Hawking radiation in Lorentz violating gravity

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Based on:

JHEP 12 (2023) 094 (ArXiv:**2310.01472**)

In collaboration with:
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Outline

- LV gravity and Black Holes' structures
- Modes' structure
- The role of KH
- Quantum state
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Lorentz Violating gravity

- A "good way" to break LLI is to assume an inhomogeneous scaling behavior between time and space:

arXiv:**0901.3775**, P.Horava , 2009

$$\vec{x} \to b\vec{x}, \quad \tau \to b^3\tau$$

- One can introduce a Stueckelberg vector field, the Aether, that parametrizes the time direction:

$$U_\mu = \frac{\partial_\mu \tau}{\sqrt{g^{\alpha\beta}\partial_\alpha\tau\partial_\beta\tau}} \qquad S[g,\tau] = -\frac{1}{16\pi G}\int_{\mathcal{M}} \sqrt{-g}\,(R + c_\theta\theta^2 + c_\sigma\sigma_{\mu\nu}\sigma^{\mu\nu} + c_\alpha a_\mu a^\mu)$$

Matter Fields

- The theory allows the presence of higher (spatial) derivative operators:

$$S_m[\phi] = \frac{1}{2}\int_{\mathscr{M}} \sqrt{-g}\ \phi\left[\nabla_\mu \nabla^\mu - \sum_{j=2}^{n} \frac{\beta_{2j}}{\Lambda^{2j-2}}(-\Delta)^j\right]\phi \qquad \Delta = \nabla_\mu \gamma^{\mu\nu} \nabla_\nu$$

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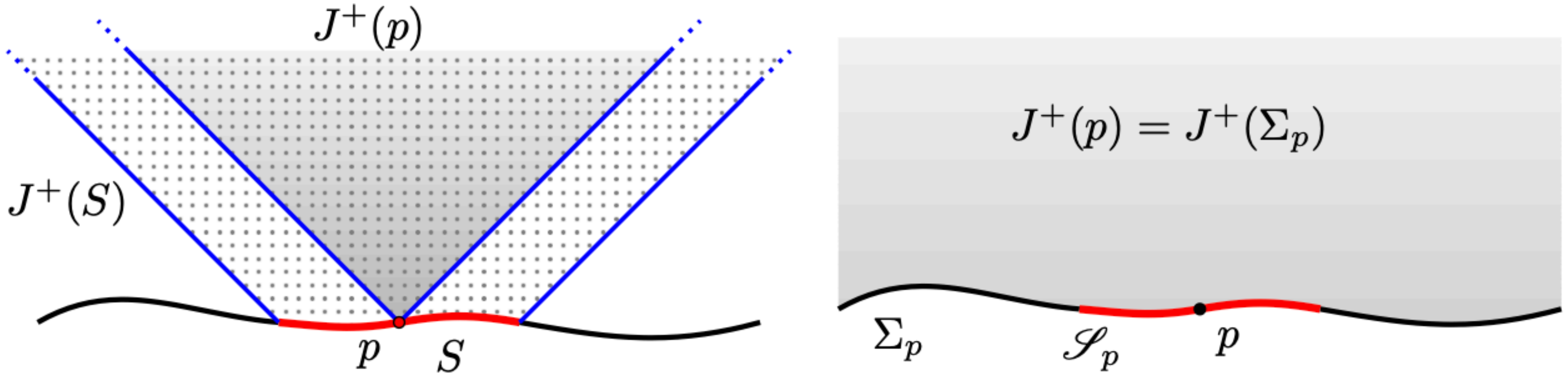
Superluminal particles:

$$\omega^2 = k^2 + \beta_4\frac{k^4}{\Lambda^2} + \ldots + \beta_{2n}\frac{k^{2n}}{\Lambda^{2(n-1)}}$$

Different notion of causality:



J.Bhattacharyya, M.Colombo, T.P. Sotiriou, ArXiv: 1509.01558

Horizons

- Killing Horizons are no more causal boundaries! What is a Black Hole?

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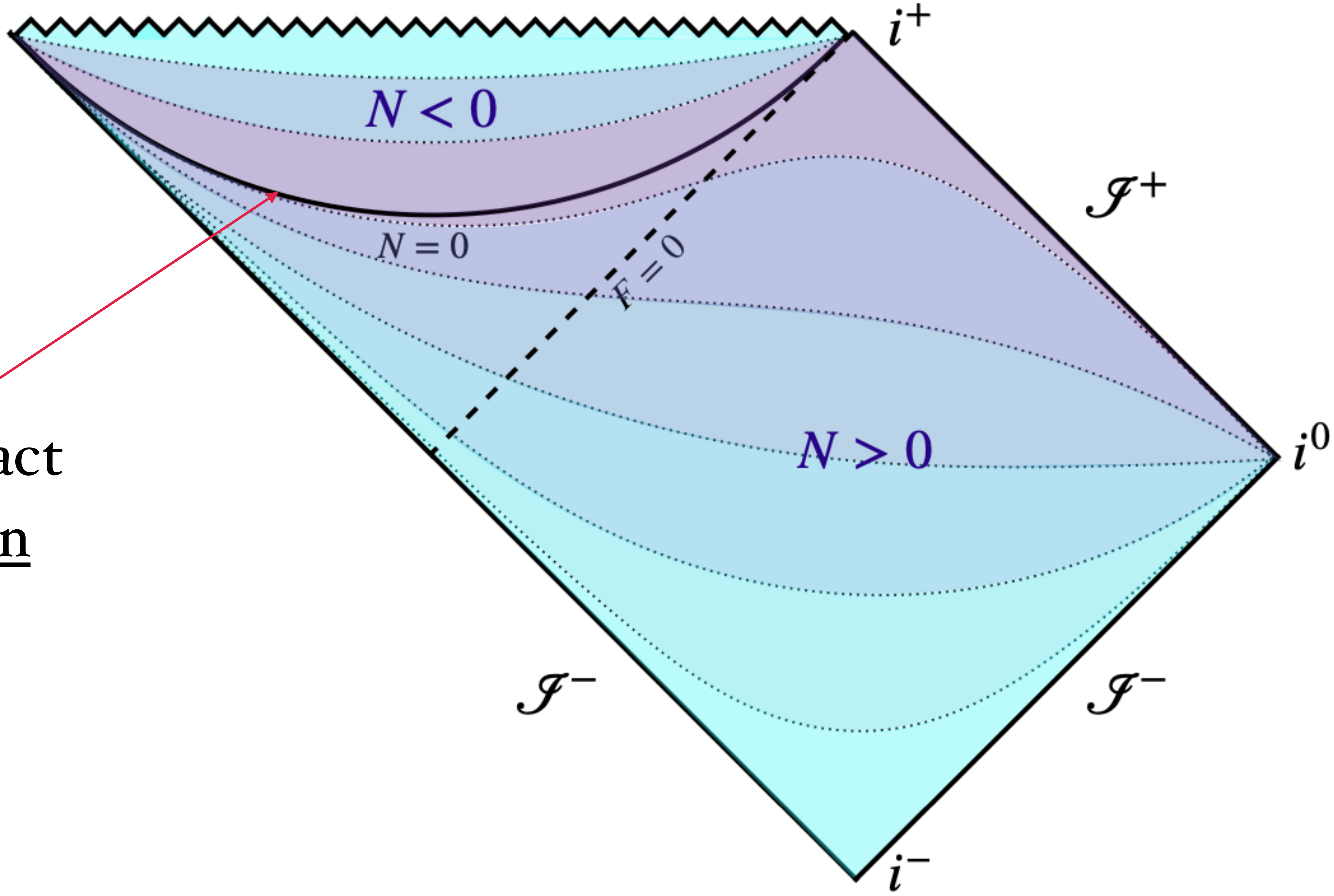

If U_μ becomes orthogonal to a compact surface, we have a Universal Horizon

Universal Horizon

- In principle the UH is a global concept. If however the spacetime is stationary and asymptotically flat, namely

$$\mathcal{L}_\chi U_\mu = \mathcal{L}_\chi g_{\mu\nu} = 0 \qquad\qquad (U\cdot\chi)\ _{i^0} = 1$$

Then the definition of the Universal Horizon can be given locally

$$UH = \{(\chi\cdot U) = 0\,, \quad (\chi\cdot a) \neq 0\}$$

$$2\kappa_{UH} = (\chi\cdot a)\ _{UH}$$

UH in Schwarzschild

- We will consider a Schwarzschild solution of the theory:

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 dS_2$$

$$U_\mu dx^\mu = \left(1 - \frac{M}{r}\right)dt + \frac{M}{r - 2M}dr$$

$$\chi^\mu \partial_\mu = \frac{\partial}{\partial t}$$

- The Universal Horizon is located at:

$$UH = \left\{1 - \frac{M}{r} = 0\right\} \implies \{r = M\}$$

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Matter fields

- Let us take a massless scalar field on a BH geometry: $\left[\nabla_\mu \nabla^\mu - \sum_{j=2}^{n} \frac{\beta_{2j}}{\Lambda^{2j-2}}(-\Delta)^j\right]\phi = 0$

- In the (t, r) plane we have the dispersion relation in the eikonal approximation:

$$\begin{cases} iU^\mu \partial_\mu \phi = \omega\phi \\ iS^\mu \partial_\mu \phi = k\phi \end{cases} \implies \omega^2 = k^2 + \sum_{j=2}^{n} \frac{\beta_{2j}}{\Lambda^{2j-2}} k^{2j}$$

- And a conservation equation: $\Omega = \omega N + kV \qquad N = (U \cdot \chi) \qquad V = (S \cdot \chi)$

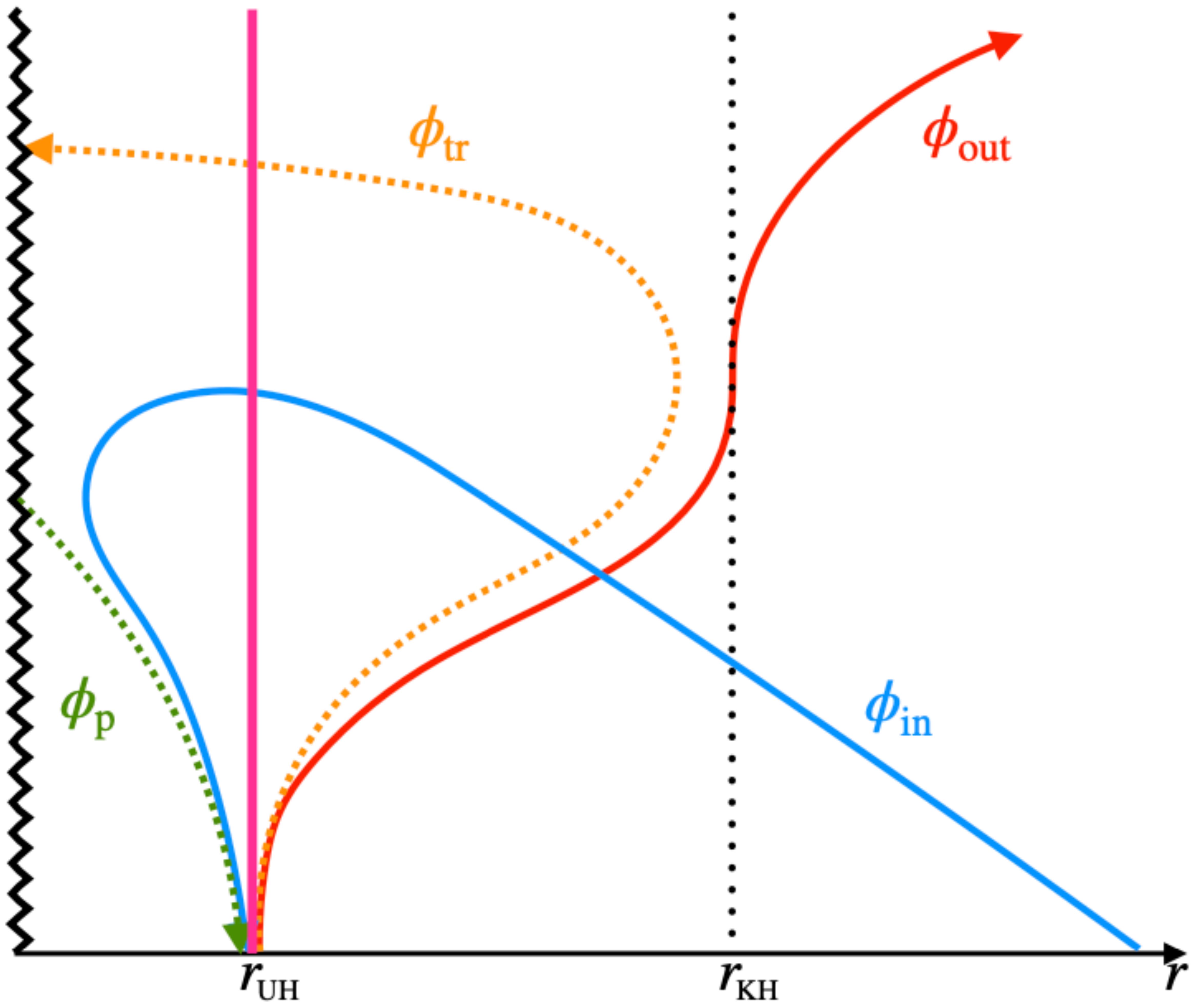
Modes' structure

- We will follow a wavepacket ψ_{Ω_0}

$$\psi_{\Omega_0}(r) = \int_0^\infty \frac{d\omega}{\sqrt{2\pi\sigma}}\, \phi_\Omega(r)\, e^{-\frac{(\omega-\omega_0)^2}{2\sigma}}$$

- For superluminal dispersion relations:

$$c_g(r,\alpha) = \frac{d\omega}{dk} \neq \frac{\omega}{k} = c_p(r,\alpha)$$

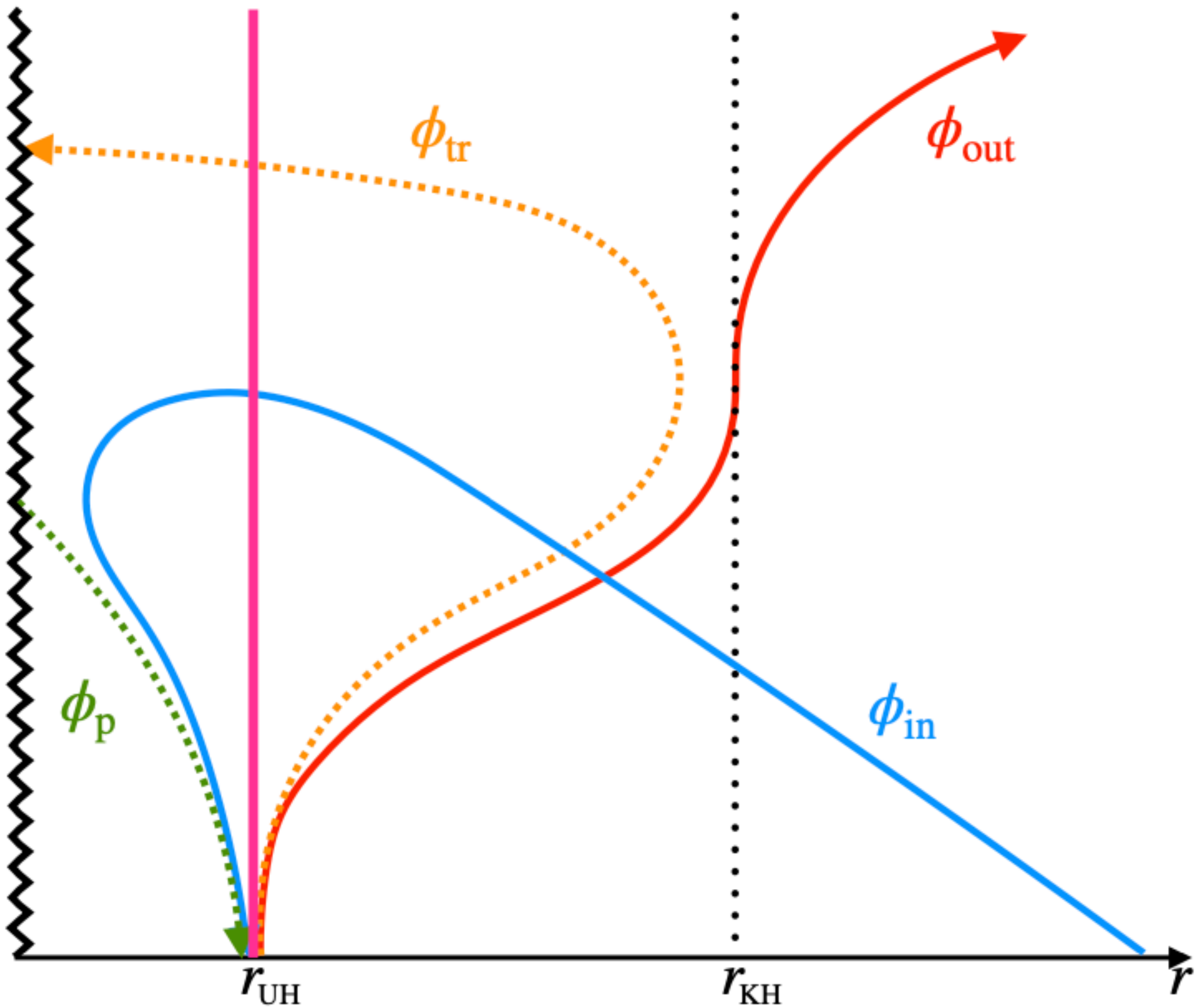


FDP, S.Liberati, M.Herrero-Valea, M.Schneider
Phys.Rev.D 106 (2022)

Modes' structure

- k is determined by the equation

$$\frac{k^4}{\Lambda^2}+\left(\frac{N^2-V^2}{N^2}\right)k^2+\frac{2\Omega V}{N^2}k-\frac{\Omega^2}{N^2}=0$$



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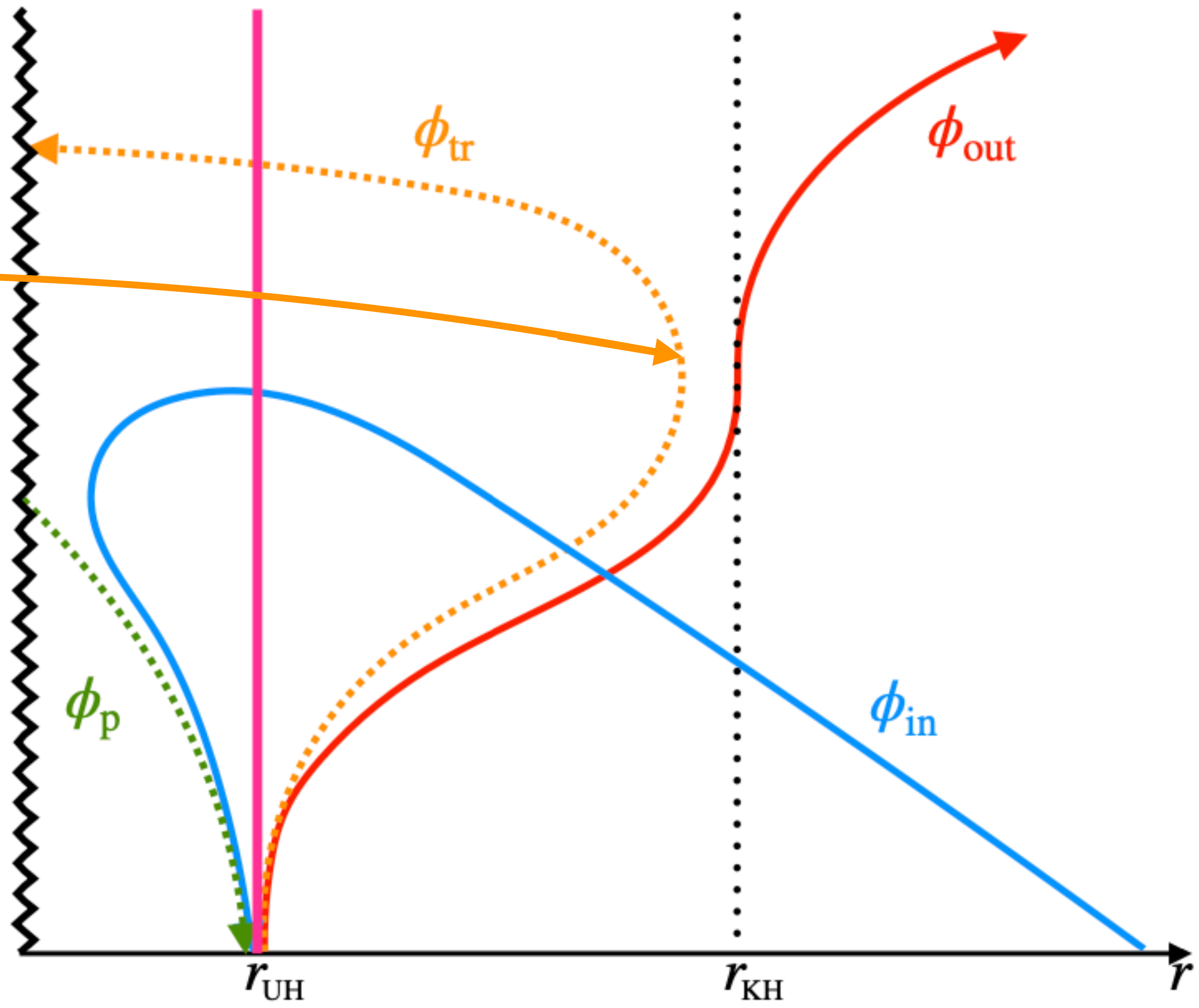

- When $\chi^2 = N^2 - V^2$ changes sign, we have a degeneracy in the solutions

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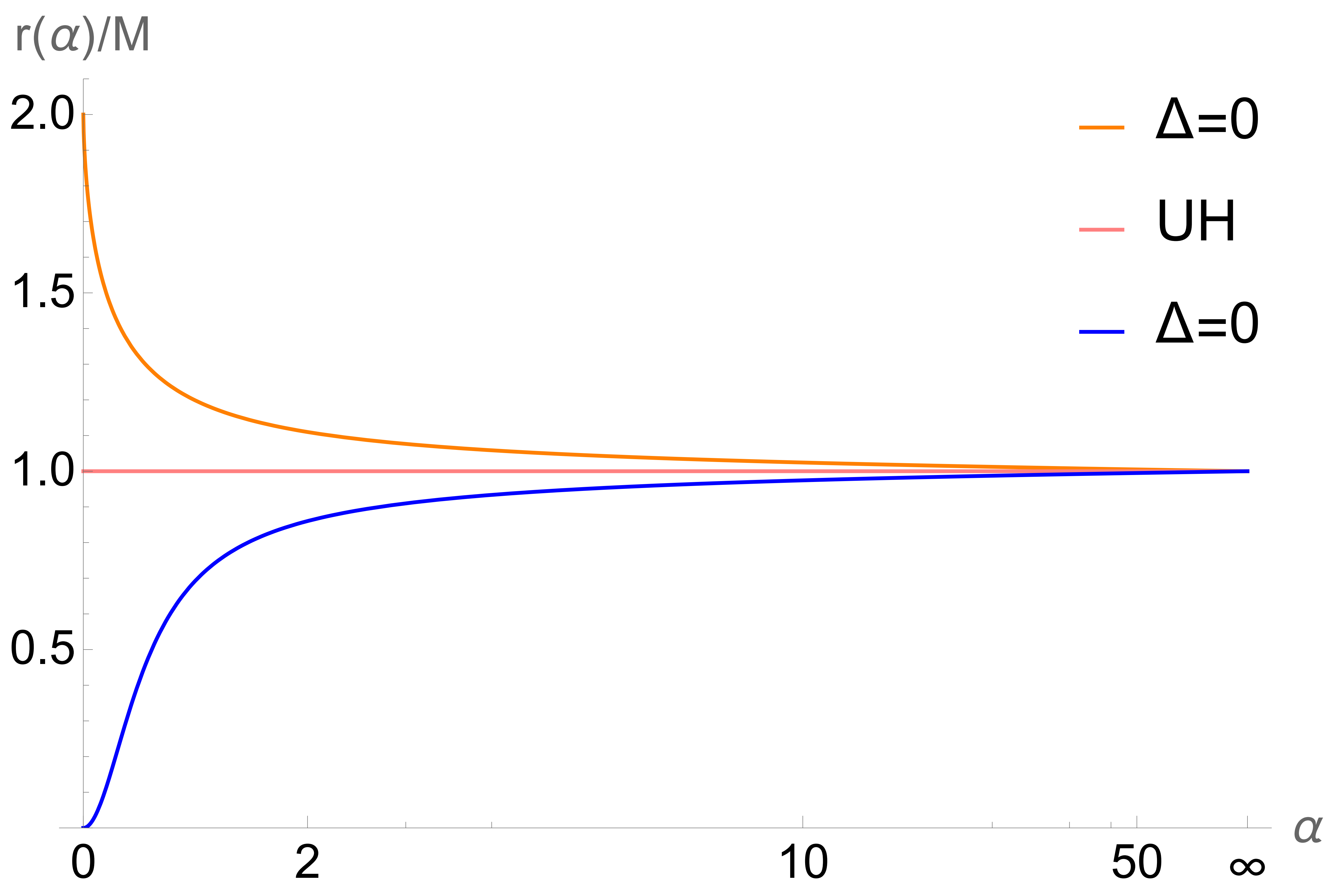
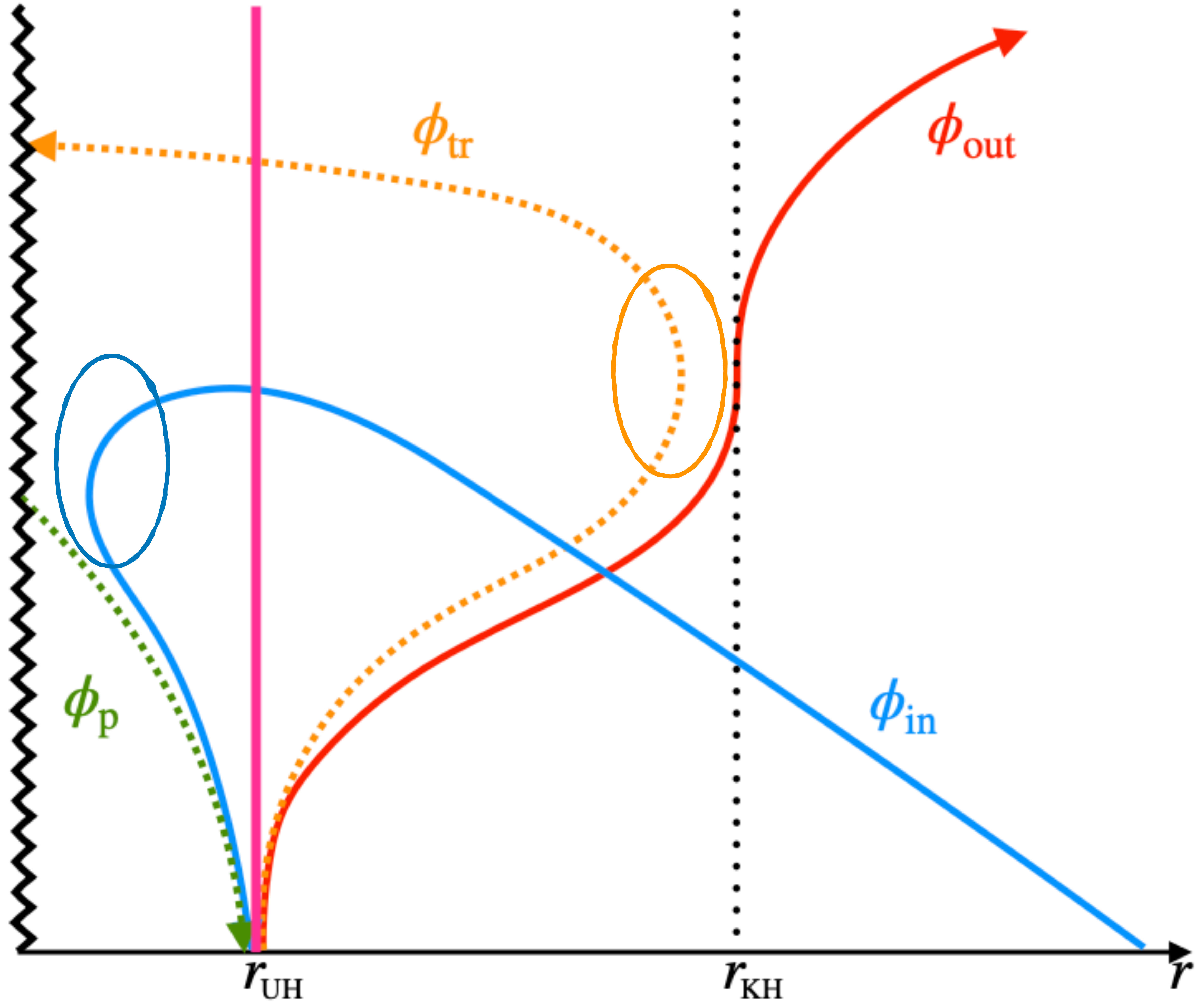
Turning points

- We can take the discriminant $\Delta(r,\alpha)$ of the polynomial and study the roots





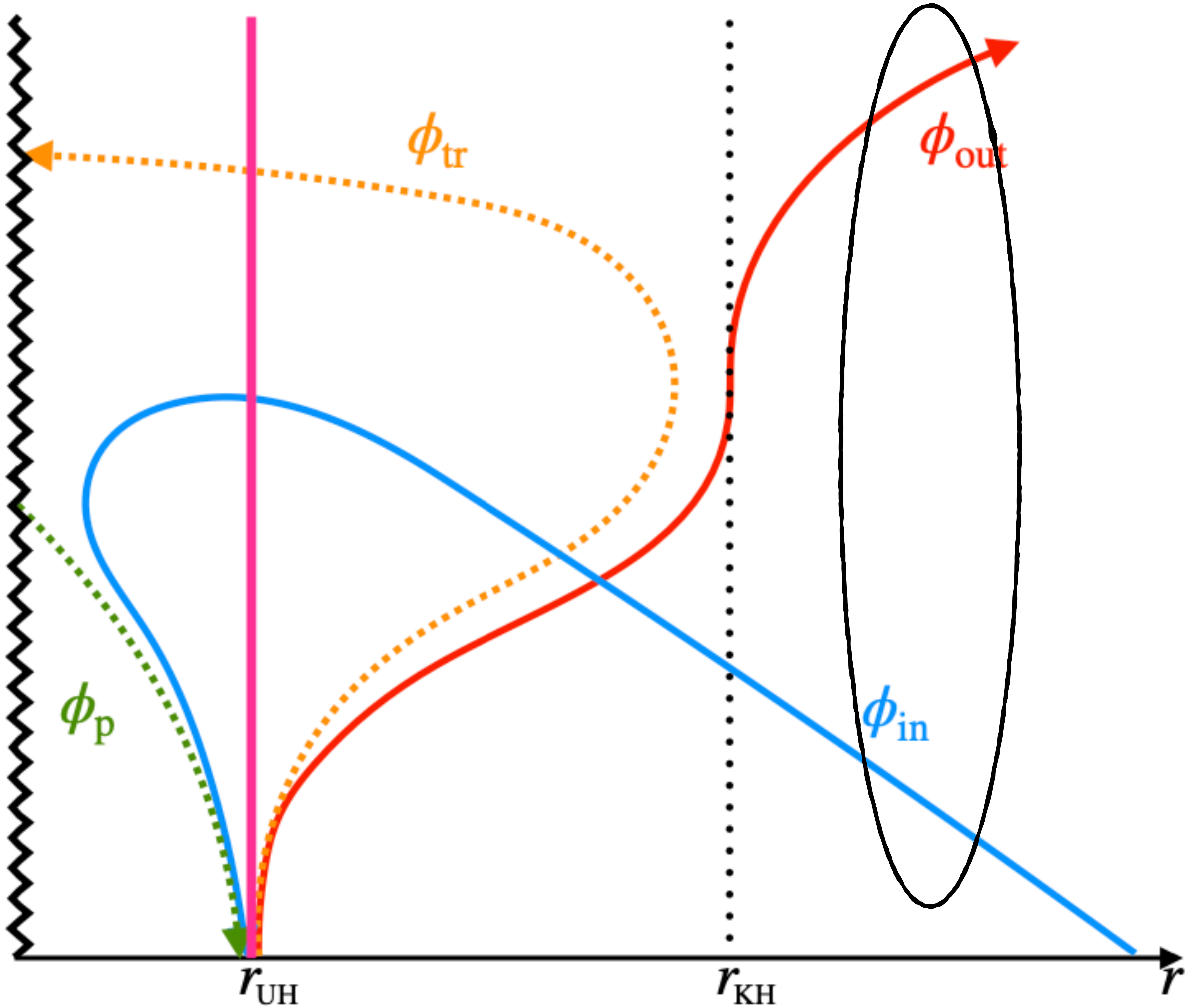
Limits

- When $r \to \infty$ ($N \to 1$), we have only two real solutions

$$\omega = \Omega > 0$$

$$k_{red} > 0\,, \qquad k_{blue} < 0$$

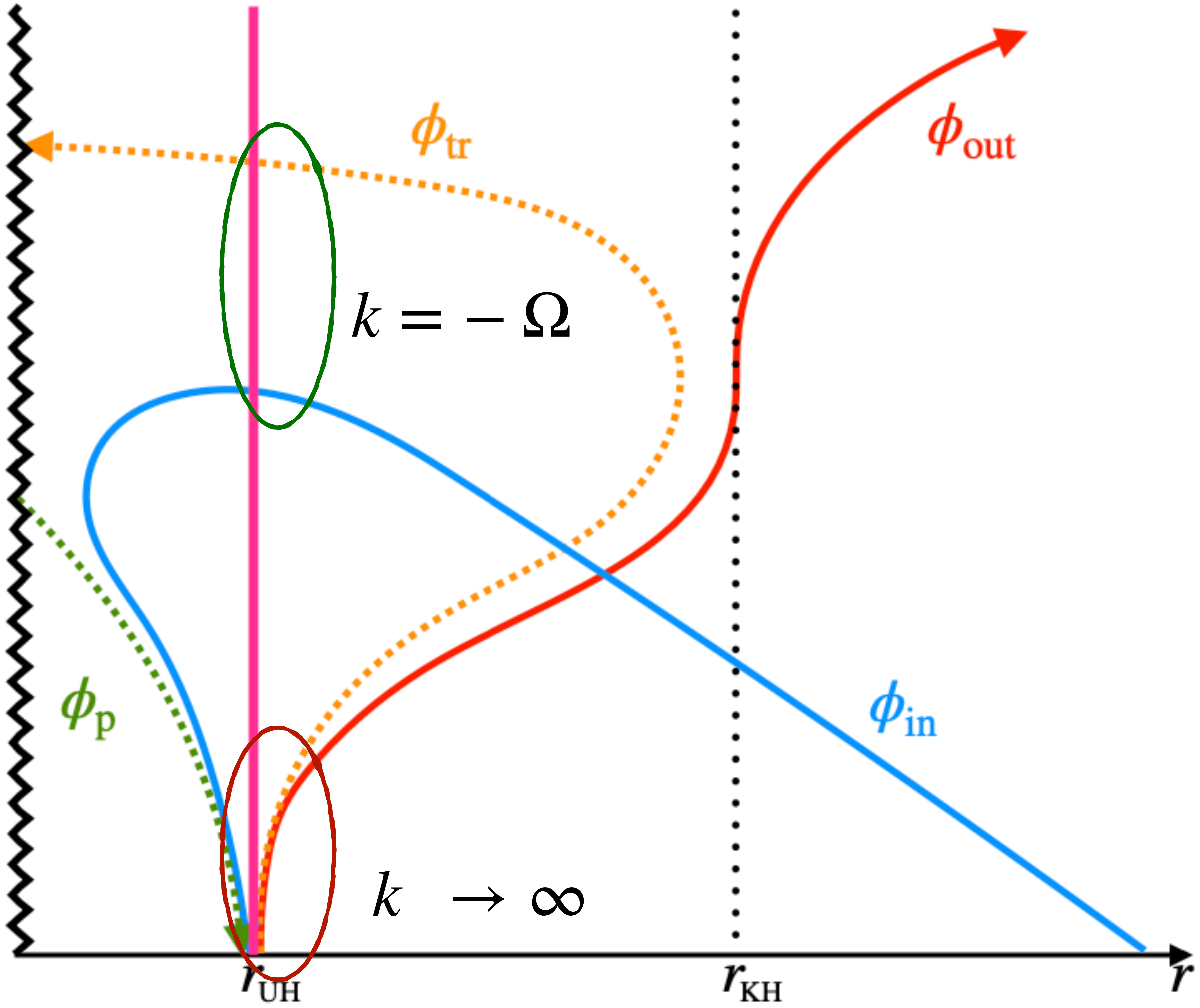
$$\lim_{r\to\infty} \psi_{\Omega_0}(r) = \int_0^{+\infty} \frac{d\Omega}{\sqrt{2\pi\sigma}}\,\phi_\Omega(r)\,e^{-\frac{(\Omega-\Omega_0)^2}{2\sigma}}$$



- In the following we will "turn off" the blue (ingoing) mode

Limits

- When $r \to r_{UH}$ ($N \to 0$), we have four real solutions, two *soft* and two *hard* ones



Hard solutions near UH

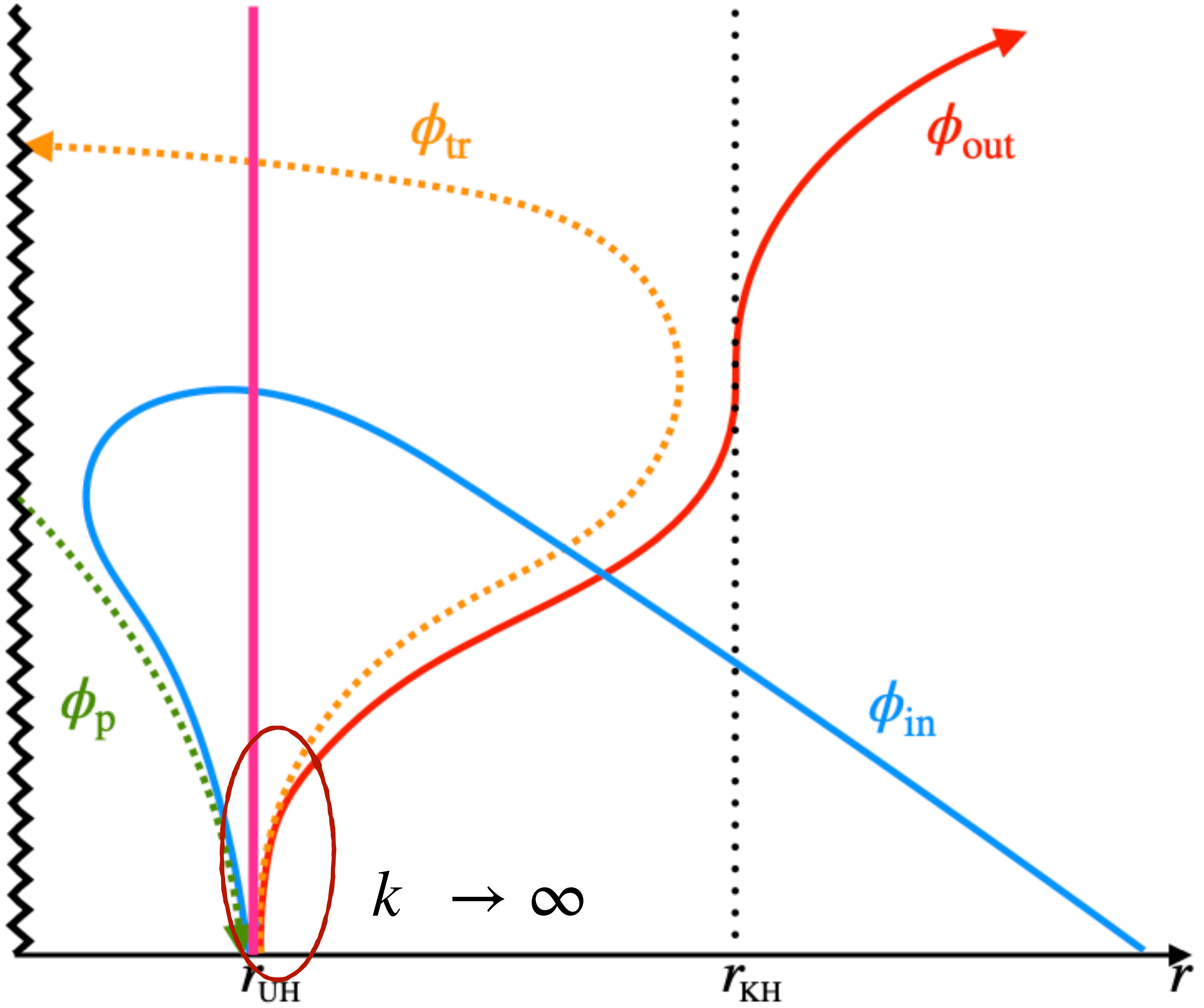
- When $r \to r_{UH}$ ($N \to 0$), the hard solutions are squeezed onto a single mode

$$d\omega = 2\frac{d\Omega}{N} \qquad \omega - \omega_0 = 2\frac{\Omega - \Omega_0}{N}$$

$$\lim_{N\to 0^+} \frac{2}{\sqrt{2\pi\sigma}N} e^{-\frac{4(\Omega-\Omega_0)^2}{2\sigma N^2}} = \delta(\Omega - \Omega_0)$$

$$\lim_{r\to r_{UH}^+} \psi^{hard}_{\Omega_0} = \phi_{\Omega_0}$$

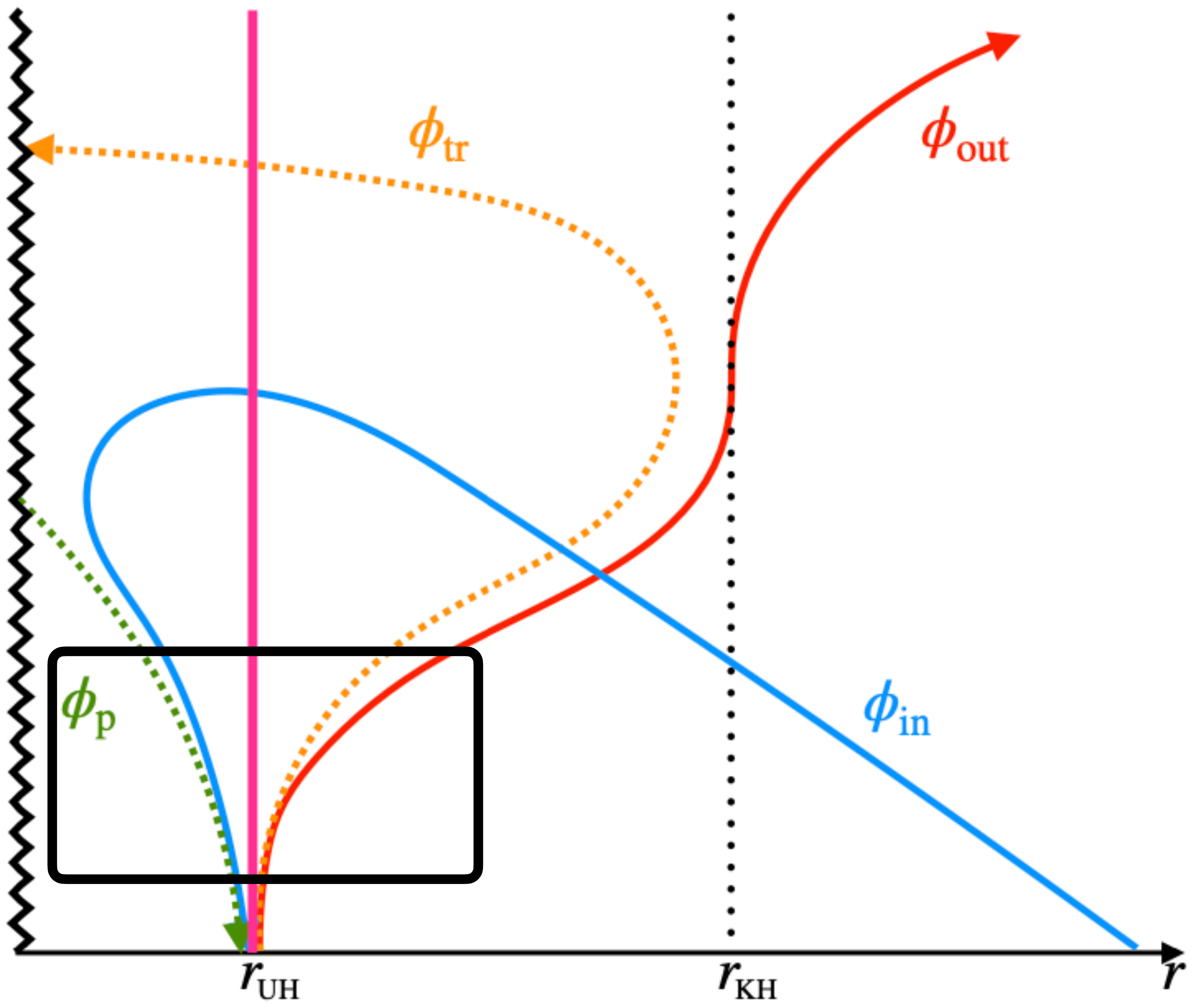
- The propagation happens with the phase velocity $c_p = \omega/k$



UH emission

- Although classically forbidden, we can compute the tunneling amplitude across the UH

$$\Gamma = e^{-2Im\,S_p} \qquad Im\,S_p = -\,Im\int_{r_1}^{r_2} k_r\,dr$$



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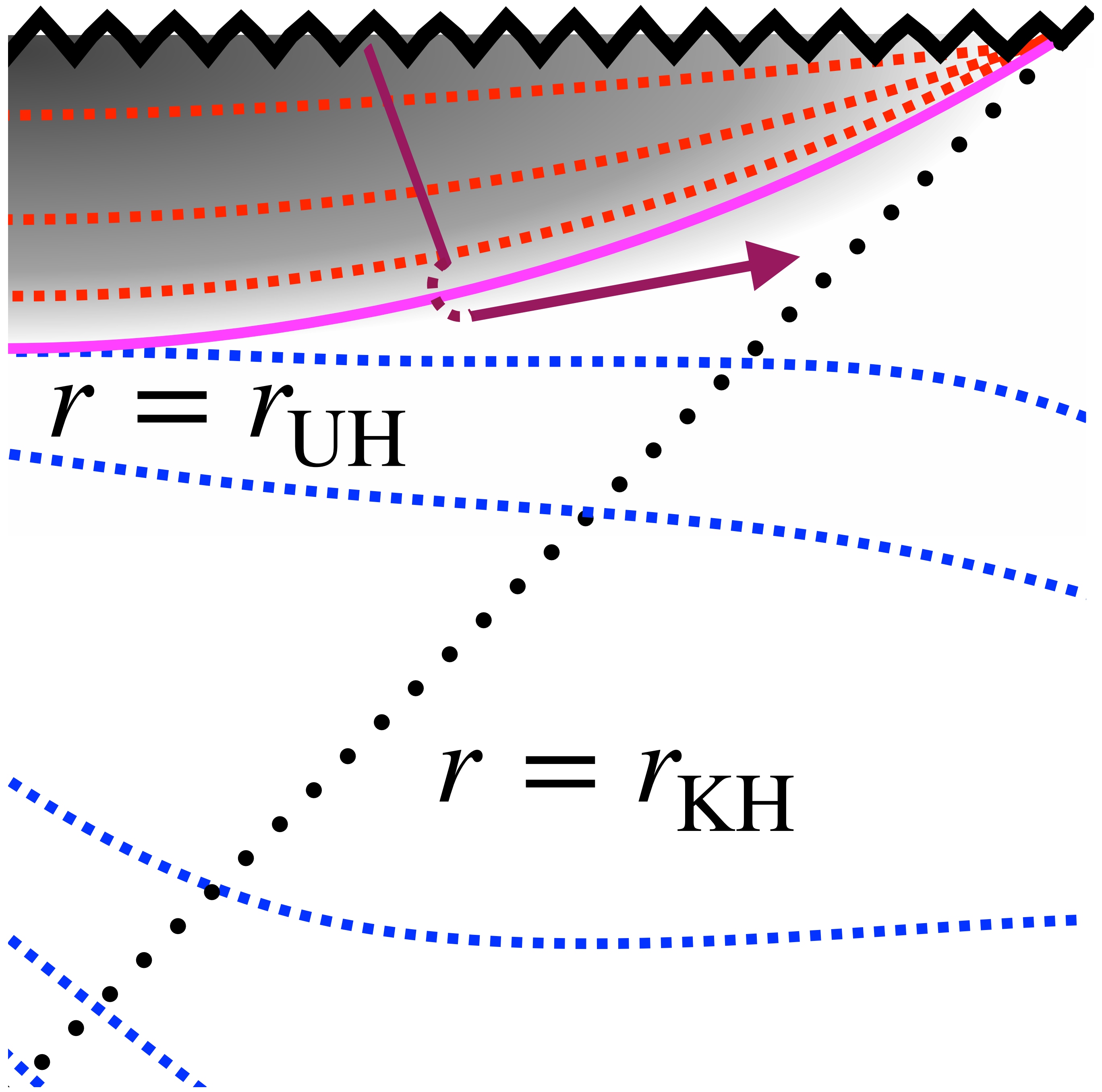

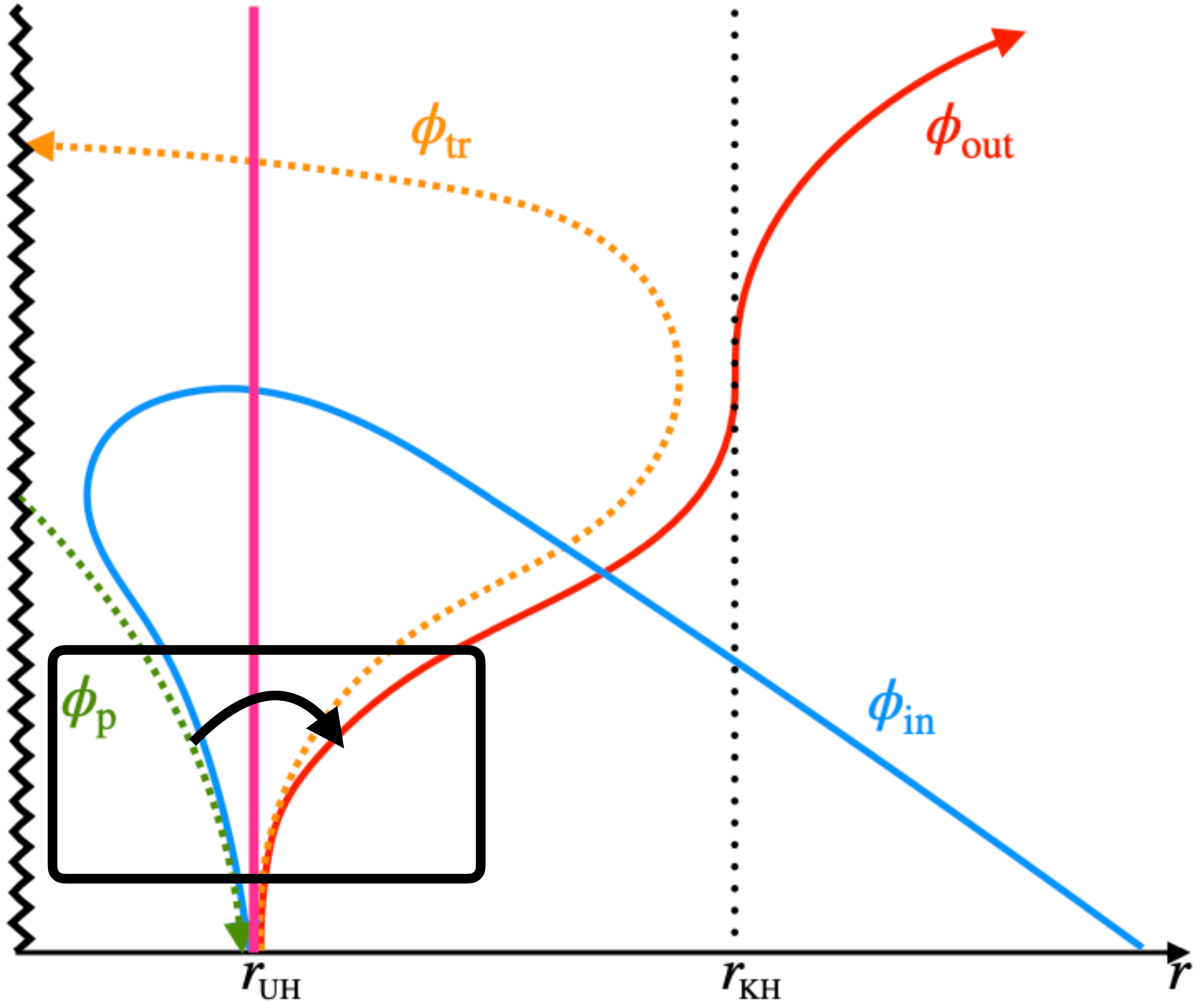

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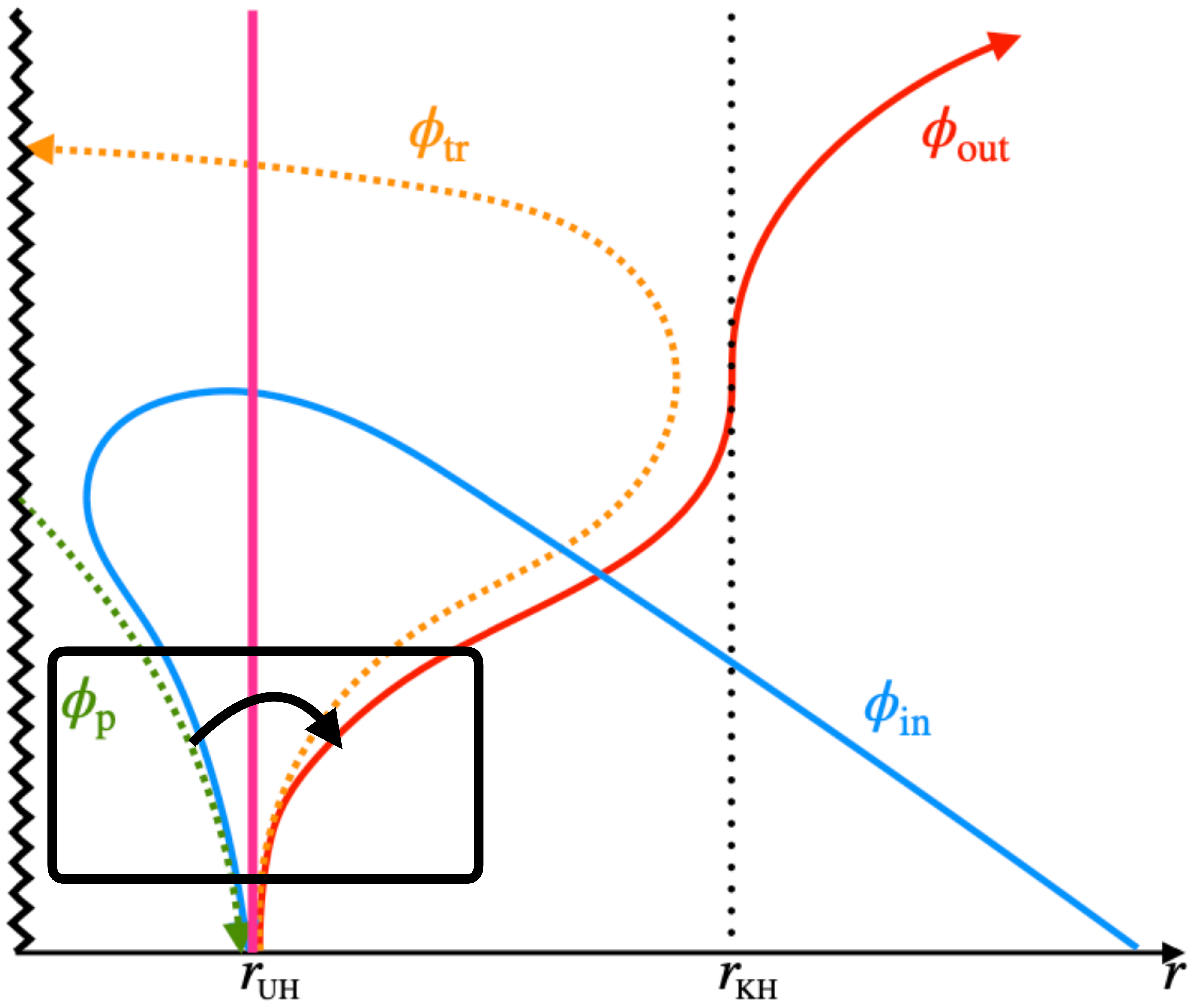
$$\Gamma = e^{-\frac{\Omega}{T_{\rm UH}}} \qquad T_{\rm UH} = \frac{\kappa_{\rm UH}}{\pi}$$

Thermal emission!



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Propagation of the outgoing ray

- The emission at the UH is insensitive to Λ. However we expect something to happen when $\Lambda \to \infty$

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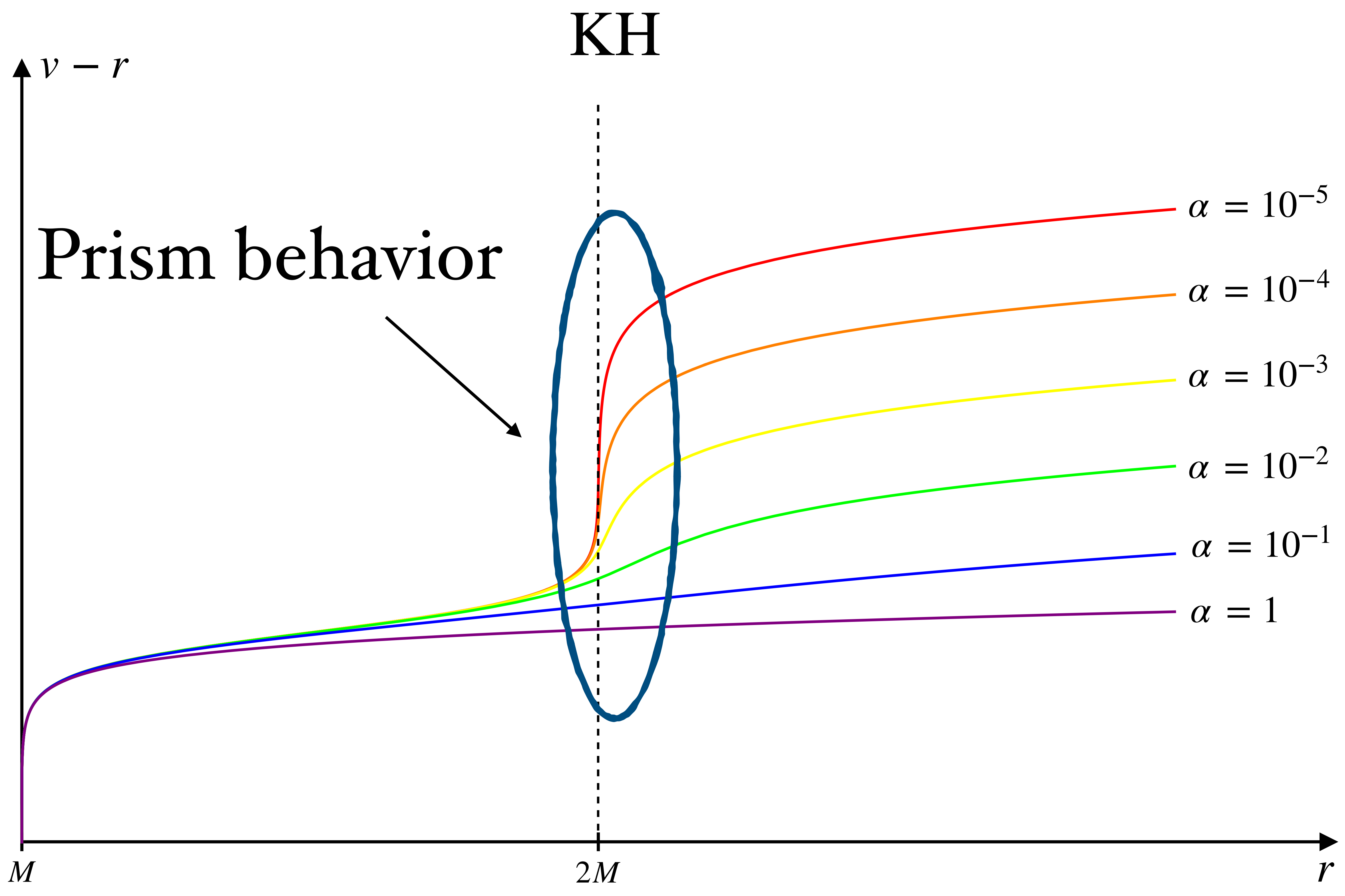
The rays for which <u>$\alpha = \Omega/\Lambda \ll 1$</u> linger at the KH for long time



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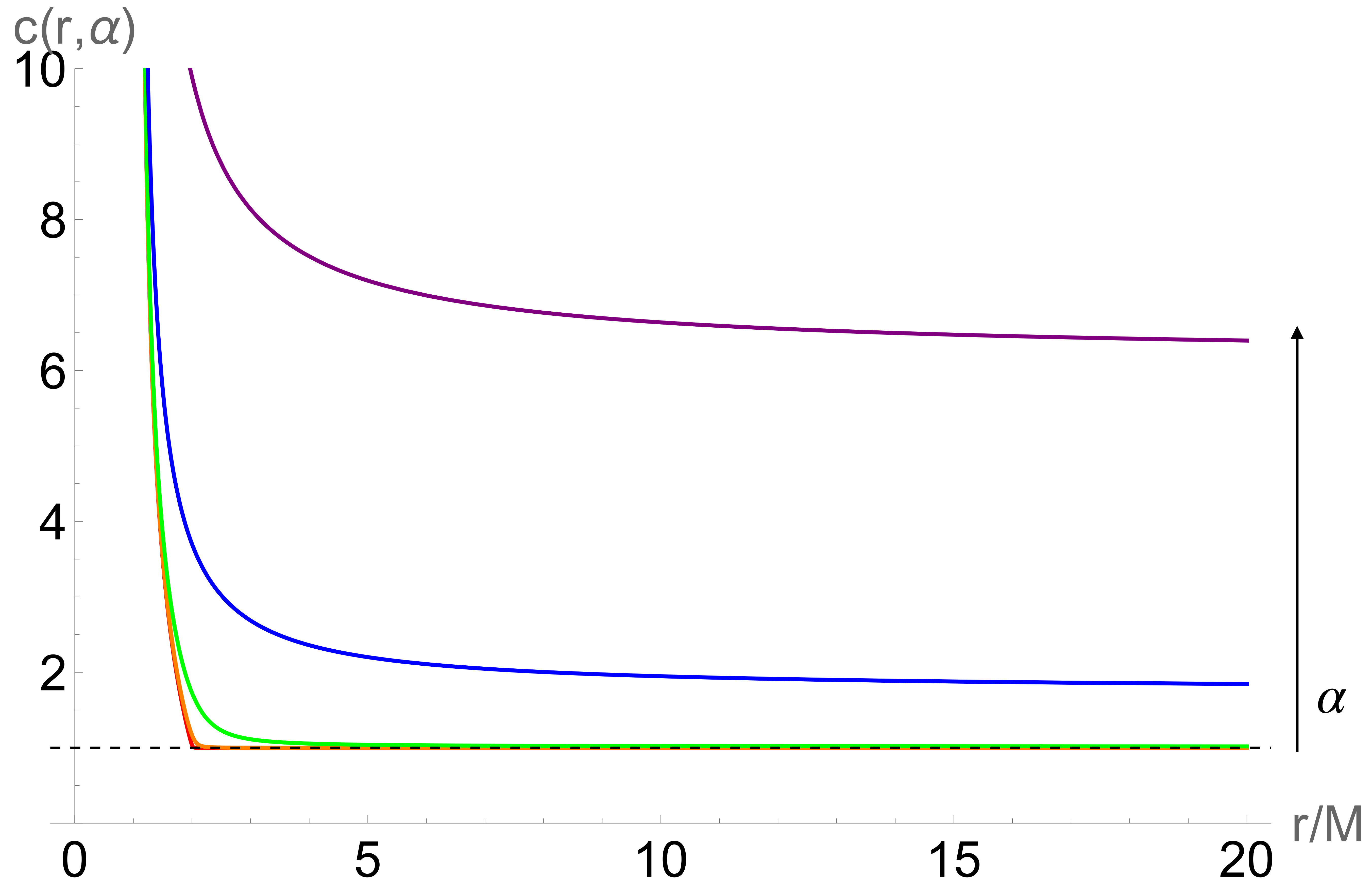
ArXiv: **2310.01472**

Propagation of the outgoing ray

- For very <u>low α</u> the red mode and the orange one assume "almost" the shape of a null trajectory

$$d\bar{u} = [c_g(r,\alpha)U_\mu + S_\mu]dx^\mu$$



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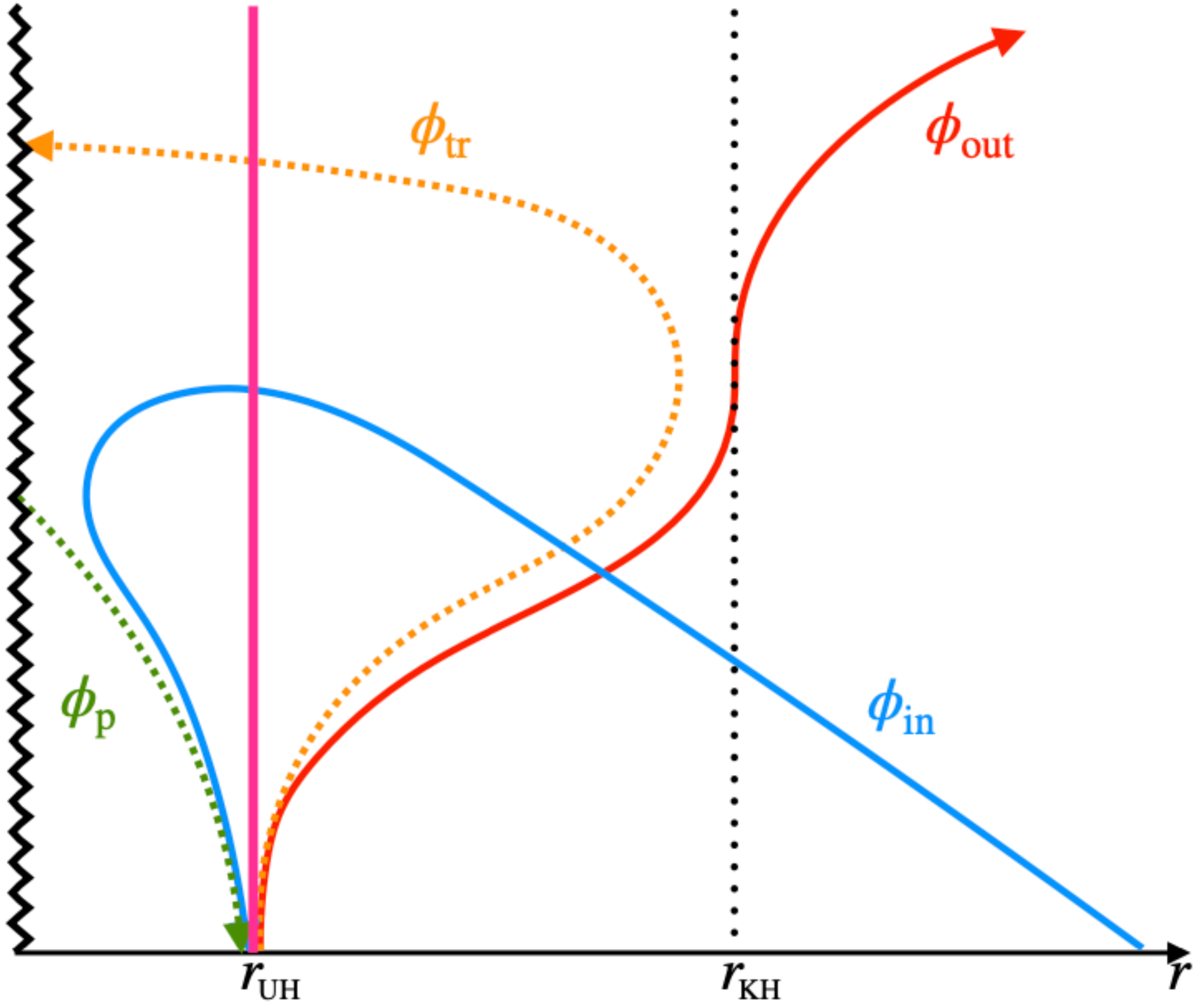
$$d\bar{u} = [c_g(r,\alpha)U_\mu + S_\mu]dx^\mu$$

- In our particle interpretation $\psi_\Omega = A\, e^{iS_p}$

$$S_p = -\int k_\mu dx^\mu \qquad k_\mu dx^\mu \propto d\bar{u}$$

$$k_\mu dx^\mu = \Omega\left[dv + \left(\frac{c(r,\alpha)U_r + S_r}{c(r,\alpha)U_v + S_v}\right)dr\right]$$



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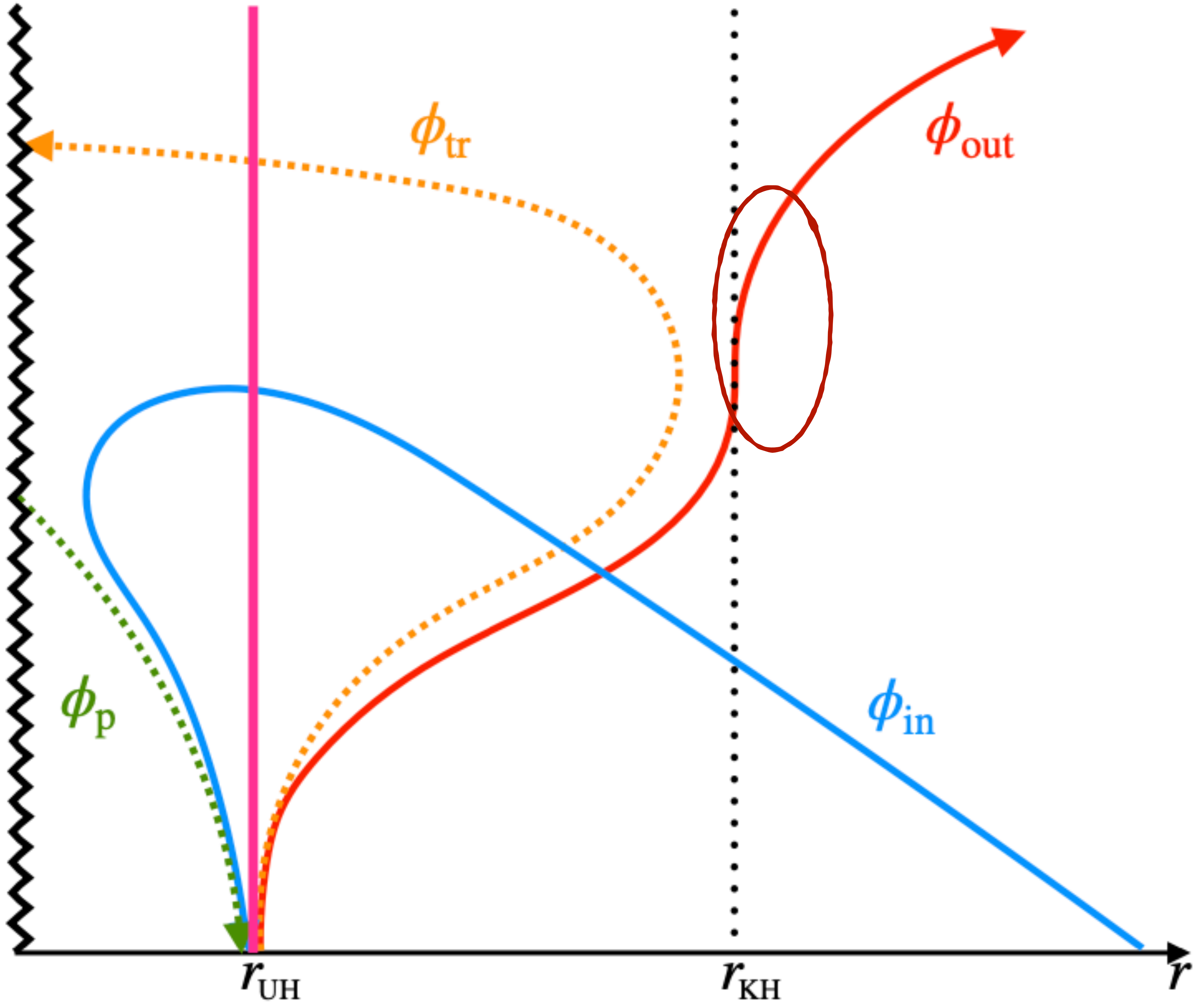
- Expanding for $\alpha \ll 1$

$$k_r = -\left[\frac{2r}{r-2M} - \frac{3r^4}{2(r-2M)^4}\alpha^2\right]\Omega$$



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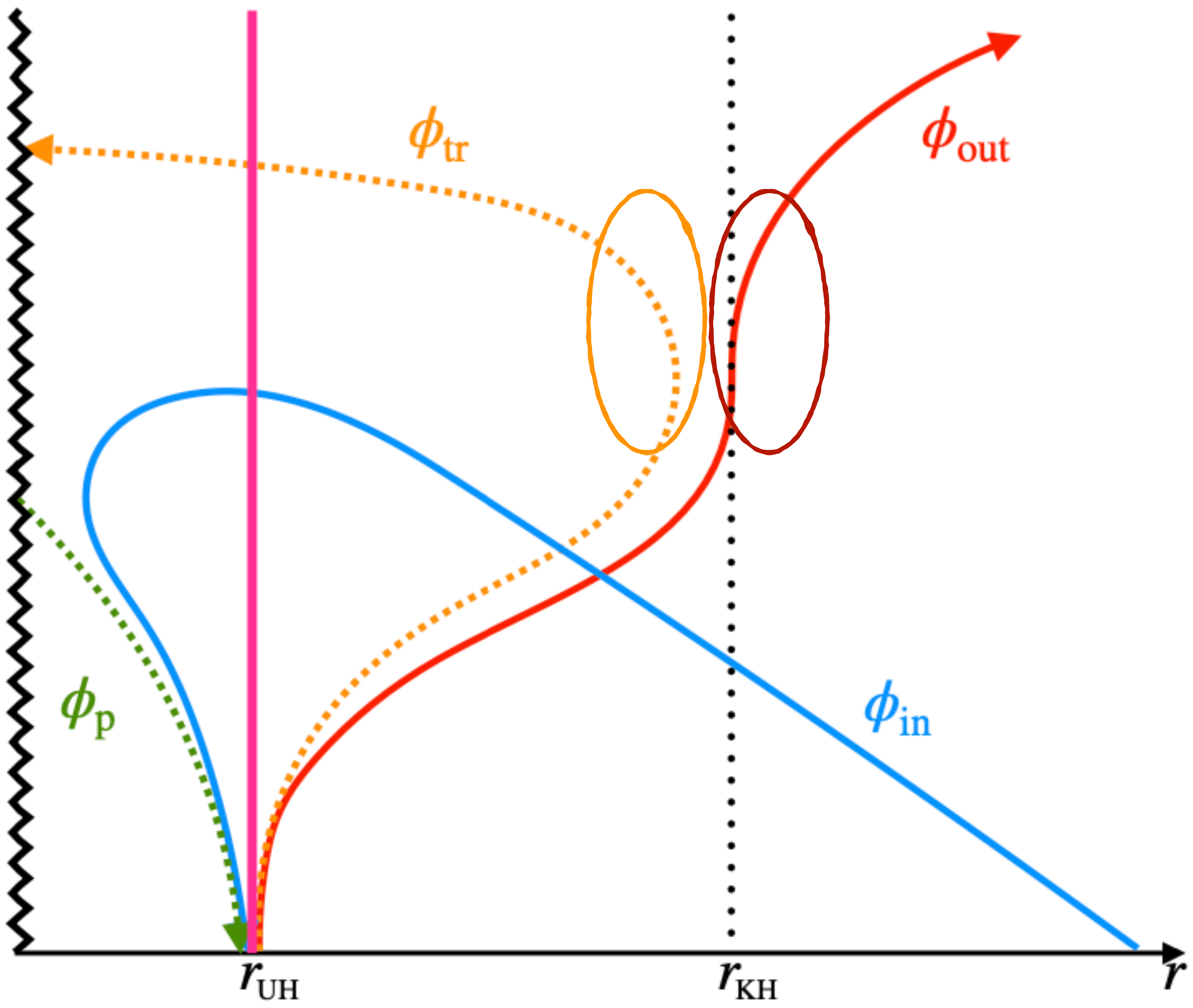
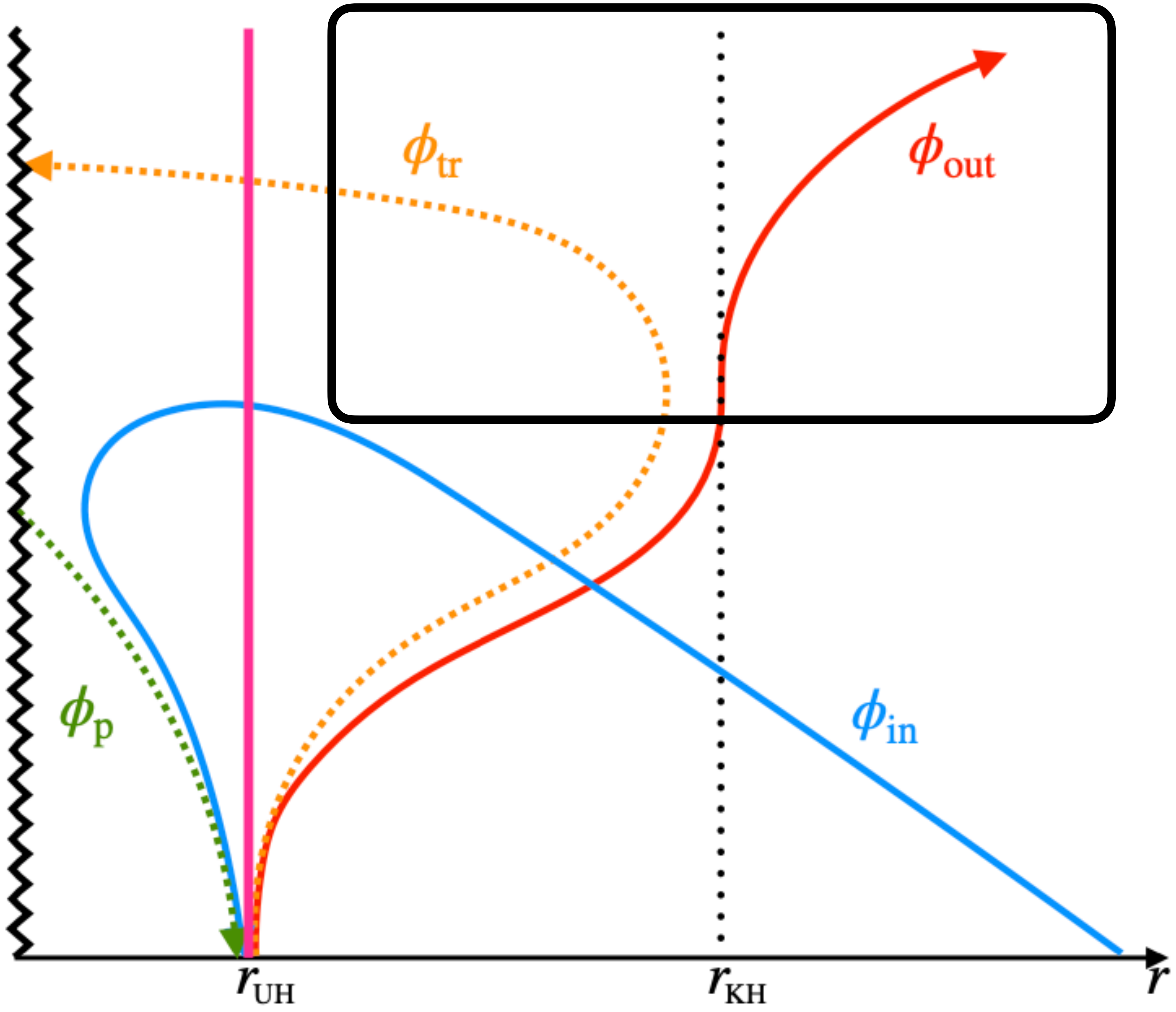
- The same shape also for the orange ray!



Propagation of the outgoing ray

- The existence of the red mode outside the KH can be interpreted as the tunnel-out of the orange mode



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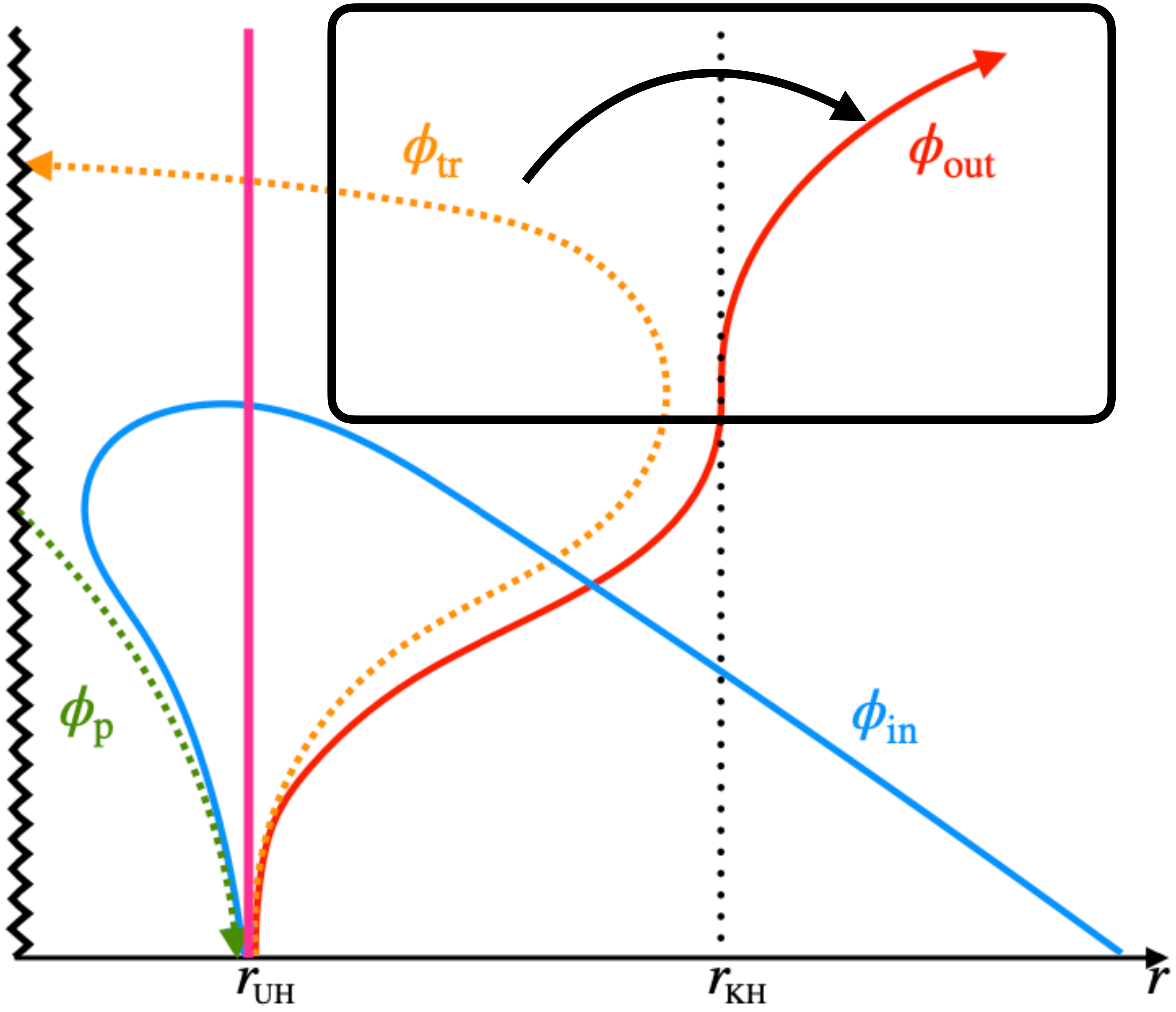
$$\Gamma = e^{-2Im\,S_p}$$

$$Im\,S_p = -\,Im\int_{r_1}^{r_2} k_r\,dr = \frac{\Omega\pi}{\kappa_{KH}}(1-3\alpha^2)$$

$$\Gamma = e^{-\frac{\Omega}{T_\alpha}} \qquad T_\alpha = \frac{T_H}{1-3\alpha^2}$$



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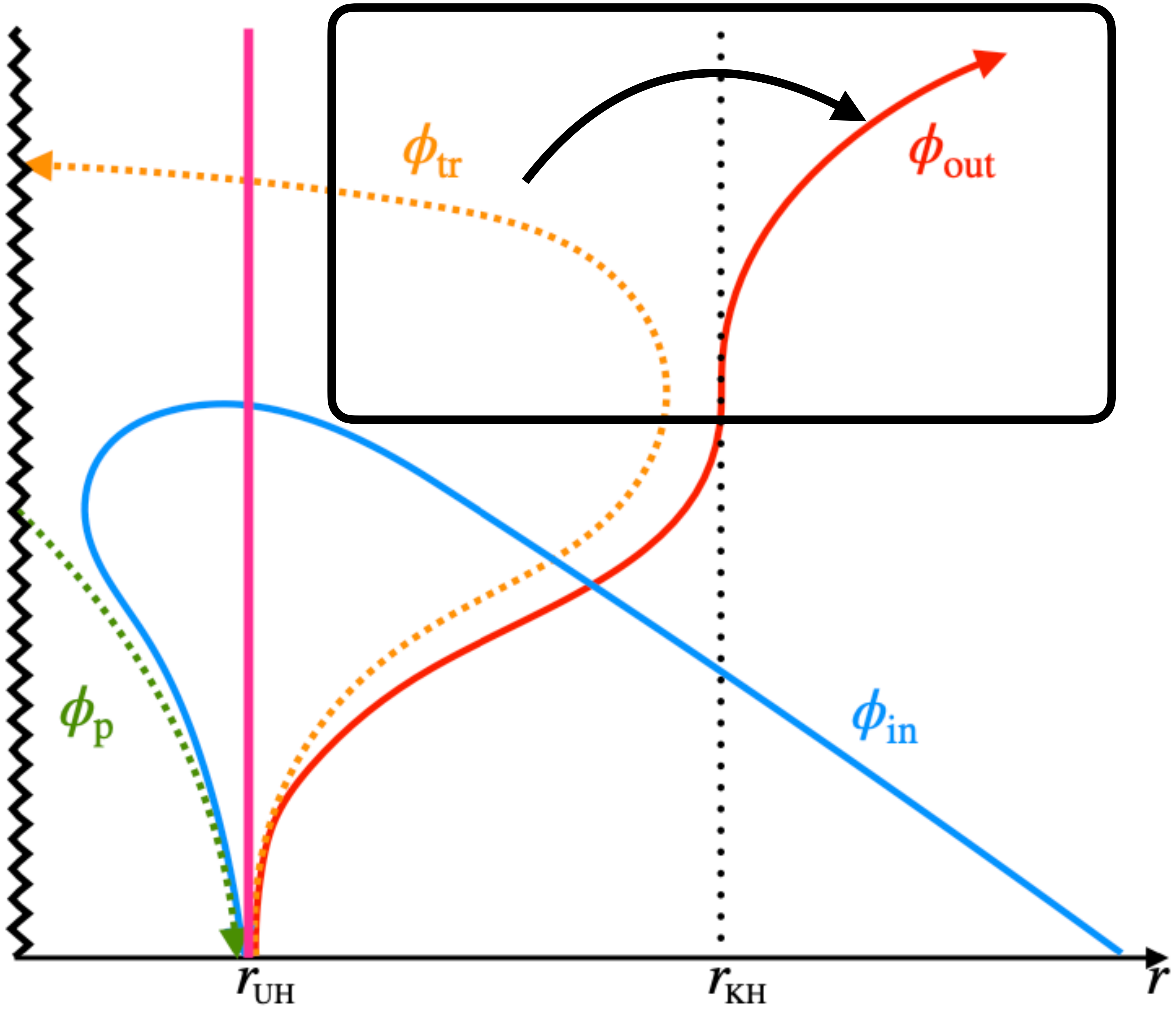
Deviation from thermality!!



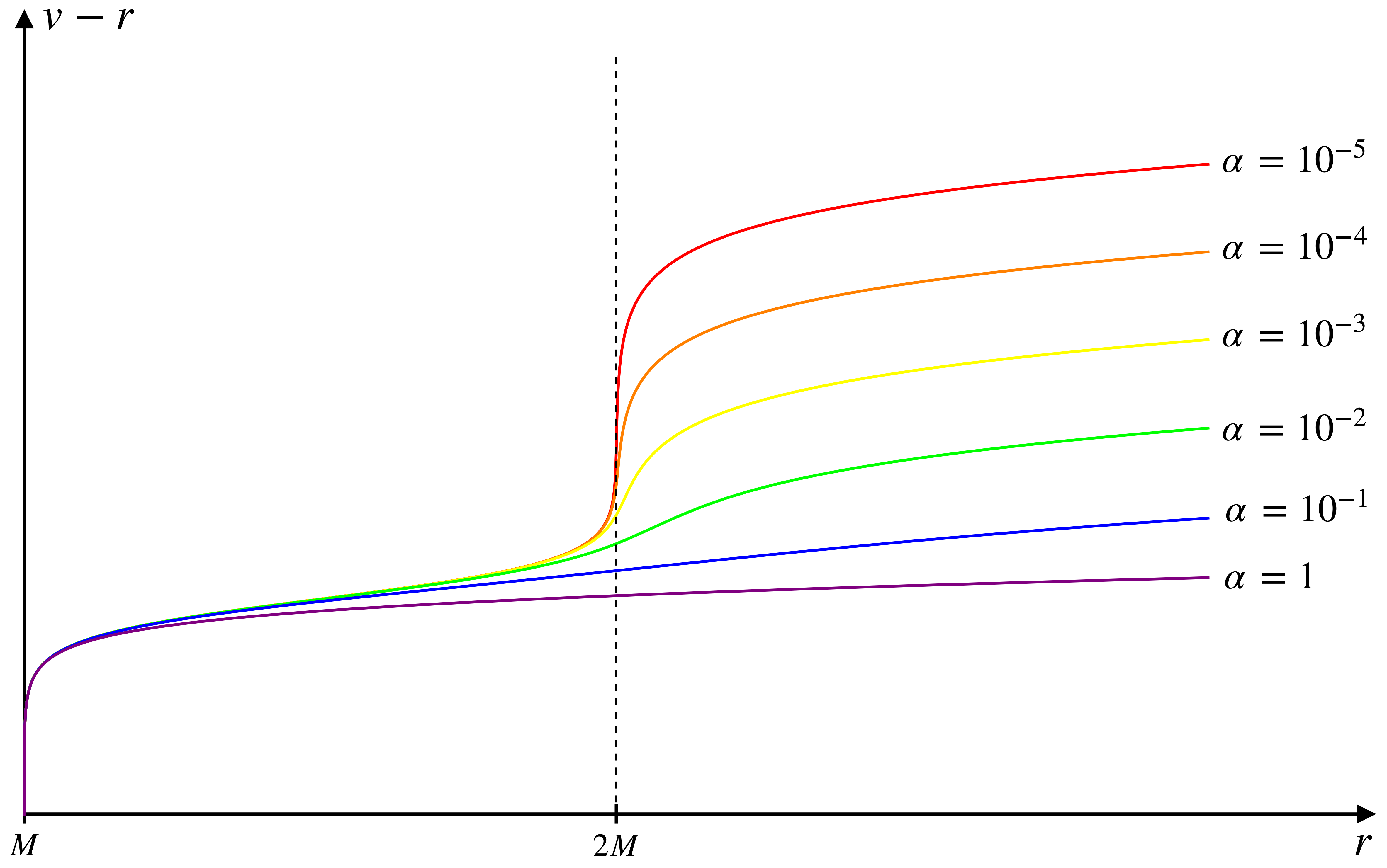
Propagation of the outgoing ray

- A necessary condition for lingering is

$$\frac{d^2 v(r)}{dr^2} = 0$$

- This way we get a bound

$$\alpha < \alpha_c \simeq 0.114$$



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Other approaches - I

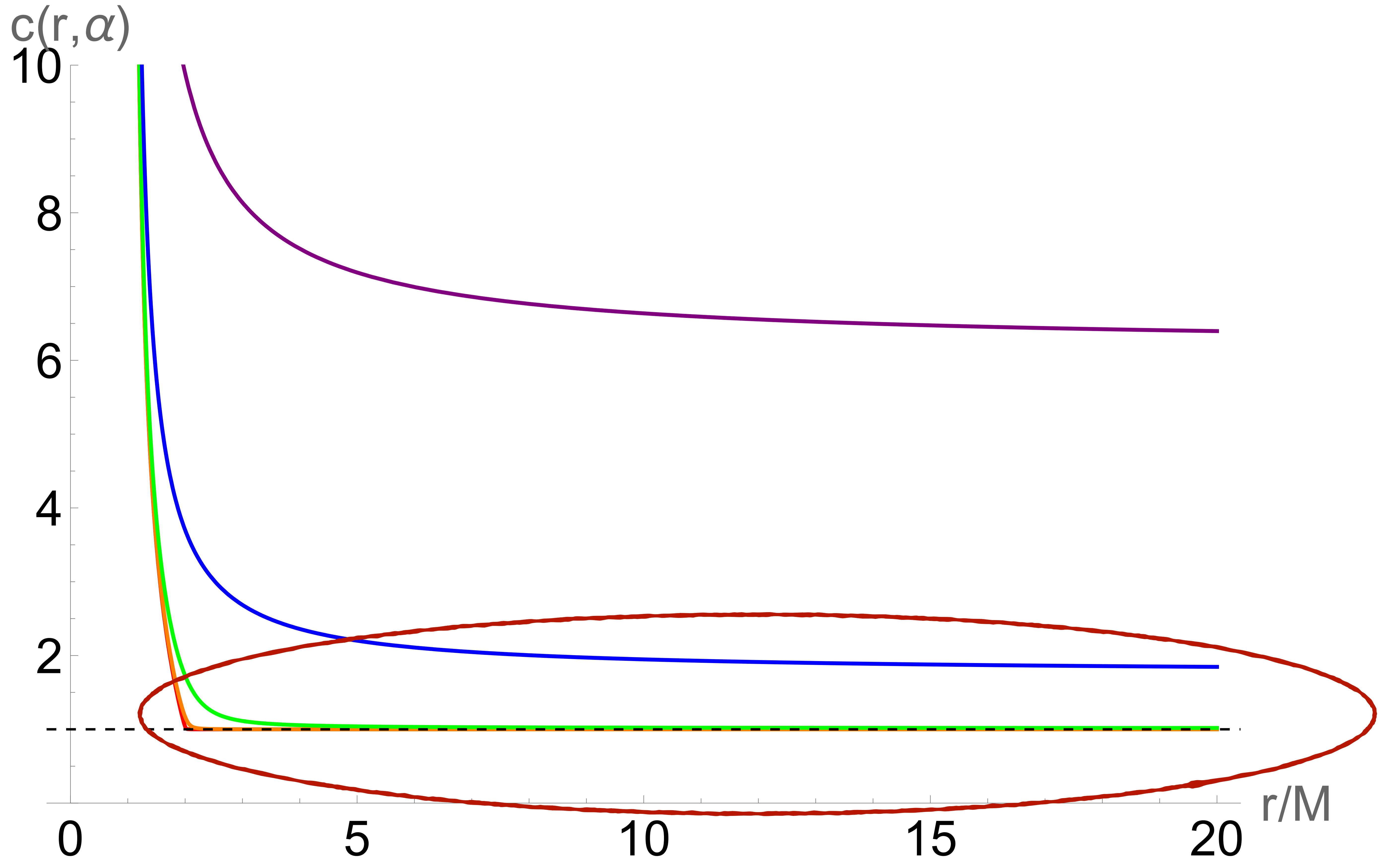
- After the KH, c_g is “almost” flat

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - (c_g^2 - 1)U_\mu U_\nu \qquad \bar{U}^\mu = \frac{1}{c_g}U^\mu$$

- The particle “seem” to come from a surface slightly inside the KH

$$r_h(\alpha) = (2 - 6\alpha^2)M + \cdots$$

$$\kappa_h(\alpha) = \frac{1}{4(1 - 3\alpha^2)M}$$

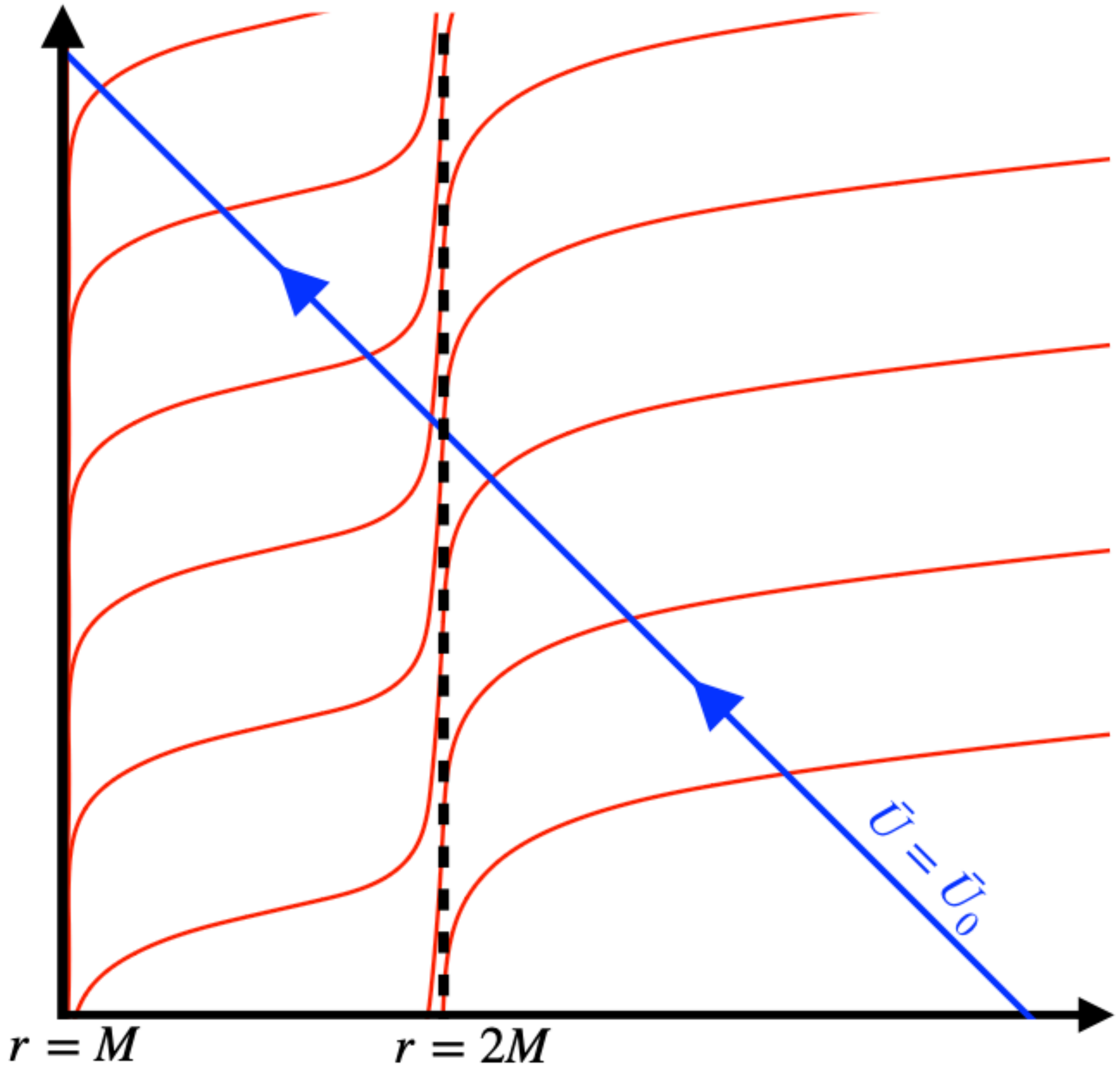


Other approaches - II

- The same result can be obtained by using the so-called “effective temperature function”, which captures the exponential peeling of the rays.

$$\kappa_{eff}(\bar{u}) = -\frac{\ddot{p}(\bar{u})}{\dot{p}(\bar{u})}$$

$$\left|\frac{\dot{\kappa}_{eff}(\bar{u})}{\kappa_{eff}(\bar{u})^2}\right| \ll 1$$



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$$\kappa_{eff}(\bar{u}) = -\frac{\ddot{p}(\bar{u})}{\dot{p}(\bar{u})}$$

$$\kappa_{eff}(\bar{u}) = \frac{1}{4M(1-3\alpha^2)} \frac{1 + 2W\left(\frac{6\alpha^2}{3\alpha^2-1} e^{-\frac{1}{4M(3\alpha^2-1)}\bar{u}}\right)}{\left[1 + W\left(\frac{6\alpha^2}{3\alpha^2-1} e^{-\frac{1}{4M(3\alpha^2-1)}\bar{u}}\right)\right]^2} \simeq \frac{1}{4M}(1+3\alpha^2) + \cdots$$

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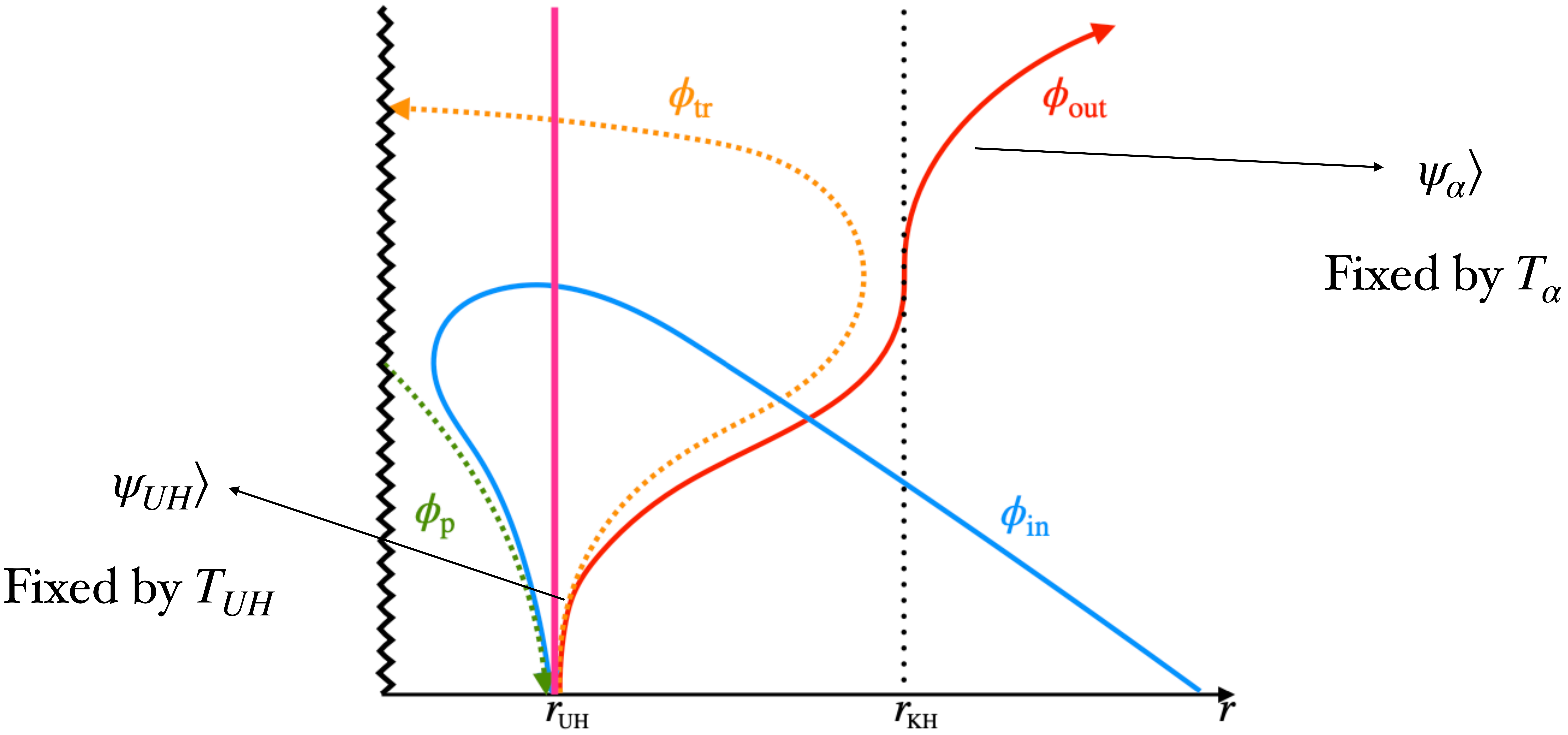
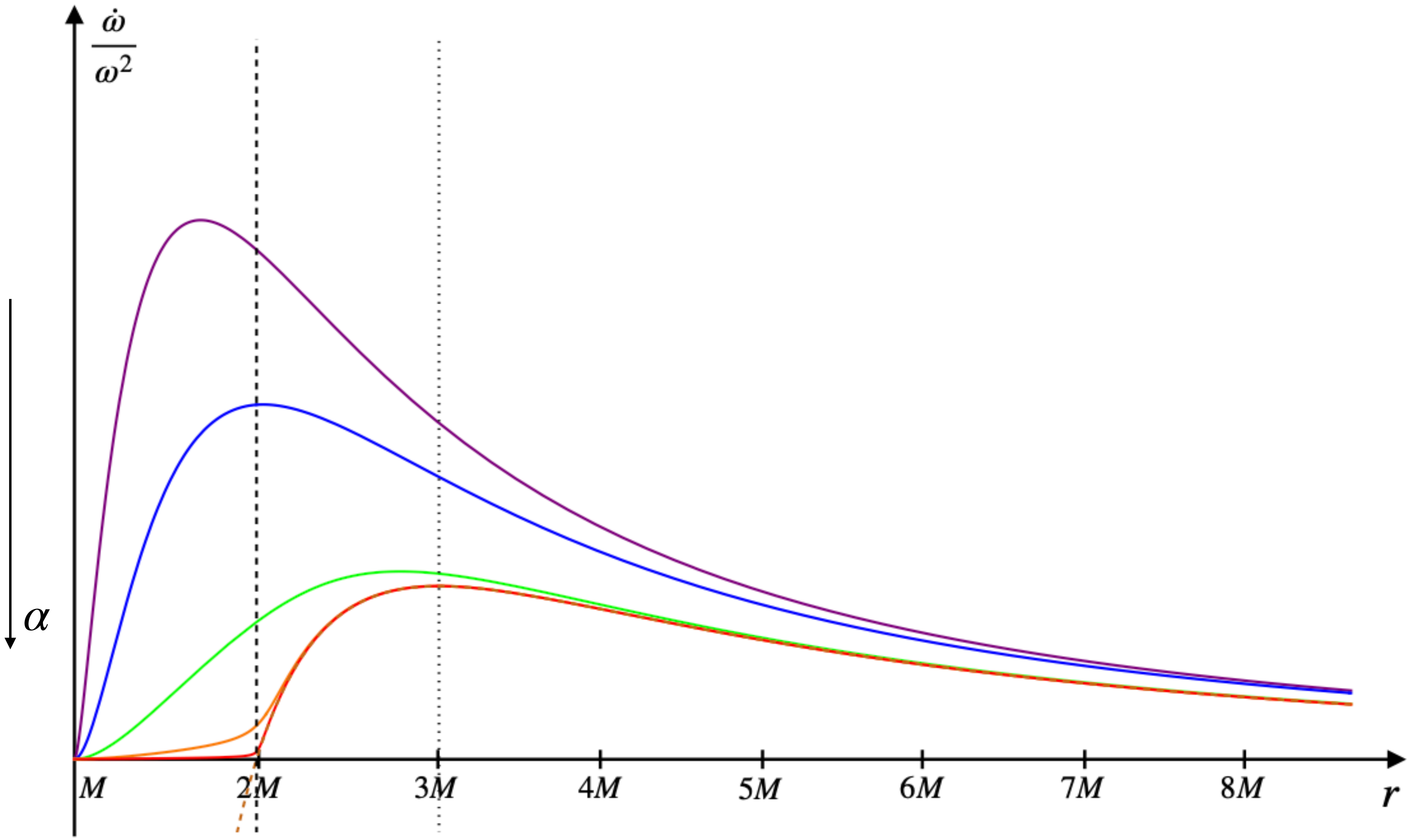
Quantum state

- We have two different shapes for ψ_Ω^{red}: do they describe the same global state?



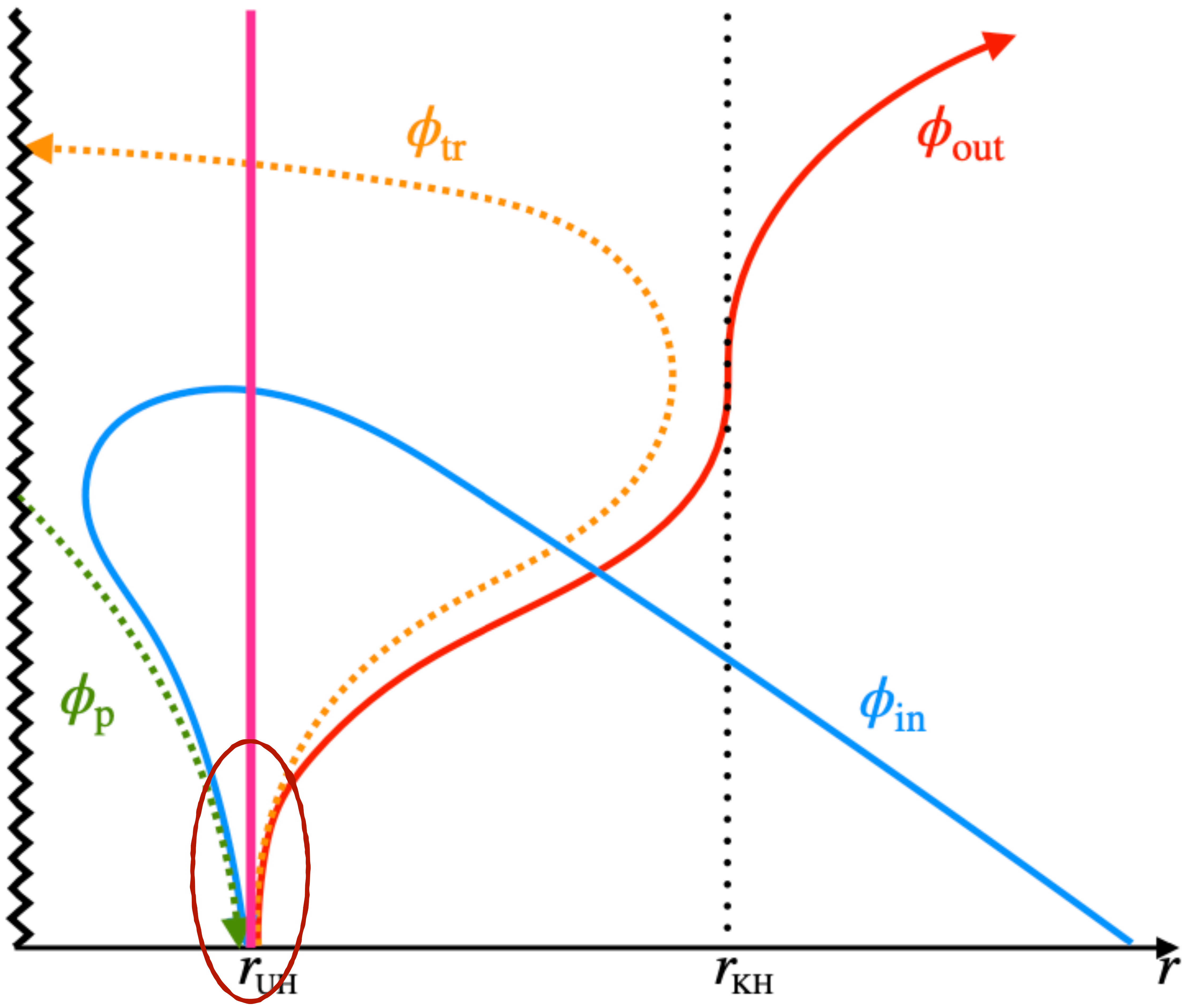
Adiabatic approximation

- If $\{\psi_\Omega^{red}\}$ are adiabatic in $[r_{UH}, r_{KH}]$, then we have compatibility between the two states



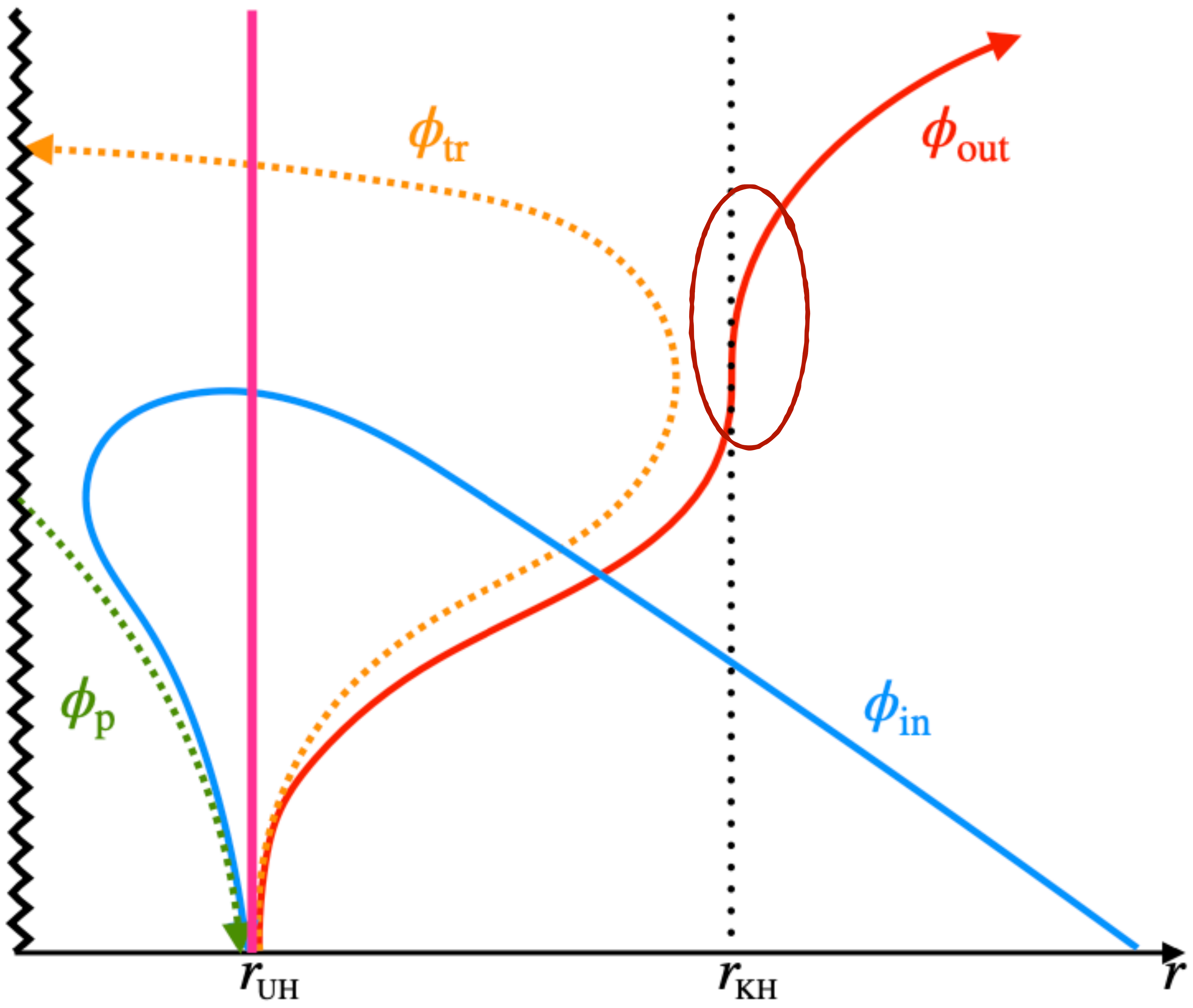
Quantum state

- So, the state is fixed by imposing the regularity at the Horizon (like in GR)



Quantum state

- So, the state is fixed by imposing the regularity at the Horizon (like in GR)
- Then, we can "evolve" the state adiabatically until the Killing Horizon, where the low-energy population is given by T_α



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- We have seen that BHs in LV Gravity enjoy radiative properties. In particular, the UH radiates with the temperature fixed by κ_{UH}

- Low-energy particles feel the KH while propagating outwards, reprocessing the modes with non-thermal deviation.

- These two features seem to describe the same global state, which defines a spectrum dominated by the UH at high energy and by the KH at low energies. This recovers HR in the limit $\Lambda \to \infty$.

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Thank you!