THEORY AND PHENOMENOLOGY OF LIGHTLIKE K-MINKOWSKI

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QUAGRAP meeting 2024

Motivation

- Fundamental localization limit from the interplay of the equivalence and uncertainty principle (Bronstein)

- Uncertainty principle allows for sharp localization of quantum fields, depending on the charge-to-mass ratio

- The equivalence principle dictates that the gravitational charge is identified with the inertial mass → intrinsic limit to localizability of the gravitational field of order $\ell_p \approx 10^{-35}m$.

G. E. Gorelik, "Matvei bronstein and quantum gravity: 70th anniversary of the unsolved problem," Physics-Uspekhi, vol. 48

Motivation

- Formal approaches to quantum gravity can be effectively described by means of noncommutative geometry: $[x^{\mu}, x^{\nu}] \neq 0$

- Motivation for investigating the noncommutative spacetime hypothesis as an effective regime of quantum gravity, regardless of the full underlying theory.

- Several phenomenological opportunities from both toy models and more formal approaches. In this talk: deviations from the Pauli exclusion principle from noncommutative QFT and effects on cold atoms from phenomenological models .

G. Amelino-Camelia “Quantum Spacetime Phenomenology”, Liv. Rev. Rel. 16 (2013) 5

PART I: MULTIPARTICLE STATES IN BRAIDED LIGHTLIKE κ-MINKOWSKI NONCOMMUTATIVE QFT

G. Fabiano and F. Mercati, "Multiparticle states in braided lightlike κ-Minkowski noncommutative QFT", *Phys.Rev.D* 109 (2024) 4, 046011

κ-Minkowski noncommutative spacetime

- Commutation relations of κ-Minkowski non-commutative space-time

$$[x^\mu, x^\nu] = i\,\ell(v^\mu x^\nu - v^\nu x^\mu)$$

- Covariance under a symmetry transformation

$$x'^\mu = \Lambda^\mu_\nu \otimes x^\nu + a^\mu \otimes 1$$

$$[x'^\mu, x'^\nu] = i\ell(v^\mu x'^\nu - v^\nu x'^\mu)$$

κ-Poincaré quantum group

- Poincaré group parameters are promoted to operators

$$[\Lambda^{\mu}_{\nu}, \Lambda^{\rho}_{\sigma}] = 0 \qquad [a^{\mu}, a^{\nu}] = i\ell(v^{\mu}a^{\nu} - v^{\nu}a^{\mu})$$

$$[a^{\gamma}, \Lambda^{\mu}_{\nu}] = i\ell[(\Lambda^{\mu}_{\alpha}v^{\alpha} - v^{\mu})\Lambda^{\gamma}_{\nu} + (\Lambda^{\alpha}_{\nu}g_{\alpha\beta} - g_{\nu\beta})v^{\beta}g^{\mu\gamma}]$$

$$\Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}g^{\alpha\beta} = g^{\mu\nu} \qquad \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}g_{\mu\nu} = g_{\alpha\beta}$$

Lightlike κ-Minkowski

- It has been shown that the covariance of N-point functions under κ-Poincaré requires $g_{\mu\nu}v^{\mu}v^{\nu} = 0$

- Lightlike κ-Minkowski commutation relations

$$[x^+, x^-] = 2i\,\ell x^- \qquad [x^+, x^j] = 2i\,\ell x^j \qquad i, j = 2,3$$

- N-point functions defined in terms of multiple copies of the lightlike κ-Minkowski algebra.

- Coordinate functions of different copies do not commute, but two-point functions only depend on coordinate differences, which are commutative!

F. Lizzi and F. Mercati, Phys. Rev. D, vol. 103, p. 126009, 2021

Lightlike κ-Poincaré Hopf Algebra

- Generalization of the Poincaré Lie algebra, dual to the κ-Poincaré group

$$[N, P_+] = iP_+ \qquad [N, P_-] = -iP_- \qquad C = P_+ P_-$$

$$\Delta P_+ = P_+ \otimes 1 + 1 \otimes P_+ + 2\ell P_+ \otimes P_+ \qquad S(P_+) = -\frac{P_+}{1 + 2\ell P_+}$$

$$\Delta P_- = P_- \otimes 1 + \frac{1}{1 + 2\ell P_+} \otimes P_- \qquad S(P_-) = -(1 + 2\ell P_+)P_-$$

$$\Delta N = N \otimes 1 + \frac{1}{1 + 2\ell P_+} \otimes N \qquad S(N) = -(1 + 2\ell P_+)N$$

Ballesteros, A. and Herranz, F.J. and del Olmo, M.A. and Santander, Physics Letters B (1995)

Noncommutative plane waves

- Plane waves serve as the basis for noncommutative functions

$$E[\xi] := e^{i\xi_- x^-} e^{\frac{i}{2}\ln(1+2\ell\xi_+)x^+}$$

$$P_\pm E[\xi] = \xi_\pm E[\xi]$$

$$E[\xi]E[\eta] = E[\xi \oplus \eta]$$

$$E^\dagger[\xi] = E[S(\xi)]$$

$$(\xi \oplus \eta)_+ = \xi_+ + \eta_+ + 2\ell\xi_+\eta_+$$

$$S(\xi)_+ = -\frac{\xi_+}{1+2\ell\xi_+}$$

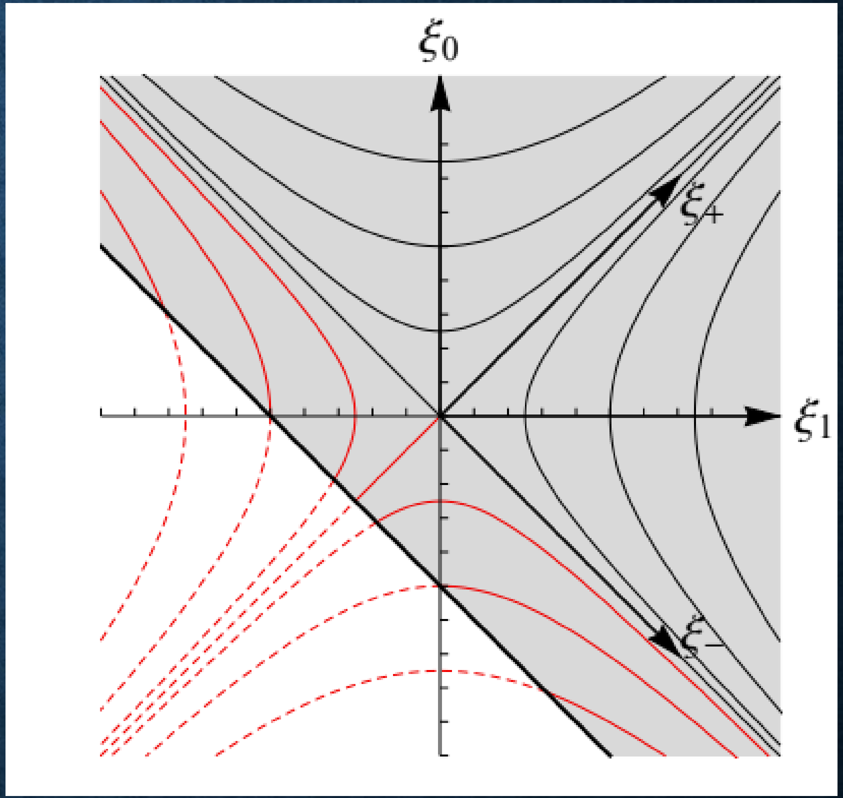
$$(\xi \oplus \eta)_- = \xi_- + \frac{\eta_-}{1+2\ell\xi_+}$$

$$S(\xi)_- = -(1+2\ell\xi_+)\xi_-$$

New type plane waves

- $E[\xi]$ with $\xi_+ < -\frac{1}{2\ell}$ are needed to parametrize the other half of momentum space

- They need to be included to construct a lightlike κ-Poincaré invariant Pauli Jordan function to perform covariant quantization



M. G. Di Luca and F. Mercati, Phys. Rev. D, vol. 107, no. 10, p. 105018, 2023

On-shell plane waves and Pauli-Jordan function

- Pauli Jordan function is constructed in terms of on-shell plane waves, with a=1,2 referring to different copies of the noncommutative algebra

$$e_a[\xi] := e^{i\frac{m^2}{\xi_+}x_a^-} e^{\frac{i}{2}\ln(1+2\ell\xi_+)x_a^+} \qquad \epsilon_a[\xi] := e^{i\frac{m^2}{\xi_+}x_a^-} e^{\frac{i}{2}\ln(1+2\ell\xi_+)x_a^+}$$

$$\xi_+ > 0 \qquad \xi_+ < -\frac{1}{2\ell}$$

- The Pauli-Jordan function turns out to be

$$\Delta_{PJ}(x_1 - x_2) = -\int_0^\infty \frac{d\xi}{2\xi} e_1(\xi) e_2^\dagger(\xi) + \int_0^\infty \frac{d\xi}{2\xi(1+2\ell\xi)} e_1^\dagger(\xi) e_2(\xi) + $$

$$-\int_{-\infty}^{-\frac{1}{2\ell}} \frac{d\xi}{2\xi} \epsilon_1(\xi) \epsilon_2^\dagger(\xi)$$

- It can be shown to be equal to the one employed in the commutative case

Covariant quantization

- Noncommutative complex scalar field

$$\phi(x_a) = \int_0^\infty \frac{d\xi}{2\xi}\left[\frac{1}{1+2\ell\xi} a(\xi) e_a^\dagger(\xi) + b^\dagger(\xi) e_a(\xi)\right] +$$
$$+ \int_{-\infty}^{-\frac{1}{2\ell}} \frac{d\xi}{2\xi} \frac{1}{1+2\ell\xi} \alpha(\xi) \epsilon_a^\dagger(\xi)$$

- Covariant quantization

$$\left[\phi(x_1), \phi^\dagger(x_2)\right] = \Delta_{\mathrm{P}J}(x_1 - x_2) \qquad [\phi(x_1), \phi(x_2)] = \left[\phi^\dagger(x_1), \phi^\dagger(x_2)\right] = 0$$

The role of the R-matrix

- The covariant quantization prescription involves products of on-shell plane waves to be reordered

$$e_2(\eta)e_1(\xi) =_{*\circ} R\left(e_1(\xi)\otimes e_2(\eta)\right) = e_1(\xi + 2\ell\eta\xi)e_2\left(\frac{\eta}{1+2\ell\xi+4\ell^2\eta\xi}\right)$$

- Where the R-matrix is a Hopf Algebra element defined as

$$R = e^{-i\ln(1+2\ell P_+)\otimes N}e^{iN\otimes\ln(1+2\ell P_+)}$$

- It will be crucial in defining multiparticle states!

G. Fabiano and F. Mercati, “Multiparticle states in braided lightlike κ-Minkowski noncommutative QFT”, *Phys.Rev.D* 109 (2024) 4, 046011

Deformed bosonic oscillator algebra

- Upon quantization, we obtain the following deformed bosonic algebra

$$a(\xi)a(\eta) = a\left(\frac{\eta}{1+2\ell\xi+4\ell^2\xi\eta}\right)a(\xi+2\ell\xi\eta)$$

$$\xi,\eta > 0$$

$$a^\dagger(\xi)a^\dagger(\eta) = a^\dagger(\eta+2\ell\xi\eta)a^\dagger\left(\frac{\xi}{1+2\ell\eta+4\ell^2\xi\eta}\right)$$

$$a(\xi)a^\dagger(\eta) - \frac{(1+2\ell\xi)(1+2\ell\eta)}{1+2\ell(\xi+\eta)}a^\dagger\left(\frac{\eta}{1+2\ell\xi}\right)a\left(\frac{\xi}{1+2\ell\eta}\right) = 2\eta(1+2\ell\eta)\delta(\xi-\eta)$$

Representation of creation and annihilation operators

- Nonlinear redefinition of creation and annihilation operators of commutative QFT

$$\left[c(\xi), c^\dagger(\eta)\right] = 2\xi\delta(\xi - \eta) \qquad [c(\xi), c(\eta)] = \left[c^\dagger(\xi), c^\dagger(\eta)\right] = 0$$

$$a(\xi) = \sqrt{1 + 2\ell\xi}\, B_{S(\xi)} c(\xi) = \sqrt{1 + 2\ell\xi}\, c(-S(\xi)) B_{S(\xi)}$$

$$a^\dagger(\xi) = \sqrt{1 + 2\ell\xi}\, c^\dagger(\xi) B_\xi = \sqrt{1 + 2\ell\xi}\, B_\xi c^\dagger(-S(\xi))$$

- where

$$B_\xi = \exp(i \ln(1 + 2\ell\xi) N)$$

Single particle states

- Definition of single particle states starting from the vacuum of the ordinary theory

$$c(\xi)|0\rangle = 0 \rightarrow a(\xi)|0\rangle = \sqrt{1+2\ell\xi}\; c(-S(\xi))B_{S(\xi)}|0\rangle = 0$$

$$\frac{a^{\dagger}(\xi)}{\sqrt{1+2\ell\xi}}|0\rangle = c^{\dagger}(\xi)B_{\xi}|0\rangle = |\xi\rangle$$

$$P_{+}|\xi\rangle = \xi|\xi\rangle \qquad P_{-}|\xi\rangle = \frac{m^2}{\xi}|\xi\rangle$$

- The single particle Fock space is equivalent to that of the commutative theory

Multi-particle states

- They are defined in the tensor product of two single-particle Fock spaces

$$|\xi\rangle \otimes |\eta\rangle = \frac{a^{\dagger}(\xi)}{\sqrt{1+2\ell\xi}}|0\rangle \otimes \frac{a^{\dagger}(\eta)}{\sqrt{1+2\ell\eta}}|0\rangle$$

$$\Delta P_{\pm}(|\xi\rangle \otimes |\eta\rangle) = (\xi \oplus \eta)_{\pm}(|\xi\rangle \otimes |\eta\rangle)$$

- In the ordinary theory, a two-boson state is of the form:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\xi\rangle \otimes |\eta\rangle + |\eta\rangle \otimes |\xi\rangle)$$

- However

$$\Delta P_{\pm}(|\eta\rangle \otimes |\xi\rangle) \neq \Delta P_{\pm}(|\xi\rangle \otimes |\eta\rangle)$$

Multi-particle states

- Observe that

$$\tilde{R}(|\xi\rangle \otimes |\eta\rangle) := R \circ \sigma(|\xi\rangle \otimes |\eta\rangle)$$
$$= e^{-2i\, ln(1+2\ell P_+)\otimes N} e^{2i\, N \otimes ln(1+2\ell P_+)} (|\eta\rangle \otimes |\xi\rangle) =$$
$$= |\eta + 2\ell\eta\xi\rangle \otimes |\frac{\xi}{1 + 2\ell\eta + 4\ell^2\eta\xi}\rangle$$

- These states are such that

$$\Delta P_\pm(|\xi\rangle \otimes |\eta\rangle) = \Delta P_\pm(\tilde{R}(|\xi\rangle \otimes |\eta\rangle))$$

- The deformed flip operator is involutive

$$\tilde{R}(|\eta + 2\ell\eta\xi\rangle \otimes |\frac{\xi}{1 + 2\ell\eta + 4\ell^2\eta\xi}\rangle) = |\xi\rangle \otimes |\eta\rangle$$

κ-Poincaré covariance of two particle states

- We can define

$$S^+ = \frac{1}{2}(1 \otimes 1 + \tilde{R}) \qquad (S^+)^2 = S^+$$

- Two-particle states can be written as

$$\sqrt{2} S^+(|\xi\rangle \otimes |\eta\rangle) = \frac{1}{\sqrt{2}}(|\xi\rangle \otimes |\eta\rangle + |\eta + 2\ell\eta\xi\rangle \otimes |\frac{\xi}{1 + 2\ell\eta + 4\ell^2\eta\xi}\rangle)$$

- The construction is κ-Poincaré covariant since it can be shown that

$$\tilde{R} e^{i\tau\Delta N} = e^{i\tau\Delta N} \tilde{R}$$

Distinguishability of identical particles

- Decay process of a particle with momentum $\left(\Pi, \frac{M^2}{\Pi}\right)$ into two identical particles of mass m

$$\Pi = \xi + \eta + 2\ell\xi\eta \qquad \frac{\mathrm{M}^2}{\Pi} = \frac{m^2}{\xi} + \frac{1}{1+2\ell\xi}\frac{m^2}{\eta}$$

- Final two-particle state

$$|L1\rangle \otimes |R1\rangle + |L2\rangle \otimes |R2\rangle$$

$$L1 = \frac{\Pi}{2}\left(1 + \sqrt{1 - \frac{4m^2}{M^2}}\right) \qquad R1 = \frac{\Pi}{2}\left(1 - \sqrt{1 - \frac{4m^2}{M^2}}\right) - \frac{2\ell m^2}{M^2}\Pi^2 + O(\ell^2)$$

$$L2 = \frac{\Pi}{2}\left(1 - \sqrt{1 - \frac{4m^2}{M^2}}\right) \qquad R2 = \frac{\Pi}{2}\left(1 + \sqrt{1 - \frac{4m^2}{M^2}}\right) - \frac{2\ell m^2}{M^2}\Pi^2 + O(\ell^2)$$

Deviations from the Pauli exclusion principle

- In the ordinary theory

$$\left(\frac{1-\sigma}{2}\right)|\xi\rangle \otimes |\xi\rangle = 0$$

- In the deformed theory, which are the states such that

$$S^{-} = \frac{1}{2}(1 \otimes 1 - \tilde{R})$$

$$S^{-}(|\xi\rangle \otimes |\eta\rangle) = 0? \rightarrow \eta = -S(\xi) = \frac{\xi}{1+2\ell\xi}$$

- These are Lorentz-covariant, in the sense that

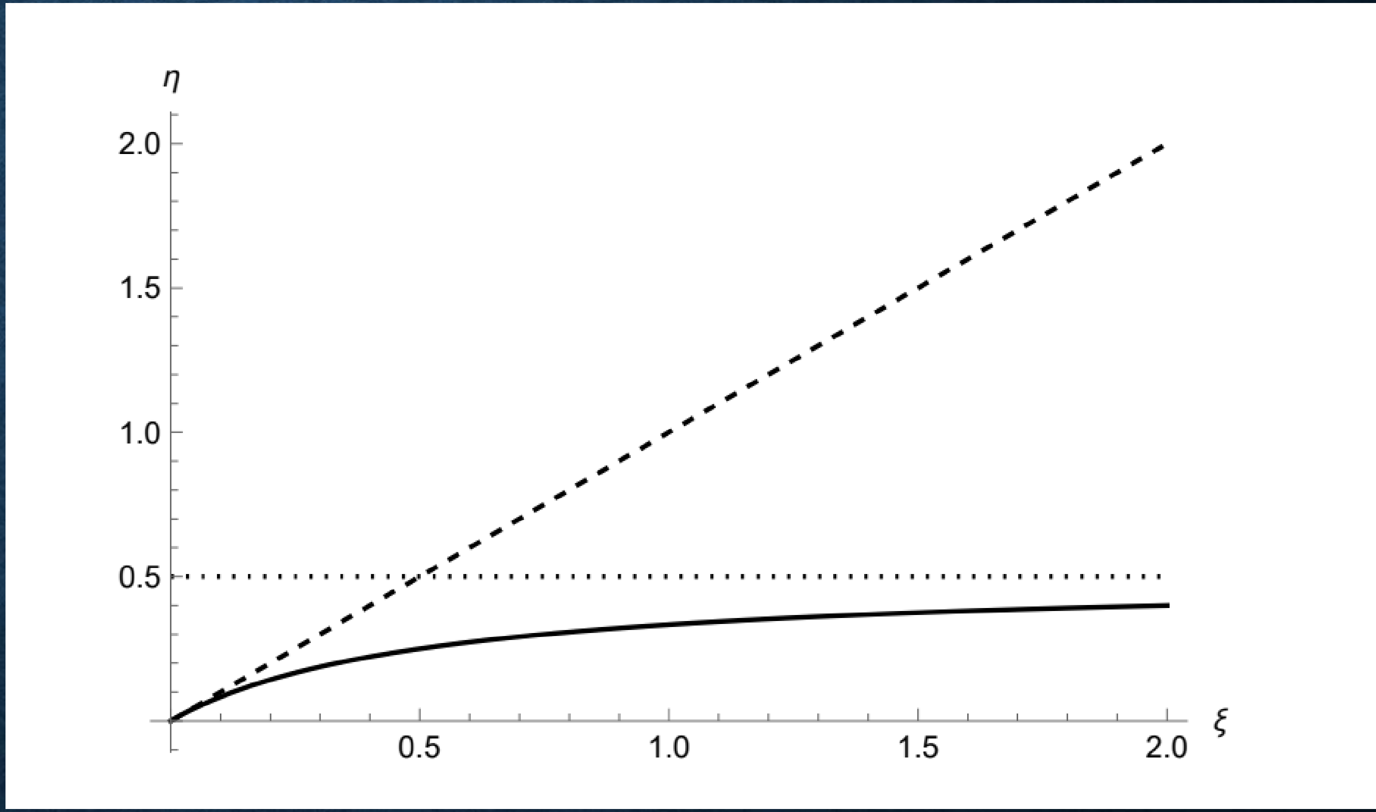
$$e^{i\tau\Delta N}(|\xi\rangle \otimes |{-S(\xi)}\rangle) = |e^{\tau}\xi\rangle \otimes |-S(e^{\tau}\xi)\rangle$$

Deviations from the Pauli exclusion principle

- States of the form

$$S^{-}(|\xi\rangle \otimes |\xi\rangle)$$

are allowed!



PART II: A QUANTUM SPACETIME MODEL WITH IR/UV MIXING AND ITS COLD ATOM PHENOMENOLOGY

G. Amelino-Camelia, G. Fabiano, D. Frattulillo and F. Mercati, in preparation

κ-lightlike Minkowski spacetime

- The coordinate noncommutativity is expressed by

$$[x^0, x^1] = i\ell(x^1 - x^0) \qquad [x^0, x^i] = i\ell x^i \qquad [x^1, x^i] = i\ell x^i \qquad i = 2,3$$

- Ordinary spatial isotropy is spoiled. The model is still relativistic, but with a deformed notion of spatial isotropy (interpretation?)

Ballesteros, A. and Herranz, F.J. and del Olmo, M.A. and Santander, M., Physics Letters B (1995)

Blaut,Daszkiewicz,Kowalski-Glikman –Mod.Phys.Lett. A 18 (2003)

IR/UV mixing

- Mechanism first appeared in studies of non-commutative field theories on Moyal non-commutative space-time

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

- In ϕ^4 theories in $d = 4$, corrections to the propagator yield contributions of the type

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\tilde{p}\right)\frac{k^3}{k^2+m^2} \qquad \tilde{p}_\mu = \theta_{\mu\nu}p^\nu$$

Minwalla,Raamsdonk,Seiberg JHEP 02 (2000)

- In the $\Lambda \gg |\tilde{p}|^{-1}$ limit,

$$\int_0^\Lambda dk \cos\left(\frac{1}{2}k\tilde{p}\right)\frac{k^3}{k^2+m^2} \simeq \frac{1}{2}\left(\frac{2}{|\tilde{p}|^2}\right) - \frac{1}{2}m^2 \ln\left(1 + \frac{\left(\frac{2}{|\tilde{p}|}\right)^2}{m^2}\right)$$

- The UV cutoff scale introduces a dependence of the integral on $\frac{1}{|\tilde{p}|}$, which yields divergence when $|\tilde{p}| \to 0$, in the IR regime. Example of dynamical IR/UV mixing.

κ-lightlike Minkowski kinematics

- The coproduct inspired composition laws, at first order in ℓ, are given by

$$(p \oplus k)_0 = p_0 + k_0 - \frac{\ell}{2}(p_0 + p_1)(k_0 + k_1)$$

$$(p \oplus k)_1 = p_1 + k_1 + \frac{\ell}{2}(p_0 + p_1)(k_0 + k_1)$$

$$(p \oplus k)_i = p_i + k_i(1 - \ell(p_0 + p_1))$$

- The mass-shell relation inspired by the Casimir element, at first order reads

$$m^2 = (p_0 - p_1)^2\left(1 + \frac{\ell(p_0 + p_1)}{2}\right) - (\mathrm{p}_2^2 + \mathrm{p}_3^2)\big(1 + \ell(p_0 + p_1)\big)$$

Kinematical IR/UV mixing

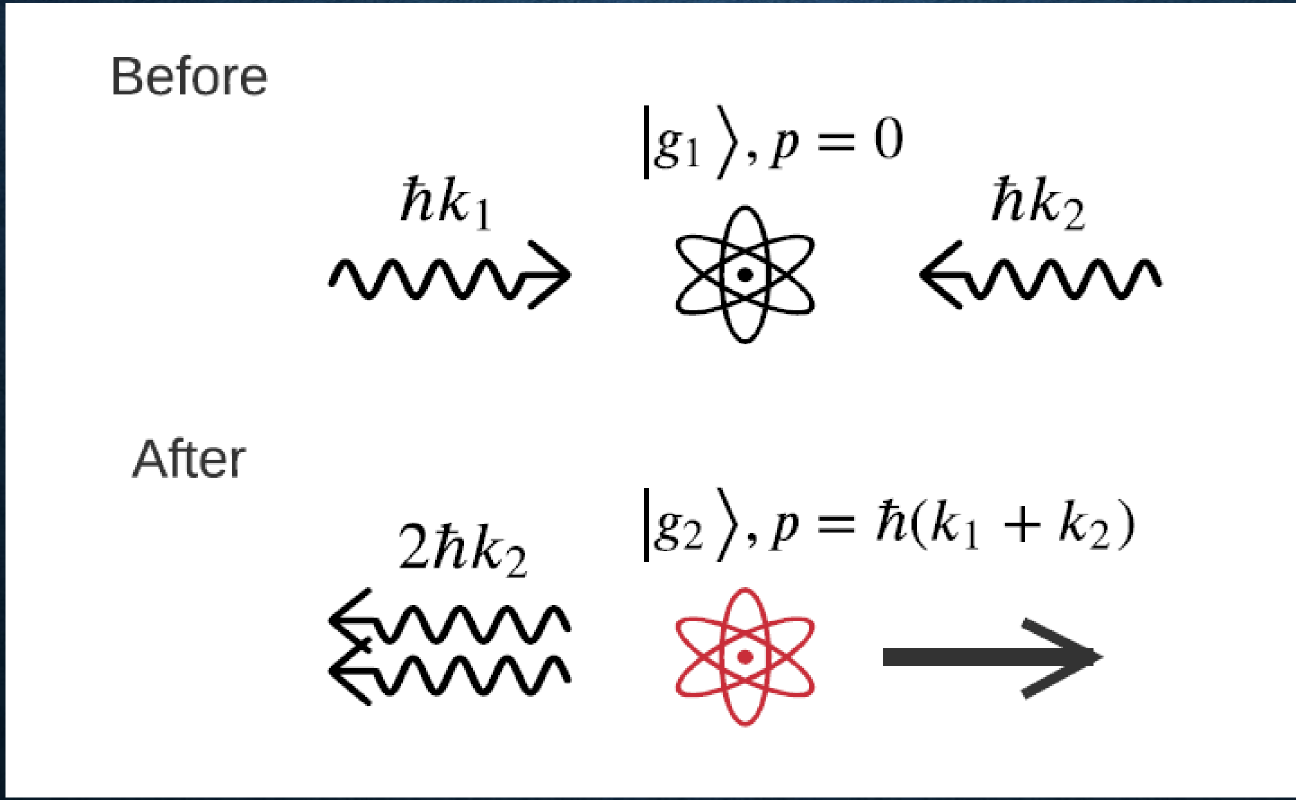
- On-shell relation in the limit $p \ll m$

$$p_0 = m + \frac{\vec{p}^2}{2m} - \frac{\ell}{4}\left[(\boldsymbol{m p_1} - p_2^2 - p_3^2) - \frac{p_1}{2m}(p_1^2 + 3p_2^2 + 3p_3^2)\right]$$

- The UV scale ℓ entails a correction to the energy-momentum dispersion relation which has a term $\propto \ell \boldsymbol{m p_1}$ that is dominant in the IR regime, when $p_1 \to 0$.

Cold atoms phenomenology

- Recoil of an atom in a two photon Raman transition: in cold-atom experiments it is possible to impart momentum to an atom by absorption of a photon of frequency ν and stimulated emission of a photon of frequency ν'.



Cold atoms phenomenology

- By employing standard energy-momentum conservation, and taking into account the resonance frequency for the atom, one can estimate the ratio $\frac{h}{m}$

$$\frac{\Delta\nu}{2\nu_*\left(\nu_* + \frac{p}{h}\right)} = \frac{h}{m}$$

with $\Delta\nu = \nu - \nu'$ and p is the modulus of the atom's initial momentum

- Laser frequencies are well controlled in lab experiments, leading to very precise determinations of the ratio $\frac{h}{m}$.

- Opportunity to implement quantum spacetime inspired corrections to the $\frac{h}{m}$ formula.

Cold atoms phenomenology: deformation

- Working at first order in ℓ allows for a simple parametrization of the corrections

$$\frac{\Delta\nu}{2\nu_*\left(\nu_* + \frac{p}{h}\right)}(1+\ell\alpha) = \frac{h}{m}$$

- α is an unknown function possibly depending on the mass of the atoms, the laser frequencies and the atom's initial momentum.

- We model the process as an interaction where in the initial state we have an atom and the photon that is to be absorbed and in the final state we have the accelerated atom and the emitted photon

Results

- Leading order corrections depending on the ordering in the interaction process

$$A + \gamma \to A' + \gamma' \qquad \ell\alpha \approx -\frac{3}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$\gamma + A \to \gamma' + A' \qquad \ell\alpha \approx \frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$\gamma + A \to A' + \gamma' \qquad \ell\alpha \approx -\frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

$$A + \gamma \to \gamma' + A' \qquad \ell\alpha \approx -\frac{1}{4}(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

Amplification

- Correction terms of the type

$$\ell \alpha_i \approx k_i(\ell m)\left(\frac{m}{p + h\nu_*}\right)\cos(\phi)\sin(\theta)$$

- Amplification factor stemming from kinematical IR/UV mixing term in the dispersion relation
- First DSR result with an amplification factor of this type

Amelino-Camelia,Lammerzahl,Mercati,Tino, Phys. Rev. Lett.103 (2009) Arzano, Kowalski-Glikman, EPL 90 (2010)

Interpretation

- We have no well-established dynamical framework to weigh the various channels of the interaction in the total correction

- The correction depends on the direction of the atom

- Average over channels and angles

$$\langle \alpha \rangle \approx \frac{1}{16\pi} \int_{S^2} d\Omega \sum_i \alpha_i = 0$$

- Variance

$$(\Delta\alpha)^2 \approx \frac{1}{16\pi} \int_{S^2} d\Omega \sum_i \alpha_i^2 \propto \left(\frac{m^2}{p + h\nu_*} \right)^2$$

Conclusions

- Exciting phenomenological opportunities from spacetime non-commutativity, both from more formal and toy models
- Possibility of putting bounds on the deformation scales both from underground experiments and atom interferometry
- Future directions: phenomenological analyses for both quantum gravity corrected atom interferometry and tests of the Pauli exclusion principle; noncommutative perturbation theory

THANKS FOR THE ATTENTION