Phenomenology of spacetime discreteness:
“Dark energy from energy diffusion"

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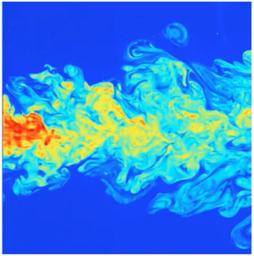
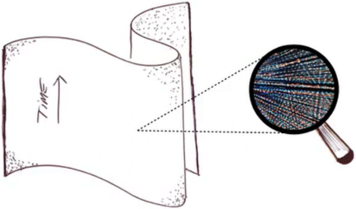
Diffusion from Planckian discreteness

Mathematical description of fluids
(**Navier-Stokes**)



Continuous fluid description breaks down at molecular scales.

Mathematical description of gravity
General Relativity



The continuum spacetime description breaks down at the Planck scale.

Effective violation of energy conservation!

Diffusion in other theories

Energy non-conservation has been claimed to arise in different scenarios:

- **Causal Set theory**[1]

  diffusion in mom. space — discrete causal set $\Rightarrow$ random fluctuations of p^μ

  prob. density diffusion — $\frac{df}{dt} = -\frac{p^i}{E}\partial_i f - (k_1 + k_2)\frac{\partial f}{\partial E} + k_1 E \frac{\partial^2 f}{\partial E^2}$

- **Non-unitary QM**[2]

  Kossakowski-Lindblad eq. — $\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\sum_\alpha \lambda_\alpha \left[\hat{K}_\alpha, \left[\hat{K}_\alpha, \hat{\rho}\right]\right]$

  average energy — $\langle E \rangle \equiv \mathrm{Tr}[\hat{\rho}\hat{H}] \quad \Longrightarrow \quad \langle \dot{E} \rangle \neq 0$

- **Stochastic Gravity**[3]

  2^{nd} order geometry — $T(\mathscr{M}) \longrightarrow T_2(\mathscr{M}) \ni v = v^\mu \partial_\mu + v^{\mu\nu}\partial_\mu \partial_\nu$

  Einstein Anomaly — $\nabla_\mu T^{\mu\nu} \neq 0$

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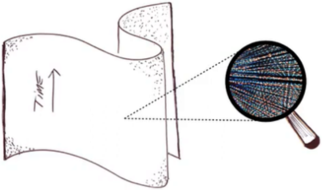
[1] D. Sorkin *et al.* “Energy momentum diffusion from spacetime discreteness” (2009)

[2] A. Kossakowski, “On quantum statistical mechanics of non-Hamiltonian systems” (1972)

[3] F. Kuipers, “Spacetime Stochasticity and Second Order Geometry” (2022)

Brownian Hypothesis:



smooth spacetime description breaks at some scale

$\Downarrow$

effective dissipation

Question:

how to accommodate **effective dissipation** in a metric theory of gravity?

- In **General Relativity** by construction $\nabla_\mu T^{\mu\nu} = 0$
- We need some wise modification..

Unimodular Gravity

UG is a mild modification of GR,[4] energy conservation not guaranteed $J_\mu \equiv \nabla^\nu T_{\mu\nu} \neq 0$

- UG (1919)

$$\boxed{R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T}$$

The equations of UG can be rearranged in an Einstein-like fashion:[5] a Λ_{eff} emerges

- UG+J≠0 (2016)

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R + \underbrace{\left[\Lambda_0 + \int_\ell J\right]}_{\Lambda_{eff}} g_{\mu\nu} = T_{\mu\nu}$$

That's ***Unimodular Gravity with diffusion***: energy dissipation feeds a dynamical Λ_{eff}

- if no *diffusion mechanism* is at play it reduces to GR with a CC not put by hand
- new interpretation of *dark energy*: register of energy non-conservation in the past!
- another nice feature: in UG vacuum fluctuations do not gravitate!

[4] A. Einstein, "Do gravitational fields play an essential role in the structure of elementary particles?" (1919)
[5] T. Josset, A. Perez, D. Sudarsky "Dark Energy from Violation of Energy Conservation" (2016)

Unimodular Cosmology

In cosmology UG predicts a **time-dependent cosmological constant**:

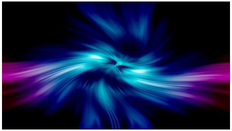
$$\Lambda(\tau) \equiv \Lambda_{in} + \int_{\tau_{in}}^{\tau} J_0(t)dt$$

UG gives two **cosmological equations** in 3 unknown functions $\rho(\tau), a(\tau), \Lambda(\tau)$

$$\begin{cases} 3\left(\frac{a'}{a}\right)^2 = \rho + \Lambda(\tau) & \textit{unimodular Friedmann eq.} \\ \rho' + 3(1+\omega)\frac{a'}{a}\rho = \Lambda'(\tau) & \textit{unimodular continuity eq.} \end{cases}$$

To obtain a specific model of **Unimodular Cosmology** we need **diffusion equation**

- encodes *energy non-conservation* $J_0 \neq 0$ or $\Lambda' \neq 0$
- that's the big deal: *QG-phenomenology*



Addressing cosmological problems

The main literature of Dissipative Unimodular Cosmology has followed this recipe:

1) **Choose model**
    - inspired by more fundamental physics (CS, MQM)
    - or just guessed (phenomenological models)
2) **Derive dynamics**
    - combine diffusion eq. with unimodular eqs
    - solve for $\rho(\tau), a(\tau), \Lambda(\tau)$
3) **Solve problems**
    - set appropriate initial conditions
    - tune free parameter(s) to fix ΛCDM

Some phenomenological models have been proposed recently, addressing:

CC problem [6], *H_0-tension* [7], *Inflation with no inflaton* [8]

[6] A. Perez and D. Sudarsky, "Dark energy from quantum gravity discreteness" (2019)
[7] A. Perez, D. Sudarsky, E. Wilson-Ewing, "Resolving the H0 tension with diffusion" (2021)
[8] L. Amadei, A. Perez, "Planckian discreteness as seeds for cosmic structure" (2022)

Experimental tests of Quantum Gravity?

Key hypothesis: the low energy limit of QG is not GR, but UG with dissipation

$$\text{fundamental QG} \quad \longrightarrow \quad \text{GR} + \text{DDE (diffusion)}$$

QG-phenomenology: if so, one might be able to test QG by cosmological observation!

$$\text{QG} \longrightarrow \text{effective dissipation} \longrightarrow \text{dynamical } \Lambda \longrightarrow \text{imprints on } (\Lambda_{obs}, j_0, H_0, ..)$$

Constraints from j_0

The **luminosity distance** can be expanded in redshift (all model independent)

$$d_L(z) = H_0^{-1} z + \frac{1}{2H_0}(1-q_0)\,z^2 - \frac{1}{6H_0}\left(1 - q_0 - 3q_0^2 + j_0\right) z^3 + \dots$$

deceleration parameter

$$q_0 \equiv -\frac{1}{H_0^2 a_0}\frac{d^2 a(\tau)}{d\tau^2}\bigg|_{\tau=\tau_0}$$

jerk parameter

$$j_0 \equiv \frac{1}{H_0^3 a_0}\frac{d^3 a(\tau)}{d\tau^3}\bigg|_{\tau=\tau_0}$$

In **Unimodular Cosmology** with DE dominating on $P = 0$ matter (model dependent)

$$q_0 = -1 + \frac{3}{2}\Omega_M^0$$

$$\boxed{j_0 = 1 + \frac{\dot{\Lambda}(0)}{2H_0^3}}$$

constraint via jerk: if QG-induced dissipation is active today then $j_0 \neq 1$ (seems so[9])

[9] E.Lusso, "Tension with ΛCDM model from H-diagram of supernovae, quasars, and gamma-ray bursts"(2019)

Constraints from Λ_{obs}

- ***Causal set approach***[10] (CS)
  - diffusion m.less particles $\Lambda'(\tau) = -\xi_{\mathrm{CS}} \rho_\gamma(\tau)$
  - effective CC today $\Lambda_0 \cong \Lambda_{ls} + \frac{\xi_{\mathrm{CS}}}{6\times 10^{-19}\ \mathrm{s}^{-1}} \Lambda_{\mathrm{obs}}$

constraint via Λ_{obs}: $\boxed{\xi_{\mathrm{CS}} \sim 10^{-19} \mathrm{s}^{-1}}$

- ***Continuous spontaneous localization***[10] (CSL)
  - diffusion of barions $\Lambda'(\tau) = -\xi_{\mathrm{CSL}} \rho_b(\tau)$
  - effective CC today $\Lambda_0 \cong \Lambda_h + \frac{\xi_{\mathrm{CSL}}}{4.3\times 10^{-31} \mathrm{s}^{-1}} \Lambda_{\mathrm{obs}}$

constraint via Λ_{obs}: $\boxed{\xi_{\mathrm{CSL}} \sim 10^{-31}\ \mathrm{s}^{-1}}$

- ***Modified geodesic frictional force*** [11] (MG)
  - diffusion m.s. particles $J_\nu = m \int F_\nu f_T \mathrm{D}p \mathrm{D}s_r$
  - effective CC today $\Lambda_0 \cong \Lambda_{ew} + 4\alpha_{\mathrm{MG}} \Lambda_{obs}$

constraint via Λ_{obs}: $\boxed{\alpha_{\mathrm{MG}} \sim 1}$

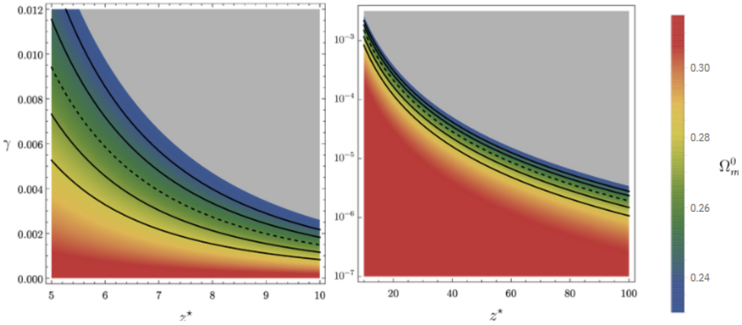
[10] T. Josset, A. Perez, D. Sudarsky "Dark Energy from Violation of Energy Conservation" (2016)

[11] A. Perez, D. Sudarsky, "Dark energy from quantum gravity discreteness" (2019)

Constraints from H_0

- ***Anomalous Decay of matter density*** [12]
  - diffusion matter density $\Omega_M(z) = \Omega_M^0 (1+z)^3 \left[\theta_+(z-z^\star) + \left(\frac{1+z}{1+z^\star}\right)^\gamma \theta_-(z-z^\star)\right]$
  - inferred Hubble constant $H_0 = \frac{\theta_s}{r_s} \int_0^{z_{\rm LS}} \frac{\mathrm{d}z}{\sqrt{\Omega_R(z)+\Omega_M(z)+\Omega_\Lambda(z)}}$

constraint via H_0: $(z^\star, \gamma)$ *such that*



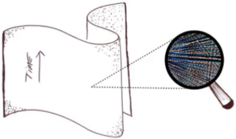
[12] A. Perez, D. Sudarsky, E. Wilson-Ewing, "Resolving the H_0 tension with diffusion" (2021)

Recap and conclusions

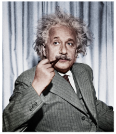
Three motivations of **Unimodular Gravity with diffusion**:

- ***New interpretation of dark energy*** register non-conservation in the past history
- ***New cosmology*** $\Lambda(t)$***-CDM*** potentially fixes issues of Λ-*CDM*
- ***New channels QG-phenomenology****?* small local effects, important changing CC

$$\text{QG} \longrightarrow \text{effective dissipation} \longrightarrow \text{dynamical } \Lambda \longrightarrow (\Lambda_{obs}, j_0, H_0, ..)$$



brownian hp



unimodular gravity

computations

The effective CC is related to the total energy of the 'matter+geometry' system!

- *Phase-space parameterization of unimodular cosmology:* $a \longrightarrow x \equiv a^3$

geometry $S_{geo} = V_0 \int 3a^4 \dot{a}^2 dt$

matter $S_{mat} = -\frac{1}{2} V_0 \int a^6 \dot{\phi}^2 dt$

$$H = \underbrace{\left(\frac{1}{3} V_0 \dot{x}^2\right)}_{H_{geo}} - \underbrace{\left(\frac{1}{2} V_0 x^2 \dot{\phi}^2\right)}_{H_{mat}}$$

- *Proper definition of effective cosmological constant:*

Friedmann eq. $H = E \quad \Longleftrightarrow \quad 3\left(\frac{a'}{a}\right)^2 = \rho + \Lambda$

$$\boxed{\Lambda_{eff} \equiv \frac{E}{V_0}}$$

If we want a dynamical Λ_{eff} we need extra terms in the Hamiltonian!!

The formal recipe of diffusion

We must postulate **hidden degrees of freedom**:

- *Hidden Hamiltonian*

$$H_{hidden} = \underbrace{H_{hid}}_{\text{store energy}} + \underbrace{V_{mat\leftrightarrow hid}}_{\text{interaction}} + \underbrace{V_{geo\leftrightarrow hid}}_{\text{interaction}}$$

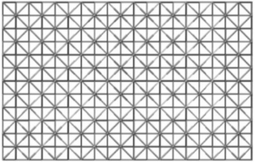
  - *Tot Hamiltonian* $H_T = H_{geo} + H_{mat} + H_{hidden}$
  - *Effective lambda* $\Lambda_{eff} \equiv \frac{1}{V_0}(E - H_{hidden})$

Theorem

Diffusion in UG requires a hidden Hamiltonian, but this can, at most, be of the form

$$\begin{aligned} H_{hidden}(\cancel{x, \dot{x}}, \phi, \dot{\phi}, Q_\alpha, \dot{Q}_\alpha, t) = H_{hid}(Q_\alpha, \dot{Q}_\alpha) + \\ + V_{hid\leftrightarrow mat}(Q_\alpha, \dot{Q}_\alpha, \phi, \dot{\phi}, t) + \\ \cancel{+ V_{hid\leftrightarrow geo}(Q_\alpha, \dot{Q}_\alpha, x, \dot{x}, t)} \end{aligned}$$

QG-defects not the fundamental ones! Just another type of emergent low-energy dofs!



Fundamental Discreteness

Quantum Gravity Defects

Smooth Spacetime

"Collective deviations from perfect smoothness, capture only effectively underlying discreteness"

Key hypothesis: the low energy limit of QG is not GR, but UG with dissipation

$$\text{fundamental QG} \quad \longrightarrow \quad \text{GR} + \text{DDE (diffusion)}$$

UG with dissipation: the tot Hamiltonian is $H_T = H_{geo} + H_{mat} + H_{def} + V_{mat \leftrightarrow def}$

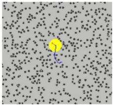
$$\text{QG dof} \quad \longrightarrow \quad \text{GEO dof} + \text{MAT dof} + \text{DEF dof}$$

The toy model of ohmic diffusion

Ideal model used in complex systems to study Brownian motion and diffusion

- defects as a ***bath of harmonic oscillators*** for matter

$$H_{hidden} = \sum_\alpha \left(\frac{1}{2} \dot{Q}_\alpha^2 + \frac{1}{2} \omega_\alpha^2 Q_\alpha^2 \right) + \sum_\alpha b_\alpha(t)\ \phi\ Q_\alpha$$



- the EoM for ϕ is Langevin-like $\Rightarrow$ ***diffusion equation***

$$x^2 \ddot{\phi} + 2x\dot{x}\dot{\phi} - m_p \beta \dot{\phi} + M^2 \phi = \xi(t)$$

The ohmic model yields a ***well defined set of equations for unimodular cosmology***!

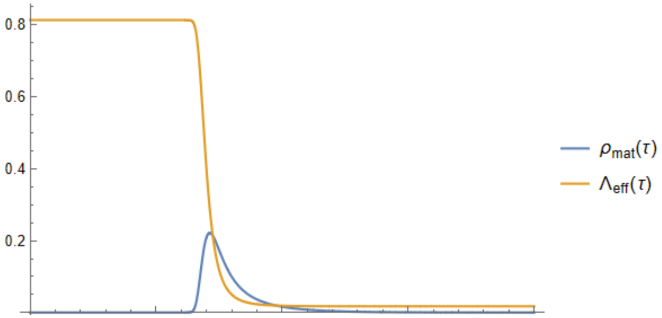
- *Diffusion equation* $\boxed{\Lambda'_{eff} = -\beta\ \rho / a^3}$

- *Diffusion parameter* $\beta = b \frac{V_{Planck}}{V_{defects}}$

'de Sitter de better': inflation and late-time acceleration

The solutions suggest an interesting modified cosmology

- **Inflation with no inflaton**: Λ_{eff} relaxation gives a suitable inflation era
- **Late-time acceleration**: the dynamics reduces to the standard one
- **No singularity**: it's avoided, $a(\tau) \to 0$ in the past only asymptotically



Further study of solutions and general idea QG-defects in a work-in-progress paper [13]

[13] P. Pellecchia, A. Perez "Diffusive effects of Planckian discreteness: the thermal bath of the QG-defects" (expected June 2024)

Conclusions

Three motivations of **Unimodular Gravity with diffusion**:

- ***New interpretation of dark energy*** register non-conservation in the past history
- ***New cosmology*** $\Lambda(t)$-*CDM* potentially fixes issues of Λ-*CDM*
- ***New channels QG-phenomenology*** small local effects, important changing CC

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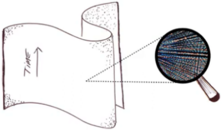

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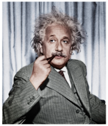
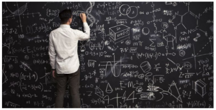


unimodular gravity



computations

The thermal bath of the **Quantum-Gravity defects**:

- ***Formal paradigm for diss. in UC*** register non-conservation in the past history

$$\text{QG dof} \longrightarrow \text{GEO dof} + \text{MAT dof} + \text{DEF dof}$$

Thanks for your attention

Diffusion from deformed symmetries

Conservation of charges is related to spacetime symmetries via the Noether theorem.

Macroscopic regime: classical theory with no length scale

- time and spacial translations $\Longrightarrow$ p^0,p^i of a particle are the charges
- conservation law for a 2-particles process: $p^\mu_{out} + q^\mu_{out} = p^\mu_{in} + q^\mu_{in}$

Microscopic regime: fundamental theory with a length scale ℓ

- deformed symmetries (e.g. κ-Poincarè) $\Longrightarrow$ generalized Noether theorem
- modified conservation law: $p^\mu_{out} \oplus q^\mu_{out} = p^\mu_{in} \oplus q^\mu_{in}$

Mesoscopic regime: classical theory plus dissipation effects

- *bicrossproduct composition* $\quad \vec{p} \oplus \vec{q} \equiv \vec{p} + e^{\ell p^0}\, \vec{q} = \vec{p} + \vec{q} + \ell p^0 \vec{q} + \mathscr{O}(\ell p^0)^2$
- *(non) conservation law* $\quad \vec{p}_{out} + \vec{q}_{out} = \vec{p}_{in} + \vec{q}_{in} - \ell p^0_{in} \left(\frac{p^0_{out}}{p^0_{in}} \vec{q}_{out} - \vec{q}_{in} \right)$

Interpretation: the term in ℓp^0_{in} can be 'lost' into microscopic dof (*effective violation*)