From Bjorken scaling to scaling violations

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At the beginning of '50 people were losing hope of a microscopic theory of strong interactions. In 1953 Dyson after spending nearly one year with his students on some computation in perturbative pion nucleon theory went to Fermi to ask an opinion:

With the pseudoscalar meson theory there is no physical picture, and the forces are so strong that nothing converges. To reach your calculated results, you had to introduce arbitrary cut-off procedures that are not based either on solid physics or solid mathematics.

In desperation I asked Fermi whether he was not impressed by the agreement between our calculated numbers and his measured numbers. He replied, "How many arbitrary parameters did you use for your calculations?" I thought for a moment about our cut-off procedures and said, "Four." He said, "I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

Agnostic theories: symmetries, dispersion relations, S-matrix, Regge pole, superconvergence sum rules, bootstrap but not field theories. As remarked by David Gross *until 1973 it was not thought proper to use field theories without apologies.*

(Gell-Mann Telegdi 1965) We use the method of abstraction from a Lagrangian field theory model. In other words, we construct a mathematical theory of the strongly interacting particles, which may or may not have anything to do with reality, find suitable algebraic relations that hold in the model, postulate their validity, and then throw away the model. We compare this process to a method sometimes employed in French cuisine: a piece of pheasant meat is cooked between two slices of veal, which are then discarded.

Around 1960 Geoffrey Chew proposed the *boostrap* philosophy.

There were no elementary constituents all particles were supposed to be on the same footing. This approach became extremely popular and also Murray Gell-Mann was not taking quarks seriously: he was considering more as a mathematical model to implement $SU(3)$ symmetry. Field theory was discredited and some authors suggested that a good knowledge of quantum field theory was detrimental to the comprehension of bootstrap.

This was a somewhat strange position because dispersion relations were a cornerstone of bootstrap and they were firstly derived in the context of local field theory. Ironically the bootstrap approach was instrumental to the birth of string theories which are among the most sophisticated quantum field theories.

The absence of elementary point-like objects suggested that hadrons were extremely soft.

This viewpoint was confirmed by the very fast decay of the proton form factor. This viewpoint was confirmed by the exponential suppression of particle productions at large momentum transfer and by Hagedorn theory where a maximum temperature somewhat less than 200 MeV was supposed to be present in nature: also a very energetic hadronic fireball would emit hadrons of no more than few hundredth MeV.

There was also a different viewpoint. Electromagnetism, Fermi interactions, and the V-A theories for weak interaction were based on local currents and quantum field theory. The purely leptonic electro-weak was described by a field theory that was non-renormalizable, however, there was some hope that the introduction of heavy vector Bosons could make the theory renormalizable.

The semi-leptonic weak interaction was mediated by a hadronic current, and the resulting current algebra (based on local commutators) was crucial to normalizing the weak interaction vertices, which played a fundamental role in Cabibbo's theory of weak interaction.

This quantum field theory approach to physics was strongly pushed in Europe: there were strong collaborations among different scientific institutions that were later formalized in the Triangular Meetings (Paris-Rome-Utrecht).

Crucial steps to Bjorken scaling

- (1958) V-A theory for $\Delta S = 0$ semileptonic transitions (Feynman, Gell-Mann)
- (1963) Cabibbo theory for $\Delta S = 1$ semileptonic transitions
- (1965) Current algebra, local commutator (Dashen, Gell-mann)
- (1965) Local commutators imply sum rules, infinite momentum frame, and very important paper (Fubini, Furlan)
- (1967) Current algebra at small distances (Bjorken)
- (1968) Callan and Gross sum rule for deep inelastic scattering, generalized by Cornwall and Norton.
- (1968) Bjorken scaling paper based on previous sum rules
- (1969) Feynman parton model
- (1969) Deep inelastic scattering in the parton model (Bjorken and Paschos)

Deep Inelastic scattering

The process is

$$electron{+}nucleon \rightarrow e{+}hadrons \qquad virtualGamma{+}nucleon \rightarrow hadrons$$

The mass squared of the virtual photon is $-q^2$ and its energy is ν/M. One is interested in the region of both large, away from elastic scattering. One defines

$$x = \frac{q^2}{\nu}, \qquad \omega = x^{-1} = \frac{\nu}{q^2}$$

We can write the cross-section in terms of two functions of these two variables $(F_1,\, F_2)$.

Bjorken scaling

Let us concentrate on the function $F_2(q^2, x)$.

Roughly speaking, if one generalizes current algebra and assumes that some equal-time commutators of currents and their derivatives have a non zero vanishing elements, one finds

$$\lim_{q^2\to\infty} \int_0^1 dx F_2(q^2, x) x^n = m_{n+1}$$

with non-zero m_n. The case $n = 0$ is related to standard equal-time commutators of currents.

It is natural to assume that

$$\lim_{q^2\to\infty} F_2(q^2, x) = F_2^\infty(x).$$

Bjorken scaling and Partons (Bjorken and Paschos)

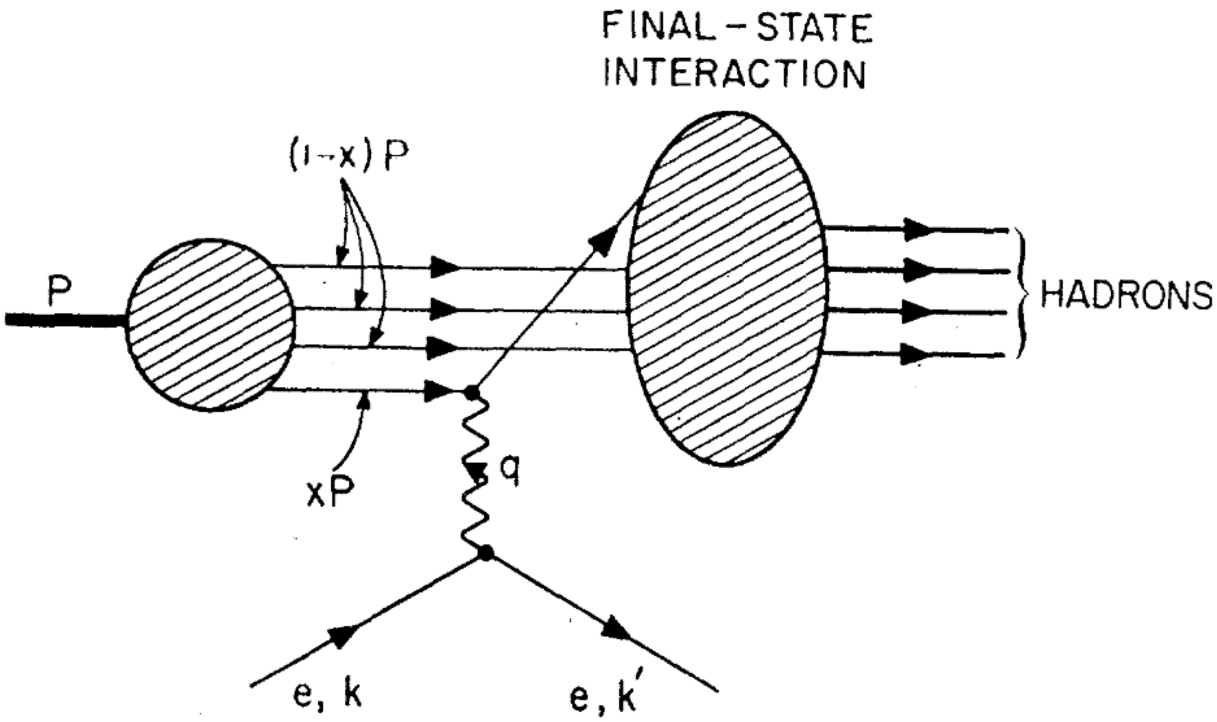


FIG. 1. Kinematics of lepton-nucleon scattering in the parton model.

The variable x defined before is also the fraction of the momentum of the parton with respect to the proton in the $P = \infty$ frame.

The data used by Bjorken and Paschos

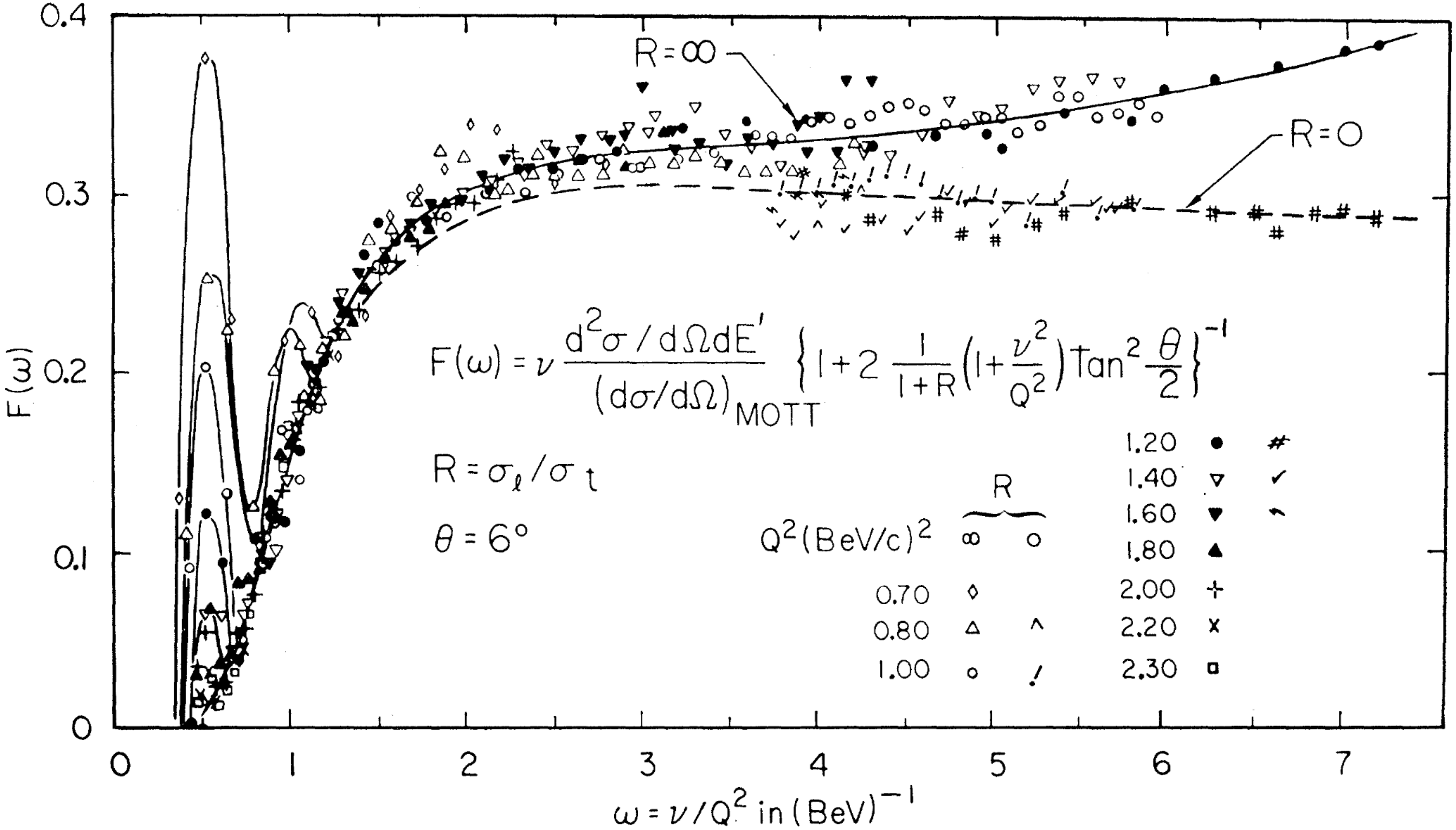
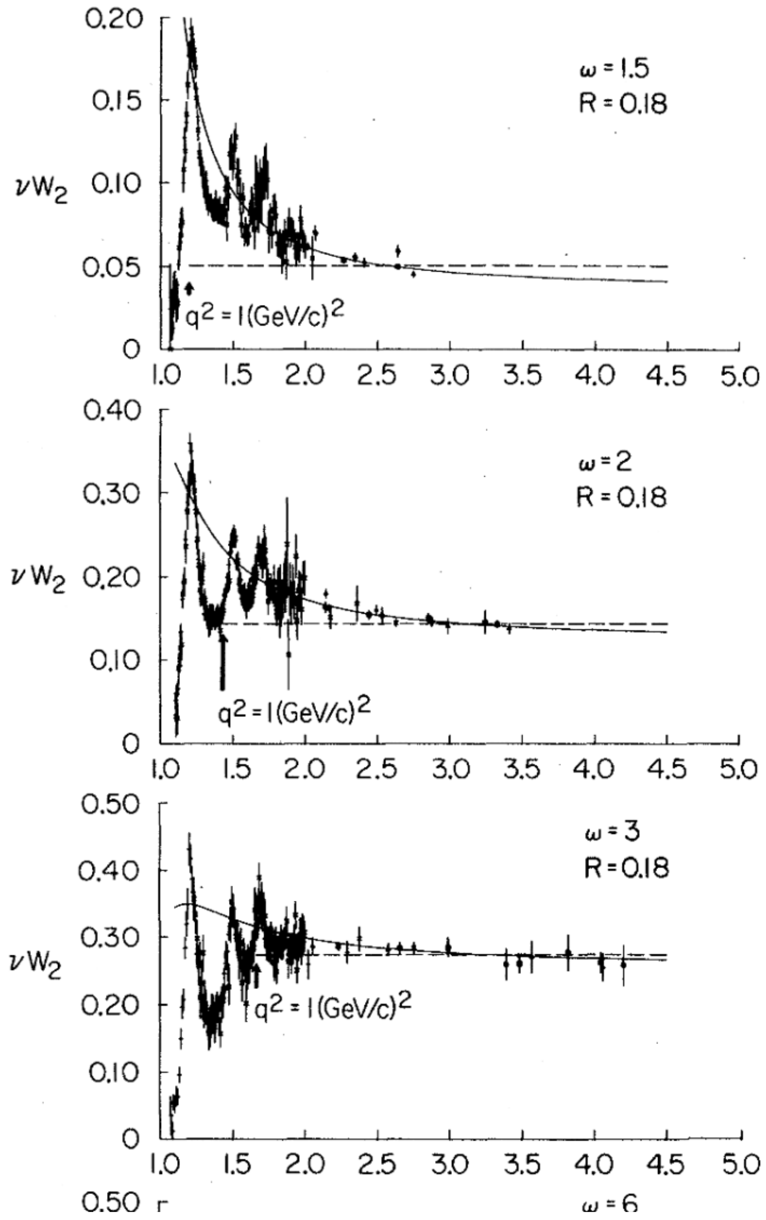
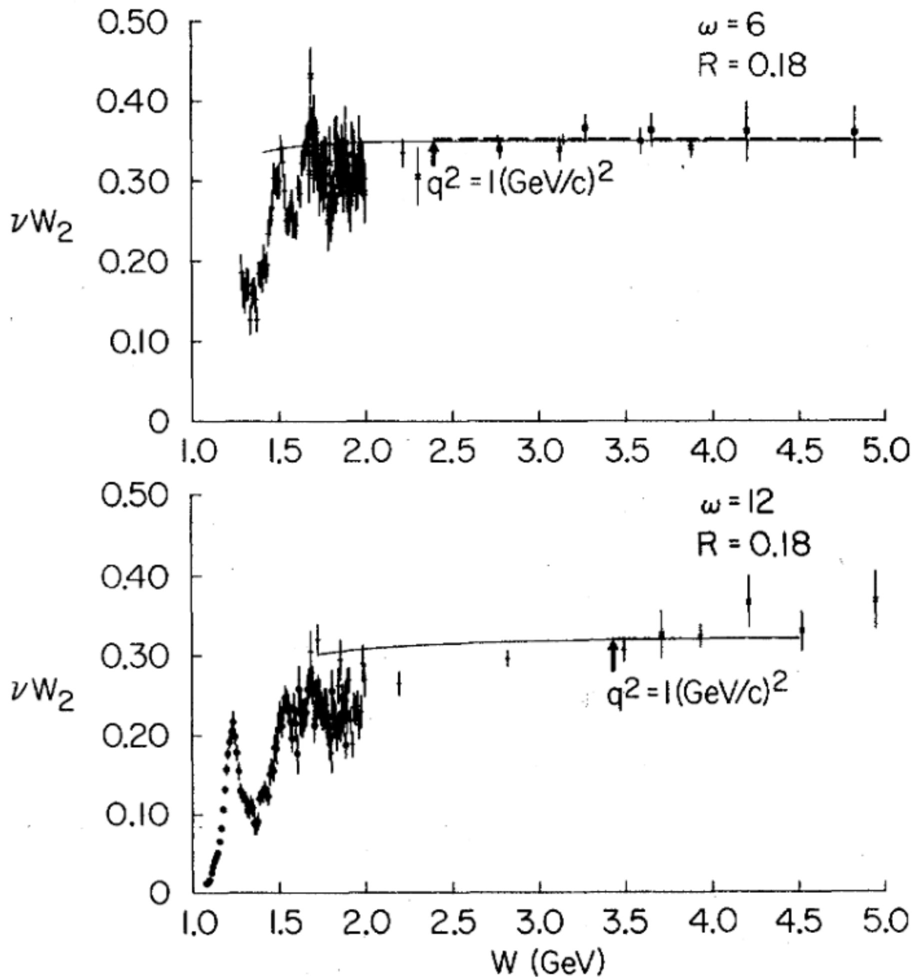


FIG. 2. Plot of the data as a function of ν/Q^2.

$$\omega = 1/x$$

The data three years later (1972)





Renormalization group

In the sixties, the renormalization group was not well understood.

Bjorken and Drell in the **book** wrote that in electrodynamic if (in modern language) the function $\beta(e^2)$ has a zero at e_0^2: the bare charge is given by

$$e_0{}^2 = F^{-1}(\infty) \tag{19.165}$$

We are presented with a dilemma. All our arguments in this section have been based on the renormalization program in perturbation theory, which permits us to expand propagators, vertex functions, etc., in power series in both the renormalized and bare charges e and e_0. However, in (19.165) we have come up with a result that puts a condition on the value of e_0 indicating that it cannot be chosen arbitrarily. This behavior is completely foreign to the perturbation development and forces us to conclude that at least one of our assumptions along the way has been wrong.[2] In particular, we suspect our

summed. Therefore, conclusions based on the renormalization group arguments concerning the behavior of the theory summed to all orders are dangerous and must be viewed with due caution.

So is it with all conclusions from local relativistic field theories.

New interest in 1970 with the Callan Symanzik equation.

It was much more easy to understand. It seemed less paradoxical the the renormalization group. It worked very well with the massive theory. In the 1971 paper, Symanzik proves in one particular case the Wilson operator expansion for the scalar ϕ^4 theory.

The changeover: Wilson operator expansion *On Products of Quantum Field Operators at Short Distances* (1964) Cornell Report LNS-64-15 Cited firstly by Brandt in 1967. Second citation 1970 (34 citations now) Similar ideas were rediscovered in the context of phase transition by Migdal and Polyakov: operator fusion. Wilson short distance operator product expansion states that in the limit of small x the product of two operators can be written as

$$A(x)B(0) \to_{x\to 0} \sum_{C} C(0)|x|^{-d_A-d_B+d_C} \,.$$

The leading terms come from the operators C with the lowest dimensions. The dimensions of the operators were the canonical ones in free theory where everything was clear.

Wilson ideas became popular much later, we the magnificent paper: *Non Lagrangian models of current algebra* (1969) has more that 3000 citations. *The strong interactions become scale-invariant at short distances. This was proposed by Kastrup and Mack.* And it was taken for granted that it was not a free theory at short distances.

Wilson is speaking of fields (e.g. the pion field), and their dimensions ... Strong interaction are described by a field theory: we have to find which one.

Brant Preparata. Light cone expansion (1970)

- Deep inelastic scattering corresponds in configuration space to the singular behavior near the light cone $x^2 = 0$ of the function

$$\langle p|J(x)J(0)|p\rangle$$

- The equal time commutators of the derivatives of the current are replaced by the Wilson expansion.

Bjorken Scaling follows if the lightcone singularities are the same as in free theory, i.e. if the operators that enter into the Wilson expansion have canonical dimensions. Light cone expansion:

$$J(x)J(0) \to_{x^2\to 0} \frac{O(x_0,0)}{x^2},$$

where $O(x_0,0)$ is a bilocal operator. In free field theory, the naive light cone expansion can be derived by the Wilson expansion by doing a Taylor expansion at $x_0 = 0$. An infinite number of terms in the Taylor expansion have to be considered, each term corresponding to a different moment of the experimental structure-function. In this way, the Wilson short-distance operator product becomes deeply related to the experimentally observed approximate Bjorken scaling.

What happens in an interacting theory?

Christ, Hasslacher, and Muller (1972) computed the coupling-dependent anomalous dimensions of the operators relevant for deep inelastic scattering (the so-called twist-two operators) for a [seudo scalar theory and for a theory with a simple vector interaction If we neglect the dependence of the running coupling constant on the momenta, one has something like

$$M_n(q^2) \equiv \int_0^1 dx x^{n-1} F(x, q^2) \ ; \quad M_n(q^2) = C_n \exp(\gamma_n(\alpha) \log(q^2)) \, . \quad (1)$$

The dependence on the running coupling constant can be trivially added.

Which theory at large momenta?

A strong interacting theory or a free theory like the $\lambda\phi^4$ with negative coupling constant proposed by Symanzik.

All stable theories without gauge fields were not asymptotically free (Coleman 1972, Landau 1955).

Asymptotic freedom was discovered in

- In 1969 Iosif Khriplovich with a beautiful computation that could be done on the back of the envelope.
- In 1972 Gerald 't Hooft (Marseille conference)
- In 1973 Gross, Politzer and Wilczek

In 1973 Georgi and Politzer, Gross, and Wilczek computed the Christ-Hasslaker-Muller formulae to QCD, we get a simple result if we consider only valence quark. When we take care of the running coupling constant, we get the final formulae:

$$\frac{\partial M_n(q^2)}{\partial \log(q^2)} = \gamma_n(\alpha(q^2)) \,,$$

where the linear term in $\alpha(q^2)$ of is obtained from the Christ-Hasslaker-Muller formulae (at order α). /nuova

If we use the standard properties of the Mellin transform and of the inverse Mellin we finally get

$$\frac{\partial F(x,q^2)}{\partial \log(q^2)} = \int_x^1 \frac{dy}{y} F(x,q^2) P_{q,q}(x/y, \alpha(q^2)) \,,$$

where the *quark fragmentation function* $P_{q,q}(z, \alpha(q^2))$ is the inverse

Mellin transform of the anomalous dimensions $\gamma_n(\alpha(q^2))$.

However, only later we understood that this function is the *quark fragmentation function*.

Using the first order in α for $\gamma_n(\alpha)$ we finally get for the non singlet contribution:

$$P_{q,q}(z) = \frac{8}{3}\frac{\alpha(q^2)}{4\pi}\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(z-1)\right)$$

Using only Mellin and inverse Mellin transform one can compute the violations of scaling without any physical interpretation of the formulae (Parisi Petronzio 1976) (for $\alpha = 0.4($.

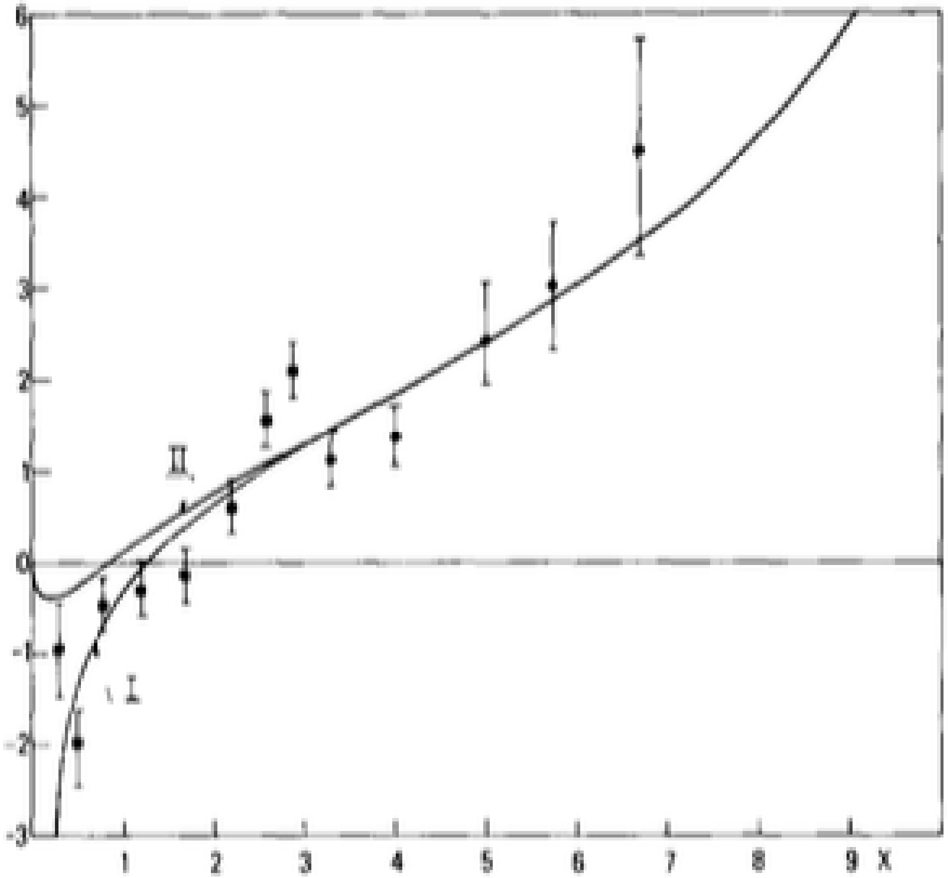


Fig. 3 Curve I is our prediction for $d \ln F_2^p(x, q^2)/d \ln q^2$ compared with experimental data taken from ref. [15] (●) and ref. [16] (■): the points taken from ref. [15] are $[d \ln W_1/d \ln q^2 + d \ln W_2/d \ln q^2]/2$ which correspond to $(c_1 + c_2)/2$ of their table 1. Curve II is obtained retaining only the octet operators in the operator expansion.

$$\frac{dN_{q_i}(x, L)}{dL} = \frac{\alpha}{4\pi} \int_x^1 \frac{dy}{y} \left[p_{qq}(x/y) N_{q_i}(y, L) + p_{qg}(x/y) N_g(y, L) \right]$$

$$\frac{dN_g(x, L)}{dL} = \frac{\alpha}{4\pi} \int_x^1 \frac{dy}{y} \left[p_{gg}(x/y) N_g(y, L) + p_{gq}(x/y) \sum_{i=1}^{8} N_{q_i}(y, L) \right]$$

Despite the relative simplicity of the final results, their derivation, although theoretically rigorous, is somewhat abstract and formal, being formulated in the language of renormalization group equations for the coefficient functions of the local operators which appear in **the light cone expansion for the product of two currents**.

The parton model revised

However, if you want to study

$$p + p \to \mu^+\mu^- + hadrons$$

no Wilson operator expansion works for that problem.

The solution was the appropriate generalization of the equivalent photon (equivalent electron) approximation in quantum electrodynamics (Weizsaker-Williams .... Cabibbo-Rocca). This was done in the paper with the Altarelli Parisi equation.

An alternative derivation of all results of current interest for the Q^2 behavior of deep inelastic structure functions is possible. In this approach all stages of the calculation refer to parton concepts and offer a very illuminating physical interpretation of the scaling violations. In our opinion the present approach, although less general, is remarkably simpler than the usual one since all relevant results can be derived in a direct way from the basic vertices of QCD, with no loop calculations being involved (the only exception is the lowest order expression for the running coupling constant which we do not rederive).

The paper was very successful.

The paper was very clearly written: it was written by Guido, not by myself ;-) It was really pedagogic.

The most important result of the paper was not the construction of a practical way to compute scaling violations in deep inelastic scattering. It was already done using the Mellin transformation. The most important point was to shift the focus from Wilson operator expansion to resolution dependent effective number of partons.

It was more than a computation: it was a shift in the language we use.

The Drell-Yan process $pp \to \mu^+\mu^- + \cdots$ could be not studied by a Wilson operator expansion. This is also true for jet production in hadronic collisions. They can be studied by factorizing the amplitude for the process in a part containing the effective parton distribution at the relevant energy and the hard scattering that could be treated in perturbation theory, but this is another story.