Born-OPPENHEIMER approximation in QCD

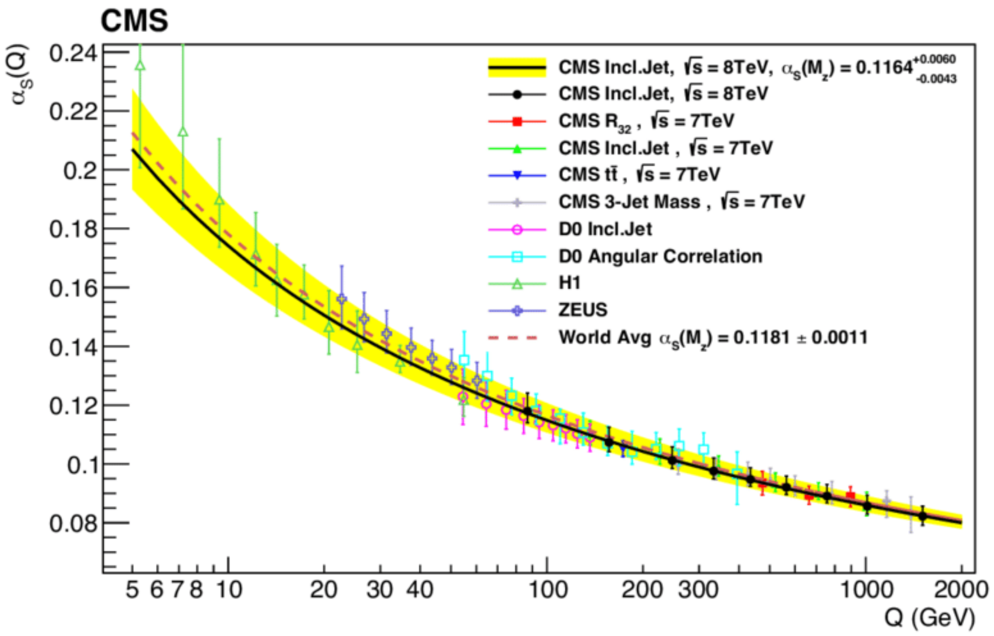
Davide Germani

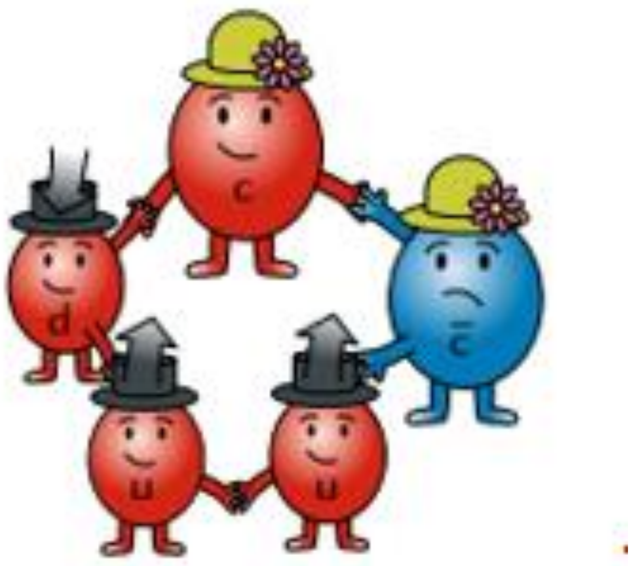
Outline

- QCD Phenomenology;
  - Asintotic freedom and confinement
  - Exotic Hadrons
  - Why Study Exotic Hadrons?
- Born-Oppenheimer Approximation
  - H_2 molecule
  - What about a QCD H_2?
- Conclusions

QCD Phenomenology

QCD is the theory that studies the interactions between quarks mediated by the gluons.



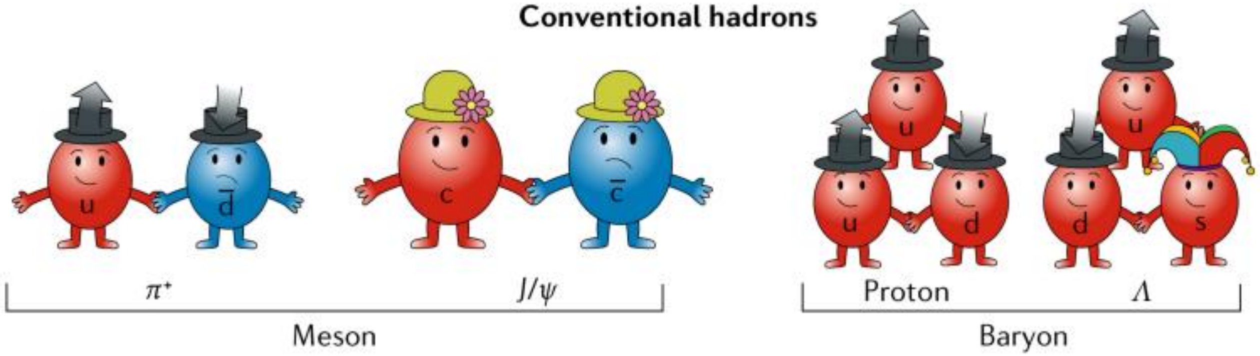


Asymptotic Freedom: Coupling grows at low energy (large distance)

Confinement: Asintotically we observe color singlet. NO QUARKS IN FINAL STATE

QCD Phenomenology

- **Standard hadrons: Mesons and Baryons;**



- **Exotic hadrons:**

Exotic hadrons are subatomic particles composed of quarks and gluons, but which consist of more than three valence quarks or have an explicit valence gluon content.

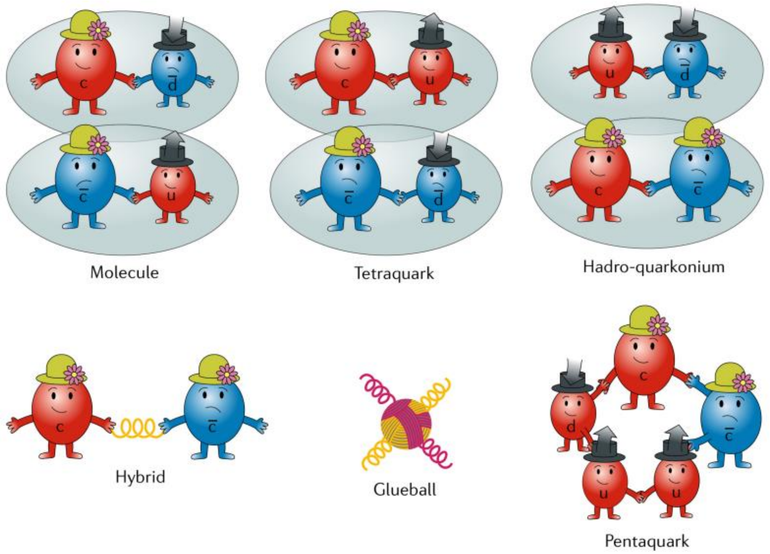
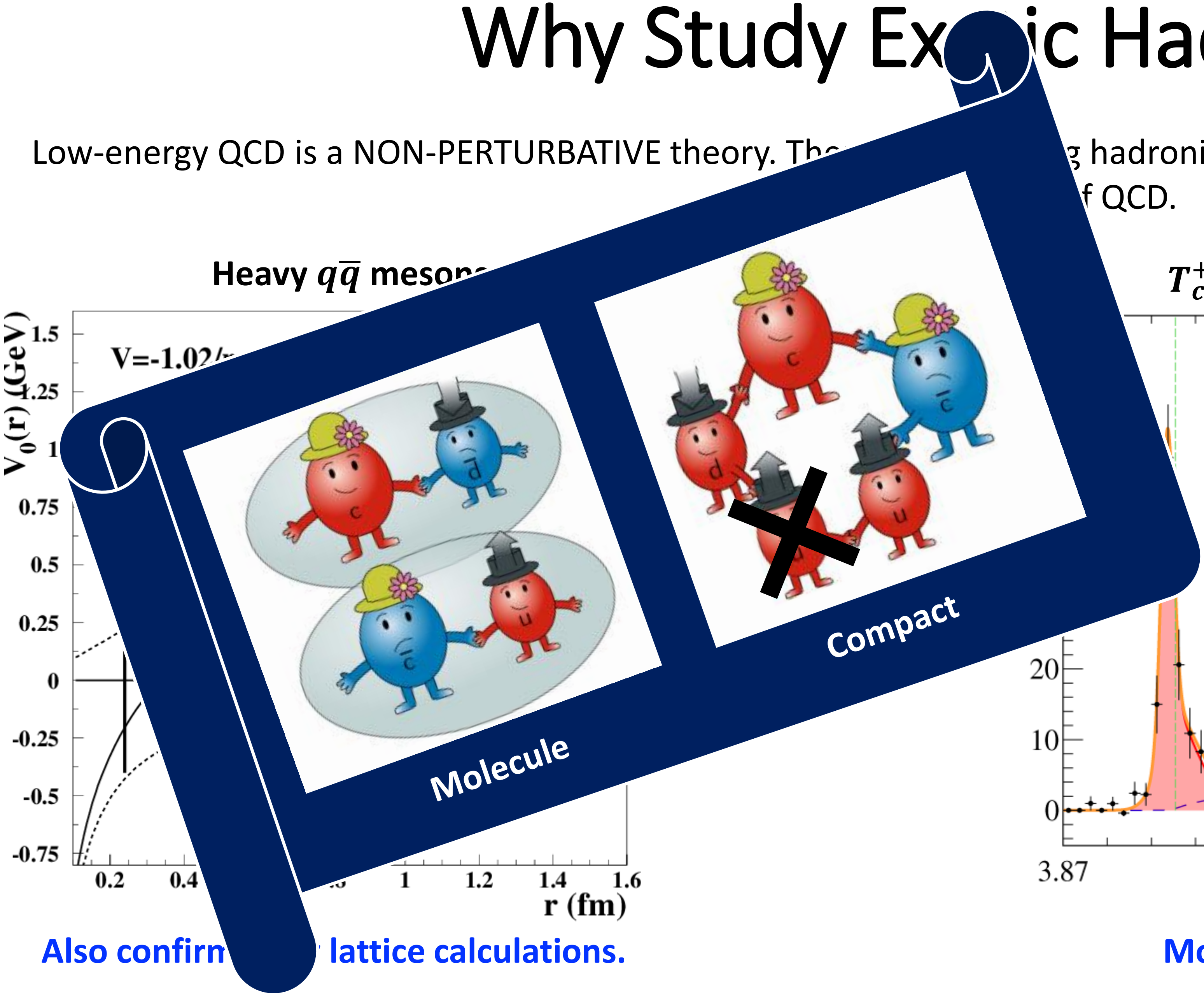


Figure from Nat Rev Phys 1, 480-494 (2019)

Why Study Ex ic Hadrons?

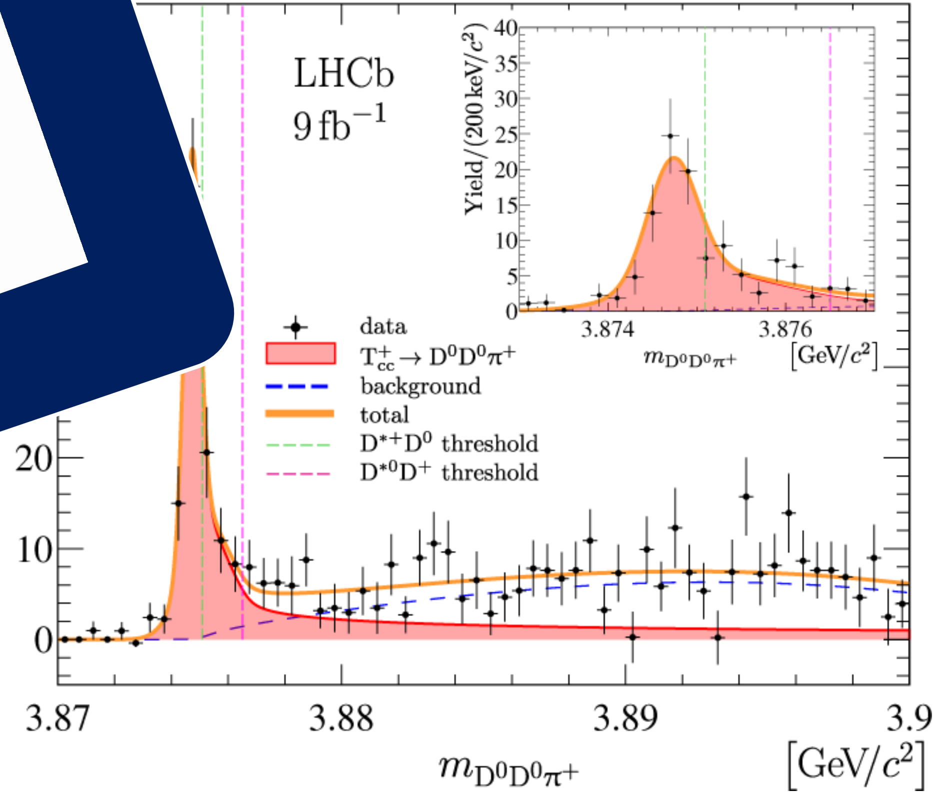
Low-energy QCD is a NON-PERTURBATIVE theory. The g hadronic resonances provides insight into the non- f QCD.

Heavy $q\bar{q}$ meson



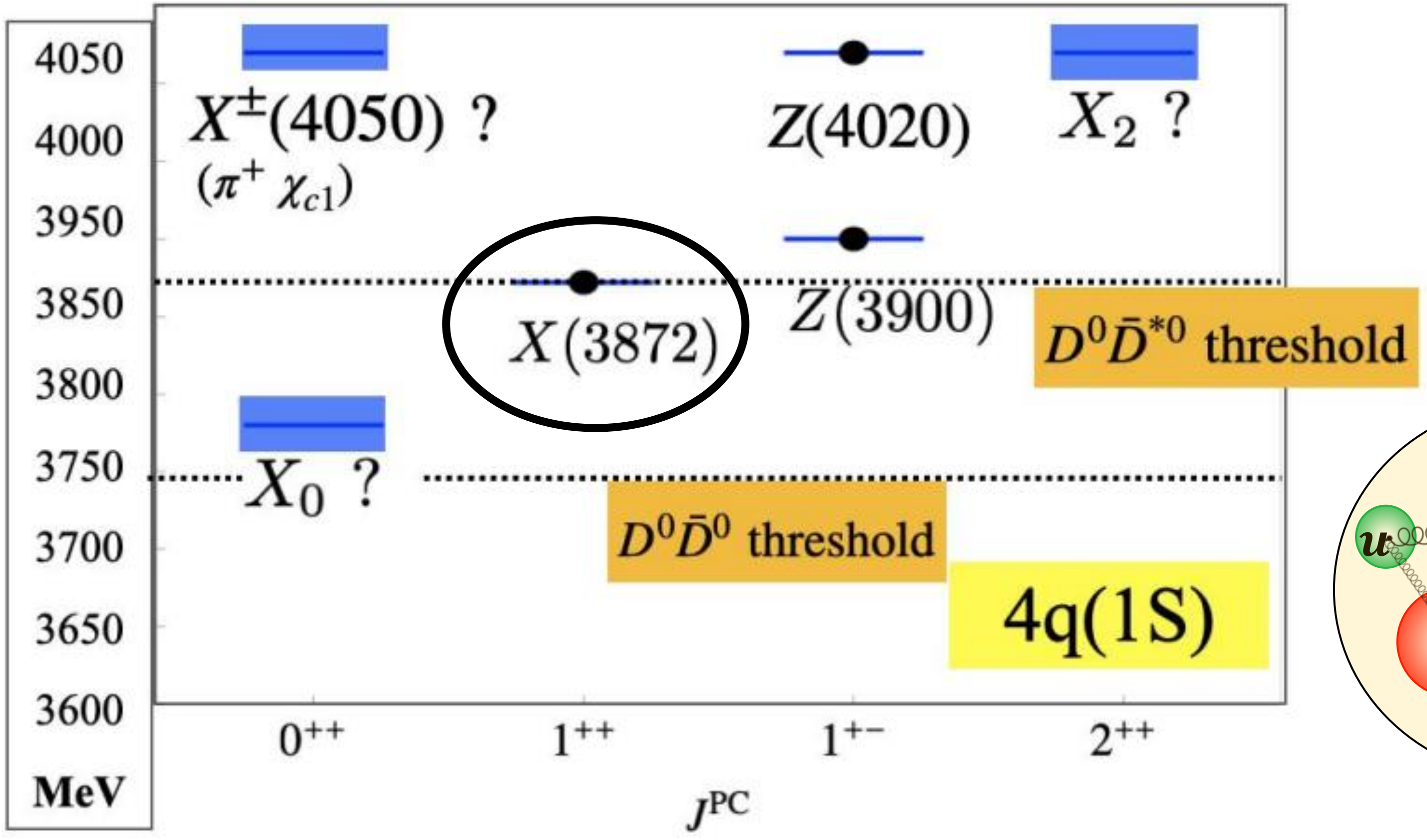
Also confirm lattice calculations.

T_{cc}^+ tetraquark lineshape



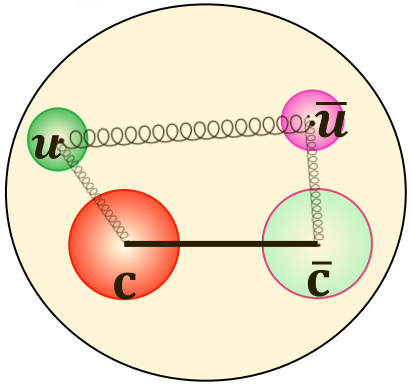
Molecule or compact object?

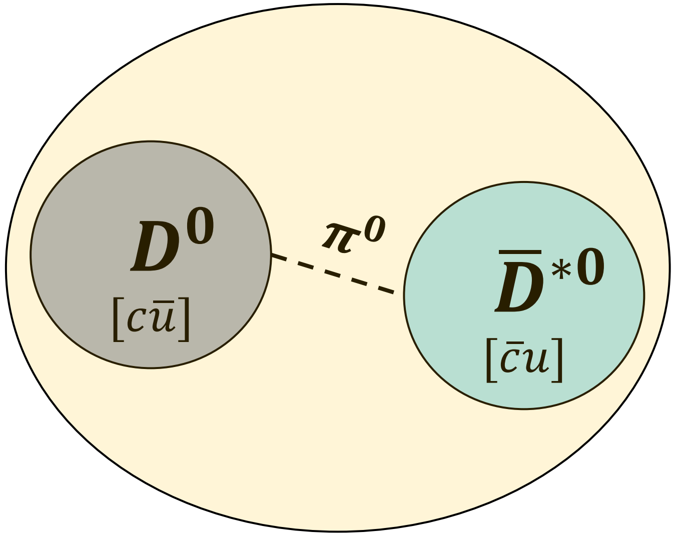
A more emblematic case



- $M_{DD^*} - M_X \leq 100$ **keV (binding energy)**
- **Strong decay seems to violate isospin:**

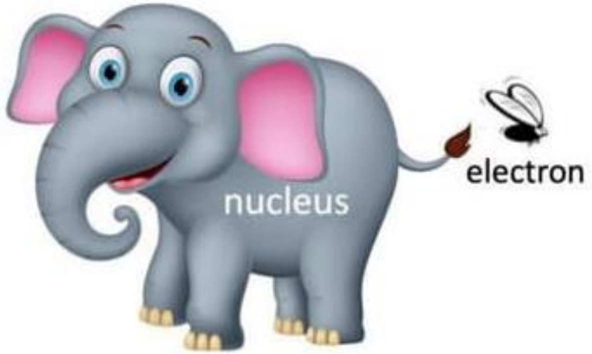
$$\frac{\mathrm{Br}(X \to J/\psi\,\rho)}{\mathrm{Br}(X \to J/\psi\,\omega)} \approx 1$$

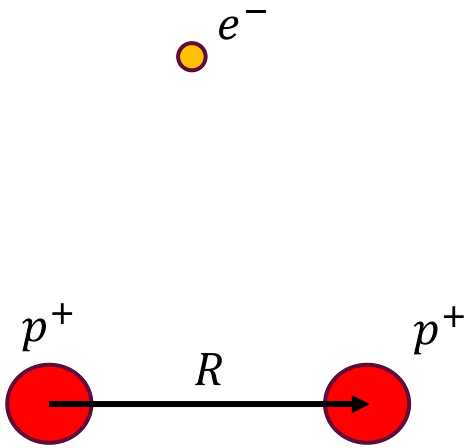




Born-Oppenheimer Approximation

In QED the BO approximation is used because the nucleus is way heavier then the electron: $m_p \gg m_e$





We can decouple the motion of the light degrees of freedom from the heavy ones

The coordinate of the nucleai enters parametrically

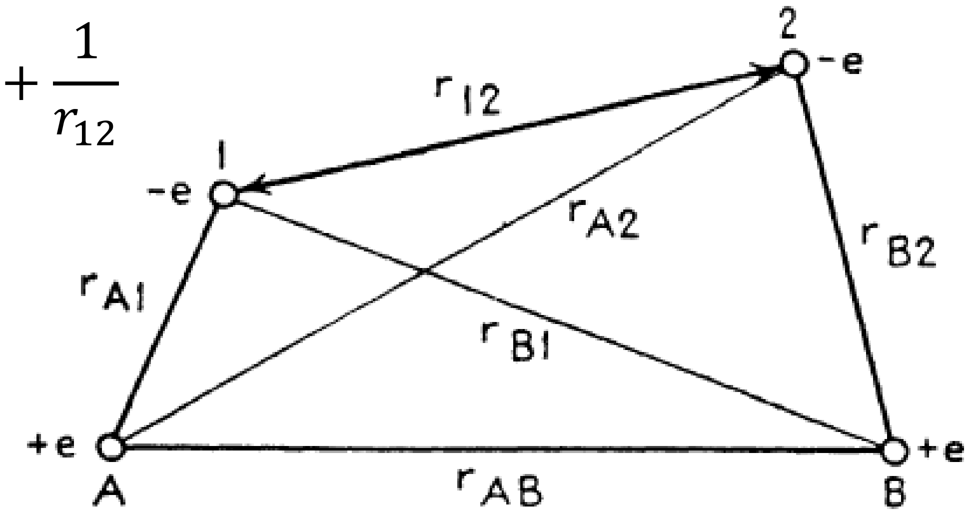
$$H_{q\bar{q}}\Psi^i_{q\bar{q}}(\mathbf{r}_q,\mathbf{r}_{\bar{q}}) = (K_{q\bar{q}} + V_{q\bar{q}}(\mathbf{r}_q,\mathbf{r}_{\bar{q}};R))\Psi^i_{q\bar{q}}(\mathbf{r}_q,\mathbf{r}_{\bar{q}}) = \delta E_i(R)\Psi^i_{q\bar{q}}(\mathbf{r}_q,\mathbf{r}_{\bar{q}})$$

BO potential

$$(H_{c\bar{c}} + \delta E_i(R))\,\Psi^i_{c\bar{c}}(R) = E_i\Psi^i_{c\bar{c}}(R)$$

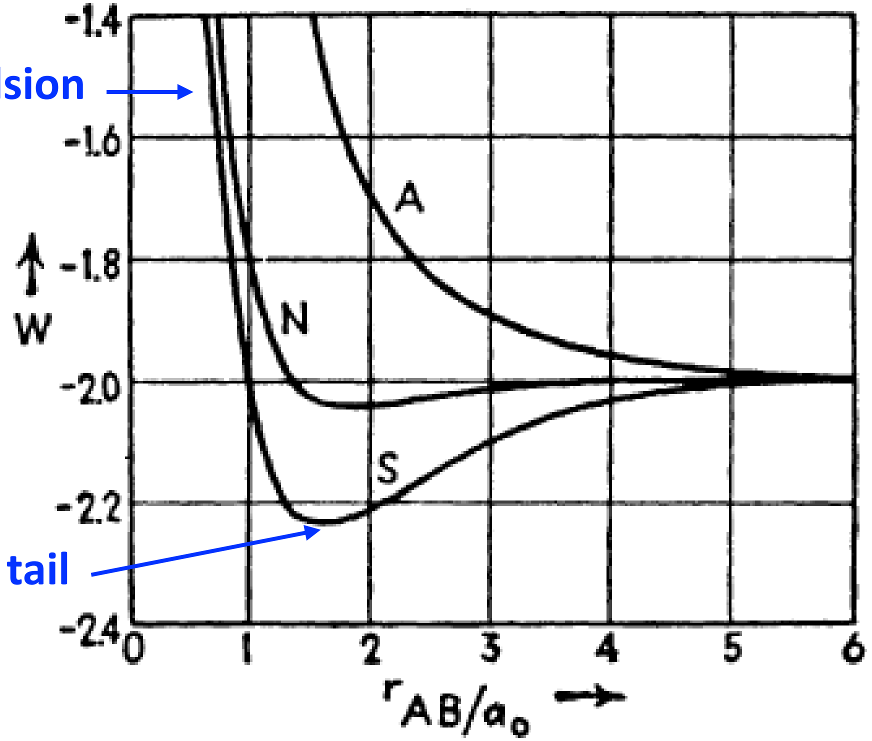
The $\mathbf{H_2}$ molecule

Potential for the electrons: $V_{ee}(\boldsymbol{r_1}, \boldsymbol{r_2}) = -\frac{1}{r_{A1}} - \frac{1}{r_{B1}} - \frac{1}{r_{B2}} - \frac{1}{r_{B1}} + \frac{1}{r_{12}}$



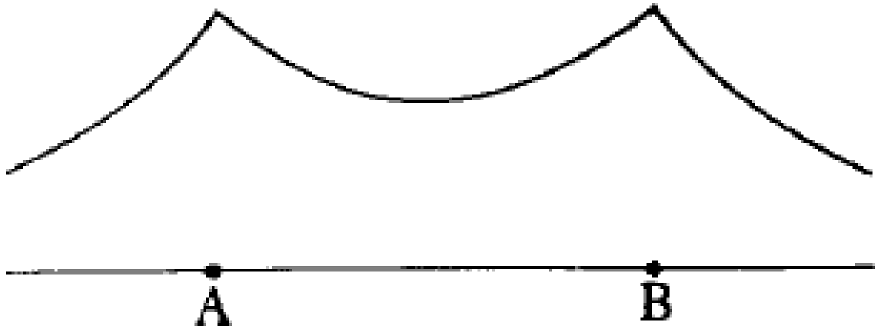
Potential for the protons:

Coulombian repulsion



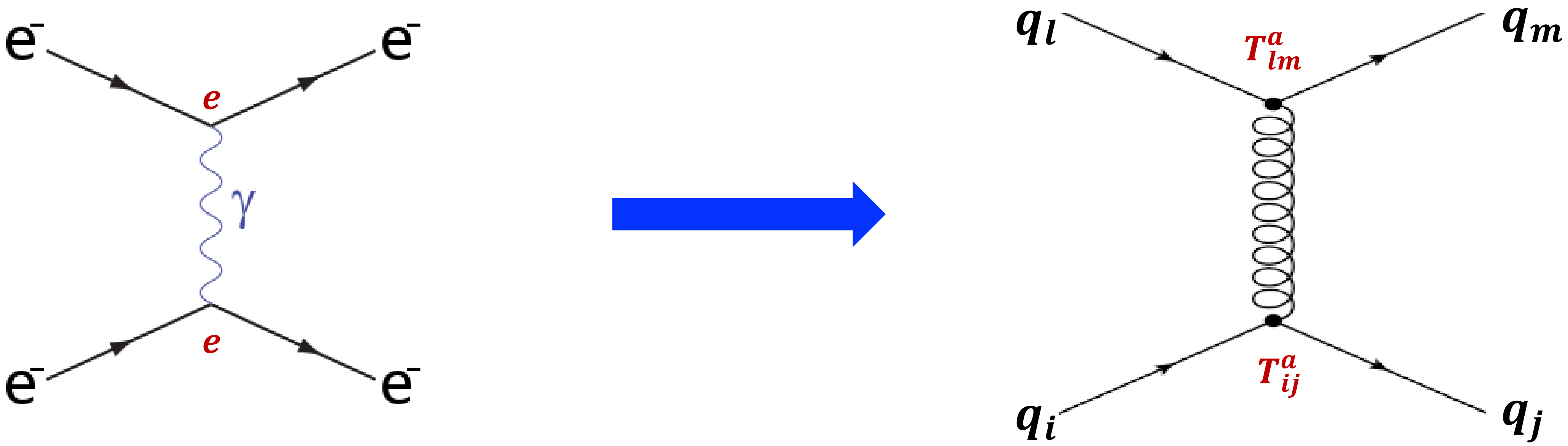
Attractive tail

Wave function of the electron



What about a QCD $\mathbf{H_2}$?

- First problem: **COLOR FACTOR**



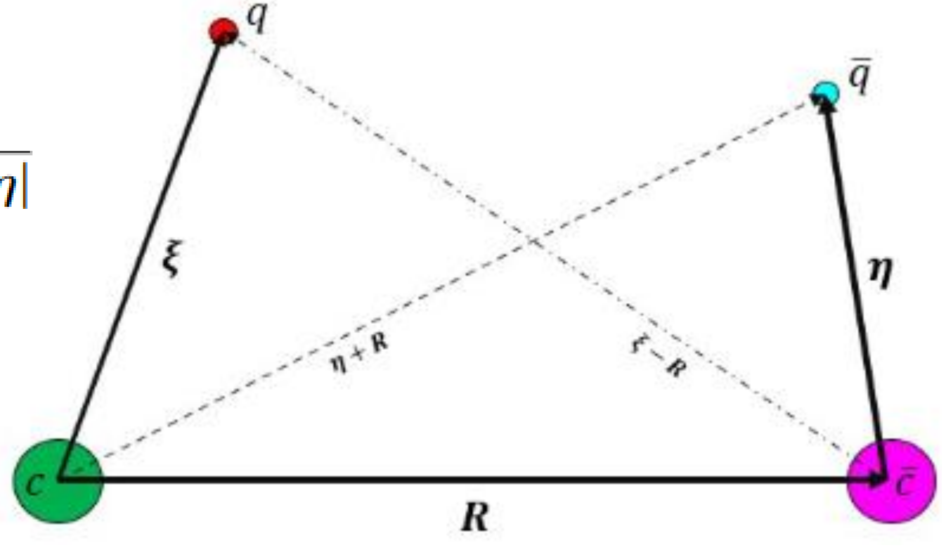
Depending on the color representation, the interaction can be attractive or repulsive (as in QED), but the strength of attraction (repulsion) also depends on the representation.

MORAL: I NEED TO SPECIFY THE COLOR REPRESENTATION OF THE QUARKS IN THE HADRON AS WELL.

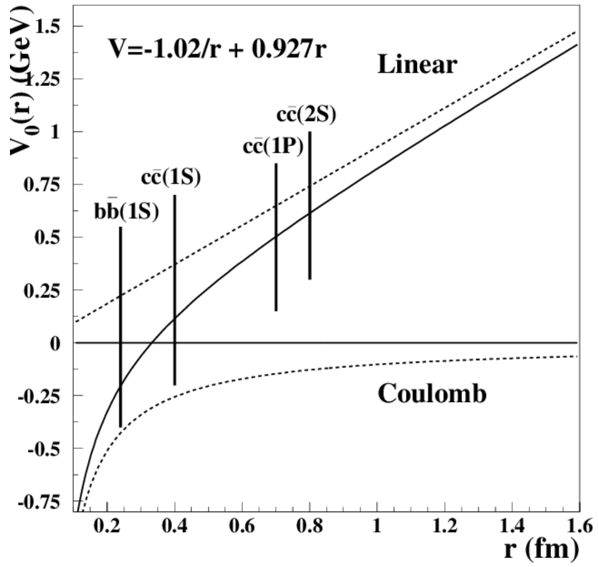
What about a QCD $\mathbf{H_2}$?

✓ First problem: **COLOR FACTOR**

$$V_{q\bar{q}}(\xi,\eta;R) = -\frac{1}{3}\alpha_s\left(\frac{1}{\xi}+\frac{1}{\eta}\right) - \frac{7}{6}\alpha_s\left(\frac{1}{|\xi - R|}+\frac{1}{|\eta + R|}\right) + \frac{1}{6}\alpha_s\frac{1}{|\xi-\eta|}$$



- Second problem: **CONFINEMENT**



$$V_{q\bar{q}}(\xi,\eta;R) = \underbrace{-\frac{1}{3}\alpha_s\left(\frac{1}{\xi}+\frac{1}{\eta}\right) - \frac{7}{6}\alpha_s\left(\frac{1}{|\xi - R|}+\frac{1}{|\eta + R|}\right) + \frac{1}{6}\alpha_s\frac{1}{|\xi-\eta|}}_{\text{'Coulomb' part}} + V_{\text{conf}}$$

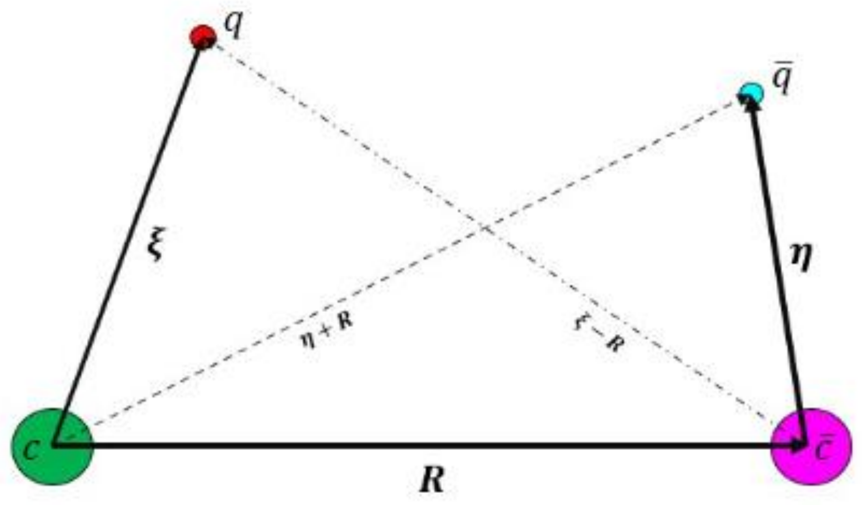
Linear term

'Coulomb' part

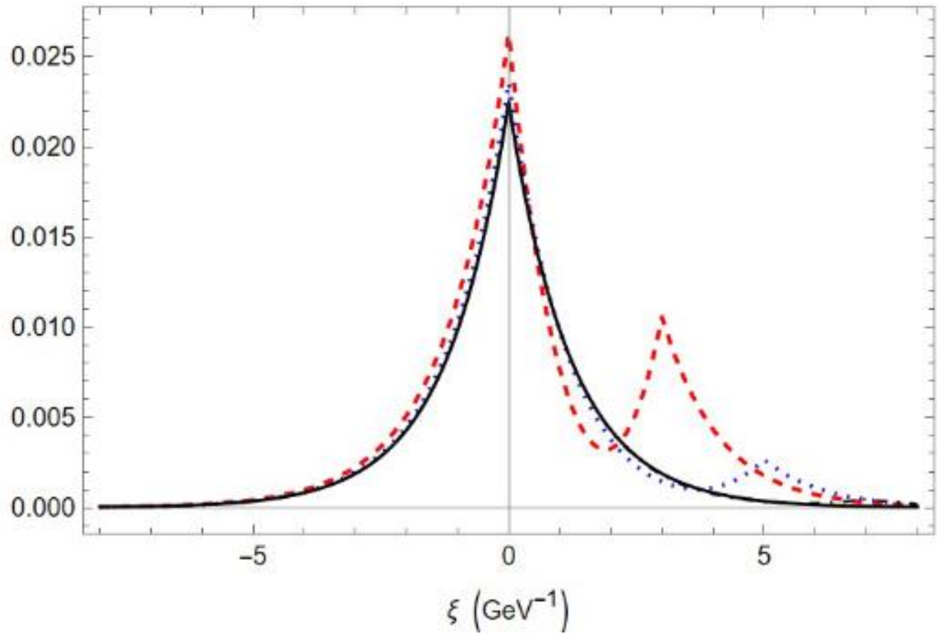
What about a QCD $\mathbf{H_2}$?

- ✓ Second problem: **CONFINEMENT**

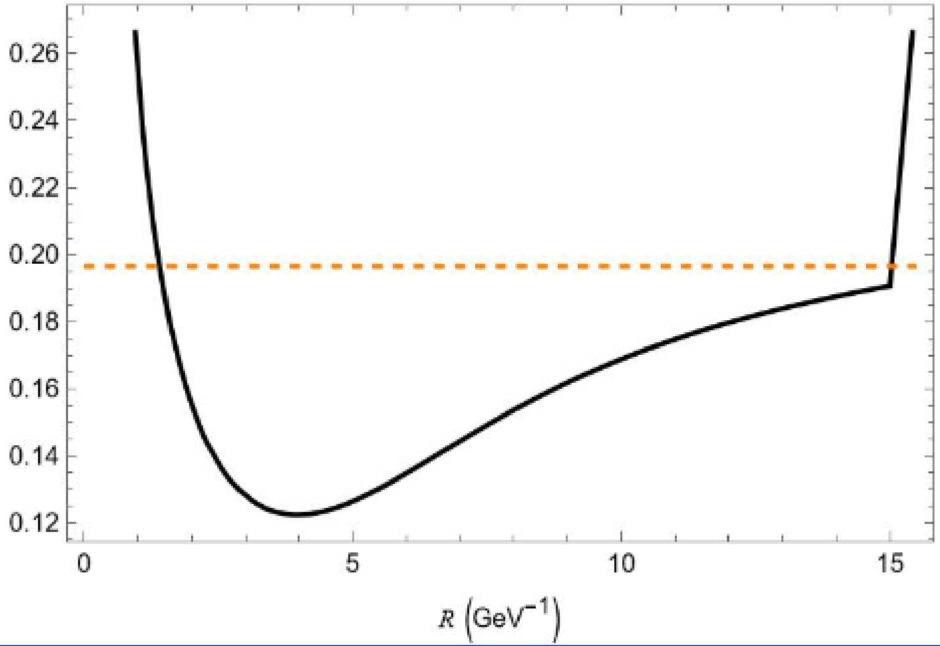
$$V_{qq}(\boldsymbol{\xi},\boldsymbol{\eta};\boldsymbol{R}) = -\frac{1}{3}\alpha_s\left(\frac{1}{\xi}+\frac{1}{\eta}\right) - \frac{7}{6}\alpha_s\left(\frac{1}{|\boldsymbol{\xi}-\boldsymbol{R}|}+\frac{1}{|\boldsymbol{\eta}+\boldsymbol{R}|}\right) + \frac{1}{6}\alpha_s\frac{1}{|\boldsymbol{\xi}-\boldsymbol{\eta}|} + V_{\text{conf}}$$



Wave function of the quark q



Potential for the $c\bar{c}$:



Conclusions

- The Born-Oppenheimer approximation can be used to study the confinement dynamics of exotic hadrons composed of two heavy quarks (charm or bottom);

- As in QED, we can use the Born-Oppenheimer approximation to predict the mass spectrum of these new particles. This work has been done for tetraquarks $\bar{q}\bar{q}cc(bb)$ for example, by Maiani et al. in arXiv:2208.02730v2 [hep-ph];

- In a recent work, Grinstein et al. (arXiv:2401.11623v1 [hep-ph]) used the Born-Oppenheimer approximation to study the radiative decays (with photon emission) of the $X(3872)$;

- The Born-Oppenheimer approximation is a very useful tool not only in molecular physics but also in other fields. Its study within the framework of QCD is still ongoing and provides many interesting insights for theoretical research activities.


THANK
YOU!