Fundamental decoherence and neutrino oscillations

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Based on *Commun. Phys.* 6 (2023) 1, 242 with G. Gubitosi and M. Arzano
and arXiv:2306.14778 with Giulia Gubitosi

Contents

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- Quantum-to-classical transition mechanism.

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Quantum Decoherence

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$$\rho_S(t) = \mathrm{Tr}_E \left\{ U(t)\rho_{S,E}(0)U^\dagger(t) \right\} .$$

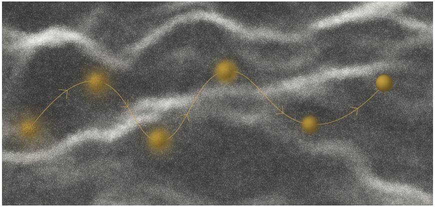
- Evolution of quantum systems described by master equations (**Lindblad**, ...)

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i[H_S, \rho_S] + D\Big[\rho_S(t)\Big] = \mathfrak{L}[\rho_S(t)] ,$$

$$\boxed{\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i[H_S, \rho_S(t)] - \frac{1}{2}\sum_\mu \lambda_\mu \left\{ L_\mu^\dagger L_\mu \rho_S(t) + \rho_S(t) L_\mu L_\mu^\dagger - 2L_\mu \rho_S(t) L_\mu^\dagger \right\}, \lambda_\mu \geq 0} \, .$$

Decoherence in quantum gravity

Gravitational field (or spacetime) as an **omnipresent** environment [*A. Bassi et al, Class. Quant. Grav. 34, 193002 (2017).*].



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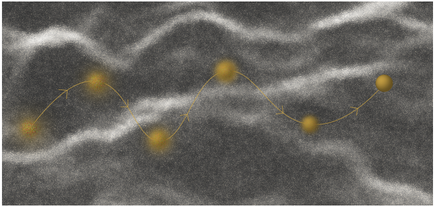

Several frameworks: Metric fluctuations [*H.P. Breuer et al,Class. Quant. Grav. 26, 105012 (2009)*], Gravitational time dilation [*I. Pikovski et al, Nat. Phys. 11 (2015) 668-672*], fluctuating minimal length [*L. Petruzziello and F. Illuminati, Nat.Comm. 12, 4449 (2021)*],. . .

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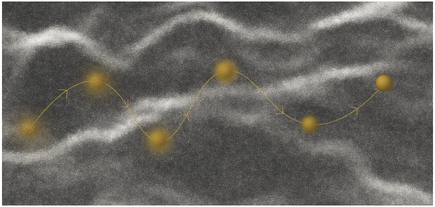

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Phenomenology: Neutron interferometry and neutral kaons [*J. Ellis et al, Nucl.Phys.B 241, 381 (1984)*], neutrino oscillations [*VDE and G. Gubitosi, arXiv:2306.14778*], optomechanical cavities [*C. Pfister et al, Nature Communications 7, 13022 (2016)*].

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Deformed symmetries are described with **Hopf algebras**

Non-linear algebraic sector	Coalgebra becomes non-trivial
$[X_i, X_j] = f(\boldsymbol{X})$	$\Delta : \mathbb{H} \to \mathbb{H} \otimes \mathbb{H}$ (coproduct)
	$S : \mathbb{H} \to \mathbb{H}$ (antipode)

Deformed symmetries in QM

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Trivial coalgebra

$$\Delta P_\mu\Big(|p\rangle\otimes|q\rangle\Big)=(p_\mu+q_\mu)\,|p\rangle\otimes|q\rangle \tag{1}$$

$$P_\mu\,\langle q| := \langle q|\,S(P_\mu) = -q_\mu\,\langle q| \tag{2}$$

Deformed coalgebra

$$\Delta P_\mu\Big(|p\rangle\otimes|q\rangle\Big)=(p\oplus q)_\mu\,|p\rangle\otimes|q\rangle \tag{3}$$

$$P_\mu\,\langle q| := \langle q|\,S(P_\mu) \neq -q_\mu\,\langle q| \tag{4}$$

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Trivial coalgebra	Deformed coalgebra
$\Delta P_\mu \Big(\|p\rangle \otimes \|q\rangle \Big) = (p_\mu + q_\mu) \|p\rangle \otimes \|q\rangle$ (1)	$\Delta P_\mu \Big(\|p\rangle \otimes \|q\rangle \Big) = (p \oplus q)_\mu \|p\rangle \otimes \|q\rangle$ (3)
$P_\mu \langle q\| := \langle q\| S(P_\mu) = -q_\mu \langle q\|$ (2)	$P_\mu \langle q\| := \langle q\| S(P_\mu) \neq -q_\mu \langle q\|$ (4)

Deformation of time translations generator

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\kappa} P_n \otimes P^n \; , \; S(P_0) = -P_0 + \frac{1}{\kappa} \mathbf{P}^2 \qquad (5)$$

Adjoint Action

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Hopf-algebraic definition of adjoint action [*H. Ruegg & V. N. Tolstoy, Lett.Math.Phys. 32, 85–101 (1994)*].

In QM

$$\mathrm{ad}_G(O) := (id \otimes S)\Delta G \diamond O \tag{6}$$

If coalgebra structures are trivial

$$\mathrm{ad}_G(O) = (id \otimes S)\underbrace{(G \otimes \mathbb{1} + \mathbb{1} \otimes G)}_{\Delta G} \diamond O = (G \otimes \mathbb{1} - \underbrace{\mathbb{1} \otimes G}_{S(G)=-G}) \diamond O = -i[G,O] \tag{7}$$

Density operator time evolution

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- **Adjoint action** of P_0 gives the time evolution of ρ.
- Time evolution preserves the **hermiticity** of ρ (required to interpret the eigenvalues of ρ as **probabilities**).

These requirements are met with

$$\boxed{\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{2}\left\{\mathrm{ad}_{P_0}(\rho) + \left[\mathrm{ad}_{P_0}(\rho)\right]^{\dagger}\right\}} \tag{8}$$

leading to

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\left[P_0, \rho\right] - \frac{1}{2\,\kappa}\left(\rho\,\boldsymbol{P}^2 + \boldsymbol{P}^2\,\rho - 2\,P_n\rho\,P^n\right) \tag{9}$$

Decoherence time and mass constraint

Free particle solution in momentum basis is

$$\langle \boldsymbol{p}|\rho|\boldsymbol{q}\rangle := \rho_{pq}(t) = \rho_{pq}(0)\exp\left\{-it[E(p)-E(q)] - \frac{t}{2\kappa}(\boldsymbol{p}-\boldsymbol{q})^2\right\} \tag{10}$$

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Off-diagonal phase coherence vanishes after a **decoherence time**

$$\boxed{\tau_D = \frac{2\kappa}{(\delta \boldsymbol{p})^2}} \quad (11)$$

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"Lifetime" of a state of a quantum system [*J. Hilgevoord APJ 64, 1451 (1996)*]

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With $E = (2m)^{-1}\boldsymbol{p}^2$, from (13) and for **momentum localized** states, $\frac{\delta p}{p} \ll 1$

$$\boxed{m \lesssim \kappa} \tag{14}$$

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Decoherence effects on neutrino oscillations

Decoherence modifies neutrino oscillations

- Damping factor in oscillation probability

$$P(\beta \to \alpha) \propto e^{-D}$$

- Quenching of neutrino fluxes

$$\Phi(\nu_e, \nu_\mu, \nu_\tau) \propto e^{-D}$$

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Oscillation probability

States

$$|\nu_\gamma\rangle = \sum_i U^*_{\gamma i}\,|\psi_i\rangle \otimes |\nu_i\rangle = \sum_i U^*_{\gamma i} \int \mathrm{d}^3 p\, \psi_i(\boldsymbol{p})\,|\boldsymbol{p}\rangle \otimes |\nu_i\rangle \tag{15}$$

Probability given by

$$P(\beta \to \alpha; t) = \mathrm{Tr}\{\rho(t)\,|\nu_\alpha\rangle\langle\nu_\alpha|\}\,, \;\; \rho(0) = \left|\nu_\beta\right\rangle\!\left\langle\nu_\beta\right| \tag{16}$$

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$$\underbrace{\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho]}_{\text{Standard QM}} \longmapsto \underbrace{\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathfrak{L}[\rho(t)]}_{\text{Decoherence}} \tag{17}$$

$$\rho(t) = \sum_{i,j} U^*_{\beta i} U_{\beta j} \int \mathrm{d}^3 p\, \mathrm{d}^3 q\ \psi_i(\boldsymbol{p})\psi^*_j(\boldsymbol{q}) e^{-it\left[E_i(\boldsymbol{p})-E_j(\boldsymbol{q})\right]} e^{-t\mathscr{L}_{ij}(\boldsymbol{p},\boldsymbol{q})}\,|\boldsymbol{p}\rangle\langle\boldsymbol{q}| \otimes \big|\nu_i\big\rangle\big\langle\nu_j\big| \tag{18}$$

Oscillation Probability

Assumptions: one-dimensional reduction, wave-packets peaked around mean momenta p_i, only retain up to first order in Δm^2 terms.

$$P_{QG}(\beta \to \alpha; L) \propto \sum_{i,j} U^*_{\beta i} U_{\beta j} U^*_{\alpha j} U_{\alpha i} \, e^{i\phi_{ij}} \frac{2\pi}{v_{g_{ij}}} \int_{-\infty}^{+\infty} \mathrm{d}p \, e^{ip(1-r_{ij})L} \, .$$

$$\cdot G_{ij}(p, r_{ij}\, p + \Delta E_{ij}/v_{g_j}) e^{-D_{ij}\left(p+p_i, r_{ij}p+p_j - v_{g_j}^{-1}\Delta E_{ij}; L\right)} \tag{19}$$

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$$D_{ij}(p,q;L) = v_{g_{ij}}^{-1} L \mathscr{L}_{ij}(p,q),\; r_{ij} = v_{g_i} \cdot v_{g_j}^{-1},\; \phi_{ij} = -L\frac{\Delta m^2_{ij}}{2\,p_{ij}}.$$

Two flavours probability

Propagation coherence condition

$$L \ll l_{\text{coh}} = \sigma_X \frac{v_{g_{ij}}}{\Delta v_{g_{ij}}} \tag{20}$$

Interaction coherence condition

$$\Delta E_{ij} \frac{\sigma_X}{v_{g_{ij}}} \ll 1 \tag{21}$$

Probability simplifies to

$$P_{QG}(\beta \to \alpha; L) = \sum_{i,j} U^*_{\beta i} U_{\beta j} U^*_{\alpha j} U_{\alpha i}\, e^{i\phi_{ij}} e^{-D_{ij}(p_i, p_j; L)} \tag{22}$$

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Considering two flavours oscillations, probability further simplifies

$$P_{QG}(\alpha \to \alpha) = e^{-D} P_{\text{std}}(\alpha \to \alpha) + \left(1 - e^{-D}\right)\left(1 - \frac{1}{2}\sin^2 2\theta\right) \qquad (23)$$

with $P_{\text{std}}(\alpha \to \alpha) = 1 - \sin^2 2\theta \sin^2 \frac{\phi}{2}$.

Fundamental decoherence QG models

Deformation of symmetries: $D = \frac{L\left(\Delta m^2\right)^2}{8 v_g E_{QG}\, p^2}$

[*M. Arzano et al, accepted on Communication Physics*].

Fluctuating minimal length: $D = \frac{16 L E^4 (\Delta m)^2}{v_g E_{QG}^5}$

[*L. Petruzziello et al, Nat. Commun. 12, 1, 4449, (2021)*].

Stochastic metric fluctuations: $D = \frac{L E^6 \left(\Delta m^2\right)^2}{4 v_g E_{QG}\, m_i^4 m_j^4}$

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Degenerate mass eigenstates: $m_i m_j \sim m^2$, $\Delta m^2 \sim (\Delta m)^2$.

Non-degenerate mass eigenstates: $m_i m_j = m_{\min}\sqrt{m_{\min}^2 + \Delta m^2}$,

$(\Delta m)^2 = \left(-m_{\min} + \sqrt{m_{\min}^2 + \Delta m^2}\right)^2$

Neutrino oscillations regimes

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Astrophysical neutrinos.

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Solar neutrinos.

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Reactor and accelerator neutrinos.

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Standard probability damping for astrophysical neutrinos

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Astrophysical neutrinos

Standard probability damping for astrophysical neutrinos
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With identical Gaussian wave-packets at production and detection probability reads

$$P(\beta \to \alpha) \propto e^{-\frac{L}{l_{\text{coh}}}} \tag{24}$$

Oscillations are washed out by propagation over such huge distances.

Solar neutrinos

$\cos\phi$ rapidly oscillating. $\Rightarrow$ Averaged probability is observed.

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$$\langle P_{\text{std}}(\alpha \to \alpha)\rangle = 1 - \frac{1}{2}\sin^2 2\theta \Rightarrow \langle P_{QG}(\alpha \to \alpha)\rangle = \langle P_{\text{std}}(\alpha \to \alpha)\rangle \quad (25)$$

Averaged oscillations **not sensitive** to QG-induced decoherence.

Neutrino oscillations regimes

~~Astrophysical neutrinos.~~

~~Solar neutrinos.~~

Atmospheric neutrinos.

Reactor and accelerator neutrinos.

Sensitivity to QG models

Observable effect when $D \gtrsim 1$.

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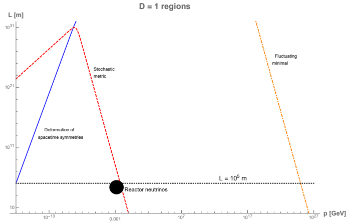

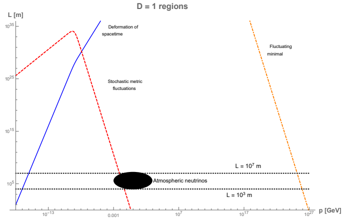

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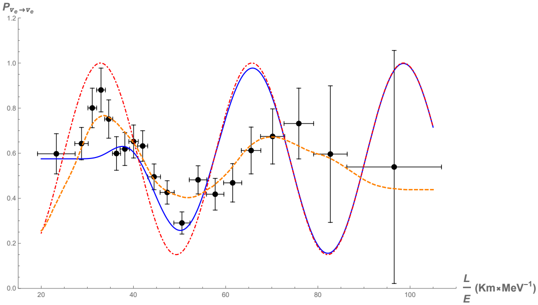

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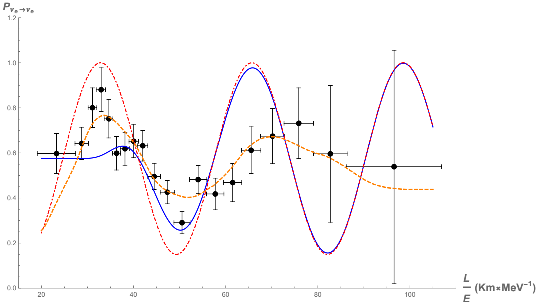
KamLAND



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Several reactors $\Rightarrow$ Focus on high energies ($\frac{L}{E} < 45\,\frac{\text{Km}}{\text{MeV}}$).

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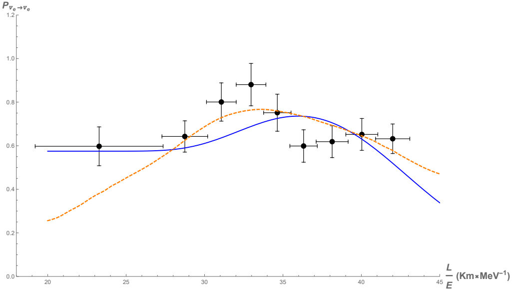

QG decoherence **not stronger** than matter decoherence (conservative choice) $\Rightarrow E_{QG} > E_{QG}^{\chi^2} \, : \, \Delta\chi^2 = \chi^2_{QG} - \chi^2_{\mathrm{KL}} = 2.7.$

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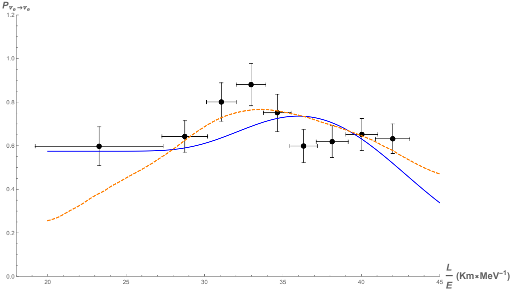

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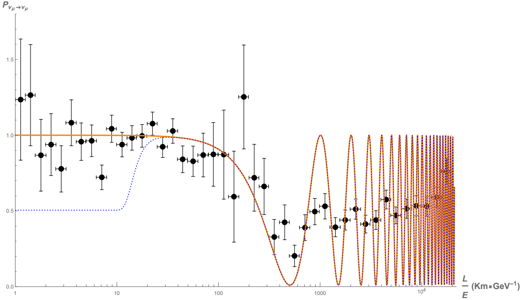
$$E_{QG} > 2.6 \times 10^{34}\,\mathrm{GeV} \tag{26}$$

Super-Kamiokande



Data from [*SK, Phys. Rev. Lett. 93, 221803 (2004)*]. $L = 10\,\text{Km}$, $m_{\text{min}} = 1\,\text{eV}$ (both conservative choices), $E_{QG} = 10^{49}\,\text{GeV}$. $\Delta m^2 = 2.45 \times 10^{-3}\,\text{eV}^2$, $\sin^2 2\theta = 0.99$ [*PDG coll., Rev. of Part. Phys., PTEP 2022 (2022) 083C01*].

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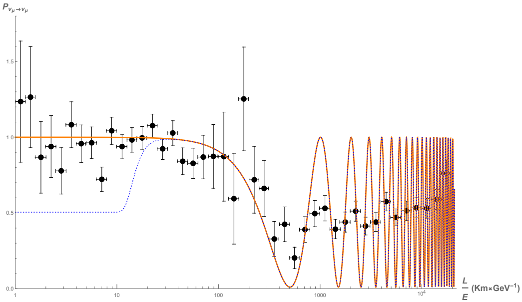

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Fast oscillations $\Rightarrow$ Focus on high energies ($\frac{L}{E} < 1000\,\frac{\mathrm{Km}}{\mathrm{GeV}}$).

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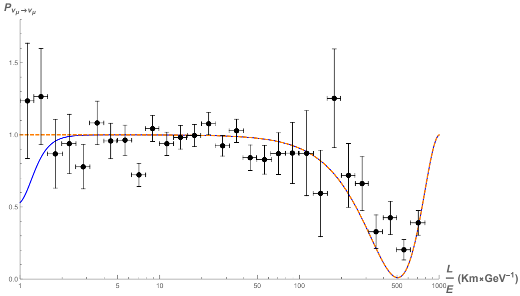
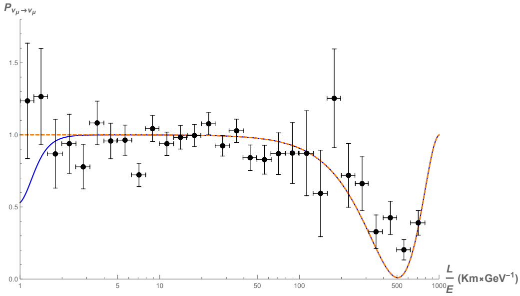


No significant damping from QG decoherence ⇒

$$E_{QG} > E_{QG}^{\chi^2} \;:\; \Delta\chi^2 = \chi^2_{QG} - \chi^2_{\text{std}} = 2.7.$$

Super-Kamiokande



No significant damping from QG decoherence ⇒
$E_{QG} > E_{QG}^{\chi^2}$: $\Delta\chi^2 = \chi^2_{QG} - \chi^2_{\text{std}} = 2.7.$

$$E_{QG} > 2.5 \times 10^{55}\,\text{GeV} \tag{27}$$

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- **Strong constraints** on the stochastic metric fluctuations scale from long baseline reactor and atmospheric neutrinos.

Thank you!

Relativistic model for metric perturbations

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \dot{\tau} H |\psi\rangle \ , \ \dot{\tau} = \frac{1}{c}\sqrt{-g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}t}\frac{\mathrm{d}x^\nu}{\mathrm{d}t}} \ . \tag{28}$$

Writing the metric as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $|h_{\mu\nu}| \ll |\eta_{\mu\nu}|$

$$\dot{\tau} = \gamma^{-1}\sqrt{1 - \frac{h_{\mu\nu}p^\mu p^\nu}{m^2c^2}} \sim \gamma^{-1}\left(1 - \frac{h_{\mu\nu}p^\mu p^\nu}{2m^2c^2}\right) , \tag{29}$$

Considering $\langle h_{ij}(t)h_{mn}(t')\rangle = \tau_c \delta_{ij}\delta_{mn}\delta(t-t')$ we get

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H_0,\rho] - \frac{1}{E_{QG}}\left[\frac{\boldsymbol{p}^2}{2m^2}H_0, \left[\frac{\boldsymbol{p}^2}{2m^2}H_0,\rho\right]\right] , \tag{30}$$

where H_0 is the free Hamiltonian in the lab frame.

Interaction with the Environment

von Neumann measurement:

$$|o_n\rangle \otimes |R\rangle \mapsto |o_n\rangle \otimes |a_n\rangle \Rightarrow |\psi\rangle \otimes |R\rangle \mapsto \sum_n c_n |o_n\rangle \otimes |a_n\rangle \tag{31}$$

$|a_n\rangle \in \mathscr{H}_A$, $|o_n\rangle \in \mathscr{H}_S$.
Two-level system: $|\psi_i\rangle \otimes |E_0\rangle \mapsto |\psi_i\rangle \otimes |E_i\rangle$, $i = 1, 2$. Environment-system entanglement emerges dynamically

$$|\psi\rangle \otimes |E_0\rangle = \frac{1}{\sqrt{2}}\Big(|\psi_1\rangle + |\psi_2\rangle\Big) \otimes |E_0\rangle \mapsto \frac{1}{\sqrt{2}}\Big(|\psi_1\rangle \otimes |E_1\rangle + |\psi_2\rangle \otimes |E_2\rangle\Big) \tag{32}$$

The density operator of the system is

$$\rho_S = \mathrm{tr}_E\,\{\rho\} = \frac{1}{2}\Big(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2|\langle E_1\rangle E_2 + |\psi_2\rangle\langle\psi_1|\langle E_2\rangle E_1\Big) \tag{33}$$

For macroscopic systems $|E_i\rangle = \prod_{\alpha=1}^{N} |e_\alpha^{(i)}\rangle$, $\langle e_\alpha^{(1)}\rangle e_\beta^{(2)} = \varepsilon \lesssim 1$, thus

$$\boxed{\langle E_1\rangle E_2 \sim \varepsilon^N \sim 0}$$

Example of system-environment interaction

Two-level system $\{\, |0\rangle\,, |1\rangle \,\}$; environment composed of N two-level systems $\{\, |\uparrow\rangle_i\,, |\downarrow\rangle_i \,\}$.

$$\hat{H}_{\text{int}} = \frac{1}{2}\hat{\sigma}_z \otimes \hat{E} \ \ , \ \ \hat{E} := \sum_{i=1}^{N} g_i\, \hat{\sigma}_z^{(i)} \tag{34}$$

$2^N - 1$ energy levels for E given by $|n\rangle = |\uparrow\rangle_i |\downarrow\rangle_i \ldots |\uparrow\rangle_i$ with $\varepsilon_n = \sum_{i=1}^{N} (-1)^{n_i} g_i$.

$$e^{-it\hat{H}_{\text{int}}}\, |\psi\rangle = e^{-it\hat{H}_{\text{int}}} \Big(a\, |0\rangle + b\, |1\rangle \Big) \otimes \sum_{i=1}^{2^N-1} c_n\, |n\rangle = a\, |0\rangle\, |\varepsilon_0(t)\rangle + b\, |1\rangle\, |\varepsilon_1(t)\rangle \tag{35}$$

with $|\varepsilon_0(t)\rangle = |\varepsilon_1(-t)\rangle = \sum_{i=1}^{2^N-1} c_n e^{-it\,\frac{\varepsilon_n}{2}}$.
The decoherence parameter

$$r(t) = \langle \varepsilon_1(t) \rangle\, \varepsilon_0(t) = \sum_n |c_n|^2 e^{-i\,\varepsilon_n t} \ \Rightarrow \ \boxed{\langle |r(t)| \rangle \sim 2^{-N}} \tag{36}$$

Recurrence time τ_{rec} exists since N is always finite. $g_i = g\ \forall\, i$ and $\sum_{i=1}^{2^N-1} c_n\, |n\rangle = \bigotimes_{i=1}^{N} \frac{1}{\sqrt{2}} \Big(|\downarrow\rangle_i + |\uparrow\rangle_i \Big)$ give $r(t) = \big[\cos(gt)\big]^N$ with $\tau_{\text{rec}} = \frac{\pi}{g}$.
Highly improbable, typically $\tau_{\text{rec}} \propto N!$

Environmental superselection

Interaction with the environment selects the preferred basis.

$$|\psi_\pm\rangle \otimes |E_0\rangle \mapsto \frac{1}{\sqrt{2}}\Big(|\psi_1\rangle \otimes |E_1\rangle \pm |\psi_2\rangle \otimes |E_2\rangle\Big) \tag{37}$$

$|\psi_\pm\rangle$ get entangled with the environment.
Superselected states are the ones that get **least entangled** with the environment ($|\psi_i\rangle$).
Typically $\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$.
Quantum measurement limit: $\hat{H} \approx \hat{H}_{\text{int}}$. Typically $\hat{H}_{\text{int}} = \hat{S} \otimes \hat{E}$, thus eigenstates of $\hat{S}$ get selected

$$e^{-it\,\hat{S}\otimes\hat{E}}\,|s_i\rangle \otimes |E_0\rangle = |s_i\rangle \otimes e^{-it\,\lambda_i\,\hat{E}}\,|E_0\rangle := |s_i\rangle \otimes |E_i(t)\rangle \tag{38}$$

Quantum limit: $\hat{H} \approx \hat{H}_S$. Constants of motion of S (energy) get selected.

Details of the contraction

The contraction

$$\boldsymbol{N} \mapsto c^{-1}\boldsymbol{N} \ , \ \boldsymbol{P} \mapsto c^{-1}\boldsymbol{P} \ , \ \kappa \mapsto c^{-2}\kappa \tag{39}$$

gives

$$\Pi_0 = \frac{1}{c^2\kappa}P_0 + \sqrt{\mathbb{1} - \frac{1}{c^4\kappa^2}\left(P_0^2 - c^2\boldsymbol{P}^2\right)} \to \mathbb{1} \ , \ \Pi_0^{-1} = \frac{-\frac{1}{c^2\kappa}P_0 + \sqrt{\mathbb{1} - \frac{1}{c^4\kappa^2}\left(P_0^2 - c^2\boldsymbol{P}^2\right)}}{\mathbb{1} - \frac{1}{c^4\kappa^2}\left(P_0^2 - c^2\boldsymbol{P}^2\right)} \to \mathbb{1} \tag{40}$$

thus

$$\Delta P_0 = P_0 \otimes \Pi_0 + \Pi_0^{-1} \otimes P_0 + \frac{1}{c^2\kappa}cP_n\Pi_0^{-1} \otimes cP^n \to P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\kappa}P_n \otimes P^n \tag{41}$$

$$S(P_0) = -P_0 + \frac{1}{c^2\kappa}c^2\boldsymbol{P}^2\Pi_0^{-1} \to -P_0 + \frac{1}{\kappa}\boldsymbol{P}^2 \ , \ cS(P_i) = -cP_i\Pi_0^{-1} \Rightarrow S(P_i) = -P_i$$

Adjoint Action

In standard QM evolution of density operators ρ is given by the adjoint action of **time translations generator**.

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Abstract adjoint action defined as [*H. Ruegg & V. N. Tolstoy, Lett.Math.Phys. 32, 85–101 (1994)*]

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In QM

$$\mathrm{ad}_G(O) := (id \otimes S)\Delta G \,\bar{\diamond}\, O \;,\; (G_1^n \otimes G_2^m)\,\bar{\diamond}\, O := (-i)^{n+m} G_1^n\, O\, G_2^m \tag{43}$$

If coalgebra structures are trivial

$$\mathrm{ad}_G(O) = (id \otimes S)\underbrace{(G \otimes \mathbb{1} + \mathbb{1} \otimes G)}_{\Delta G} \diamond O = (G \otimes \mathbb{1} - \underbrace{\mathbb{1} \otimes G}_{S(G)=-G}) \diamond O = -i[G, O] \tag{44}$$