# Siena/Ferrara laboratory for birefringence measurements of substrates and reflective coatings

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*F. Della Valle, University of Siena and INFN Pisa*

Group composition:

| | |
|---|---|
| Guido Zavattini | Staff Ferrara, optics |
| Max Aurélie Mailliet | Post-Doc Ferrara, optics |
| Alina Soflau | Master's student Ferrara |
| Andrea Mazzolari | Staff Ferrara, silicon |
| Giovanni Di Domenico | Staff Ferrara, tomography |
| Federico Della Valle | Staff Siena, optics |
| Carmen Marinelli | Staff Siena, optics |
| Emilio Mariotti | Staff Siena, optics |

# Summary

- Brief background
- Birefringence noise from high finesse mirrors
- Birefringence measurements in transmission
- Birefringence measurements in reflection

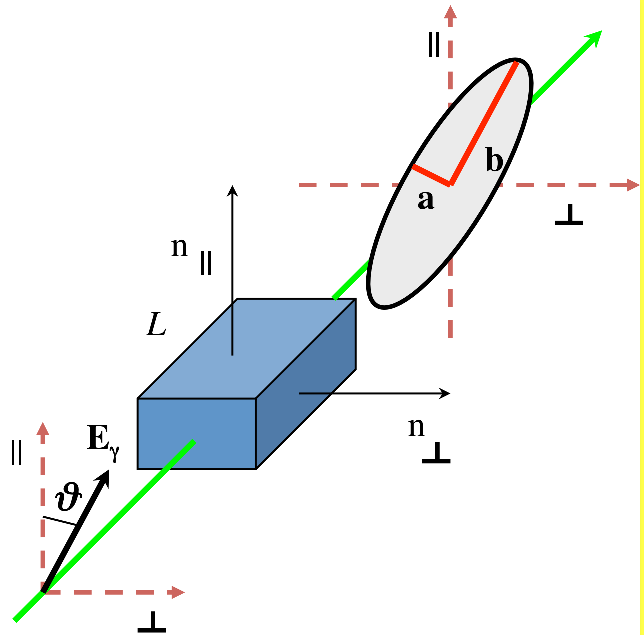
# La Fisica: birifrangenza ed ellitticità

- In un mezzo birifrangente $n_{\parallel} \neq n_{\perp}$
- Attraversando un mezzo birifrangente un fascio linearmente polarizzato acquisisce un'ellitticità $\psi = \pm a/b$ (il segno distingue i due versi di rotazione di $\mathbf{E}_\gamma$)

$$\mathbf{E}_\gamma = E_\gamma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Delta\phi = \frac{2\pi(n_{\parallel} - n_{\perp})L}{\lambda}$$

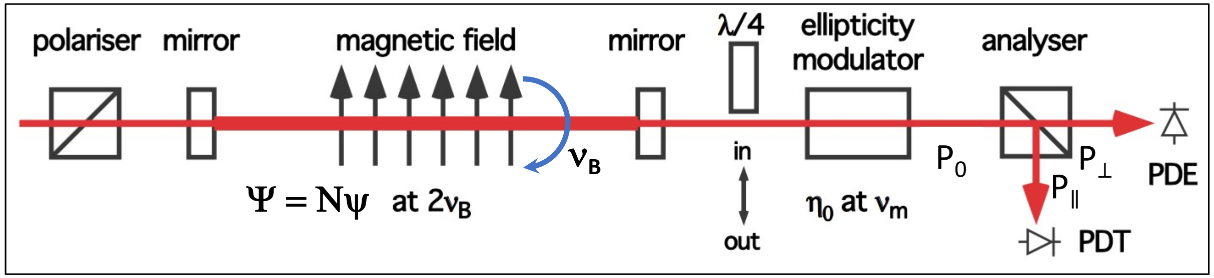
$$\mathbf{E}'_\gamma = E_\gamma \begin{pmatrix} 1 + i\frac{\Delta\phi}{2}\cos 2\vartheta \\ i\frac{\Delta\phi}{2}\sin 2\vartheta \end{pmatrix}, \qquad \Delta\phi \ll 1$$

$$\psi = \pm\frac{a}{b} \approx \frac{\Delta\phi}{2}\sin 2\theta = \frac{\pi(n_{\parallel} - n_{\perp})L}{\lambda}\sin 2\vartheta$$




$$\Delta n \approx 10^{-7},\ L \approx 10\ \text{cm},\ N \approx 10,\ \lambda = 1064\ \text{nm} \ \longrightarrow\ \Delta\phi \approx 0.6\ \text{rad} \approx 34^\circ$$

# PVLAS general scheme




F. Della Valle et al. Eur. Phys. J. C (2016) 76:24
A. Ejlli et al. Physics Reports 871 (2020) 1–74

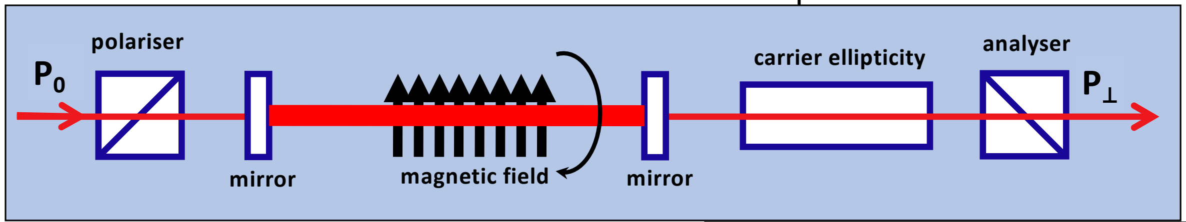
@ $B_{ext}$ = 2.5 T
$\Delta n = 2.5 \times 10^{-23}$

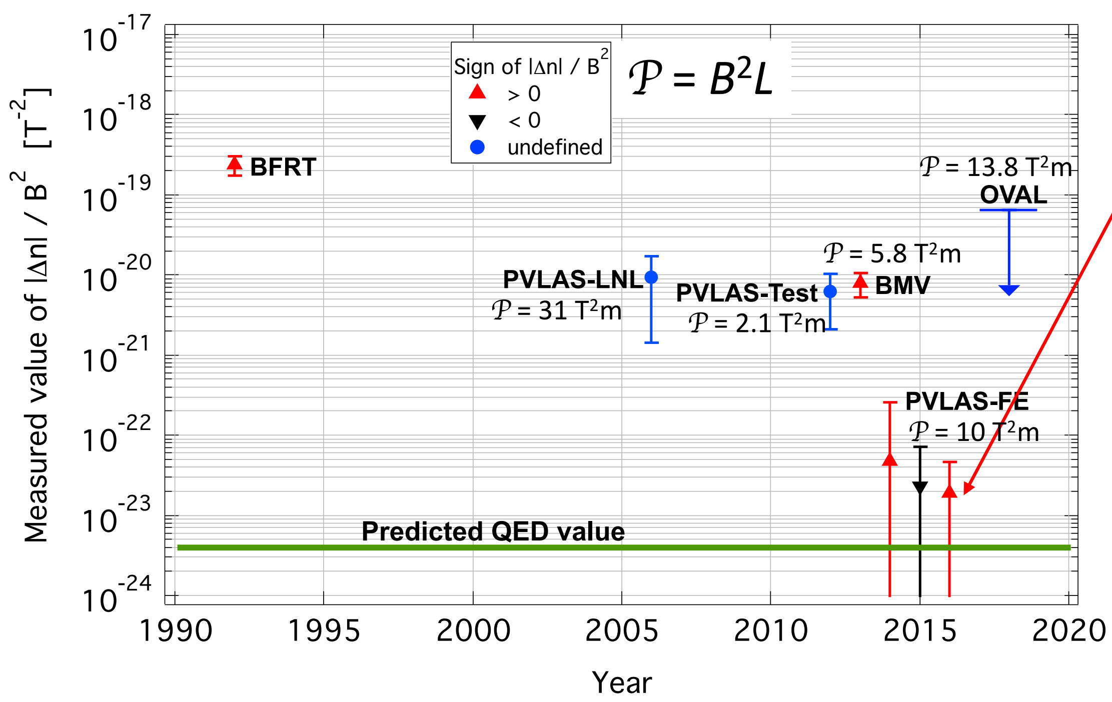
- *L* is the length of the birefringent medium (in PVLAS experiment $\Delta n_{\rm B} \propto B^2$)
- Single pass ellipticity: $\psi = \dfrac{\pi \Delta n_{\rm B} L}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t)$
- The Fabry-Perot cavity amplifies $\psi$ by a factor $N = 2\mathcal{F}/\pi$. We had $\mathcal{F} = 7 \times 10^5$.
- The ellipticity modulator allows heterodyne detection which linearizes the ellipticity $\psi$ to be measured and allows the distinction between a rotation and an ellipticity. The insertion of the $\lambda/4$ wave plate allows measuring rotations.
- The rotating magnetic field modulates the desired signal due to VMB.

$$\Rightarrow I_{\rm out} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t) N \psi(t) + 2\eta(t)\Gamma(t) + \ldots \right\}$$

# State of the art

General scheme: modulated or pulsed field



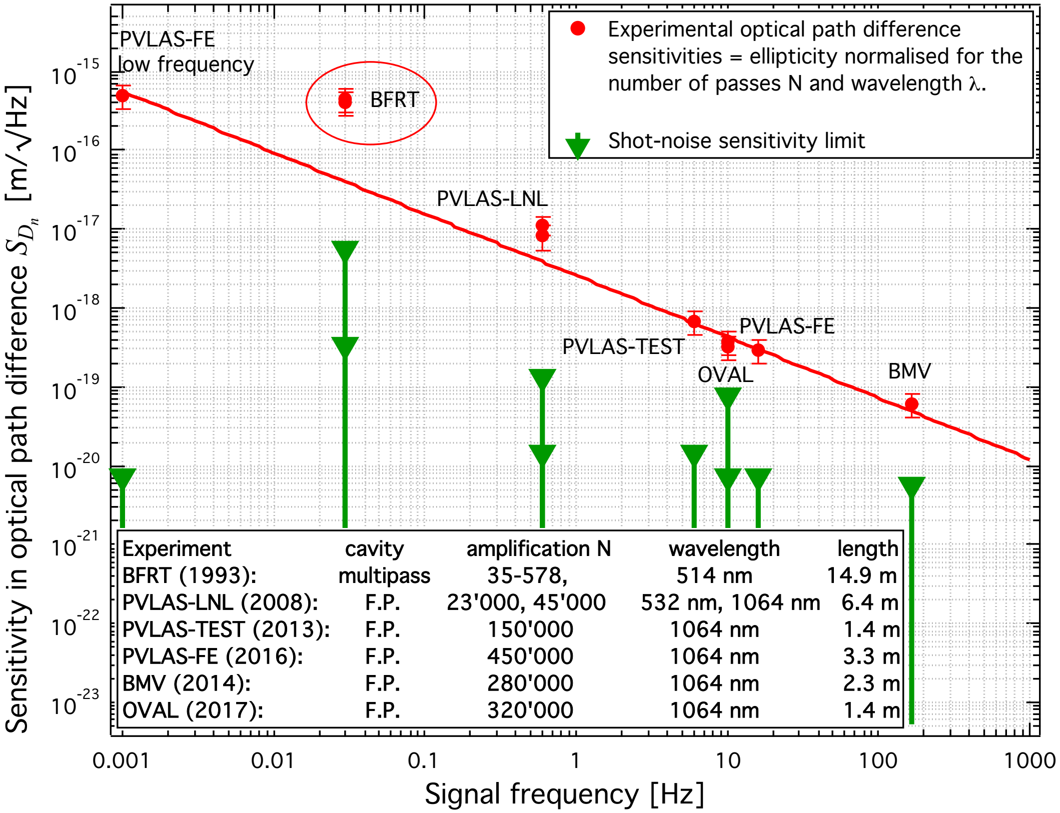




Physics Reports 871 (2020) 1–74

- The PVLAS - FE result remains the most sensitive measurement yet performed: $\Delta n/B^2 = (1.9 \pm 2.7)\times10^{-23}$ $T^{-2}$ with 2.5 T
- Permanent magnets allowed careful debugging of systematics: $B^2L = 10$ $T^2m$
- Optical path difference sensitivity: $S_{OPD} = 4\times10^{-19}$ m/√Hz @ ≈ 16 Hz
- Cavity amplification was $N \approx 4.5\times10^5$
- Intrinsic noise from the mirrors limited the sensitivity and the SNR
- Measured noise: x50 shot-noise @ 16 Hz

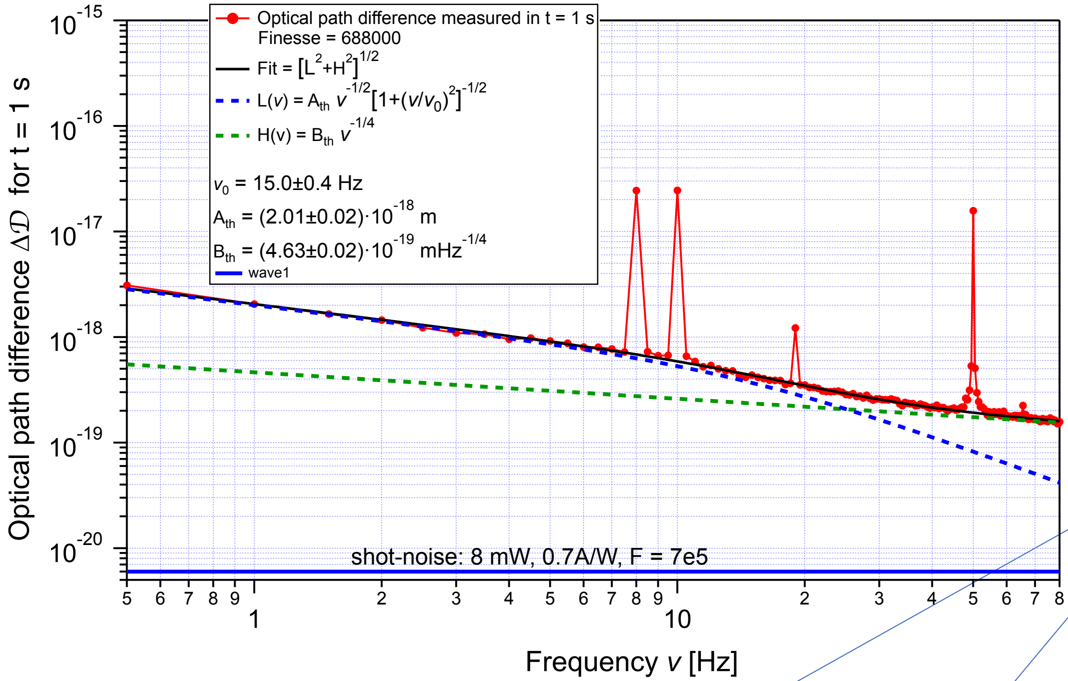
# Intrinsic mirror birefringence noise

## Limits in the sensitivity of a polarimeter




- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- This noise seems to be an intrinsic property of the cavity mirrors
- With a low finesse cavity one does reach shot-noise. The limit is not the method.

# Intrinsic mirror birefringence noise

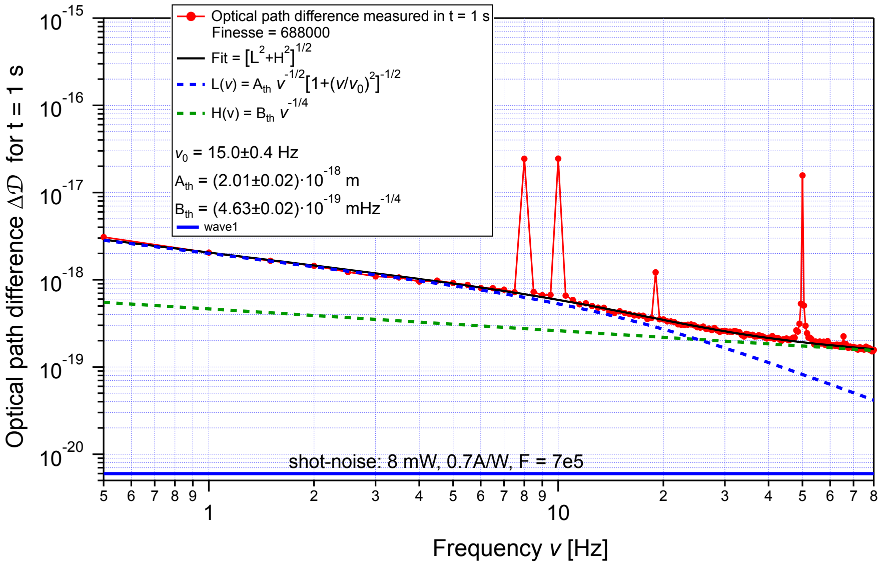



- Typical PVLAS-FE optical path difference noise
- Finesse = 6.88 x $10^5$
- Peaks at 8 Hz and 10 Hz represent Cotton-Mouton calibration signals from 850 µbar Argon gas.
- The peak at 19 Hz is generated by a Faraday rotation leakage due to the total cavity static birefringence from the mirrors.
- Brownian? Why the cut-off?
- Thermo-elastic model points to tantala.
- For ET we can measure new coatings. Finesse must be F ≥ 5e4 (R ≥ 99.995%): the amplified mirror noise must be greater than shot-noise.
- Will be testing crystaline GaAs/AlGaAs mirrors.

$$S_{\rm OPD}(\nu) = \sqrt{\left(\frac{A_{\rm th}\nu^{-1/2}}{\sqrt{1+(\nu/\nu_0)^2}}\right)^2 + \left(B_{\rm th}\nu^{-1/4}\right)^2}$$

$$A_{\rm th} = (2.01 \pm 0.02) \times 10^{-18}\ {\rm m}, \quad \nu_0 = (15.0 \pm 0.4)\ {\rm Hz}, \quad B_{\rm th} = (4.63 \pm 0.02) \times 10^{-19}\ {\rm m/Hz}^{1/4}$$

# Intrinsic mirror birefringence noise




- Estimated the thermoelastic birefringence noise in reflection (Physics Reports 871 (2020) 1–74)
- $C_{SO}$ = stress optic coefficient
- Y = Young's modulus
- $\alpha_T$ = thermal expansion coefficient
- $r_0$ = beam radius on mirror
- $C_T$ = specific heat capacity
- $\rho$ = density
- $\lambda_T$ = themal conductivity

Temperature spectral density

$$S_T(\nu) = \sqrt{\frac{8k_B T^2}{\pi r_0^2\sqrt{\pi\rho C_T\lambda_T\nu}}} \propto \nu^{-1/4}$$

**Optical path difference spectrum**

$$S_{\Delta\mathcal{D}} = 2d_e\sqrt{2}C_{SO}Y\alpha_T S_T(\nu)$$

Fused silica

$$S_{\Delta\mathcal{D}}^{(FS)} \sim 4\times 10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

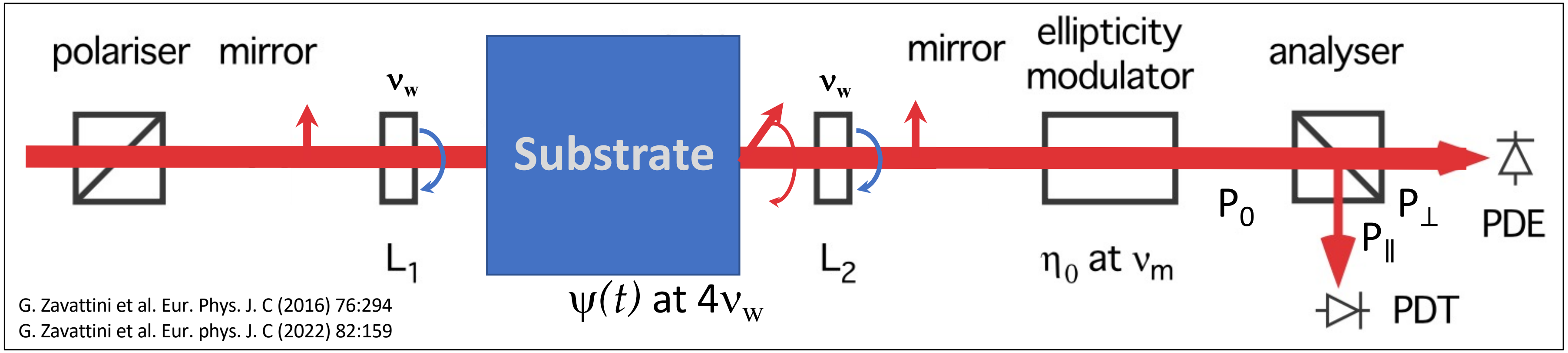
Tantala

$$S_{\Delta\mathcal{D}}^{(Ta)} \sim (1 \div 5)\times 10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Compatible with $B_{th} = (4.63 \pm 0.02)\times 10^{-19}\ \mathrm{m}/\mathrm{Hz}^{1/4}$

# Substrate birefringence measurements

- Single pass ellipticity: $\psi(t) = \frac{\pi \int \Delta n \, dL}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t).$
- Here $\vartheta$(t) is the angle between the polarisation and the birefringence axis. $\phi$(t) is the HWP angle: $\vartheta(t) = 2\phi(t)$




$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

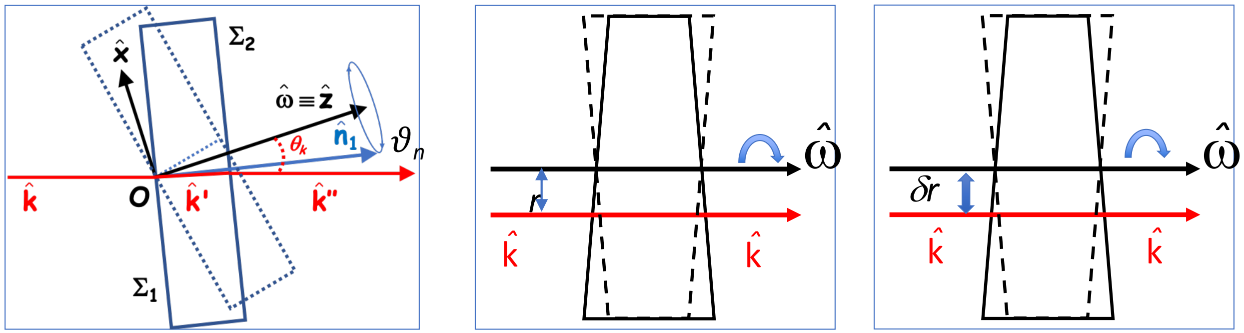
- $\alpha_{1,2}$ are the residual retardations from $\pi$ of the HWPs. The modulator's frequency is $\nu_m$ = 50 kHz.
- The detected intensity is **demodulated** at the modulator's frequency $\nu_m$ to obtain the ellipticity spectrum.
- The ellipticity spectrum includes the desired signal, systematic effects and noise

$$I_{\rm out} \simeq I_0 \left\{\eta^2(t) + 2\eta(t)\psi(t) + 2\eta(t)\Gamma(t) + \ldots\right\}$$

# Generation of spurious harmonics from rotating HWPs

$$\alpha_{1,2}(\phi, \mathrm{T}, r) = \alpha_{1,2}^{(0)}(\mathrm{T}) + \alpha_{1,2}^{(1)} \cos\phi(\mathrm{t}) + \alpha_{1,2}^{(2)} \cos 2\phi(\mathrm{t}) + \dots$$

Temperature dependence of $\alpha_{1,2}^{(0)}(\mathrm{T}) = \frac{2\pi}{\lambda}\int \Delta\mathrm{n}\ \mathrm{dL}$




ALIGNMENT — WEDGE $\beta$ — WEDGE + OSCILLATION @ $\nu_w$

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda}\Delta n \frac{D}{n^2}\vartheta_n\vartheta_k$$

$$\alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda}\Delta n \frac{D}{4n^2}\vartheta_n^2\vartheta_k^2$$

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda}\Delta n\ \Delta r_0\ \beta$$

$$\alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda}\Delta n\ \delta r\ \beta$$

Generate 4th harmonic but can be controlled to < $10^{-5}$ level corresponding to an optical path difference $\int \Delta n\ dL \lesssim 10^{-12}$ m

✓ The HWPs can be aligned separately using a frequency doubled laser @ 532 nm

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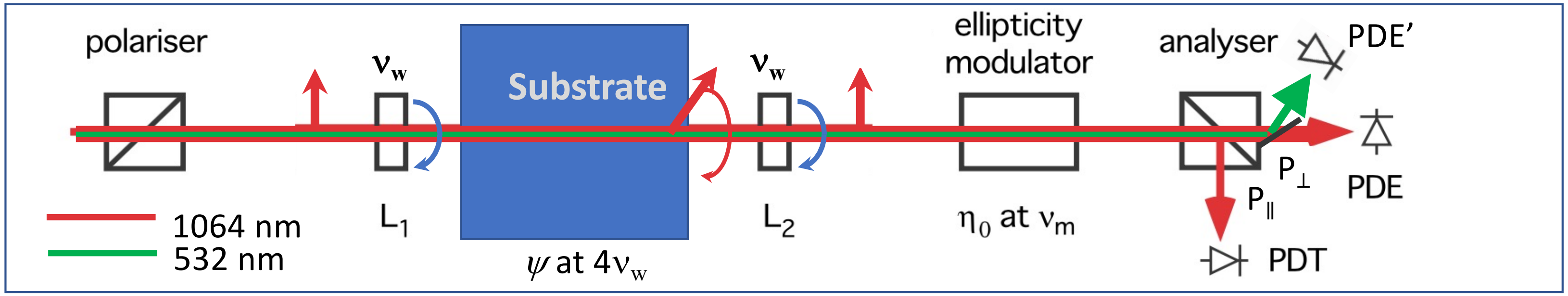
# Baseline scheme for substrate birefringence measurements




$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

Signal @ $4\nu_w$

Spurious signals
Contain harmonics of $\nu_w$

Relative rotation phase error
Degrades extinction

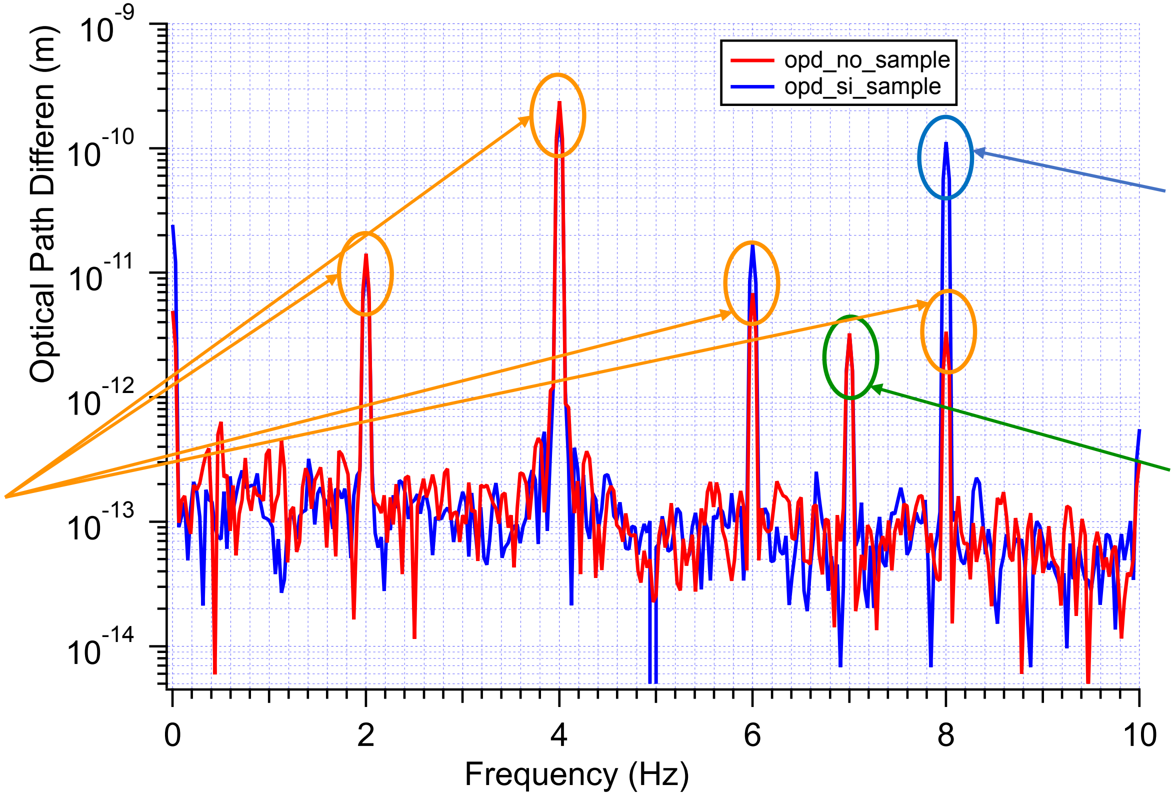
$\alpha_{1,2}$ are the phase errors from π of the two HWPs and $\phi(t)$ is their rotation angle

- ✓ 532 nm beam (HWP -> FWP) allows independent alignment of the rotating HWPs to reduce 1st, 3rd and 4th harm.
- ✓ At 1064 nm, control the temperature of the wave-plates to reduce the dominating 2nd harmonic
- ✓ Reduced systematic peaks such that $\alpha_{1,2}^{(1,2,3)} \lesssim 10^{-4}$ at all relevant harmonics and in particular, for the 4th harmonic, $\alpha_{1,2}^{(4)} \lesssim 10^{-5}$. Can be subtracted vectorially ➔ Ellipticity sensitivity $\psi_0 \approx 10^{-6}$
- ✓ Can produce X-Y 'maps' of the static average birefringence of a substrate: $\Delta n = \frac{\psi_0 \lambda}{\pi L}$
- ✓ Optical path difference sensitivity $S_{OPD} \lesssim 10^{-12}$ m
- ✓ Calibration with the Cotton-Mouton effect in air using a rotating 2.5 T permanent magnet

# Example: spectrum of a 1-mm thick Si sample

$$OPD = \Delta n L = \frac{\psi_0 \lambda}{\pi}$$

Integration time = 32 s; Hanning window.



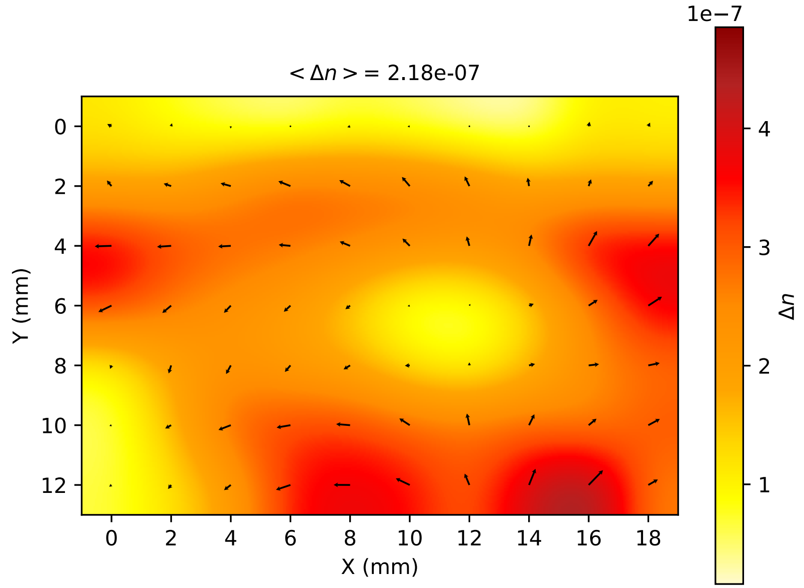

- Spurious harmonics from temperature and misalignment.

- Peak due to silicon birefringence: $\Delta n = 1.1 \times 10^{-7}$; L = 1 mm

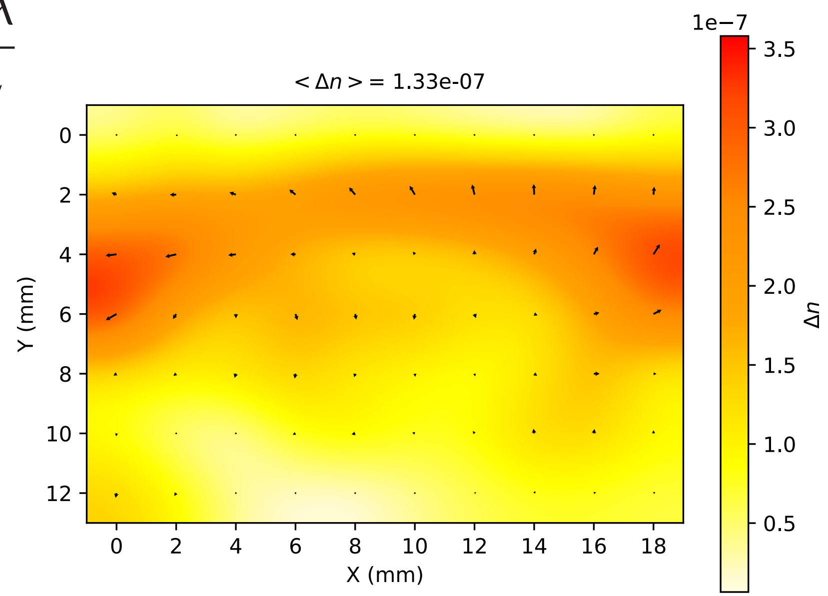
- Calibration Cotton-Mouton peak of air. $\Delta n = 3.9 \times 10^{-12}$; L = 0.84 m

# Example of birefringent map: first samples

- Silicon crystal samples (100), L = 1-mm thick, 2.5 cm x 2.5 cm, cut in house from larger sample
- Measurements using 1064nm (significant absorption). Will be repeated with 1550nm
- Subtracted vectorially the waveplate contribution (small effect)
- Held with clamp from bottom edge (left): extra stress can be seen due to clamp.
- Held without clamp (right). Upper half maintains same optical path difference.
- <u>Non uniform birefringence</u>.



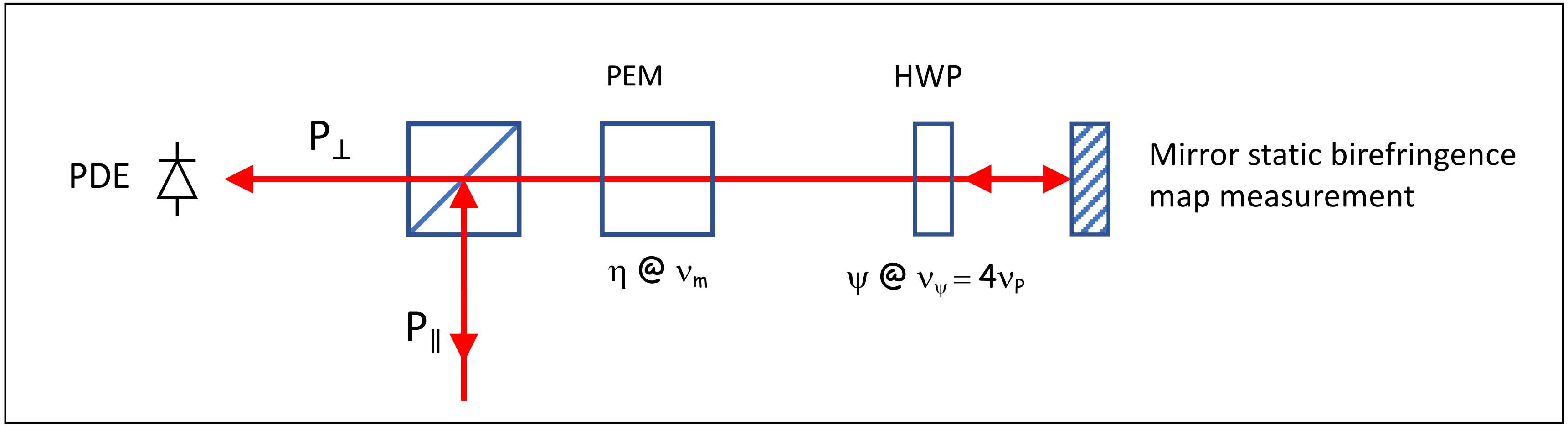

$$\Delta n = \frac{\psi_0 \lambda}{\pi L}$$



# Reflective coating birefringence measurements

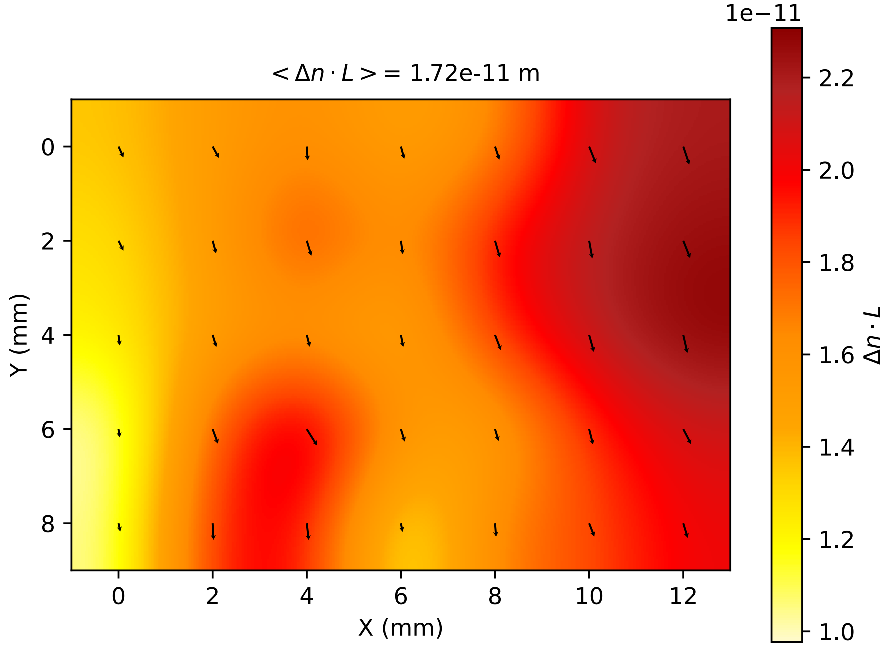
Reflection scheme for static birefringence maps of reflective coatings:

- At present we have a 1064 nm beam aligned.
- With a silver mirror the induced ellipticity is minimum and is, at present, associated to the rotating HWP .
- Will implement a 532 nm beam to distinguish the rotating HWP effect from the mirror effect.
- Will also introduce a rotating magnet for calibration.

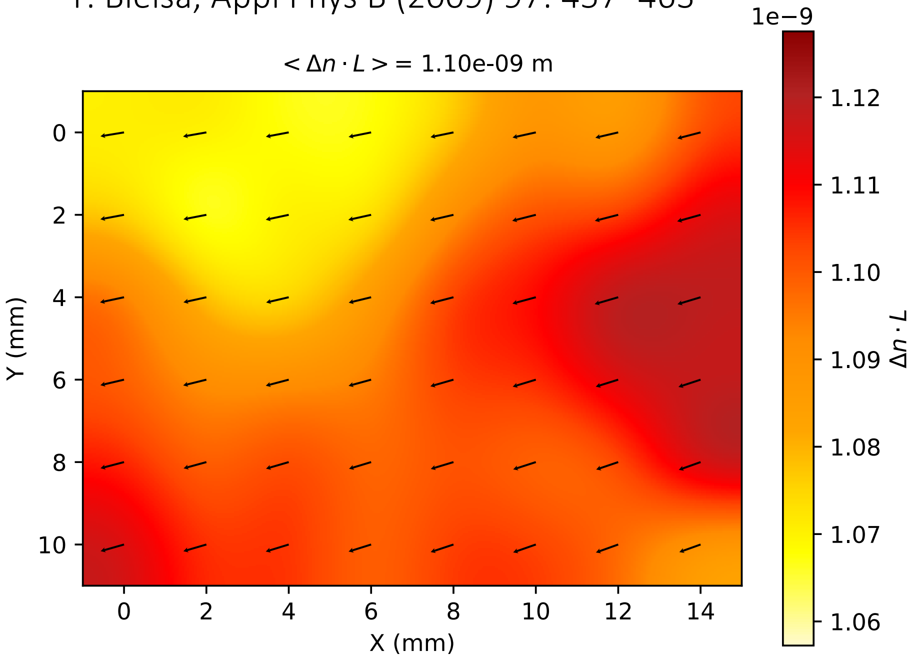


# Example of birefringent map of coatings: first samples

- Silver mirror.
- Very low birefringence.
- Measured ellipticity is dominated by the rotating half-waveplate.



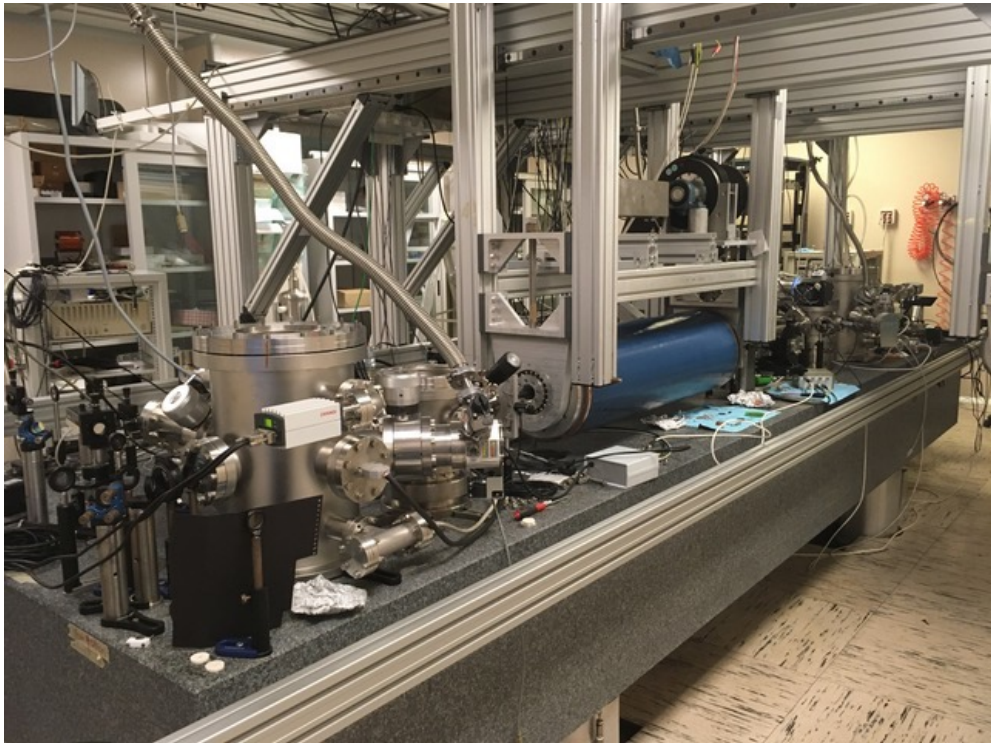

- Dielectric mirror with T ≈ $10^{-3}$. 'Uniform'.
- Polarization can be aligned in cavities.
- Higher reflectivity, lower birefringence. For F ≈ $10^5$, Δn·L ≈ 3 x $10^{-13}$ m.
- Brandi et al. Appl. Phys. B 65, 351–355 (1997); F. Bielsa, Appl Phys B (2009) 97: 457–463



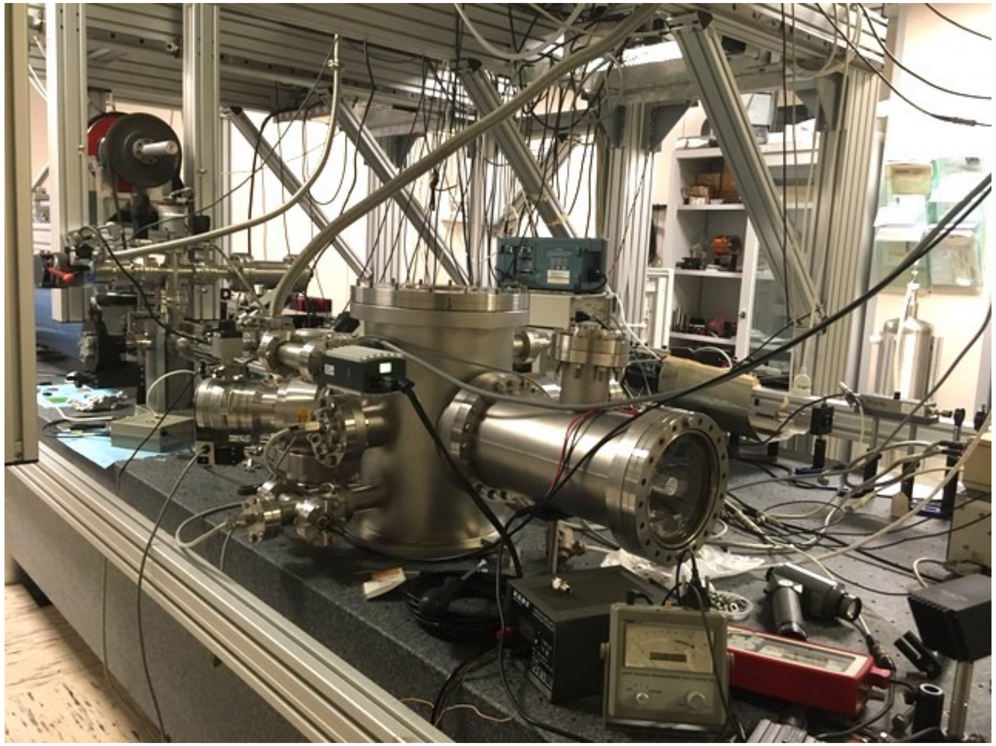
# Pictures

At present being used with rotating HWPs.

General view from input side

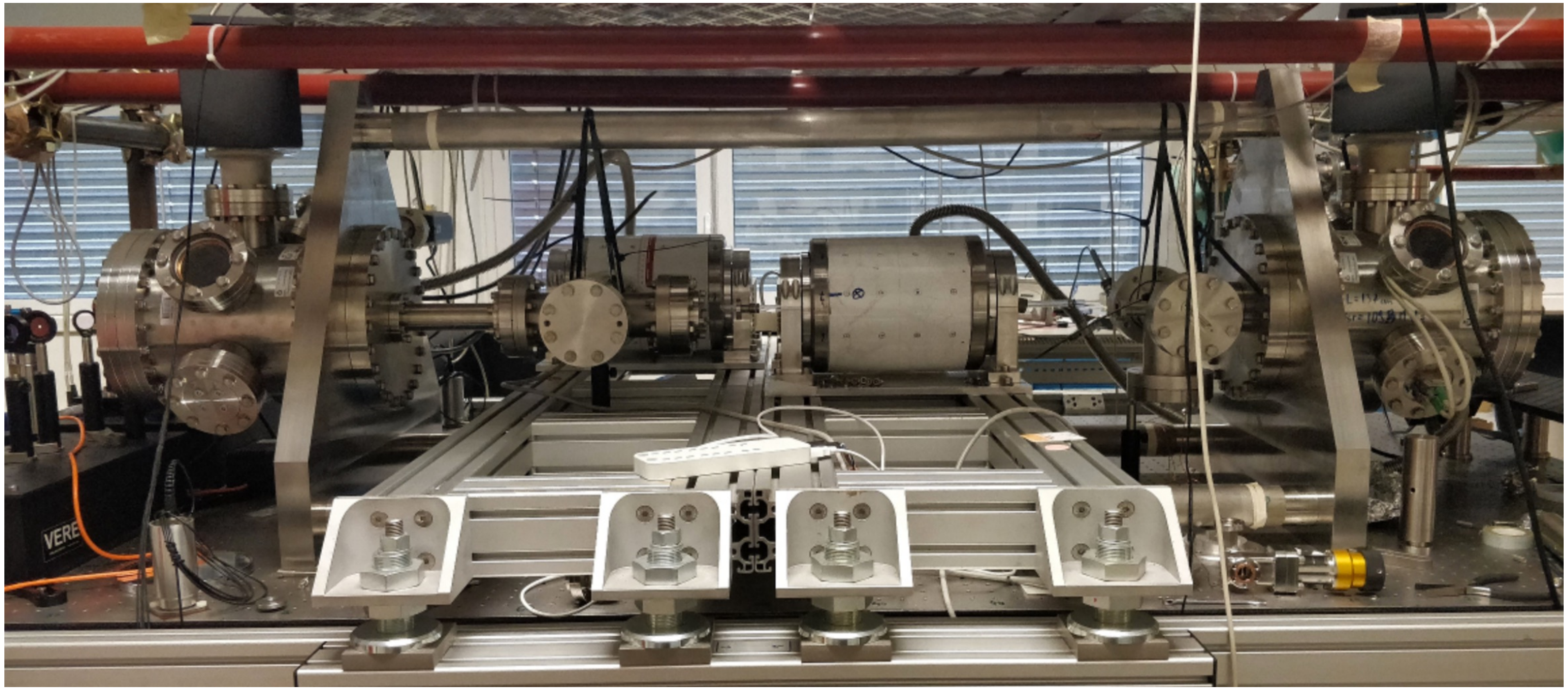


General view from output side



# Pictures: lab2

Polarimeter at present being used with a low finesse cavity (F ≈ 3000).

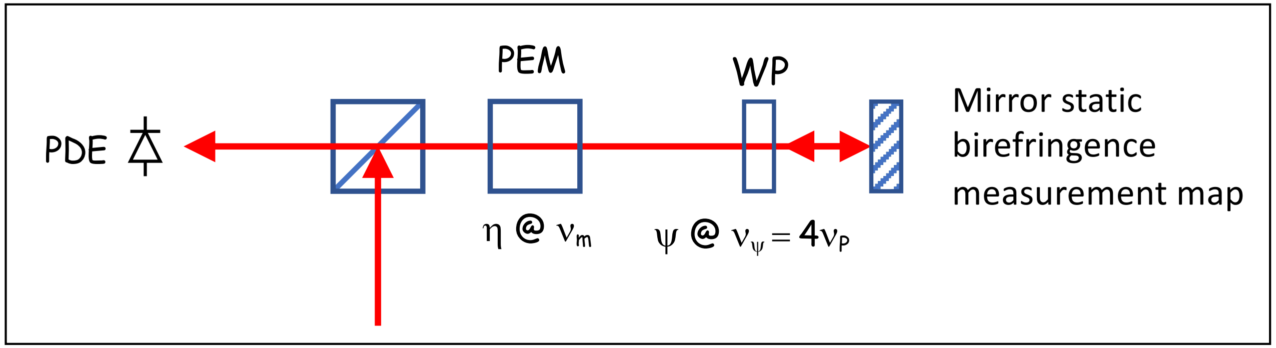


Near future: will be dedicated to birefringence measurements with the rotating HWPs at 1064nm and 1550nm.

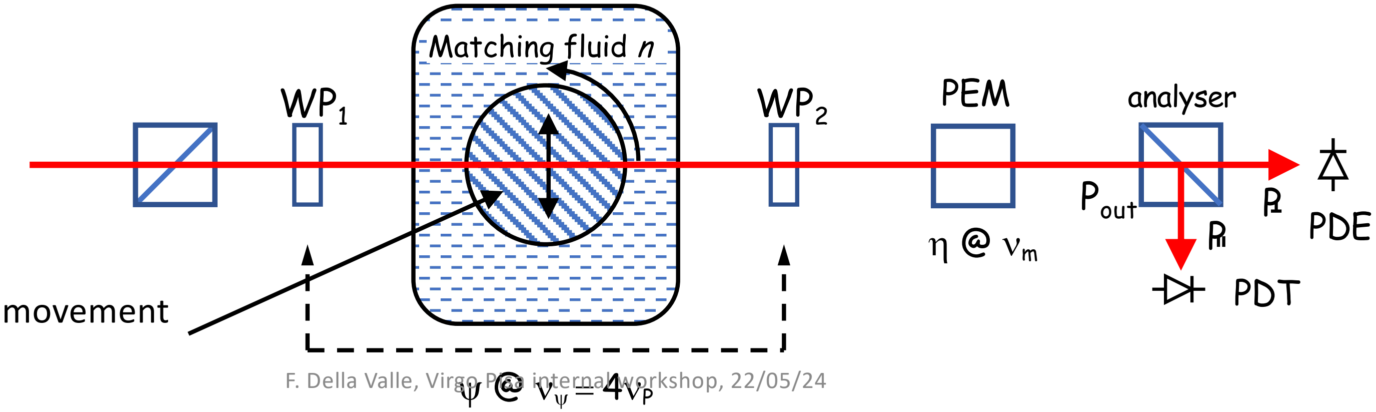
Thank you

# To be implemented

Very near future:
Reflection scheme
for coatings



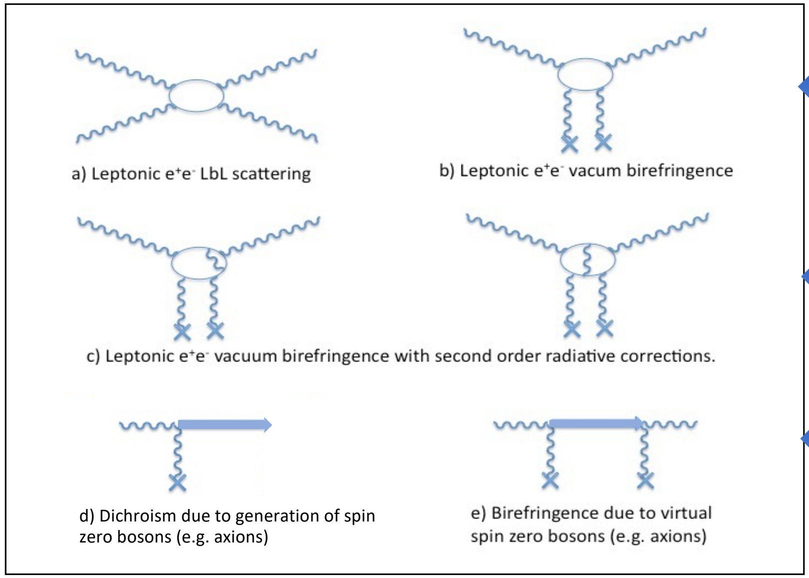

Near future: Birefringence measurements as a function of depth?
Is birefringence tomography possible?




Also vertical movement
for ‘depth’

# Background work in sensitive polarimetry

Experimental study of the induced birefringence by an external magnetic field in vacuum




Light-by-light interaction and vacuum magnetic birefringence.

Must be there: $\Delta n = 4\text{X}10^{-24}\ B^2$ with B in Tesla.

Includes MCPs

Radiative correction 1.45%

Contributions from hypothetical neutral light particles coupling to two photons: ALPs

Euler-Kockel-Heisenberg Lagrangian predicts VMB

$$\mathcal{L}_{\text{EK}} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[1\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots$$

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}{}_e^3}{m_e c^2} = 1.32\times 10^{-24}\ \text{T}^{-2}$$

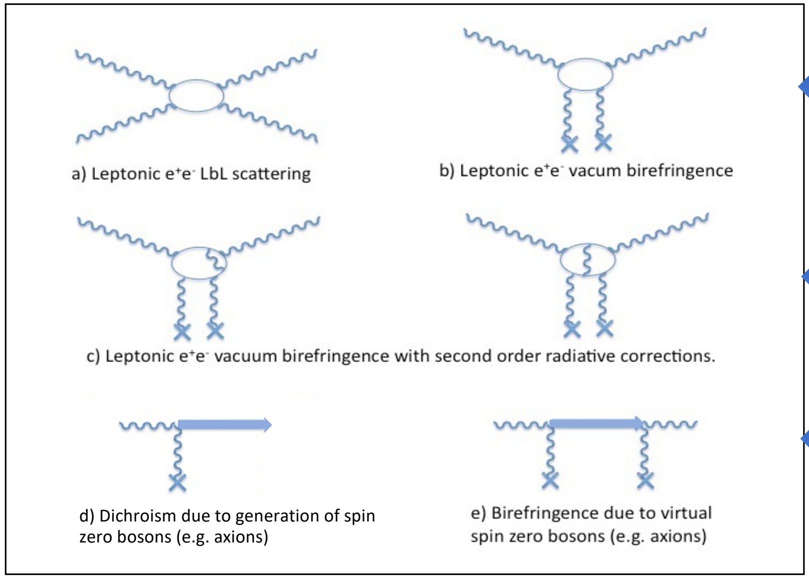
$$\Delta n = 3A_e B_{\text{ext}}^2$$

@ $B_{ext}$ = 2.5 T

$\Delta n = 2.5 \cdot 10^{-23}$

# Background work in sensitive polarimetry

## Experimental study of the speed of light in an external magnetic field in vacuum




Light-by-light interaction and vacuum magnetic birefringence.
Must be there: $\Delta n = 4\text{X}10^{-24}\ B^2$ with B in Tesla.
Includes MCPs

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Euler-Kockel-Heisenberg Lagrangian predicts VMB

$$\mathcal{L}_{\rm EK} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[1\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots$$

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}{}_e^3}{m_e c^2} = 1.32\times 10^{-24}\ \mathrm{T}^{-2}$$

$$\Delta n = 3A_e B_{\rm ext}^2$$

@ $B_{ext}$ = 2.5 T

$\Delta n = 2.5 \cdot 10^{-23}$

# Comments and questions: 1

## KAGRA

- Birefringence $\Delta n \approx 10^{-6}$ with 15 cm thick sapphire substrate. Projected 2D map
- Non uniform birefringence map of substrate (amplitude and direction). Phase shifts of 4 rad effect
- $\Delta n \approx 10^{-7}$ in silicon. Non uniform here too. For ET the desired thickness is 67 cm.
- ➔ Total phase shift ≈ 1 rad
- Is $\Delta n \approx 10^{-7}$ still too large? If uniform, align polarization with axis of system birefringence. If non uniform...

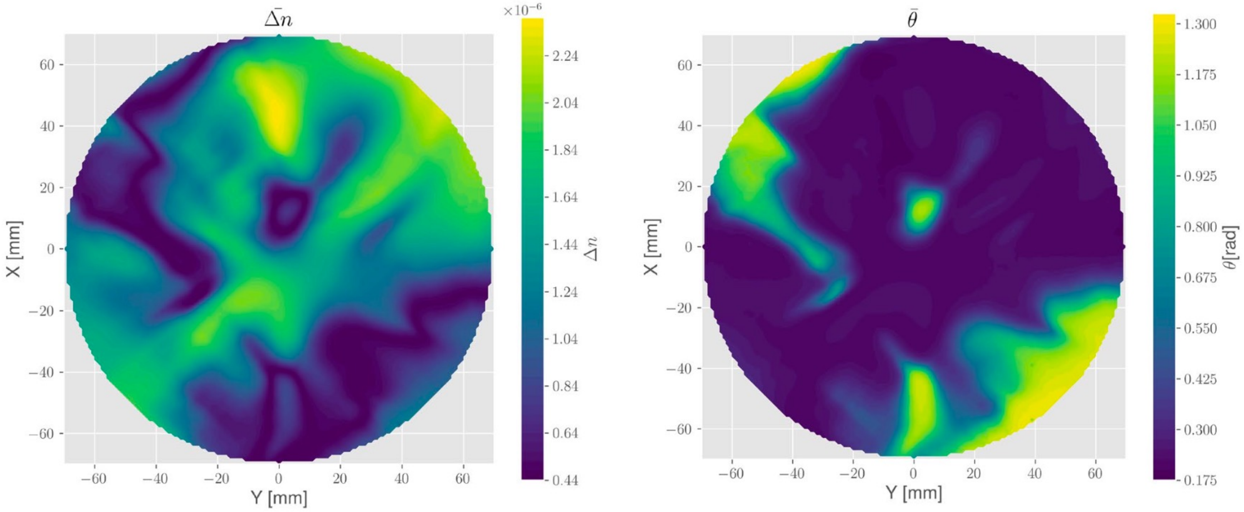



**Figure 4.** Mean distribution of both birefringence $\Delta n$ and $\theta$-angle, calculated from the six input-polarization combinations which led to no miscalculations.



https://doi.org/10.1038/s41598-023-45928-0

# Comments and questions: 2

## MIRRORS

- Our experience and other's too (Toulouse BMV group) have found that the static birefringence of coatings:

$$\Delta n_{\text{high finesse}} < \Delta n_{\text{low finesse}}$$

- There seems to be a 'more' uniform map compared to substrates (over ≈ few centimeters).
- Origin not clear. C. Rizzo's, Toulouse, group attribute to first layer near substrate (F. Bielsa, Appl Phys B (2009) 97: 457–463).
- With stoichiometry of silicon nitride coatings one can control stress on silicon. Maybe birefringence of mirrors with silicon nitride?
- In our Fabry-Perot based polarimeters the static mirror birefringences were oriented to subtract each other and the polarisation aligned to the axis of the cavity as a whole. In this way the two eigenmodes of the cavity are almost superimposed.

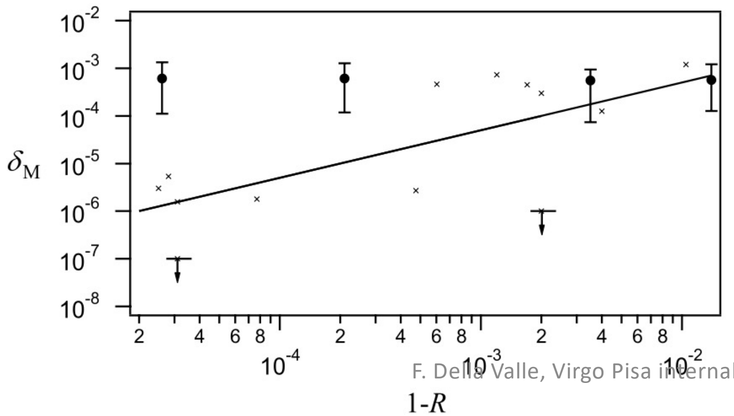
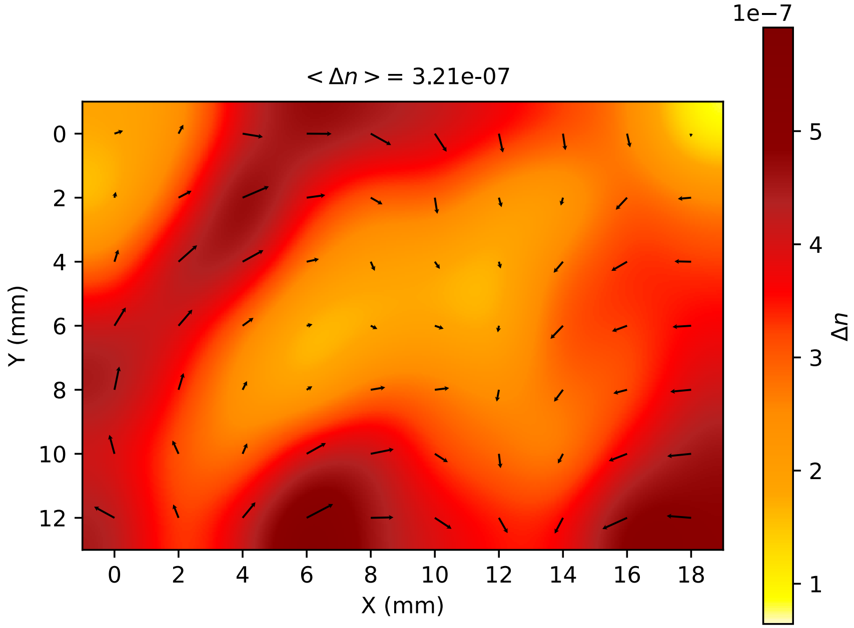



**Fig. 6** Two different numerical calculations for the induced phase retardation per reflection as a function of $(1 - R)$. *Solid curve*: birefringence only for the first layer just after the substrate. *Dots with error bars*: calculation with random birefringence per each layer. *Crosses*: measurements plotted in Fig. 3

# Example of birefringent map: first examples

- Silicon crystal samples (100), L = 1 mm thick, 3x3 cm, cut in house
- Measurements using 1064nm (significant absorption). Will be repeated with 1550nm
- Held from bottom edge: extra stress can be seen due to clamp like in the previous sample.
- Residual stress at edges from cutting of samples?
- This particular smaple had a broken corner. Other than the clamp effect (bottom) residual stress is seen.




$$\Delta n = \frac{\psi_0 \lambda}{\pi L}$$

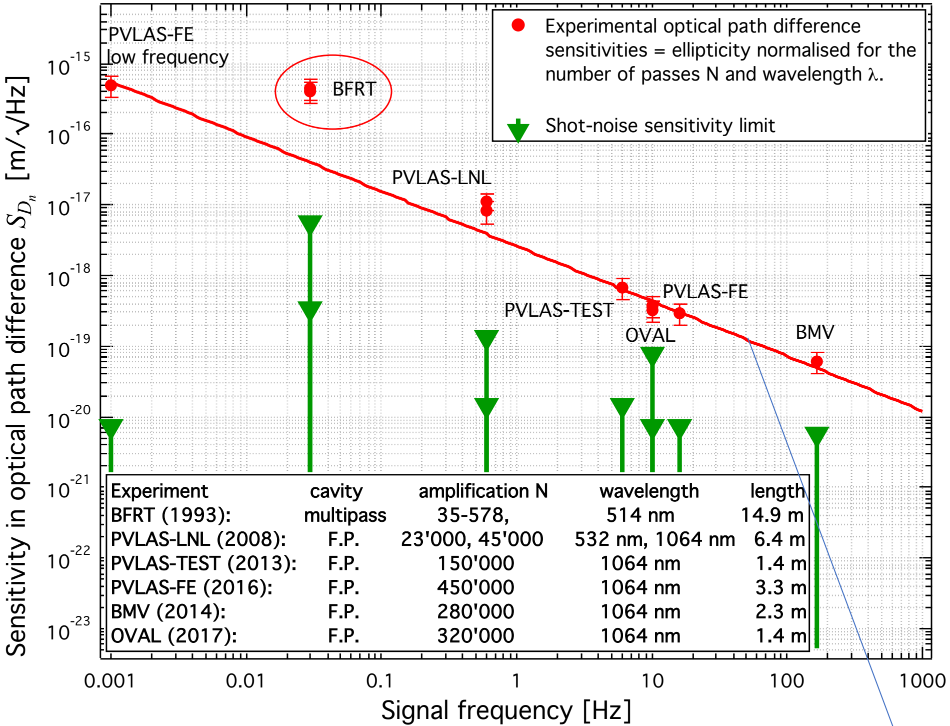
# Induced birefringence from stress

- Residual stress will generate a (static) birefringence map inside the sample
- External stress will also generate a birefringence

$$\Delta n = C_{\mathrm{SOC}} \left( \sigma_1 - \sigma_2 \right)$$

- $C_{\mathrm{SOC}}$ = Stess optic coefficiente $[Pa^{-1}]$, $\sigma_1$ and $\sigma_2$ stress along perpendicular directions $[Pa]$
- Typical values of stress optic coefficient: $C_{\mathrm{SOC}} \approx 10^{-12}\ Pa^{-1}$
- Fused silica: $3.4 \times 10^{-12}\ Pa^{-1}$
- Crystalline Silicon (axes): $(0.6 \div 1) \times 10^{-12}\ Pa^{-1}$
- Some initial work done for stress induced birefringence in Silicon as ET-LF substrate: C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012
- Sapphire: could not find a value for $C_{\mathrm{SOC}}$.

# Intrinsic mirror birefringence noise

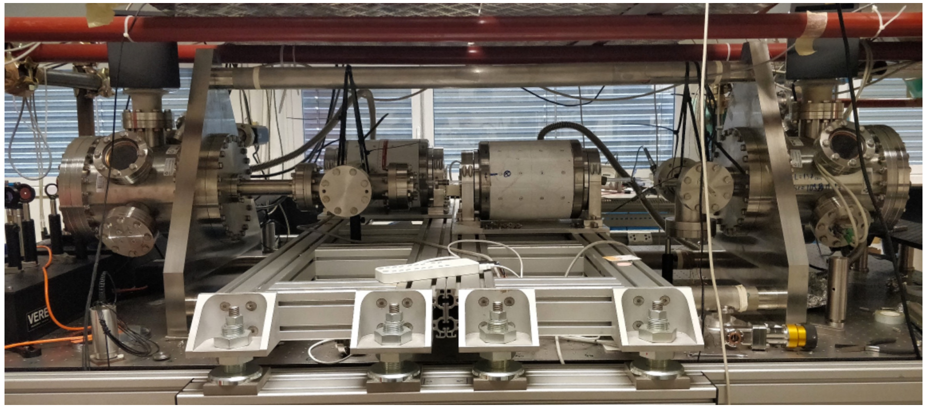



- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- This noise seems to be an intrinsic property of the cavity mirrors (thermal noise in the tantala layers)
- With low finesse one does reach shot-noise. The limit is not the heterodyne method

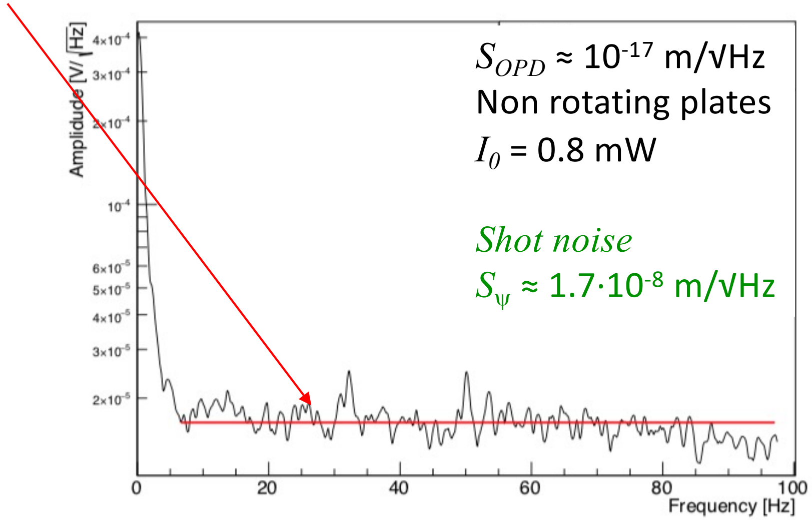
$$S_{\mathrm{OPD}} \approx 2.6 \times 10^{-18} \nu^{-0.77}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad \text{intrinsic noise}$$

# Noise with non-rotating HWPs inside the F.P.

- Important issue: Could a static birefringence from the HPWs degrade the sensitivity?
- Laser locking worked normally
- Measured a finesse of *F* = 850
- Sensitivity did not degrade with the presence of the HWPs and was compatible with shot-noise



Mirror birefringence ≈ $10^{-6}$ /reflection




$OPD_{mirrors}$ ≈ $10^{-12}$ m per reflection (≈ 1 μm thick)
$OPD_{intrinsic}$ in experiments > $10^{-19}$ m/√Hz
➡ $OPD_{intrinsic}/OPD_{mirrors}$ > $10^{-7}$ 1/√Hz

$\Delta n_{quartz}$ = 0.01: thickness ≈ 1 mm ➔ $OPD_{quartz}$ ≈ $10^{-5}$ m ➔ $S_{OPD}$ ≈ $(OPD_{intrinsic}/OPD_{mirrors})\cdot OPD_{quartz}$ ≈ $10^{-13}$ m/√Hz

<u>If the OPD noise was proportional to the absolute OPD ➡ sensitivity would have been ≈ $10^{-13}$ m/√Hz</u>

# HWP defect issues: temperature and alignment

$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

Generating 4th harmonic from $\alpha_{1,2}(t)$ in $\psi(t)$ : Expansion of the intrinsic HWP defects $\alpha_{1,2}(t)$:

$$\alpha_{1,2}(\phi, \mathrm{T}, r) = \alpha_{1,2}^{(0)}(\mathrm{T}) + \alpha_{1,2}^{(1)}(\mathrm{r}(\mathrm{t})) \cos\phi(\mathrm{t}) + \alpha_{1,2}^{(2)} \cos 2\phi(\mathrm{t}) + \ldots$$

- $\alpha^{(0)}_{1,2}$ (from manufacturer) depends on TEMPERATURE T and appears @ 2nd harmonic in $\psi(t)$
- $\alpha^{(1)}_{1,2}$ depends on WEDGE of wave-plates and their ALIGNMENT: appears @ 1st and 3rd harmonic in $\psi(t)$
- $\alpha^{(2)}_{1,2}$ depends on ALIGNMENT generating 4th harmonic in $\psi(t)$ just like a birefringence signal.
- Time modulation of $\alpha^{(1)}_{1,2}$ due to transverse axis oscillation will also generate a 4th harmonic in $\psi(t)$

$$r(t) = r_0 + \delta r \cos(\phi(t) + \phi_{\delta r})$$

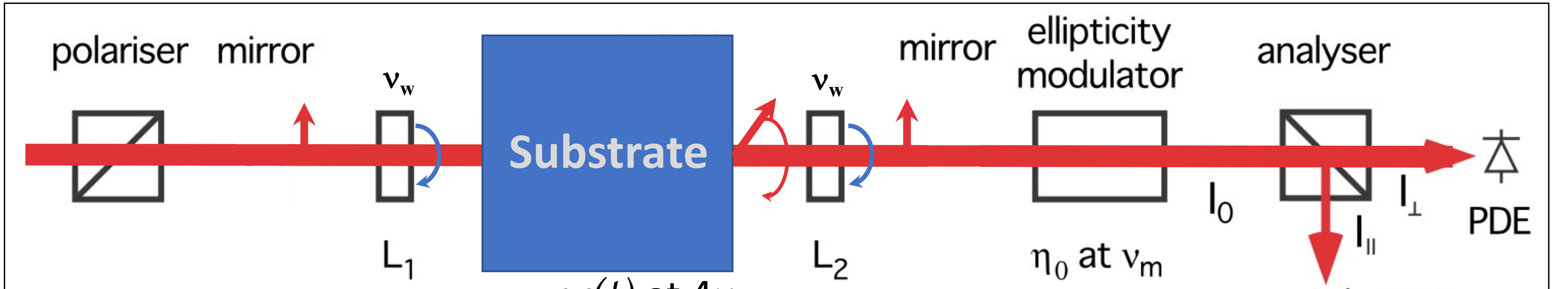
The resulting ellipticity is the combination of the two HWPs.

✓ They can be aligned separately using a frequency doubled laser @ 532 nm

# Substrate birefringence measurements

## Polarisation modulation scheme

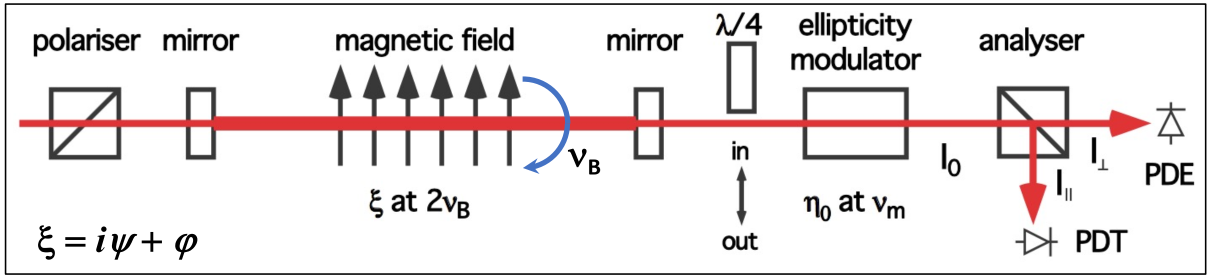
- Method: rotate polarisation inside the substrate. Developed for the VMB@CERN experiment
- Insert two co-rotating half wave plates @ $\nu_w$ with a fixed relative angle $\Delta\phi$
- Heterodyne detection linearizes the ellipticity $\psi(t)$ to be measured.
- We have 1064 nm working system and are buying a new 1550 nm laser (Thorlabs ULN15TK)




G. Zavattini et al. Eur. Phys. J. C (2016) 76:294
G. Zavattini et al. Eur. phys. J. C (2022) 82:159

$$I_{\text{out}} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t)\psi(t) + 2\eta(t)\Gamma(t) + \ldots \right\}$$

# General scheme




F. Della Valle et al. Eur. Phys. J. C (2016) 76:24
A. Ejlli et al. Physics Reports 871 (2020) 1–74

- *L* is the length of the birefringent medium (in our experiment $\Delta n_{\rm B} \propto B^2$ )
- Single pass ellipticity: $\psi = \dfrac{\pi \Delta n_{\rm B} L}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t)$
- The Fabry-Perot cavity amplifies $\psi$ by a factor $N = 2\mathcal{F}/\pi$. We had $\mathcal{F} = 7 \times 10^5$.
- The ellipticity modulator allows heterodyne detection which linearizes the ellipticity $\psi$ to be measured and allows the distinction between a rotation and an ellipticity. The insertion of the $\lambda/4$ wave plate allows measuring rotations.
- The rotating magnetic field modulates the desired signal due to VMB

$$I_{\rm out} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t)\psi(t) + 2\eta(t)\Gamma(t) + \ldots \right\}$$