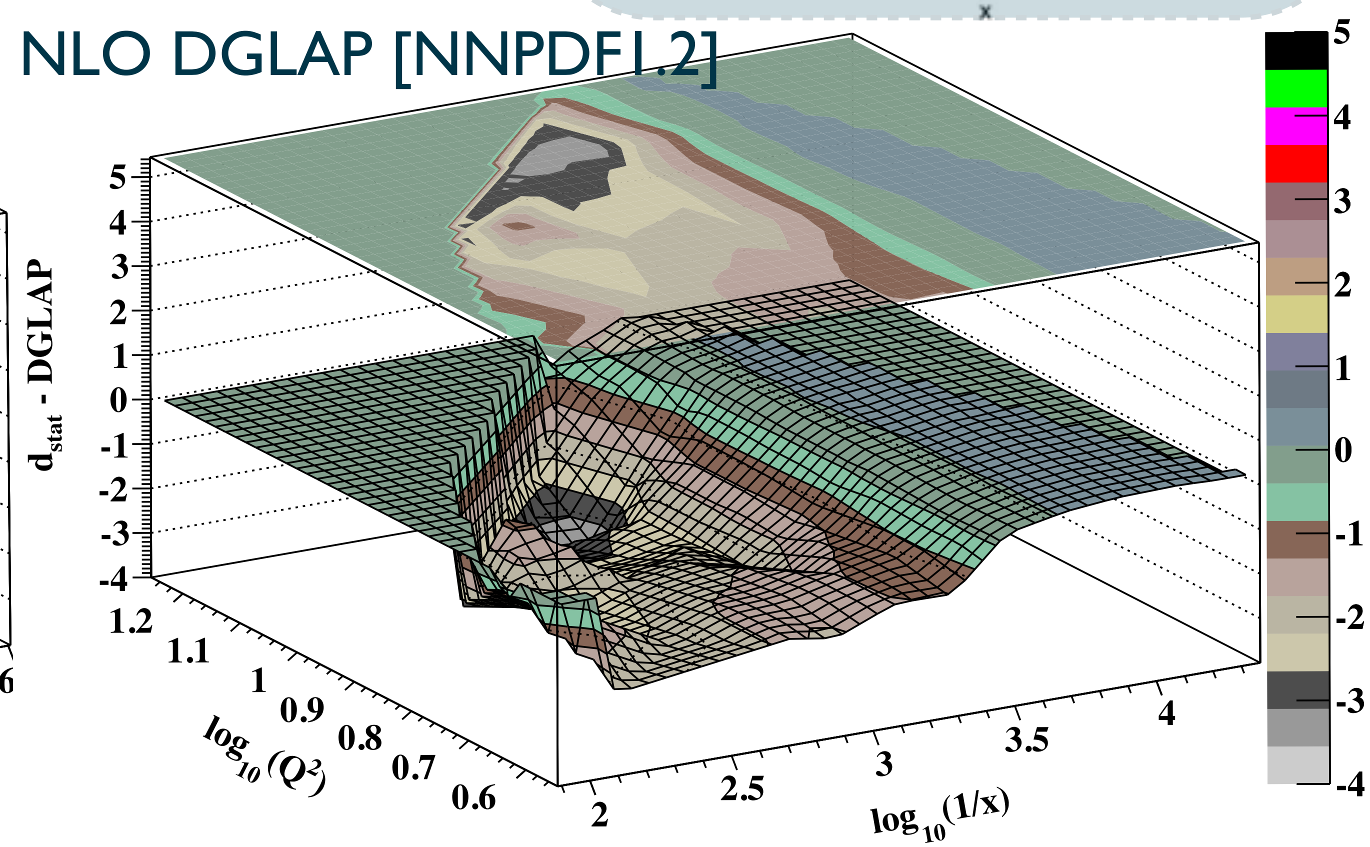
rcBK [AAMQS]



theoretical errors underestimated

⟨d_{stat}^{rcBK}⟩ =0.3±9

NLO DGLAP [NNPDF1.2]



⟨d_{stat}^{DGLAP}⟩ =-0.8± 1.1

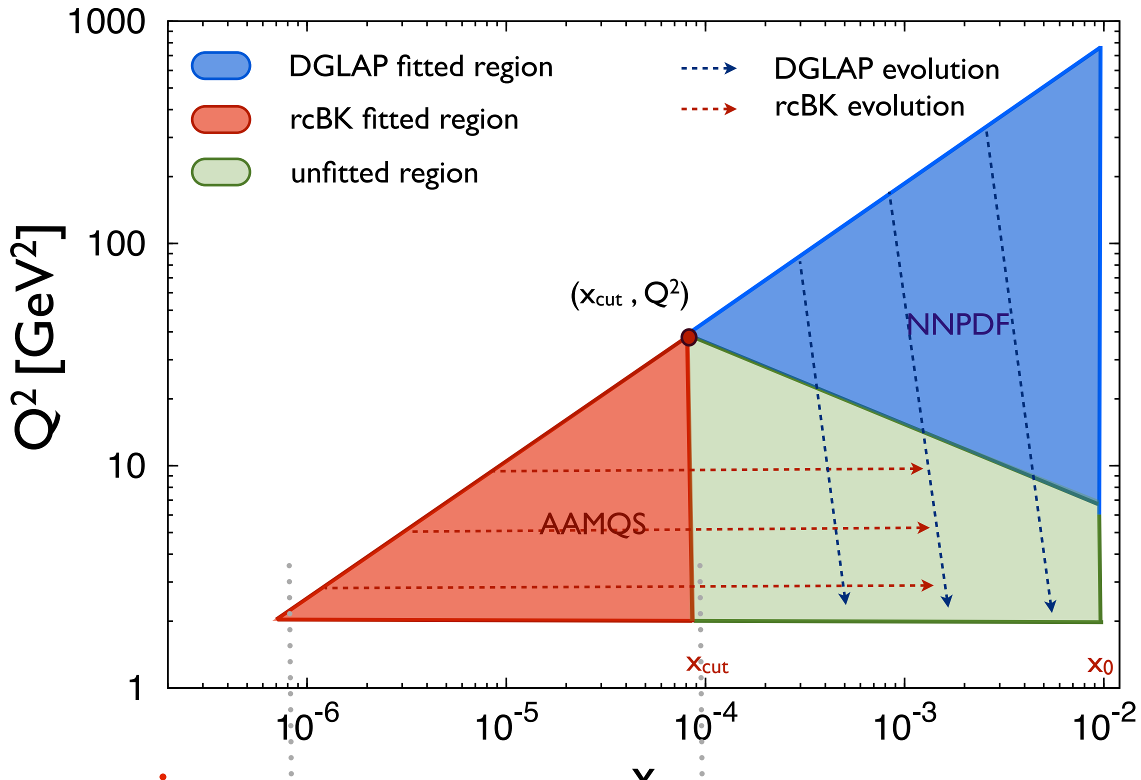
results - rcBK (AAMQS) low-x extrapolation

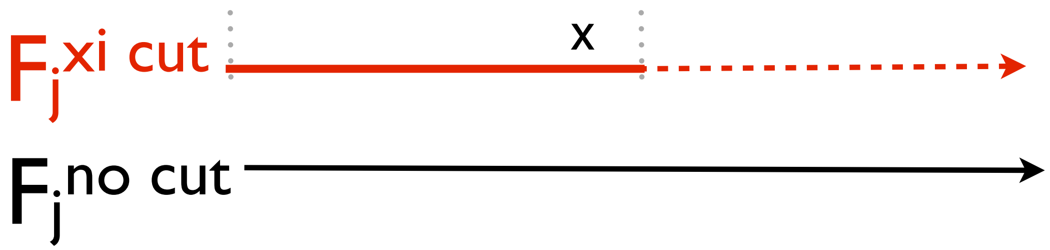
Need to test:

- predictive power of rcBK approach
- (un)sensitivity to boundary effects encoded in different i.c. for evolution under inclusion/exclusion of data subsets

} extrapolate results for $F_2(x,Q^2)$ & $F_L(x,Q^2)$ to tiny values of x [smaller than currently available]

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$





$$\boxed{F_j^{x^i_{cut}} / F_j^{no\ cut}, \quad \mathrm{j} = 2, \mathrm{L}}$$

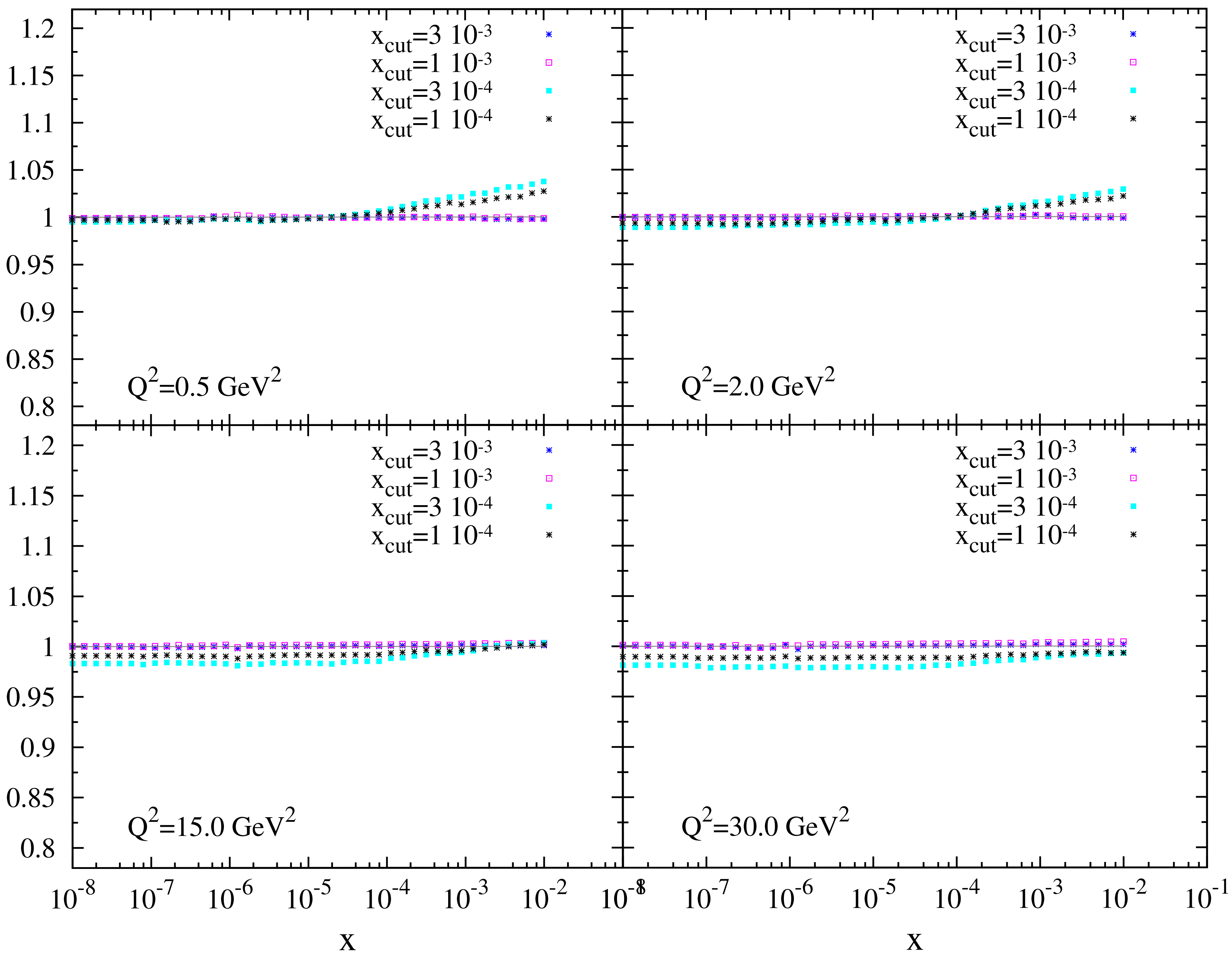
ratio of:
- structure function extrapolated to low-x from on a fit with cut
- to the one extrapolated from the fit to all available data

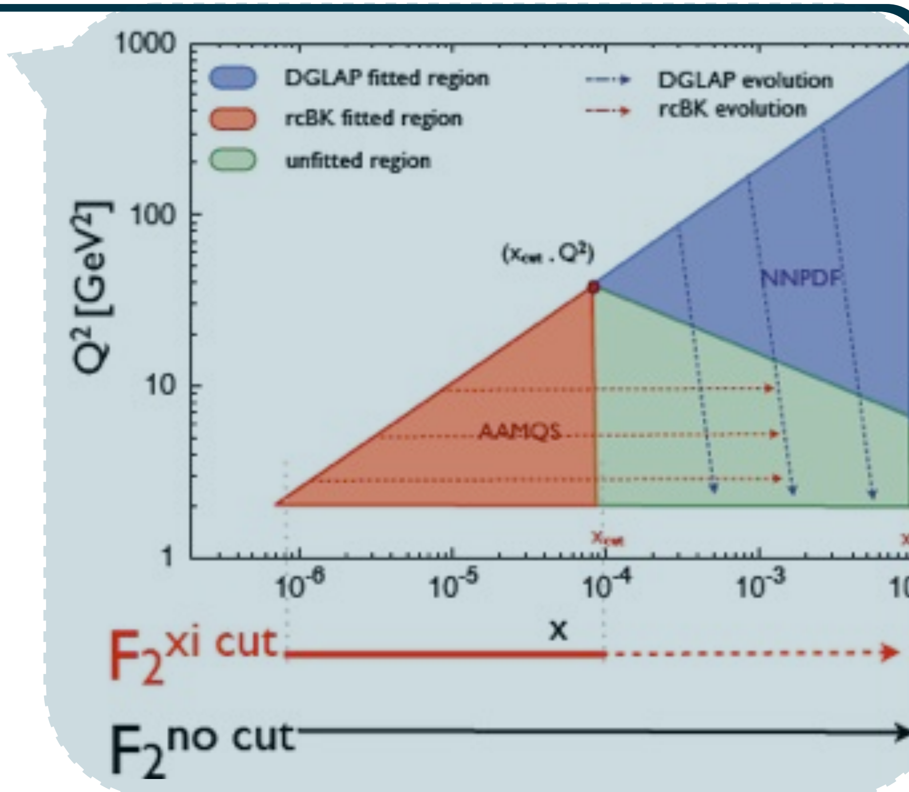
results - rcBK (AAMQS) low-x extrapolation

Total structure function $F_2(x,Q^2)$

$$x_{cut} = 3\cdot 10^{-3},\ 1\cdot 10^{-3},\ 3\cdot 10^{-4},\ 1\cdot 10^{-4}$$

$$F_2^{x^i_{cut}} / F_2^{no\ cut}$$





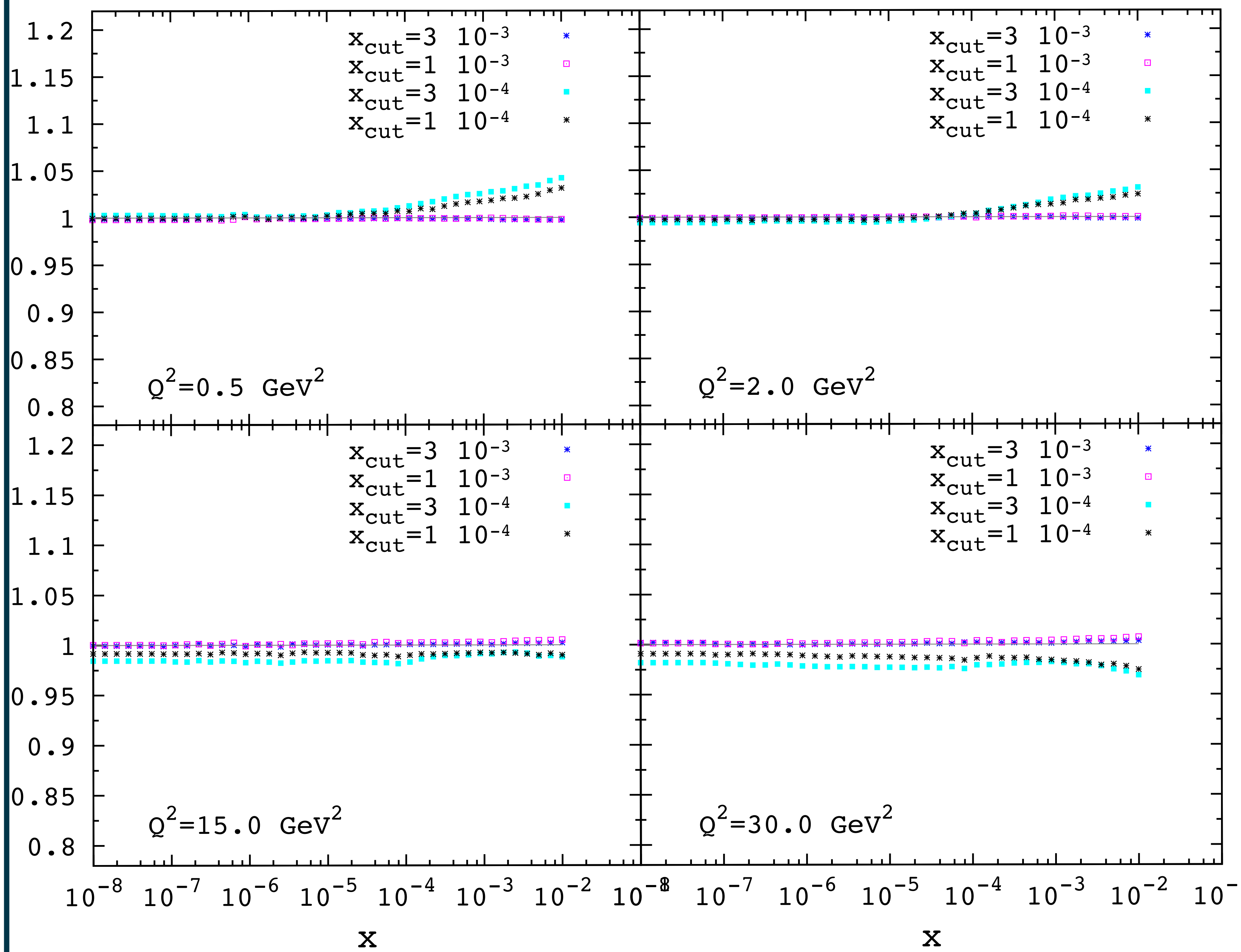
- Predictions from different fits: converge $x\sim10^{-4}$ [independently of the cut] within 1%

results - rcBK (AAMQS) low-x extrapolation

Longitudinal structure function $F_2(x,Q^2)$

$$x_{cut} = 3\cdot 10^{-3},\ 1\cdot 10^{-3},\ 3\cdot 10^{-4},\ 1\cdot 10^{-4}$$

$$F_L^{x^i_{cut}}/F_L^{no\ cut}$$



no F_L data included in any fit [calculated from AAMQS param]

converge $x\sim 10^{-4}$ within 1% [independently of the cut]

- Convergence: rcBK admit asymptotic solutions independent of i.c.

This predictions could be experimentally verified [LHeC or EIC]

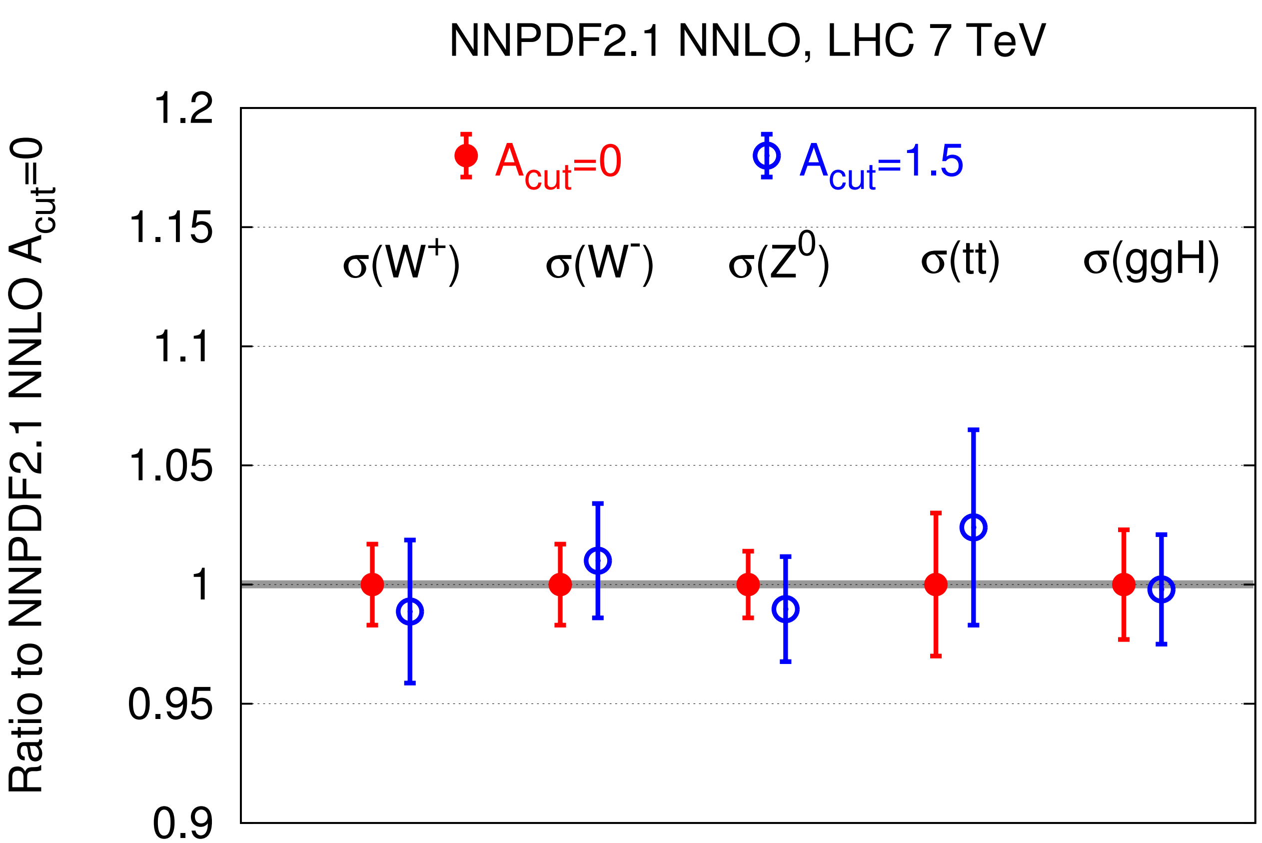
Implications for LHC phenomenology

- deviations from linear evolution => data should be excluded from DGLAP analysis
- estimate theoretical uncertainty rendered from potential deviations in DGLAP fits
- calculate benchmark LHC cross sections using PDF sets obtained through

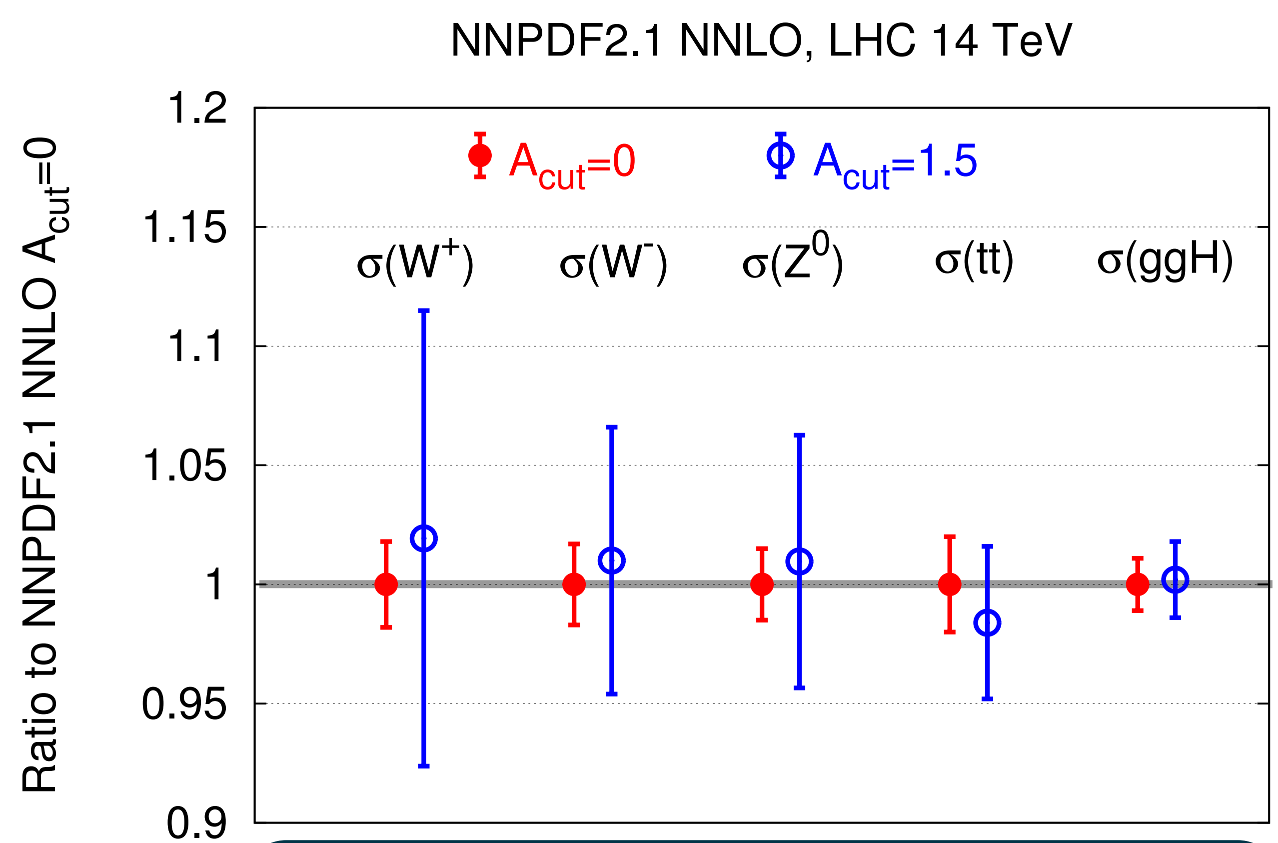
  1) fit to all data (without small-x kinematical cuts: $A_{cut}=0$)

  2) fit excluding small-x data (with small-x kinematical cuts: $A_{cut}=1.5$)

Only PDF uncertainties considered



moderate impact at $\sqrt{s}=7$ TeV



quite significant impact at $\sqrt{s}=14$ TeV
[smaller values of x are probed]

Implications for LHC phenomenology

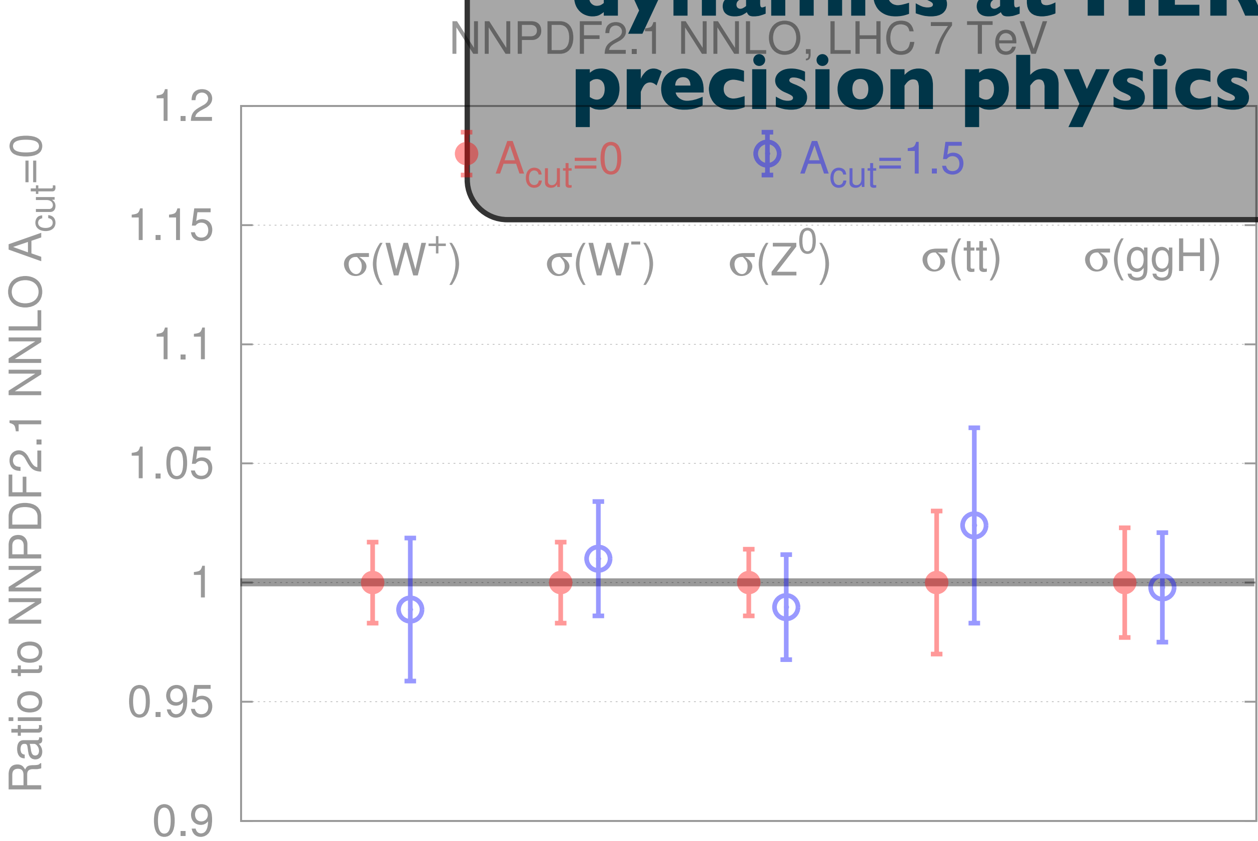
- deviations from linear evolution => data should be excluded from DGLAP analysis
- estimate theoretical uncertainty rendered from potential deviations in DGLAP fits
- calculate benchmark LHC cross sections using PDF sets obtained through

  1) fit to all data (without small-x kinematical cuts: $A_{cut}=0$)

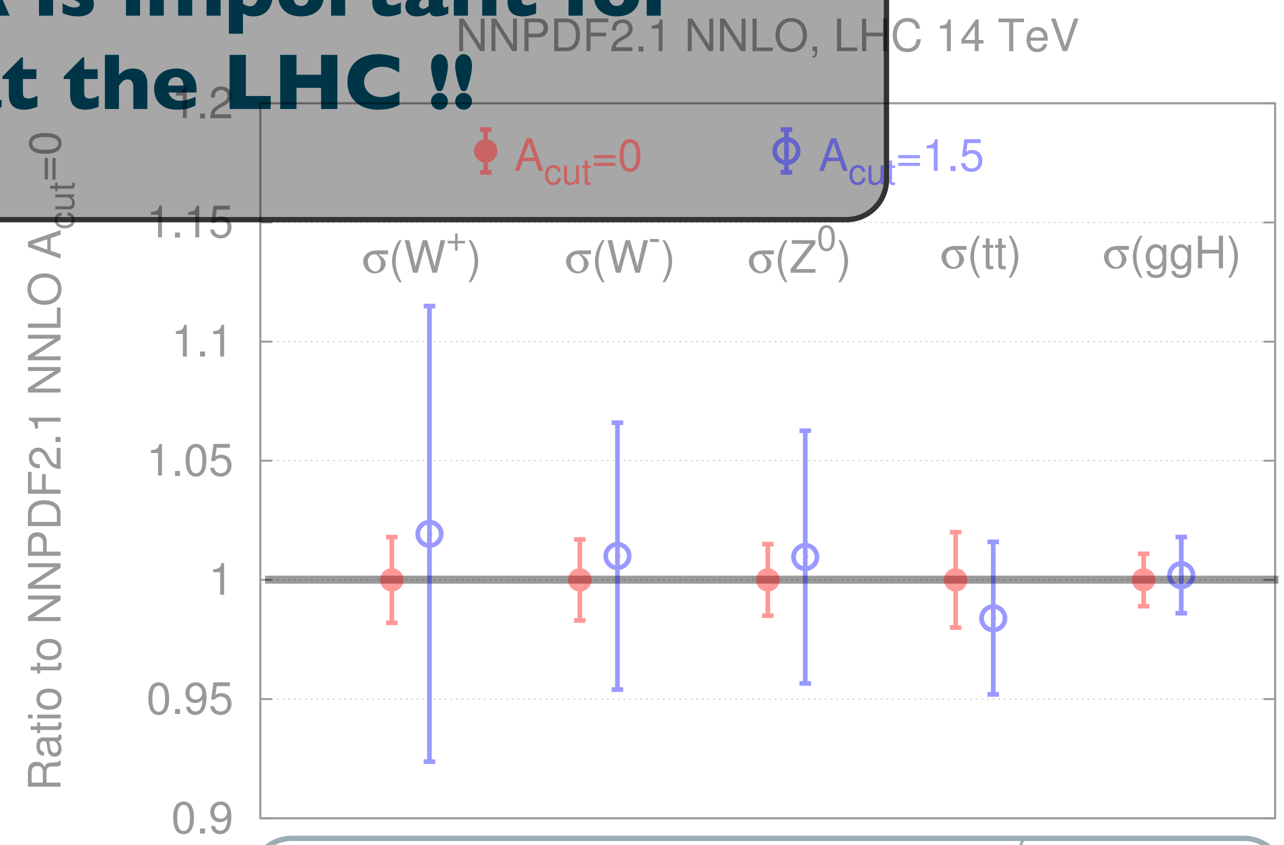
  2) fit excluding small-x data (with small-x kinematical cuts: $A_{cut}=1.5$)

Understanding small-x and Q^2 dynamics at HERA is important for precision physics at the LHC !!

Only PDF uncertainties considered



moderate impact at $\sqrt{s}=7$ TeV



quite significant impact at $\sqrt{s}=14$ TeV
[smaller values of x are probed]

Conclusions

Precision study: suitability of rcBK and DGLAP approaches to describing HERA data in moderate (x,Q^2) region. Setting common test ground: selected kinematic cuts to both fitting procedures and perform systematic comparisons

- DGLAP fits: sensitivity to exclusion of small-x data sets

  suggests novel physics obscured by its encoding in the freedom of i.c.(?)

- rcBK fits: robust agains exclusion of data above some x_{cut} (with x_{cut} as low as 10^{-4})

- Predictive power at low-x of the approach:

  - rcBK has predictive power towards low x: yields robust predictions at small-x

    [can be confronted with data from LHeC and EIC]

  - DGLAP has no predictive power : uncertainties grow very fast for low x outside data region

- The saturation line can be delineated: kinematic regions where DGLAP and rcBK differ substantially can be identified

- Exclusion of small-x data from DGLAP: significant increase on theoretical uncertainty for standard production cross sections at the LHC

Thank you!

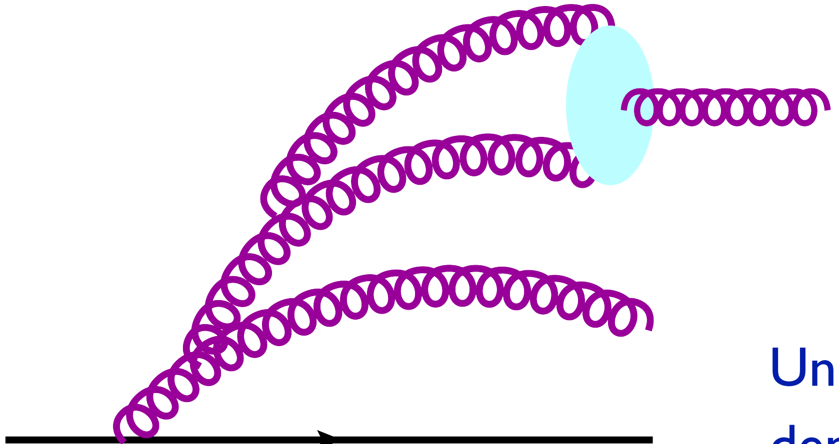
Backup slides

Introduction

- Knowledge of partonic structure of the proton at all relevant scales: crucial role in analysis of data from HE colliders => acquired by phenomenological parton fits to existing data [perturbative QCD based]

- Different QCD approaches for the description of the scale dependence of the parton distribution functions [strategy of resuming to all orders large logarithms]

In the limit of small Bjorken-x [HE]:



deviations from standard collinear perturbation theory are expected on account of large gluon densities => non-linear processes become relevant

Unitarity sets upper limit on the growth rate of gluon densities: realized by inclusion of recombination processes

highly probable in high density environment

"BK-JIMWLK"

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k_t})}{\partial \ln(\mathbf{x_0}/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k_t}) - \phi(\mathbf{x}, \mathbf{k_t})^2$$

the **C**olor **G**lass **C**ondensate is the correct framework in which to address small-x physics

Interplay between radiation and recombination processes => dynamical transverse momentum scale: the saturation scale Q_s [onset of non-linear corrections]

once non-linearities are included: a dynamical scale is generated and this immediately means collinear factorization does not hold

Interplay between the two approaches

- Need for systematic studies comparing both approaches
- Natural procedure to elucidate wether interesting dynamics is hidden in boundary conditions:
  - systematically displace the boundaries & check stability of both approaches under such changes
  - Sensitivity of the fits to changes in boundary conditions:
    - PDFs (DGLAP)
    - UDG (rcBK)

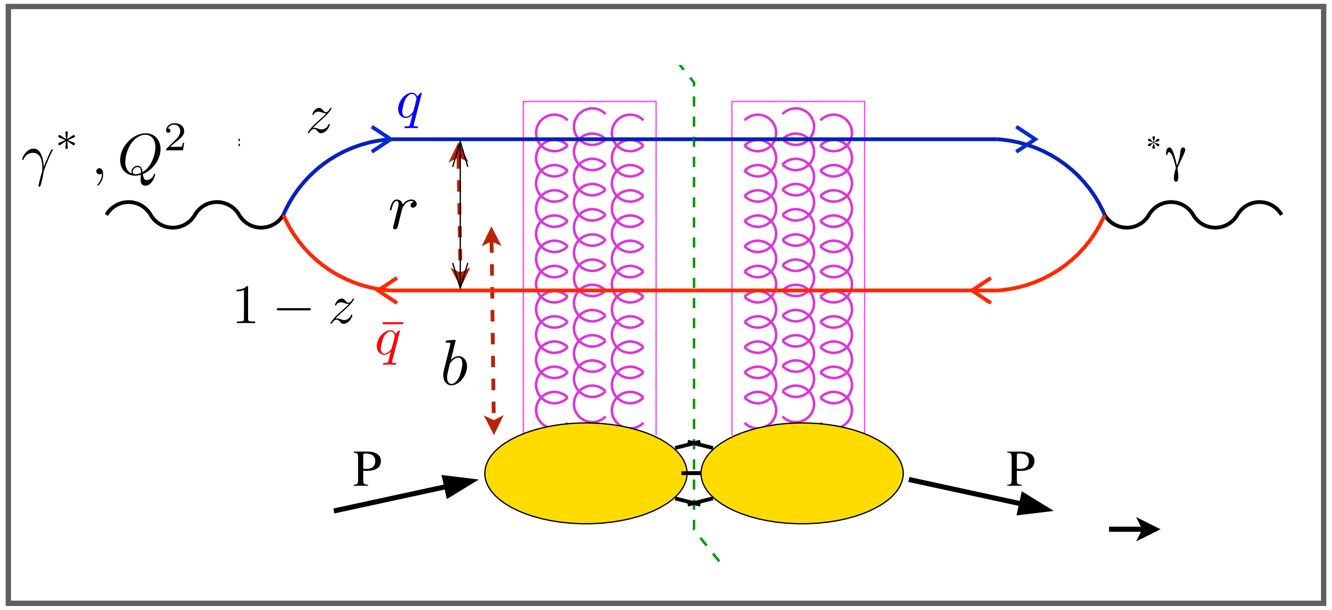
contaminated by physics effects beyond the dynamical content of the evolution equation

non-linear approach - rcBK: AAMQS implementation

Albacete, Armesto, Milhano, Quiroga, Salgado (AAMQS) arXiv:1012.4408[hep-ph]

- Dipole model formulation of e-p scattering process: virtual photon-proton cross section

$$\boxed{\sigma_{T,L}(x,Q^2) = 2\sum_f \int_0^1 dz \int d^2\mathbf{b}d^2\mathbf{r}|\Psi^f_{T,L}(e_f,m_f,z,Q^2,\mathbf{r})|^2\mathcal{N}(\mathbf{b},\mathbf{r},x)}$$



light-cone wave function for the virtual photon to fluctuate into a q-qbar dipole of quark flavor f

- Observables of interest related to the γ*-proton cross section

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}(\sigma_T + \sigma_L)$$

$$F_L(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_L$$

$$\boxed{\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)}$$

non-linear approach - rcBK: AAMQS implementation

- Initial conditions [for the rcBK evol. eq. $\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)}$] in AAMQS global fits to data

$$\boxed{\mathcal{N}^{MV}(r,x_0) = 1 - e^{-\left(\frac{r^2 Q_{s,0}^2}{4}\right)^{\gamma} \ln\left(\frac{1}{r\Lambda_{QCD}}\right)}}$$

- 2 fit parameters:
  - initial saturation scale [at x_0=0.01]
  - anomalous dimension [steepness of the dipole amplitude fall-off with decreasing r]

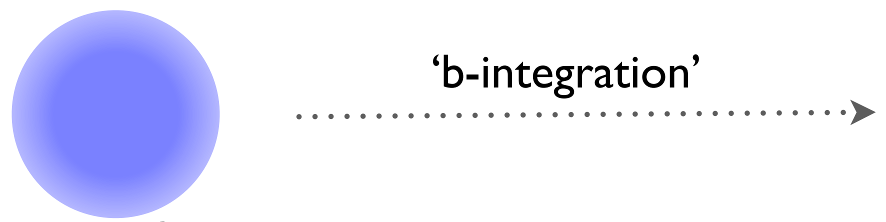
Dumitru and Petreska arXiv:1112.4760[hep-ph]

- the anomalous dimension follows from taking higher corrections in the MV semiclassical calculation. $\gamma \sim 1 + \# A^{2/3}$

- results for dipole amplitude match AAMQS fits to proton data

non-linear approach - rcBK: AAMQS implementation

- b-dependence of dipole amplitude $\mathcal{N}$(b,r,x): governed by long-distance non-perturbative phenomena [extra model input]: AAMQS resorts to translational invariance approximation



$\mathcal{N}(\mathbf{b}, \mathbf{r}, x)$ $\qquad$ $\sigma_0 \mathcal{N}(\mathbf{r}, x)$

average over impact parameter

$$2 \int d\mathbf{b} \to \sigma_0$$

[average transv. area of quark distrib. in transv. plane]

- regularization of the coupling: phase space for all dipoles explored [arbitrarily large] => need to regulate in the IR

$$\begin{cases} \alpha_s(r^2 < r^2_{fr}) = \dfrac{12\pi}{(11N_c - 2n_f)\ln\left(\frac{4C^2}{r^2\Lambda_{QCD}}\right)} \\ \alpha_s(r^2 \geq r^2_{fr}) = \alpha_{fr} \end{cases}$$

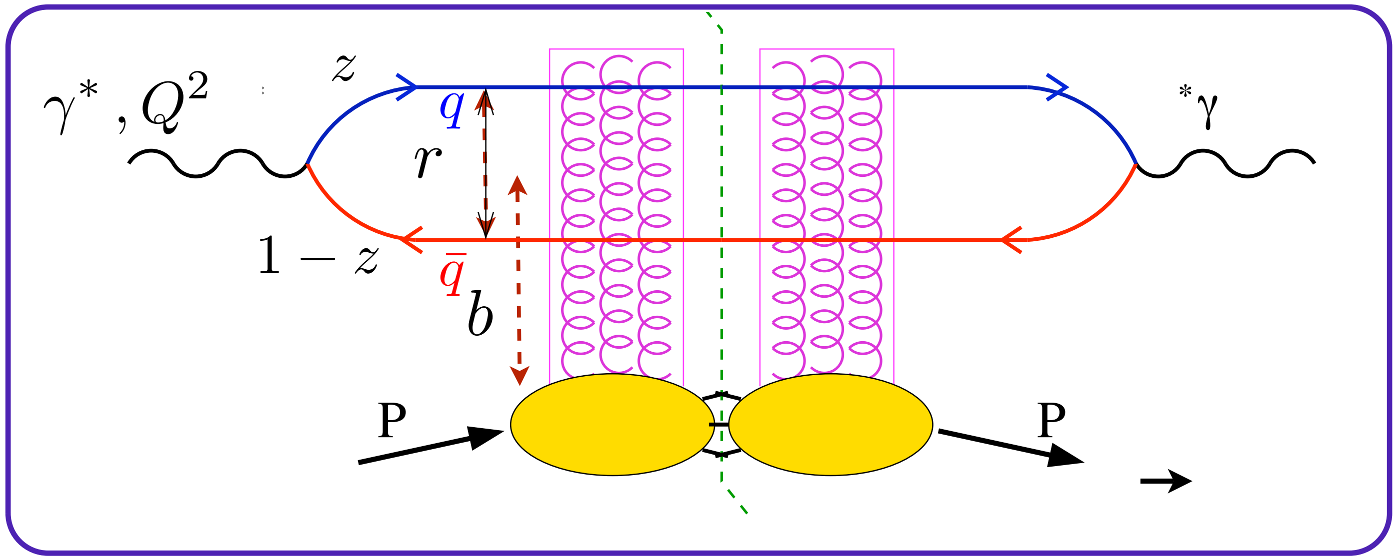
momentum space [calculation of the quark part of ß]
Fourier transform
coordinate space

$\Lambda_{QCD} = 0.241\mathrm{GeV}$

- AAMQS global fits to HERA e-p data: calculate σ_r and F_2 according to the dipole model with small-x dependence described by rcBK equation. MV initial condition for the dipole amplitude

- 4 free parameters: $\sigma_0,\ C^2,\ Q^2_{s,0},\ \gamma$

AAMQS setup. Dipole model formulation of e+p scatt. + rcBK eq.



- dipole model formulation of the e-p scattering process

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$

x<<1

$$\begin{cases} F_2(x,Q^2) = \dfrac{Q^2}{4\pi^2\alpha_{em}}(\sigma_T + \sigma_L) \\ F_L(x,Q^2) = \dfrac{Q^2}{4\pi^2\alpha_{em}}\sigma_L \end{cases}$$

virtual photon-proton cross section [long. & trans. polarization of γ^*]

$$\sigma_{T,L}(x,Q^2) = 2\sum_f \int_0^1 dz \int d^2\mathbf{b} d^2\mathbf{r} |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}(\mathbf{b}, \mathbf{r}, x)$$

[light-cone wave function for γ^*
to fluctuate into a q-qbar dipole]

Im. part of dipole-target scatt. amplitude
[all strong interaction and x dependence]

AAMQS setup. Dipole model formulation of e+p scatt. + rcBK eq.

- small-x dynamics of the dipole scattering amplitude described by rcBK equation

$$\frac{\partial N(r,x)}{\partial \ln(x_0/x)} = \int d^2 r_1 \mathbf{K}^{run}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2)[N(r_1,x) + N(r_2,x) - N(r,x) - N(r1,x)N(r_2,x)]$$

non-linear term

evolution kernel including rc corrections:

Balitsky, Phys.Rev.D75:014001,2007

$$K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

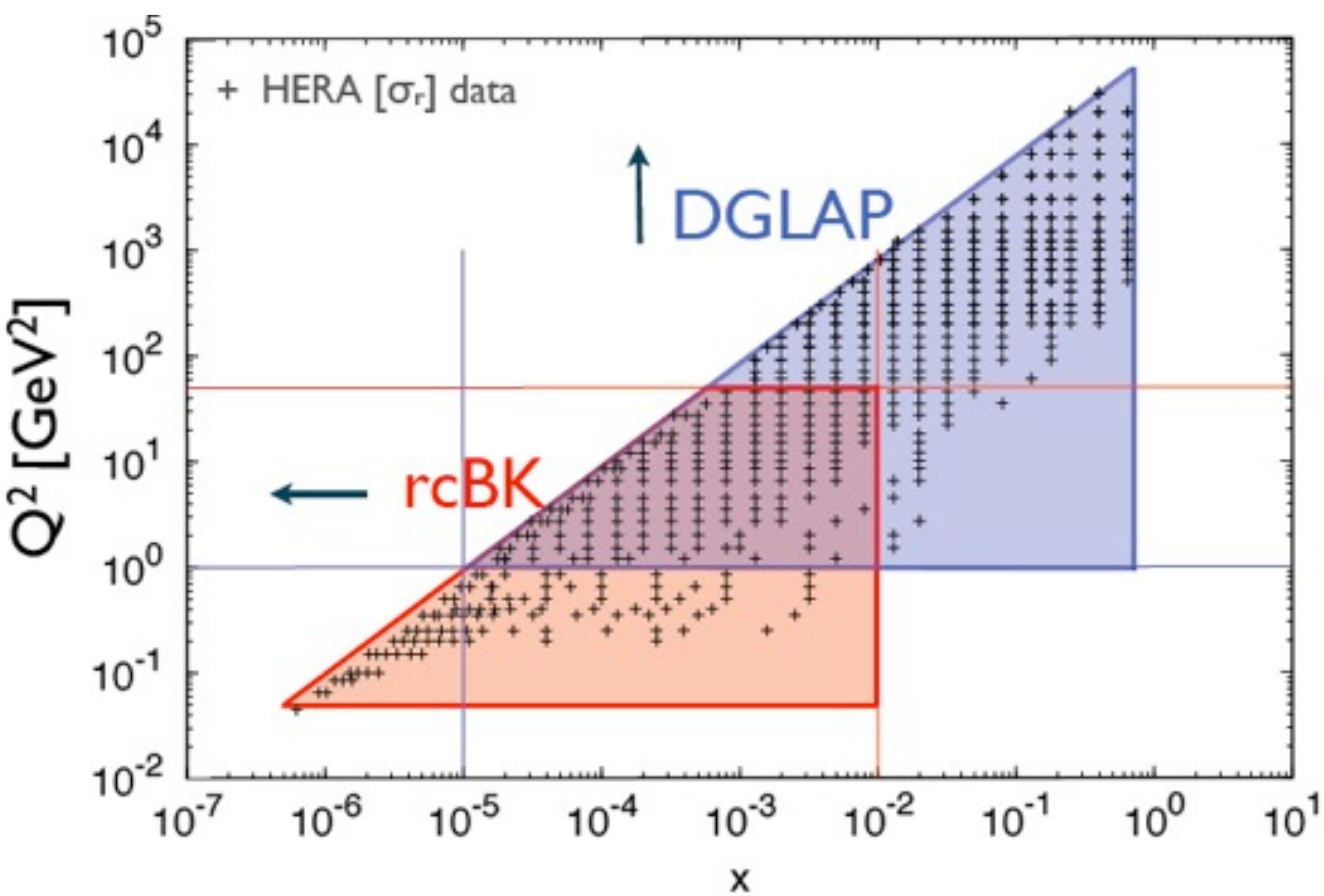
- Regularization of the coupling: phase space for all dipoles sizes explored [arbitrarily large] => need to regulate in the IR

$$\alpha_s(r^2 < r_{fr}^2) = \frac{12\pi}{(11N_c - 2n_f)\ln\left(\frac{4C^2}{r^2\Lambda_{QCD}^2}\right)} \qquad \alpha_s(r^2 \geq r_{fr}^2) = \alpha_{fr}$$

Fourier transform: momentum to coordinate space

comparison of evolutions

	Equation	Evolution variable	Predictive power		Initial conditions	Implementation	range of applicability (x,Q^2)
			low x	high Q^2			
DGLAP	linear	Q^2	✗	✔	$xf(x,Q_0^2)$	NNPDF	(>10^{-5} , >1-4)
rcBK	non-linear	x	✔	✗	$\mathcal{N}(r,x_0)$	AAMQS	(<10^{-2} , < 50)



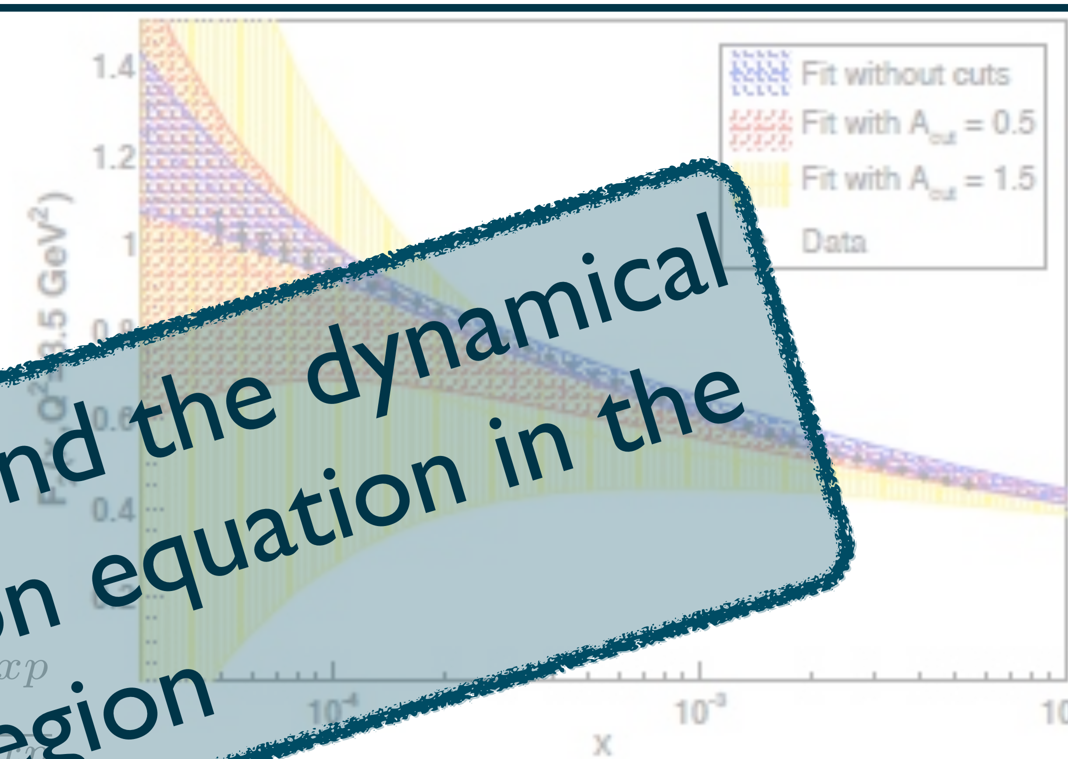
both approaches **coexist** in a region

(Non-linear?) deviations from NLO DGLAP evolution

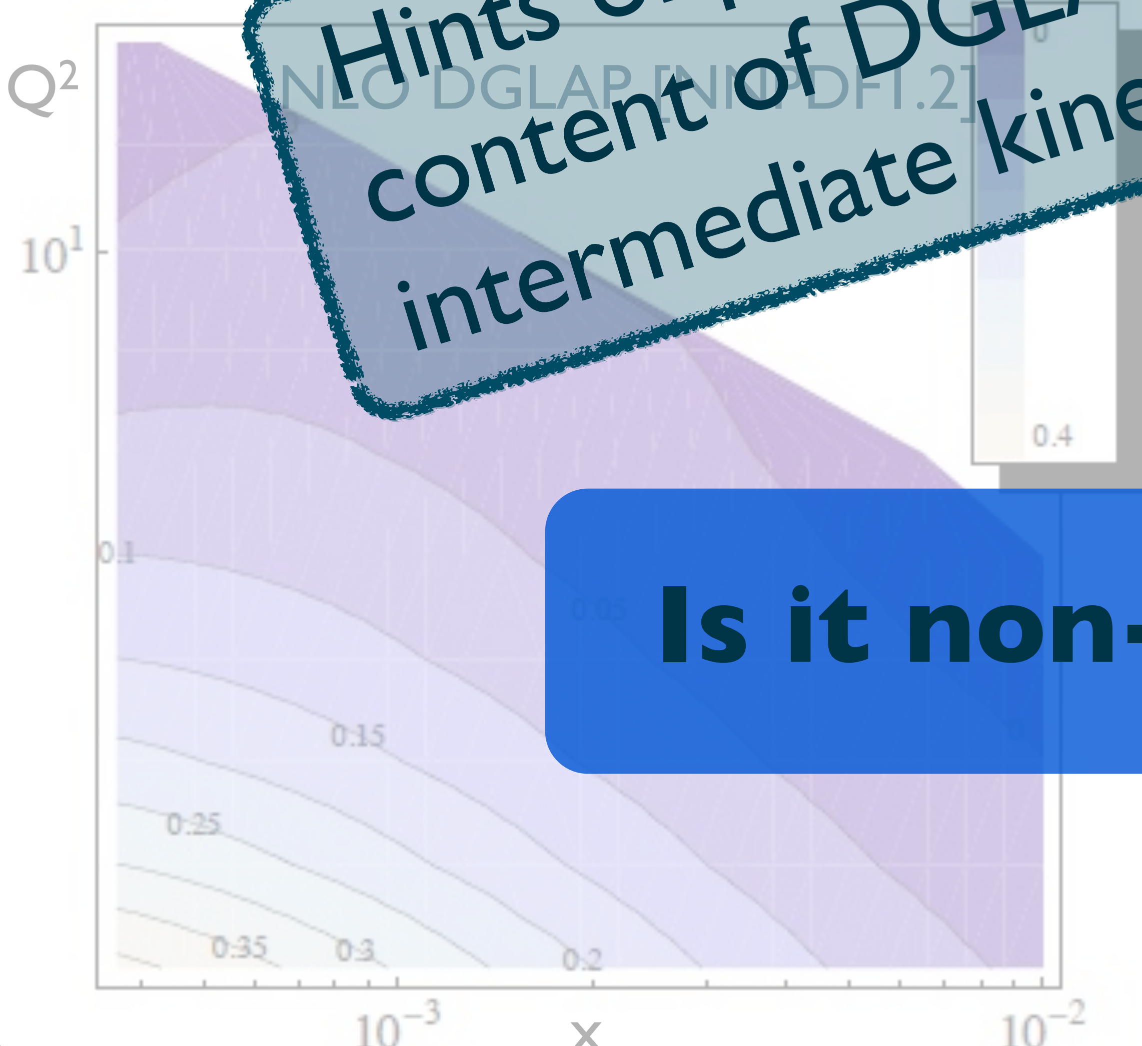
- NNPDF: fits with cuts $Q^2 > Q_{cut}^2 = A_{cut} x^{-\lambda}$

Caola, Forte, Rojo, PLB 686, 2010

fits tend to systematically underestimate the data



- Quantifying the deviations $d_{rel}(x, Q^2) = \dfrac{F_2^{th} - F_2^{exp}}{F_2^{th} + F_2^{exp}}$



NLO DGLAP: deviations as large as 35% !!
[at low x and low Q^2]

- not corrected by
  - NNLO corrections
  - improved treatment of heavy quark effects

Hints of physics effects beyond the dynamical content of DGLAP evolution equation in the intermediate kinematical region

Is it non-linear effects??

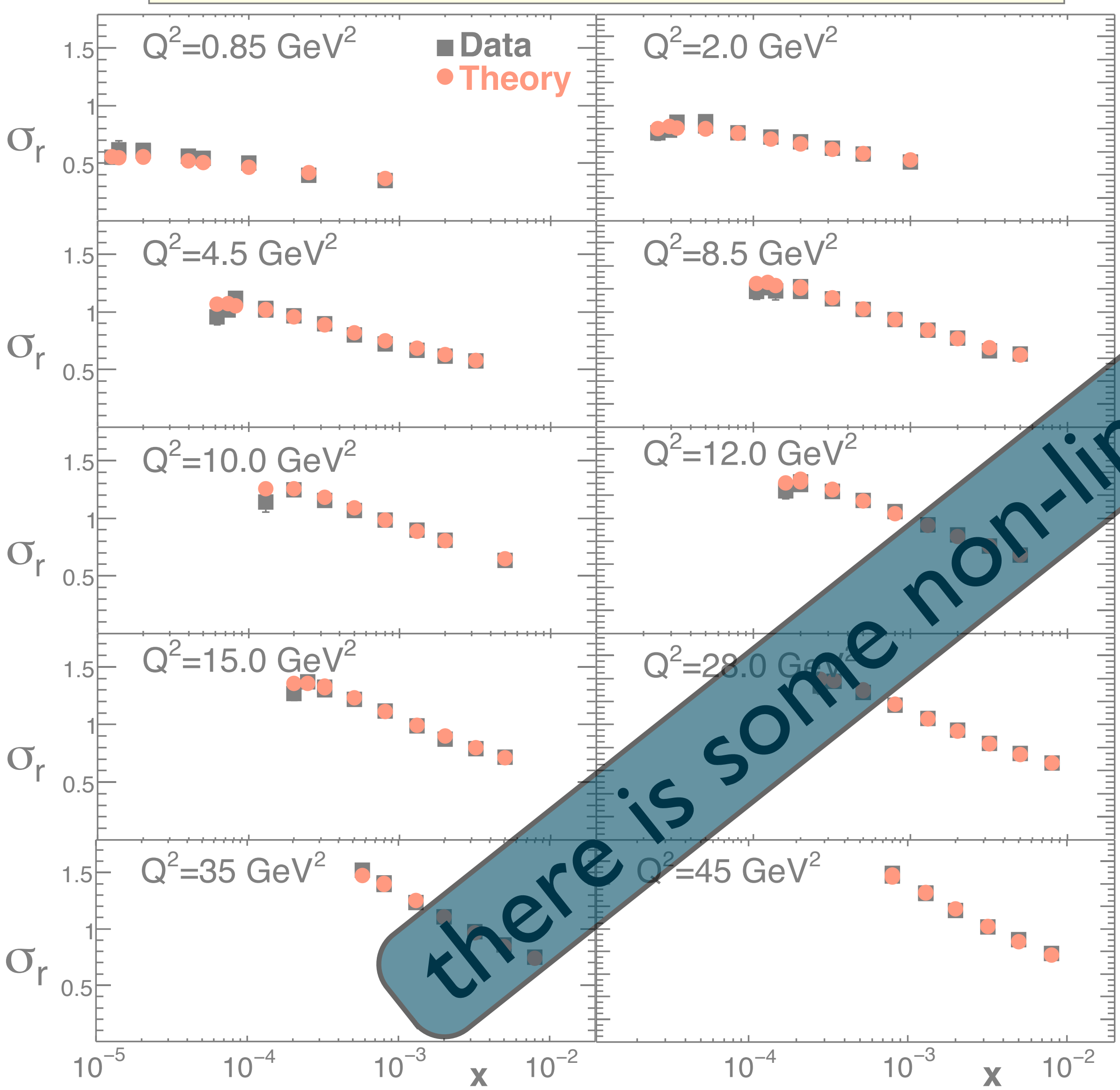
non-linear approach - rcBK

- Similarly good fits to DGLAP + naturally accommodates geometric scaling
- AAMQS implementation: does a very good job describing HERA data
  - especially latest data (combined H1-ZEUS analysis)

arXiv:1012.4408
arXiv:0902.1112

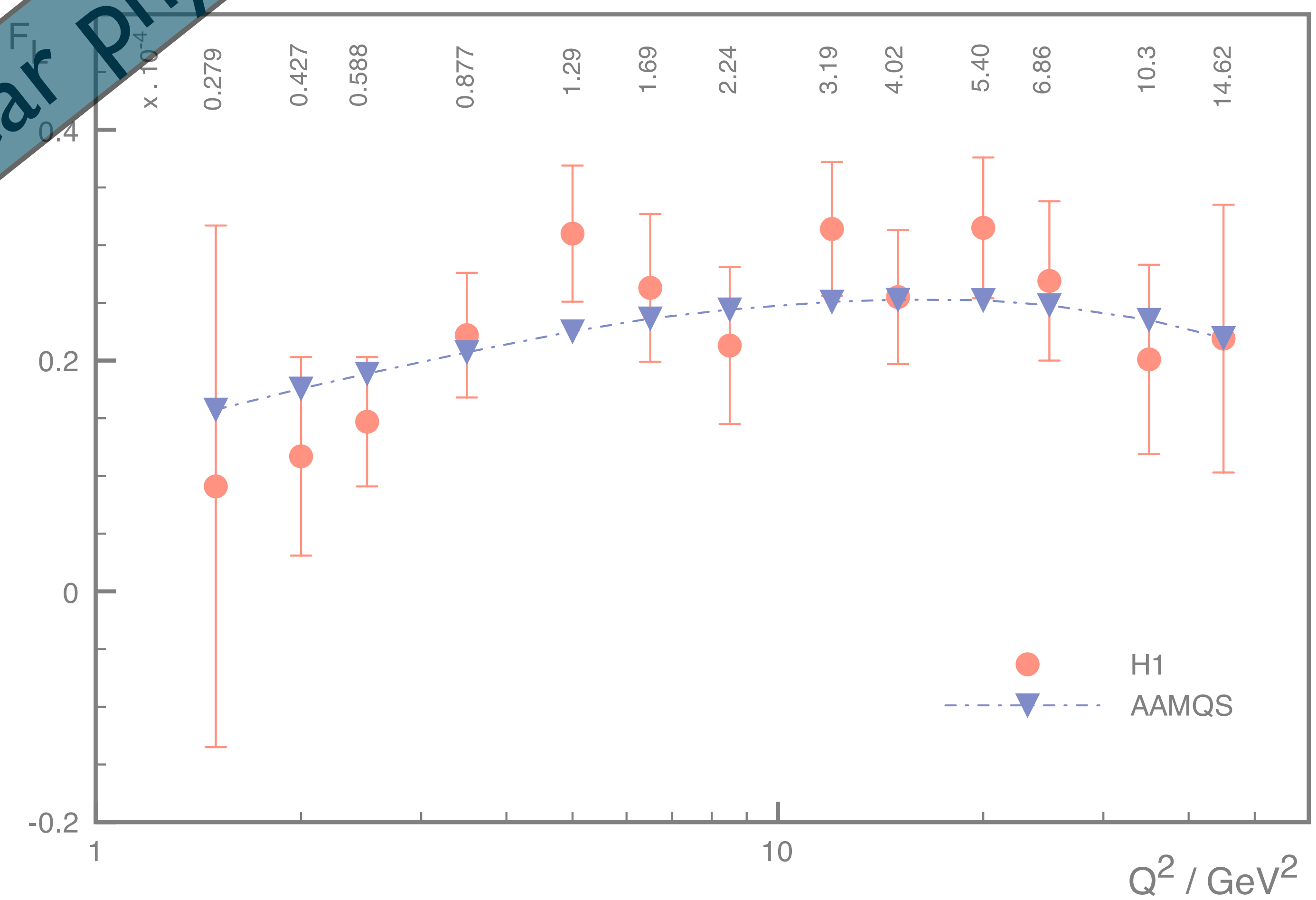
quite challenging!

Fit including heavy quarks



AAMQS calculation of F_L vs latest data

[independent test of the method]

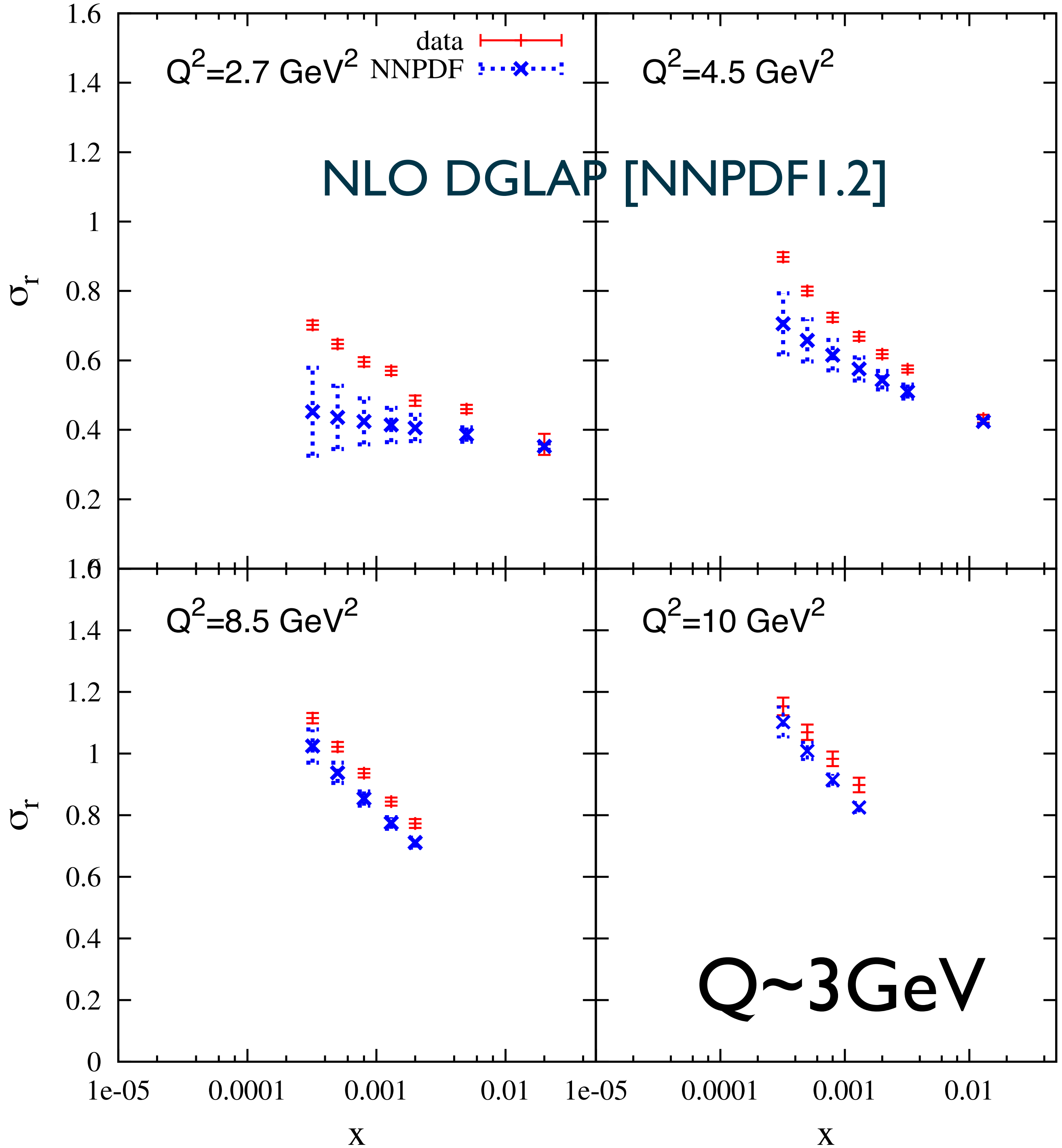


there is some non-linear physics going on here

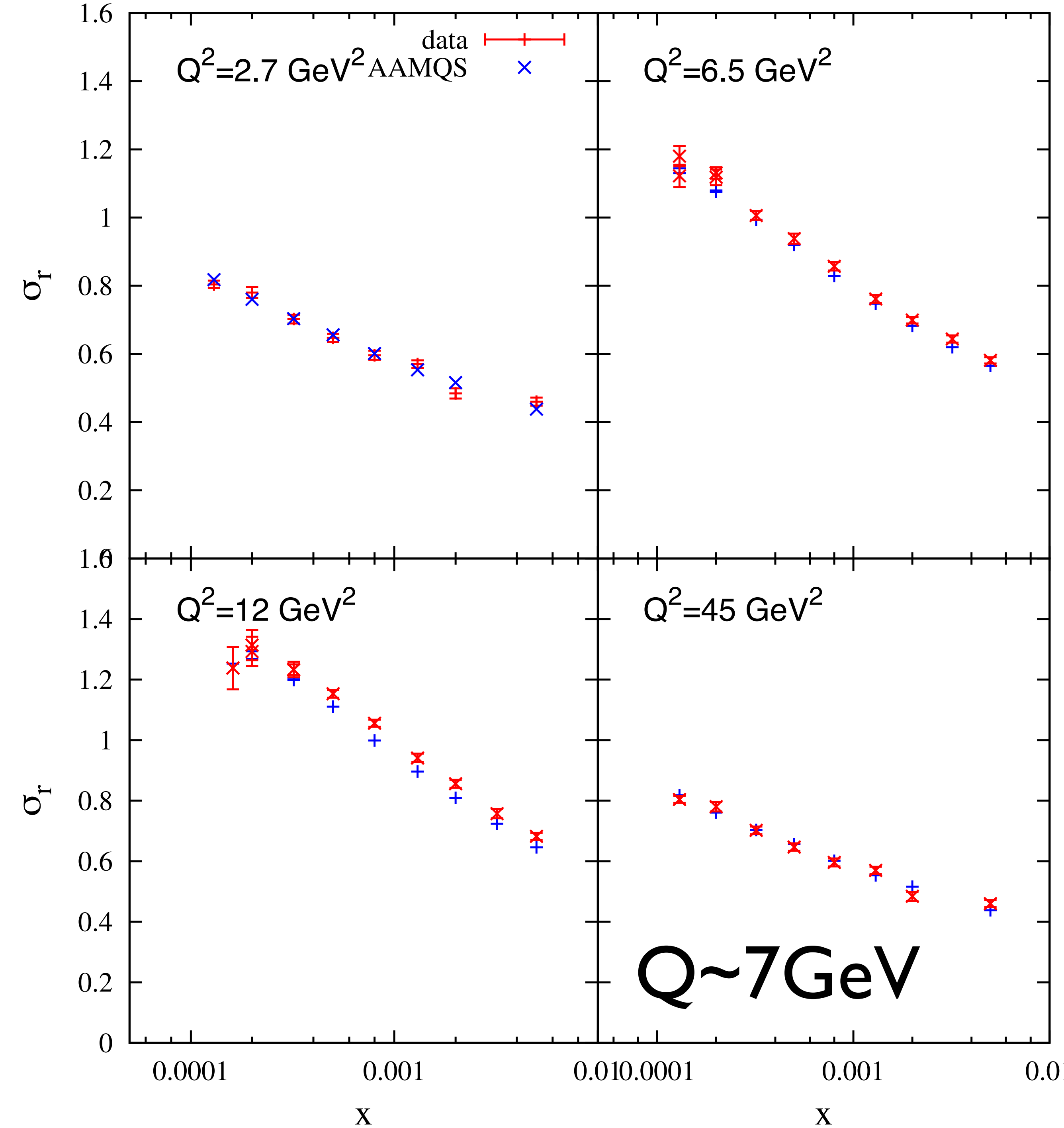
results - NLO DGLAP & rcBK fits with cuts

- NNPDF and AAMQS extrapolation to the common unfitted region

NNPDF A_{cut}=1.5



AAMQS x_{cut}=10^{-4}



- deviation from data at low x and low Q^2
- very good description of data even with the more restrictive cut

results - measuring the deviations

- **Relative distance** between theoretical and experimental results: measures the absolute size of deviations

$$d_{rel}(x,Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\frac{1}{2}\left(\sigma_{r,th} + \sigma_{r,exp}\right)}$$

  - theoretical predictions from DGLAP ($\sigma_{r,\,DGLAP}$) and rcBK ($\sigma_{r,\,rcBK}$) and experimental data ($\sigma_{r,\,exp}$): values of the reduced cross section in the common extrapolated region

- **Statistical distance** between theoretical and experimental results: statistical significance of the deviation in units of standard deviation

$$d_{stat}(x,Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\sqrt{\left(\Delta\sigma^2_{r,th} + \Delta\sigma^2_{r,exp}\right)}}$$

meaningless when large theory errors

  - the theoretical error for rcBK (AAMQS), $\Delta\sigma^2_{r,\,rcBK}$: estimated as maximal difference among the theoretical predictions corresponding to fits with different cuts [probably underestimated => values of d_{stat}^{rcBK} overestimated]

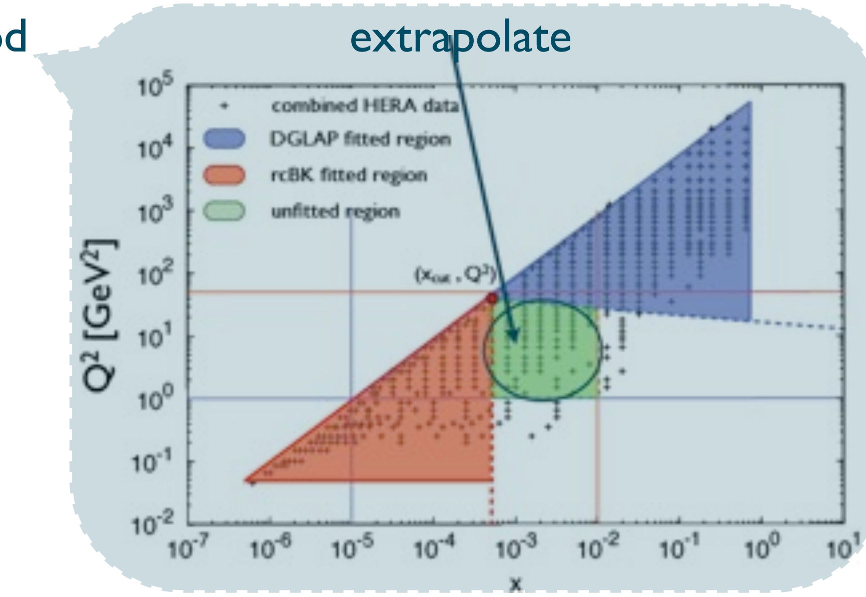
  - for DGLAP (NNPDF) full information on correlated systematics is taken into account

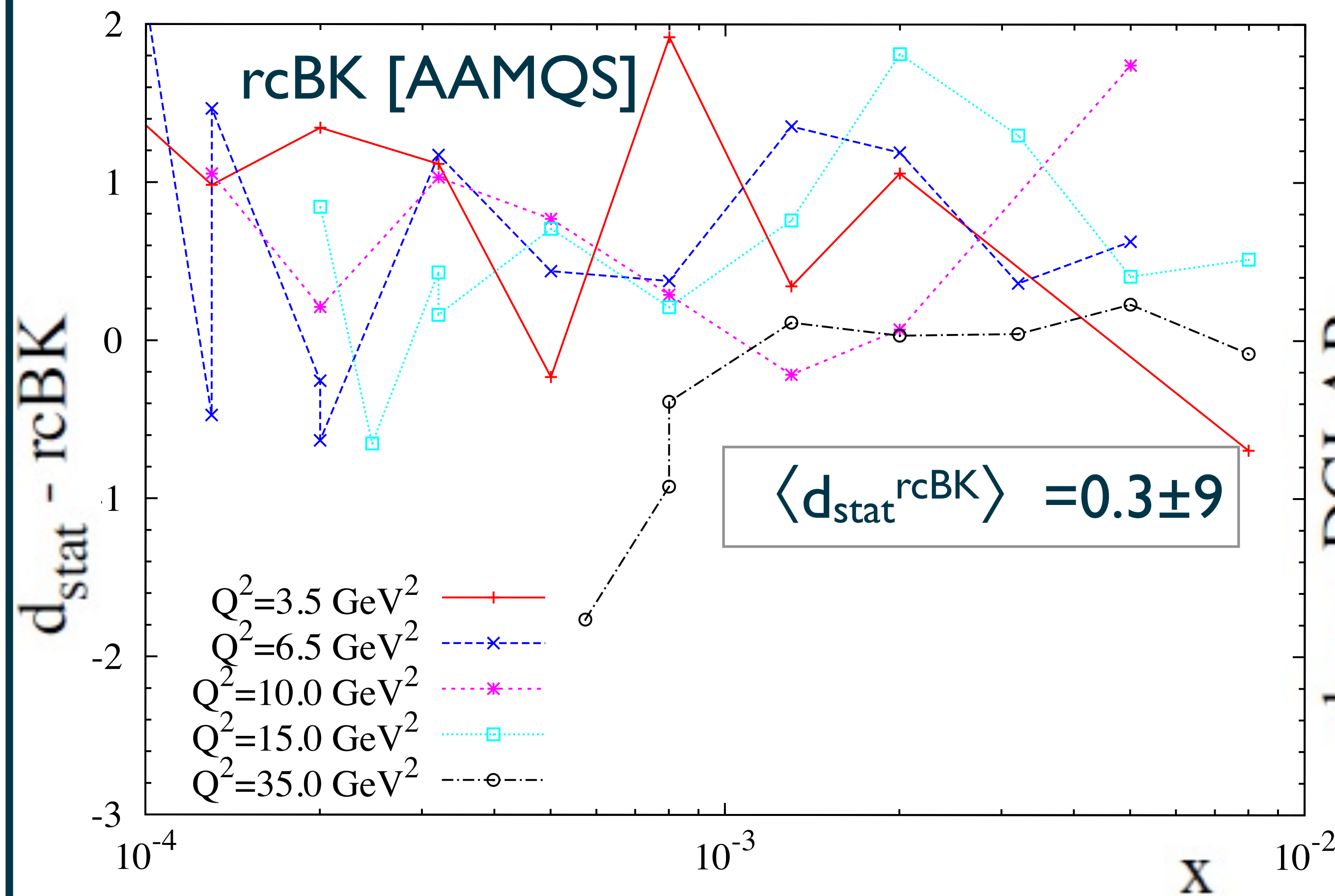
results - measuring the deviations

- Statistical distance between theoretical and experimental results: measures statistical significance of the deviation in units of standard deviation

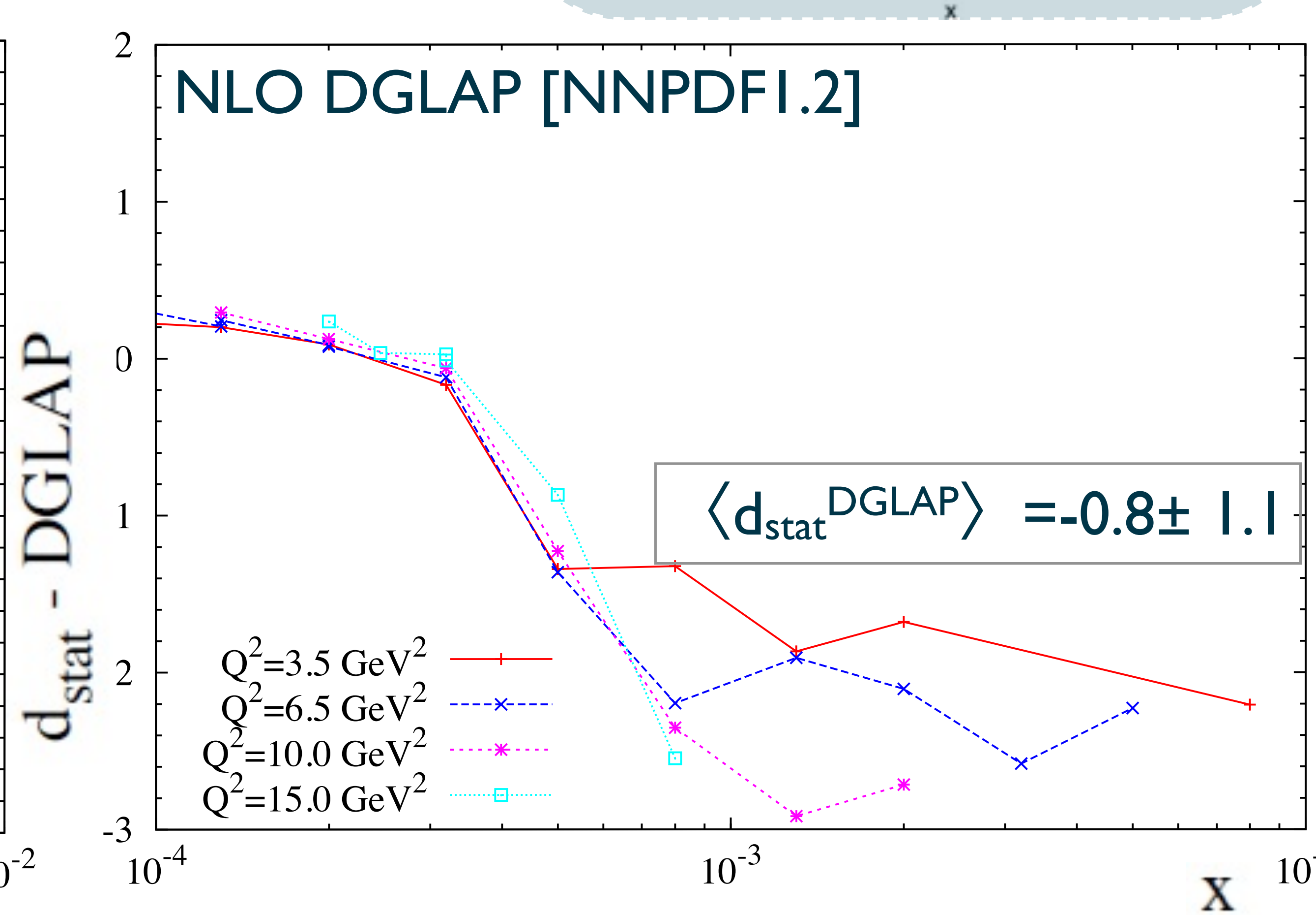
$$d_{stat}(x,Q^2) = \frac{\sigma_{r,th} - \sigma_{r,exp}}{\sqrt{\left(\Delta\sigma^2_{r,th} + \Delta\sigma^2_{r,exp}\right)}}$$

method





theoretical errors underestimated



huge theoretical uncertainty at low-x