Phenomenology of in-vacuo dispersion in FLRW spacetime

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QUAGRAP Meeting 2024, Naples

JCAP 01 (2024) 070, in collaboration with Amelino-Camelia, Gubitosi, Rosati, Bedic

Quantum Gravity Phenomenology

- Search of possible signatures of quantum gravity at scales lower than the heuristically expected Planck energy.

- Studies of in-vacuo dispersion are the most active area of quantum-gravity phenomenology. Energy dependent speed of light $\mathrm{v} \approx 1 + \eta \frac{E}{E_{Pl}}$ (where η is a dimensionless parameter).

- Difference in the time of flight of photons are amplified by cosmological distances $\Delta t \approx \eta\, L \frac{\Delta E}{E_{Pl}}$ and they can be within the reach of our current sensitivity.

"Tests of quantum gravity from observations of gamma-ray bursts", Amelino-Camelia,Ellis, Mavramatos, Nanopoulos,Sarkar, *Nature* 393 (1998) 763-765

Modified dispersion relations

- Many approaches to the Quantum Gravity suggest the possibility of having modified dispersion relations:

$$E^2 - P^2 - m^2 + f(E, P, m, E_{QG}) = 0$$

- where E_{QG} is an UV scale expected to be of the order of the Planck energy. We will consider only deformations linear in $\frac{E}{E_{QG}}$ since it is enough for phenomenological purposes.

LIV vs DSR

- LIV

- Deformed dispersion relations but undeformed Poincaré transformations to connect inertial reference frames

→Relativistic invariance is broken → preferred reference frame.

- DSR

- Deformed dispersion relations, deformed composition laws of momenta and deformed Poincaré transformations to connect inertial reference frames in order to accommodate a new energy scale $\sim E_{Pl}$ →

The relativity principle is preserved.

"Relativity in space-times with short distance structure governed by an observer independent (Planckian) length scale", Amelino-Camelia, *Int.J.Mod.Phys.D* 11 (2002) 35-60

1+1-D MINKOWSKI SPACETIME

Deformed dispersion relations (LIV)

- Deformed dispersion relation at first order in $\frac{1}{E_{Pl}}$:

$$m^2 = E^2 - P^2 + \frac{1}{E_{Pl}}(\alpha E^3 + \beta E P^2)$$

- Undeformed Poincaré transformations to connect inertial reference frames

$$[E, P] = 0\,; \qquad [N, E] = P; \qquad [N, P] = E$$

- Energy dependent time of flight of photons:

$$m = 0 \rightarrow v = \frac{\partial E}{\partial P} = 1 - \frac{1}{E_{Pl}} P(\alpha + \beta) \rightarrow \Delta t = \frac{1}{E_{Pl}} \mathrm{L}\, \Delta E(\alpha + \beta)$$

Deformed dispersion relations(DSR)

- Deformed dispersion relation at first order in $\frac{1}{E_{Pl}}$:

$$m^2 = E^2 - P^2 + \frac{1}{E_{Pl}}(\alpha E^3 + \beta E P^2)$$

- Deformed Poincaré transformations to connect inertial reference frames

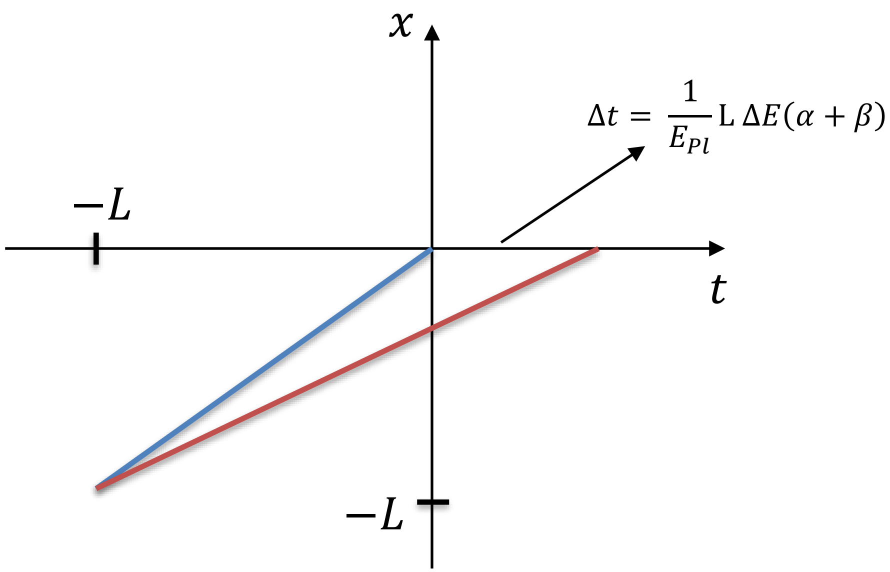
$$[E, P] = 0\,; \quad [N, E] = P - \frac{1}{E_{Pl}} EP(\alpha + \beta); \quad [N, P] = E + \frac{1}{2\,E_{Pl}}\alpha E^2 + \frac{1}{2E_{Pl}}\beta P^2$$

- Energy dependent time of flight of photons:

$$m = 0 \rightarrow \Delta t = \frac{1}{E_{Pl}} \mathrm{L}\, \Delta E(\alpha + \beta)$$

"Taming Nonlocality in Theories with Planck-Scale Deformed Lorentz Symmetry", Amelino-Camelia,Matassa, Mercati, Rosati, *Phys.Rev.Lett.* 106 (2011) 071301

Time delay



- In blue the worldline of a soft photon for which we neglect Planck scale corrections.
- In red the worldline of an hard photon.

CURVED SPACETIME

Jacob and Piran formula in FLRW spacetime

$$\Delta t = \frac{\eta_1 D(z)}{E_{Pl}} \Delta E = \frac{\eta_1}{E_{Pl}} \Delta E \int_0^z \frac{d\,x}{H(x)} (1+x)$$

- This formula can be obtained

  1) assuming that the comoving distance, traveled by both particles and emitted from the same source, is the same.
  2) using the relation $E = \frac{E_0}{1+z}$ (where E_0 is the energy at source and E is the measured energy).

- ΔE is the measured energy difference .

- z is the redshift of the source and $H(z) = \mathrm{H}_0 \sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}$.

“Lorentz-violation-induced arrival delays of cosmological particles”, Jacob, Piran, *JCAP* 01 (2008) 031

LIV generalization of the time delay formula in FLRW

- Symmetries in a LIV model can be broken in many ways, therefore the most general LIV formula for the time delay can contain a very large number of terms.

- An example of time delay formula that generalizes the Jacob and Piran result is:

$$\Delta t = \frac{\widetilde{D}(z)}{E_{Pl}} \Delta E = \frac{1}{E_{Pl}} \Delta E \int_0^z \frac{d\,x}{H(x)} \left(\eta_1 (1+x) + \eta_2 + \eta_3 (1+x)^2 + \eta_4 \frac{1}{(1+x)} + \cdots \right)$$

" Planck-scale-modified dispersion relations in FRW spacetime" , Rosati, Amelino-Camelia, Marciano, Matassa, *Phys.Rev.D* 92 (2015) 12, 124042

DSR in 1+1-D de Sitter spacetime

- The most general deformation of the de-Sitter dispersion relation at first order in $\frac{1}{E_{Pl}}$ is:

$$m^2 = E^2 - P^2 - 2HNP + \frac{1}{E_{Pl}}(\alpha\, E^3 + \beta\, EP^2 + 2\gamma\, HNEP + 4\mu\, H^2N^2E)$$

- The algebra of symmetry generators that leaves the previous dispersion relation invariant is:

- $$[E,p] = Hp - \frac{1}{E_{Pl}}HE[\,(\alpha+\gamma-\sigma)p + 4\mu\, HN]$$
- $$[N,E] = p + HN - \frac{1}{E_{Pl}}E[(\alpha+\beta-\sigma)p + HN(\alpha+\gamma-\sigma)]$$
- $$[N,p] = E + \frac{1}{2E_{Pl}}[(\alpha+2\sigma)E^2 + \beta\, p^2 + 2\gamma\, HNp + 4\mu\, H^2N^2].$$

Deformed composition laws

- An important ingredient of DSR models concerns the conservation law of energy-momenta that must be deformed in order to be compatible with the previous deformed symmetry algebra.

- $E_{tot} = E_1 + E_2 + \frac{1}{E_{Pl}}\left((2\sigma - \beta - a - b)P_1P_2 + (c - \gamma + \sigma)H\,(N_1P_2 + P_1N_2) - \alpha\, E_1E_2 + 2(c - 2\mu)H^2N_1N_2\right)$

- $P_{tot} = P_1 + P_2 + \frac{1}{E_{Pl}}\left((\sigma - b)E_1P_2 + (\sigma - a)E_2P_1 + cH(N_1E_2 + E_1N_2)\right)$

- $N_{tot} = N_1 + N_2 + \frac{1}{E_{Pl}}(aE_1N_2 + bE_2N_1)$

Slicing technique

- We extend our result to FLRW space-time by describing particle propagation in FLRW as a sequence of N infinitesimal steps of propagation in de Sitter space-time.

- Propagation of the signal in each slice is described using deformed de Sitter kinematics and the full trajectory is reconstructed by suitably matching the results in each slice and considering the limiting procedure $N \to \infty$.

Most general DSR formula for time delays in FLRW

$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{d\,x}{H(x)} (1+x) \left(\eta_1 + \eta_2 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^2 \right) + \eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^4 \right) \right)$$

- $\eta_1 = (\alpha + \beta)$, $\eta_2 = (-\alpha - \gamma + \sigma + 2\mu)$, $\eta_3 = -\mu$.
- ΔE is the measured energy difference.
- z is the redshift of the source and $H(z) = \mathrm{H}_0\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}$.
- The most general DSR formula contains the Jacob e Piran term (η_1) and just two new terms.

“ Planck-scale-modified dispersion relations in FRW spacetime” , Rosati, Amelino-Camelia, Marciano, Matassa, *Phys.Rev.D* 92 (2015) 12, 124042

“Amelino-Camelia, Frattulillo, Gubitosi, Rosati, Bedic, *JCAP* 01 (2024) 070“

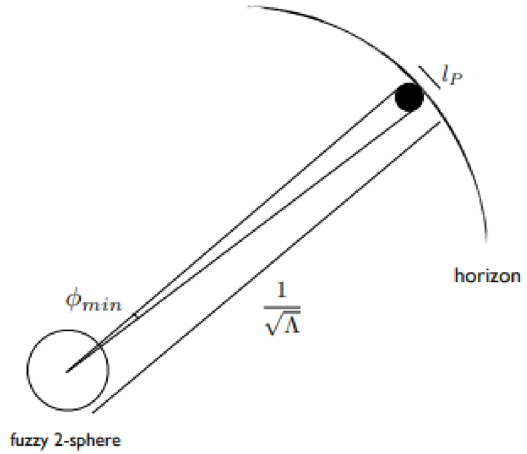
SOME NOTEWORTHY SPECIAL CASES

Curvature induced scenario (1)

- We look for scenarios in which quantum gravity effects are triggered by space-time curvature, motivated by quantum groups' studies and considerations arising from loop-quantum gravity research.

- If we live in a 3-sphere with large radius $L = \frac{1}{\sqrt{\Lambda}}$, or in a Lorentzian space-time with a horizon at that distance, we will never see a sphere of radius ℓ under an angle smaller than $\frac{\ell}{L}$. Suppose in addition that no object with size smaller than $\ell \equiv \ell_P$ exists in the universe, then we will never see anything having angular size smaller than:

- $$\phi_{min} = \ell_P \sqrt{\Lambda}$$



"A note on the geometrical interpretation of quantum groups and non-commutative spaces in gravity" Bianchi,Rovelli, *Phys.Rev.D* 84 (2011) 027502

Curvature induced scenario (2)

- A scenario with interesting phenomenological implications is the one where in-vacuo dispersion occurs only in combination with space-time curvature so that when curvature is negligible there is no expected time delay.

- Only terms involving powers of z higher than 1 contribute to curvature-induced time-delay effects.

Considering the leading order expansion in terms of the redshift of the time delay expression, we obtain

$$\Delta t = \frac{\Delta E}{E_{Pl}} \frac{1}{H_0} \eta_1 \, z + O(z^2),$$

thus, we have to impose η_1 =0.

" Phenomenology of curvature induced quantum gravity effects" , Amelino-Camelia, Rosati, Bedic, *Phys.Lett.B* 820 (2021) 136595

Scenarios with undeformed addition of energy

- In DSR models a desirable feature is that the addition law of particle energies remains undeformed.

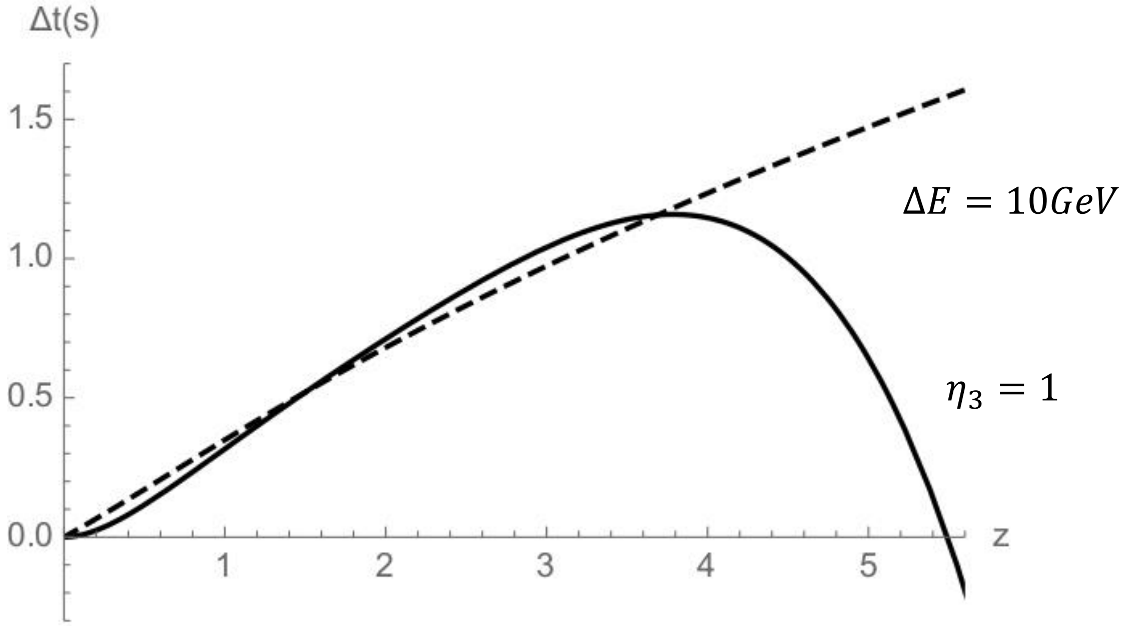
1. Preserving the linearity of addition of energies is advantageous from the point of view of the interpretation of the results.

2. Undeformed composition law of energies would correspond (in the framework of quantum groups) to a “primitive coproduct” for time-translation generators, that is necessary for having a “time-like” q-deformation of de Sitter symmetries.

- In order to have an undeformed addition of energy we have to impose $\eta_2 = 0$.

A one-parameter scenario: curvature-induced and undeformed addition of energy

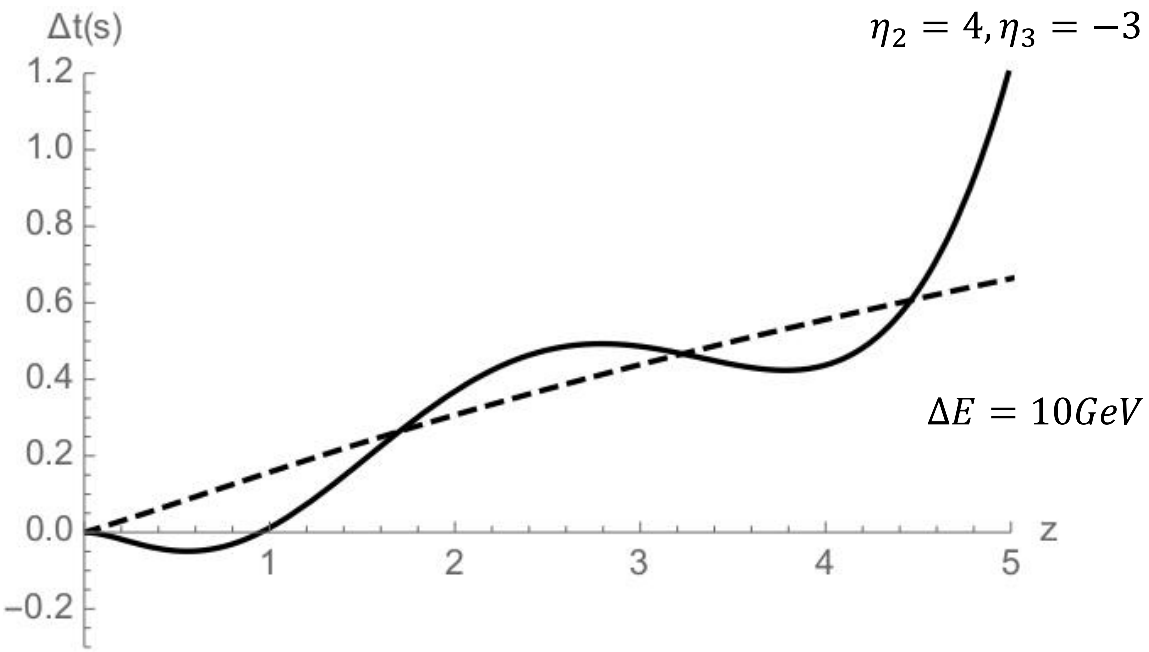
$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{d\,x}{H(x)} (1+x) \left(\eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{d\,y}{H(y)} (1+y) \right)^4 \right) \right)$$



The dashed line represents the expected time delay for the “Jacob-Piran” case and it is normalized imposing that the two lines cross at z = 1.5

Scenarios with time delay changing sign

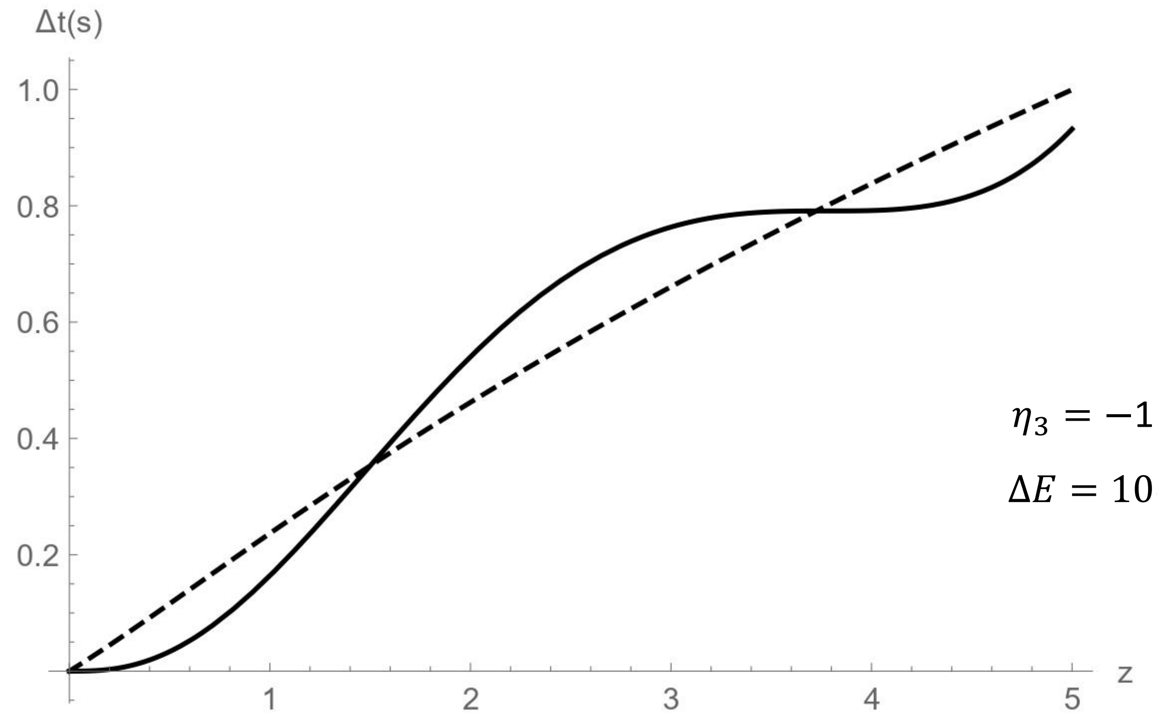
$$\Delta t = \frac{\Delta E}{E_{Pl}} \int_0^z \frac{dx}{H(x)} (1+x) \left(\eta_2 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{dy}{H(y)} (1+y) \right)^2 \right) + \eta_3 \left(1 - \left(1 - \frac{H(x)}{1+x} \int_0^x \frac{dy}{H(y)} (1+y) \right)^4 \right) \right)$$



The dashed line represents the expected time delay for the "Jacob-Piran" case, and it is normalized imposing that the two lines cross at z = 1.7

Another one-parameter scenario (curvature-induced and monotonic time delay)

$$\Delta t = \frac{\Delta E}{E_{Pl}} \eta_3 \int_0^z \frac{d\,x}{H(x)}(1+x)\left(-2\left(1-\left(1-\frac{H(x)}{1+x}\int_0^x \frac{d\,y}{H(y)}(1+y)\right)^2\right)+\left(1-\left(1-\frac{H(x)}{1+x}\int_0^x \frac{d\,y}{H(y)}(1+y)\right)^4\right)\right)$$



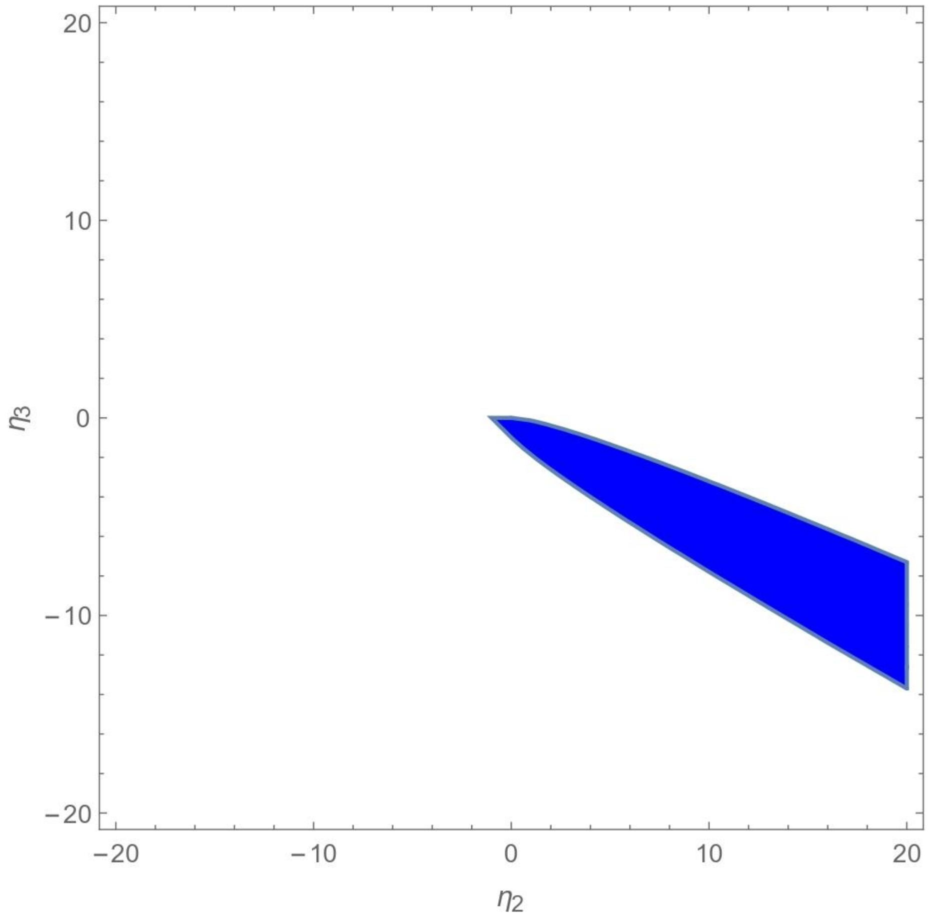
$\eta_3 = -1$

$\Delta E = 10 GeV$

$\eta_2 = -2\,\eta_3$

The dashed line represents the expected time delay for the “Jacob-Piran” case, and it is normalized imposing that the two lines cross at z = 1.5

Monotonicity for $\eta_1 \neq 0$



- The blue region identifies the range of parameters η_2, η_3 such that the time delay depends monotonically on redshift when $\eta_1 = 1$.

TESTING IN-VACUO DISPERSION WITH GRBs

Gamma Ray Bursts (GRBs)

- GRBs are the main sources in the search for quantum gravity effects on particle propagation due to their high redshift distances and wide energy-spectrum of emission.

- At present intrinsic temporal dependencies of the spectrum are not well understood and it is difficult to isolate those from QG effects.

- Suitable statistical methods can compensate our lack of knowledge on the emission mechanism.

- Many interesting results analyzing time of arrival of both photons and neutrinos have been obtained to date encouraging further anaysis when more data will be soon available.

“ Quantum gravity phenomenology at the dawn of the multi-messenger era- A review ” Addazi et al, *Prog.Part.Nucl.Phys.* 125 (2022) 103948

Conclusions

- We derived the most general DSR formula for time delays in FLRW spacetimes.

- This formula contains the Jacob and Piran term and just two new terms are allowed.

- We analyzed some interesting scenarios which present deviations from the famous formula proposed by Jacob and Piran for the LIV scenario, paving the way for novel phenomenological studies, exploring for instance the possibility of having a non-monotonic time delay.

Thank you!