Semiclassical gravity and black hole mimickers

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Based on:

JA, C. Barceló, R. Carballo-Rubio, L. J. Garay arXiV:2310.12668

JA, S. Liberati, V. Vellucci (in preparation)

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Introduction

Black holes (BHs) are the most well-accepted candidates for the dark and compact objects observed

Observational fits, such as:

- Long-baseline interferometry (EHT)
- Gravitational-wave detection (LIGO, VIRGO, KAGRA)

Theoretical arguments

- Stability limits of neutron stars
- Absence of regular stars beyond certain compactness $C_R = \frac{2M}{R}$

Buchdahl's limit

[1] Buchdahl (1959)

Consider a spherically symmetric star described by

$$T^{\mu}_{\ \nu} = \textbf{diag}\left(-\rho, p_R, p_T, p_T\right)$$

Buchdahl's theorem establishes that, if:

1. The interior metric matches the Schwarzschild solution at the surface
2. The energy density is monotonously decreasing: $\rho \geq 0, \quad d\rho/dr \leq 0$
3. Pressures are not highly anisotropic: $p_T \leq p_R$

Regular stars have a maximum compactness limit [1]

Buchdahl's limit

In the particular case of isotropic stars

$$T^{\mu}_{\ \nu} = \mathbf{diag}\left(-\rho, p, p, p\right)$$

With the equation of state

$$\rho(r) \equiv \rho = \mathrm{const}\,.$$

The hypotheses behind the theorem are saturated and we obtain the bound

$$\boxed{C_{\mathrm{R}} = 8/9}$$

In general relativity, pressure gravitates and, beyond certain threshold, it contributes towards collapse instead of preventing it.

Buchdahl’s limit

The physical vacuum of quantum fields in a stellar spacetime is not the GR vacuum, but the Boulware vacuum

This quantum vacuum is “less empty”, acting as an additional source of stress-energy

A star approaching the Buchdahl limit will polarize the Boulware vacuum appreciably

Do we find a compactness limit when the vacuum backreacts?

Semiclassical gravity

Theory that takes into account the backreaction of zero-point energies of quantum fields on a classical spacetime

$$G^{\mu}_{\nu} = 8\pi\left(T^{\mu}_{\nu} + \hbar\langle\hat{T}^{\mu}_{\nu}\rangle\right) \rightarrow$$

We treat this theory as a modified gravity

- The RSET is a function of the metric, field modes, and their derivatives
- It encodes both vacuum polarization and particle creation effects
- Both SETs couple through the spacetime geometry only

Semiclassical gravity

[2] Anderson, Hiscock, Samuel (1995)

We want to solve the backreaction problem in spherical symmetry

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2 \quad \text{with} \quad h(r) = [1 - C(r)]^{-1} = \left[1 - \frac{2m(r)}{r}\right]^{-1}$$

Anderson, Hiscock and Samuel [2] obtained this RSET via point-splitting regularisation

$$\langle \hat{T}^{\mu}_{\nu} \rangle^{\text{ren}} = \langle \hat{T}^{\mu}_{\nu} \rangle^{\text{AHS}} + \langle \hat{T}^{\mu}_{\nu} \rangle^{\text{num}}$$

Two covariantly conserved tensors:

$\langle \hat{T}^{\mu}_{\nu} \rangle^{\text{AHS}}$ → Analytical. Depends on metric and higher-order derivatives. Contains a renormalization ambiguity

$\langle \hat{T}^{\mu}_{\nu} \rangle^{\text{num}}$ → Numerical. Depends on integrals over the frequency and multipole number of field modes

Semiclassical gravity

The AHS-RSET is a well-motivated approximation since it captures the properties of the Boulware state

$$G^{\mu}_{\ \nu} = 8\pi \left(T^{\mu}_{\ \nu} + \hbar \langle \hat{T}^{\mu}_{\ \nu} \rangle^{\mathrm{AHS}} \right)$$

A fixed-background (no backreaction) computation of the AHS-RSET is illustrative

Consider the metric of the constant-density star:

$$ds^2 = -\frac{1}{4}\left(3\sqrt{1 - C_R} - \sqrt{1 - r^2 C_R / R^2}\right)^2 dt^2 + \left(1 - r^2 C_R / R^2\right)^{-1} dr^2 + r^2 d\Omega^2$$

Semiclassical gravity

Now, take a star with $C_R = \frac{8}{9} - \epsilon, \quad \epsilon \to 0^+$

Evaluating the AHS-RSET at $r = 0$

$$\langle \hat{\rho} \rangle = - \langle \hat{T}^t_{\ t} \rangle^{\mathrm{AHS}} \Big|_{r=0} \propto \frac{l_{\mathrm{P}}^2}{R^4} \left(\xi - \frac{1}{6} \right)^2 \frac{\log \epsilon}{\epsilon^2} + \mathcal{O}\left(\epsilon^{-2}\right)$$

$$\langle \hat{T}^r_{\ r} \rangle^{\mathrm{AHS}} \Big|_{r=0} = \langle \hat{T}^\theta_{\ \theta} \rangle^{\mathrm{AHS}} \Big|_{r=0} = \frac{1}{3} \langle \hat{T}^t_{\ t} \rangle^{\mathrm{AHS}} \Big|_{r=0}$$

$\langle \hat{\rho} \rangle \simeq - \rho$ for $\epsilon \simeq \mathcal{O}\left[\left(\frac{l_{\mathrm{P}}}{R} \right) \log \left(\frac{R}{l_{\mathrm{P}}} \right) \right]$

Potential violation of the Buchdahl theorem by vacuum polarization effects!

Reduction of order

[3] Simon, Parker (1993)

The higher-derivative nature of the AHS-RSET introduces problems

- We require additional initial conditions to specify a solution

- Some solutions might be spurious or unphysical

Following Simon and Parker [3] we apply a reduction of order to the AHS-RSET

Reduction of order is only valid when semiclassical effects are small. We use it outside its regime of validity following a modified gravity logic

Reduction of order

(tt) equation:

$$\frac{h(1-h)-rh'}{h^2r^2} = -8\pi\rho + \mathcal{O}(\hbar)$$

(rr) equation:

$$\frac{rf'+f-fh}{fhr^2} = 8\pi p + \mathcal{O}(\hbar)$$

Solving for h' and f', and differentiating, we obtain the relations

$$h^{(n)} = \mathscr{H}_n\left(h, f, \rho, p\right) \qquad f^{(n)} = \mathscr{F}_n\left(h, f, \rho, p\right)$$

and replace them in the AHS-RSET and obtain the angular components through conservation

$$\nabla_\mu \langle \hat{T}^\mu{}_r \rangle = \partial_r \langle \hat{T}^r{}_r \rangle + \frac{2}{r}\left(\langle \hat{T}^r{}_r \rangle - \langle \hat{T}^\theta{}_\theta \rangle\right) + \frac{f'}{2f}\left(\langle \hat{T}^r{}_r \rangle - \langle \hat{T}^t{}_t \rangle\right) = 0$$

Reduction of order

Semiclassical stellar equations:

$$\frac{h(1-h)-rh'}{h^2r^2} = -8\pi\rho + \hbar\langle \hat{T}^t{}_t\rangle^{\rm OR}\big(h,f,\rho,p\big)$$

$$\frac{rf'+f-fh}{fhr^2} = 8\pi p + \hbar\langle \hat{T}^r{}_r\rangle^{\rm OR}\big(h,f,\rho,p\big)$$

$$p' + \frac{f'}{2f}\big(\rho+p\big) = 0$$

$$\rho = \textbf{const.}$$

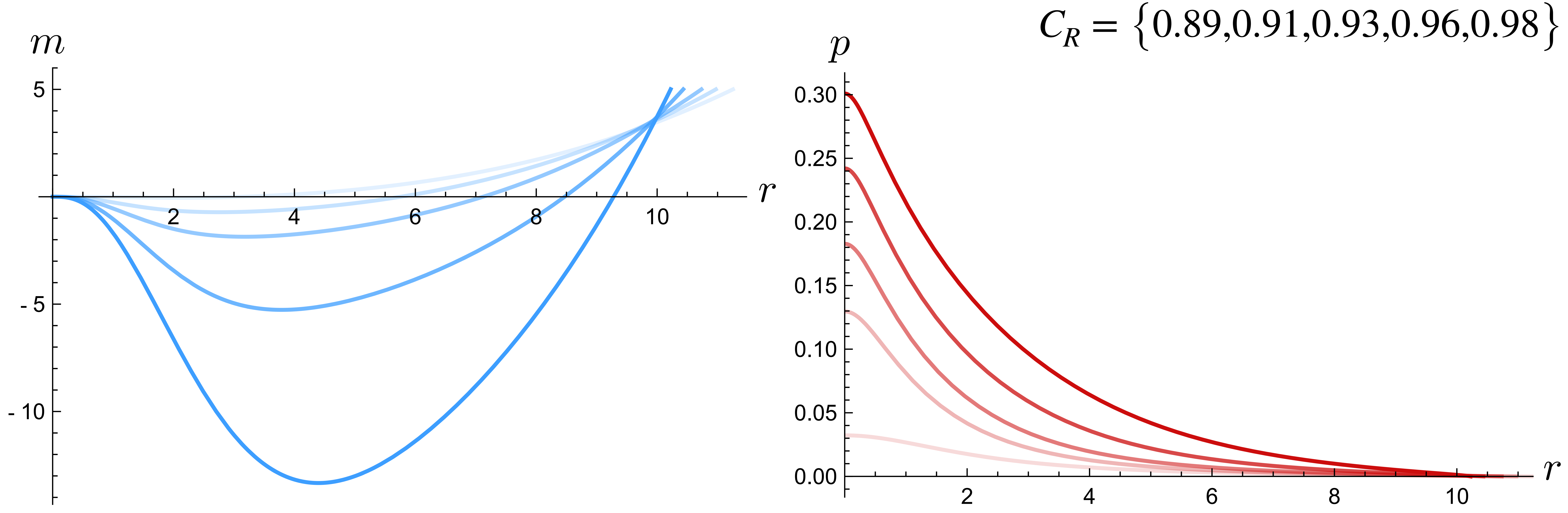
1. Integrate the vacuum equations from an asymptotically flat region inwards

2. Integrate the stellar equations as a boundary-value problem varying C_R and ρ

3. Search for regular solutions

Stellar solutions: Properties

We find regular stellar solutions that surpass the Buchdahl limit

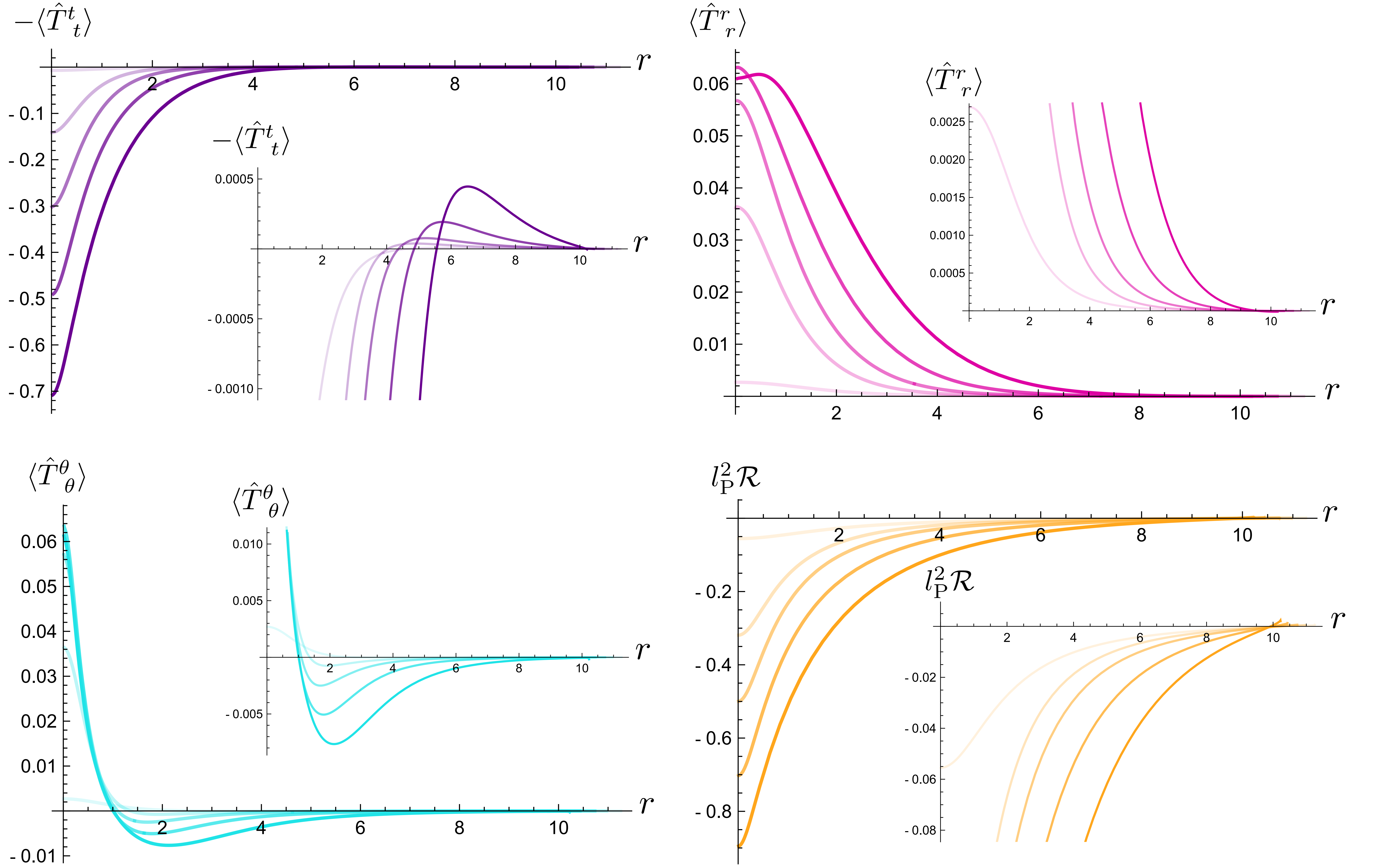


Misner-Sharp mass and classical pressure of semiclassical stars surpassing the Buchdahl limit

As C_R is increased, a negative mass interior emerges

Stellar solutions: Properties

$C_R = \{0.89, 0.91, 0.93, 0.96, 0.98\}$

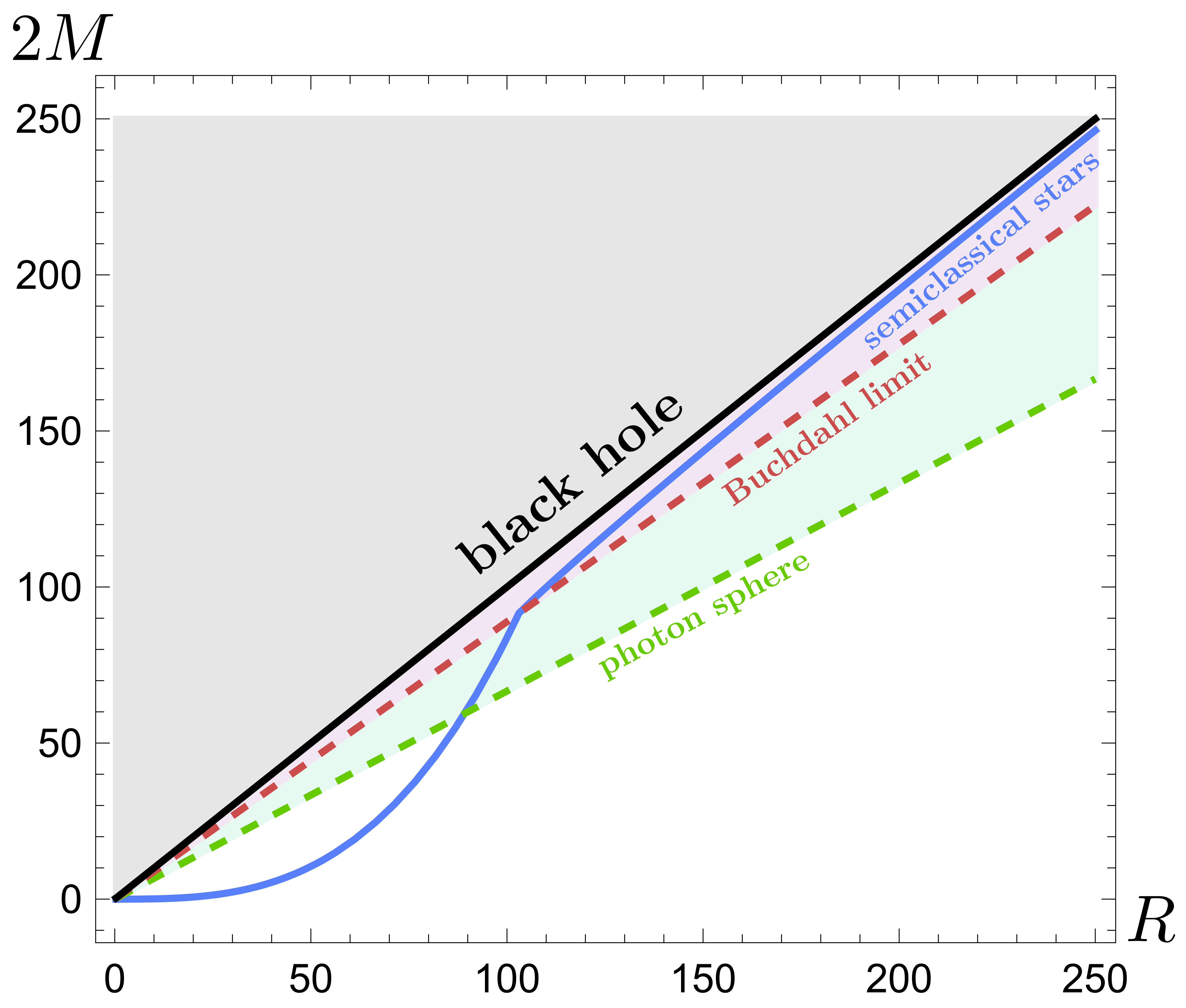


Order-Reduced RSET components

The RSET has $\langle \hat{\rho} \rangle < 0$ and positive pressures at $r = 0$

Ricci scalar is negative and finite

Stellar solutions: Properties

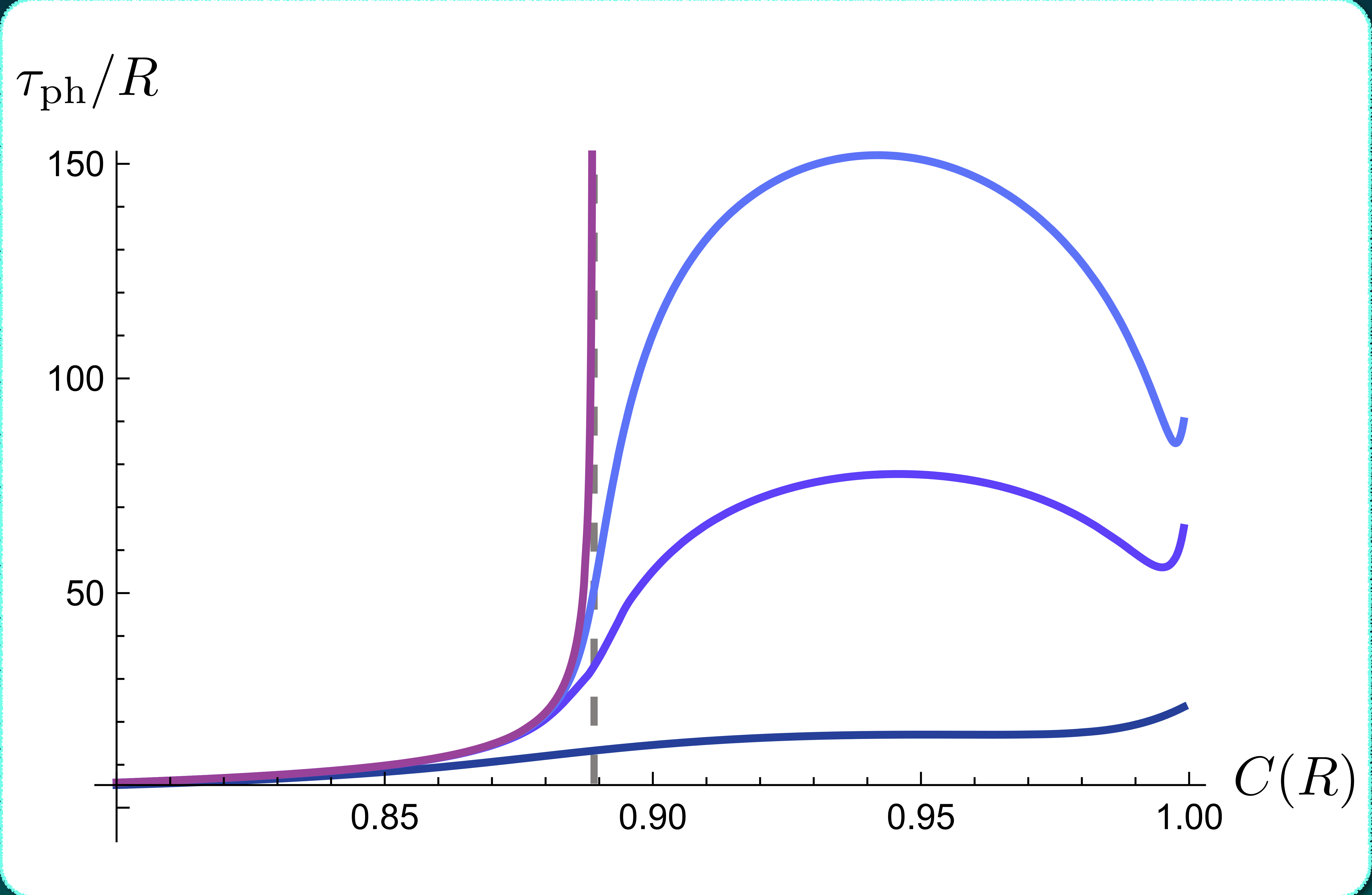


Mass-to-Radius diagram of semiclassical stars

Three regimes:

- Sub-Buchdahl: perturbatively corrected constant-density stars
- Buchdahl: negative energies build up near the center, supporting the structure
- Super-Buchdahl: stars with negative mass interiors that can approach the BH compactness

Stellar solutions: Properties



Comparison of the crossing time of classical and semiclassical solutions

The crossing time for null rays

$$\tau_{ph} = 2\int_0^{r_{ph}} \left(h/f\right)^{1/2} dr$$

stays finite across the Buchdahl threshold

With a large separation of scales:

$$\tau_R/R \propto \left(M/l_{\rm P}\right)$$

Stellar solutions: Properties

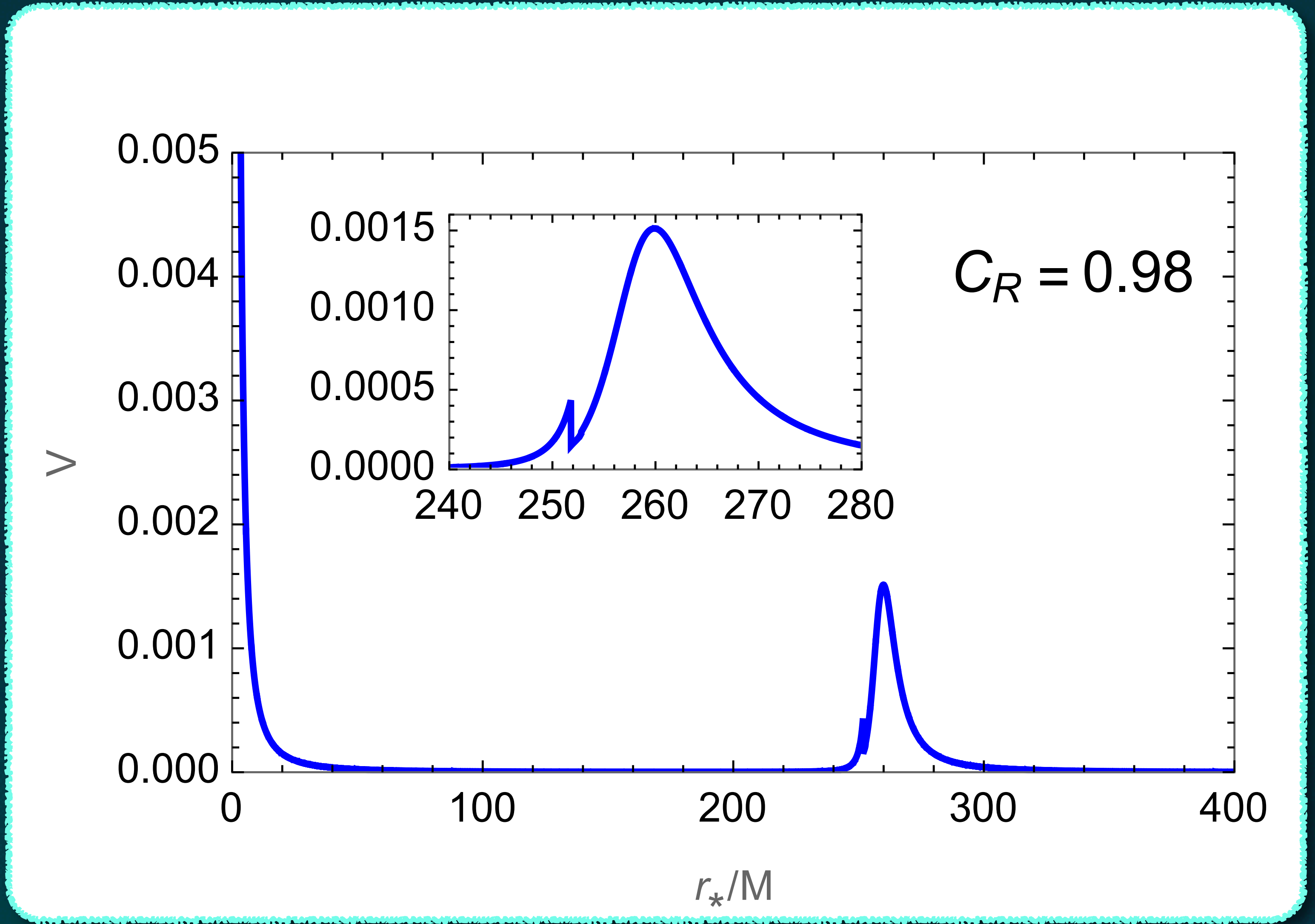
[4] Medved, Martin, Visser (2004)

[5] Chandrasekhar, Detweiler (1975)

Consider the axial perturbations of an $s = 2$ test field [4]

$$\frac{d^2\psi_s}{dr_*^2} + \left(\omega^2 - V_s\right)\psi_s = 0$$

$$V_s = \frac{\left[l\left(l+1\right) - 2\right]f}{r^2} + \frac{2f}{r^2 h} - \frac{hf' - fh'}{2rh^2}$$



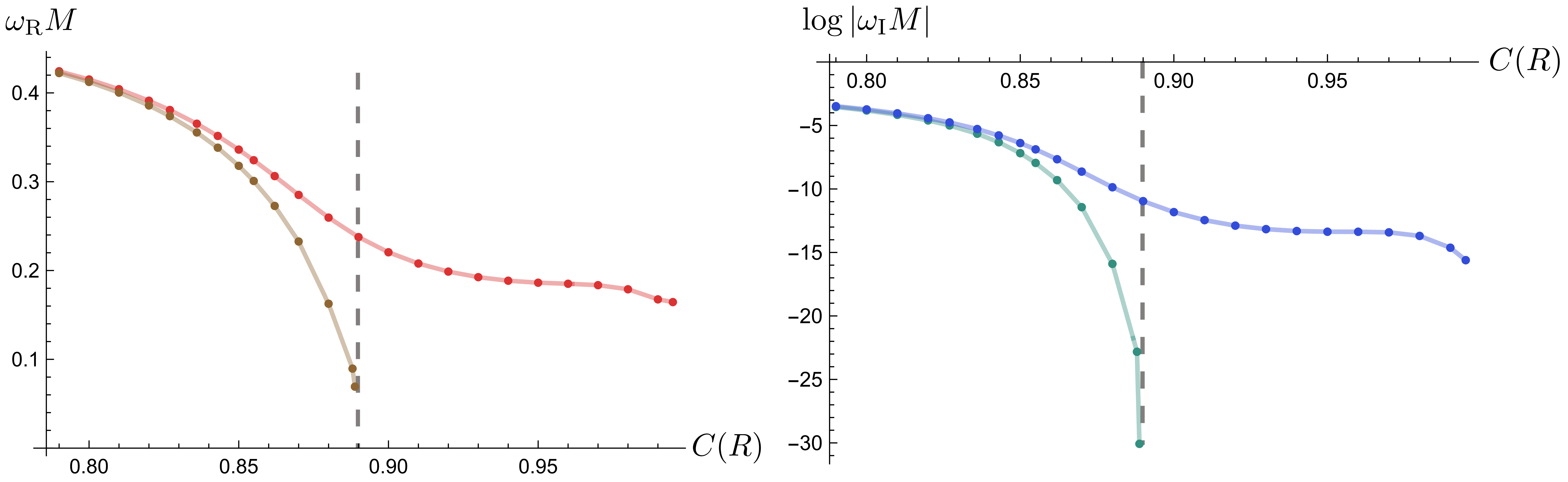
Quasinormal modes correspond to solutions satisfying the boundary conditions

$$\psi_s \simeq e^{i\omega r_*}, \quad r_* \to \infty$$

$$\psi_s \simeq r^l, \quad r \to 0$$

We obtain the frequencies $\omega = \omega_R + i\omega_I$ of the fundamental $l = 2$ mode through the direct integration method [5]

Stellar solutions: Properties



Comparison of QNM frequencies: classical and semiclassical solutions

Beyond the Buchdahl limit, the fundamental QNM frequencies remain (almost) constant.

The imaginary part is small, indicating the presence of long-lived modes.

Numerical evolution of test fields displays echoes when compactness is large

Stellar solutions: Properties

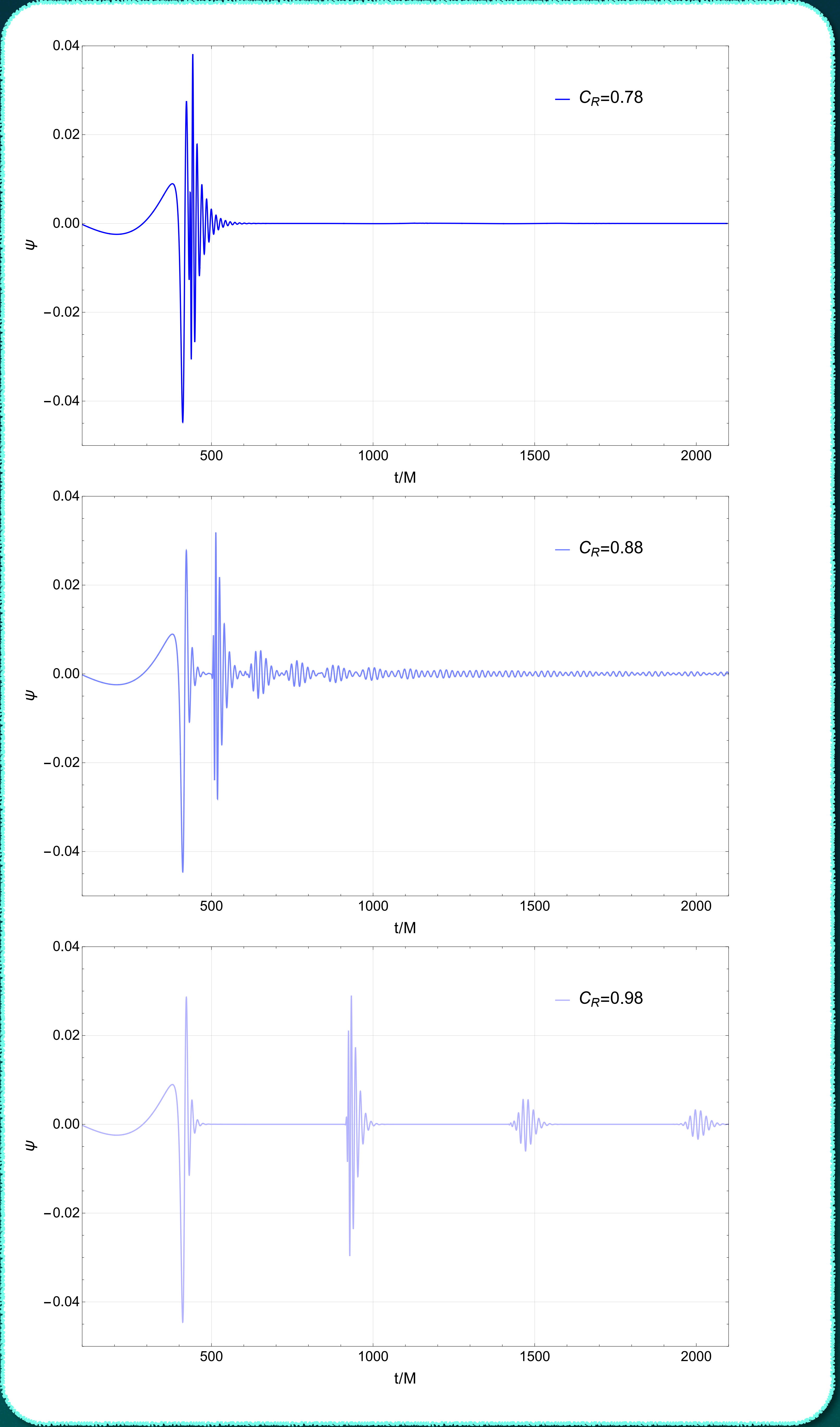
Now we perform a time domain analysis for $l = 2$ modes

$$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial r_*^2} + V\psi = 0$$

Taking as initial condition for the field a Gaussian pulse

$$\psi(r,0) = \psi_0 \exp\left(-\frac{\left(r_* - r_*^c\right)^2}{8M^2}\right), \quad \frac{\partial \psi(r,0)}{\partial t} = 0$$

Echoes are produced as the star goes ultracompact, but large scale separation will produce a huge time delay

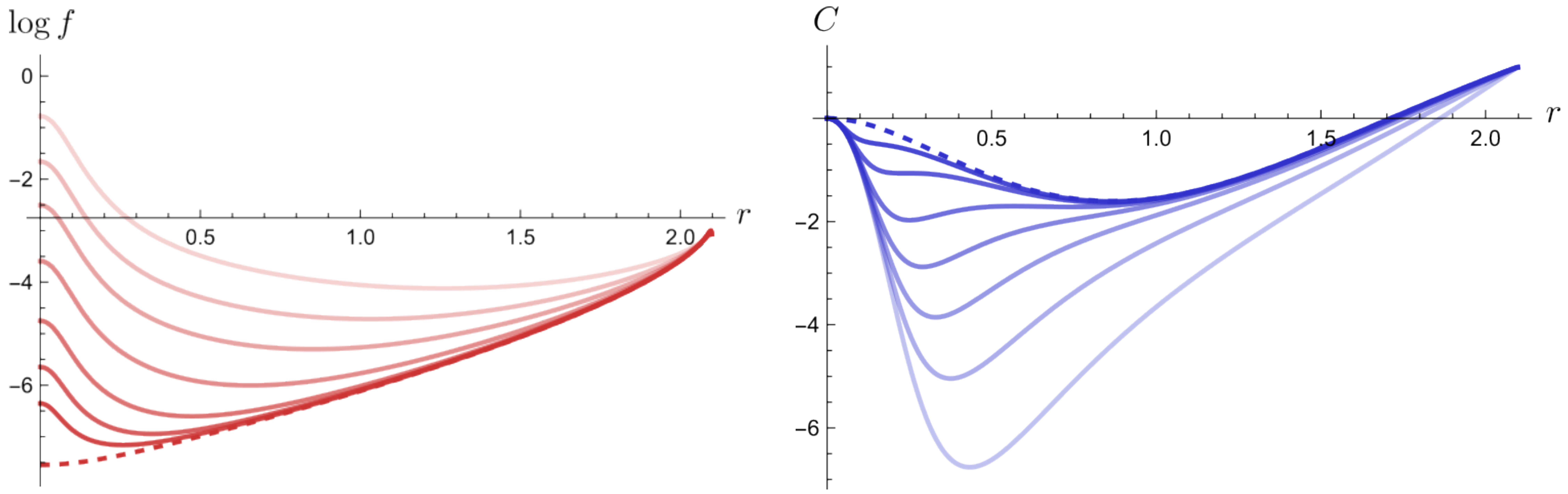


Stellar solutions: Properties

Is there no hope of semiclassical stars producing observable echoes?

The space of solutions allows for super-critical stars with shorter crossing times.

Their exterior geometry is identical

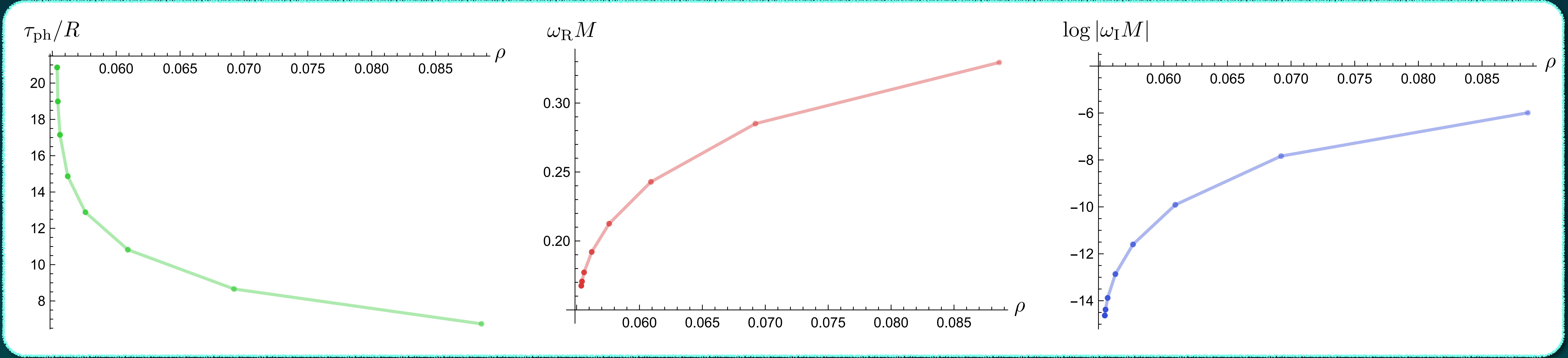


Metrics of super-critical semiclassical stars with $C(R) = 0.99$ and $M = 1$. These are obtained by increasing ρ

Stellar solutions: Properties

QNM frequencies increase as we go super-critical

Changes in the crossing time have a clear impact on QNMs



Crossing time and quasinormal mode frequencies of super-critical semiclassical stars with $C(R) = 0.99$

We observe analogous changes in the echo waveforms in time domain

Future directions

Backreaction from vacuum polarization allows to surpass the Buchahl limit

The semiclassical star model serves as a well-motivated alternative to BHs

Exact RSETs

Numerical computation of the RSET in stellar interiors

Other RSET approximations (Polyakov) predict semiclassical stars

Stability

Deriving effective EoSs that reproduce the physics of semiclassical stars

Formation mechanisms

Role of the inner horizon during gravitational collapse?

Lifetime of trapped regions?

Axisymmetry?

In the future…