20+ YEARS OF INCLUSIVE V_{cb}

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35 years swinging in flavour physics
Roma Tre, 17.05.2024

ELSEVIER

NUCLEAR PHYSICS B

Nuclear Physics B 527 (1998) 21–43

Next-to-leading QCD corrections to $B \to X_s\gamma$: Standard Model and two-Higgs doublet model

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NUCLEAR PHYSICS B

Nuclear Physics B 534 (1998) 3–20

Next-to-leading QCD corrections to $B \to X_s\gamma$ in supersymmetry

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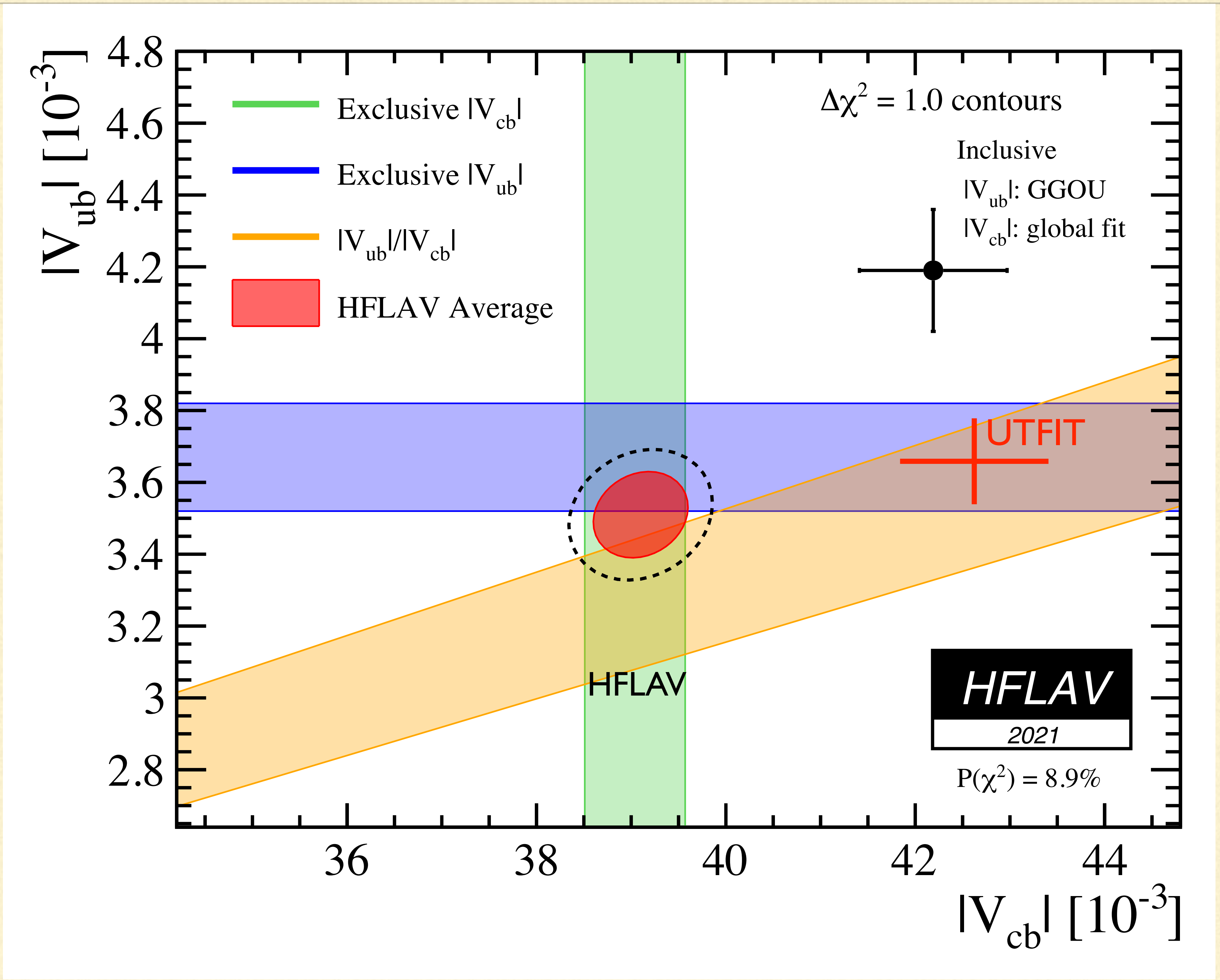
Received 17 June 1998; accepted 6 July 1998

Off-shell matching…

$$BR(B \to X_s\gamma) \propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

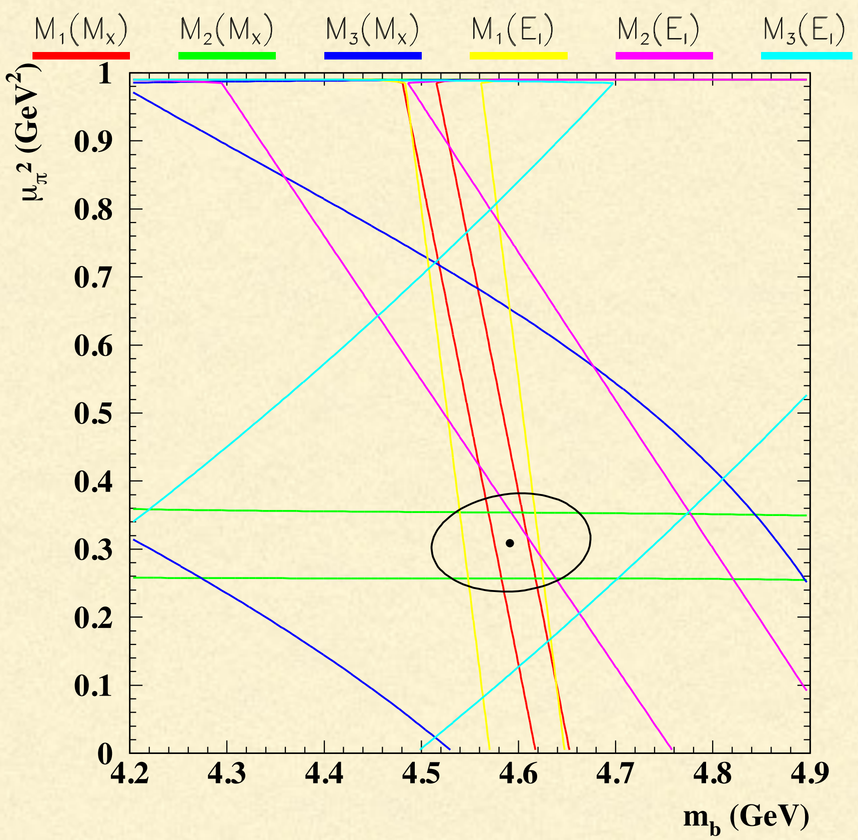
Since many years the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ diverge



Do we believe these errors?

DELPHI 2002

hep-ph/0210319



Heavy Quark Parameters and $|V_{cb}|$ from Spectral Moments in Semileptonic B Decays

Marco Battaglia[1], Marta Calvi[2], Paolo Gambino[1], Arantza Oyanguren[3], Patrick Roudeau[4], Laura Salmi[5], Jose Salt[3], Achille Stocchi[4], Nikolai Uraltsev[2,6,7]

Fit Parameter	Fit Values	Fit Uncertainty	Syst. Uncertainty	
$m_b(1\text{ GeV})$	4.59	$\pm$ 0.08	$\pm$ 0.01	GeV
$m_c(1\text{ GeV})$	1.13	$\pm$ 0.13	$\pm$ 0.03	GeV
$\mu_\pi^2(1\text{ GeV})$	0.31	$\pm$ 0.07	$\pm$ 0.02	GeV^2
ρ_D^3	0.05	$\pm$ 0.04	$\pm$ 0.01	GeV^3

$$|\mathrm{V_{cb}}| = 0.0419 \times \left(1 \pm 0.016|_{meas} \pm 0.015|_{fit} \pm 0.010|_{pert}\right)$$

2.4% uncertainty on V_{cb}

OPE for inclusive B decays set-up in 1990s

Chay,Georgi,Grinstein, Bigi,Shifman, Uraltsev,Vainshtein…

Here first analysis without an expansion in $1/m_c$

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

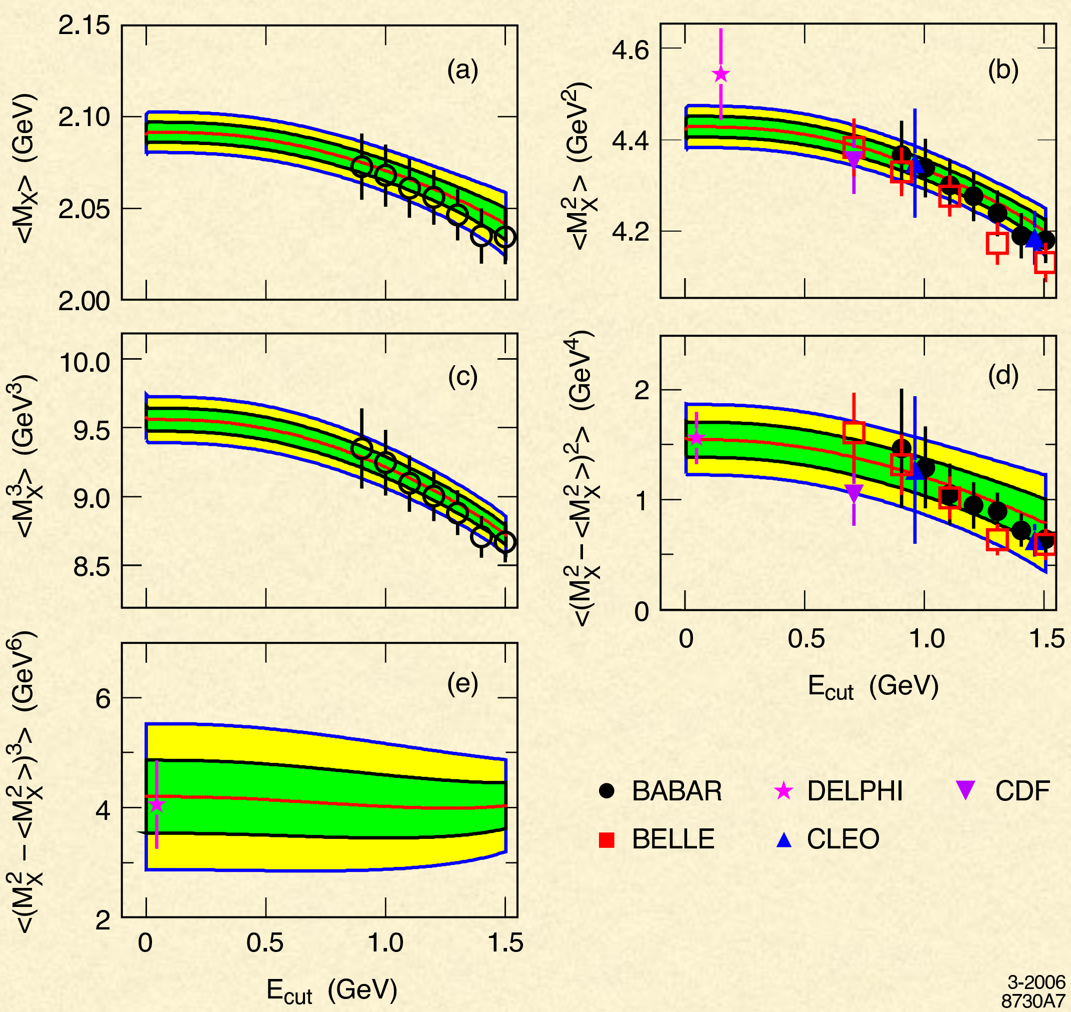
Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

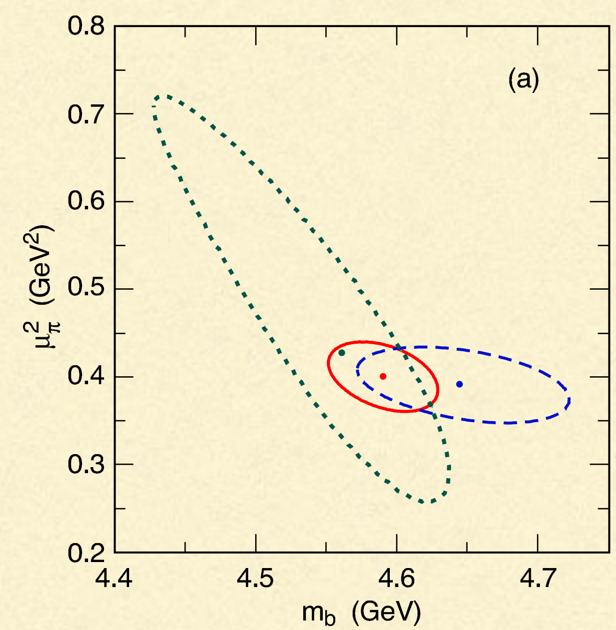
Reliability of the method depends on our control of higher order effects.
Quark-hadron duality violation would manifest itself as inconsistency in the fit.

BABAR+BELLE 2005

Buchmuller&Flaecher 0507253



Combined Fit	OPE FIT RESULT: χ^2/N_{dof} =19.3/44					
	$\lvert V_{cb}\rvert \times 10^{-3}$	m_b (GeV)	m_c (GeV)	μ_π^2 (GeV^2)	ρ_D^3 (GeV^3)	μ_G^2 (GeV^2)
RESULT	41.96	4.590	1.142	0.401	0.174	0.297
Δ exp	0.23	0.025	0.037	0.019	0.009	0.024
Δ HQE	0.35	0.030	0.045	0.035	0.022	0.046
$\Delta\ \Gamma_{SL}$	0.59					



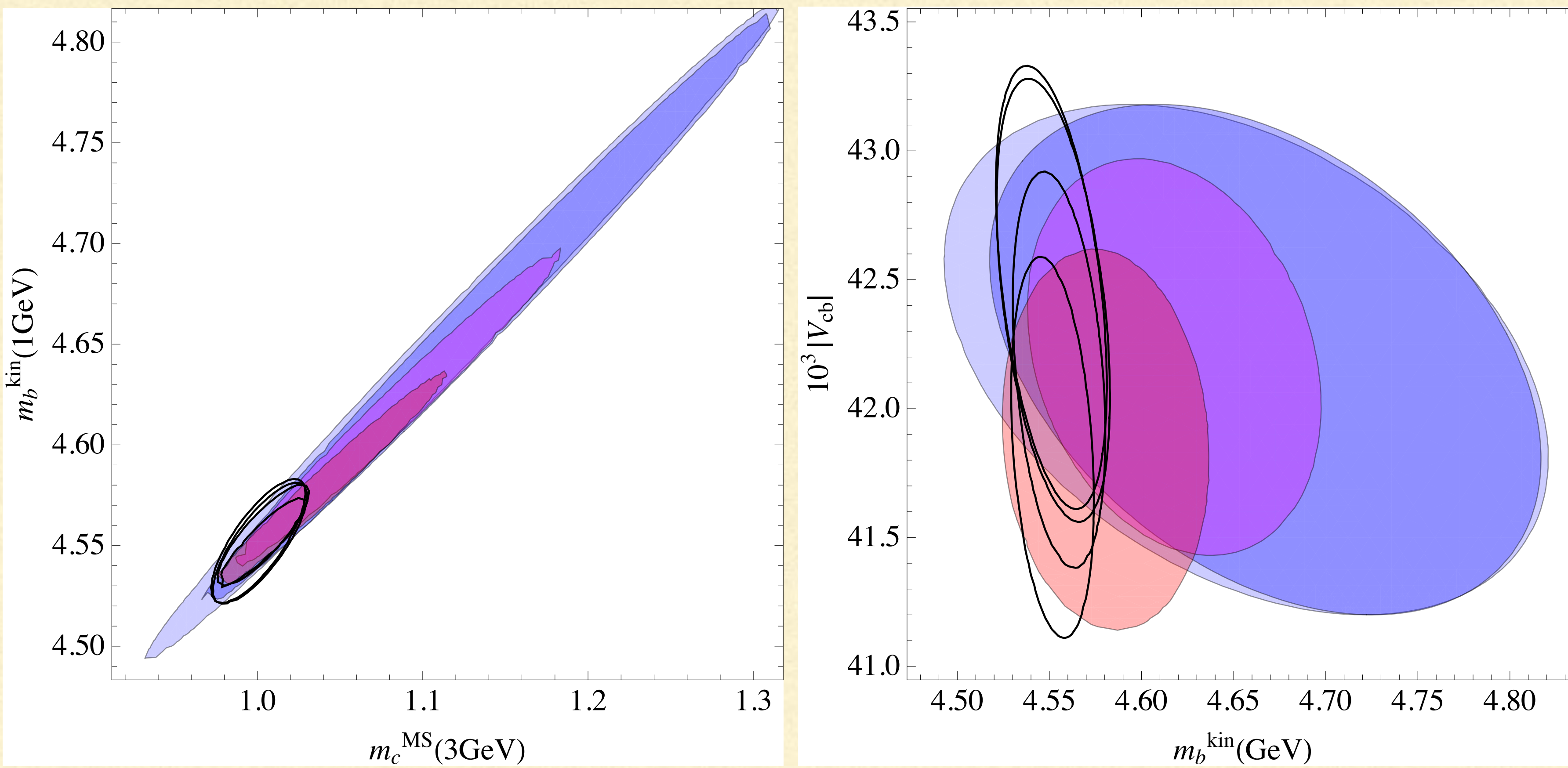
1.7% uncertainty on V_{cb}

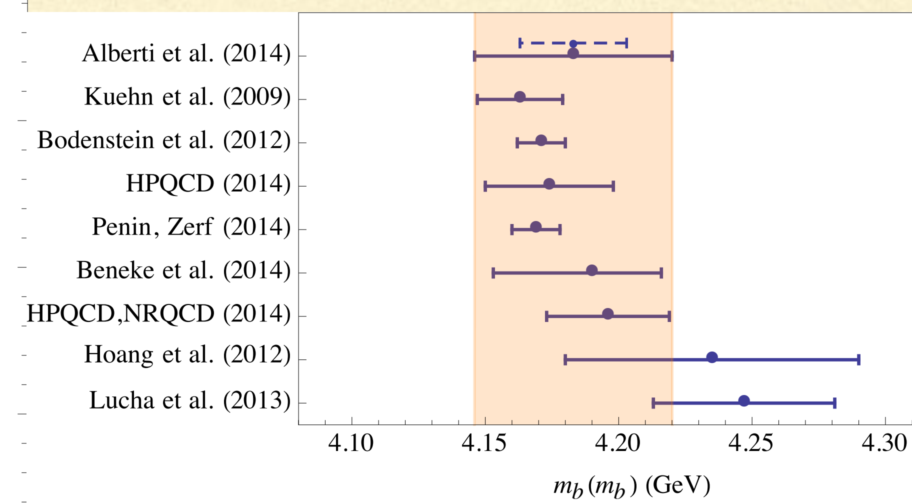
Combined $B \to X_c \ell \nu$, $B \to X_s \gamma$ fit

unknown $O(\alpha_s \Lambda/m_b)$

HEAVY QUARK MASSES AND THEORETICAL CORRELATIONS

C. Schwanda, PG 2013





$$|V_{cb}| = (42.42 \pm 0.86)\ 10^{-3}$$

Fig. 3. Two-dimensional projections of the fits performed with different assumptions for the theoretical correlations. The orange, magenta, blue, light blue 1-sigma regions correspond to the four scenarios considered in [58]. The black contours show the same regions when the m_c constraint of Ref. [59] is employed.

State-of-the art

- **Total Rate**
  - NNLO   Czarnecki, Pak, *Phys.Rev.D* 78 (2008) 114015, *Phys.Rev.Lett.* 100 (2008) 241807
  - N3LO ($b \to c$)   **MF**, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003.
  - N3LO ($b \to u$) **NEW**   **MF**, Usovitsch, hep-ph/2310.03685
- **M_X and E_l moments**
  - NLO differential rate   Aquila, Gambino, Ridolfi, Uraltsev, Nucl.Phys.B 719 (2005) 77
  - NNLO for moments with $E_{\text{cut}} < E_l$, numerical results for specific E_{cut} and $\rho = m_c/m_b$ Biswas, Melnikov, JHEP 02 (2010) 089; Gambino, JHEP 09 (2011) 055. Gambino, JHEP 09 (2011) 055.
  - NLO for μ_π^2 and μ_G^2 Alberti, Gambino, Nandi, *Nucl.Phys.B* 870 (2013) 16, *JHEP* 01 (2014) 147
  - N3LO for moments without cuts   **MF**, Schönwald, Steinhauser, JHEP 08 (2022) 039.
- **q^2 moments with a lower cut on q^2**
  - NLO up to ρ_D^3 Moreno, Mannel, Pivovarov, *Phys.Rev.D 105 (2022) 5, 054033*
  - NNLO for moments with $q^2_{\text{cut}} \leq q^2$ **NEW**   **MF**, Herren, in preparation
- **QED effects**

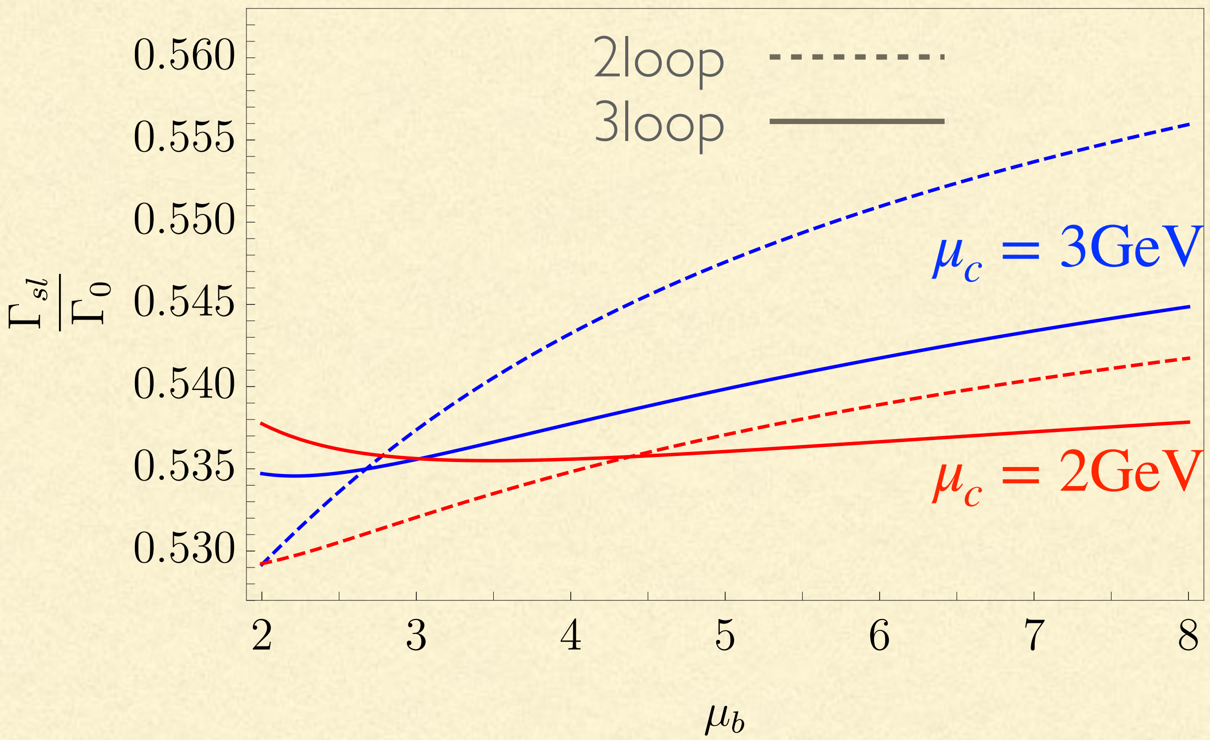
  Bordone, Gambino, Haisch, Piccione, hep-ph-2309.02849

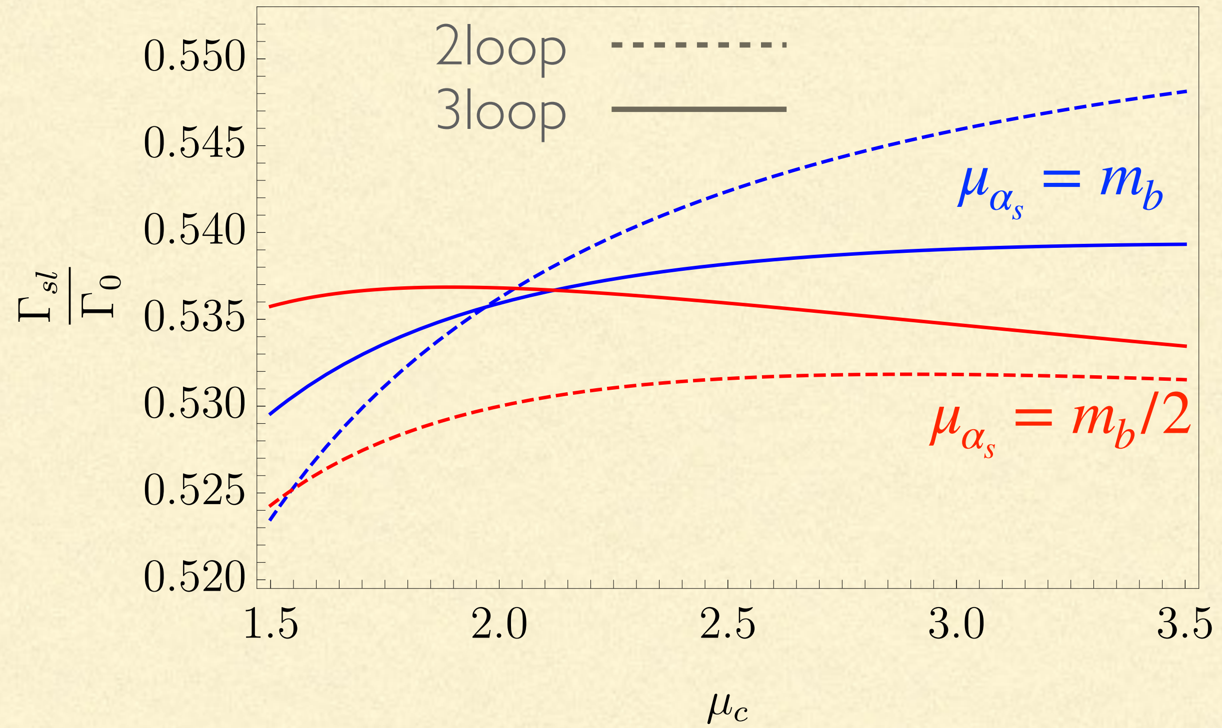
Also important: 3loop relation kinetic-pole mass

Calculations based on new sophisticated methods

RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604





Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty $\pm 0.7\,\%$ (max spread), central values at $\mu_c = 2\text{GeV}, \mu_{\alpha_s} = m_b/2$.

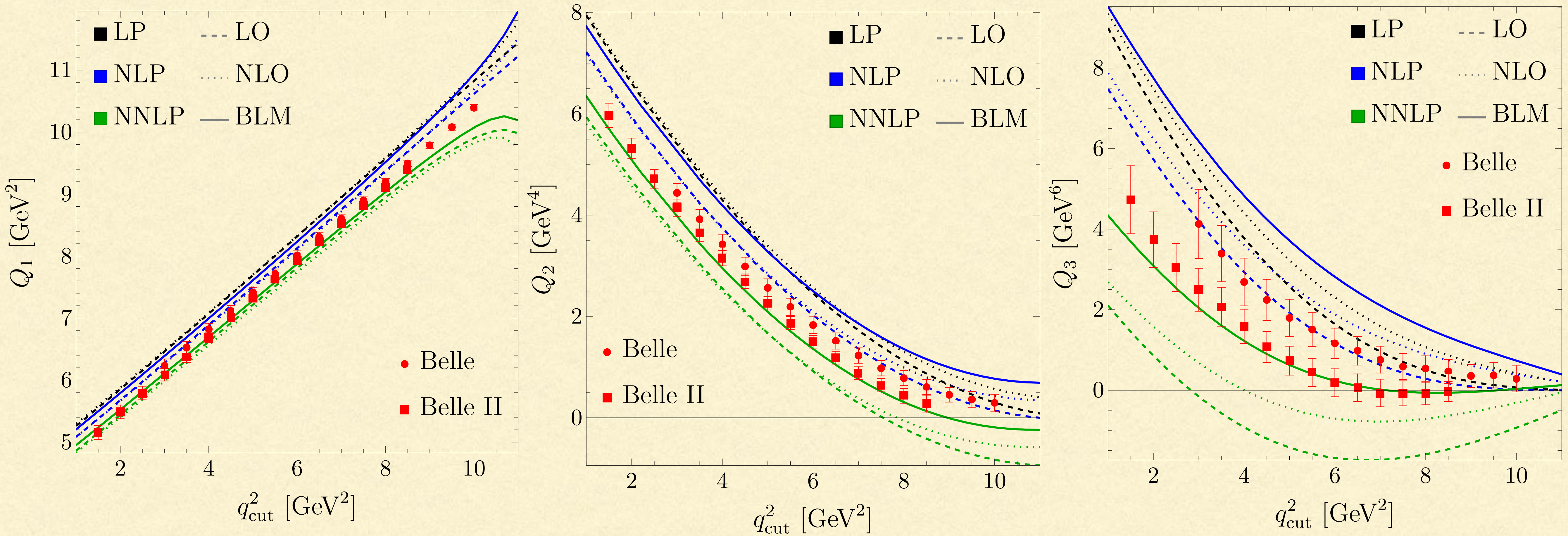
$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3 m_c)$, duality violation.

Conservatively: 1.2% overall theory uncertainty in Γ_{sl}

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$

$O(\alpha_s^2\beta_0)$ CORRECTIONS TO q^2 MOMENTS

Finauri, PG 2310.20324



sizeable for 2nd and 3rd moments

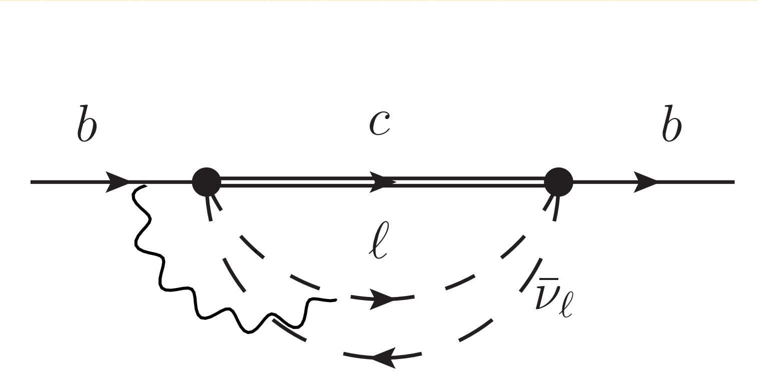
Belle and Belle II moments differ by $\sim 2\sigma$

New $O(\alpha_s^2)$ calculation Fael and Herren 2403.03976

QED CORRECTIONS

Bigi, Bordone, Haisch, Piccione PG
2309.02849

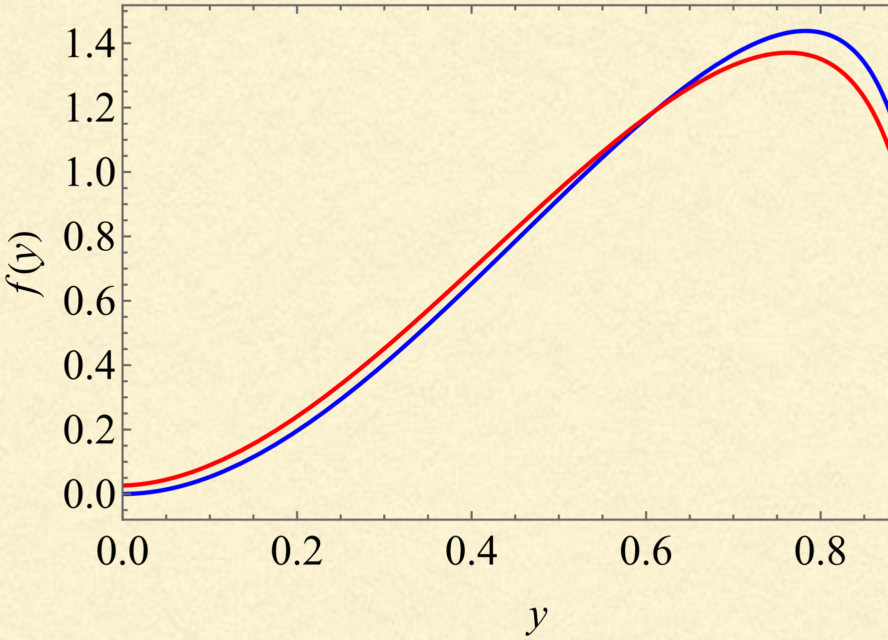
In the presence of photons, ***OPE valid only for total width*** and moments that do not resolve lepton properties (E_ℓ, q^2). Expect mass singularities and $O(\alpha\Lambda/m_b)$ corrections.



Leading logs $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)}\left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$

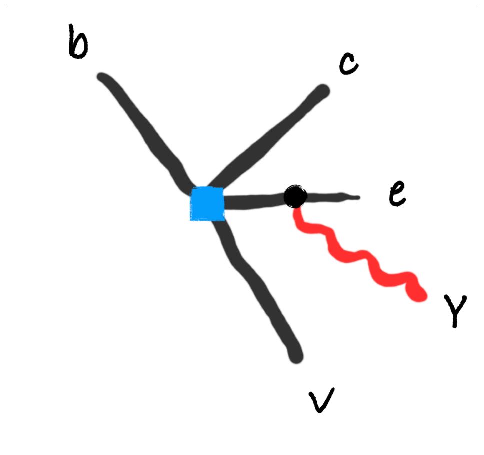
$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+$$



Electron energy spectrum

QED Leading contributions

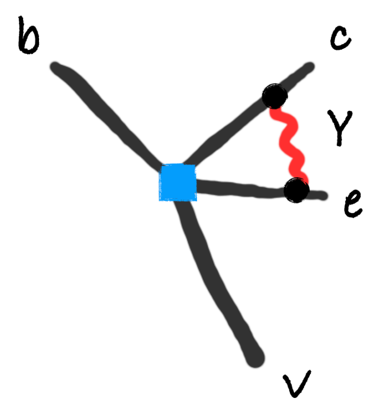
1. **Collinear logs**: captured by splitting functions



also at subleading power!

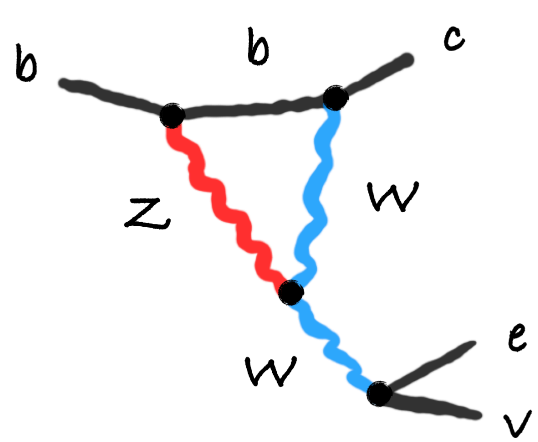
$$\sim \frac{\alpha_e}{\pi} \log^2 \left(\frac{m_b^2}{m_e^2} \right)$$

2. **Threshold effects** or Coulomb terms



$$\sim \frac{4\pi\alpha_e}{9}$$

3. **Wilson Coefficient**



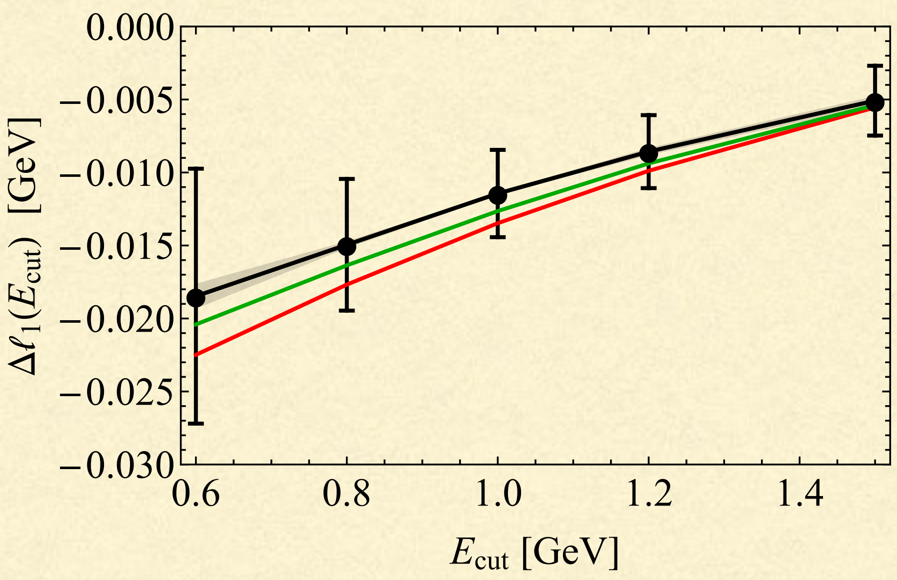
$$\sim \frac{\alpha_e}{\pi} \left[\log \left(\frac{M_Z^2}{\mu^2} - \frac{11}{6} \right) \right]$$

COMPLETE $O(\alpha)$ EFFECTS IN LEPTONIC SPECTRUM

We assume the true E_e determined by the B-factory experiments corresponds to the energy before the electron enters the detector, and after all final-state radiation associated to the hard process has taken place.

Small but non-negligible differences with PHOTOS in BaBar leptonic moments

$E_{\rm cut}$	$\delta{\rm BR}_{\rm incl}^{\rm BaBar}$	$\delta{\rm BR}_{\rm incl}^{\rm LL}$	$\delta{\rm BR}_{\rm incl}^{\rm NLL}$	$\delta{\rm BR}_{\rm incl}^{\alpha}$	$\delta{\rm BR}_{\rm incl}^{1/m_b^2}$	$\delta{\rm BR}_{\rm incl}$	σ
0.6	−1.26%	−1.92%	−1.95%	−0.54%	−0.50%	−0.45%	+0.34
0.8	−1.87%	−2.88%	−2.91%	−1.36%	−1.29%	−1.22%	+0.30
1.0	−2.66%	−4.03%	−4.04%	−2.38%	−2.26%	−2.15%	+0.25
1.2	−3.56%	−5.43%	−5.41%	−3.65%	−3.43%	−3.27%	+0.14
1.5	−5.22%	−8.41%	−8.26%	−6.37%	−5.73%	−5.39%	−0.09



The black curve corresponds to the correction obtained by BaBar using PHOTOS, while the red (green) curve corresponds to our QED prediction including the LL terms (all QED corrections). The grey band represents the systematic uncertainty on the PHOTOS bremsstrahlungs corrections that BaBar quotes, while the black error bars correspond to the total uncertainties of the QED corrected BaBar results.

~0.2% reduction in V_{cb}

A GLOBAL FIT

Finauri, PG 2310.20324

m_b^{kin}	$\overline{m}_c(2\,\text{GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

Includes all leptonic, hadronic, and q^2 moments

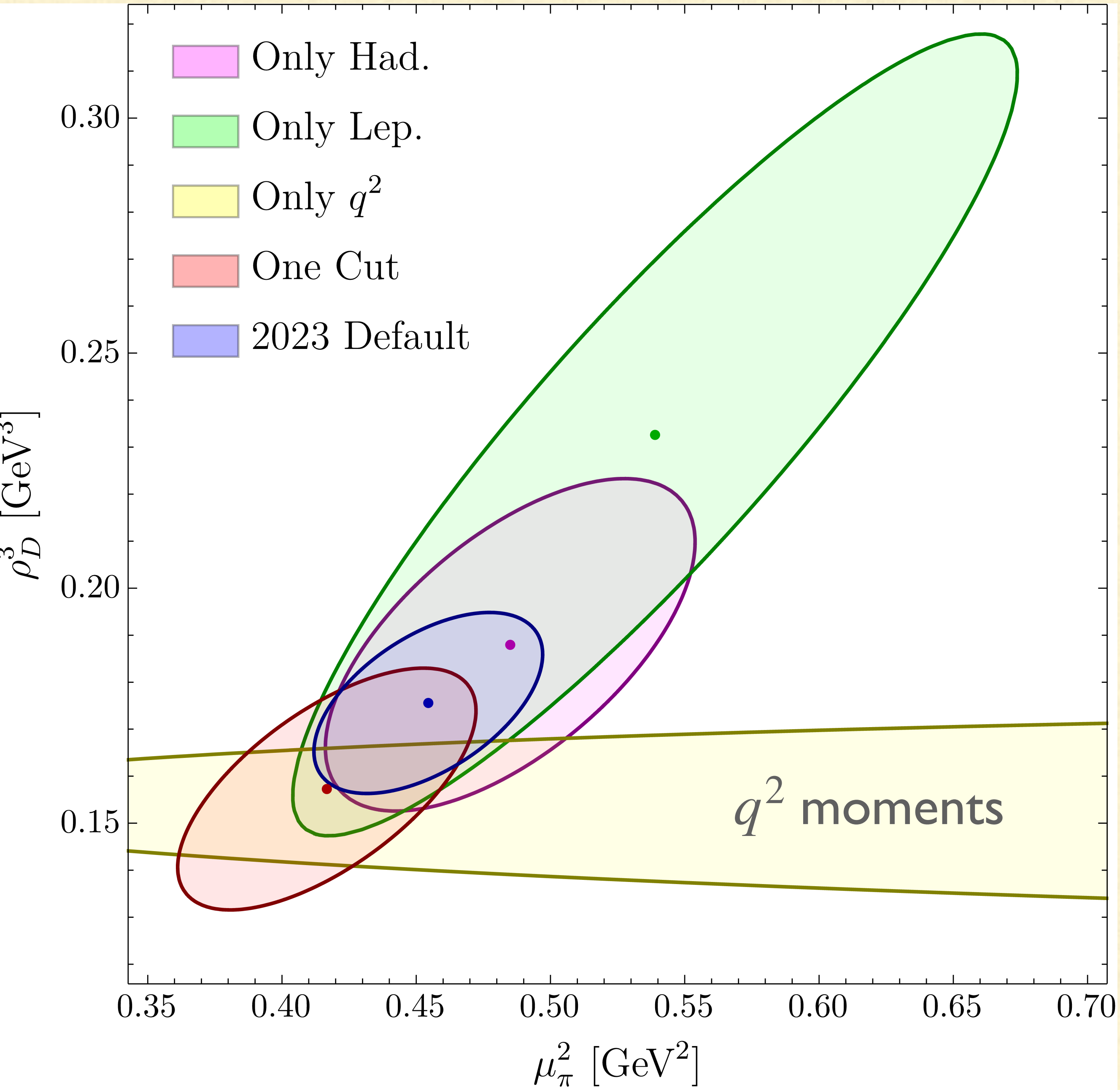
Up to $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$, $O(1/m_b^3)$ for M_X, E_ℓ moments
Up to $O(\alpha_s^2\beta_0)$, $O(\alpha_s/m_b^3)$ for q^2 moments
Subtracts QED effects beyond those computed by PHOTOS (only BaBar BR and lept moments) $\delta|V_{cb}| \sim -0.2\,\%$

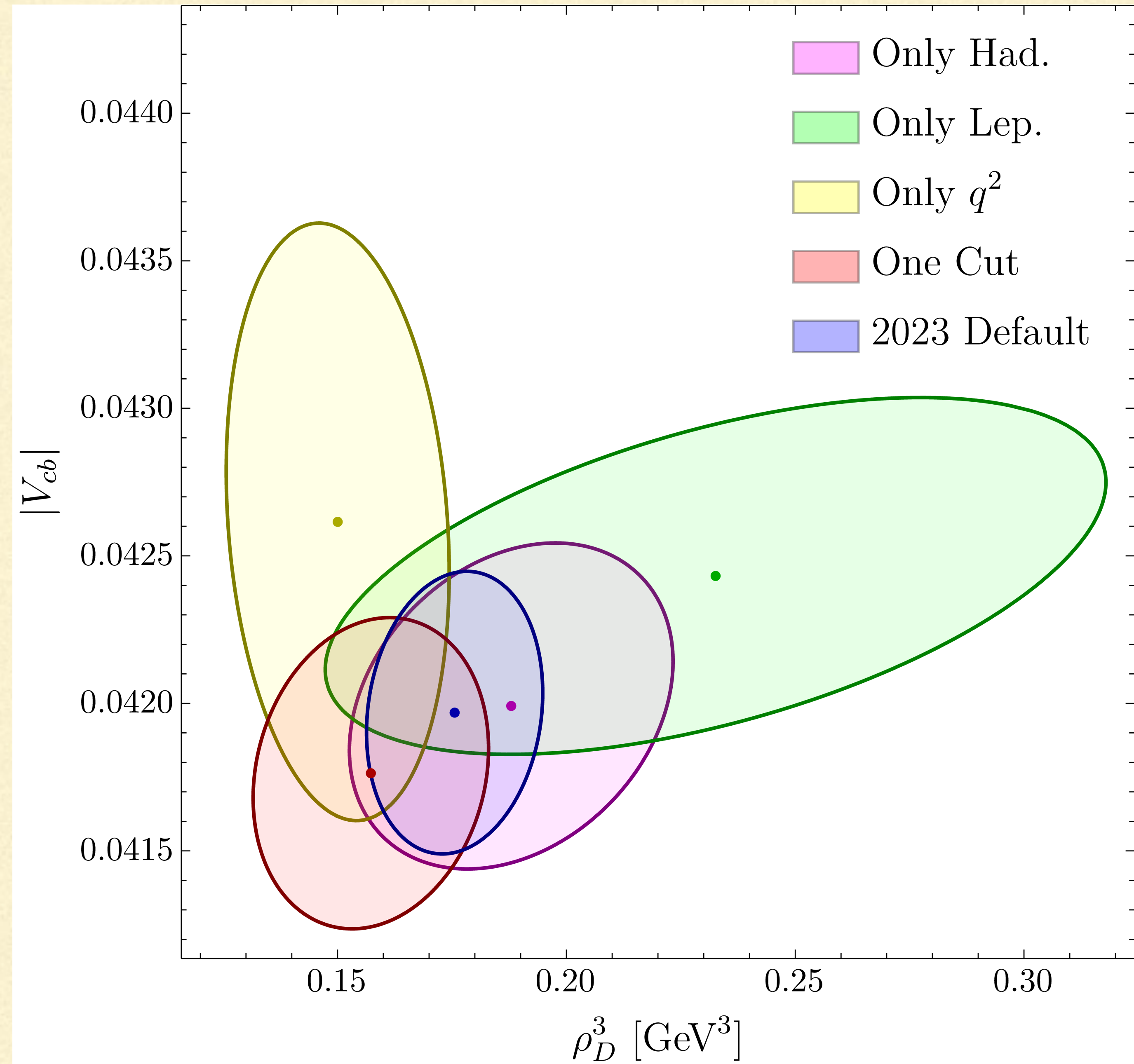
Employs $\overline{m}_b(\overline{m}_b) = 4.203(11)\text{GeV}$ and $\overline{m}_c(3\text{GeV}) = 0.989(10)\text{GeV}$ (FLAG)
$\chi^2_{min}/dof = 0.55$

$$|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_{\Gamma}) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}$$

comparison of different datasets

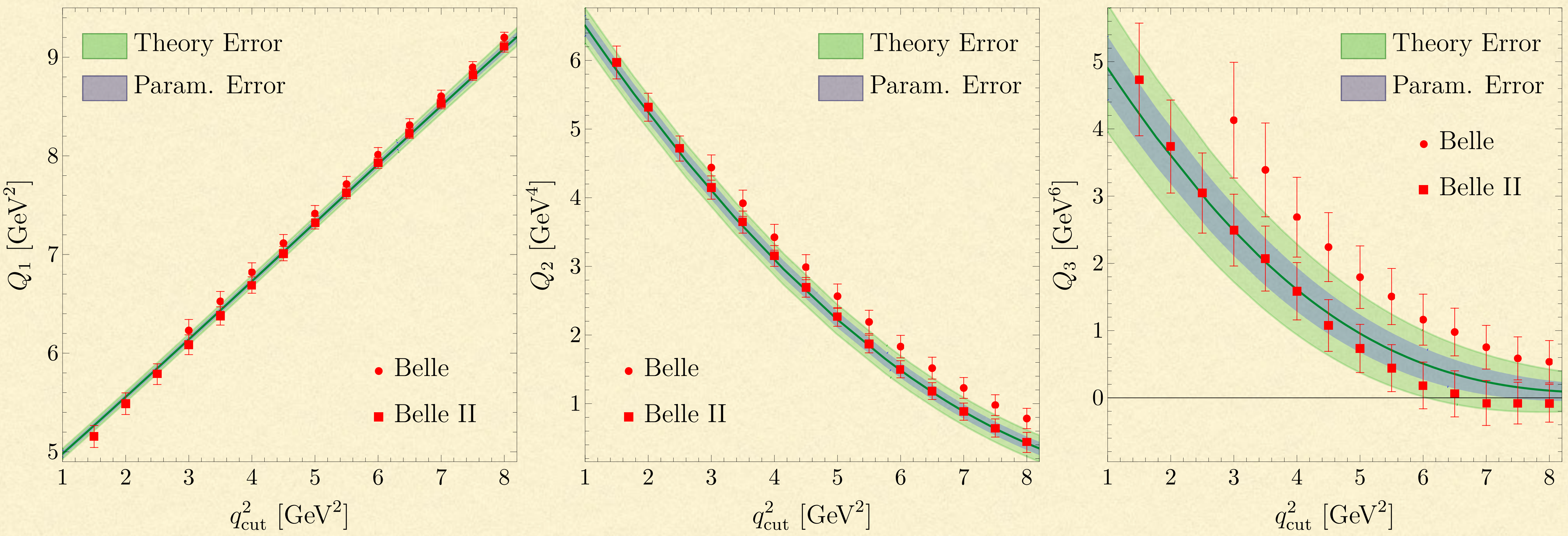
Finauri, PG 2310.20324





Theory correlations are no longer an issue

MINOR TENSIONS IN HIGHER q^2 MOMENTS



HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$ Mannel,Turczyk,Uraltsev 1009.4622

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\, \mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\, \mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

ϵ excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

still without q^2 moments!

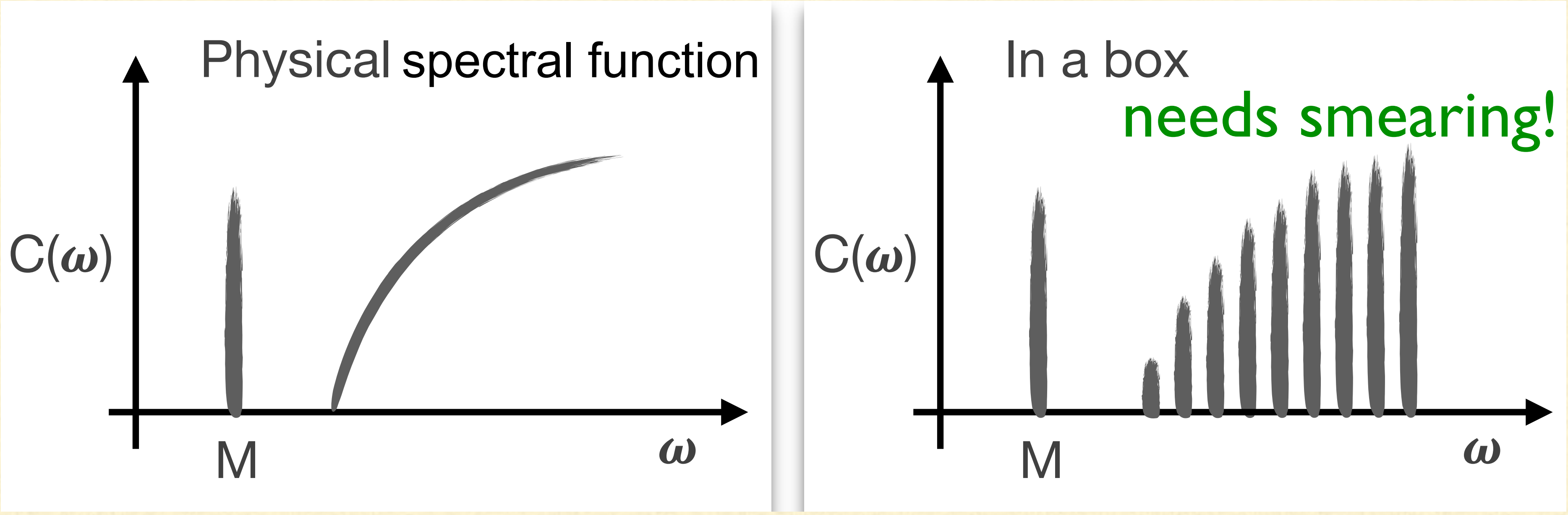
$$|V_{cb}| = 42.00(53)\times 10^{-3}$$

Bordone, Capdevila, PG, 2107.00604
Update of 1606.06174

INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes *impractical* to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \to$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

- While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa



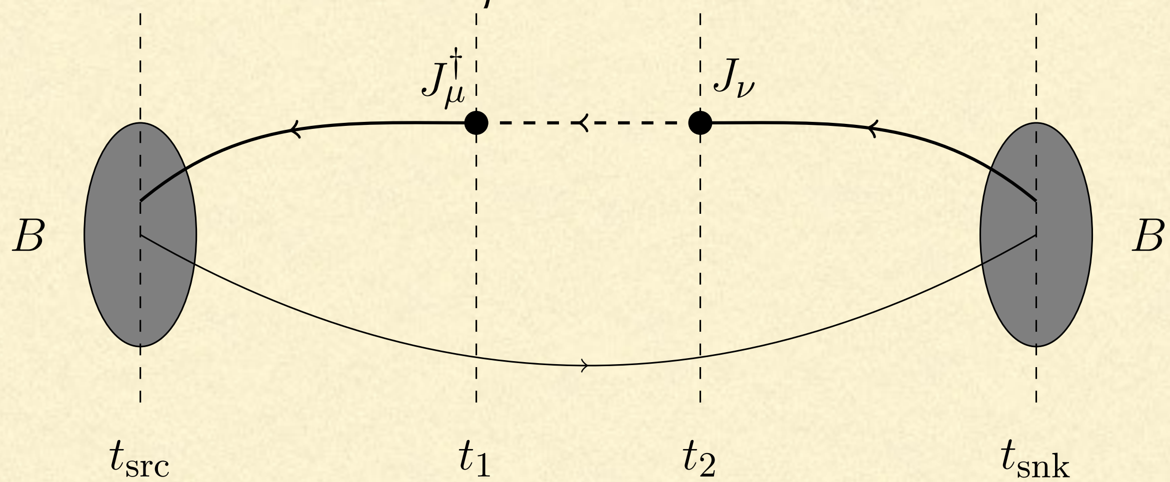
W. Jay @Snowmass workshop

A PRACTICAL APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

$$\sim \langle B | J^\dagger_\mu(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle$$

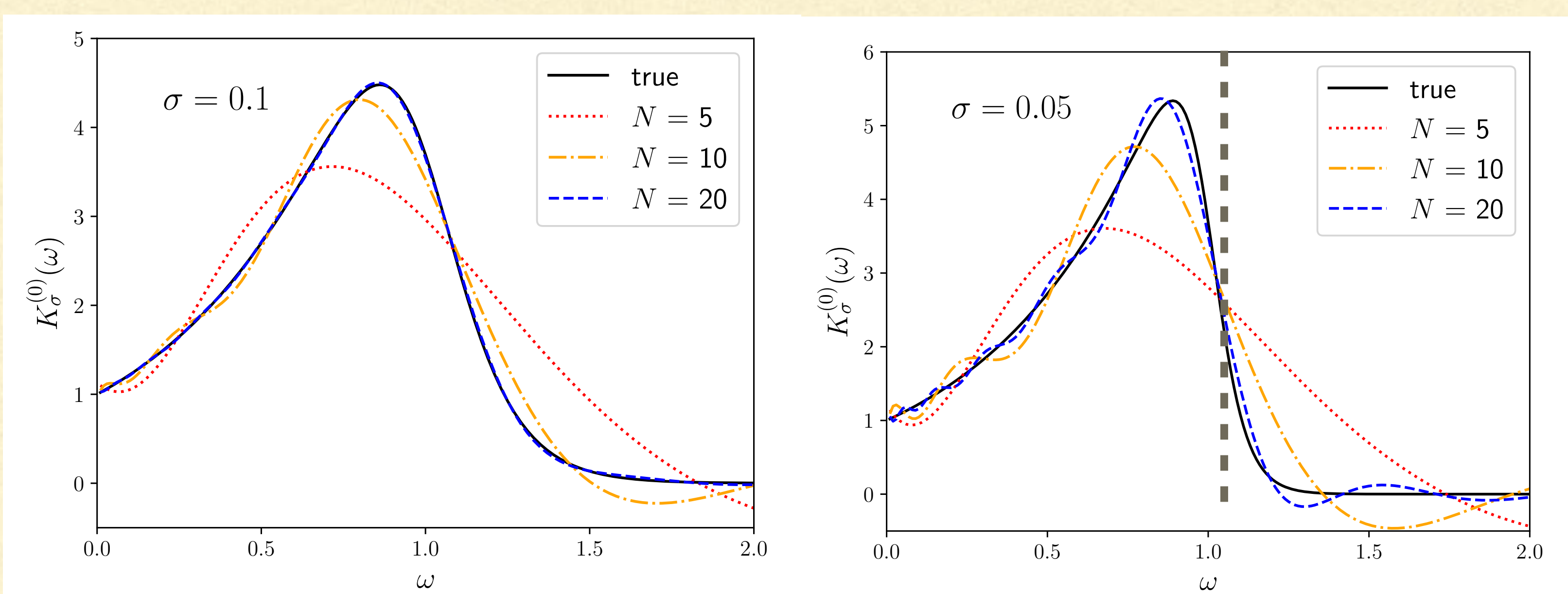


$$d\Gamma \sim L^{\mu\nu} W_{\mu\nu}, \quad W_{\mu\nu} \sim \sum_X \langle B | J^\dagger_\mu | X \rangle \langle X | J_\nu | B \rangle$$

$$\int d^3x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_B} \langle B | J^\dagger_\mu(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle \sim \int_0^\infty d\omega W_{\mu\nu} e^{-t\omega}$$

smearing kernel $f(\omega) = \sum_n a_n e^{-na\omega}$

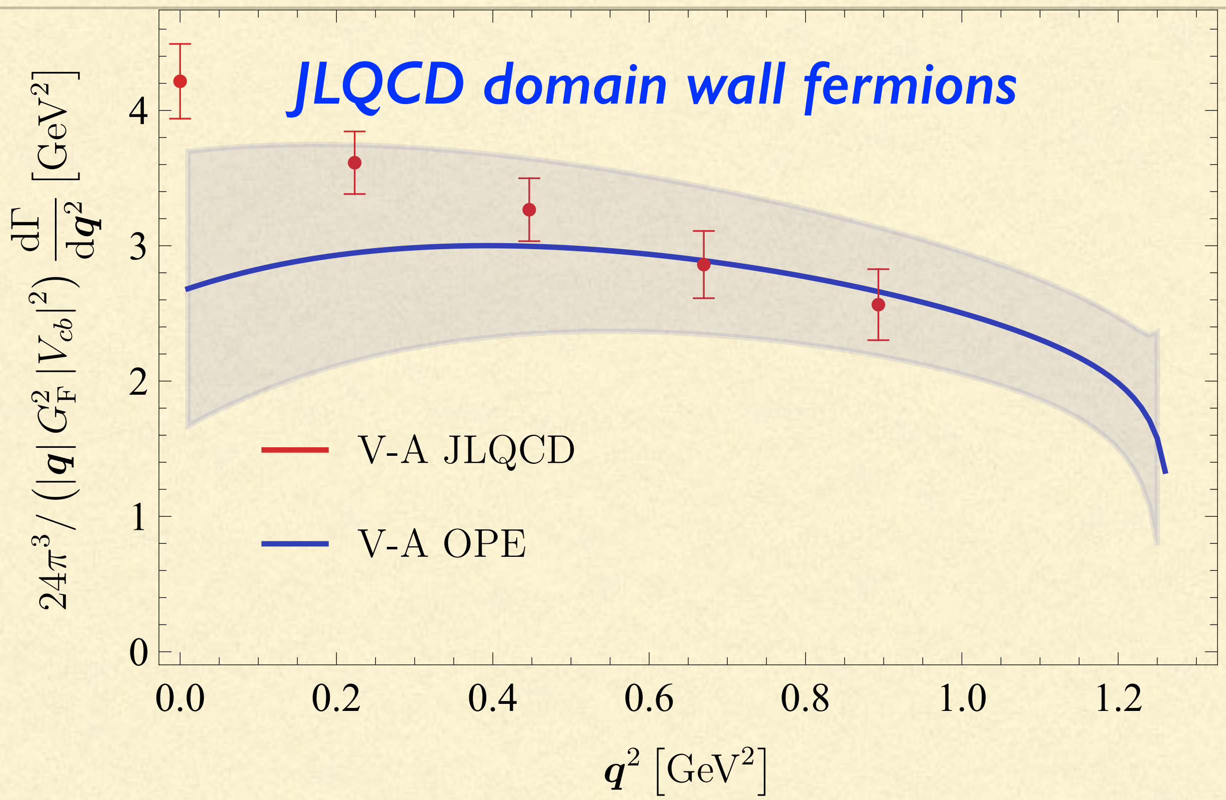
The necessary smearing is provided by phase space integration over the hadronic energy, which is cut by a θ with a sharp hedge: sigmoid $1/(1+e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$ for $\sigma \to 0$. Larger number of polynomials needed for small σ

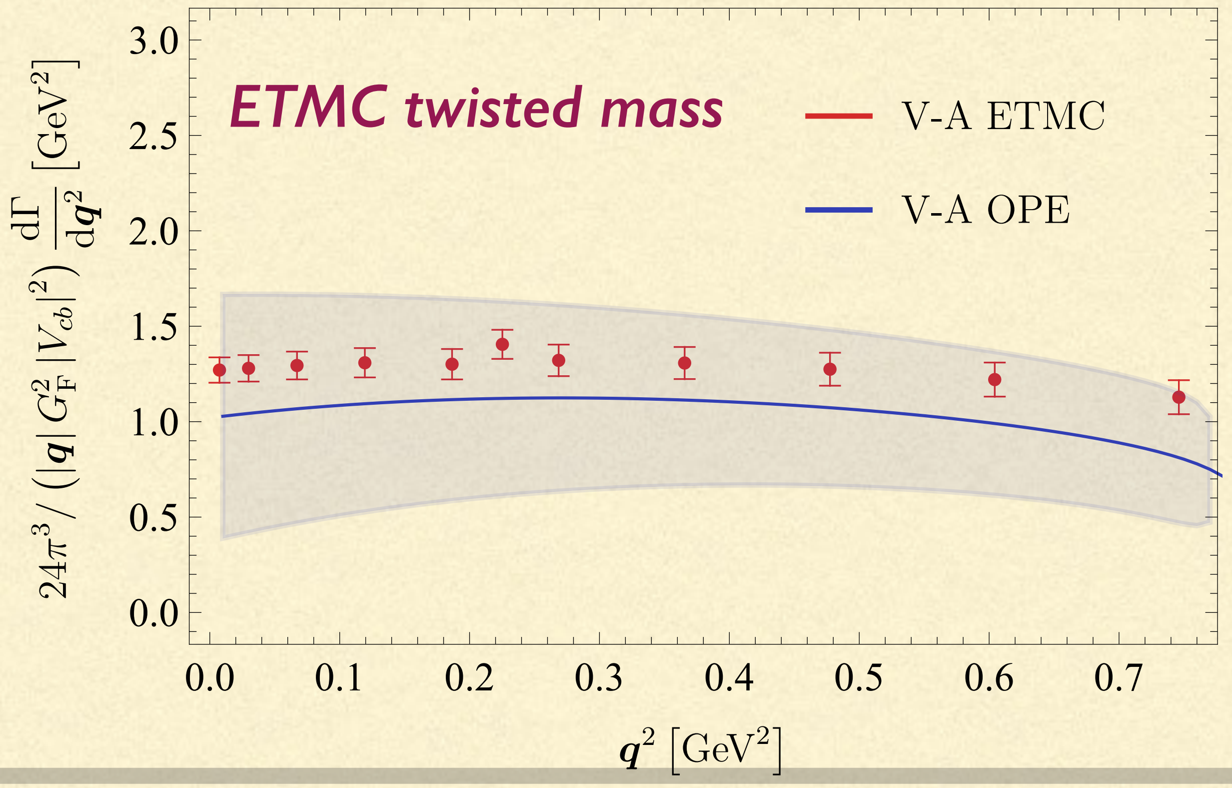


Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

$$\lim_{\sigma\to 0} \lim_{V\to\infty} X_\sigma$$

LATTICE VS OPE





m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2\text{ GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2\text{ GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2\text{ GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2+\mathbf{q}^2} \sim 1-1.5\,\text{GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

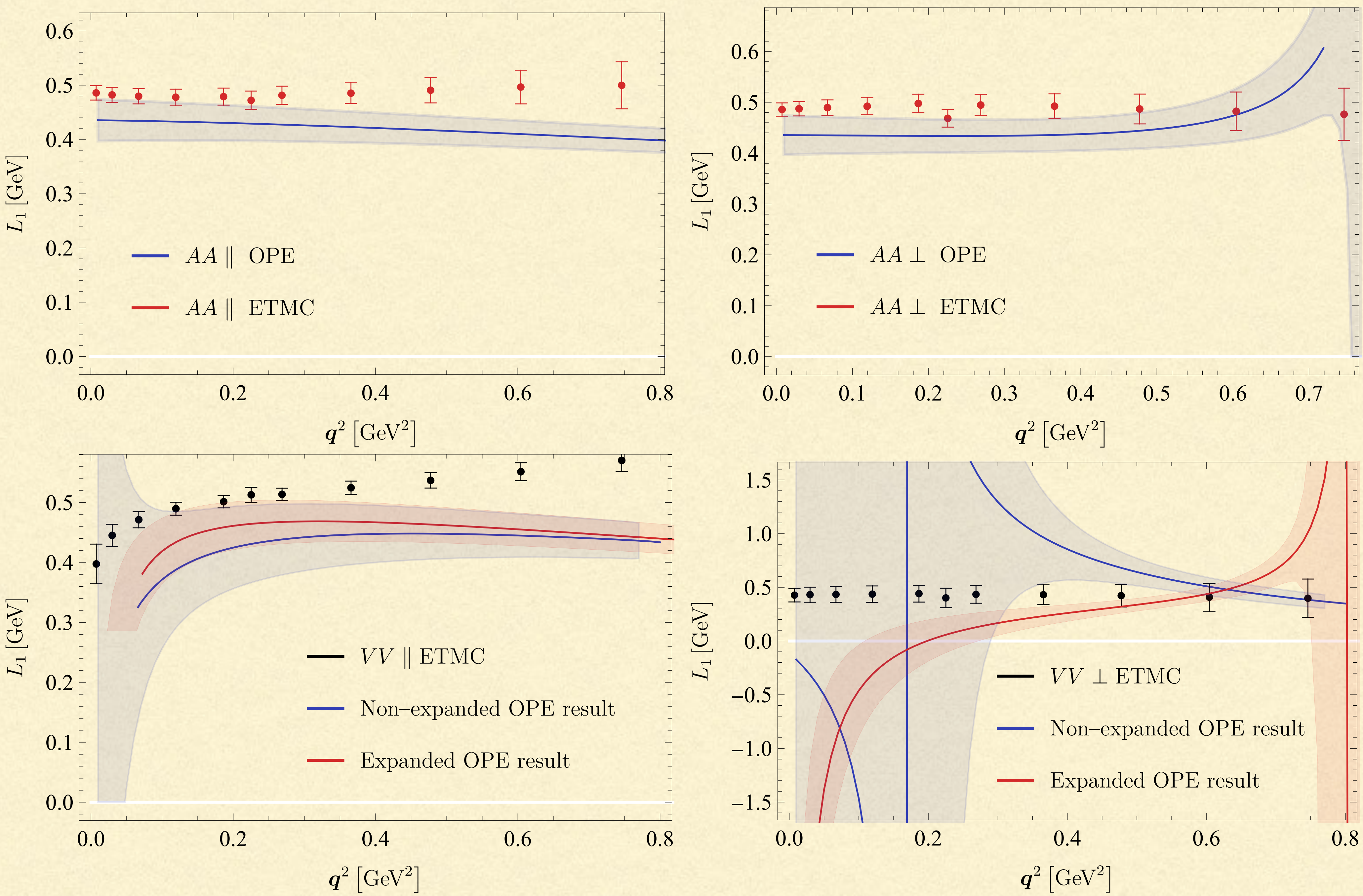
Twisted boundary conditions allow for any value of $\vec{q}^2$

Smaller statistical uncertainties

MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

$$L_1 = \langle E_\ell(\mathbf{q}^2) \rangle$$



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

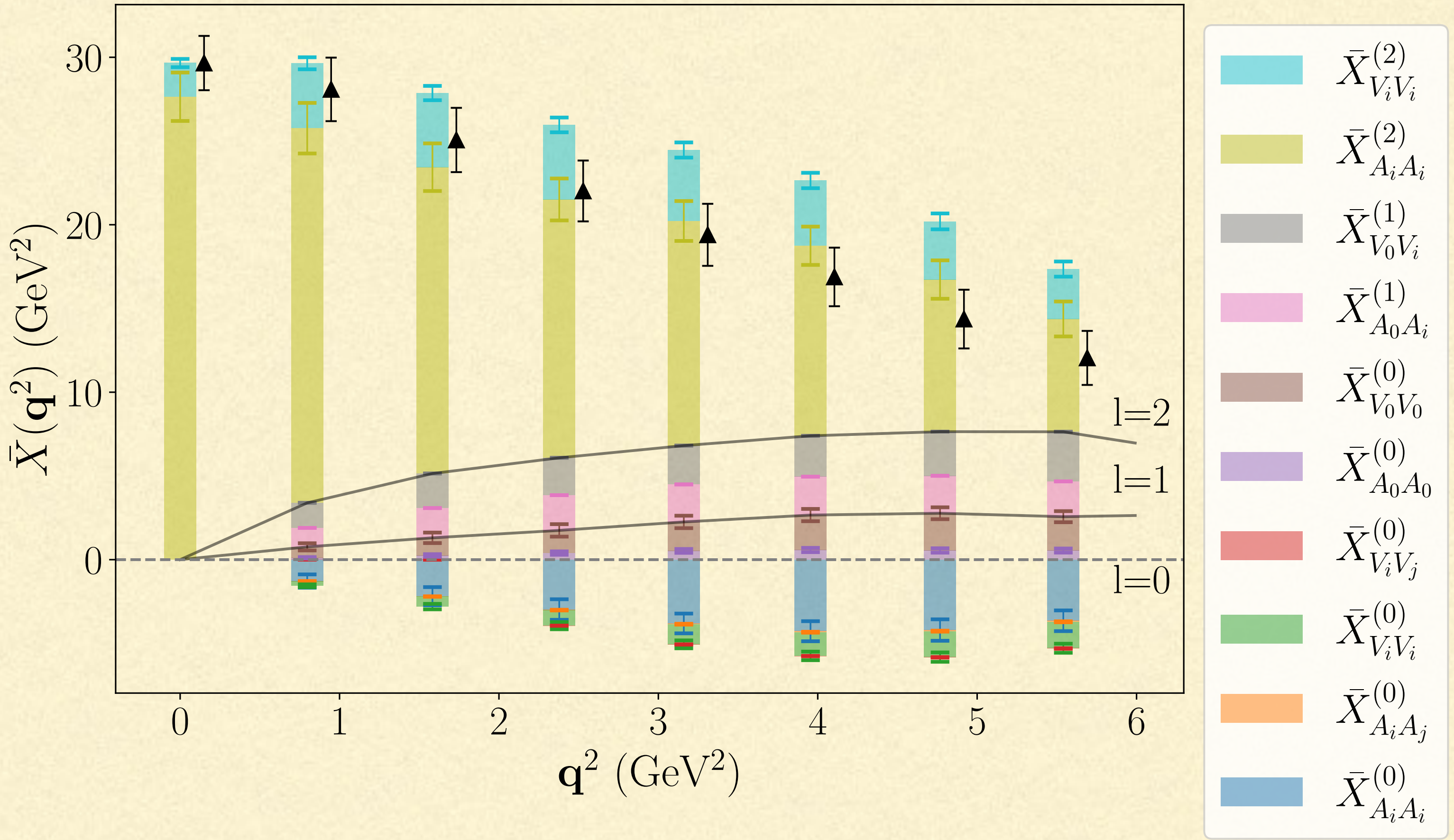
First results at the physical *b* mass

Relativistic heavy quark effective action for b

B_s decays, domain wall fermions, improved implementation of Chebychev polynomials and Backus-Gilbert

qualitative study ~5% statistical uncertainty on total width

possibly better to compare with partial width at low $\vec{q}^2$



Barone, Hashimoto, Juttner, Kaneko, Kellermann, 2305.14092

Ongoing work on semileptonic D_s decays by two collaborations

CONCLUSIONS

- *Inclusive $b \to c$ is robust: q^2 moments consistent with leptonic and hadronic ones; $O(\alpha_s^3)$ effects show perturbation theory OK; higher powers appear small. But don't dream of going below 1%…*
- *Calculations of inclusive semileptonic meson decays on the lattice have started. Will they reach a competing precision? Certainly they will validate and complement the OPE approach*
- *Precision physics may look like a Sisyphus' effort… but it's become the main avenue to search for New Physics. It has to be done with care and passion, following Marco's example*

Auguri Marco!