Thermalization, the Glasma and Strong Interactions through Field Coherence

L. McLerran, 16/9/11

Outline:

The sQGP Paradigm and a little Heavy Ion Lore

Saturation and Coherent Fields in the Hadron Wavefunction

The Formation of the Glasma

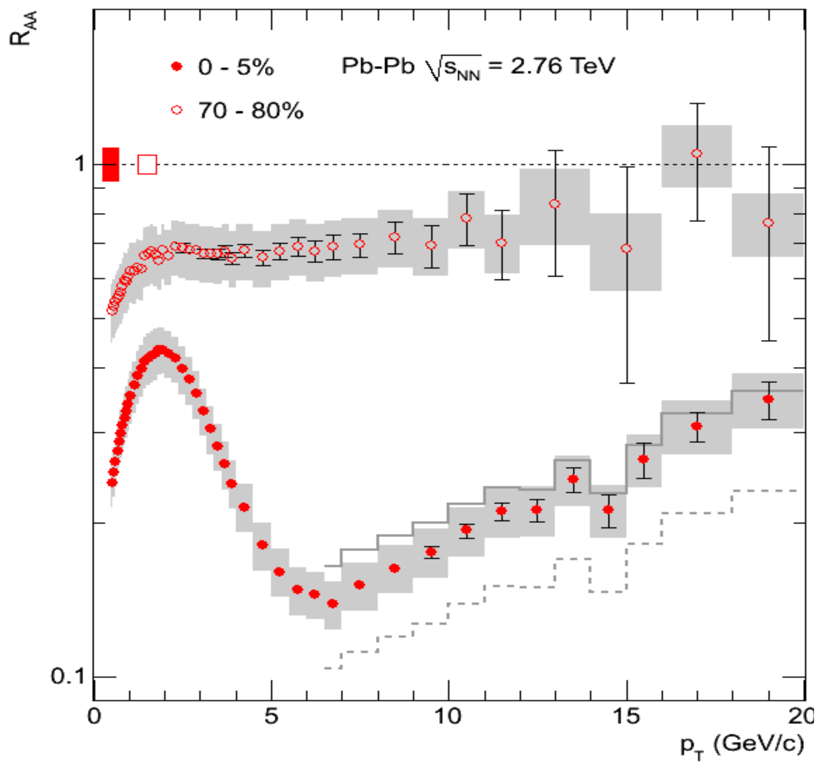
Early Time Evolution of the Glasma

Thermalization and Transport

Implications

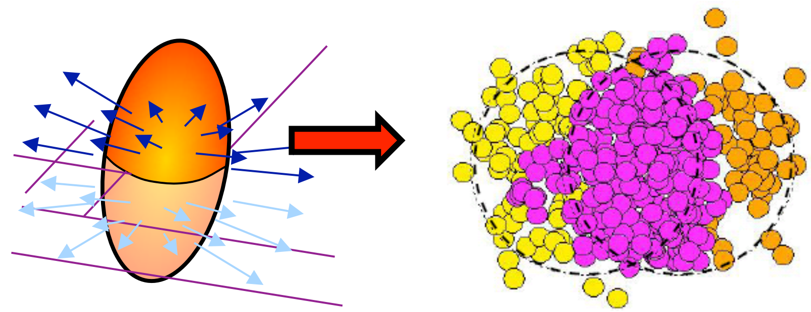
RHIC and LHC Data on heavy Ions

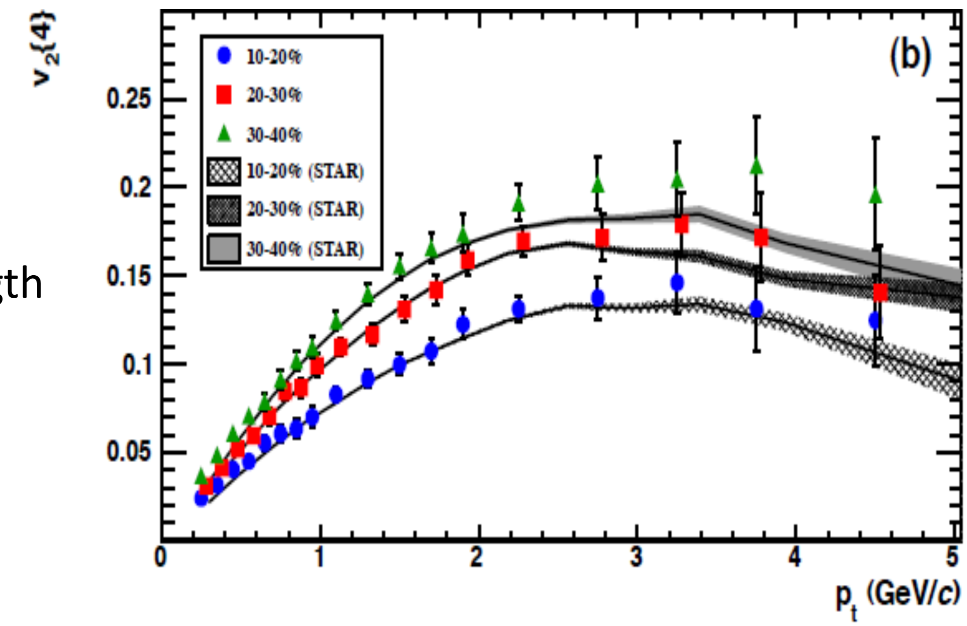
PLB 696 (2011) 30-39



"Jet quenching" => Large Energy Loss /Length

Very Strong Collective Flow Patterns
Consistent with Perfect Fluid
Hydrodynamics
Small Viscosity to Entropy

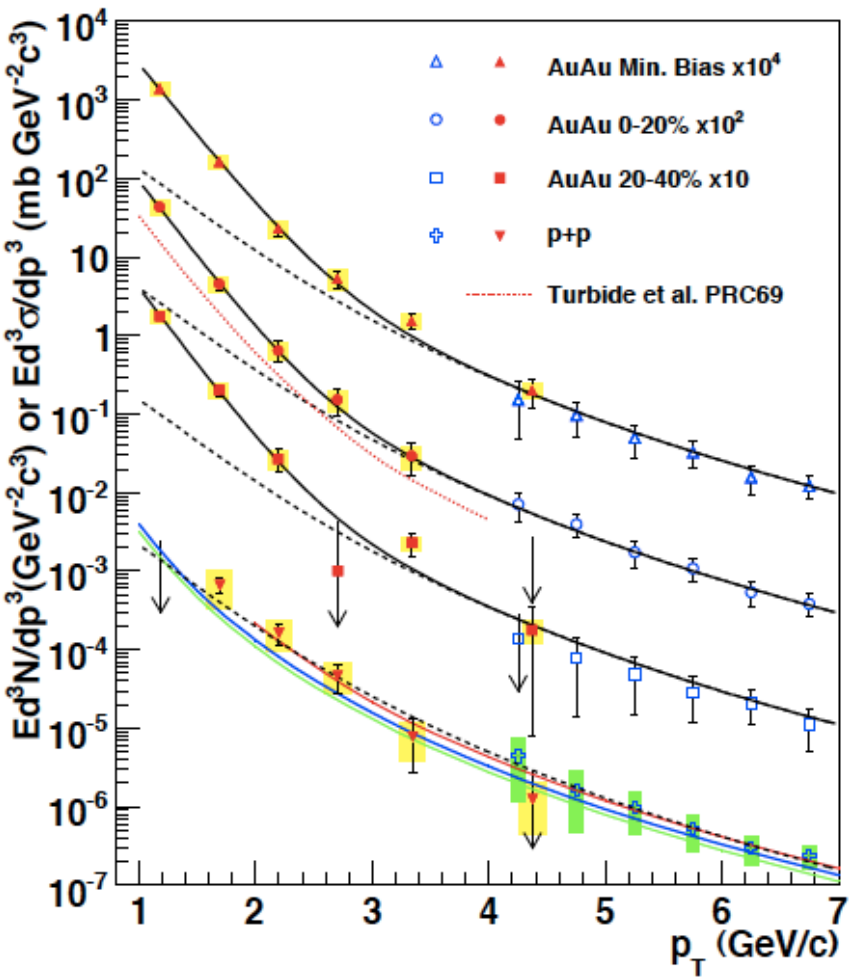




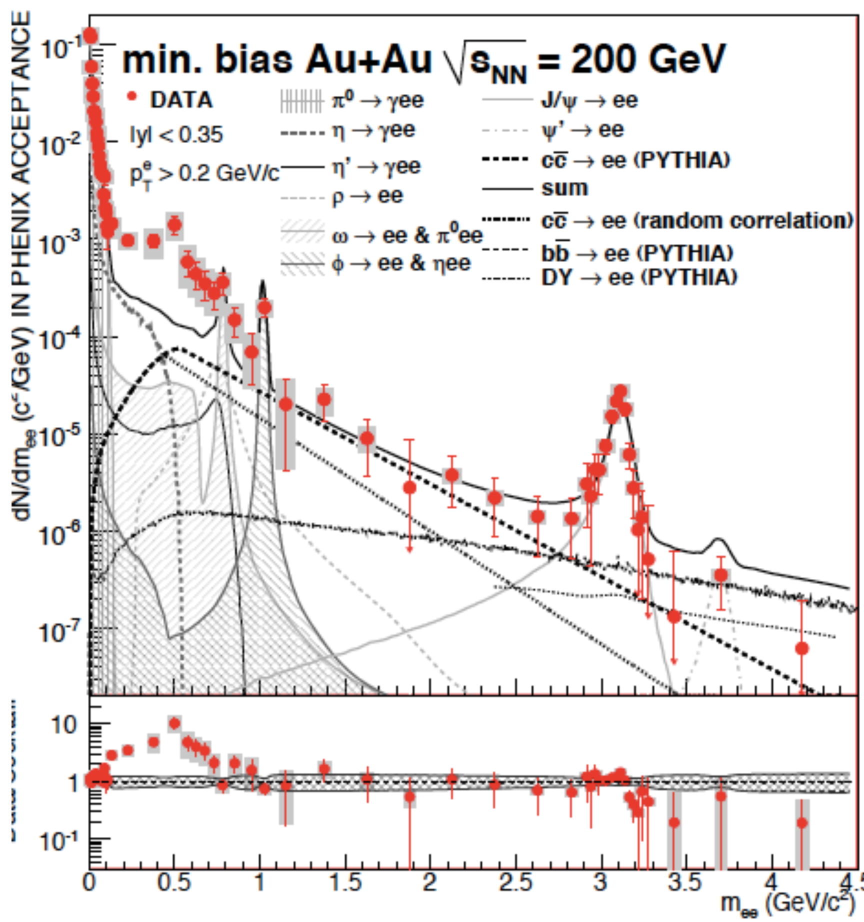
Strongly Interacting QGP Paradigm:
Well thermalized Quark Gluon
Plasma

Photons and Dileptons

QGP & Hydro:
Magnitude and shape
Flow not explained



QGP & Hydro
Magnitude and shape not explained
Dilepton have p_T slope of 100 MeV



Saturation and Coherent Fields:

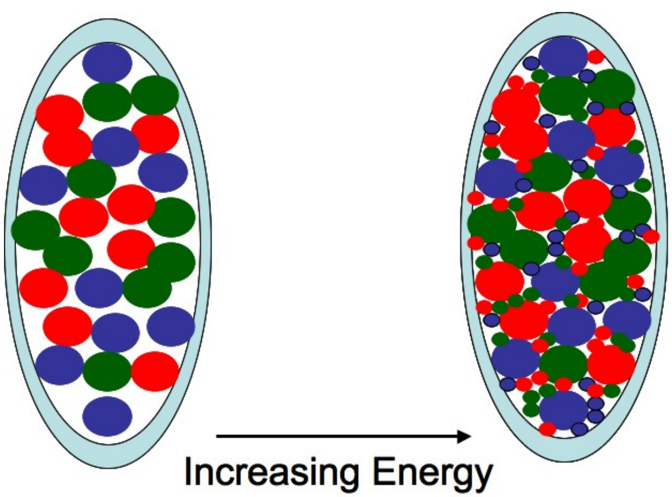
Color Glass Condensate:

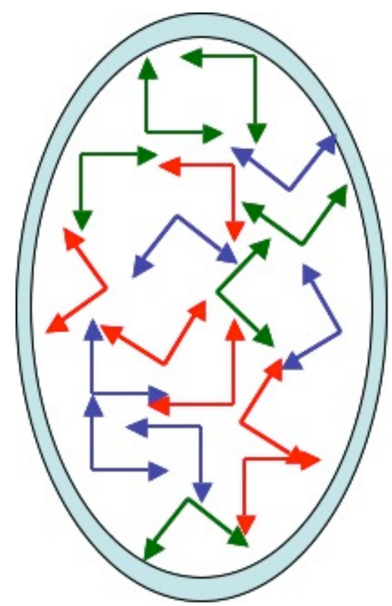
High density gluon fields:

$$\frac{dN}{dy d^2 r_T d^2 p_T} \sim \frac{1}{\alpha_s}$$

Parton distributions replaced by ensemble of coherent classical fields
Renormalization group equations for sources of these fields

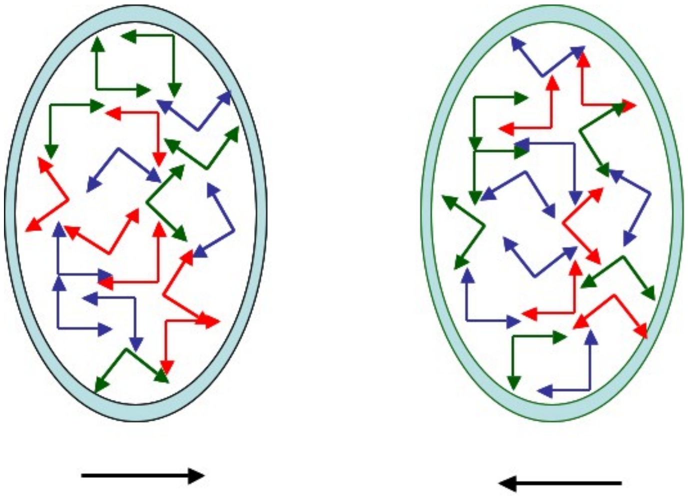
$$Q_{sat}^2 >> \Lambda_{QCD}^2$$

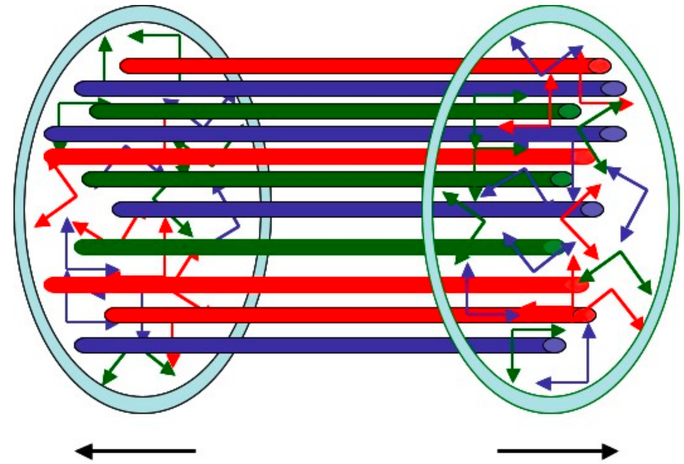




Collisions of two sheets of colored glass

Sheets get dusted with color electric and color magnetic fields





The initial conditions for a Glasma evolve classically and the classical fields radiate into gluons
Longitudinal momentum is red shifted to zero by longitudinal expansion

But the classical equations are chaotic:

Small deviations grow exponentially in time

Chaos and Turbulence:

CGC field is rapidity independent => occupies restricted range of phase space
Wiggling strings have much bigger classical phase space
A small perturbation that has longitudinal noise grows exponentially

$$A_{classical} \sim 1/g$$

$$A_{quantum} \sim 1$$

After a time

$$t \sim \frac{ln^p(1/g)}{Q_{sat}}$$

system isotropizes,

But it has not thermalized!

Thermalization naively occurs when scattering times are small compared to expansion times. Scattering is characterized by a small interaction strength.

How can the system possibly thermalize, or even strongly interact with itself?

Initial distribution:

$$\frac{dN}{d^3x d^3p} \sim \frac{Q_{sat}}{\alpha_s E} \; F(E/Q_{sat})$$

A thermal distribution would be:

$$\frac{dN}{d^3x d^3p} \sim \frac{1}{e^{E/T} - 1} \sim T/E$$

Only the low momentum parts of the Bose-Einstein distribution remain

$$E \sim Q_{sat}$$

$$\text{“}T \sim Q_{sat}/\alpha_s\text{”}$$

As dynamics migrates to UV, how do we maintain isotropy driven by infrared modes with a scale of the saturation momentum?

Phase space is initially over-occupied

$$f_{thermal} = \frac{1}{e^{(E-\mu)/T} - 1}$$

Chemical potential is at maximum the particle mass

$$\rho_{max} \sim T^3 \qquad \epsilon_{max} \sim T^4$$

$$\rho_{max}/\epsilon_{max}^{3/4} \leq C$$

But for isotropic Glasma distribution

$$\rho_{max}/\epsilon_{max}^{3/4} \leq 1/\alpha_S^{1/4}$$

Where do the particle gluons go?

If inelastic collisions were unimportant, then as the system thermalized, the ratio of the energy density and number density are conserved

$$f_{thermal} = \rho_{cond}\delta^3(p) + \frac{1}{e^{(E-m)/T} - 1}$$

One would form a Bose-Einstein Condensate

Over-occupied phase space => Field coherence in important
Interactions can be much stronger that g^2

$$N_{coh} g^2$$

Might this be at the heart of the large amount of jet quenching, and strong flow patterns seen at RHIC?

Problem we try to solve:

How does the system evolve from an early time over-occupied distribution to a thermalized distribution

We argue that the system stays strongly interacting with itself during this time due to coherence

First: Kinetic Evolution Dominated by Elastic Collisions in a Non-Expanding Glasma

$$\partial_t f(p,X) = C_p[f]$$

Blaizot, Gelis, Liao, LM, Venugopalan

$$f(p,X) = \frac{\Lambda_s(t)}{\alpha_s p} g(p/\Lambda(t))$$

$$\Lambda_s(t_i) \sim \Lambda(t_i) \sim Q_{sat}$$

Small angle approximation for transport equation:

$$\left.\frac{\partial f}{\partial t}\right|_{\rm coll} \sim \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_s}{\Lambda_s} f(p)(1+f(p)) \right] \right\}$$

$$\frac{\Lambda\Lambda_s}{\alpha_s} \equiv -\int_0^\infty dp\, p^2 \frac{df}{dp}$$

$$\frac{\Lambda\Lambda_s^2}{\alpha_s^2} \equiv \int_0^\infty dp\, p^2 f(1+f)$$

Due to coherence, the collision equation is independent of coupling strength!

There is a fixed point of this equation corresponding to thermal equilibrium when

$$T \sim \Lambda \sim \Lambda_s/\alpha_s$$

Estimates of various quantities
(Momentum integrations are all dominated by the hard scale)

$$n_g \sim \frac{1}{\alpha_s}\Lambda^2\Lambda_s \qquad \epsilon_g \sim \frac{1}{\alpha_s}\Lambda_s\Lambda^3 \qquad \frac{\epsilon_g}{n_g} \sim \Lambda$$

$$n = n_c + n_g \qquad \epsilon_c \sim n_c\, m \sim n_c\sqrt{\Lambda\Lambda_s}$$

$$m^2 \sim \alpha_s \int dp\, p^2 \frac{df(p)}{d\omega_p} \sim \Lambda\Lambda_s$$

The collision time follows from the structure of the transport equation and is

$$t_{scat} = \frac{\Lambda}{\Lambda_s^2}$$

The scattering time is independent of the interaction strength

Thermalization in a non-expanding box

$$\epsilon = \Lambda_s \Lambda^3 \sim constant \qquad t_{scat} = \frac{\Lambda}{\Lambda_s^2} \sim t$$

So that

$$\Lambda_s \sim Q_s \left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

$$n_g \sim n_0 \left(\frac{t_o}{t}\right)^{1/7} \qquad m \sim Q_s (t_0/t)^{1/7} \qquad \frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{1/7}$$

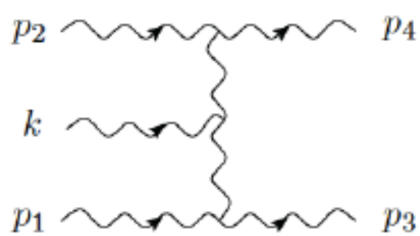
$$s \sim \Lambda^3 \sim Q_s^3 (t/t_0)^{3/7}$$

At thermalization $\Lambda_s = \alpha_s \Lambda$

$$t_{th} \sim \frac{1}{Q_s}\left(\frac{1}{\alpha_s}\right)^{\frac{7}{4}} \qquad s \sim Q_s^3/\alpha_s^{3/4} \sim T^3$$

How do inelastic processes change this?

Rates of inelastic and elastic processes are parametrically the same



$$\frac{1}{t_{scat}} \sim \alpha_s^{n+m-2} \left(\frac{\Lambda_s}{\alpha_s}\right)^{n+m-2} \left(\frac{1}{m^2}\right)^{n+m-4} \Lambda^{n+m-5}$$

$$m^2 \sim \Lambda_s \Lambda$$

$$t_{\rm scat} = \frac{\Lambda}{\Lambda_{\rm s}^2},$$

What about the condensate? Difficult to make definite statement.
In relaxation time limit, we would expect:

$$\frac{d}{dt}\rho_{cond} = -\frac{a}{t_{scat}}\rho_{cond} + \frac{b}{t_{scat}} n_{gluons}$$

Either $\rho_{cond} >> n_{gl}$ or $\rho_{cond} = \frac{b}{a} n_{gluons}$

Condensate would rapidly evaporate near thermalization time

Effect of Longitudinal Expansion

$$\partial_t f - \frac{p_z}{t}\partial_{p_z} f = \left.\frac{df}{dt}\right|_{p_z t} = C[f] \qquad \partial_t \epsilon + \frac{\epsilon + P_L}{t} = 0$$

Assume approximate isotropy restored by scattering.
Will check later that this is consistent.

$$P_L = \delta\epsilon \qquad 0 < \delta < 1/3$$

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta} \qquad \Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

$$\left(\frac{t_{th}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{\frac{7}{3-\delta}}$$

The asymmetry parameter:

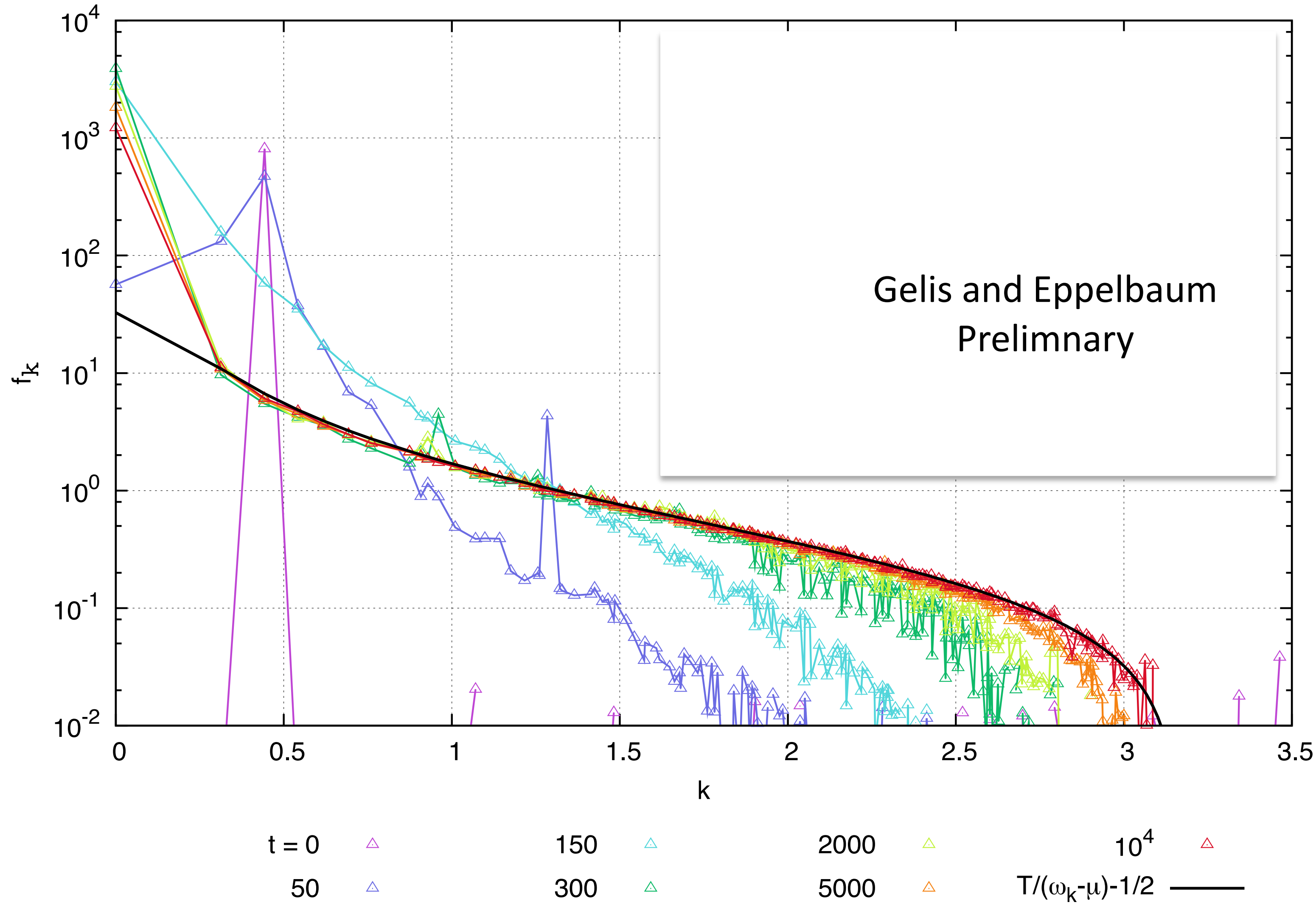
Take moments of transport equation

$$< p_x^2 > \qquad < p_T^2 >$$

Prove that the solutions for these moments reduce to constants times the ultraviolet scale at large times

$$< p_x^2 > / < p_T^2 > \sim constant$$

Need to know solutions to transport equation to determine the value of the anisotropy parameter



Gelis and Eppelbaum
Prelimnary
f_k
k
t = 0
50
150
300
2000
5000
10^4
T/(ω_k-μ)-1/2

Phenomenological Consequences:

Flow is more easily generated since system is anisotropic and also generates flow at earliest times

Jet quenching at early times should be stronger due to coherence of media

Will get more flow for photons than hydrodynamic simulations, but enough?

Contribution to di-lepton spectrum from gluons annihilating in condensate to virtual quark loop. Not suppressed by powers of interaction due to coherence. Di-lepton pair will have very small transverse momentum because condensate gluons have very small transverse momentum.

Geometric scaling of photon and di-lepton distributions lead to huge enhancements

$$\frac{dN}{dydM^2} \sim \pi R^2 \left(\frac{Q_{sat}^2}{M^2} \right)^p \quad p \sim 3-4$$