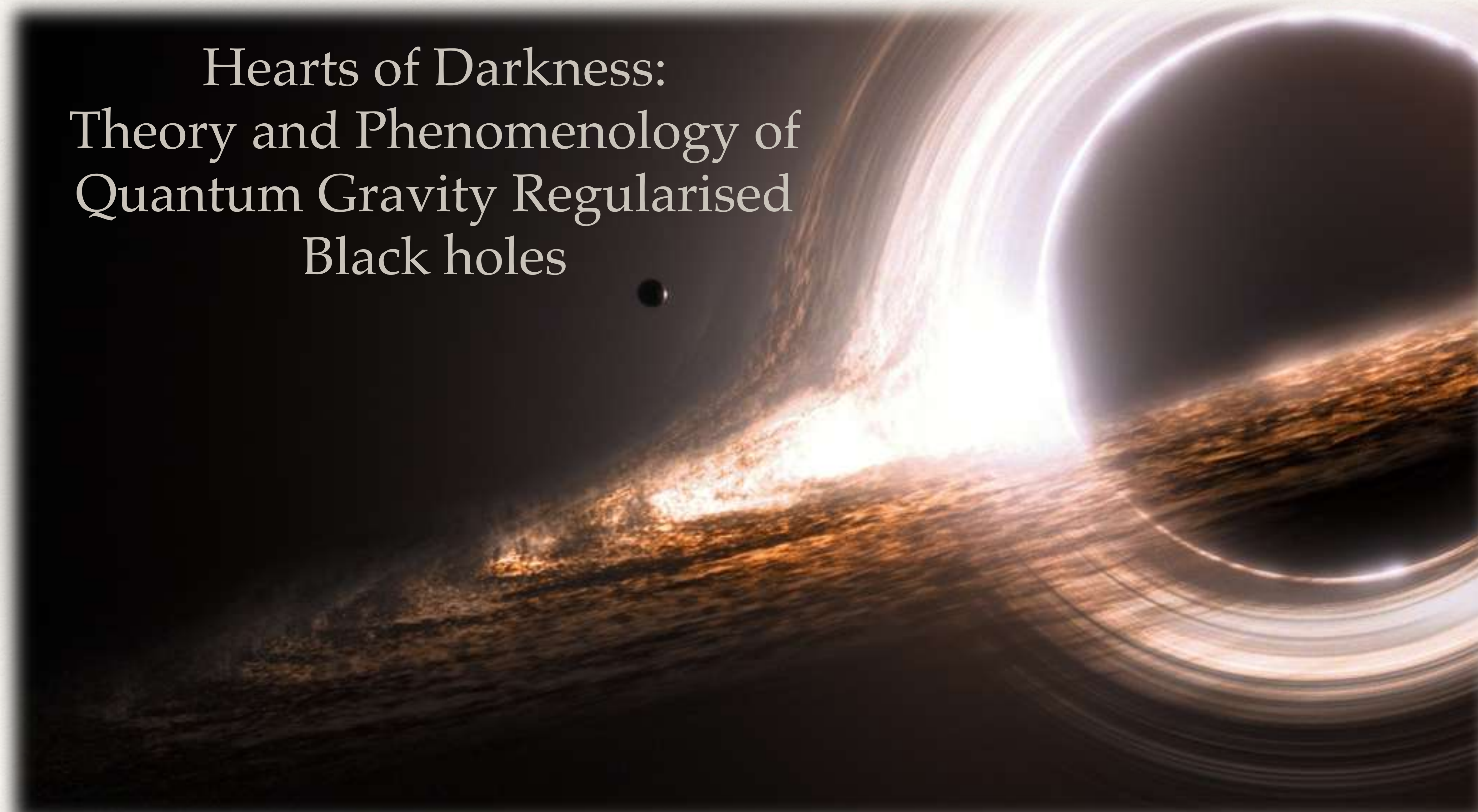
QUAGRAP Meeting, Naples 2024

Stefano Liberati

Hearts of Darkness: Theory and Phenomenology of Quantum Gravity Regularised Black holes



Work in collaboration with: R. Carballo-Rubio, F. Di Filippo, E. Franzin , J. Mazza, C. Pacilio, V. Vellucci, M. Visser

It from the cracks that light gets in... **Anthem-Leonard Cohen**

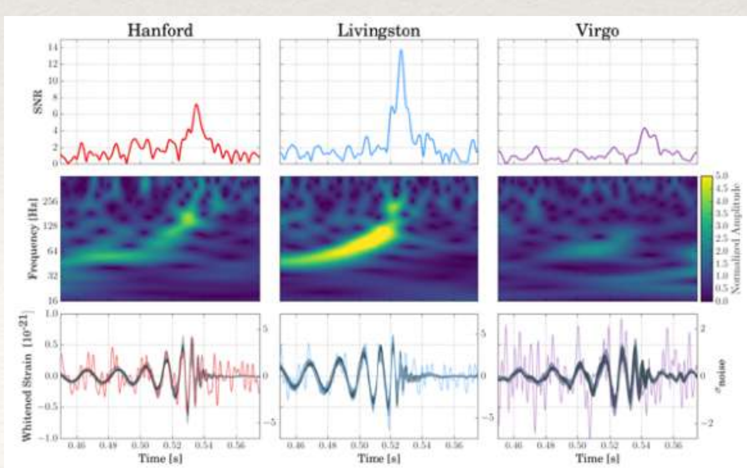
A new dawn for testing General Relativity

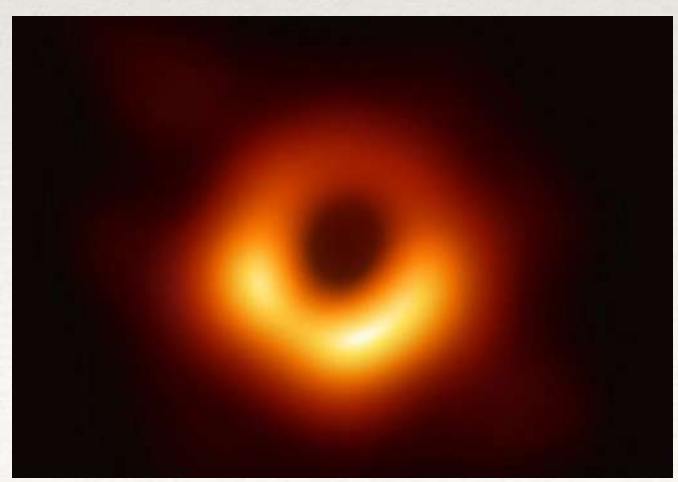
Albeit we "use" GR everyday (e.g. GPS) still it has some tantalising features and it has resisted so far any attempt to be quantised…

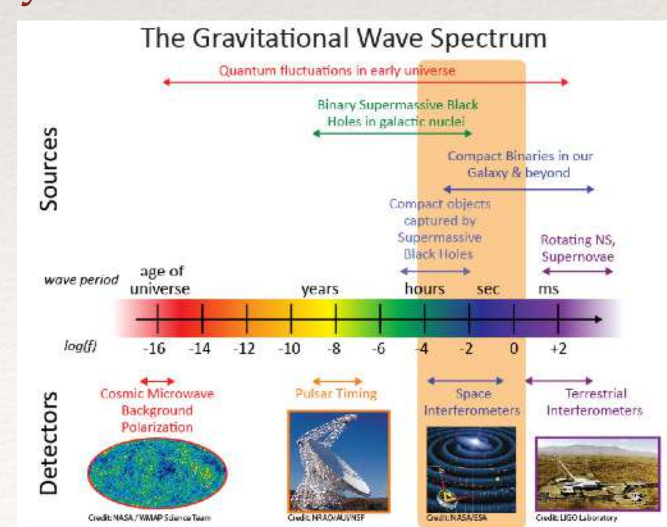
- Singularities
- Critical phenomena in gravitational collapse
- Horizon thermodynamics
- Spacetime thermodynamics: Einstein equations as equations of state.
- The cosmological constant problem
- Faster than light and Time travel solutions
- AdS/CFT duality, holographic behaviour
- Information Problem in BH Physics

There are a ubiquitous objects that are associated to most of these odd GR features:
Black Holes
Understanding them "in nature" would be key to test our understanding of gravity.
Unfortunately so fare very sparse knowledge was allowed by observations…

No more



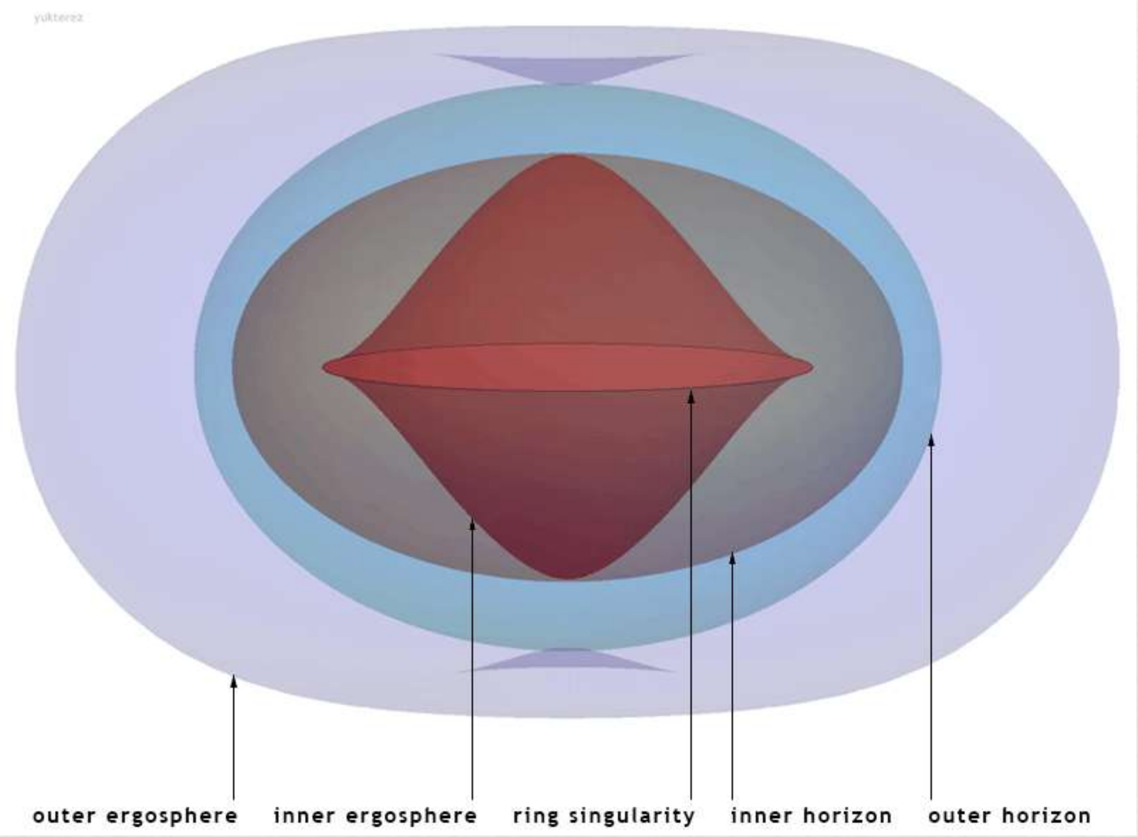




Black Holes:
the Rosetta stone of gravity

"The **black holes** of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."
Subrahmanyan Chandrasekhar

- Albeit we are nowadays familiar with the concept of Black Holes their acceptance as a physical solution of General relativity has been far from obvious.
- Even once was understood the nature of the event horizon, BH are still characterised by "hard to digest" structures
  - Singularities: infinite curvature
  - Cauchy horizons (associated to timelike singularities and time machines): end of predictability



QG is supposed to "cure" these features:
If it does so just in a hidden QG core of Planck scale then BH will be exactly as in GR.
But what if the "cure" requires long range (in time and/or space) effects?
Then maybe we could test QG using BH… could we?

Singularity

- A singularity is where General relativity is no more predictive: we cannot describe spacetime there —> missing points.
- Penrose's theorem is what makes very confident that singularities must form inside black holes generically

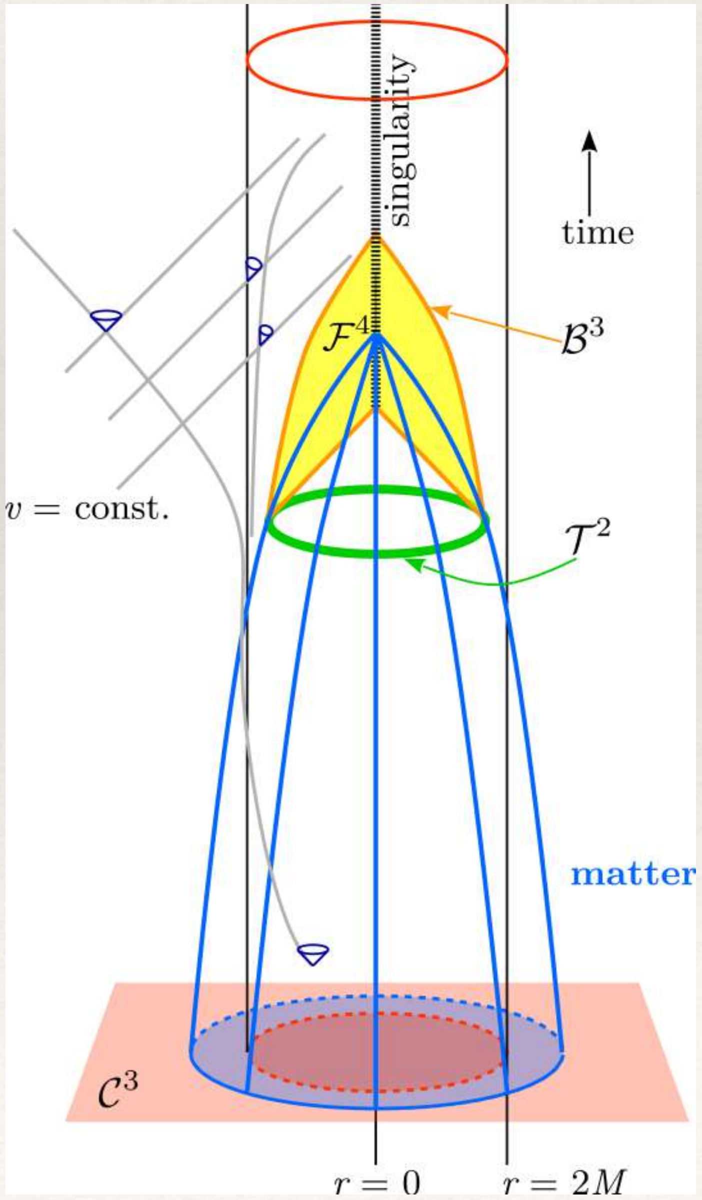
Penrose's singularity theorem

Assumptions

- The theory of gravity is GR
- The gravitational collapse becomes enough strong to have convergent light cones (trapped region)
- Matter gravitates in the standard way (no exotic/quantum matter: if p=wϱ w>-1)

Implication

Once a trapped region forms the collapse would be unstoppable and has to lead to a singularity



Avoidance of this conclusion requires at least one of the following

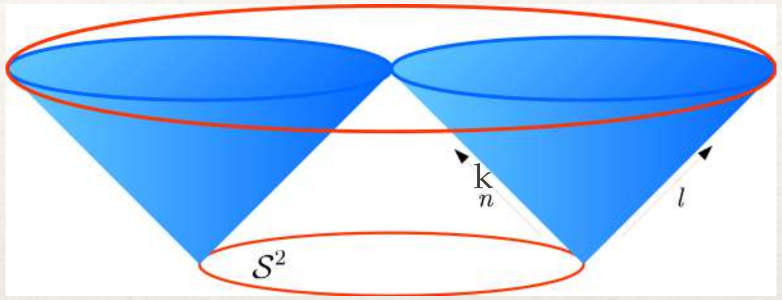
- The weak energy condition is violated.
- The Einstein field equations do not hold.
- Lorentzian geometry does not provide an adequate description of spacetime inside BHs.
- Global hyperbolicity (Cauchy evolution) breaks down.

We shall be ready to give up the first two and hold the last two…

Focussing on the focussing point

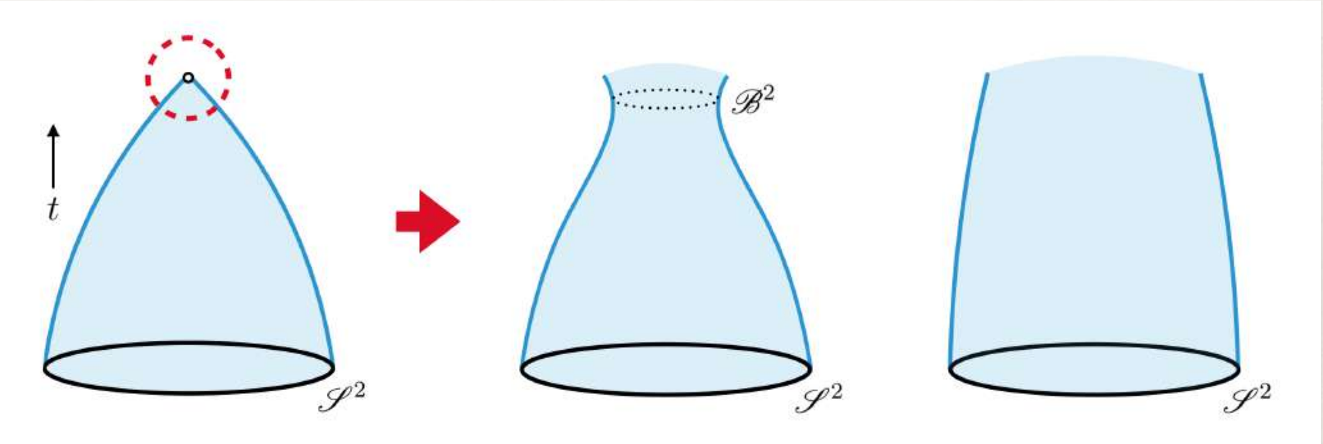
- **Let's assume that QG produces a <u>space-time which is regular and entirely predictable in the sense of a Cauchy problem</u>.**
  - **No singularities both in the sense of incomplete geodesic as well as curvature singularities (metric is at least C[2]).**

Penrose' theorem works by proving first that in a collapse a focussing point for outgoing light rays is reached and then by showing that this point (or sets of points) cannot be part of the spacetime. If QG removes such a focussing point what can happen? We can have



$$\theta^{(i)}_{\text{null}} = \frac{1}{\delta A}\frac{\delta A}{\delta \lambda}\,.$$

- **Defocusing point at a finite affine distance, $\lambda_{DEF}=\lambda_0$;**
- **Defocusing point at an infinite affine distance, $\lambda_{DEF}=\infty$;**
- **Focusing point at infinity, $\lambda_{DEF}=\emptyset$;**
  - **still singular at finite affine parameter for ingoing congruence**



Apart from the above behaviour of the outgoing light rays
we can catalogue all the possible cases by considering the radius R at which defocussing happens and the behaviour of the ingoing light rays there.
We then get only

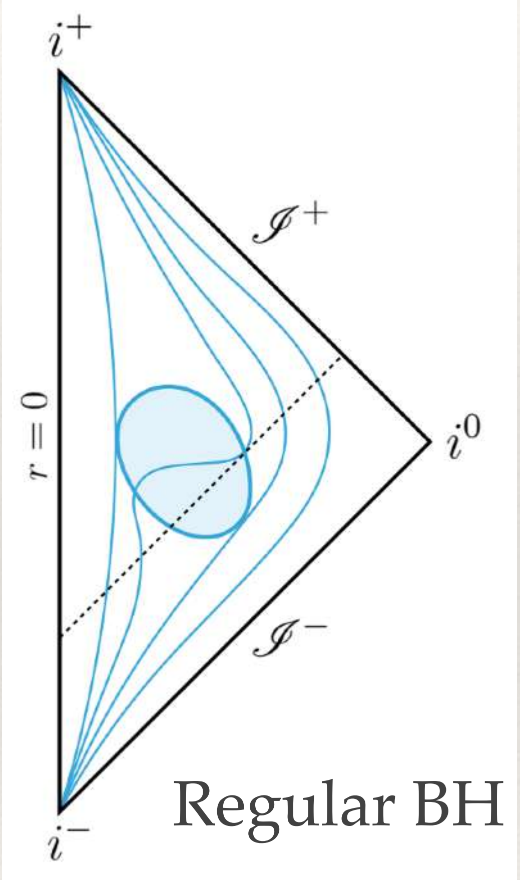
4 viable classes:

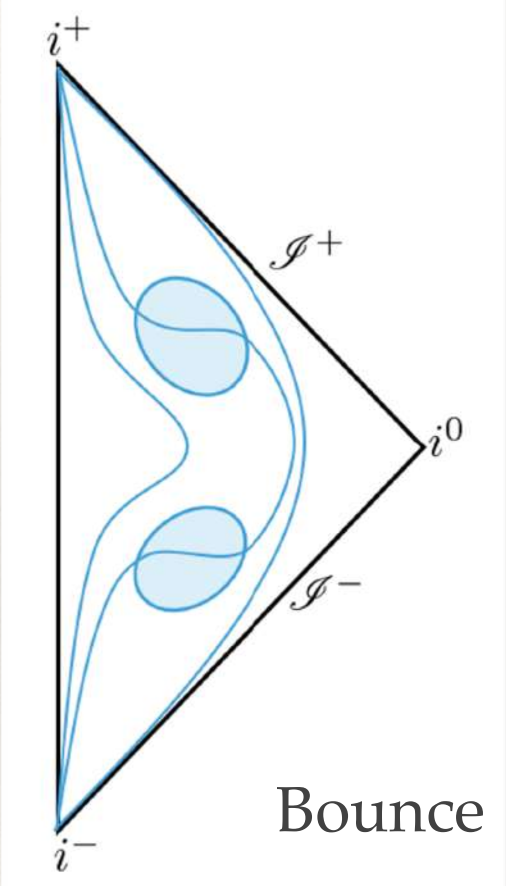
1. $(\lambda_0, R_0, \bar{\theta}^{(k)} < 0)$
2. $(\lambda_0, R_0, \bar{\theta}^{(k)} \geq 0)$
3. $(\infty, R_\infty, \bar{\theta}^{(k)} < 0)$
4. $(\infty, R_\infty, \bar{\theta}^{(k)} \geq 0)$

Class 1: Evanescent horizons

- The expansion relative to the outgoing null vector vanish and changes sign.
- The expansion of the intersecting ingoing radial null geodesics remains negative.

- We recover the geometry of an evanescent regular black hole.
- The geometry possesses an outer and an inner horizon that merge in finite time.
- This situation corresponds to a regular BH with no singularity
- Or to a bounce from a BH to a White Hole (the time reversal of a black hole)





Note: one can think of Inner Horizons as White Horizons which have been turned Inside Out

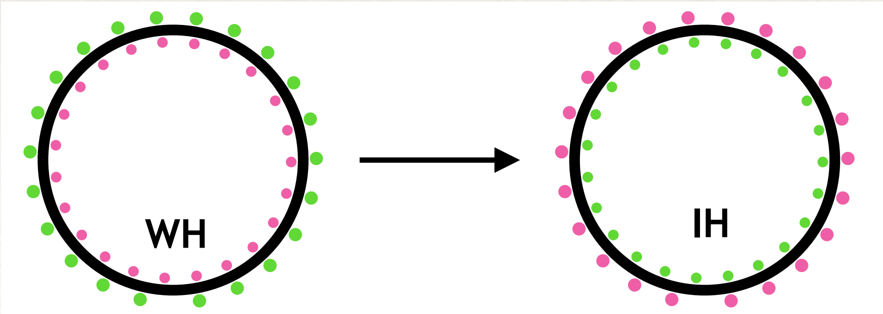
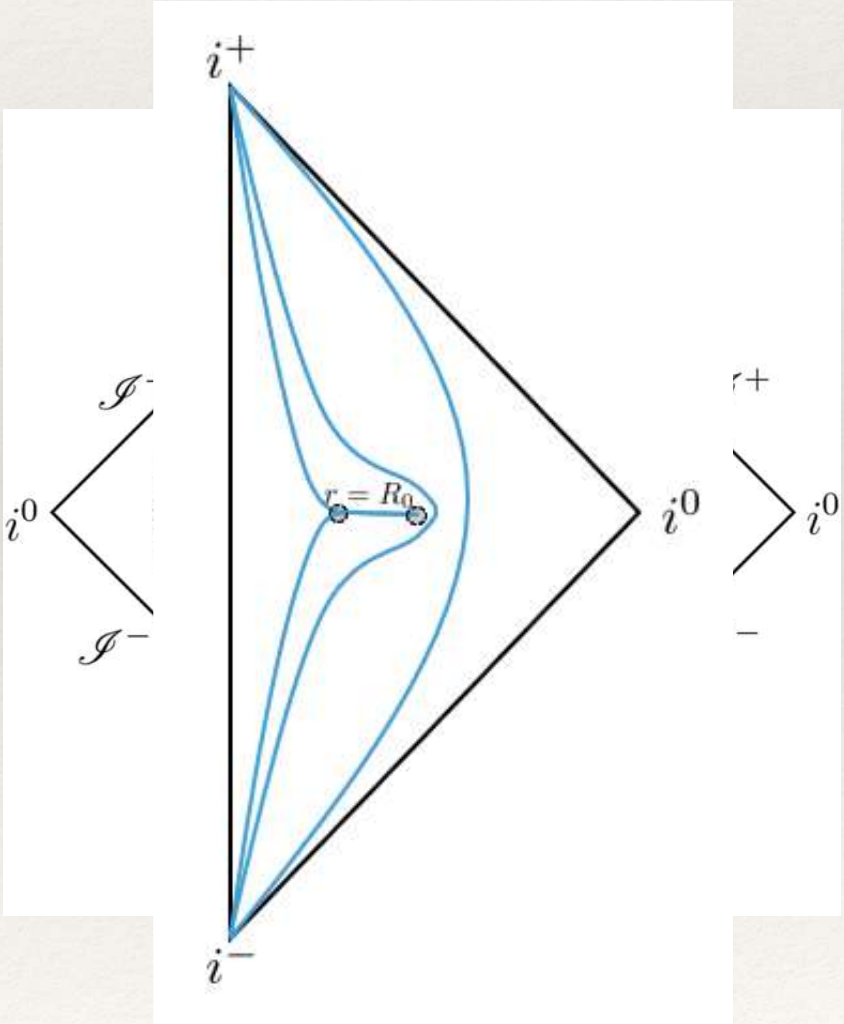


Figure by courtesy of R. Carballo-Rubio

Class 2: One way hidden wormholes

- The expansion relative to the outgoing null rays vanish and changes sign.
- The expansion of the intersecting ingoing radial null rays changes sign as well.

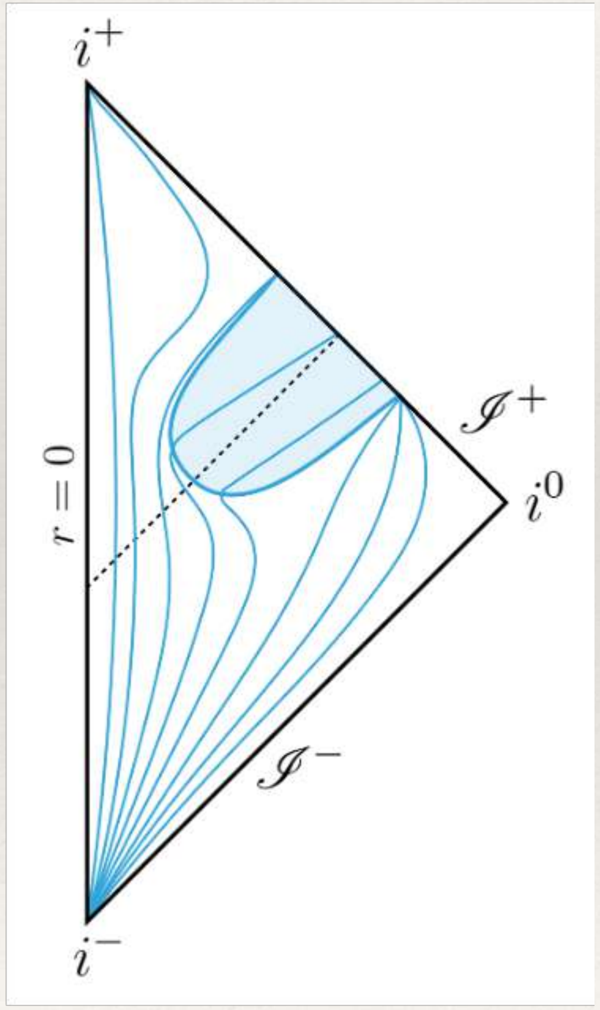
- The geometry possesses a minimum radius throat that resembles the one of a wormhole;
- The throat is inside a trapping horizon and can be traversed only in one direction.
- Problematic creation from gravitational collapse as topology change is incompatible with global hyperbolicity. However, if one gives up (at least in two points) metric analyticity requirement then possible to conceive a geometry with minimum finite radius locally.



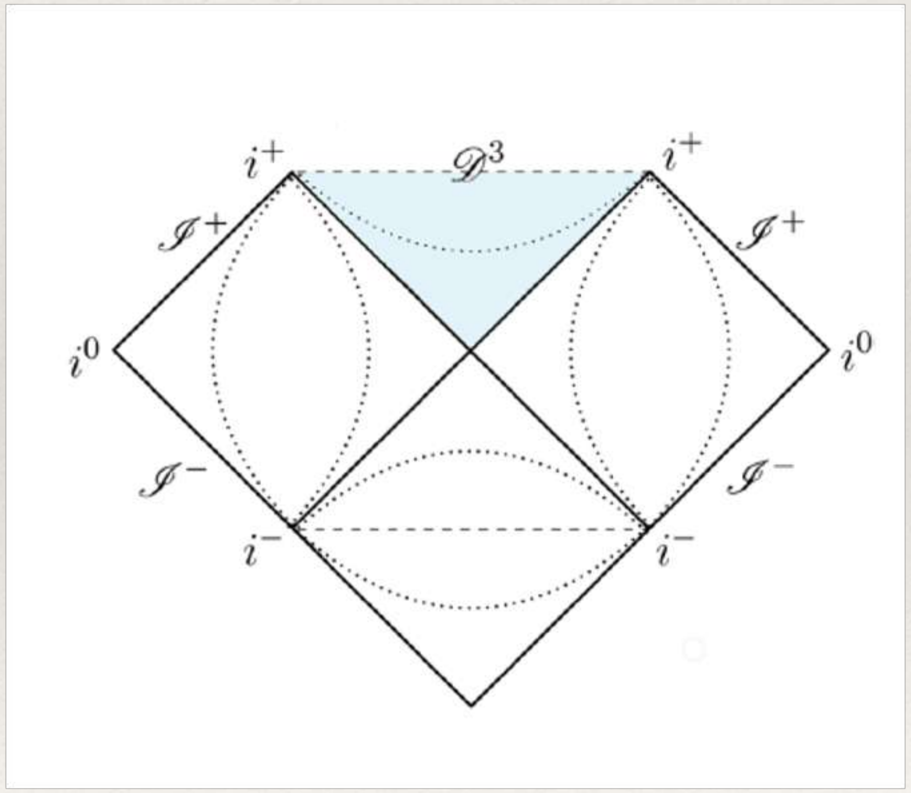
Asymptotic resolutions: Cases 3,4

- These are (idealised?) cases in which the defocussing point is pushed at infinity.

Everlasting horizons



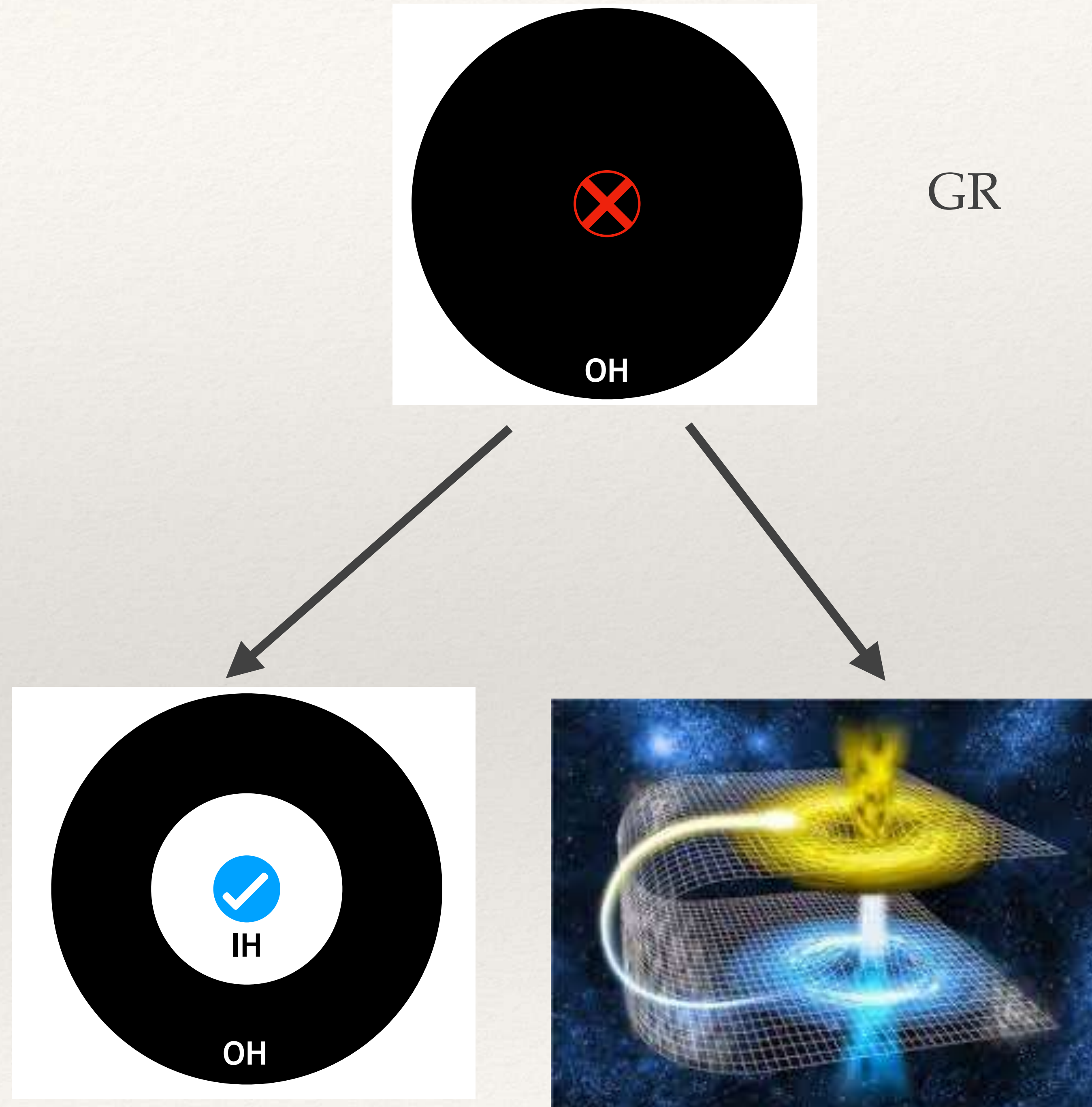
Asymptotic hidden wormholes



These are allowed but rather unphysical singularity resolutions.
We shall not deal with these asymptotic cases further…

First “take-home” message

- The analysis of the singularity resolutions tells us that substantially, once a trapping horizon forms, there are two classes of singularity free solutions (local in space and time) available:
  - Simply connected topology: Regular black holes (and bounces) with inner horizons.
  - Non-simply connected topology: Hidden Wormholes (wormholes shielded by a trapping horizons)



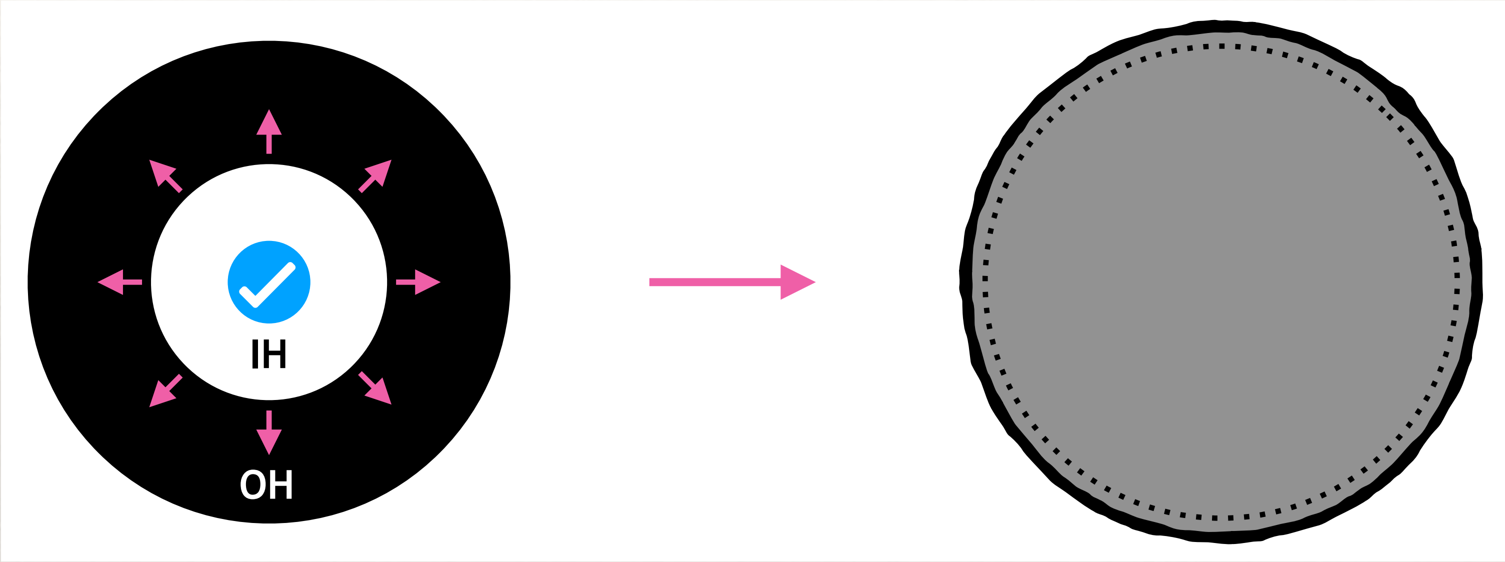
Regular BH

Hidden WH

Figures by courtesy of R. Carballo-Rubio

Limiting cases

- In both these cases one can ask what happens if $R_0 \to r_{horizon}$ and "overtakes it"
  - The answer is simple one gets two corresponding new classes of objects
    - Horizonless Quasi-BH
    - Naked wormholes



Quasi-BH

Let us define a static and spherically symmetric quasi-black hole as a spacetime satisfying:

(i) the geometry is Schwarzschild above a given radius R that is defined to be the radius of the object,
(ii) the geometry for r ≤ R is not Schwarzschild, and
(iii) there are no event or trapping horizons.

Naked Wormhole

Easy to engineer WH-mickers by "gluing" two copies of Schw. or Kerr spacetime cut just above the horizon but in general these are not correspondent to regularised solutions.

Class 1: Examples

$$ds^2 = -\left(1-\frac{2m(r)}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1-\frac{2m(r)}{r}\right)} + r^2\left[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right] .$$

m(r)=Misner-Sharp Mass

Model	$m(r)$
Bardeen [44]	$M\frac{r^3}{(r^2+\ell^2)^{3/2}}$
Hayward [45]	$M\frac{r^3}{r^3+2M\ell^2}$
Dymnikova [46]	$M\left[1-\exp\left(\frac{r^3}{\ell^3}\right)\right]$
Fan–Wang [47]	$M\frac{r^3}{(r+\ell)^3}$

- Requirements for the mass function
  $m(r) \to M$ as $r \to \infty$ and $m(r) = O(r^3)$ as $r \to 0$ (at least)
- Asymptotic flatness+Regularity at the core+Outer Horizon imply also Inner Horizon. The position of the inner and outer horizons and their surface gravity depend on m(r)
- Within GR, RBHs are non-vacuum solutions, the effective stress-energy tensor can be read off from the Einstein tensor; several interpretations in terms of non-linear electrodynamics. In general Violations of energy conditions.
- Even non-rotating RBH have inner horizons
- Rotating regular black holes (Kerr-like) can be constructed e.g. using generalised Janis–Newman procedure (albeit care is required…)

Class 1: Regular-BH limit

- Let us take Hayward RBH for concreteness: $m(r) = \dfrac{Mr^3}{r^3 + 2\ell^2 M}$, $\qquad \phi(r) = 0.$
- The effective stress energy tensor takes the form associated with an anisotropic perfect fluid

$$\rho(r) = \frac{3\ell^2}{2\pi}\left(\frac{m(r)}{r^3}\right)^2 = -p_r(r), \qquad p_t(r) = \frac{3\ell^2}{\pi}\frac{r^3 - \ell^2 M}{r^3 + 2\ell^2 M}\left(\frac{m(r)}{r^3}\right)^2 = \frac{2r^3 - 2\ell^2 M}{r^3 + 2\ell^2 M}\rho(r)\,.$$

- $2m(r) = r$ has 2 roots for $M/\ell > 3\sqrt{3}/4$ a degenerate/double root for $M/\ell = 3\sqrt{3}/4$ (at $r_* = \sqrt{3}\ell$) and no roots for $M/\ell < 3\sqrt{3}/4$

Assuming $M/\ell > 3\sqrt{3}/4$ and $M \gg \ell$ one has a RBH a ultra compact object with 4 "zones"

- The (approximately isotropic) dS core $[r \sim \ell < 2M]$:

$$\rho(\ell) \equiv -p_r(\ell) = \frac{3}{8\pi\ell^2}\left[1 - \mathcal{O}\left(\ell/M\right)\right] = -p_t(\ell)\,.$$

- The (mildly anisotropic) crust $[r \sim L_+ \equiv \sqrt[3]{2\ell^2 M}]$:

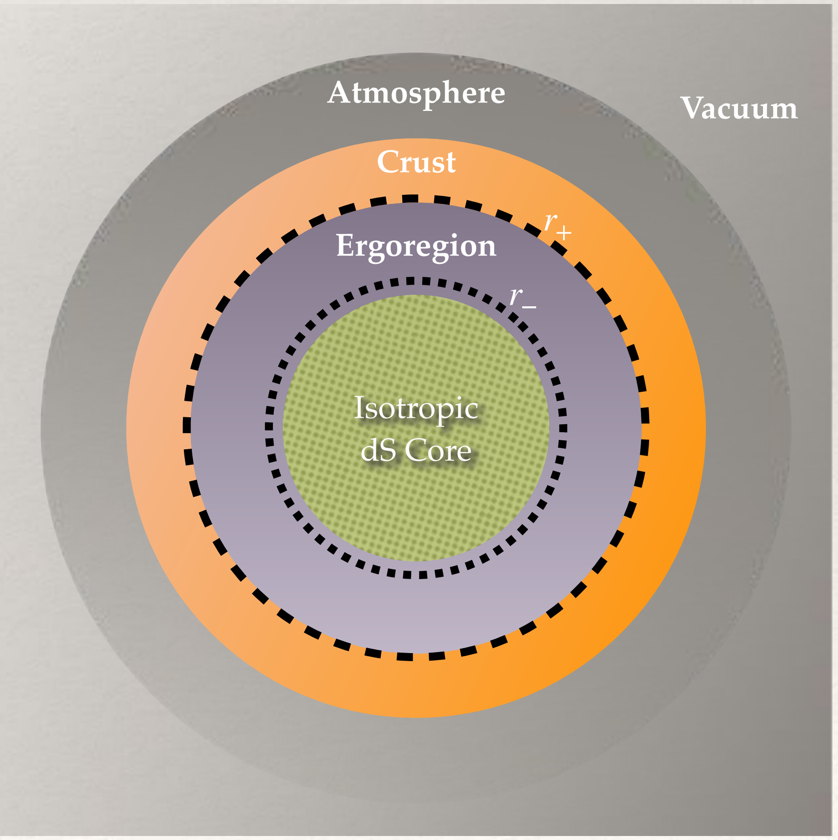
$$\rho(L_+) \equiv -p_r(L_+) = \frac{\Lambda_0}{4}\left[1 + \mathcal{O}\left(\ell/M\right)\right], \qquad p_t(L_+) = \frac{\Lambda_0}{8}\left[1 + \mathcal{O}\left(\ell/M\right)\right].$$

- The (grossly anisotropic) atmosphere $[r \sim 2M]$:

$$\rho(M) \equiv -p_r(M) = \Lambda_0\left(\frac{\ell}{2M}\right)^4\left[1 + \mathcal{O}\left(\ell^2/M^2\right)\right], \qquad p_t(M) = 2\rho(M)\left[1 + \mathcal{O}\left(\ell^2/M^2\right)\right].$$

- The (approximately vacuum) asymptotic region $[r \sim R \gg M]$:

$$\rho(R) \equiv -p_r(R) = \Lambda_0\left(\frac{\ell}{2M}\right)^4\left(\frac{2M}{R}\right)^6\left[1 + \mathcal{O}\left(\ell^2 M/R^3\right)\right], \quad p_t(R) = 2\rho(R)\left[1 + \mathcal{O}\left(\ell^2 M/R^3\right)\right].$$



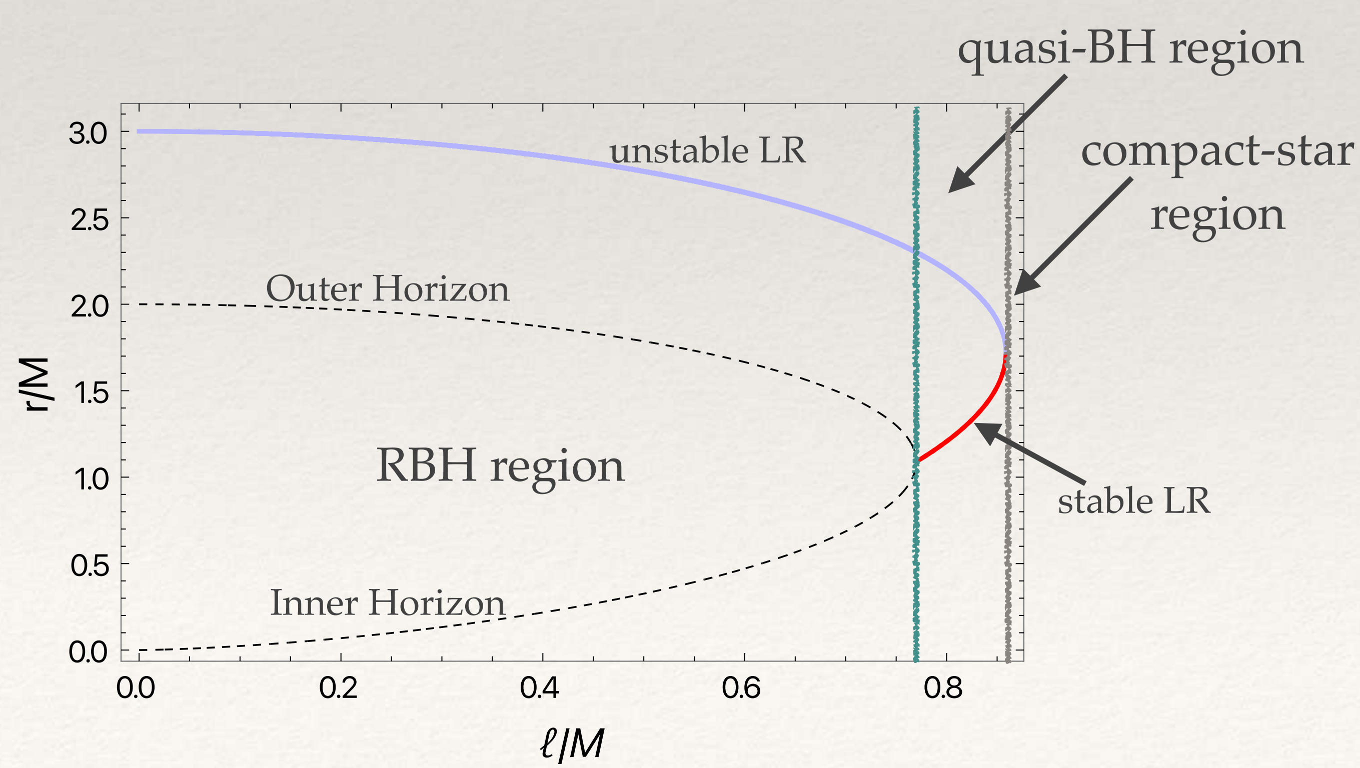
Class 1: Quasi-BH limit

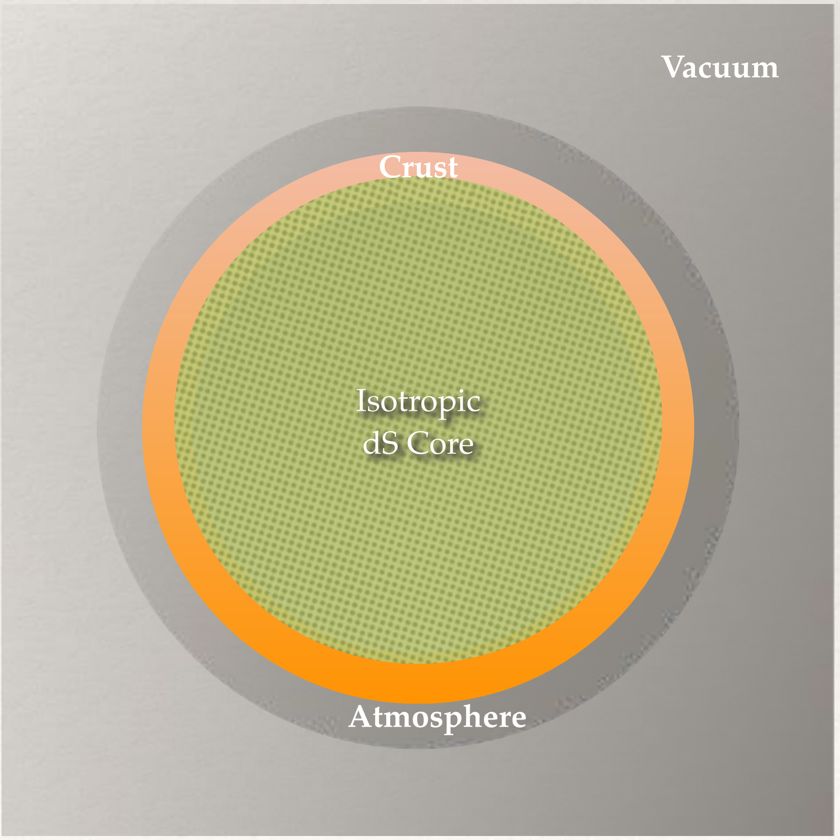
- Let us take Hayward RBH for concreteness: $m(r) = \dfrac{Mr^3}{r^3 + 2\ell^2 M}$, $\quad \phi(r) = 0.$
- The effective stress energy tensor takes the form associated with an anisotropic perfect fluid

$$\rho(r) = \frac{3\ell^2}{2\pi}\left(\frac{m(r)}{r^3}\right)^2 = -p_r(r), \qquad p_t(r) = \frac{3\ell^2}{\pi}\frac{r^3 - \ell^2 M}{r^3 + 2\ell^2 M}\left(\frac{m(r)}{r^3}\right)^2 = \frac{2r^3 - 2\ell^2 M}{r^3 + 2\ell^2 M}\rho(r)\,.$$

- $2m(r) = r$ has 2 roots for $M/\ell > 3\sqrt{3}/4$ a degenerate/double root for $M/\ell = 3\sqrt{3}/4$ and no roots for $M/\ell < 3\sqrt{3}/4$

Assuming $M/\ell \lesssim 3\sqrt{3}/4$. In this case, the different scales ℓ and M coalesce, the horizons disappear, with the dS-like core growing in size and the crust and atmosphere shrinking.

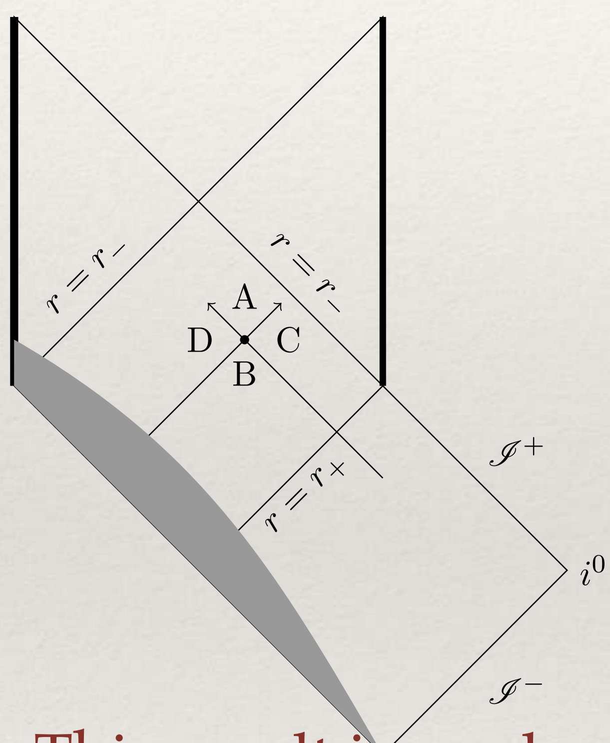




Similar structure to gravastars

Problem: Mass inflation instability

Problem: The inner horizon is generically classically unstable



$$m_A(r_0(v)\,|_{u=u_o}) \propto v^{-\gamma} e^{|\kappa_-|v}$$

Without fine tuning there is an instability at inner horizon (mass inflation) in QG time scale, while evaporation time is generically infinite.

Note also that possible cosmological constant relevant only after a time $v \sim 1/\sqrt{\Lambda}$.

Similarly, ingoing Hawking flux can become relevant (see Buonanno et al. 2022) but too late for astrophysical black holes

R.Carballo-Rubio, F.Di Filippo, SL, C.Pacilio and M.Visser,

JHEP 1807, 023 (2018). [arXiv:1805.02675 [gr-qc]].

JHEP 05 (2021) 132 • e-Print: 2101.05006 [gr-qc]

See also: [arXiv:2212.07458 [gr-qc]].

This result is can be extended to continuous incoming fluxes and to dynamical geometries (<u>it is not just linked to Cauchy horizons</u>). Even the GR Kerr one…

R.Carballo-Rubio, F.Di Filippo, SL, C.Pacilio and M.Visser e-Print: 2402.14913.

This seems to suggest that a RBH regularisation with an inner horizon cannot be the end point of the collapse… can we avoid mass inflation?

Stable regular black holes?

- Basic idea: a possible stable endpoint is a Regular BH with zero surface gravity at the IH but non zero one at the outer horizon given that mass inflation is exponential in κ_-

R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio and M. Visser, ``Regular black holes without mass inflation instability,'' JHEP 09 (2022), 118.

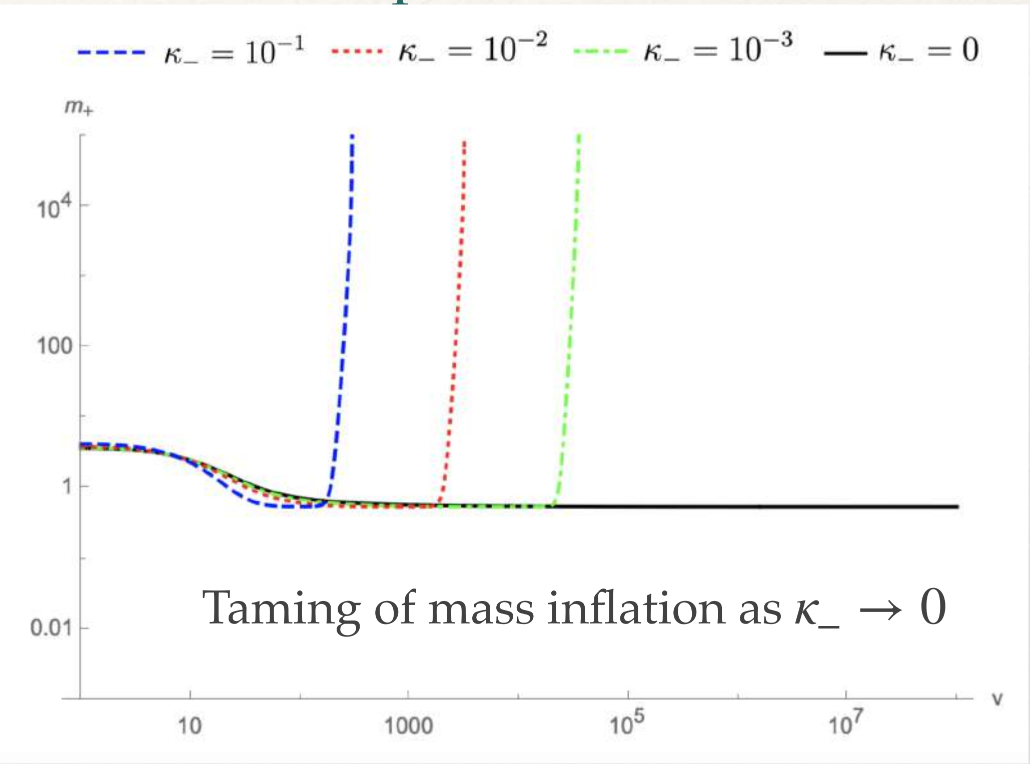
$$\mathrm{d}s^2 = -e^{-2\phi(r)}F(r)\mathrm{d}v^2 + 2e^{-\phi(r)}\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

Misner-Sharp quasi-local mass m $\quad F(r) = 1 - \frac{2m(r)}{r}.$

$$F(r) = \frac{(r-r_-)^3(r-r_+)}{(r-r_-)^3(r-r_+) + 2Mr^3 + [a_2 - 3r_-(r_+ + r_-)]r^2}, \qquad \phi(r) = 0,$$

subject to

$$r_- \ll r_+ \sim 2M; \qquad r_- \sim |r_+ - 2M|; \qquad a_2 \gtrsim \frac{9}{4}r_+r_-.$$



- Generalisation to rotating black holes.

E. Franzin, S.Liberati, J. Mazza and V. Vellucci, ``Stable Rotating Regular Black Holes,''. [arXiv:2207.08864 [gr-qc]].

$$ds^2 = \frac{\Psi}{\Sigma}\left[-\left(1 - \frac{2m(r)r}{\Sigma}\right)\mathrm{d}t^2 - \frac{4a\,m(r)r\sin^2\theta}{\Sigma}\mathrm{d}t\,\mathrm{d}\phi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 + \frac{A\sin^2\theta}{\Sigma}\mathrm{d}\phi^2\right],$$

$$\Psi = \Sigma + \frac{b}{r^3}, \quad \Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2m(r)r + a^2, \quad A = (r^2+a^2)^2 - \Delta a^2\sin^2\theta,$$

$$m(r) = M\frac{r^2 + \alpha r + \beta}{r^2 + \gamma r + \mu}.$$

$$\alpha = \frac{a^4 + r_-^3 r_+ - 3a^2 r_-(r_- + r_+)}{2a^2 M},$$
$$\beta = \frac{a^2(2M - 3r_- - r_+) + r_-^2(r_- + 3r_+)}{2M},$$
$$\gamma = 2M - 3r_- - r_+,$$
$$\mu = \frac{r_-^3 r_+}{a^2}.$$

$$r_+ = M + \sqrt{M^2 - a^2},$$

$$r_- = a^2\left[M + (1-e)\sqrt{M^2 - a^2}\right]^{-1}$$

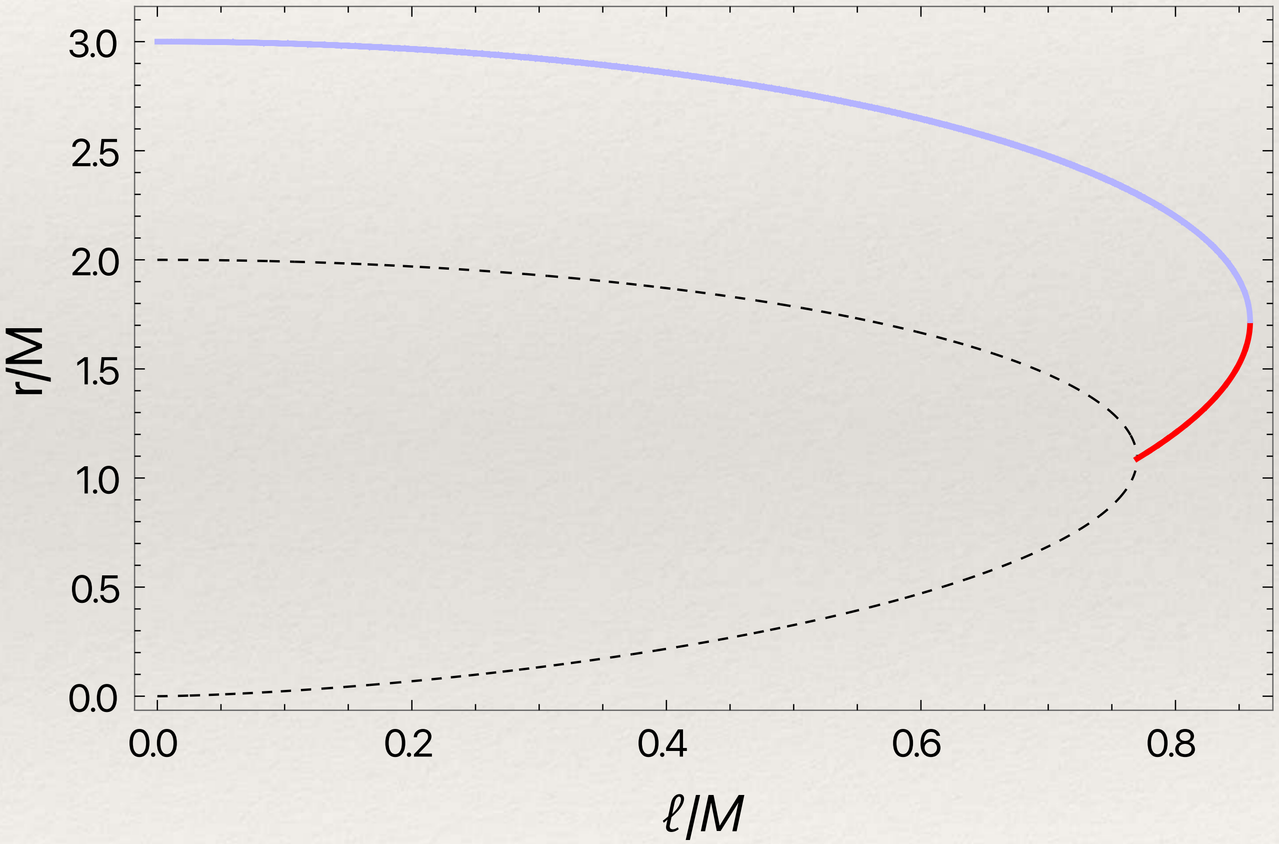
However… semiclassical instability

S. Hollands, R.M. Wald and J. Zahn,
Class. Quant. Grav. 37 (2020) no.11, 115009

T. McMaken,
Phys. Rev. D107 (2023) no.12, 125023

C. Barcelò, V. Boyanov, R. Carballo-Rubio and L.J. Garay,
Phys. Rev. D106 (2022) no.12, 124006

- Zero surface gravity at the inner horizon might not be enough to stabilise a regular black hole: there is an exponential quantum instability ruled by
$$\lim_{r\to r_-} < U|T_{uu}|U > = -\frac{1}{48\pi}\left(\kappa_-^2 - \kappa_+^2\right)$$
- Preliminary investigations seems to suggest that
    - classical mass inflation would push the inner horizon inwards
    - the quantum instability would push the inner horizon outwards and dominate.
    - The position of the IH is basically set by ℓ, so the semiclassical instability suggests that one effectively gets $\ell \to \ell(v)$
    - So, this chain of instabilities may lead the RBH to end up extremal or a quasi-BH…
    - But quasi-BH have necessarily an inner stable light ring! $\longrightarrow$ possibly unstable again?



Credits: Edgardo Franzin, Stefano Liberati, Vania Vellucci. e-Print: 2310.11990 [gr-qc]

Take home message: RBH are most probably always dynamical objects at most metastable. Compatibility of this metastability with observations is an open issue.

Class 2: The Simpson-Visser Black-Bounce

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right) dt^2 + \left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right)^{-1} dr^2 + (r^2+\ell^2)\left[d\theta^2 + \sin^2\theta d\phi^2\right],$$

A.Simpson, M.Visser. JCAP 02 (2019) 042
e-Print: 1812.07114 [gr-qc]

- a two-way, traversable wormhole à la Morris-Thorne for $\ell > 2M$,
- a one-way wormhole with a null throat for $\ell = 2M$, and
- a regular black hole, in which the singularity is replaced by a bounce to a different universe, when $\ell < 2M$; the bounce happens through a spacelike throat shielded by an event horizon and is hence dubbed "black-bounce" in [6] or "hidden wormhole" as per [4].

RN extension:
E.Franzin,SL, J.Mazza, A.Simpson, M.Visser. JCAP 07 (2021) 036.
e-Print: 2104.11376 [gr-qc]

Rotating counterpart

J.Mazza, E.Franzin, SL. JCAP 04 (2021) 082 • e-Print: 2102.01105 [gr-qc]

$$ds^2 = -\left(1 - \frac{2M\sqrt{r^2+\ell^2}}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4Ma\sin^2\theta\sqrt{r^2+\ell^2}}{\Sigma} dt d\phi + \frac{A\sin^2\theta}{\Sigma} d\phi^2 \tag{2.16}$$

with

$$\Sigma = r^2 + \ell^2 + a^2\cos^2\theta, \qquad \Delta = r^2 + \ell^2 + a^2 - 2M\sqrt{r^2+\ell^2},$$
$$A = (r^2+\ell^2+a^2)^2 - \Delta a^2 \sin^2\theta.$$

WoH traversable wormhole;

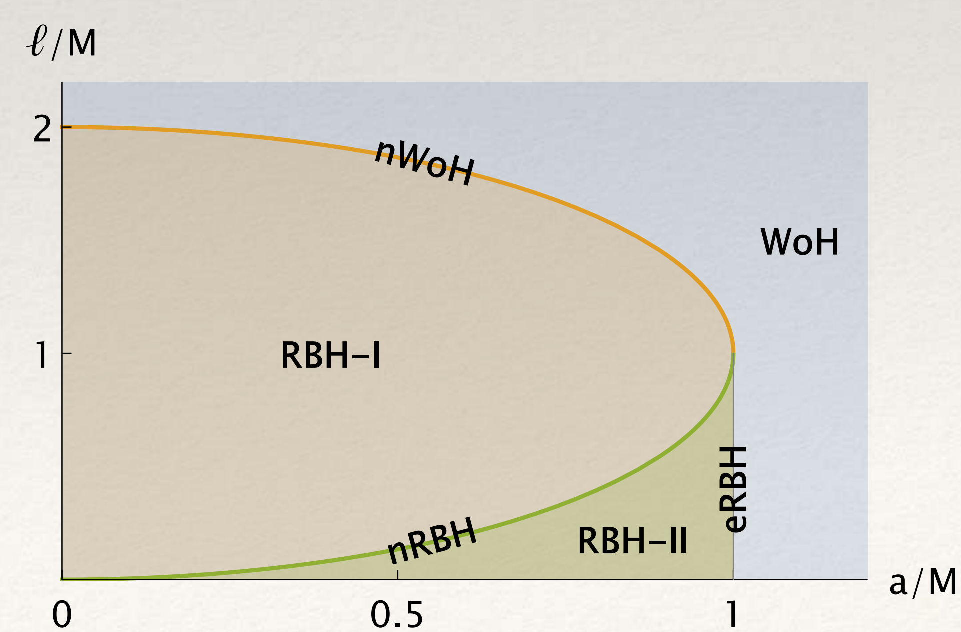
nWoH null WoH, i.e. one-way wormhole with null throat;

RBH-I regular black hole with one horizon (in the $r > 0$ side, plus its mirror image in the $r < 0$ side);

RBH-II regular black hole with an outer and an inner horizon (per side);

eRBH extremal regular black hole (one extremal horizon per side);

nRBH null RBH-I, i.e. a regular black hole with one horizon (per side) and a null throat.



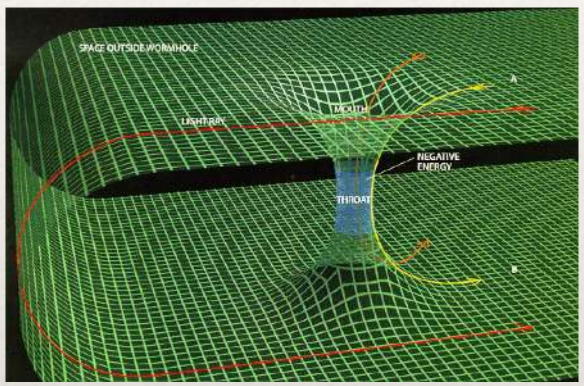
E. Franzin, S. Liberati, J. Mazza, R. Dey and S.Chakraborty,
Phys. Rev. D105, no.12, 124051 (2022)

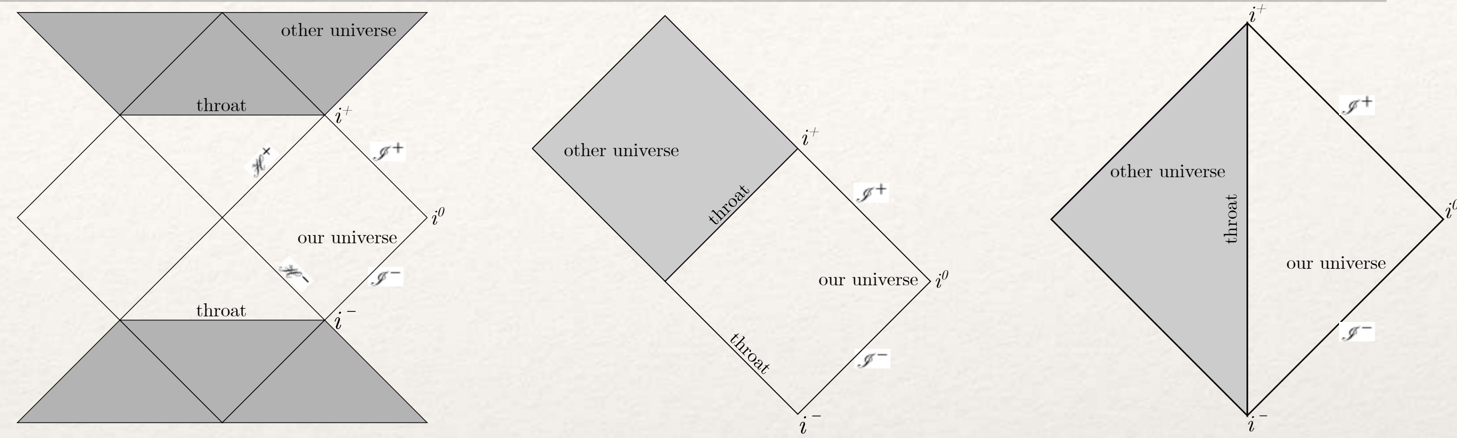
Class 2 limiting case: traversable wormholes

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right) dt^2 + \left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right)^{-1} dr^2 + (r^2+\ell^2)\left[d\theta^2 + \sin^2\theta d\phi^2\right],$$

$$\boxed{\left(1 - 2M/(\sqrt{r^2+\ell^2}\right) = 0 \text{ has no roots for } \ell > 2M}$$

(similarly for the rotating case)





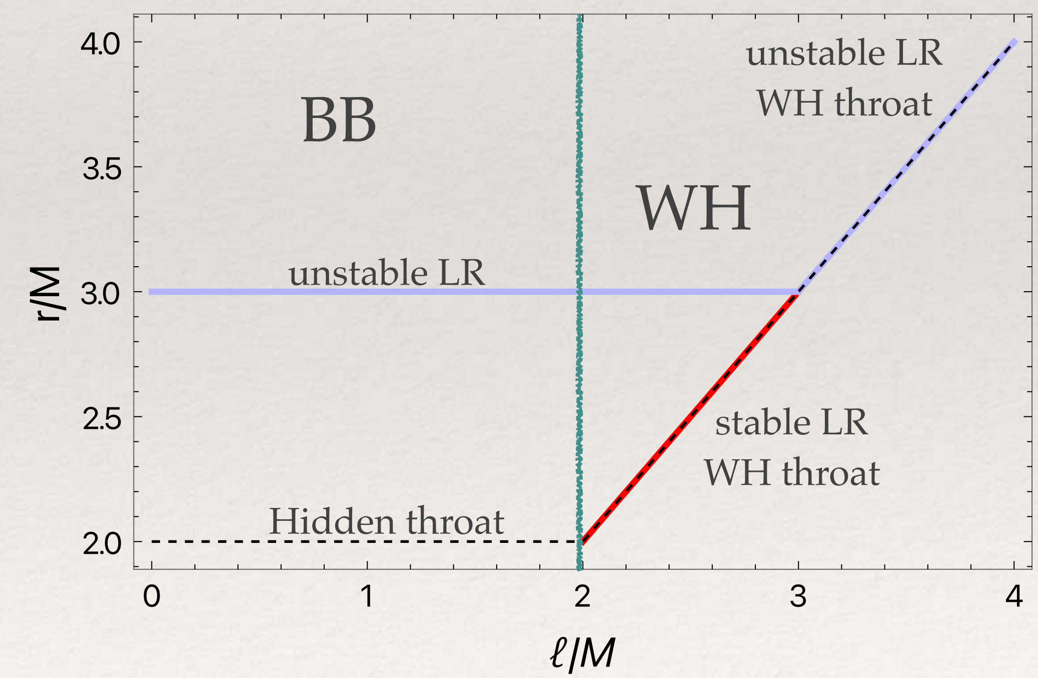
(a) The regular black hole. The maximally extended spacetime continues above and below the portion shown by repetition of this fundamental block.

(b) The null-throat wormhole. The analytically extended spacetime continues above and below by repetition of this fundamental block.

(c) The traversable wormhole.

Figure 1. Penrose diagrams of regular black hole, null-throat wormhole and traversable wormhole. The white area represents "our universe" while the gray area is the "other universe".

- Energy conditions violation at the WH throat
- Also in this case the naked WH will sport for $2M < \ell < 3M$ a stable light-ring at the WH throat.
- For $\ell > 3M$ there is only an unstable light-ring again at the wormhole throat.



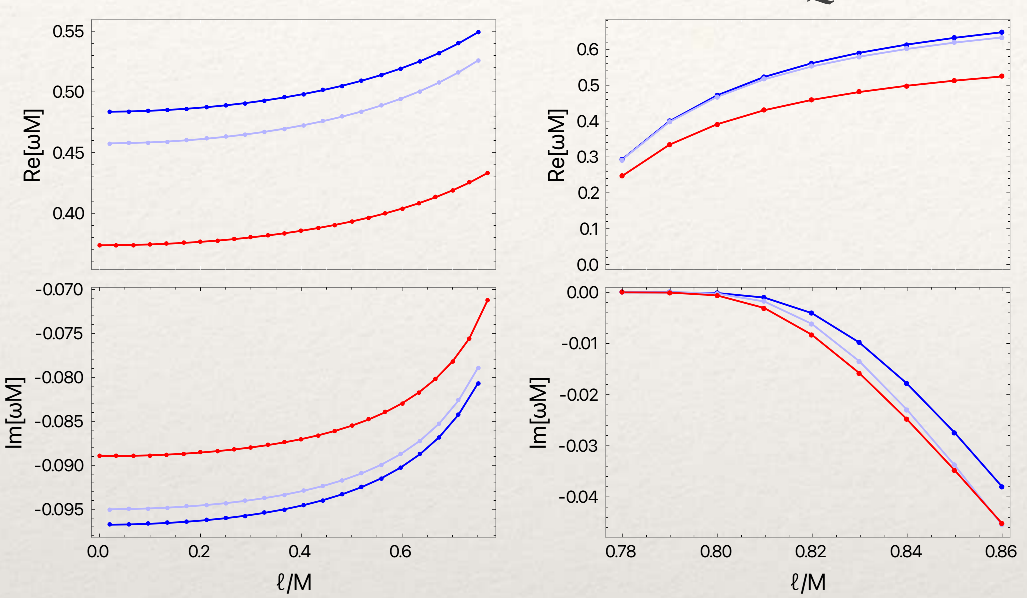
Take home message: In spite of being "more exotic" the Black Bounces appear to be less prone to instabilities (no IH instability if $\ell > r_-$).
What about the WH case?

E.~Franzin, S.~Liberati and V.~Vellucci,
``From regular black holes to horizonless objects: quasi-normal modes, instabilities and spectroscopy,''
[arXiv:2310.11990 [gr-qc]].

Cases 1 & 2: Quasi-normal modes analysis

Let us we study test- field and linear gravitational perturbations in such spacetimes,
varying the regularization parameters so to pass smoothly from RBHs to the ultracompact horizonless objects.

Bardeen RBH　　Bardeen Quasi-BH



Spin 2
Test Field
perturbation

SV-BB　　SV-WH

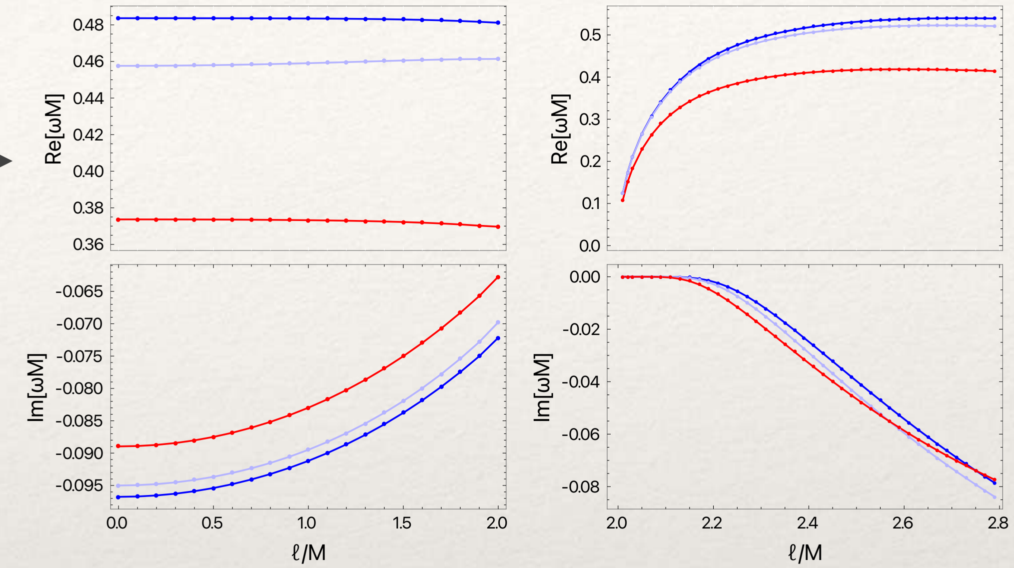


Figure 2. Quadrupolar $l = 2$ fundamental QNMs of the Bardeen metric for test-field perturbations, $s = 0$ (blue), $s = 1$ (light purple) and $s = 2$ (red). On the left results for values of ℓ in the RBH branch that is from $\ell = 0$ (Schwarzschild) to $\ell = \ell_{\rm ext} = \frac{4}{3\sqrt{3}}M$ (extremal RBH). On the right results for values of ℓ in the horizonless branch. Note that for values of the regularization parameter near the extremal case the imaginary part is extremely small and thus we have very long living modes.

Figure 3. Quadrupolar $l = 2$ fundamental QNMs of the SV metric for test-field perturbations, $s = 0$ (blue), $s = 1$ (light purple) and $s = 2$ (red). On the left results for values of ℓ in the RBH branch, that is from $\ell = 0$ (Schwarzschild) to $\ell = 2M$ (one-way wormhole with an extremal null throat). On the right results for values of ℓ in the horizonless branch. It is worth noticing the relative flatness of the real part curves which highlights weak deviations from the singular GR solution behaviour recovered for $\ell = 0$. On the left results for values of the regularization parameter near the extremal case ($\ell_{\rm ext} = 2M$): the imaginary part is extremely small and thus we have very long living modes.

Regular Black holes						
	Bardeen			Simpson–Visser		
	Test $s=2$	Axial	Polar	Test $s=2$	Axial	Polar
$\ell/M = 0.2$						
Δ_R	0.0075	−0.0012	0.0037	$-3\cdot 10^{-5}$	−0.0002	−0.0005
Δ_I	0.0045	0.0090	0.0090	0.0022	0.0044	0.0044
$\ell/M = 0.6$						
Δ_R	0.0808	0.0069	0.0297	−0.0003	−0.0163	−0.0067
Δ_I	0.0674	0.0776	0.0810	0.0236	0.0292	0.0292
$\ell/M = 1.6$						
Δ_R				−0.0053	−0.0690	−0.0428
Δ_I				0.1798	0.1854	0.1776

Horizonless compact objects		
	Bardeen	Simpson–Visser
	Test $s=2$	Test $s=2$
$\delta = 0.05$		
Δ_R	0.1380	−0.1801
Δ_I	0.9712	0.9970
$\delta = 0.10$		
Δ_R	0.3613	−0.0310
Δ_I	0.6441	0.9015
$\delta = 0.20$		
Δ_R		0.0482
Δ_I		0.5913

Table I. Relative deviations from the quadrupolar fundamental Schwarzschild frequency $\Delta_{R/I} = \frac{\omega_{R/I}-\omega^S_{R/I}}{|\omega^S_{R/I}|}$ with $\omega^S M = 0.37367 - 0.08896i$, for $s = 2$ test-field and linear gravitational perturbations, both in the axial and polar sectors, for selected valued of the regularization parameter. Results are shown for the Bardeen and SV spacetimes, on the left for the RBH branch and on the right for horizonless configurations. For the Bardeen metric there are no results for $\ell/M = 1.6$ and $\delta = 0.2$, with $\delta \equiv \ell/\ell_{\rm ext} - 1$, since for those values of compactness the spacetime not only lose the presence of the horizon but even of a photon sphere. For both spacetimes results for axial and polar gravitational perturbations are not reported for horizonless configurations because of the numerical issues present in this branch. Looking at the test field case, it is easy to see the large increment Δ_I passing from the RBH configurations to the horizonless ones for small δs.

Summary

- For $\ell \lesssim M$ both the RBH and SV BB show deviations for the Schwarzschild QNM
- SV BB tends to show smaller deviations.
- **Third generation GW detector with enough statistics might see this!**
- Quasi-BH configurations show marked longer perturbations lifetimes (tiny imaginary part) for $\ell \gtrsim \ell_{\rm ext}$
- This is a sufficient condition to expect non-linear instability and appears to be related to the presence of the inner-stable-light-ring
- However, note that the imaginary part becomes comparable with the Schwarzschild one very rapidly as the compactness decreases even before the inner-light ring disappear.
- A non-linear analysis is definitely needed…
- (and what about matter interactions?)

R. Carballo-Rubio, F. Di Filippo, S. Liberati and M. Visser,
Phys. Rev. D98, no.12, 124009 (2018)

Phenomenology: parametrising the uncertainties

Size, $R = r_S(1 + \Delta)$: the value of the radius below which the modifications to the classical geometry are O(1). $\Delta \geq 0$.
Note the compactness parameter $\mu = \Delta/(1 + \Delta)$. So for $\Delta \ll 1$ one has $\mu \simeq \Delta$

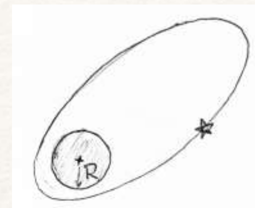
	τ_+ - Lifetime	τ_- -formation time	μ- compactness	κ-Absorption Coeff.	Γ-Elastic reflection Coeff.	$\tilde{\Gamma}$-Inelastic reflection Coeff.	$\varepsilon(r)$- Tails
Classical GR BH	∞	~10 M	0	1	0	0	0
Trapped regions (RBH+Hidden WH)	undertermined	~10 M	0	1	0	0	Non-zero
Quasi-BH	∞	Model dependent	Model dependent	Model dependent	Model dependent	Model dependent	Model dependent
Bouncing Geometries (long lived)	$\mathcal{T}^{(i)}$	Model dependent	0	1	0	0	non-zero and $r_* = O(r_S)$
Traversable Wormholes	∞	unknown	>0	Model dependent	1-κ	0	Model dependent

Note: one of the parameters is not independent: e.g. inelastic interaction parameter must satisfy $\tilde{\Gamma} = 1 - \kappa - \Gamma$

Including additional independent parameters would provide more freedom to play with the observational data but less constraining power. The set introduced is minimal, but still able to asses the observational status of black holes.

EM channels

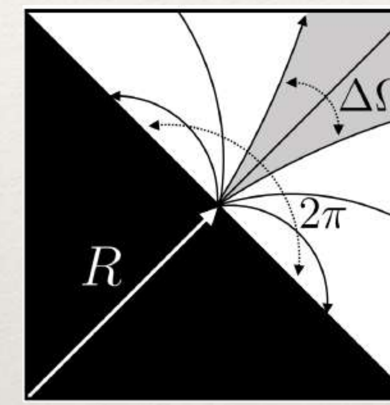
1. Stars orbiting the BH mimicker



- Tracking several stars we can determine the mass of Sgr A* and our distance from it. $M = 4\times10^6\, M_\odot$ and d = 8 Kpc
- Most close orbiting star S2 constraints the radius of Sgr A*: The periastron of S2 is 17 light hours, while the Schwarzschild radius of Sgr A* is 40 light seconds. Therefore, $\Delta \leq O(10^3)$.

2. Infalling matter.

NAIVE EXPECTATION:
STRONG CONSTRAINTS FROM ABSENCE OF THERMAL RADIATION FROM HARD SURFACE IN THE CASE OF QUASI-BH
HOWEVER QUITE GENERALLY RADIATION EMITTED AS A CONSEQUENCE OF SMASH OF MATTER ON A HARD SURFACE RATHER THAN A HORIZON WILL BE SUBJECT TO STRONG LENSING. INDEED THE ESCAPE SOLID ANGLE IS



FOR $r \to r_s$ $\quad \dfrac{\Delta\Omega}{2\pi} = \dfrac{27}{8}\mu + \mathscr{O}(\mu^2).$

THEREFORE, ONLY A SMALL FRACTION OF THE LIGHT EMITTED FROM THE SURFACE OF THE OBJECT WILL IMMEDIATELY ESCAPE TO INFINITY

- Cataclysmic events (stars disruptions)

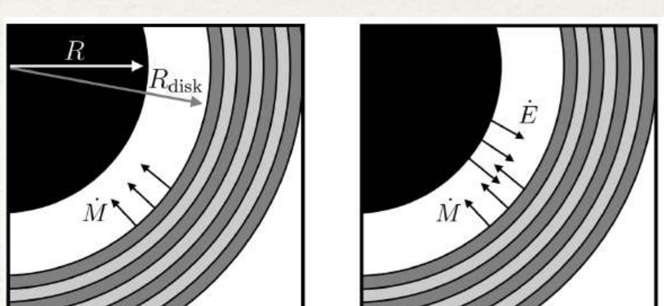
weak constraint due to complex physics $\quad \mu \leq 10^{-4}\dfrac{\kappa_{\rm T} M_\star}{4\pi r_{\rm s}^2} = \mathscr{O}(1) \times \left(\dfrac{10^8 M_\odot}{M}\right)^2.$

- Steady accretion

Claims in the past of the exclusion of horizonless objects of ANY compactness. (Narayan-Broderick, 2006).

These derivations are based mainly on two strong assumptions:
1. Thermalisation of the reemitted flux. OK thanks to strong lensing.
2. Steady state: i.e. equilibrium of ingoing (accretion) and outgoing (reemission) fluxes. Not OK due to possible absorption

Neglecting κ and *Γ* still one gets from SgrA* and IR emission 10^{-2} fainter than expected $\mu \simeq \Delta \leq \mathscr{O}(10^{-17}).$

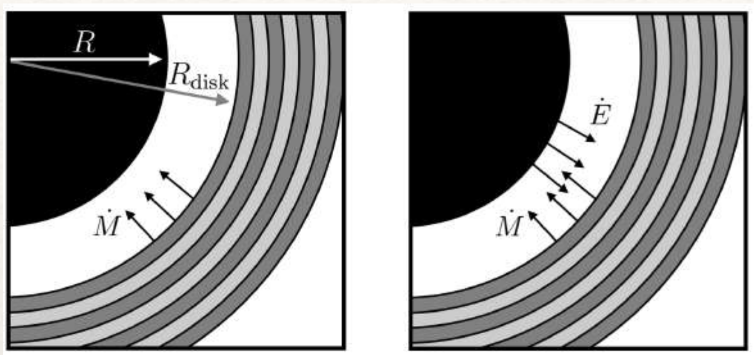


10^2 meters over a size of 10^{10} m! Still very far from Planck scale.

Let's analyse in detail the case of non-zero absorption
(i.e. simple case $\kappa \neq 0$ but $\Gamma = 0$)

R. Carballo-Rubio, F. Di Filippo, S.L. and M.Visser, JCAP08 (2022) no.08, 055.[arXiv:2205.13555 [astro-ph.HE]].

EHT Constraints from Reemission

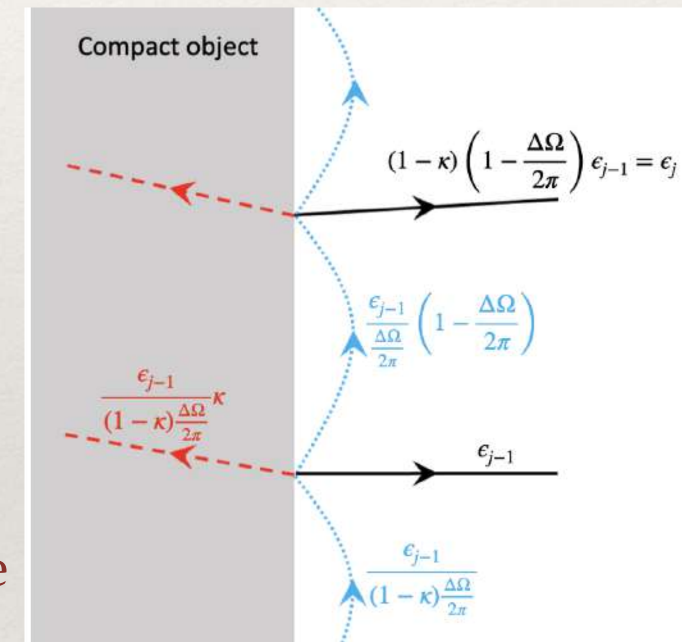


The minimum surface luminosity expected at infinity L∞ can be estimated as

$$L_\infty > \eta \dot{M} \;\text{ where }\; \eta = \dot{E}/\dot{M}$$

An upper bound on the observed luminosity can then be translated into a constraint on the η parameter. From ETH we know $\eta < 10^{-2}$

How this translates on a bound on the relative compactness $\mu = 1 - 2M/r_*$?



Assuming that all the kinetic energy of infalling matter is converted to outgoing radiation, leads to the naive result $\eta = 1 - \sqrt{\mu}$

However, this does not take into account lensing ($\Delta\Omega/2\pi = 27\mu/8 + O(\mu^2)$) and the possibility that part of the radiation is absorbed by the Quasi-BH. I.e. the case $\kappa \neq 0$.

Indeed, one can model the quasi-BH—matter interaction as a series of bounces of the radiation over the surface which have to be summed up.

The net effect is $$\eta(t) = \frac{\dot{E}}{\dot{M}} = \frac{\Delta\Omega}{2\pi}\frac{(1-\kappa)}{\kappa + \frac{\Delta\Omega}{2\pi}(1-\kappa)}\left\{1 - (1-\kappa)^{t/\tau}\left(1 - \frac{\Delta\Omega}{2\pi}\right)^{t/\tau}\right\}.$$

t=time over which SGrA* has been accreting $t \approx T_{\rm Edd} \approx 3.8 \times 10^8$ yr

τ = time for each bounce $\approx O(10M) \sim 10^2$ s

For the physical limit $\tau/T \ll \kappa < 1$ $$\eta = \frac{\Delta\Omega}{2\pi}\frac{1-\kappa}{\kappa + \frac{\Delta\Omega}{2\pi}(1-\kappa)}.$$

So e.g. for $\tilde{\Gamma} = 1 - \kappa = 10^{-5} \Longrightarrow \mu \lesssim 10^{-7}$
or for $\tilde{\Gamma} = 1 - \kappa = 10^{-2} \Longrightarrow \mu \lesssim 1$

So no meaningful upper-bound constraints can be placed for objects with large absorption coefficients

- Zulianello, Carballo-Rubio, SL, Ansoldi: *Phys.Rev.D* 103 (2021) 6, 064071 • e-Print: 2005.01837 [gr-qc]

Extension to rotating BH

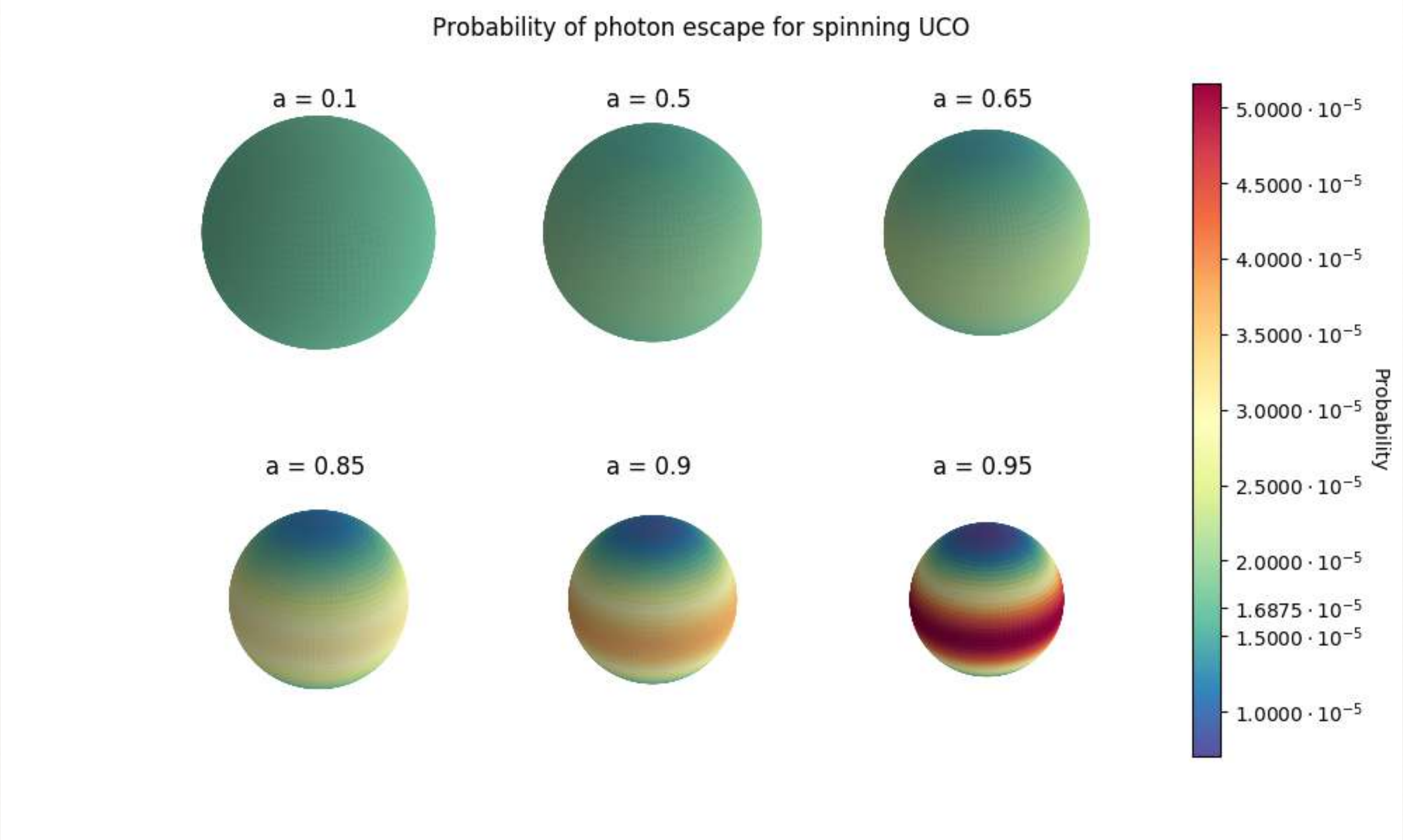


FIG. 11: Visualizations of photon escape probability for different values of a, normalized to the same color scale. The value $P = 1.6875 \cdot 10^{-5}$ corresponds to the case $a = 0$.

The re-emission of radiation can be enhanced or suppressed w.r.t. the non-rotating case if it happens respectively at the equator or at the poles, due to the dependence of the escaping angle to the azimuthal coordinate.

GW channel: Echoes

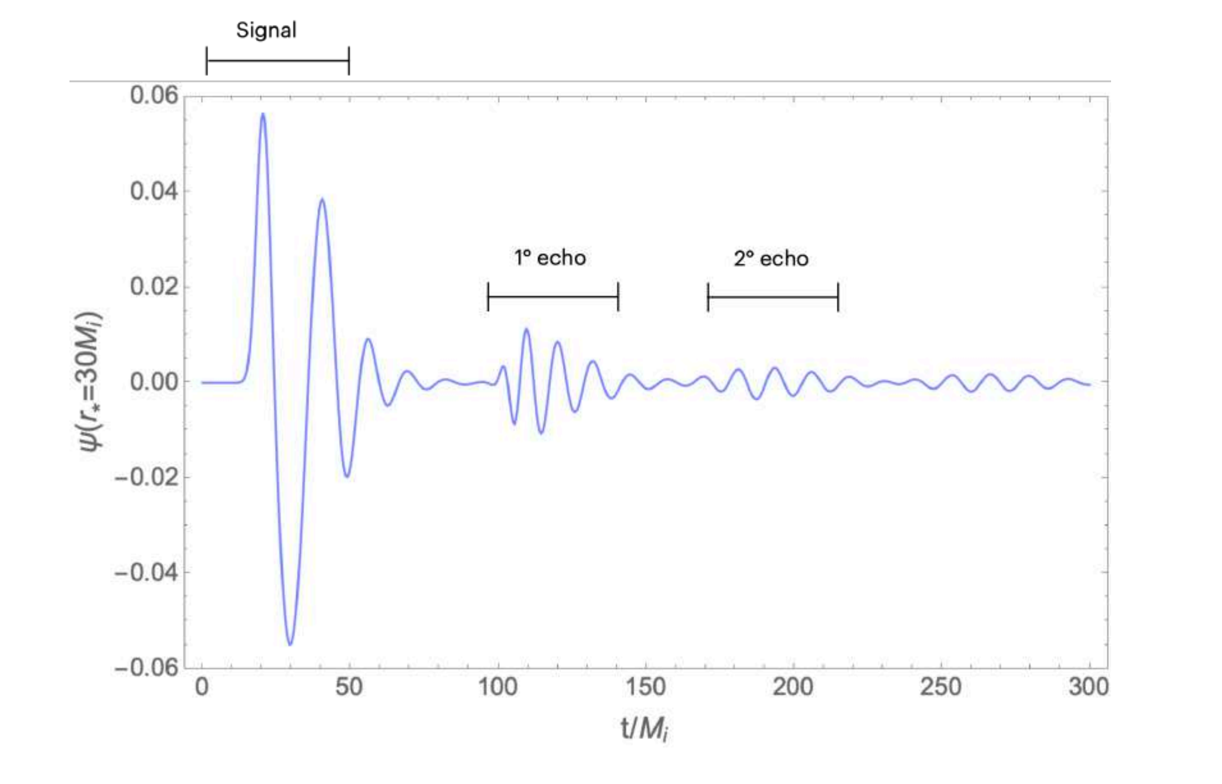
- In the case of a black hole GW scattered back at the potential barrier (usually close to the light ring) are lost inside the horizon.
- For an horizonless object (quasi-BH or traversable wormhole) instead the wave can go through the center and bounce again at the potential barrier with a part transmitted at infinity and one par reflected. This generates "echoes".

Key point: even for ultra compact objects the delay between such echoes is macroscopic (logarithmic scaling).

Time delay for an object of compactness $\Delta = r/2M_0 - 1$

$$\Delta t_{\text{echo}} = 2\int_{r_0=2M_0(1+\Delta)}^{r_{\text{peak}}\approx 3M_0} \frac{\mathrm{d}r}{1-2M_0/r} \approx 2M_0\left[1-2\Delta-2\ln(2\Delta)\right]$$

- **The amplitude of gravitational wave echoes would be proportional to Γ.**
- **A non-observation of echoes can only constrain this parameter.**
- **A positive detection of echoes could be used in order to determine also Δ.**
- **The other two parameters which are relevant for the process are τ^+, which has to be greater than the characteristic time scale of echoes (this would place a very uninteresting lower bound on this quantity), and τ_- which has to be smaller .**



So far searches for quasi-periodic signals…

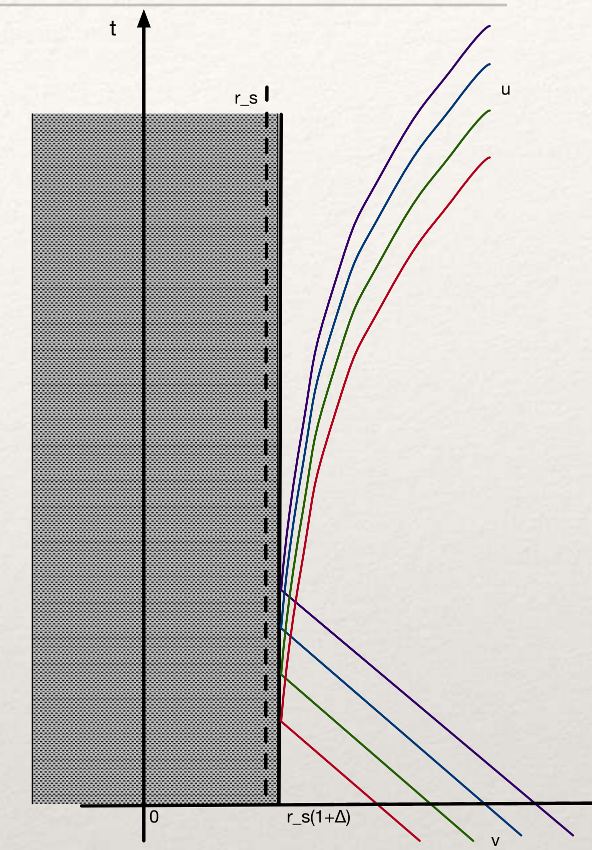
Echos and Non-linear back reaction

NON-LINEAR INTERACTIONS BETWEEN THE GW AND THE CENTRAL OBJECT

- These are neglected in extant analyses. However, this appears to be inconsistent
- **For quasi-BH even modest amounts of accretion will generate a trapped region**
- The formation of a trapping horizon might be avoided by nonlinear interactions

Example: If vacuum polarisations supports a QUasi-BH in Boulware vacuum

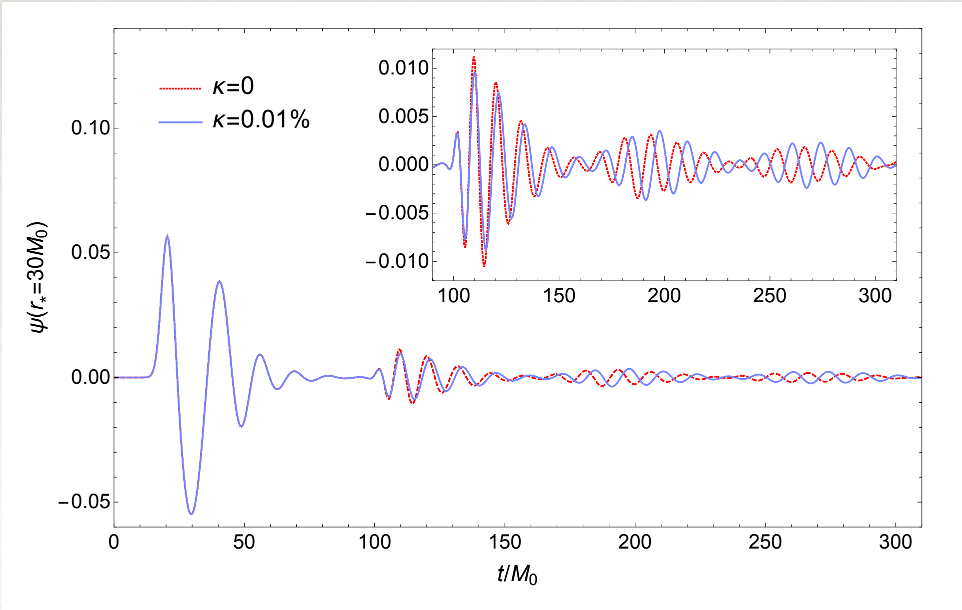
RSET $\propto -\left(1-\dfrac{2M}{r}\right)^{-2}$ so even tiny change 2M→r can generate huge back-reaction.



V. Vellucci, E. Franzin and S. Liberati, ``Echoes from backreacting exotic compact objects,'' arXiv:2205.14170 [gr-qc]].

- The more compact the central object is, the larger is the fraction of the energy stored in the gravitational waves to be transferred through nonlinear interactions. I.e. large absorption $\kappa = 1 - E_{\rm out}/E_{\rm in}$

- A model-independent outcome of these interactions has to be the expansion of the central object in order to avoid the formation of trapping horizons.



- For very compact objects, very small ΔM corresponds to large variations in the compactness.

- So, even for $\kappa \sim 0.01\,\%$ one get noticeable delays between echoes given that the compactness of the object has to increase

Closure

- BH are the new frontier for testing classical and quantum deviations from GR
- Basic arguments from Penrose singularity theorem show that regular spacetime resolutions of singularities are divide in two families depending on the absence/presence of a minimal radius
- For both these families there are related horizonfull and horizonless solutions.
- Ensuing instabilities of inner trapping horizon might lead to rapid evolution and specific long living configuration…
- In any case: avoiding the central singularity appears to generically lead to long range effects (in time or space).
- The resulting black hole mimickers are very hard to exclude with current observations but they are not hopeless and better modelling plus multimessanger astrophysics will be the key to this.

Hopefully, we might be at the dawn of a new form of QG phenomenology based on BH observations!

THANK YOU!


we have seen many ultra compact object in the sky
sure we have seen so many GR black holes!
Haven't we?