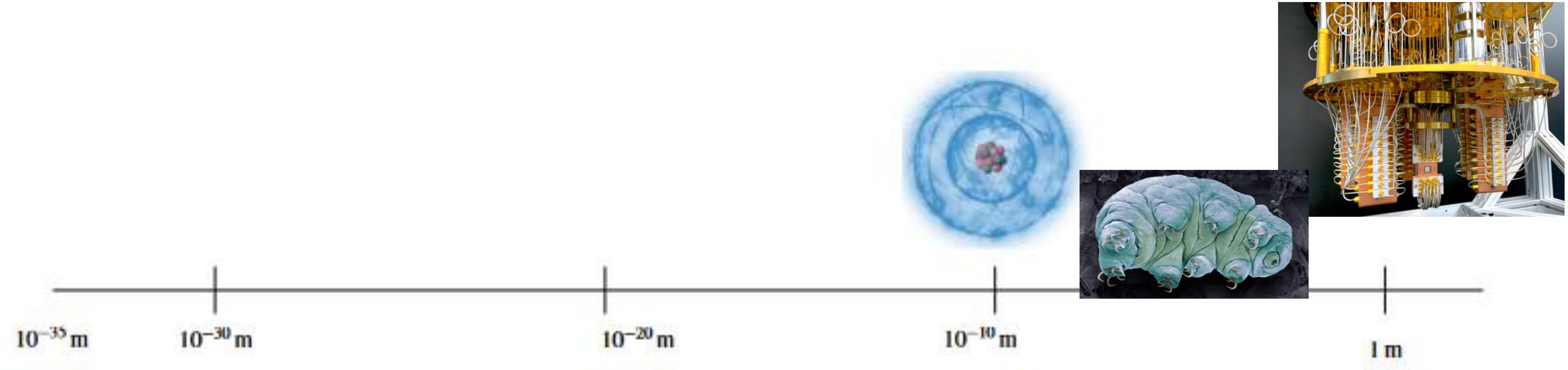
QUANTUM OBSERVABLES AT COLLIDERS

FEDERICA FABBRI

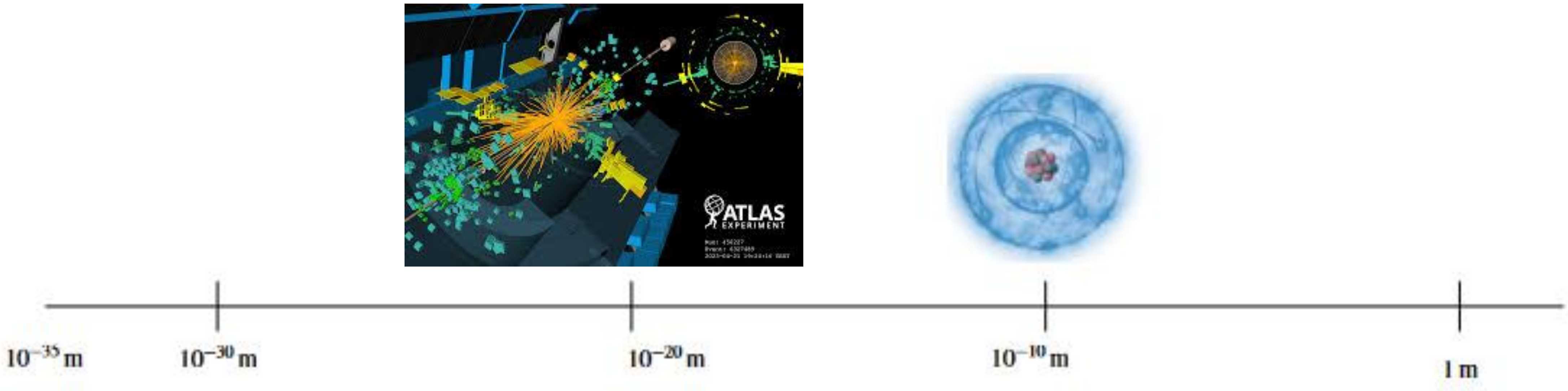
Quantum observables at colliders

- Recent years have seen several development in quantum computing and technologies.
- Quantum information ideas/concept applied to several systems: photons, atoms, molecules, tardigrade



Quantum observables at colliders

- Can we go in the other direction?
- Try to apply concepts as entanglement and Bell's inequality violation to fundamental particles generated at colliders: quarks, bosons



Motivations

Highest energy test of the quantum nature of reality

Try to complete the picture provided by the standard model of particle physics

Study of highly energetic qudit that are interacting with fundamental forces and self-decaying

Highly multidisciplinar sector, that highy mixes quantum information, theoretical particle physics and experimental particle physics

Entanglement in flavored scalar scattering

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Quantum tops at circular lepton colliders

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Probing entanglement in top quark production with the CMS detector

CMS Collaboration

- ~100 pheno paper since the beginning of the year
  - almost one paper per day
- 2 experimental papers on the topic (one ATLAS and one CMS) in the last year

sent and future particle

Entanglement suppression an

Tao-Ran Hu,[1,*] Su C

ization in jet production:

er model

Dmitri Kharzeev,[1,2,3,§]

Kwangmin Yu[6,††]

Polarization and quantum entanglement effects in $B_c^{\pm} \to J/\psi + \pi^{\pm} + \pi^0$ process

Kaiwen Chen[1], Yiqi Geng[1,2*], Yichao Jin[1†], Zhicheng Yan[1,2], Ruilin Zhu[1,3,4‡]

Outline

Introduction to QI concepts

Qubits

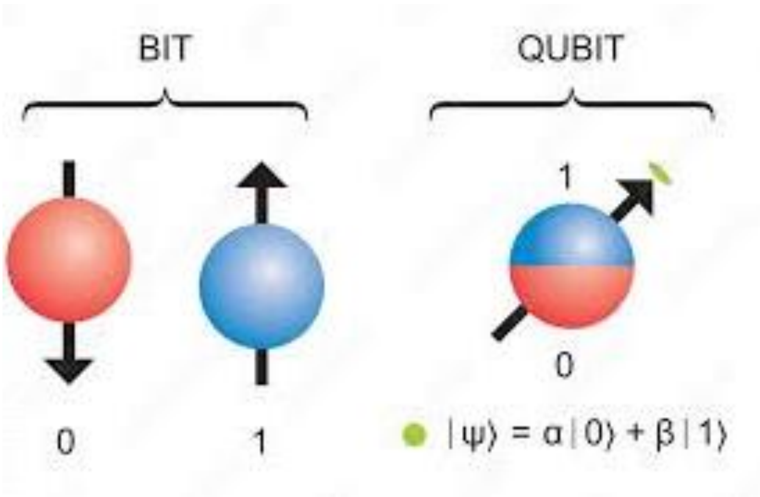
The most fundamental brick of QI is a qubit:

- When observed it can only assume 2 possible states: $|0\rangle$ and $|1\rangle$
- The qubit can be described as a super-imposition of these two states: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- The probability of measuring it in state $|0\rangle$ is $|\alpha|^2$
- The measurement causes the wave function to collapse

Fundamental particles can be treated as representation of qubits, in a specific property that can be measured: spin

- A fermion is a representation of a qubit
- A boson is a representation of a qutrit

It is possible to use different quantum properties, e.g. flavour (talk from S. Vahsen talk for Bell and Bell II collaborations)



System of qubits

- System of qubits manifest properties un-explainable in classical physics as entanglement

$$\psi_c = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- A certain state ψ_c is entangled if it cannot be written as two separate sub-states
- A pure state can be described by a single $|\psi\rangle$, while while a mixed state can be in one of the multiple quantum state $|\psi\rangle$
  - In the particle physics systems generated at colliders we never have full states: mixture of energies, processes
  - System by a spin density matrix $\rho = \sum p_i \ |\psi_i\rangle\langle\psi_i|$
  - The separability/entanglement of a state can be determined starting from the spin density matrix applying specific criteria
    - Eg Peres-Horodecki criterion [ref]
  - From this it is also possible to determine sufficient conditions for entanglement
  - Another quantity that can be derived from the spin density matrix and quantifies the entanglement is the concurrence 0<=C[ρ]<=1
- The same concepts can be extended to systems of qutrits → but everything is more difficult, including determining if a system is entangled

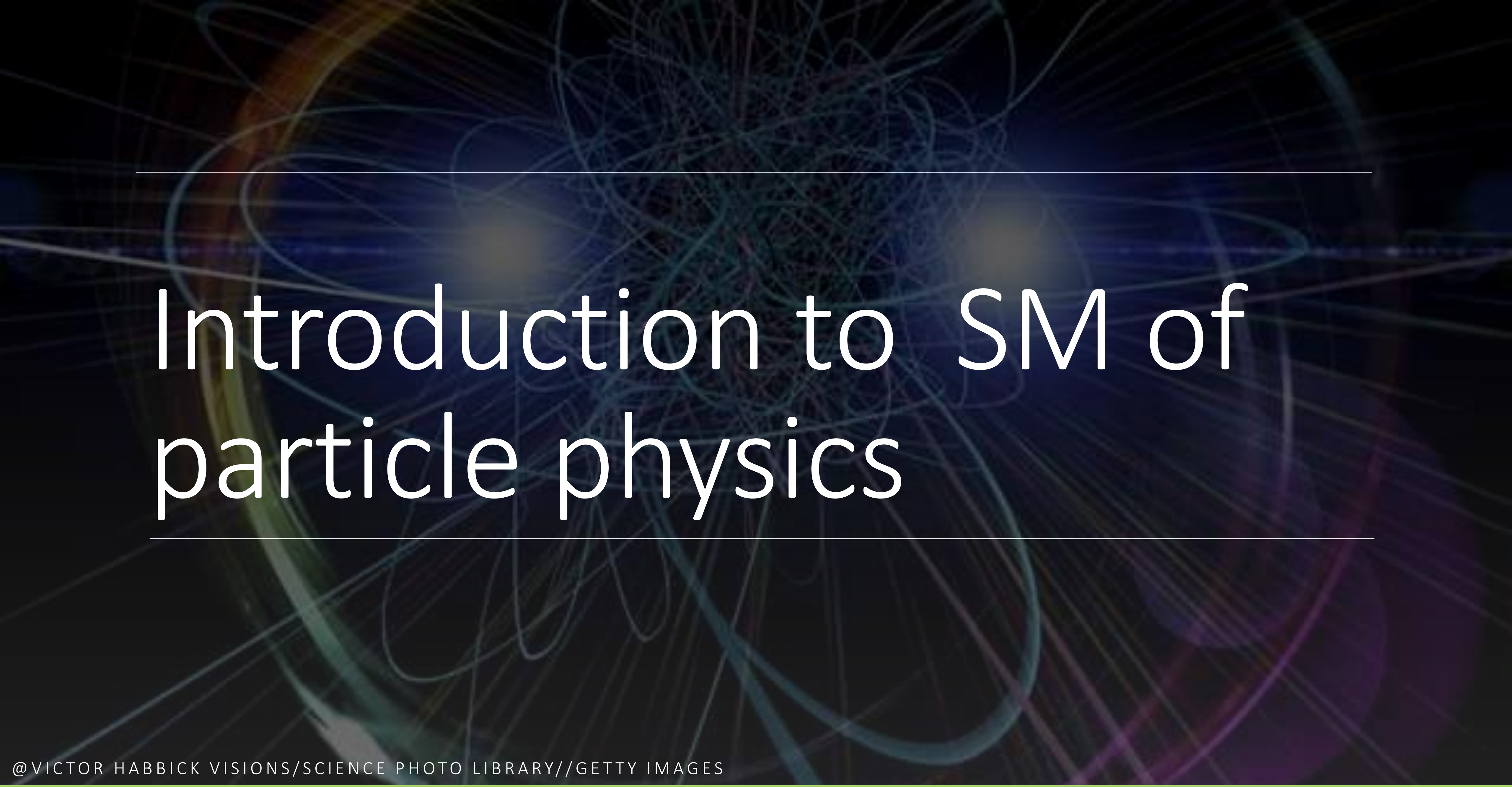
Bell's inequality

- Two different observers Alice and Bob can perform two different measurements (A,A') and (B,B') on a system composed by two qubits.
  - The measurement can only have 2 outcomes (-1,1)
- By combining the expectation value of different experiments can be written the following equation:

$$B = E(AB) + E(A'B) + E(A'B') - E(AB')$$

- "Classical" physics predicts B to be smaller than 2, and this is based on two assumptions:
  - Realism: the measured property of the qubit is defined prior to the measurement
  - Locality: the measurement performed by Alice on one sub-system cannot influence the other subsystem
- Quantum mechanics predicts that this equation can be violated
- Several measurements in multiple systems observed the violation of this inequality
- Measuring it at LHC would be the highest energy Bell's inequality test.

Introduction to SM of particle physics



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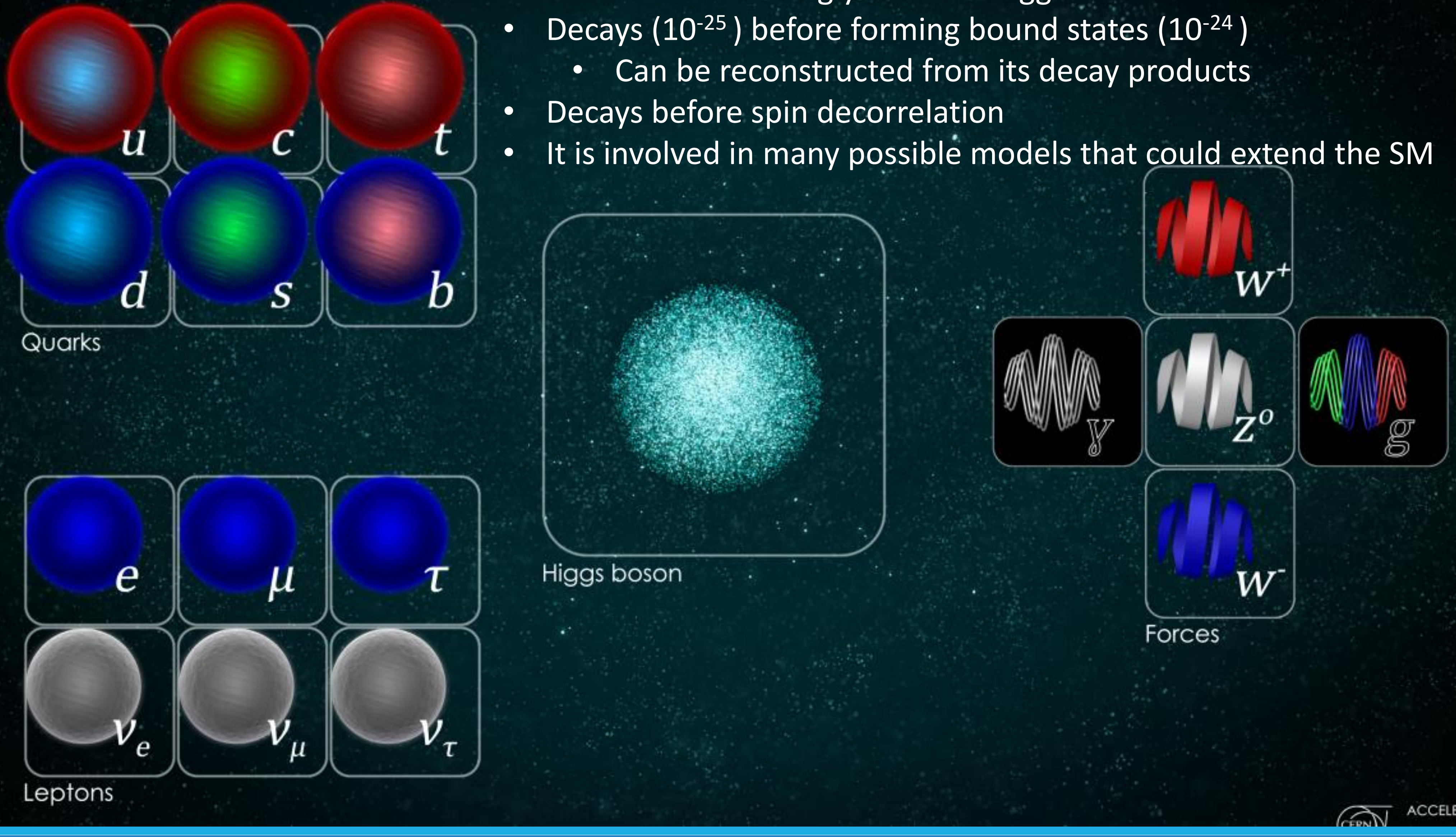
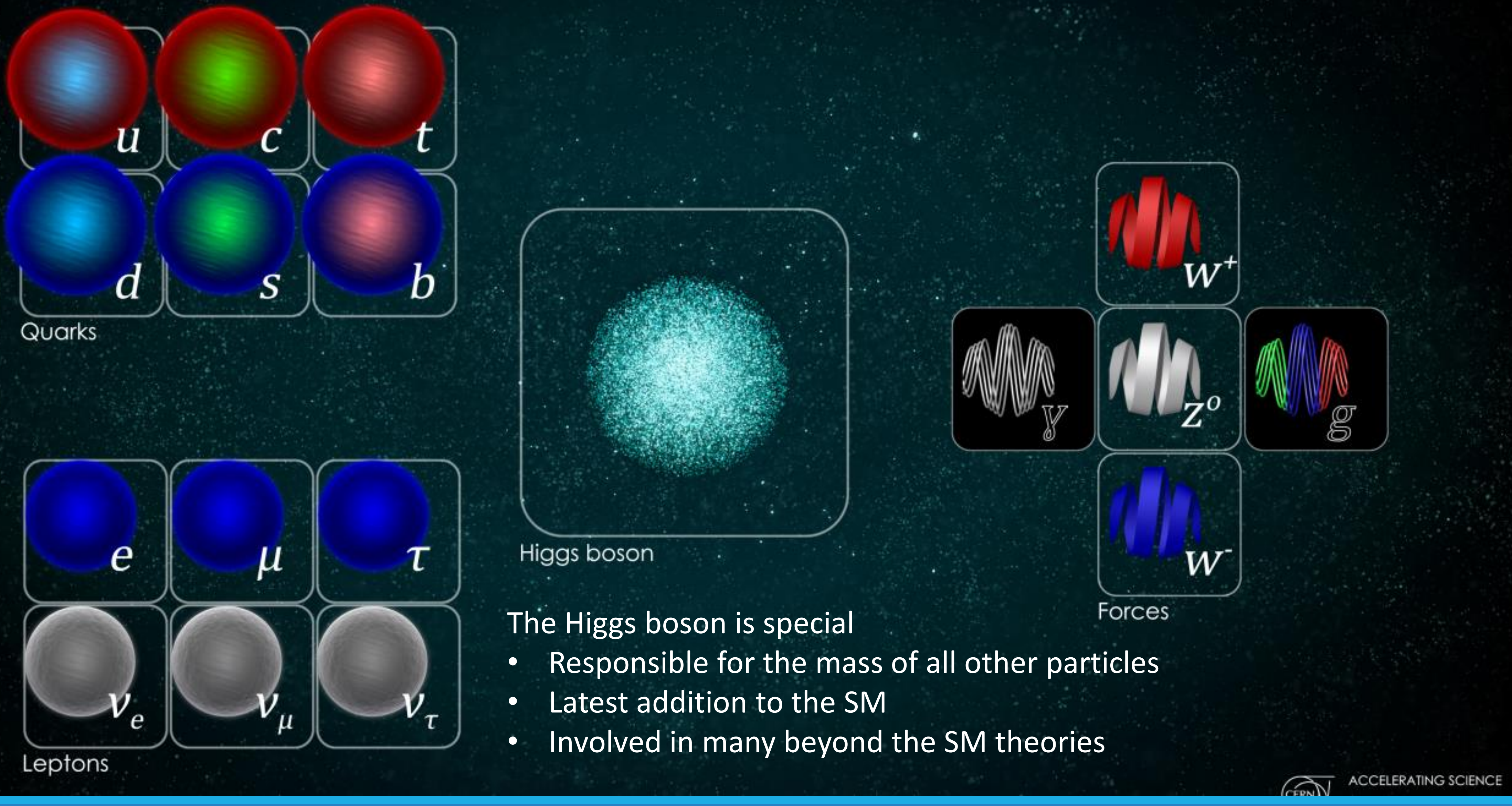
The top-quark is special:

- Heaviest particle in the SM (mass close to a gold atom)
    - Interacts strongly with the Higgs sector
- Decays (10^{-25}) before forming bound states (10^{-24})
    - Can be reconstructed from its decay products
- Decays before spin decorrelation
- It is involved in many possible models that could extend the SM



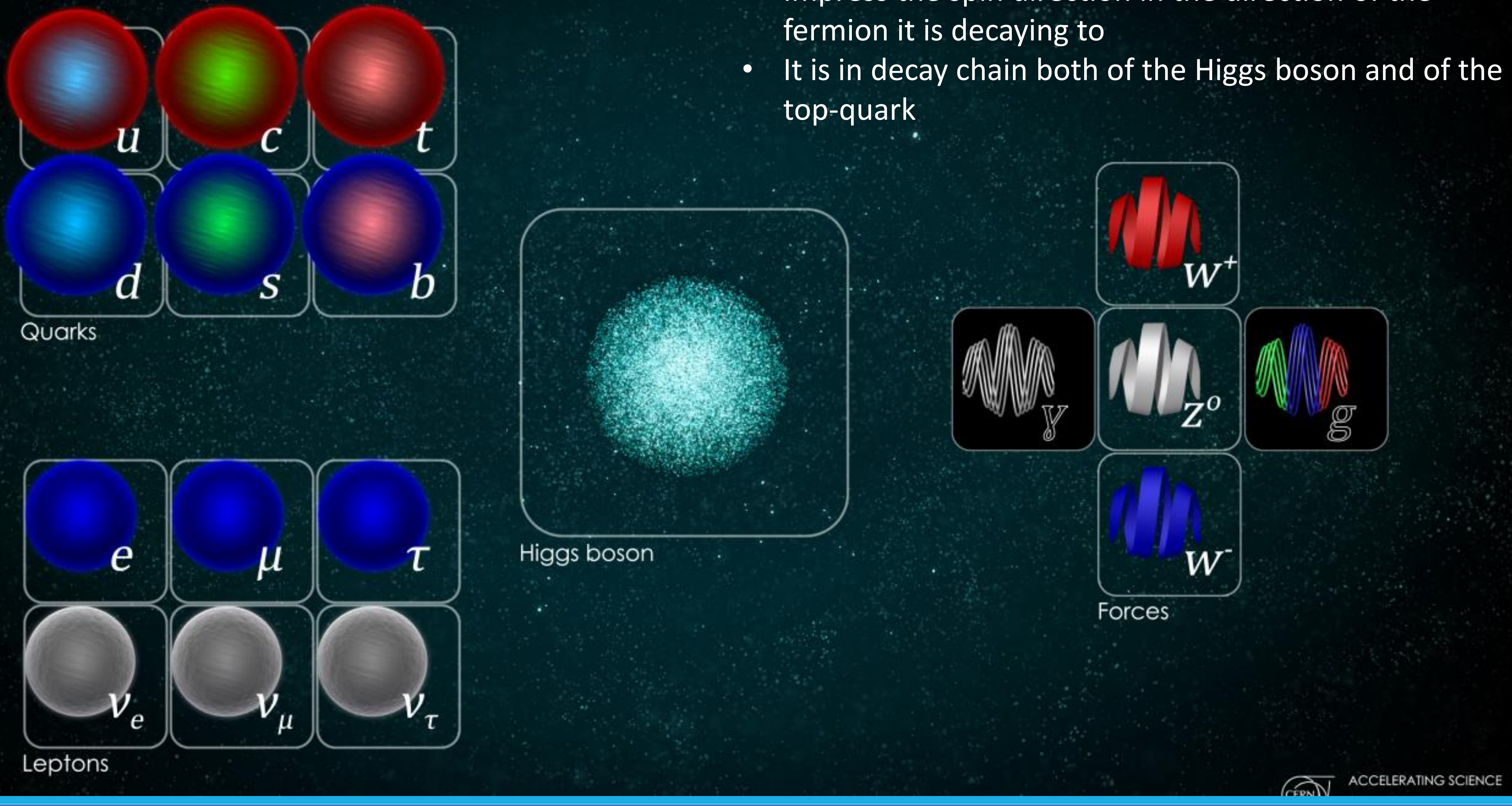


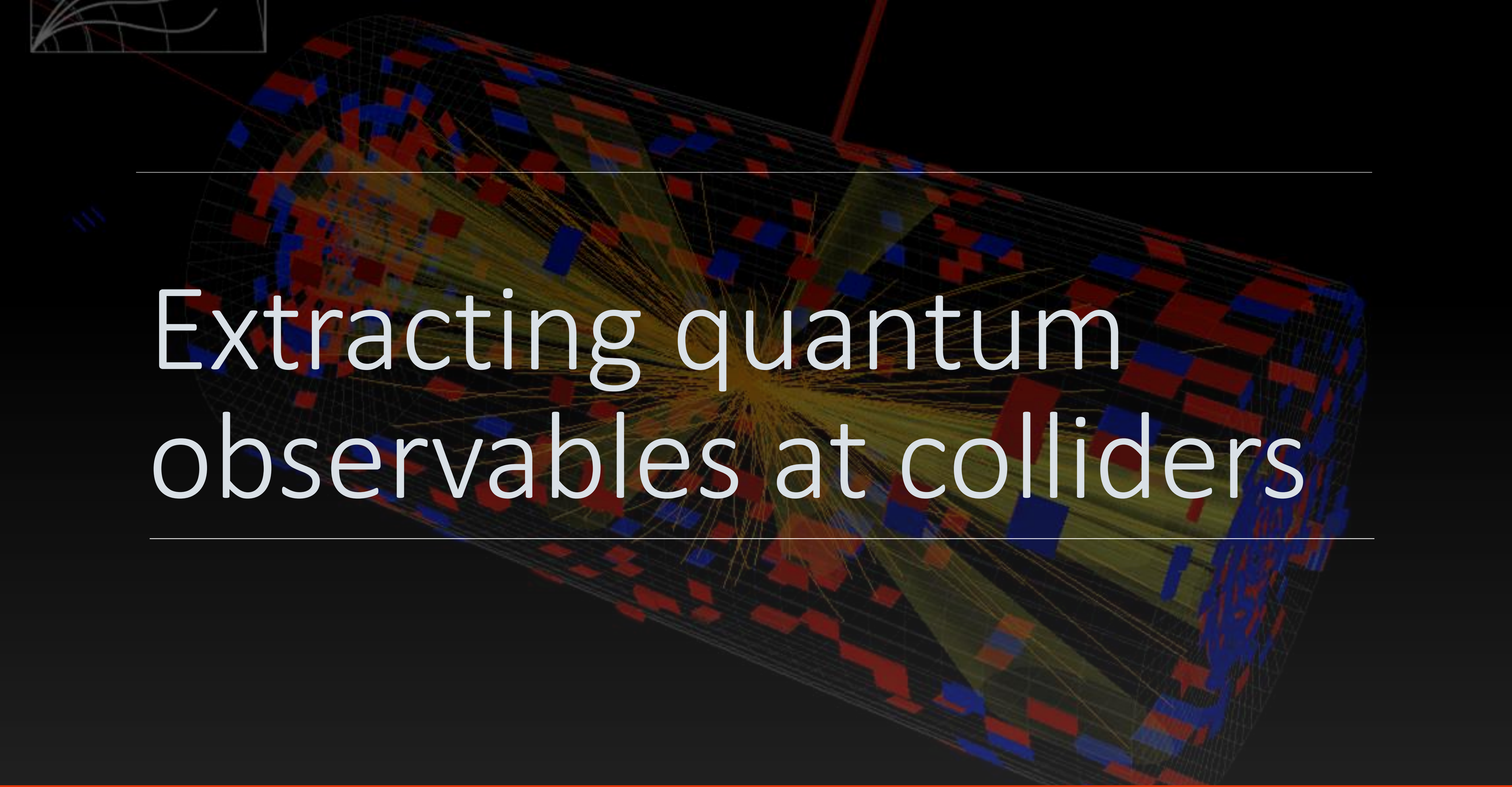
The Higgs boson is special

- Responsible for the mass of all other particles
- Latest addition to the SM
- Involved in many beyond the SM theories

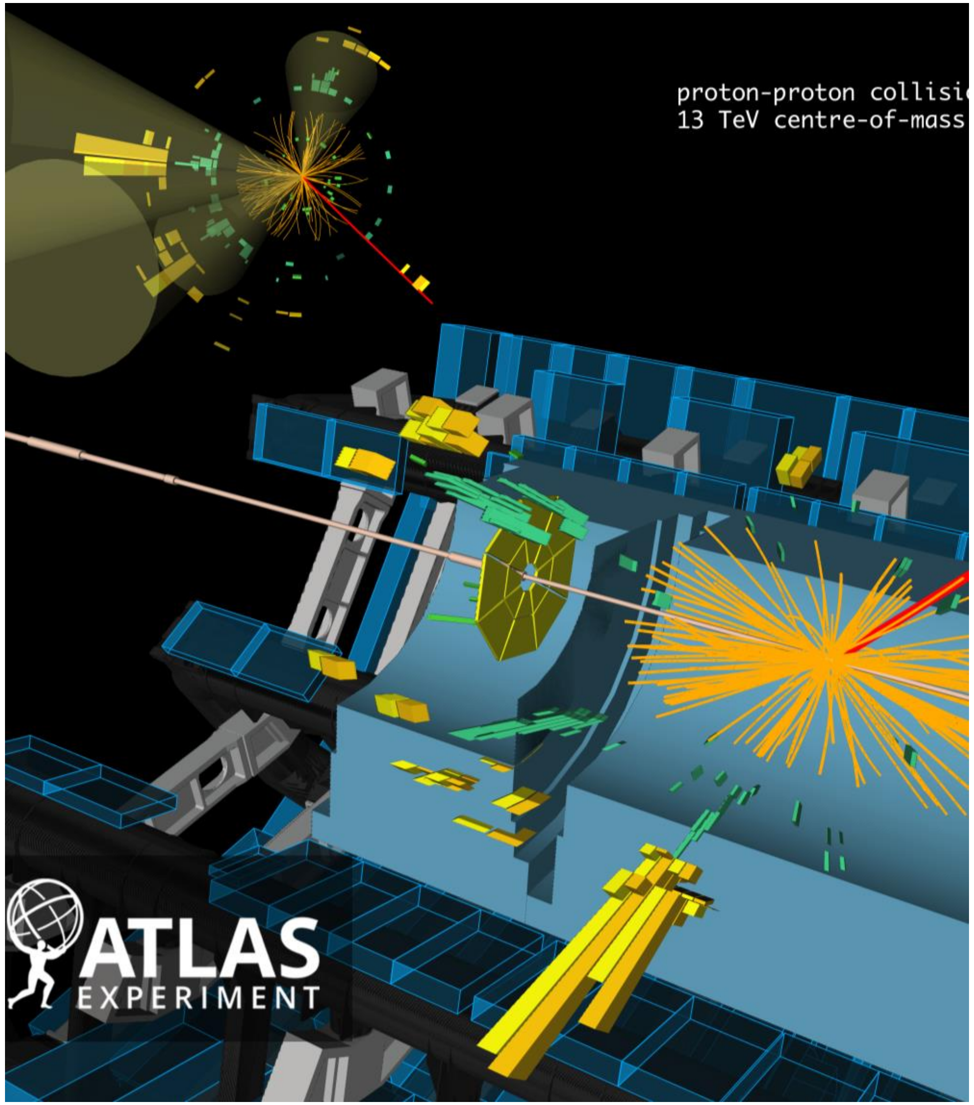
The W-boson is special:

- Electroweak interaction mediator
- Impress the spin direction in the direction of the fermion it is decaying to
- It is in decay chain both of the Higgs boson and of the top-quark





Extracting quantum observables at colliders



Quantum tomography

- Unfortunately, we cannot directly measure the spin direction of the particles created at LHC in every event
- Quantum tomography: build many similar states and extract the component of the spin density matrix as averages of specific quantities.
- Many similar states: pp collisions produced at LHC
  - Then selected with dedicated requirement
- Specific quantities: angular distributions of the decay products of the interesting particles
  - In the center of mass of the top quark/ W boson
- The relation between the direction of the decay products and the spin is due to the nature of the electroweak interaction

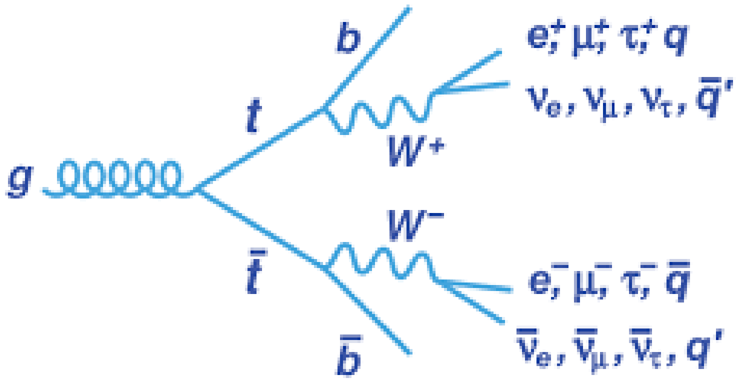
Qutrit quantum tomography

$$R = \tilde{A} I_4 + \sum_i \left(\tilde{B}_i^+ \sigma^i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \otimes \sigma^j \qquad \rho = \frac{R}{\mathrm{tr}(R)}$$

There is a relation between the angular distribution of the top quark decay product and the spin density matrix

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

$$C_{ii} = \frac{9 \left\langle \cos(\theta_i^a) \cos(\theta_i^b) \right\rangle}{\beta_a \beta_b}$$



- The β coefficients quantify the relation between the direction of the particle of interest spin and the direction of the decay product
  - For the (anti)top quark it is -1/+1 only for charged leptons and down type quarks.

Qutrit quantum tomography

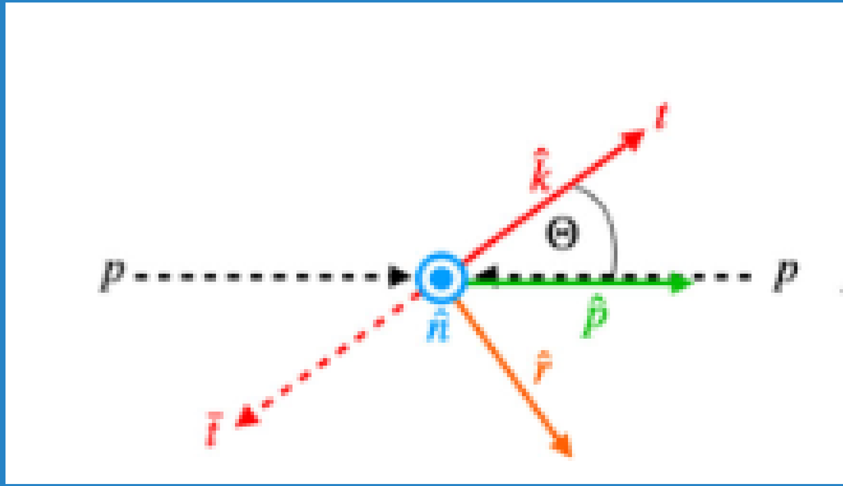
$$R = \tilde{A} I \ldots \qquad \frac{R}{\ldots(R)}$$

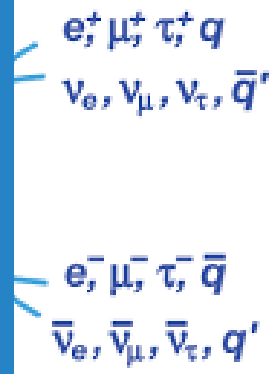
There is a rel… …e spin density matr…

$$\frac{1}{\sigma}\frac{\mathrm{d}\ldots}{\mathrm{d}\Omega_{+}\ldots}$$

$$C_{ii} = \frac{9\left\langle \cos(\theta_i^a)\cos(\theta_i^b)\right\rangle}{\beta_a \beta_b}$$

The angles are between the decay products and a specific frame, defined in the top-pair rest frame.





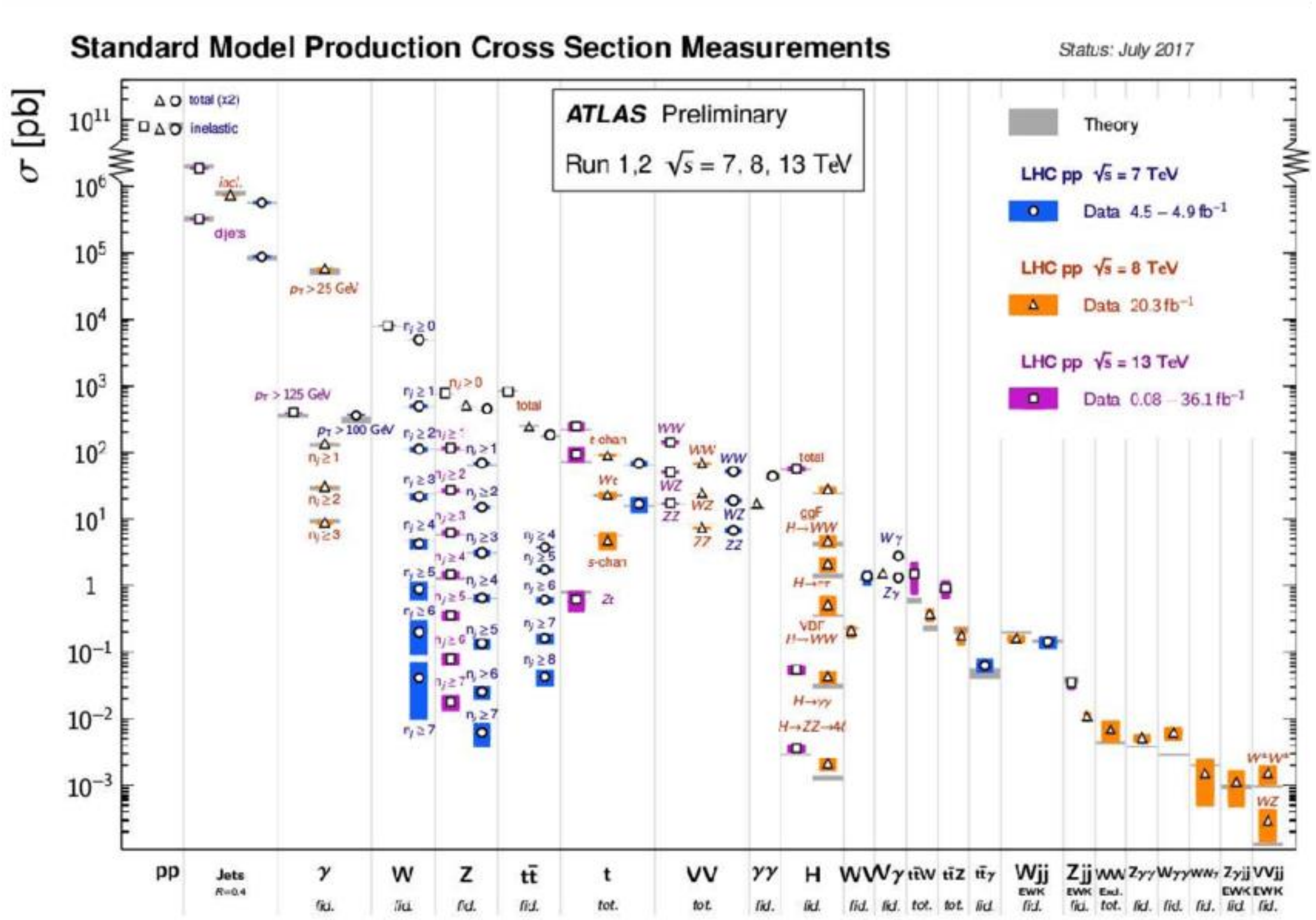
- The β co… …st spin and the direction of the decay product
  - For the (anti)top quark it is -1/+1 only for charged leptons and down type quarks.

Interesting processes

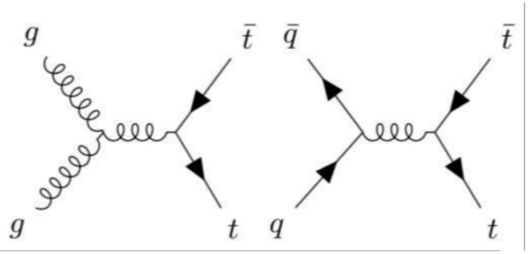
@Rafael Aoude

Standard model results

- Top pair production:
  - Very high cross-section, pure final state
- Higgs boson production:
  - Lower cross section
  - Hard to separate from the signal

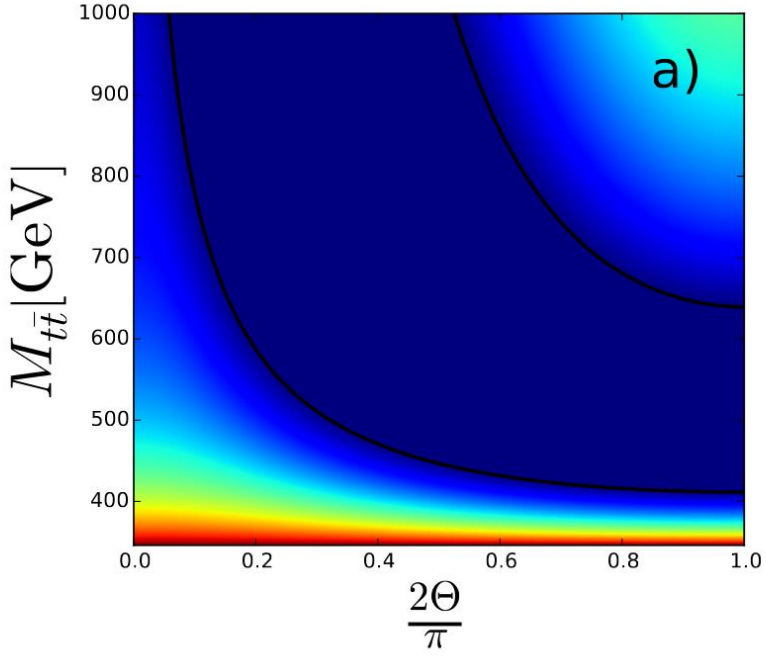


Top-quark pair production

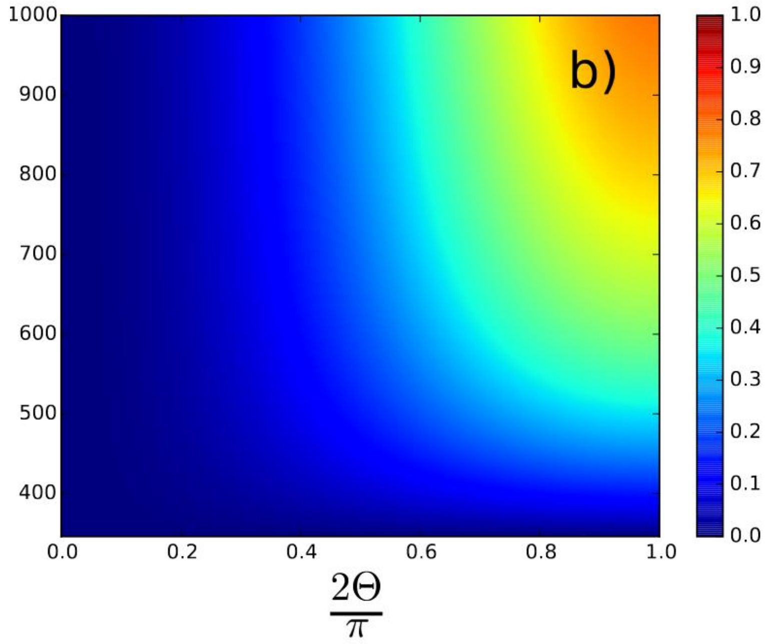


- Two different production channel at LHC (90% are gg)
- The two states are different, and so is the entanglement between the top quarks produced in the two ways

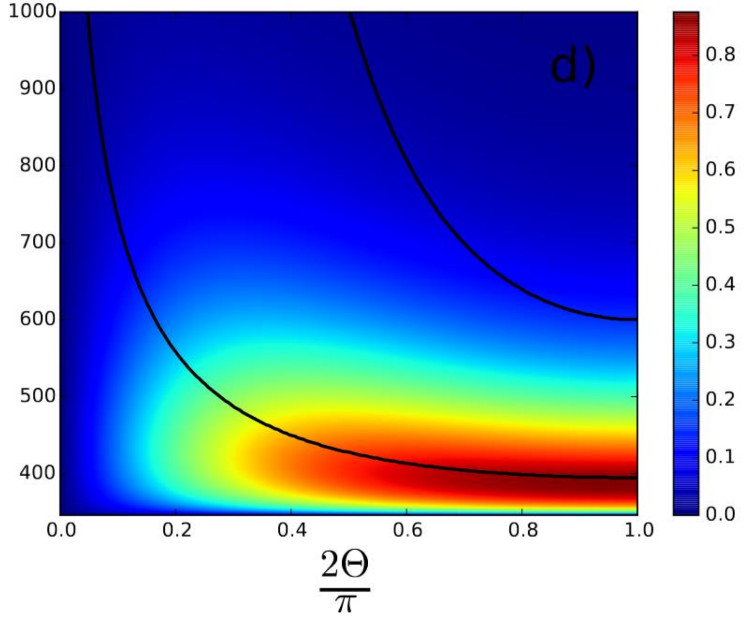
Concurrance gg



Concurrance qq



σ (cross-section)



Y. Afik, J.R.M. de Nova *Eur. Phys. J. Plus* **136**, 907 (2021)

Measuring the entanglement in tt

- From the Peres-Horodecki criterion it is possible to derive multiple sufficient conditions for entanglement based exclusively on the diagonal coefficient of the spin density matrix.
  - The best marker depends on the region of the phase space

- In the threshold region the following marker has the largest value:

$$-C_{11}-C_{22}-C_{33} > 1$$

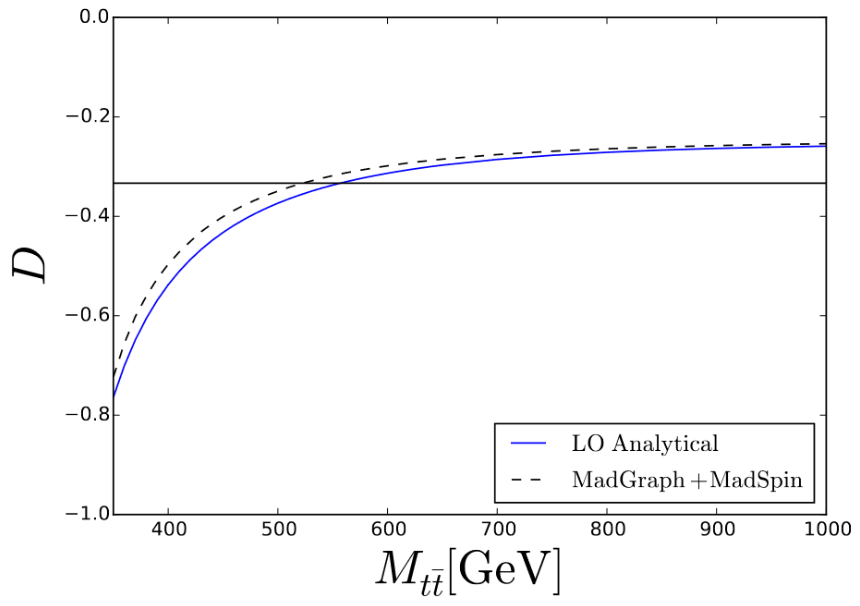
$$D = -(C_{11}+C_{22}+C_{33})/3$$

- D can be measured with a single angular distribution.

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos(\varphi)} = \frac{1}{2}\left(1+\beta_a\beta_b\, D\cos(\varphi)\right)$$

- Where φ is the angle between the two decay products of the top quark pairs

- D < -1/3 means that the top-quarks are entanglement



Y. Afik, J.R.M. de Nova *Eur. Phys. J. Plus* **136**, 907 (2021)

[Z. Dong, D. Goncalves, K. Kong, A. Navarro arXiv:2305.07075](https://arxiv.org/abs/2305.07075)

What about Bell's inequality?

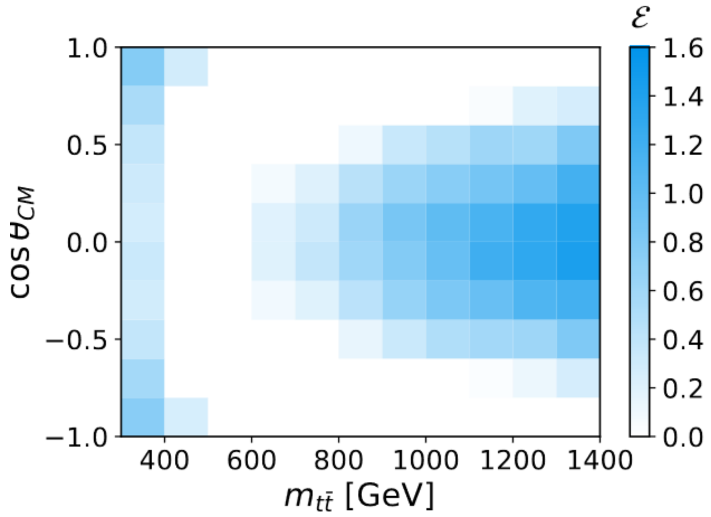
The entanglement between the two top quarks is only a necessary but not sufficient condition for the violation of Bell's inequality.

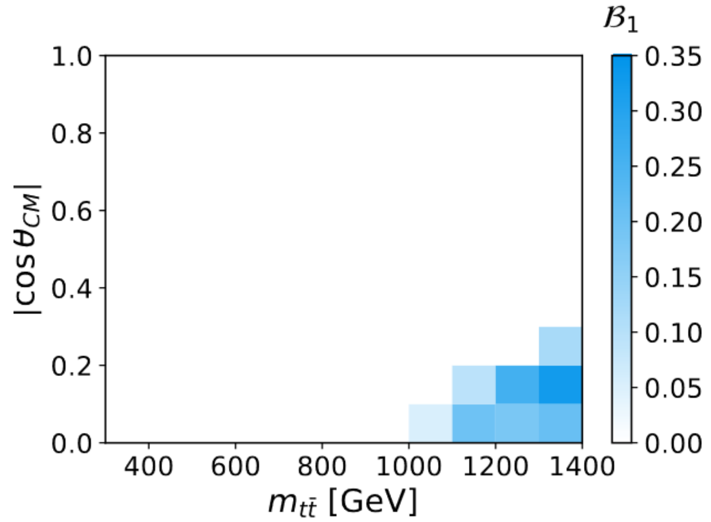
Dedicated Bell's operators applied to the spin density matrix allow to identify a relation between Bell's inequality violation and the coefficient of the spin density matrix

$\mathcal{B}_1 \equiv |Crr - Cnn| - \sqrt{2} > 0$

These will be harder to observe in LHC collisions:

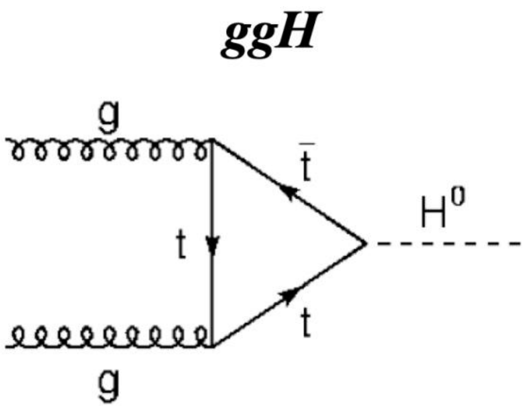
- Possible only in the high mass region
- Very limited corner of the phase space

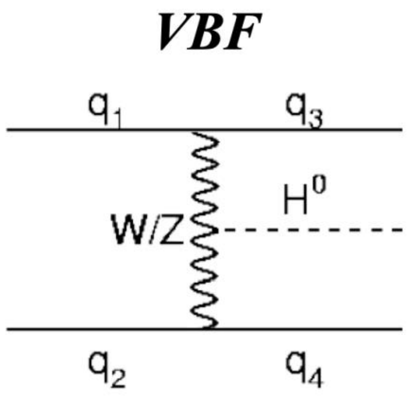


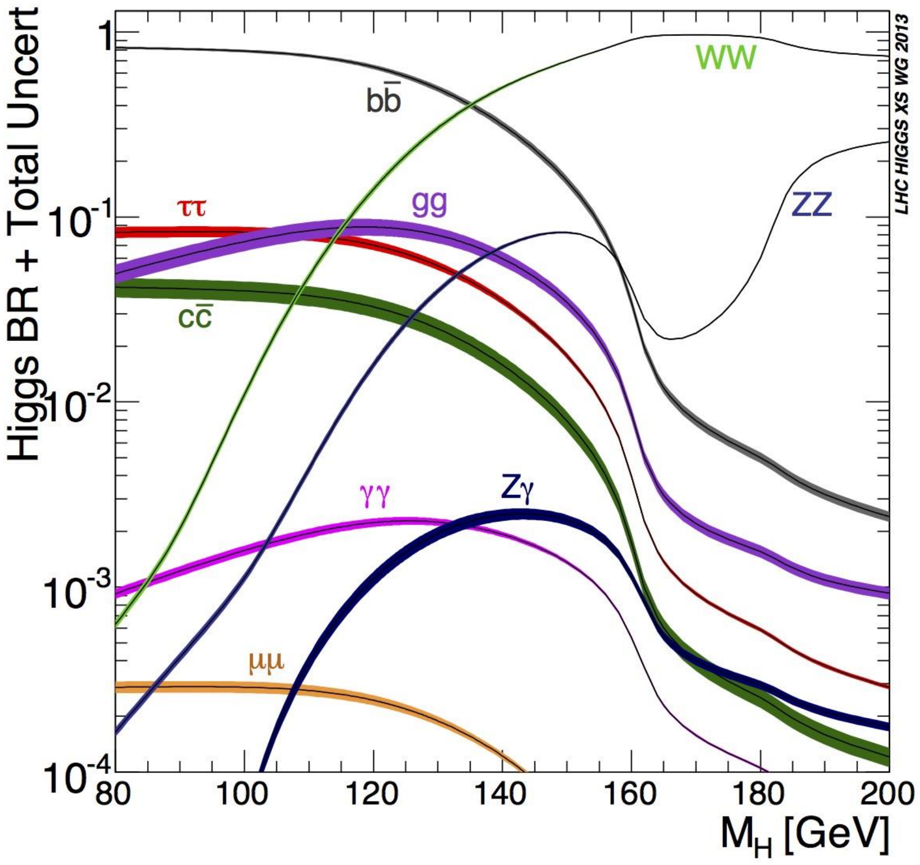


Higgs Boson – production and decay

- Higgs characteristic:
- It is a scalar (spin 0)
  - Decay products are always maximally entangled
  - Independently on the production channel
- It has a mass of 125 GeV → < that 2 times the W mass
  - One of the W bosons produced is always off-shell



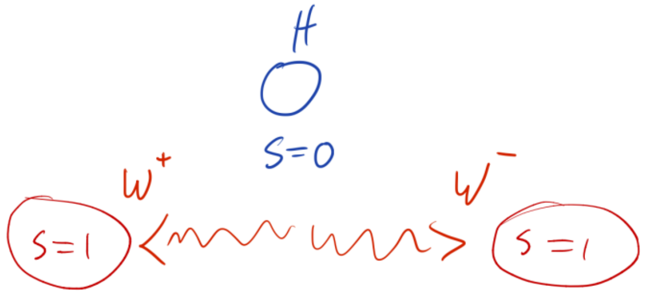


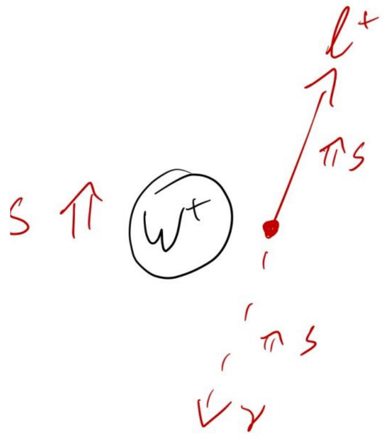


Quantum operators

- The W bosons from Higgs decay can be treated as qutrits.
- The W bosons should be always maximally entangled
  - Due to the off-shelness of one of the bosons this is not always the case, maximally entangled only at rest
- The spin analysing power for a W+ boson is +1 only for charged leptons and down-type quark.
- Also in this case it is possible to use a Bell's operator (optimal for qutrits) to test the correlations between the W bosons
  - No need to have strict phase space requirements as in the top-pair case

$m_W^<$ [GeV]	0	20	30	40	50
$\mathcal{I}_2^{\text{gen,max}}$	1.78	1.91	1.96	1.94	1.95
$\mathcal{I}_3^{\text{xyz}}$	2.62	2.76	2.81	2.82	2.77



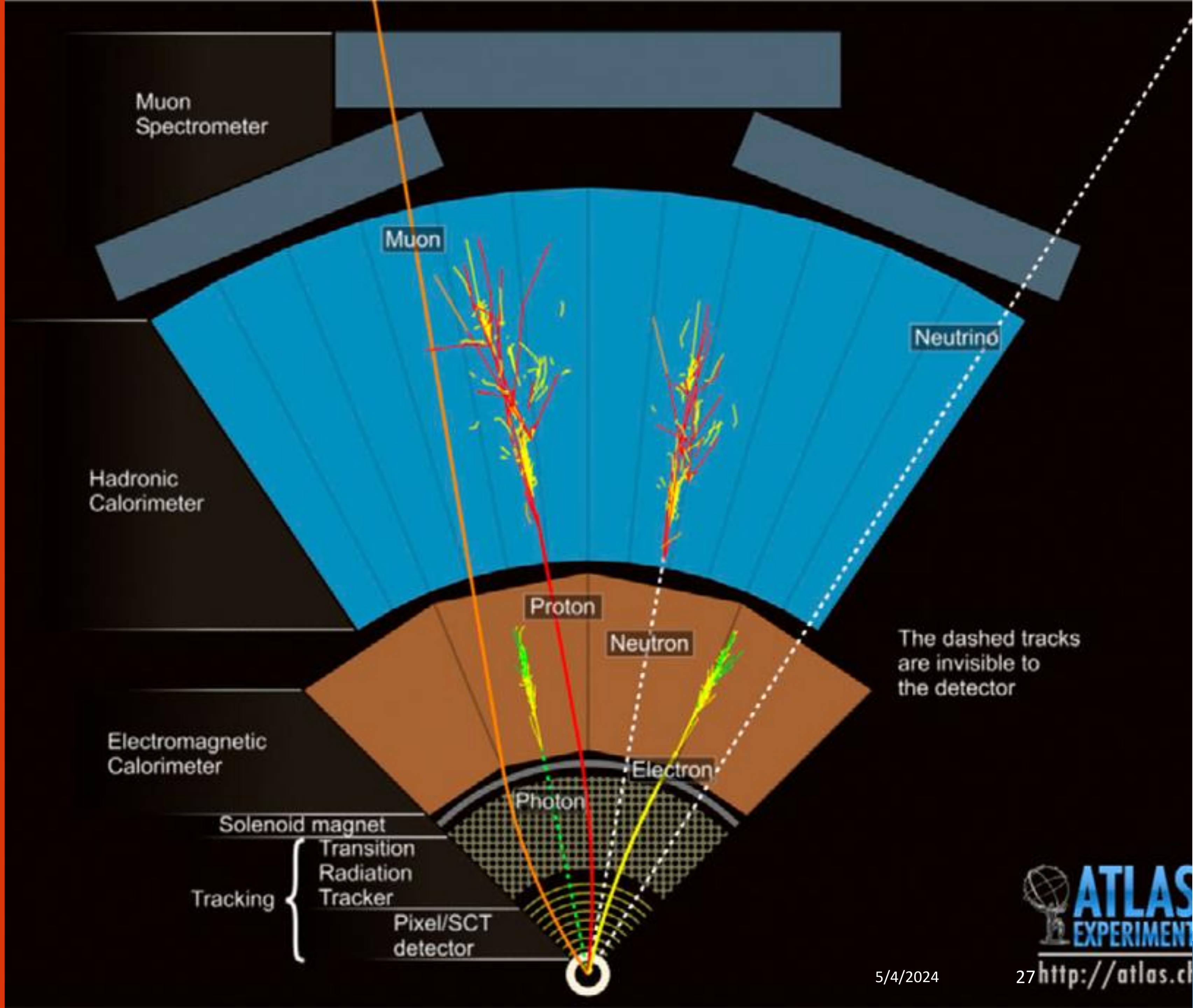


@Alan Barr

Challenges at colliders

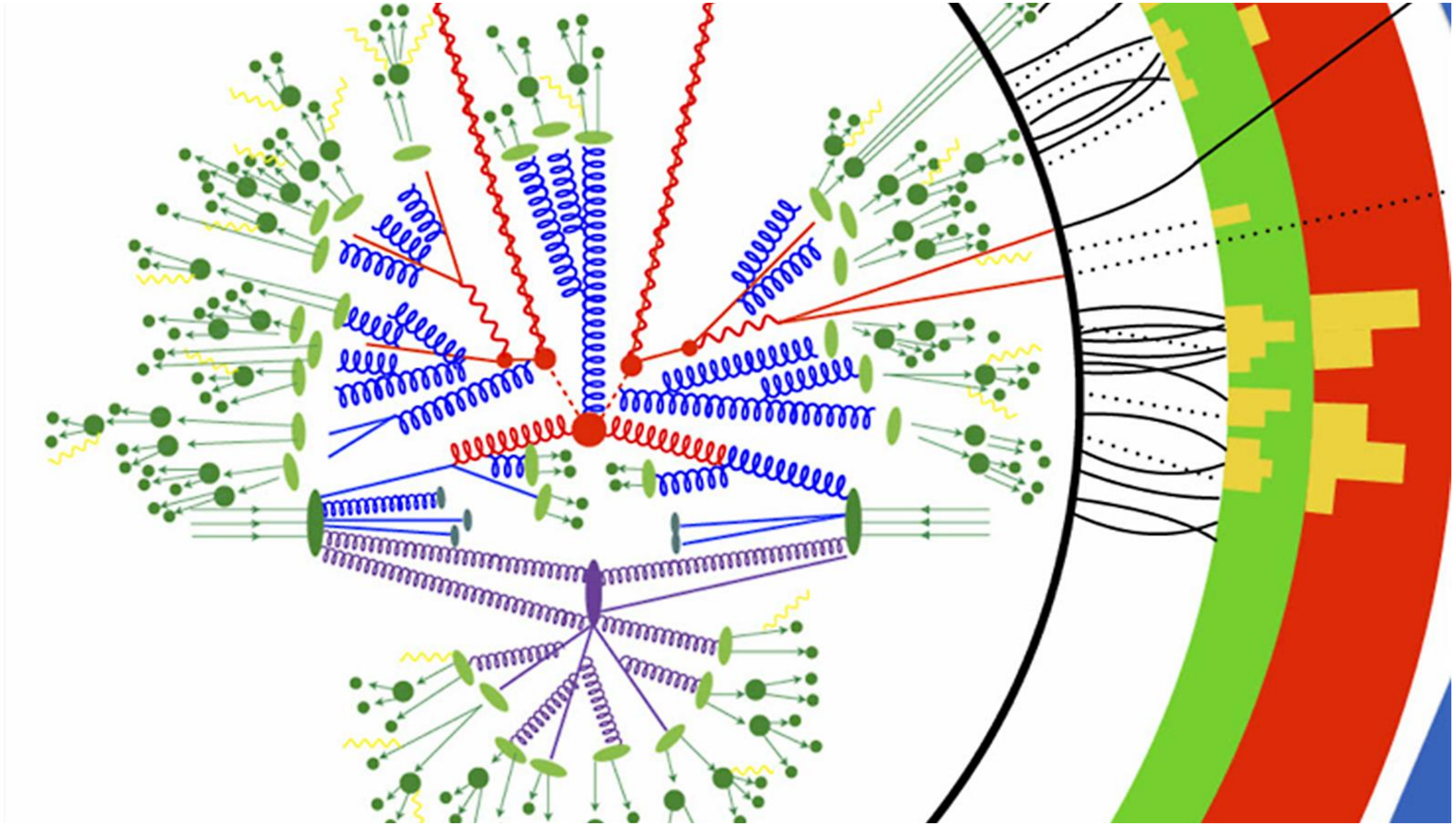
Neutrinos

- The charged leptons are measured with high precision and accuracy at ATLAS/CMS
  - maximum spin analysing power
- For the entanglement measurement angles have to be measured in the top/W rest frames
  - The system must be fully reconstructed.
  - All decay products must be measured (precisely)
- Every time the W decays with a charged lepton in the final state also a neutrino is emitted.
  - Neutrinos 4-momentum can only be inferred from missing energy on the transverse plane
  - Events with multiple neutrinos are extremely difficult



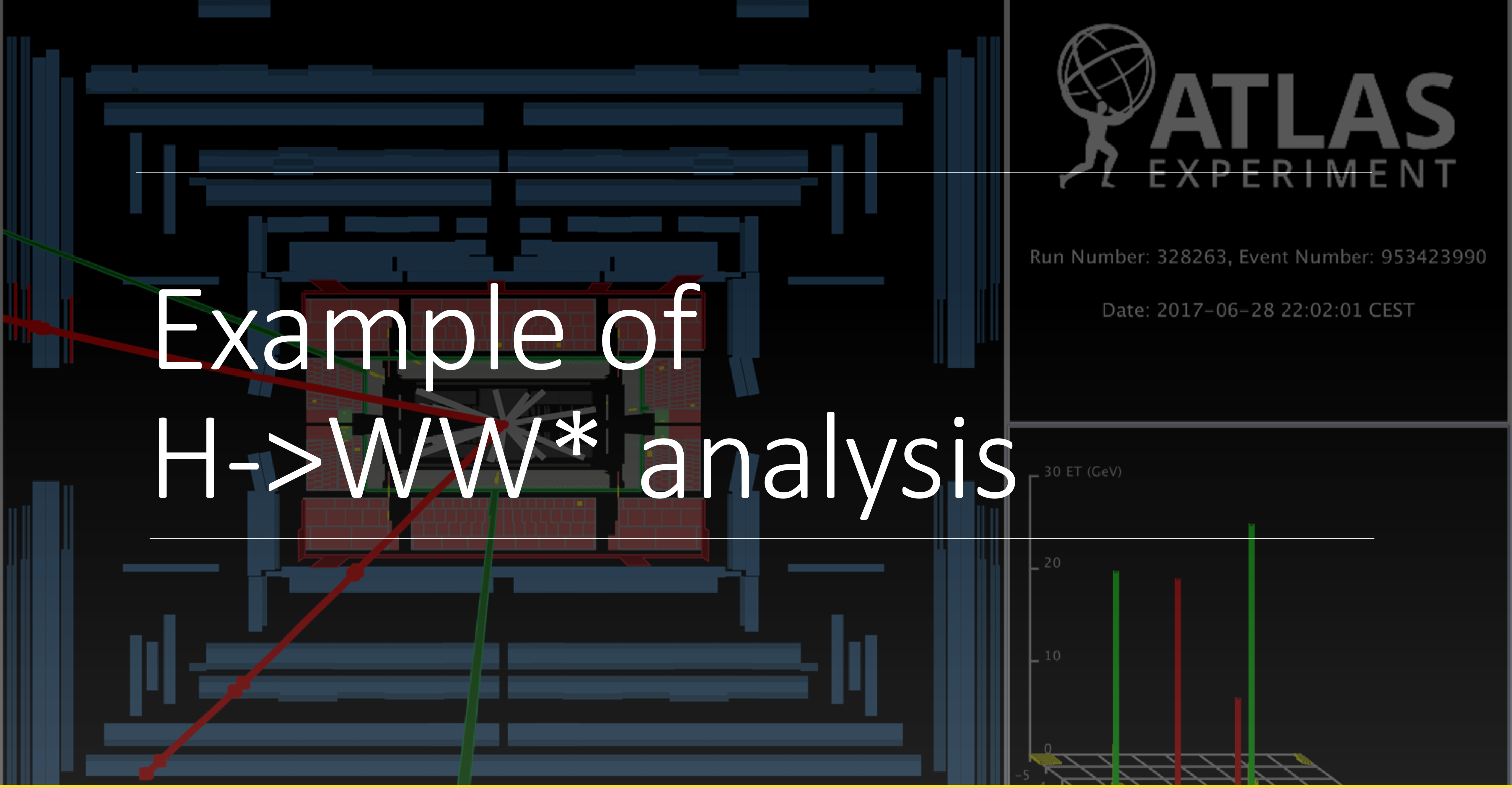
Additional challenges

- Quarks and gluons "shower" and "hadronise" before the interaction with the detector.
    - They are observed in the detector as "jets" → the energy and direction of the original quarks are smeared
- Identifying the events among all processes generated at LHC requires apply restrictions to the phase-space where the measurement is performes
- The effect of all these limitations is a distortion of the measured quantities (C coefficients, D)
    - These needs to be corrected with dedicated statistical strategies, that worsen the precision of the measurement



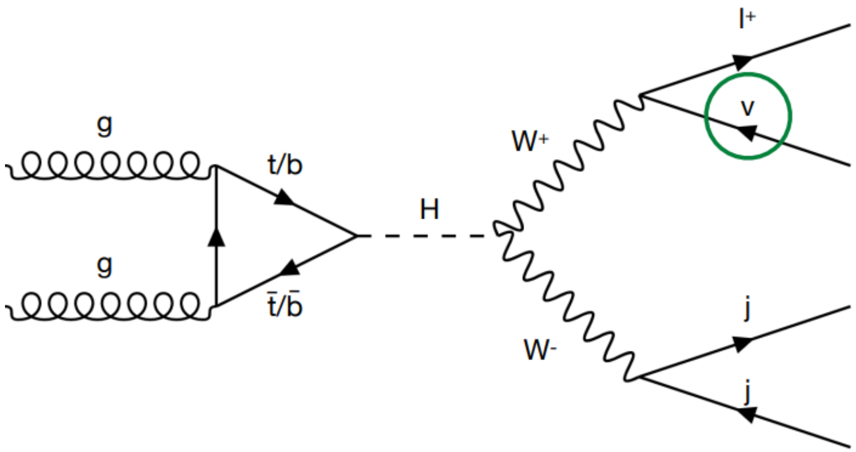
@Ben Nachman

Example of H->WW* analysis

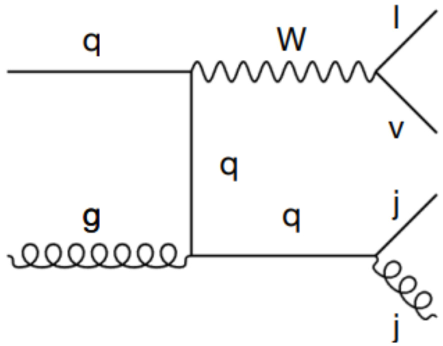


Selection of the final state

- The dileptonic final state contains 2 neutrinos
  - Very limited background
  - Under-constrained system
  - It is not possible to reconstruct with high precision the Higgs and W frames.
- The semi-leptonic final state can be fully reconstructed
  - There is a significant amount of background that needs to be reduced
  - It is hard to identify the W decay product to be used to measure the angles
    - Spin analysing power depends on the flavour of the jet
    - Can be identified only for b,c quark initiated jets



W + jets



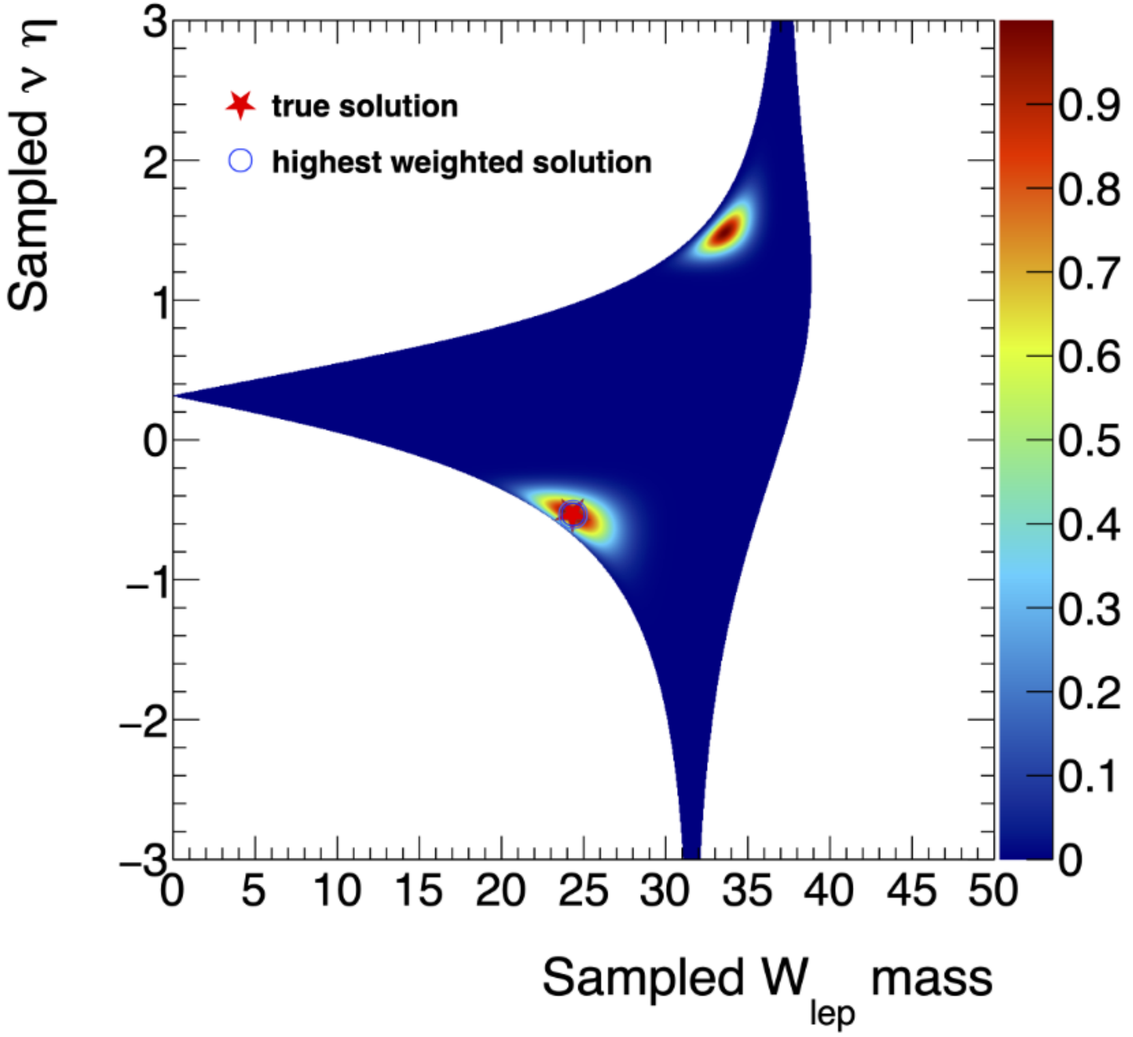
Reconstructing the final state

F. Fabbri, J. Howarth, T. Maurin *Eur. Phys. J.* C **84**, 20 (2024)

- Require that the hadronically decaying W is on-shell
- Sample the phase space of W_{lep} mass and P_z of the neutrino
  - Reconstruct the neutrino px and py
- For each point evaluate a weight as:

$$w = \exp\left(\frac{(\nu_x - P_x^{miss})^2}{\sigma_x^2}\right) \cdot \exp\left(\frac{(\nu_y - P_y^{miss})^2}{\sigma_y^2}\right)$$

- P_{miss} is the measurable missing energy on the transverse plane
- The solution with the highest weight is the preferred solution
- This technique allows also to have a handle on the background



Selection result

- This technique allows to separate the signal from the background and proceed to quantum observables measurement
- This channel has never been to measure the Higgs boson

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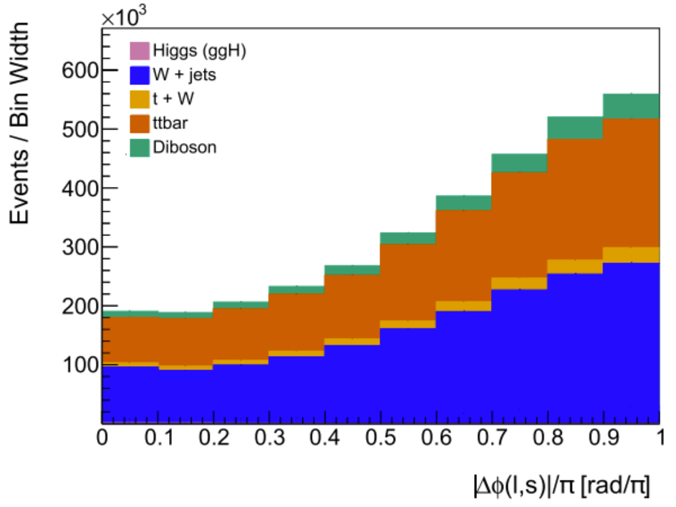
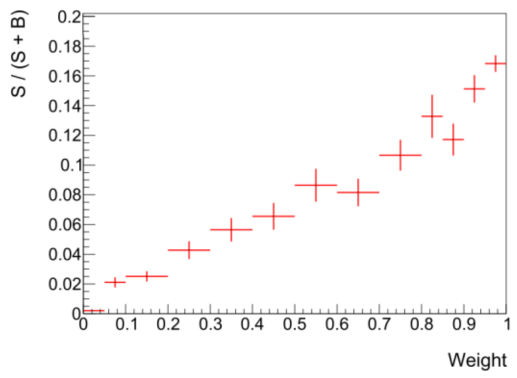
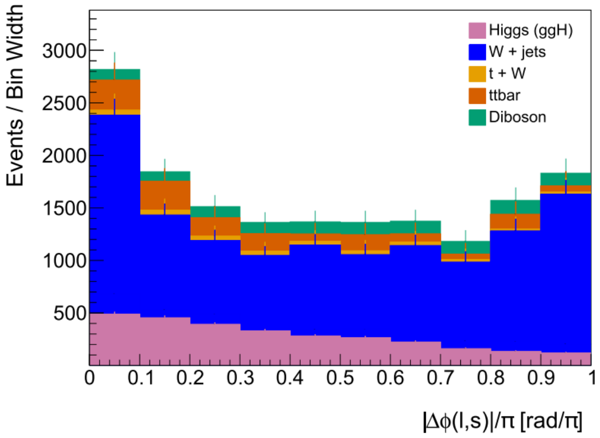




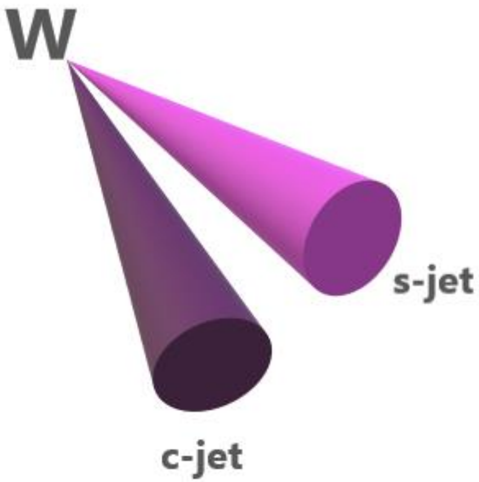
Process	Idealised	$\epsilon_c = 40\%$
W + jets	13131 ± 785	10444 ± 664
WW	2298 ± 31	1137 ± 22
$t\bar{t}$	601 ± 76	1453 ± 119
tW	217 ± 8	350 ± 11
Higgs	5967 ± 76	2843 ± 56
S/(S+B)	0.27	0.18



Measure Bell's inequality violation

There is the need to identify an hadronic W decay product with known spin analysing power

- Focus on a specific decay channel of the W, where we can identify the c-quark
- Identify the s-quark fixing the W mass to the known PDG value



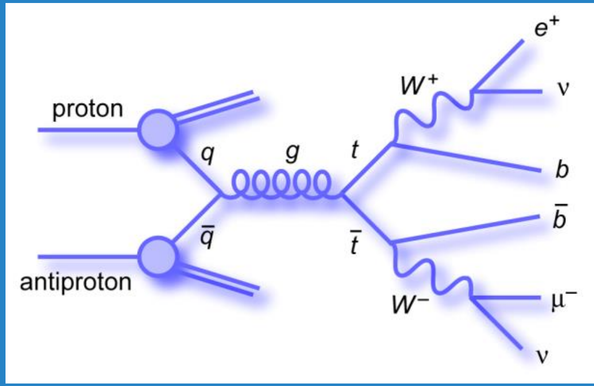
- Once the s-jet is identified it is possible to reconstruct the Bell's operator
  - Apply dedicated technique to remove smearing effects caused by selection and reconstruction
  - Analysis applied on MC simulated events

Luminosity [fb^{-1}]	$\langle \mathcal{B}^{zx}_{\mathrm{CGLMP}} \rangle$ (idealised)	Significance (idealised)
139	2.45 ± 0.25 (0.18)	1.8 (2.5)
300	2.45 ± 0.17 (0.12)	2.65 (3.75)
3000	2.45 ± 0.05 (0.04)	9.0 (11.25)

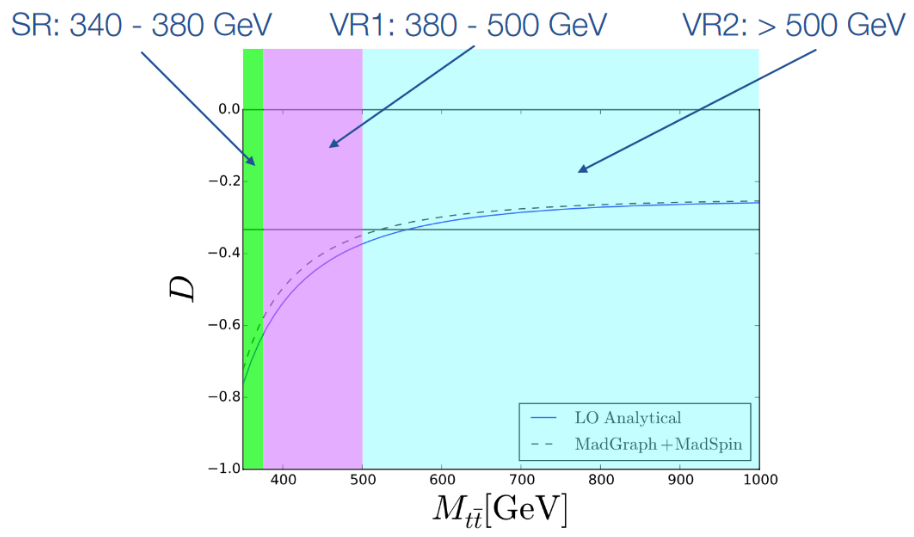
Observation of entanglement between top quarks

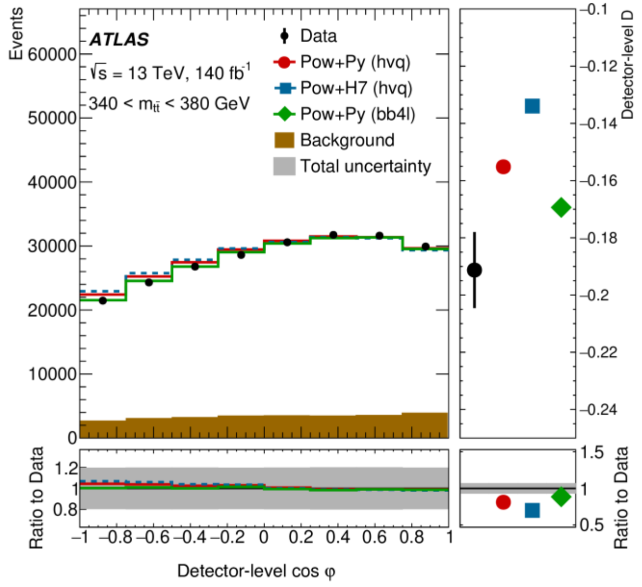
Analysis strategy

- Select events in the dilepton channel
- Select only events in the threshold region
- Reconstruct the entire final state assuming to the central value:
  - the two W masses
  - the two top-quark masses
- Measure the angle φ and extract D
- Correct for smearing effects caused from the detector and estimate the associated uncertainties



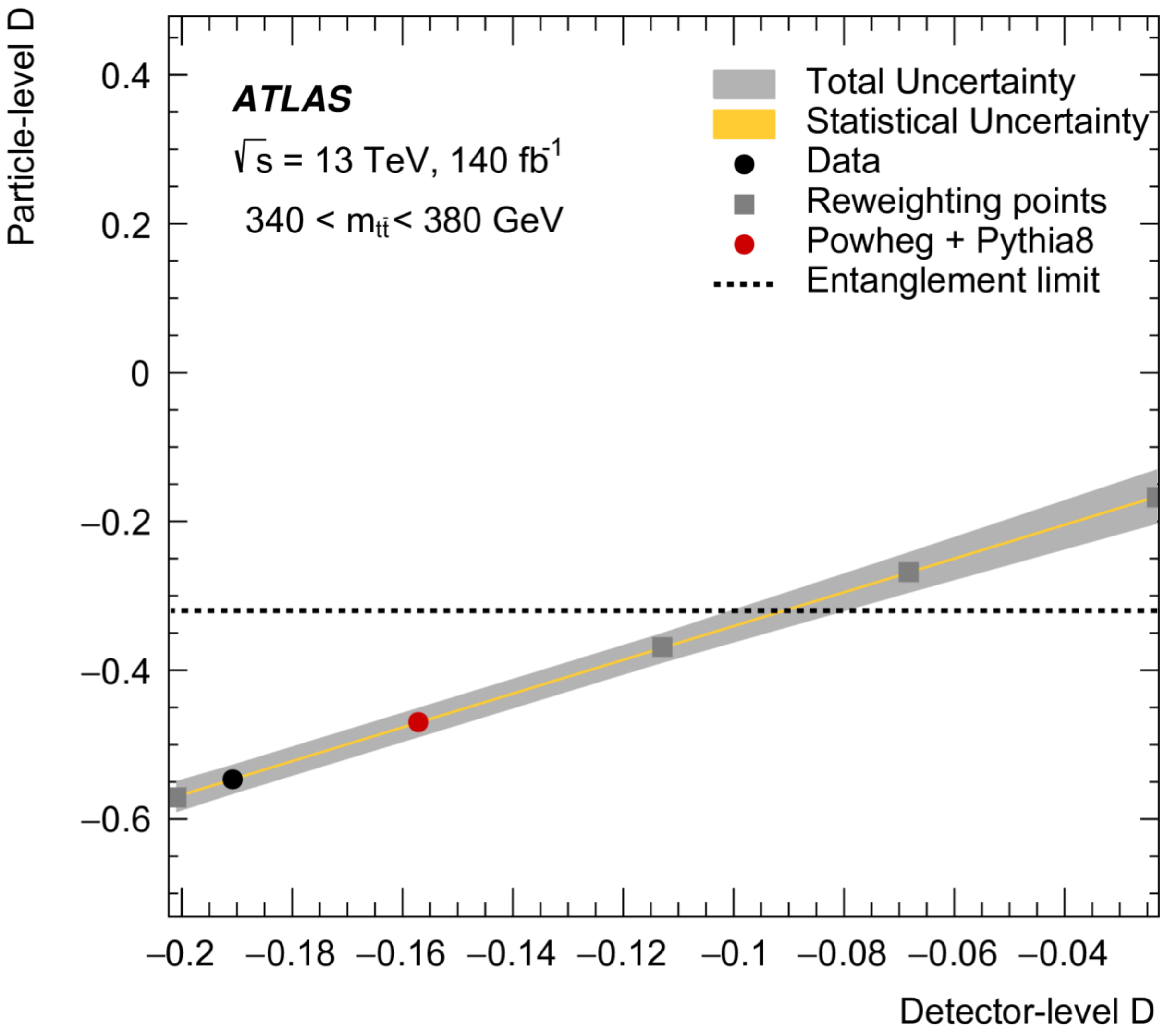
ATLAS Collaboration, arXiv:2311.07288





Removing detector effects

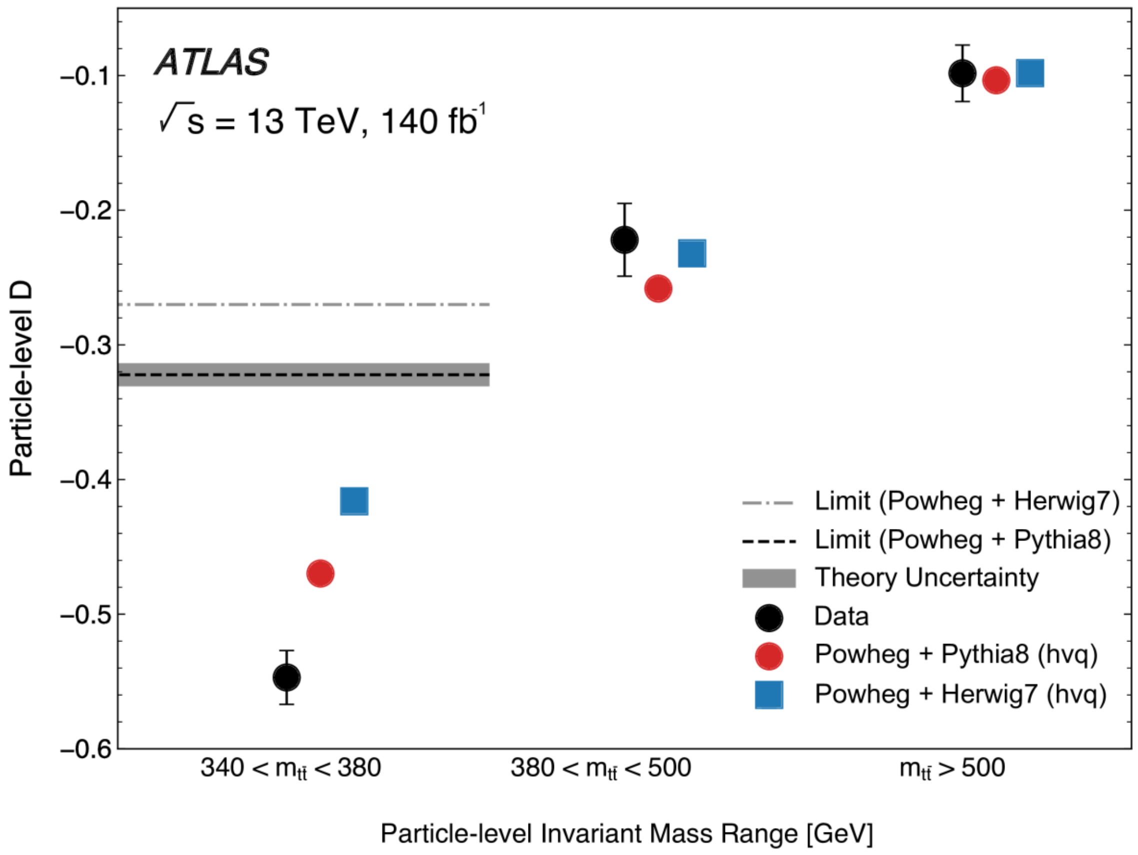
- Calibration curve connecting every measured value of D to the corresponding truth distribution.
    - Based on MC distributions
- To construct this curve we need to change the amount of entanglement in our MC.
    - We create 5 hypothesis points corresponding to the SM and 4 different reweighing points: (+20%, -20%, -40%, -60%)
- All systematic effects modify the line



ATLAS Collaboration, arXiv:2311.07288

SR $D = -0.547 \pm 0.002$ [stat.] ± 0.021 [syst.]
VR1 $D = -0.222 \pm 0.001$ [stat.] ± 0.027 [syst.]
VR2 $D = -0.098 \pm 0.001$ [stat.] ± 0.021 [syst.]

- Entanglement is observed with very high significance.
- The data show a level of entanglement higher than the SM predictions
- There are differences in the expected level of entanglement for different generators



Results

$$\textbf{SR} \quad D = -0.547 \ \pm 0.002 \ [\text{stat.}] \pm 0.021 \ [\text{syst.}]$$
$$\textbf{VR1} \quad D = -0.222 \ \pm 0.001 \ [\text{stat.}] \pm 0.027 \ [\text{syst.}]$$
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- Entanglement is observed with very high significance.
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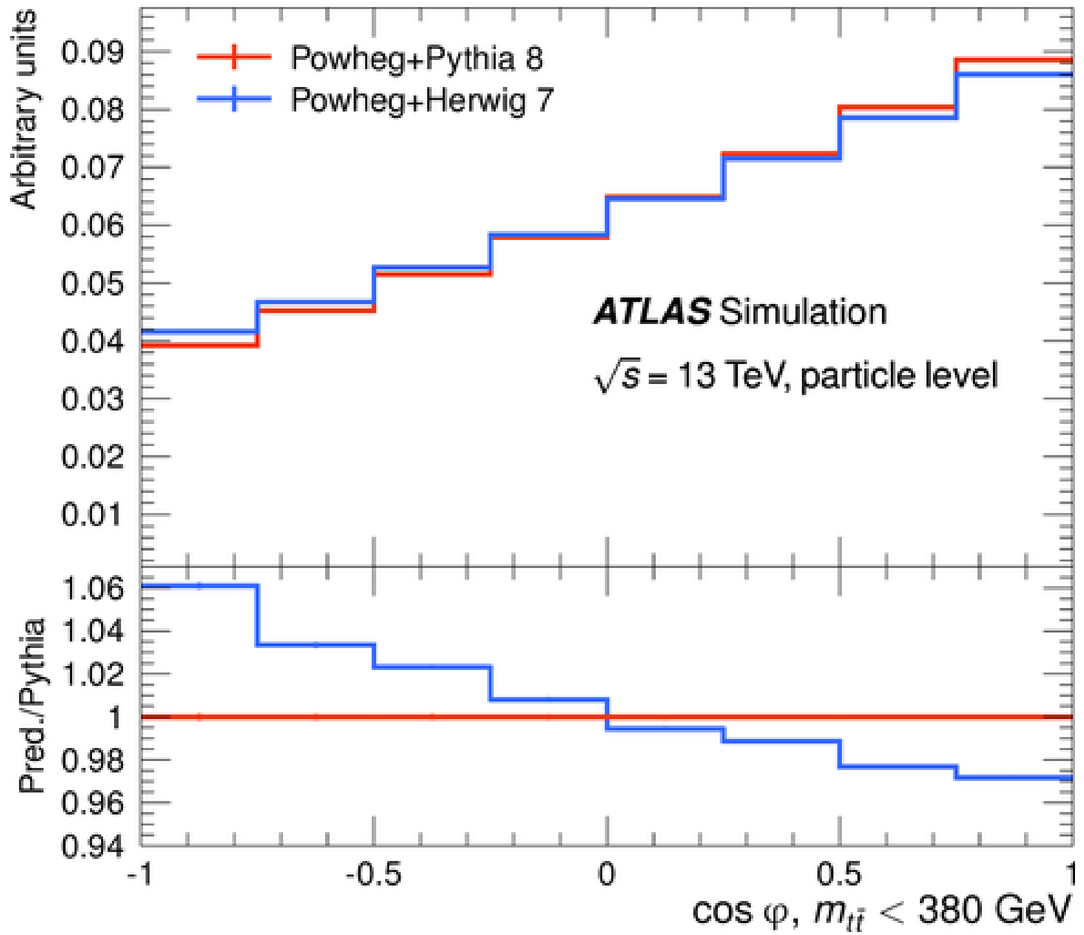
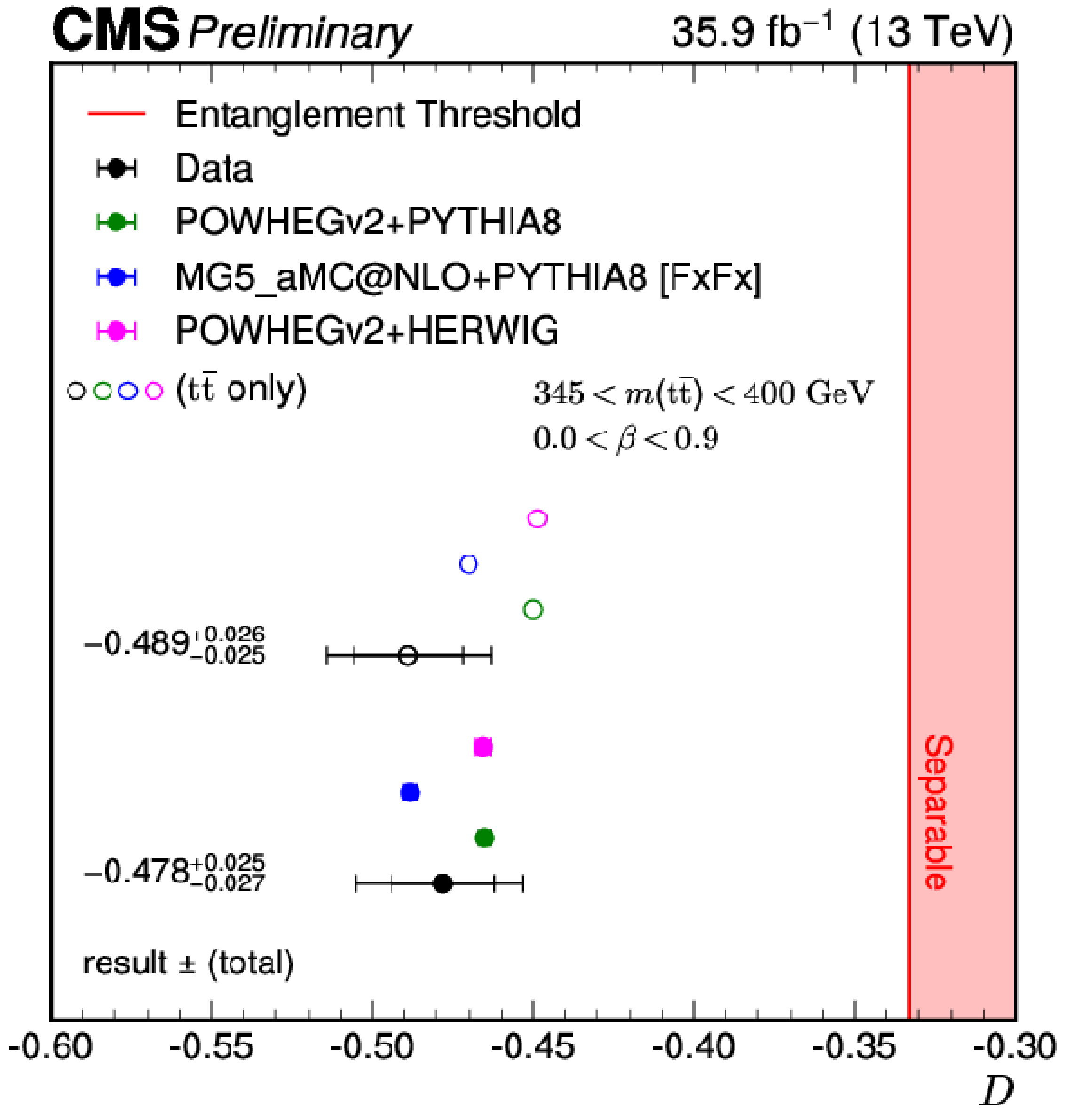


ATLAS Collaboration, arXiv:2311.07288

Results

CMS results

- Last month also the other multi-purpose LHC experiment confirmed the observation of entanglement using a technique similar to the ATLAS one.
- In this publication there is less discrepancy between the data and predictions when adding to the predictions an approximated simulation for a new particle
- This particle called toponium is a top-pair bound state predicted by the SM but never observed
  - Missing in "standard" MC
  - This new particle should enhance the entanglement between top-quark pairs at threshold



CMS Collaboration, TOP-23-001

Search for new physics

Physics beyond the standard model

Open questions:

- Nature of dark matter
- Source of the asymmetry between matter and antimatter
- Understand the origin of the mass hierarchy (why top-quark is 172.5 GeV and the up-quark is 2.2 MeV)
- Unification with gravity

Current status:

- No unexpected particles observed at LHC
- New physics must be at very high energy or light and not very interactive

Approaches to new physics search:

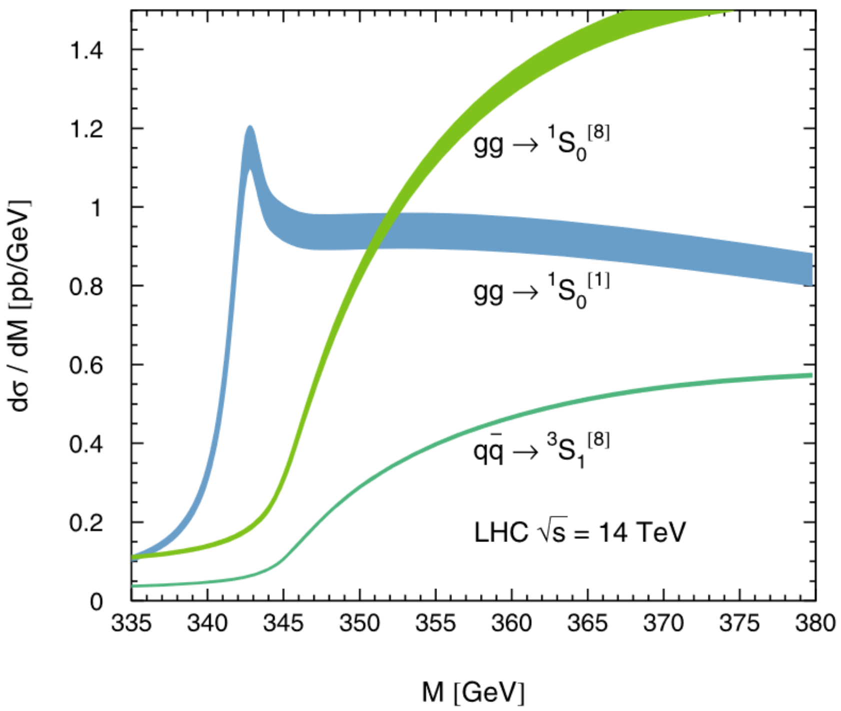
- Bump in the number of events as a function of the invariant mass
- Employ a model-independent effective theory to search for hints of new physics

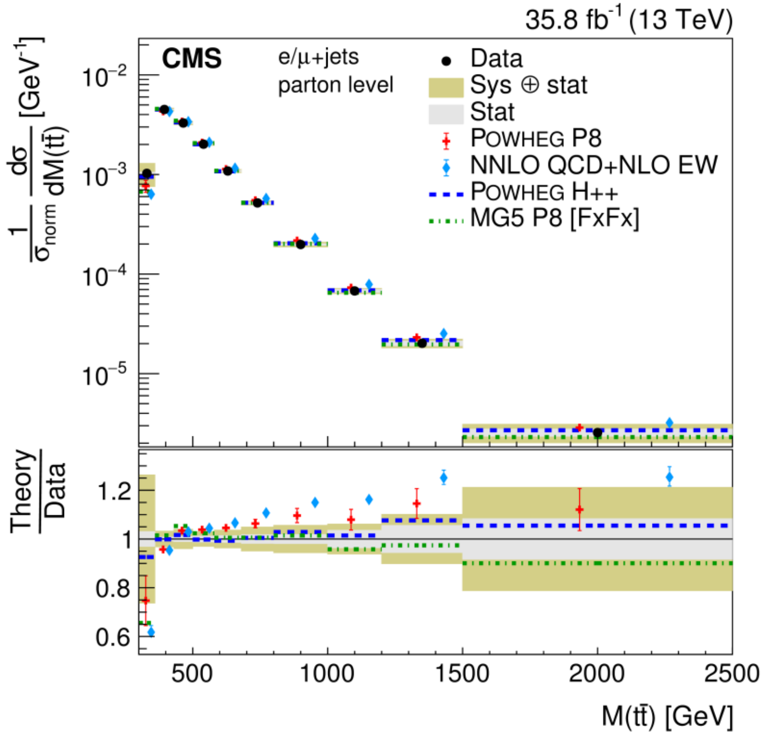
Can the quantum observables enter in the game?

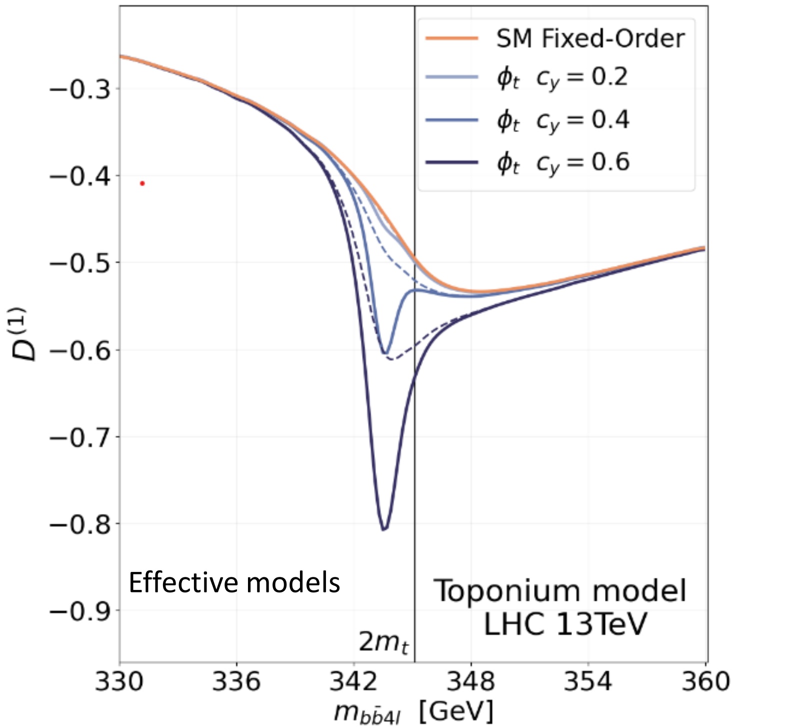
- Only if “new physics” can change the entanglement between particles https://doi.org/10.22323/1.235.0032

CMS Collaboration, Phys. Rev. D 97, 112003

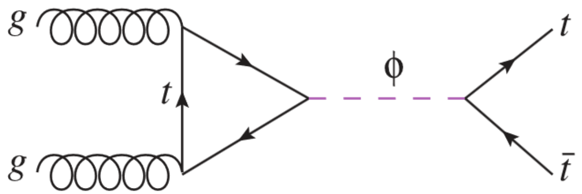
F. Maltoni, C. Severi et al JHEP03(2024)099







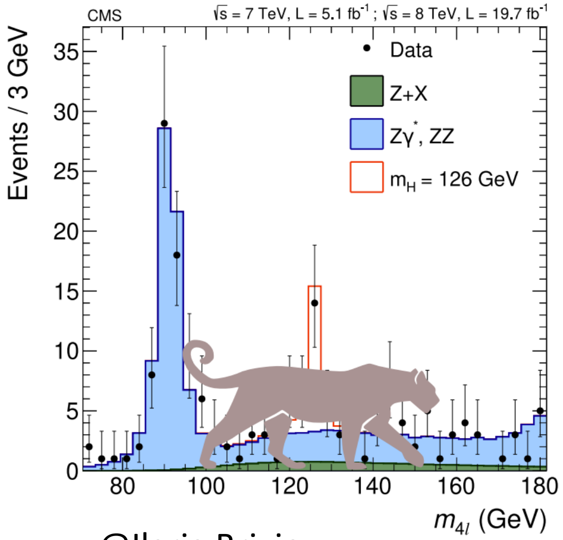
Direct searches



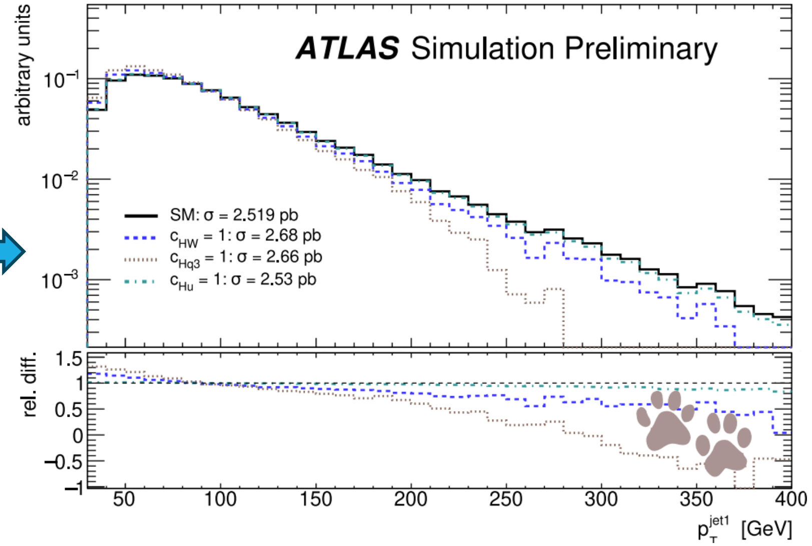
- The presence of new particles results in an enhancement in the cross section around the mass of the new particle
- To observe a shape the resolution of the measurement must be good enough, otherwise the only effect is a mild enhancement in the cross-section
- A new resonance highly modify the correlations, this feature can be used to observe a new particle
  - Hopefully toponium will be the first case
  - Not exactly a particle beyond the Standard Model, it is a pseudo-bound state predicted in the gg→tt color singlet

Effective searches

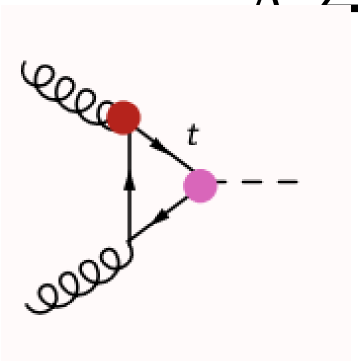
- If a particle has a too high mass to be created at LHC it will modify other distributions, that can be used as hints of the presence of new physics
- Multiple distributions originating from different processes can be combined to search physics beyond the LHC scale with an effective approach
- Extend the Lagrangian with operators not included in the SM, each weighted by a coefficient
  - Translates in a modification in the couplings
  - A global fit to various distributions is used to put constraints to these coefficients
  - Fit compatible with 0 means no new physics beyond the SM.



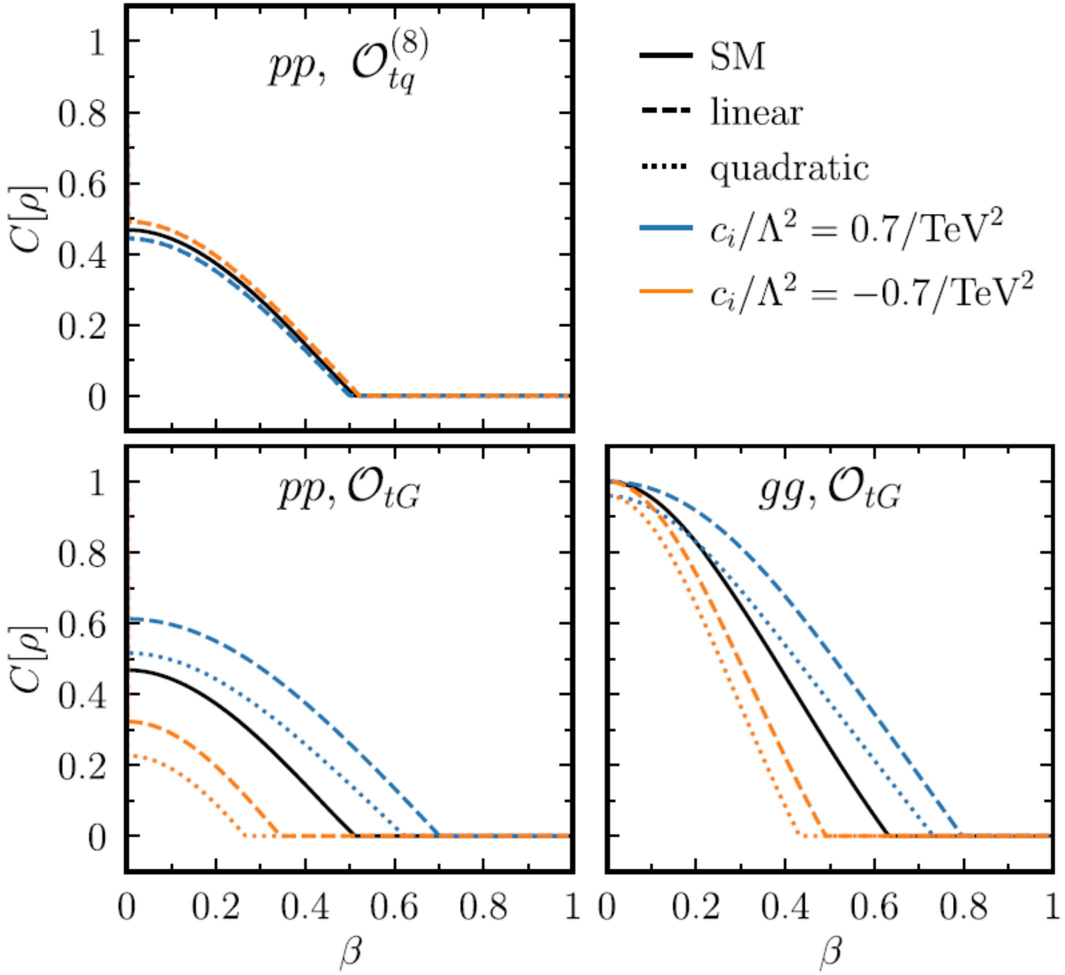
@Ilaria Brivio



$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2}\sum c_i O_i$$



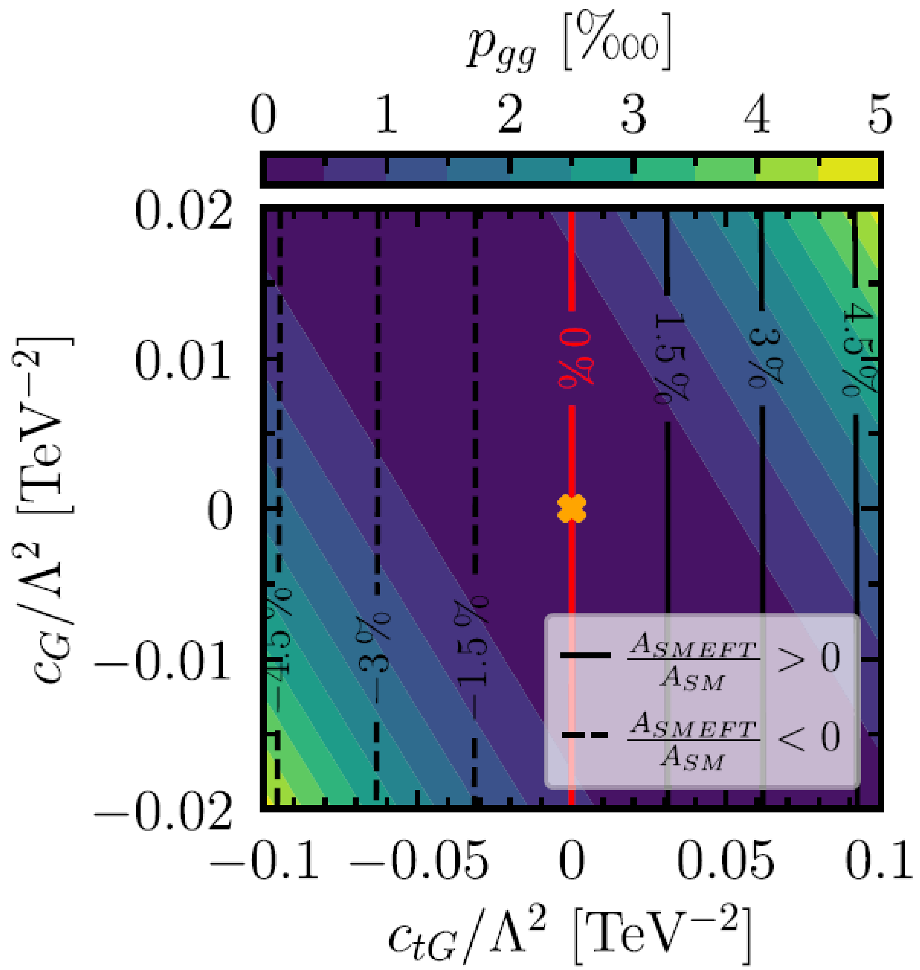
Effective searches



- The operators included in the effective Lagrangian highly modify the quantum correlations between particles
- The concurrence, D, the coefficient of the spin density matrices
- Include these quantities to the global fit will enhance the sensitivity to the presence of new physics

R. Aoude, E. Madge, F. Maltoni Phys. Rev. D 106, 055007

Effective searches



- The operators included in the effective Lagrangian highly modify the quantum correlations between particles
- The concurrence, D, the coefficient of the spin density matrices
- Include these quantities to the global fit will enhance the sensitivity to the presence of new physics
  - Effects on these quantities of EFT operators are orthogonal to effects on other quantities
  - Essential to increase the sensitivity

R. Aoude, E. Madge, F. Maltoni Phys. Rev. D 106, 055007

Ongoing activities in the group

Multiple activities ongoing in the group:

- Phenomenological studies on Higgs and Boson final states
- Including two master thesis
- Experimental activities in the ATLAS group on top-quark pairs and Higgs final states
- Including an Erasmus Internship

European Individual Marie Curie Fellowship “QUANTUM LHC” on the topic

Theory and Experimental groups working in close contact:

- Professors: M. Sioli (Unibo) F. Maltoni (Unibo)
- Researchers: D. Pagani (INFN), M. Romano (INFN), F. Fabbri (Unibo)
- Post-doc: Pryanka Lamba (Unibo)

Soon opening two PhD positions:

- Experimental study of quantum information inspired observables at the LHC (supervisor: M. Sioli, F. Fabbri)
- Theoretical study of quantum information inspired observables at the LHC (supervisor: F. Maltoni, F. Fabbri)

Conclusions

- Presented a new growing field “the measurement of quantum information inspired observables at colliders”
- Mixing quantum information concepts with experimental and theoretical particle physics.
- Quantum tomography procedures allow to reconstruct the spin density matrix(ρ) of various final states
- From (ρ) it is possible to derive criterion for entanglement and Bell’s inequality violation
- There are serious experimental challenges related to this: reconstruction of the neutrinos, identification of jet’s flavour
- Presented two examples of analyses:
  - Bell’s inequality measurement in $H \rightarrow WW^*$
  - Observation of entanglement between top-quark pairs
- Large impact of these quantities foreseen in the search for new physics
- Lots of more material on the topic: https://www.unibo.it/sitoweb/federica.fabbri26/contenuti-utili

Peres Horodecki criterion

Given the state C it is separable in the substates a and b if:

$$\rho^{T_2} = \sum_n p_n \rho_n^a \otimes (\rho_n^b)^T$$

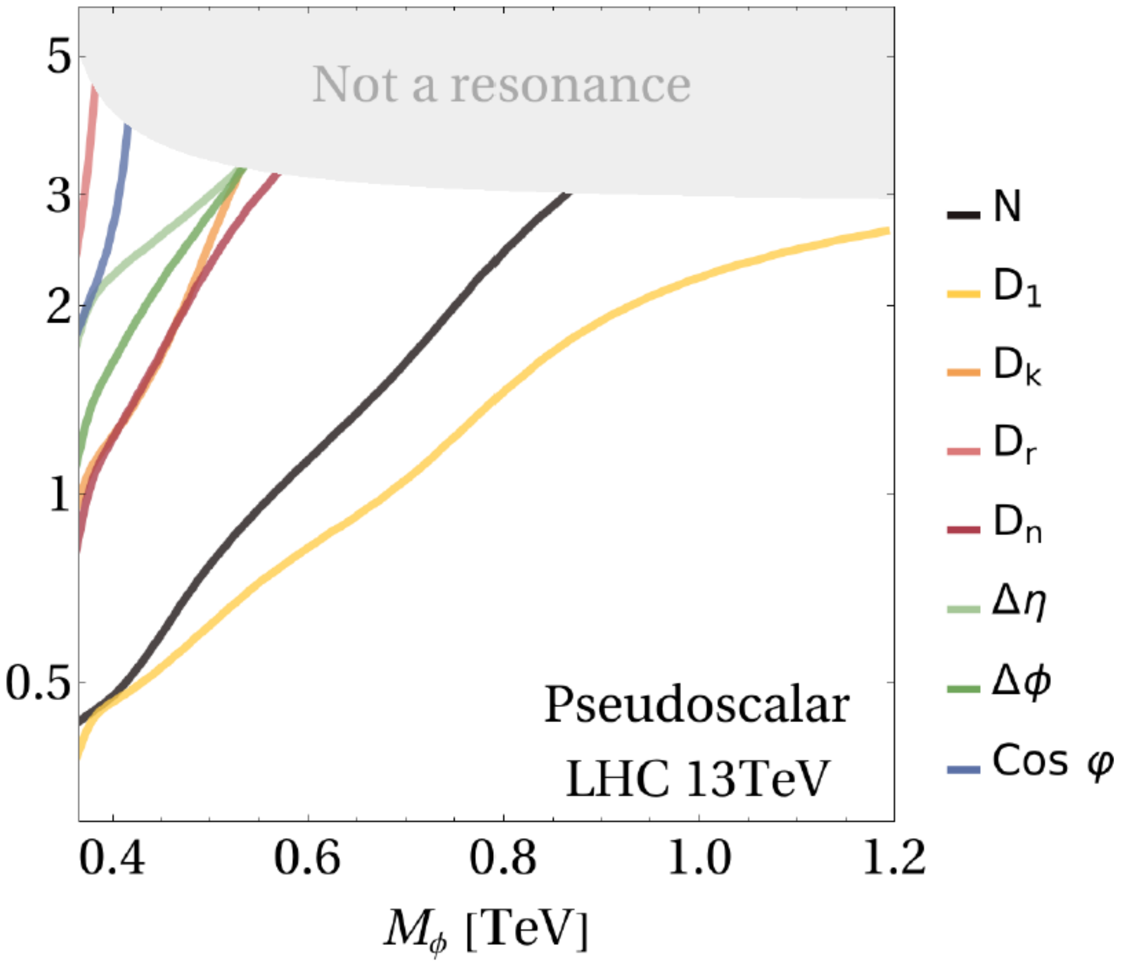
Is a valid spin density matrix:

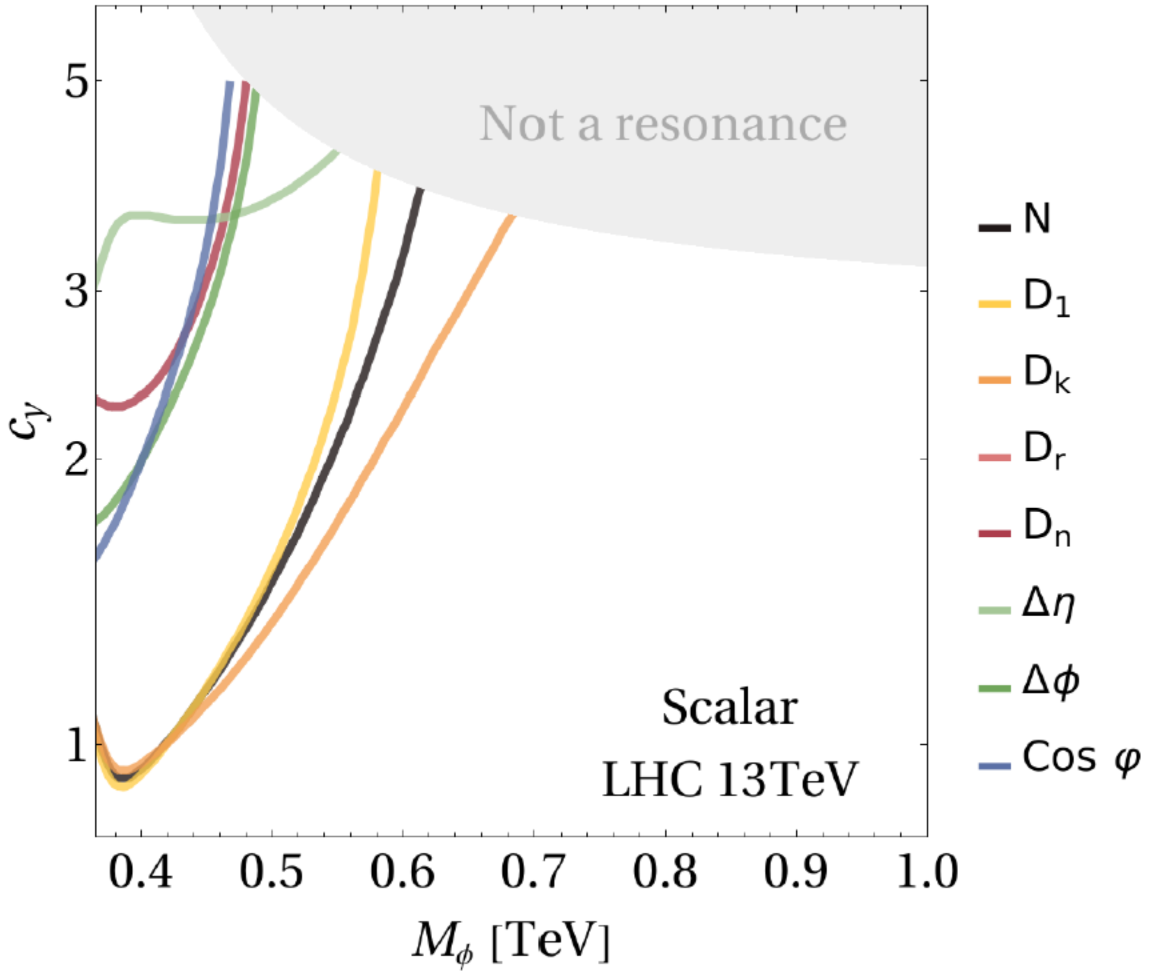
- $\rho = \rho^\dagger$
- Tr(ρ) = 1
- ρ is positive

From the last point it can be derived that is a section of the matrix is negative it is a sufficient condition for non-separability (entanglement)

F. Maltoni, C. Severi et al JHEP03(2024)099

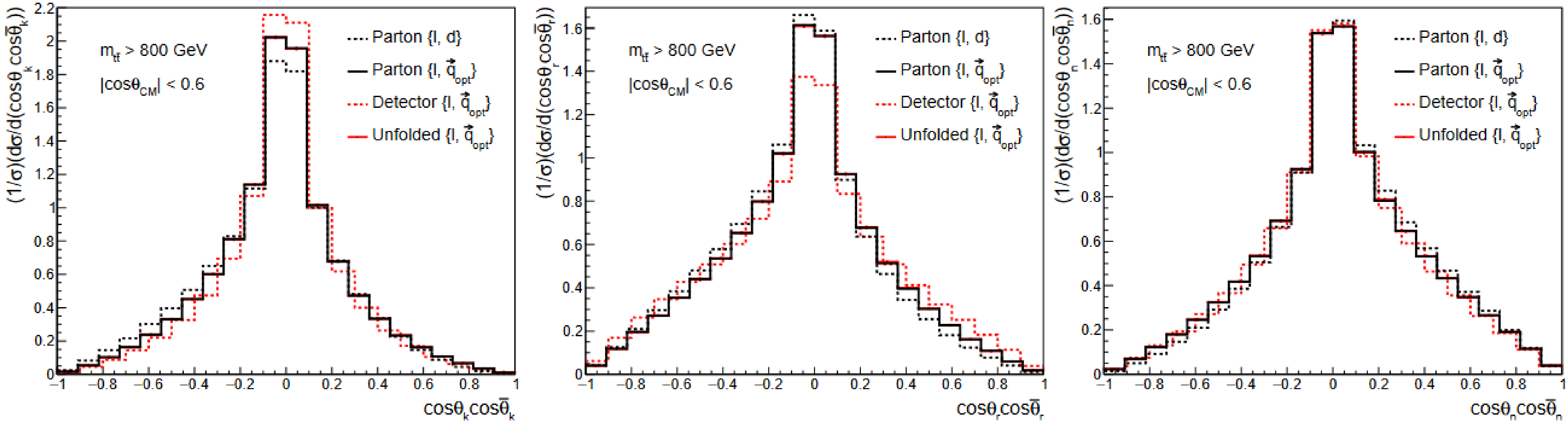
Limit setting





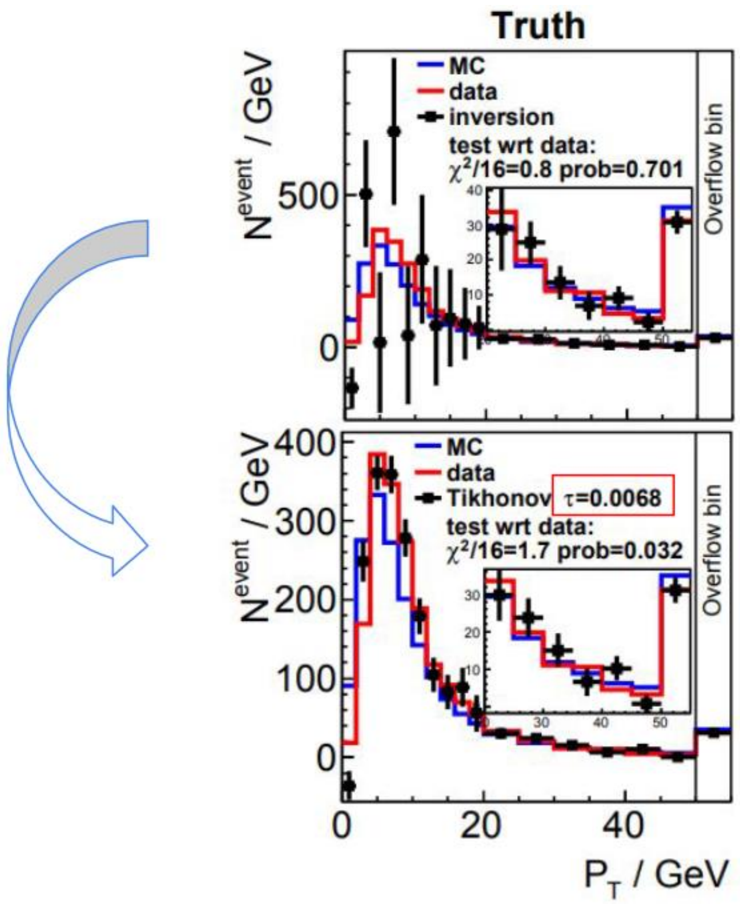
Z. Dong, D. Goncalves, K. Kong, A. Navarro arXiv:2305.07075

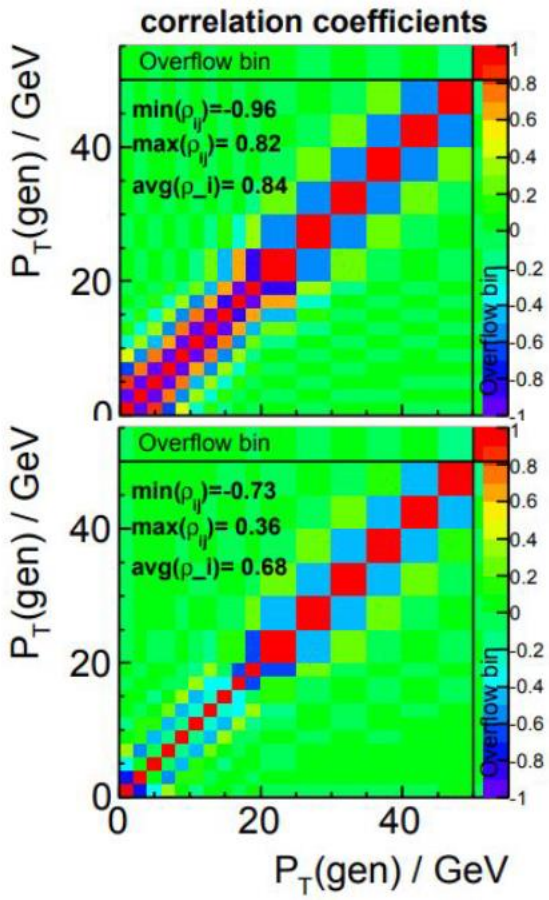
Smearing effects

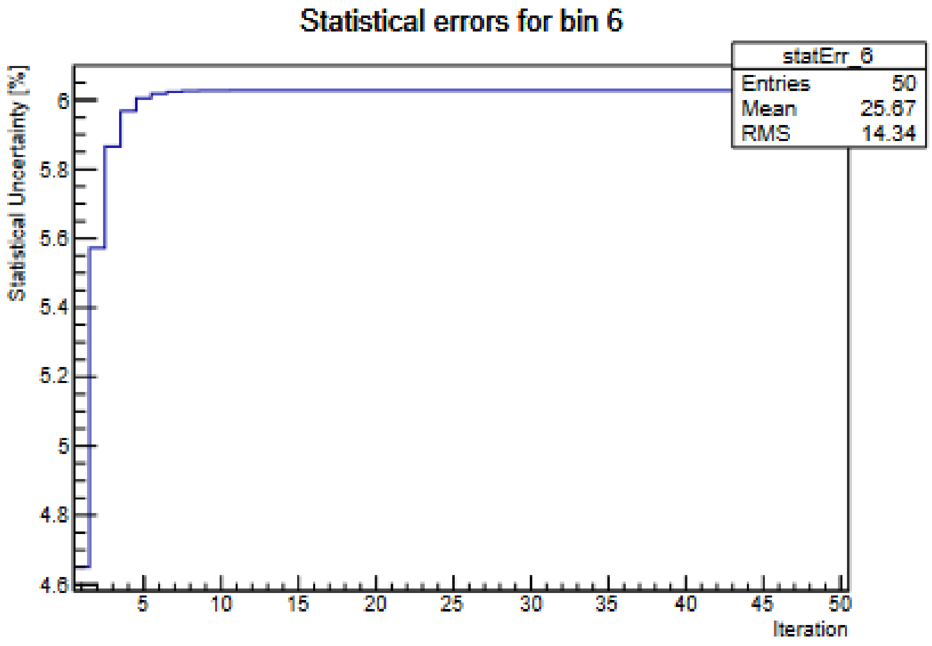


Unfolding

Proceeding by Stefan Schmitt







Quantum tomography - II

Now using this relationship, we can obtain the coefficient of the spin density matrix.

Starting from a single W:

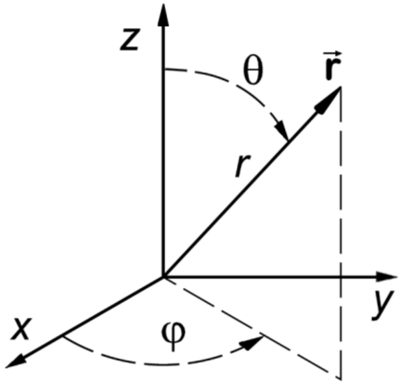
$$\langle \xi_x^{\pm} \rangle_{\rm av} = \int d\Omega\, p(\ell^{\pm}_{\hat{\mathbf{n}}}; \rho_W) \sin\theta \cos\phi = \pm \tfrac{1}{\sqrt{2}}(\Lambda_1 + \Lambda_6),$$

$$\langle \xi_y^{\pm} \rangle_{\rm av} = \int d\Omega\, p(\ell^{\pm}_{\hat{\mathbf{n}}}; \rho_W) \sin\theta \sin\phi = \pm \tfrac{1}{\sqrt{2}}(\Lambda_2 + \Lambda_7),$$

$$\langle \xi_z^{\pm} \rangle_{\rm av} = \int d\Omega\, p(\ell^{\pm}_{\hat{\mathbf{n}}}; \rho_W) \cos\theta = \pm \tfrac{1}{2}(\Lambda_3 + \sqrt{3}\Lambda_8),$$

The probability density function for a W boson with spin density matrix ρ to emit a charged lepton in the direction $\hat{n}(\theta, \varphi)$ is:
$p(l^{\pm}{}_{\hat{n}}; \rho) = \frac{3}{4\pi} tr(\rho \Pi_{\pm;\hat{n}})$, where $\Pi_{\pm;\hat{n}}$ is the projection operator $\Pi_{\pm;\hat{n}} = | \pm_{\hat{n}} \rangle \langle \pm_{\hat{n}} |$

Where $\xi_i^{\pm} = \widehat{n_i} \cdot \widehat{n_{l^{\pm}}}$, with the axes defined in the rest frame of the of the W^+ and W^- bosons



CGLMP equation - I

- We have 2 experimentalist Alice and Bob
- Each can perform two measurements A_1,A_2 or B_1,B_2
- Each measurement can have 3 outcome (-1,0,1) and we define $P(A_i = B_j + k)$

$$\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) \\ + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) \\ - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1).$$

- As in the CHSH case the limit posed for this equation by local realist theories is 2
- Similarly to the top-pair case we can translate this in a relation between the elements of the spin density matrix exploiting:
  - $P(A1 = 0) = \langle P^0_{A1} \rangle = Tr\{\rho P^0_{A1} \otimes I_3\}$, where P^0_{A1} is the projection associated to the eigenvalue 0 of A1

CGLMP equation - II

So the inequality can be written as $I_3 = \langle O_{Bell} \rangle = Tr\{\rho O_{Bell}\} \leq 2$

$$I_3^{xy} = 4(h_{44} + h_{55}) - \frac{4}{\sqrt{3}}(h_{11} + h_{16} + h_{61} + h_{66}) - \frac{4}{\sqrt{3}}(h_{22} + h_{27} + h_{72} + h_{77})$$

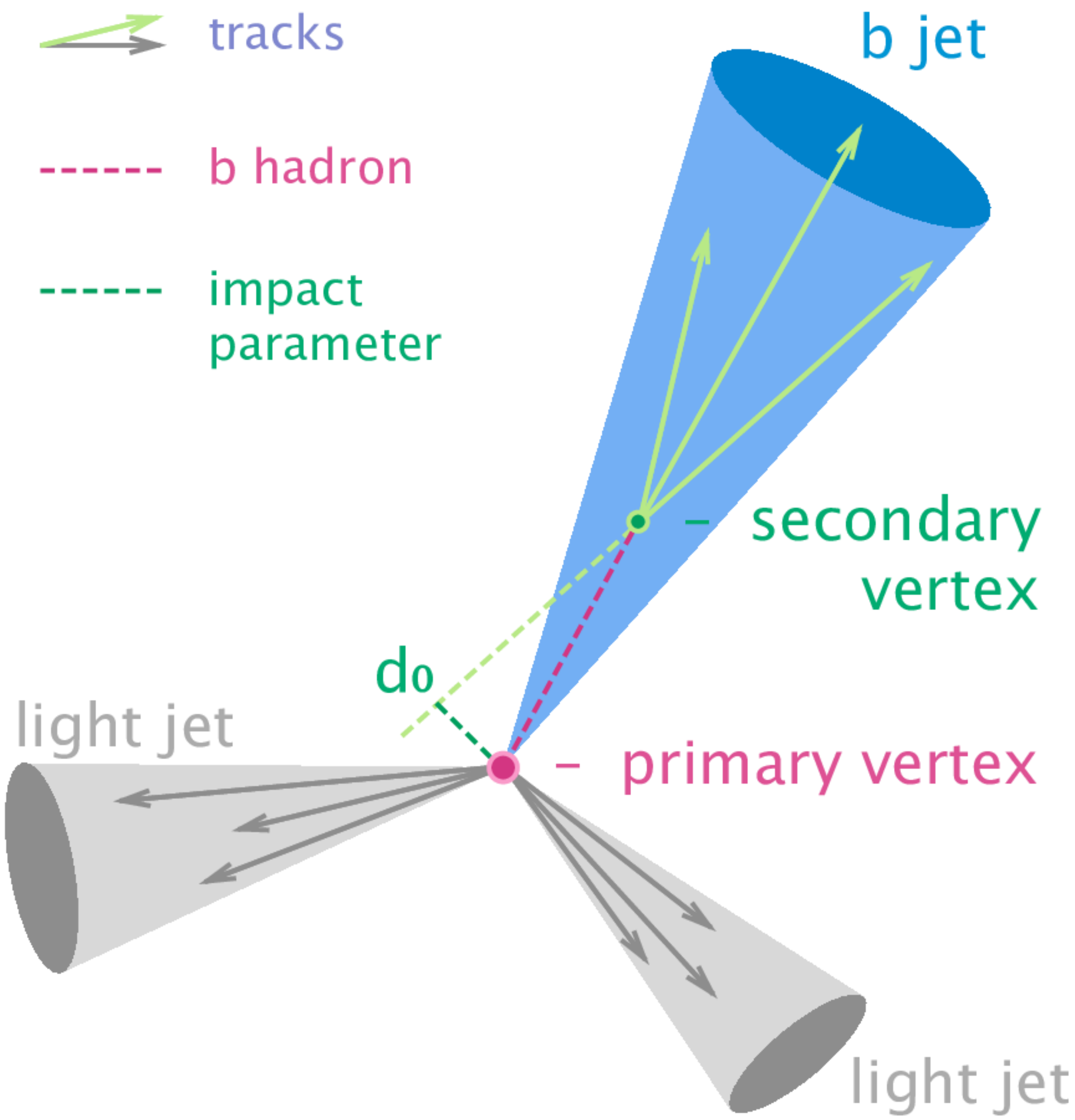
Then including the quantum tomography protocol extended to a pair of W bosons we obtain:

$$I_3^{xy} = \frac{8}{\sqrt{3}}\langle \xi_x^+ \xi_x^- + \xi_y^+ \xi_y^- \rangle + 25 \left\langle \left((\xi_x^+)^2 - \left(\xi_y^+\right)^2 \right) \left(\left(\xi_y^-\right)^2 - (\xi_x^-)^2 \right) \right\rangle + 100 \langle \xi_x^+ \xi_y^+ \xi_x^- \xi_y^- \rangle$$

These are expressed with respect to a specific pair of axes x,y defined in the W rest frame

Corresponding expectation values can also be defined for any pair of orthogonal axes, and the value of the Bell's inequality may depend on the actual ax selected, so the best definition for the CGLMP inequality is:

$$I_3^{xyz} = \max(\langle I_3^{xy} \rangle, \langle I_3^{yz} \rangle, \langle I_3^{zx} \rangle)$$



Tracks can also be used to guess the flavour of the quark/gluon that created the jet.

- A hadron that contains a “b”-quark tends to have a longer life-time than other hadrons
- The showering starting point is displaced compared to the PV

Possible to exploit tracks to identify the origin of the jets, and split these in 3 categories:

- b-quark originated jet:
  - High efficiency in identifying the jet (~85% max)
  - Low mis-tag rate, difficult to reconstruct a light jet for a b-jet, but still possible
- c-quark originated jets:
  - Low efficiency in identifying the jet (~40% max)
  - High mis-tag rate, quite easy to reconstruct a light or b-jet as a c-jet
- Light(u/s/d and gluons) flavour jets