Exploring pulsar glitches with supersolids

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The collaboration

Innsbruck





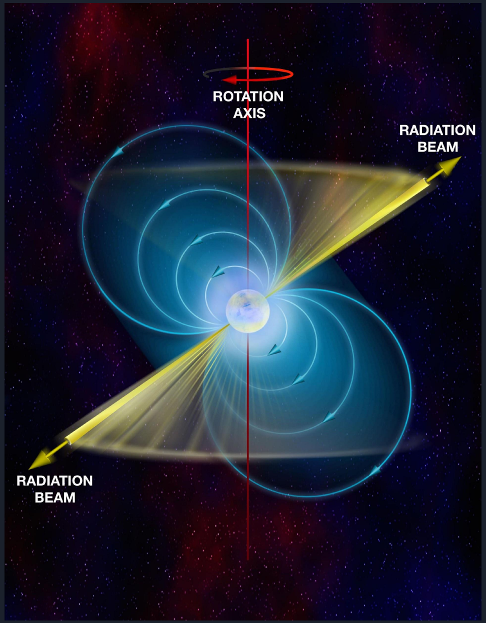
Gran Sasso


Outline

- 📌 Glitches in Neutron Stars
- 📌 Supersolidity
- 📌 Glitches with supersolids

Glitches in Neutron Stars

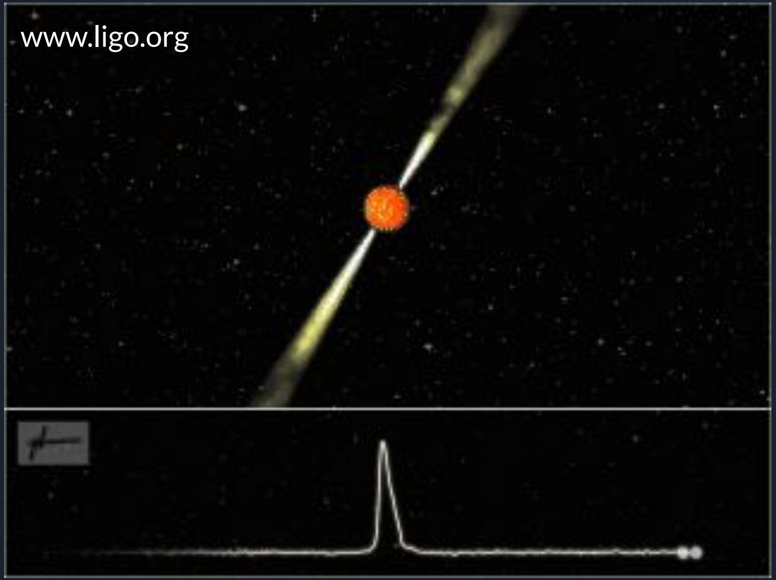
Neutron Stars



$$M \sim 1 \div 2 M_\odot$$
$$R \sim 10\text{km}$$
$$T \sim \text{keV}$$

They are typically observed as *pulsars*

The lighthouse effect



Crab pulsar (Credits NASA)

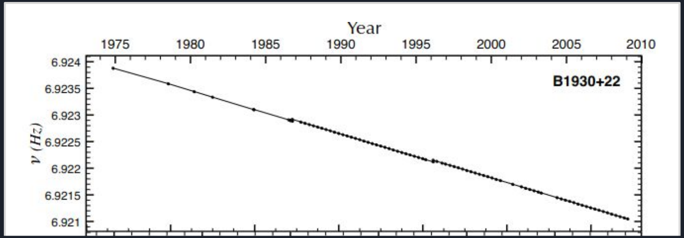
$$T \simeq 33.5\text{ms}$$

They behave *almost* like perfect clocks

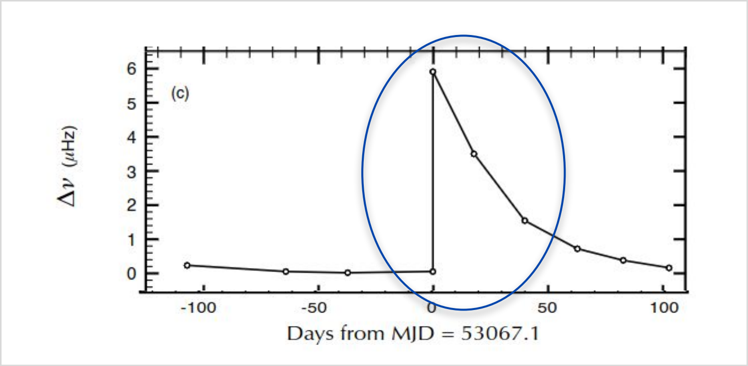
We observe that neutron stars spin-down

$$\dot{\Omega} \propto -\Omega^n$$

$$\dot{L}_{\rm tot}(t) = -N_{\rm em}$$



Glitch event !



Espinoza *et al*, MNRAS, 414, 2, 1679-1704 (2011)

Possible explanations:

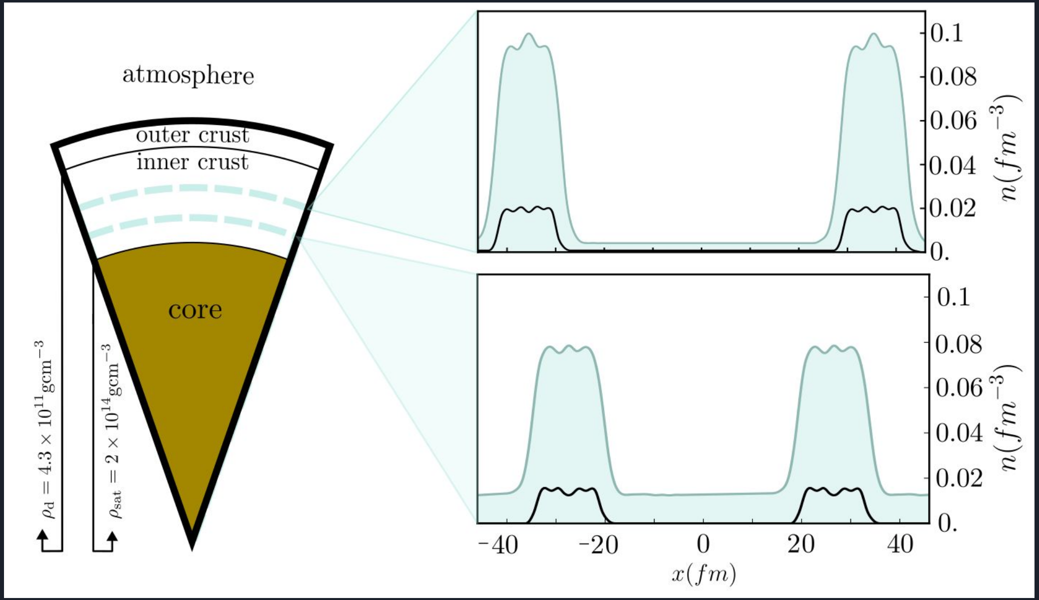
The exterior

The interior

- Star-quakes: interplay between crystalline outer crust and liquid interior

- Superfluidity inside the neutron star

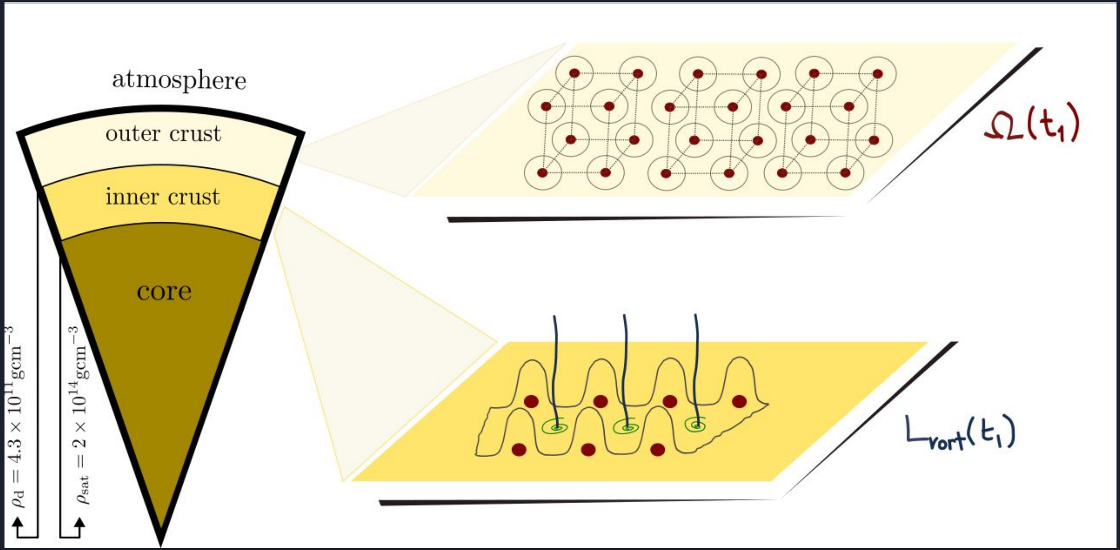
Inside a Neutron Star



Negele, Vautherin, Nucl. Phys. A207, 298 (1973)

The role of superfluidity

Vortex unpinning in the inner crust accelerates the outer crust



Warszawski *et al* MNRAS, 415,1611 (2011)

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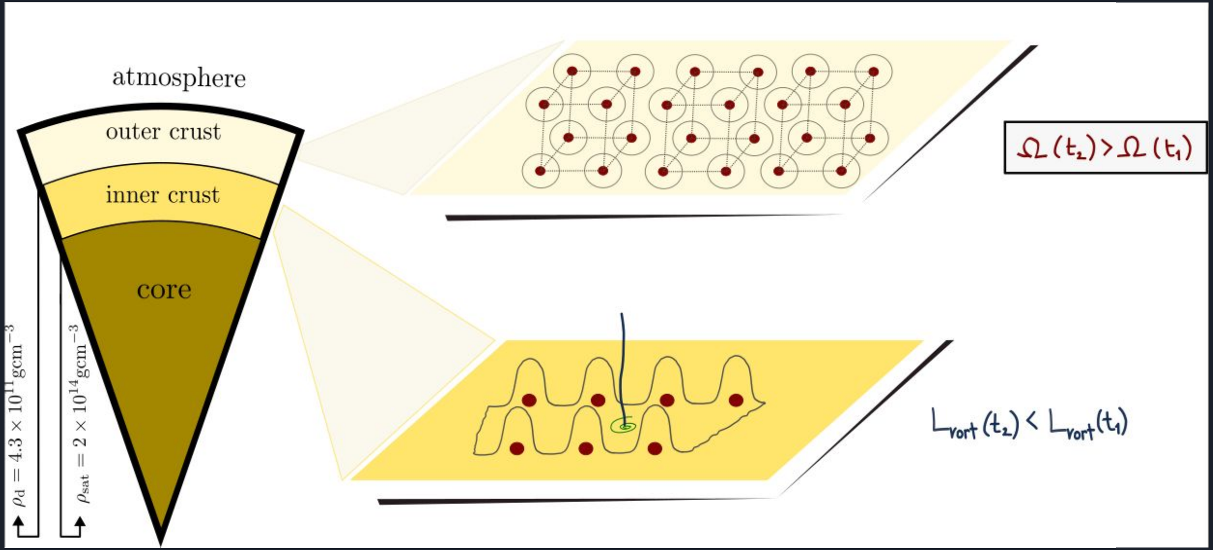


Warszawski *et al* MNRAS, 415,1611 (2011)

Glitches data

- Size $\frac{\Delta\Omega}{\Omega} \sim 10^{-12} \div 10^{-3}$ and waiting times

  Vela, J0537-6910 and J1341-6220 show quasiperiodic behaviour

- Reservoir effect ? (only in J0537-6910)
- Age and glitching activity
- ….

Haskell and Melatos, IJMPD 24, 530008 (2015)
Espinoza *et al*, MNRAS, 414, 2, 1679-1704 (2011)
Zhou *et al* arxiv:2211.13885

Supersolidity

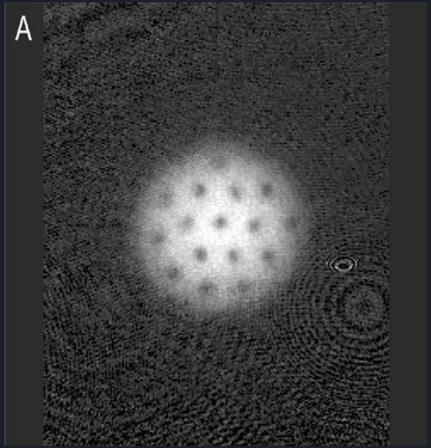
Ultracold bosons

Below the critical temperature, bosons are well described by short-range isotropic interaction

$$U_c(\boldsymbol{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\boldsymbol{r})$$

The system is *superfluid* and stores angular momentum in *vortices*

$$L_z = N\ell_z$$



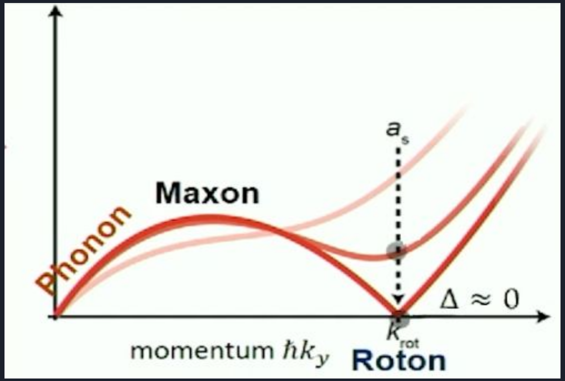
Abo-Shaeer *et al*, Science 292, 5516, 476-479 (2001)

What is a supersolid?

Speculations on Bose-Einstein Condensation and Quantum Crystals

G. V. Chester PRA 2, 256

Published 1 July 1970



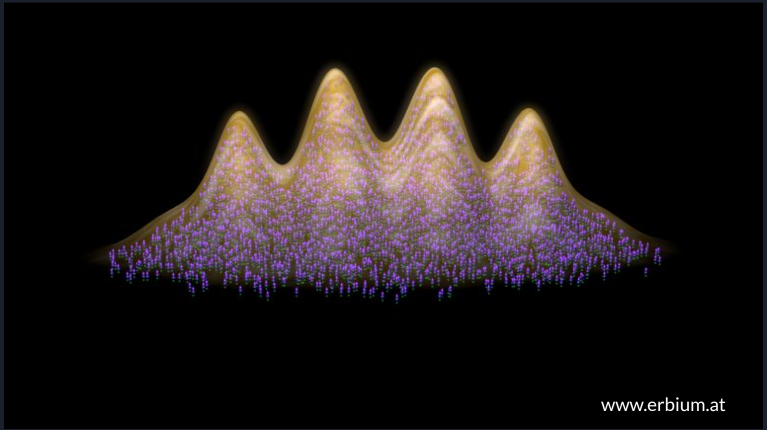
It is a ***superfluid-like*** solid

Dipolar bosons

Long-range interaction

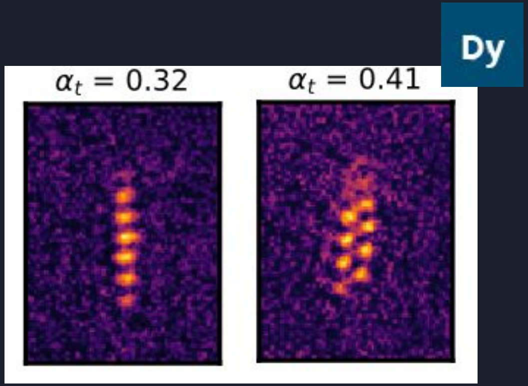
$$U_{\mathrm{dd}}(\boldsymbol{r}) = \frac{3\hbar^2 a_{\mathrm{dd}}}{m} \frac{1 - 3\cos^2\theta}{r^3} \qquad a_{\mathrm{dd}} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2}$$

$$\epsilon_{\mathrm{dd}} = \frac{a_{\mathrm{dd}}}{a_s}$$

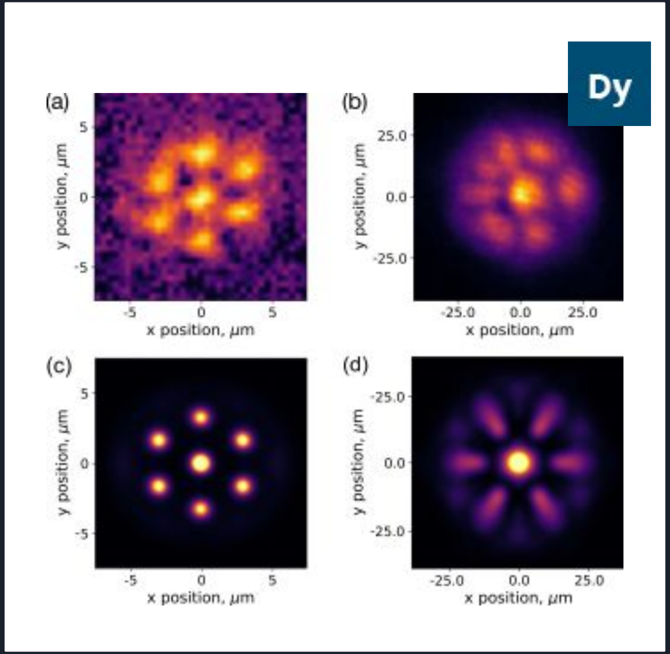


For a suitable choice of parameters, the ground-state has a *density modulation* and is *superfluid*

Supersolid phase observed in LENS, Stuttgart, Innsbruck labs with lanthanide atoms



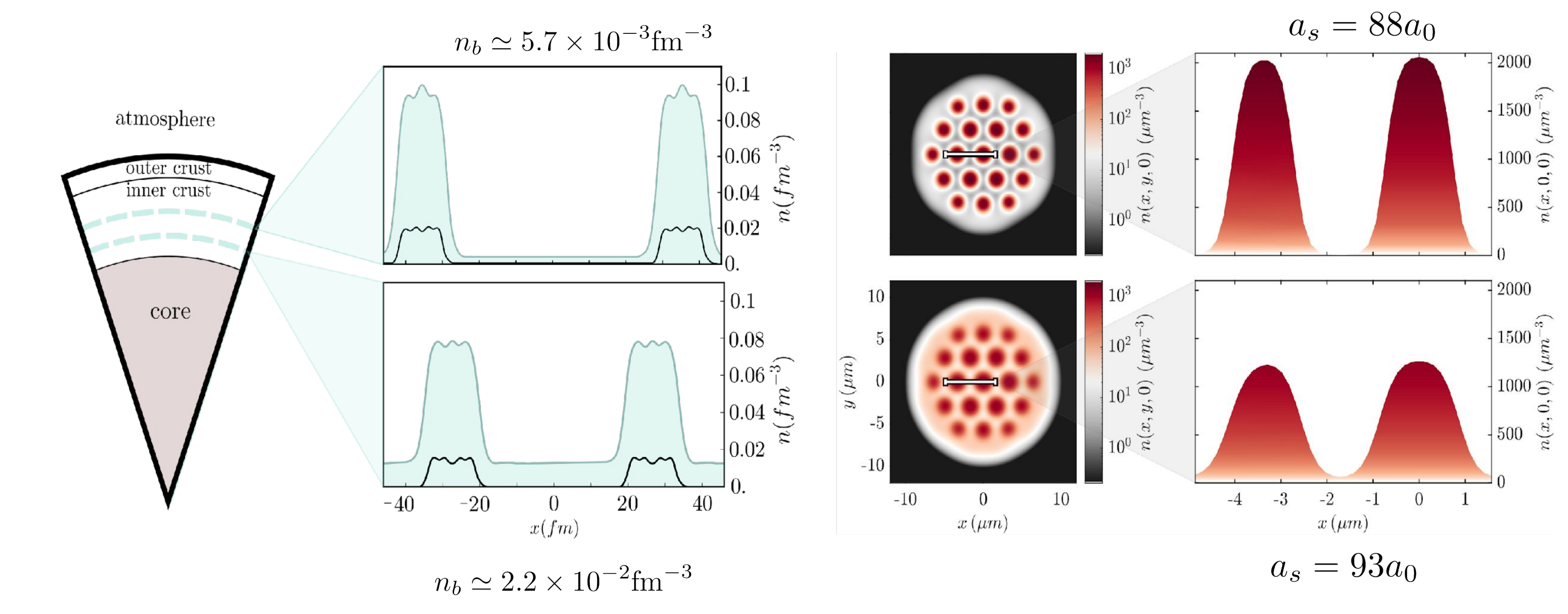
Norcia *et al*, Nature **596**, 357–361 (2021)



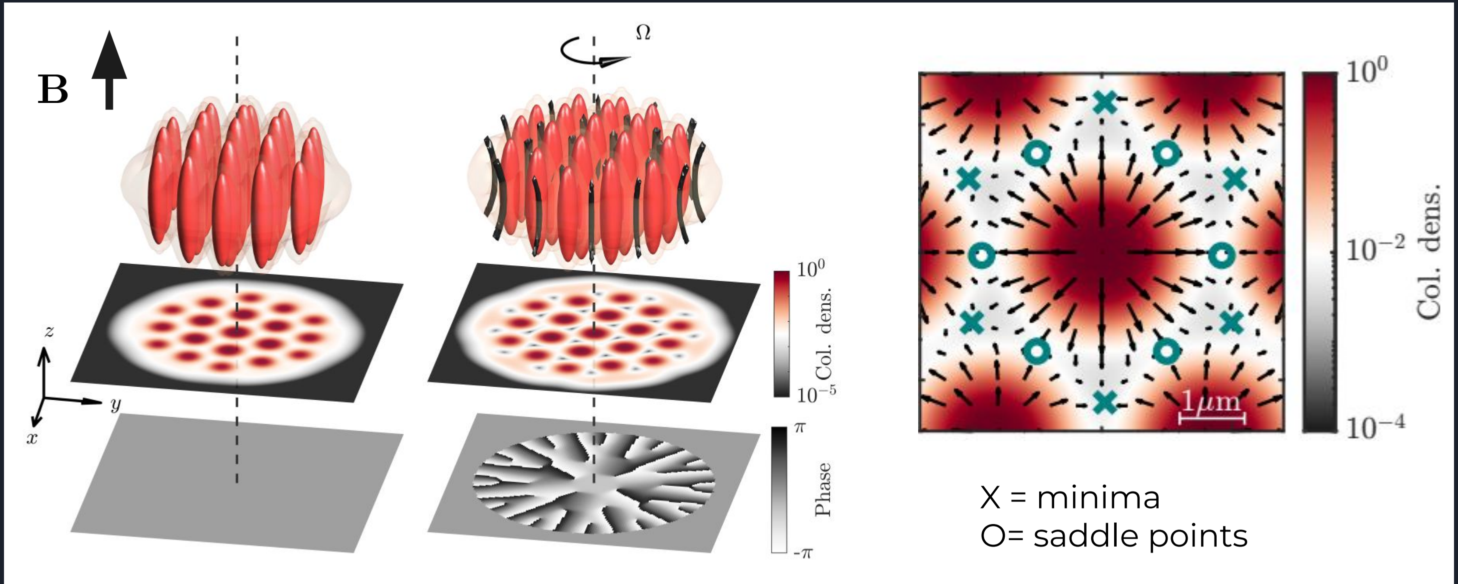
Bland *et al* PRL 128, 195302 (2022)

Glitches with supersolids

Inner crust *vs* Supersolids



In supersolids vortices are pinned in the low-density regions



Supersolid inertia

For a rigid body

$$I_{\text{mass}}(t) = \langle x^2 + y^2 \rangle_{\Psi(t)}$$

$$I_{\text{ss}}(t) = \alpha I_{\text{mass}}(t)$$

how "rigid" is a supersolid

Educated ansatz

$$L_{\text{tot}}(t) = L_{\text{ss}}(t) + L_{\text{vort}}(t)$$

$$L_{\text{ss}}(t) = I_{\text{ss}}(t)\Omega(t)$$

$$\Downarrow$$

$$I_{\text{ss}}(t)\dot{\Omega}(t) = -N_{\text{em}} - \dot{I}_{\text{ss}}(t)\Omega(t) - \dot{L}_{\text{vort}}(t)$$

Legget PRL 25, 1543 (1970)

Time evolution of the supersolid

$$i\hbar\partial_t\Psi = (1 - i\gamma)[\mathcal{H}(\Psi, a_{\mathrm{dd}}, a_{\mathrm{s}}) - \Omega(t)\hat{L}_z]\Psi$$

Schrödinger equation for the wavefunction

$$\mathcal{H}(\Psi, a_{\mathrm{dd}}, a_{\mathrm{s}}) = -\frac{\hbar^2\nabla^2}{2m} + \frac{m}{2}[\omega_r^2(x^2+y^2) + \omega_z^2 z^2] + \int d^3\mathbf{r}'[U_c(\mathbf{r}-\mathbf{r}') + U_{dd}(\mathbf{r}-\mathbf{r}')]|\Psi(\mathbf{r}',t)|^2 + \gamma_{\mathrm{QF}}|\Psi(\mathbf{r},t)|^3 - \mu$$

The numerical evolution

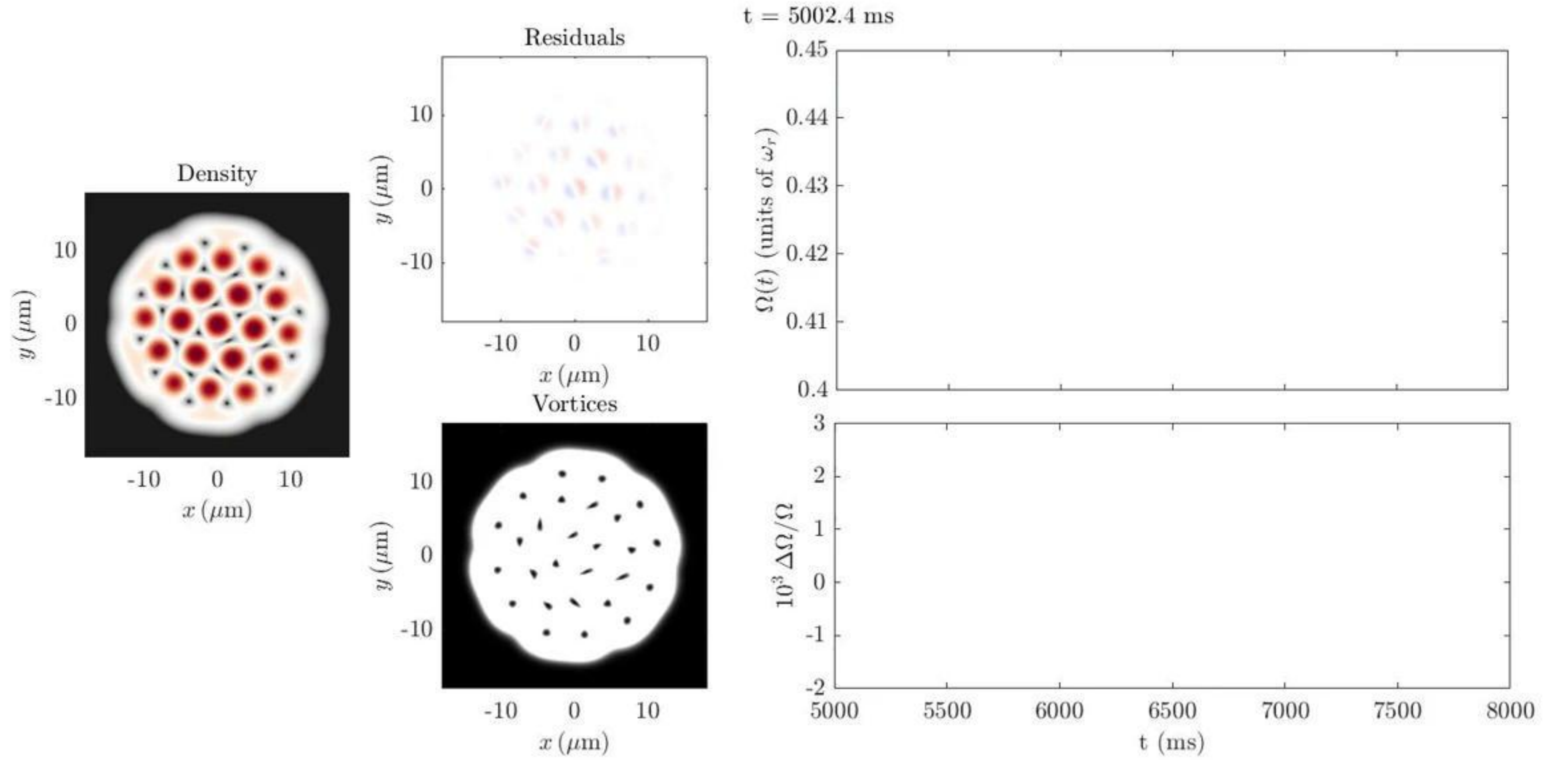
After computing the ground-state wavefunction, at each time step we solve

$$I_{\rm ss}(t)\dot{\Omega}(t) = -N_{\rm em} - \dot{I}_{\rm ss}(t)\Omega(t) - \dot{L}_{\rm vort}(t)$$

Feedback equation

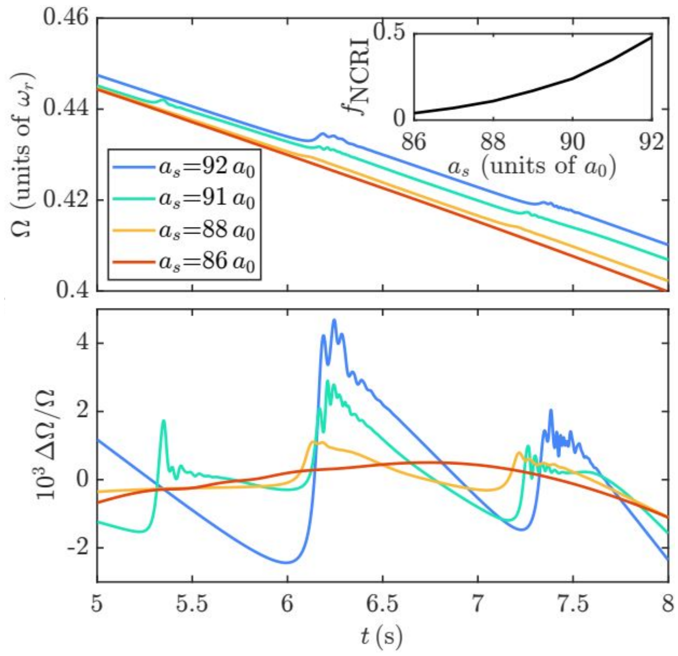
$$i\hbar\partial_t\Psi = (1-i\gamma)[\mathcal{H}(\Psi, a_{\rm dd}, a_{\rm s}) - \Omega(t)\hat{L}_z]\Psi$$

Schrödinger equation
for the wavefunction



$$N_{\rm em} = 4.3 \times 10^{-35}\text{kg m}^2/\text{s}^2\,, \quad \gamma = 0.05\,, \quad \Omega_{\rm init} = 0.5\omega_r\,.$$

Results



$$\alpha = 1 - f_{\text{NCRI}}$$

$$I_{\text{ss}}(t) = \alpha I_{\text{mass}}(t)$$

$$N_{\text{em}} = 4.3 \times 10^{-35} \text{kg m}^2/\text{s}^2\,, \quad \gamma = 0.05\,, \quad \Omega_{\text{init}} = 0.5\omega_r\,.$$

Conclusions

- Numerical observation of supersolid glitches
- Simulations of inner crust vortex interstitial pinning
- Observation of crystal oscillations and vortices percolation

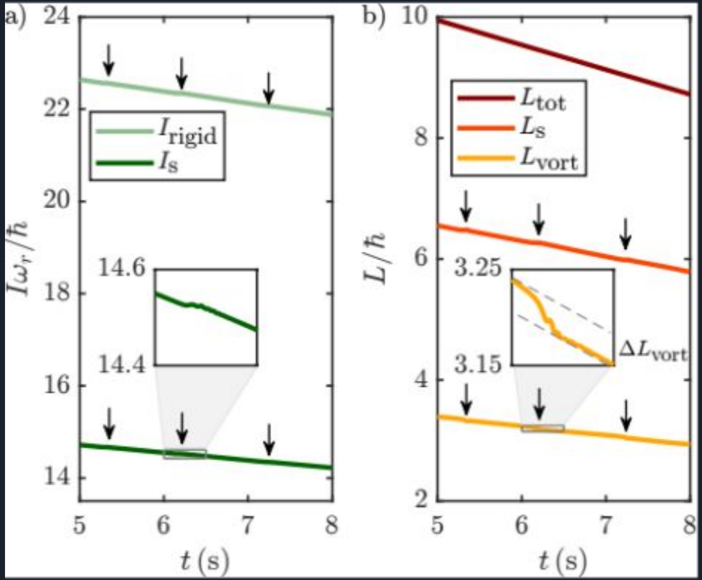
Future perspectives

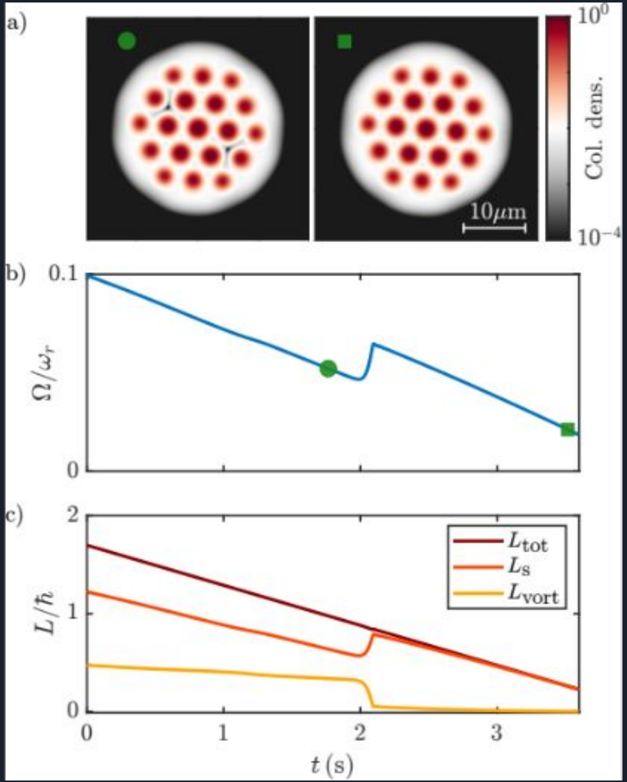
- Scalability of the pulse shape
- Connection to microphysics
- Neutron star’s heating processes
- Glitches, r-modes and gravitational waves

Thank you for listening!

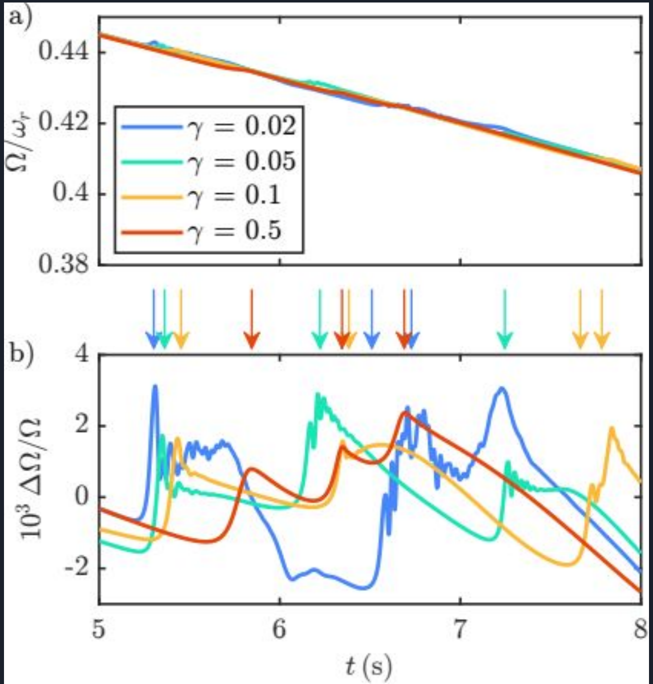
Backup slides

Testing the angular momentum model



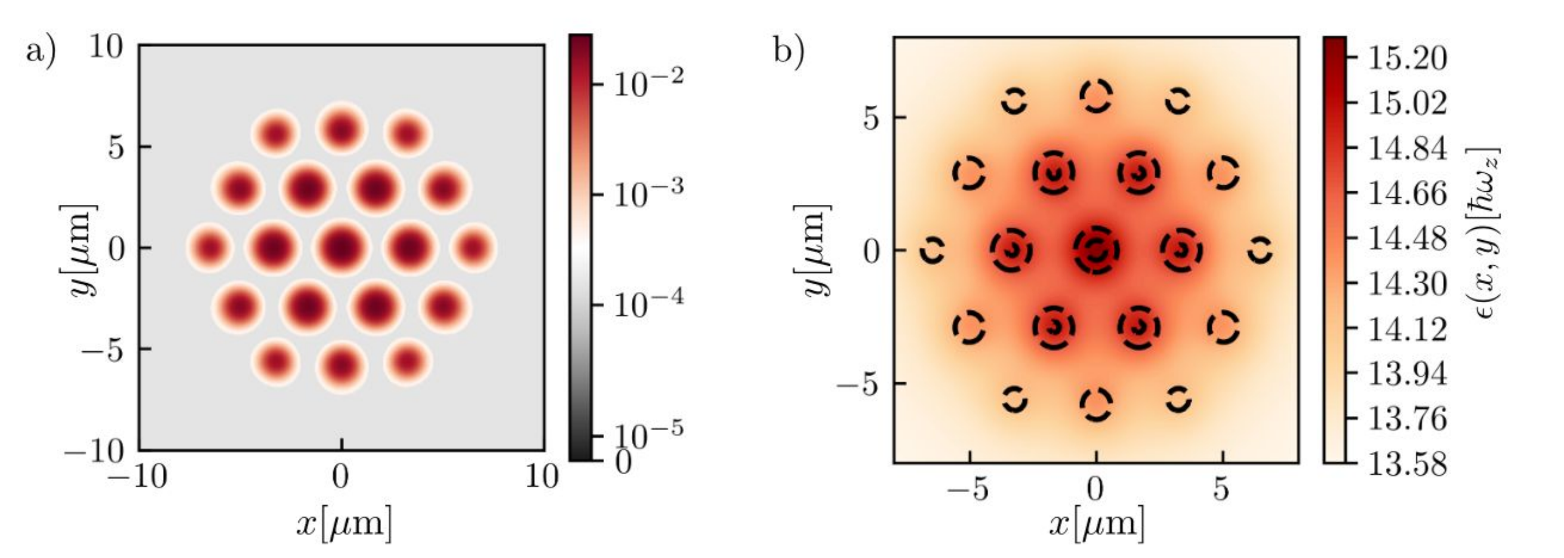


The role of dissipation



$$i\hbar\partial_t\Psi = (1 - i\gamma)[\mathcal{H}(\Psi, a_{\mathrm{dd}}, a_{\mathrm{s}}) - \Omega(t)\hat{L}_z]\Psi$$

Pinning energy



3D view