Efficient Optimization on neutral atoms Quantum Computers

Or… "The unbearable pain of running quantum algorithms in the 2020's"

Quminars @ UNIBO 16/04/2024
Simone Tibaldi, University of Bologna

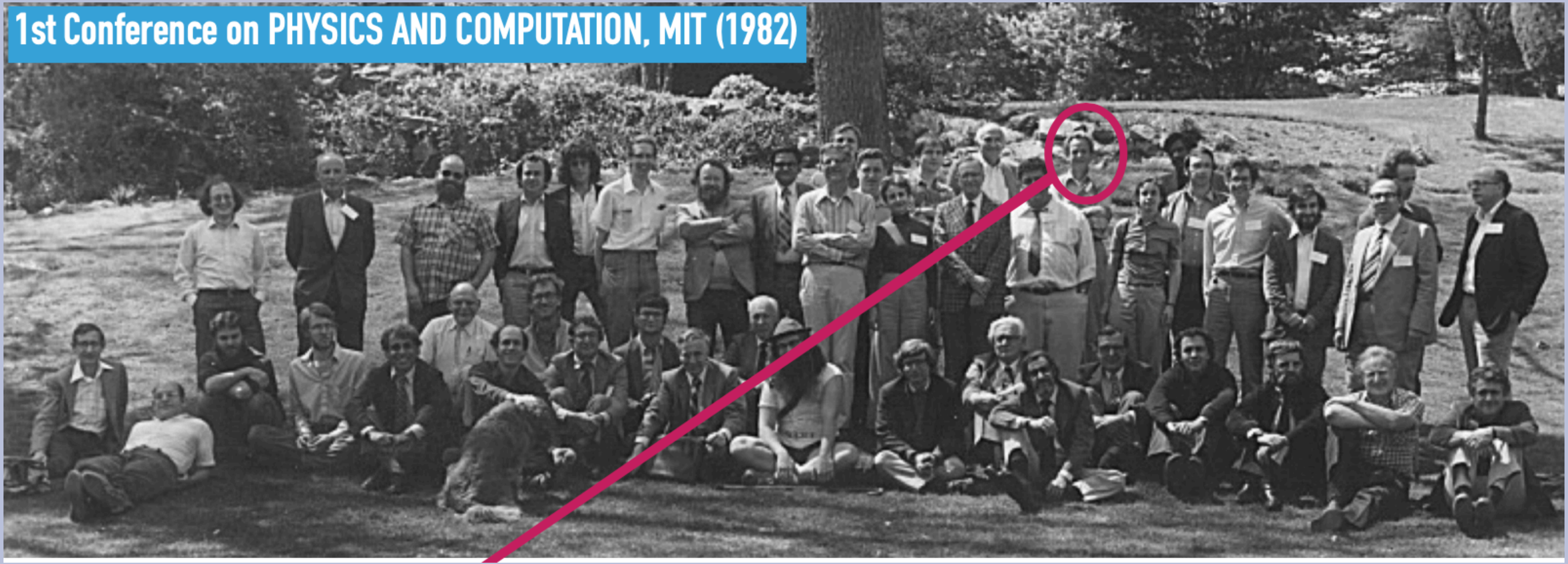
INDICO page:
https://agenda.infn.it/event/41198/

Efficient Optimization on neutral atoms Quantum Computers

Or… “The unbearable pain of running quantum algorithms in the 2020’s”

Quminars @ UNIBO 16/04/2024
Simone Tibaldi, University of Bologna

From THIS …



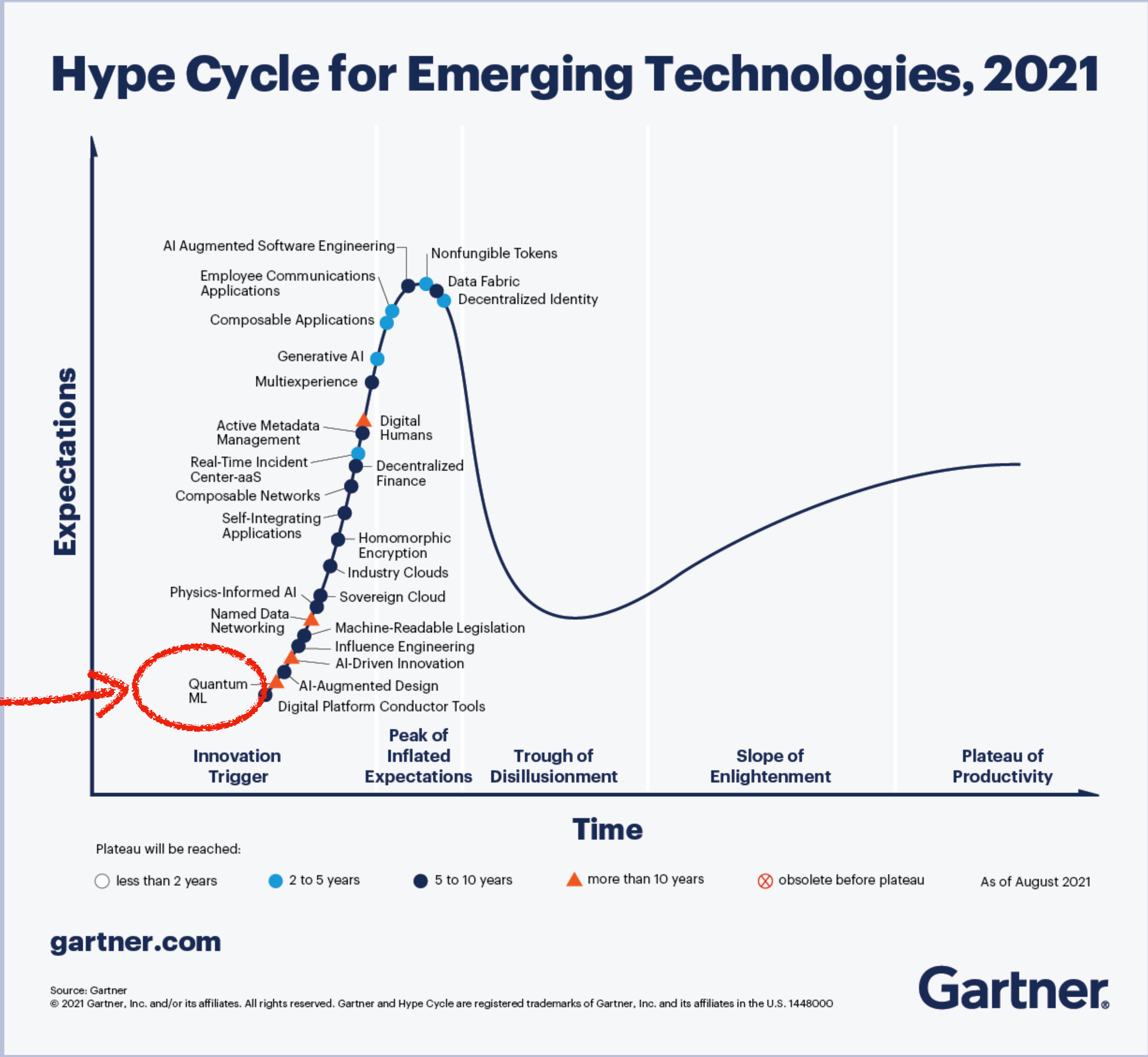
Now, what kind of physics are we going to imitate? First, I am going to describe the possibility of simulating physics in the classical approximation, a thing which is usually described by local differential equations. But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics—which is what I really want to talk about, but I'll come to that later. So what kind of simulation do I mean?

I want to talk about the possibility that there is to be an *exact* simulation, that the computer will do *exactly* the same as nature.

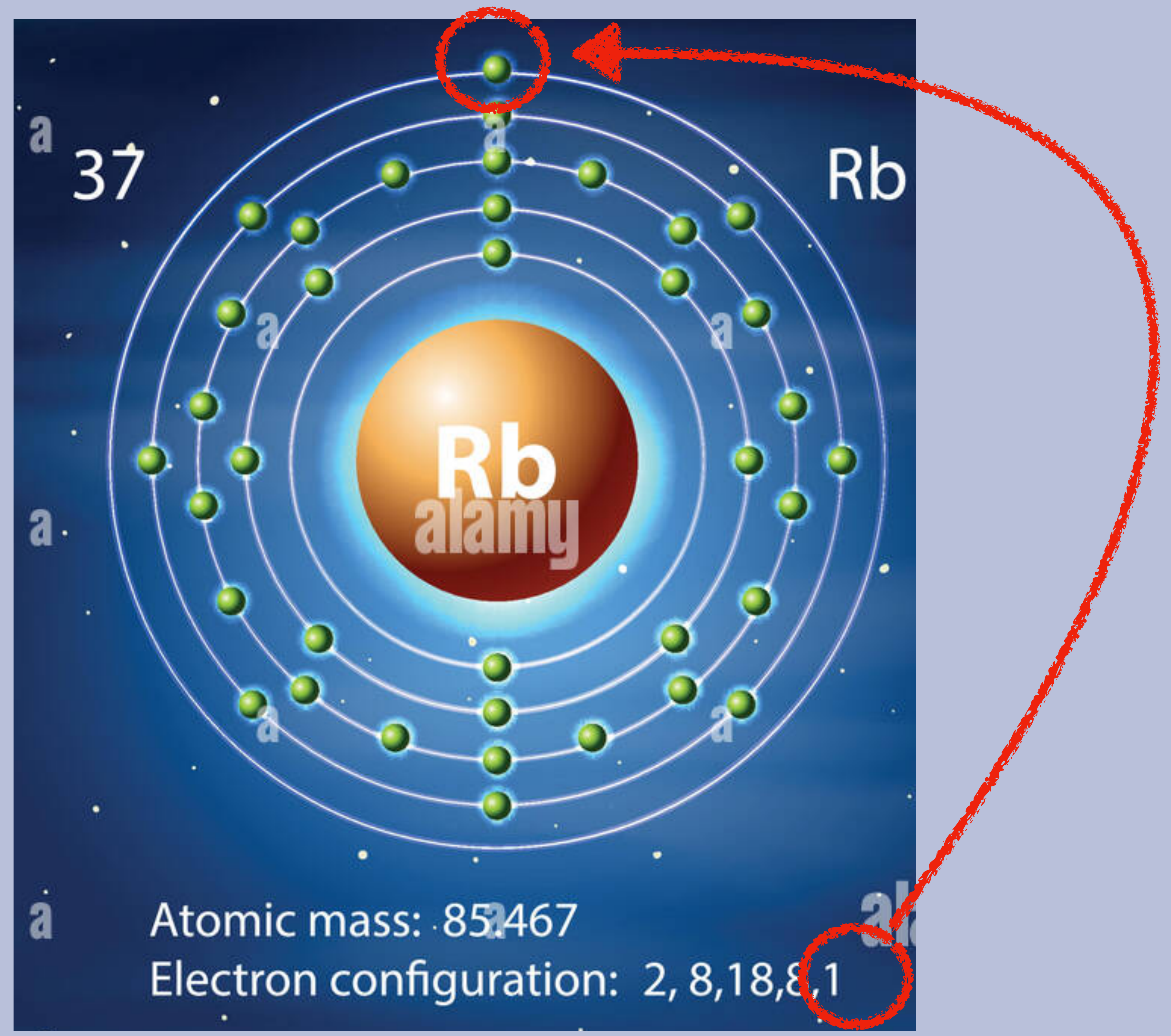
International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

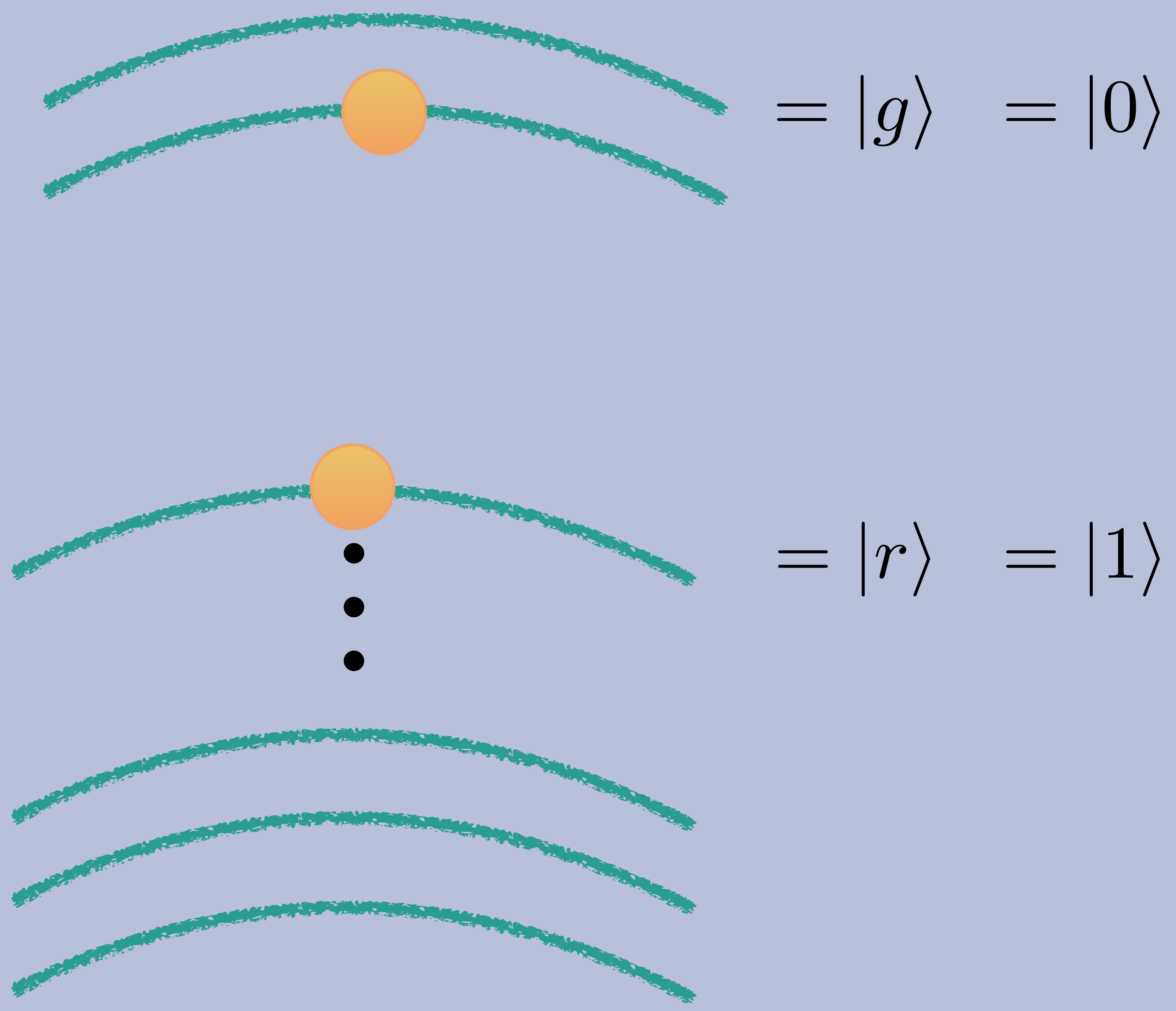
… to THIS

We made it!



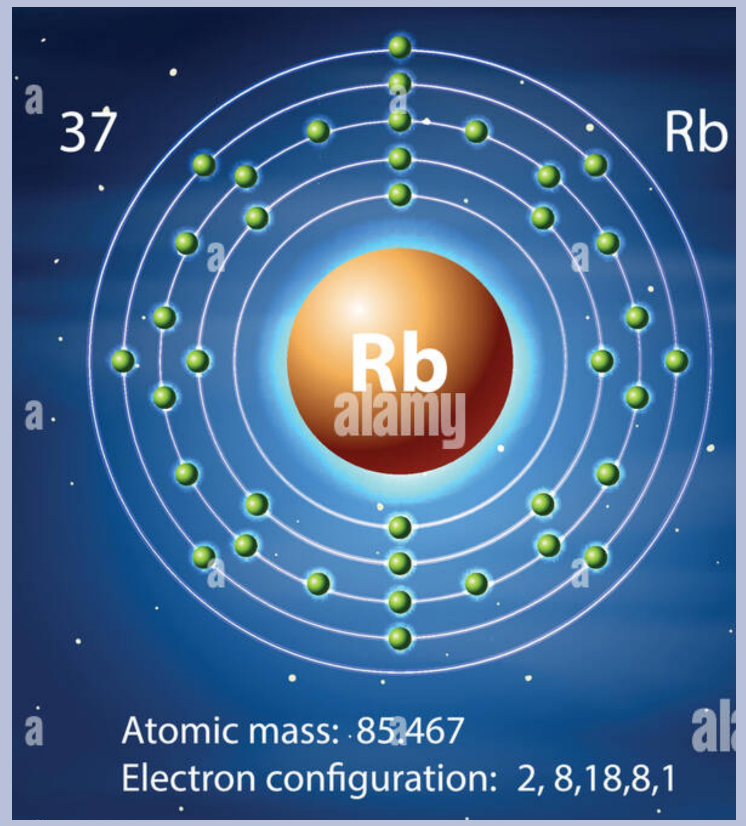
Let's create one

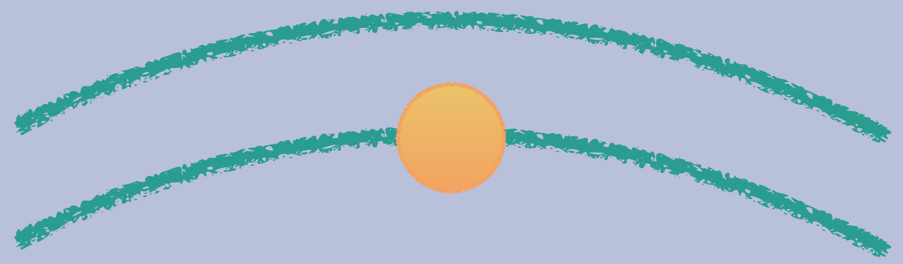




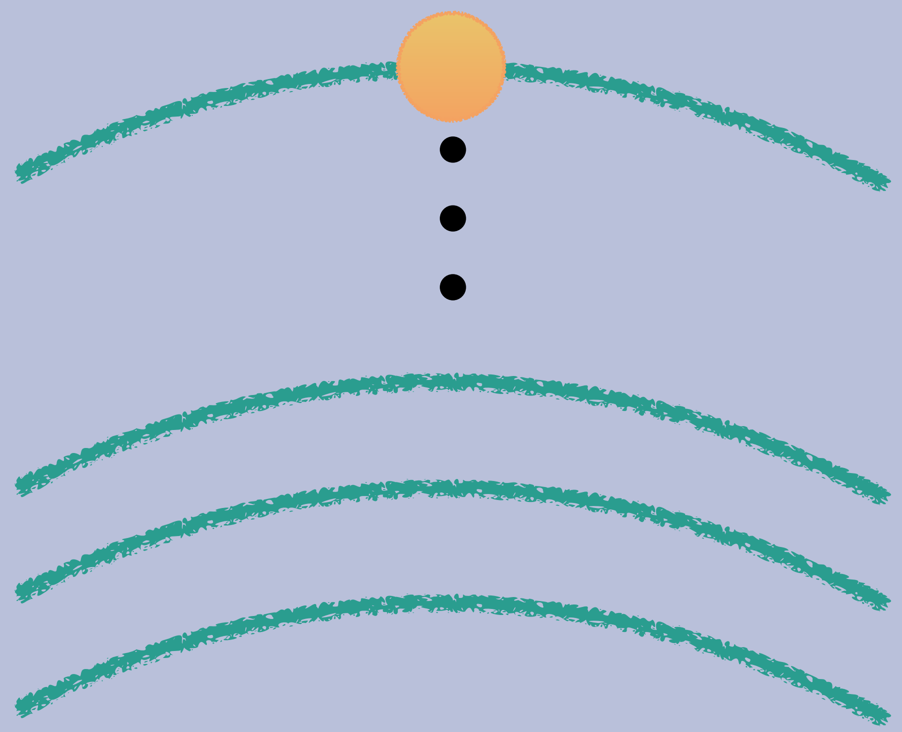
We can control the state with a laser set to a specific frequency

Let’s create one



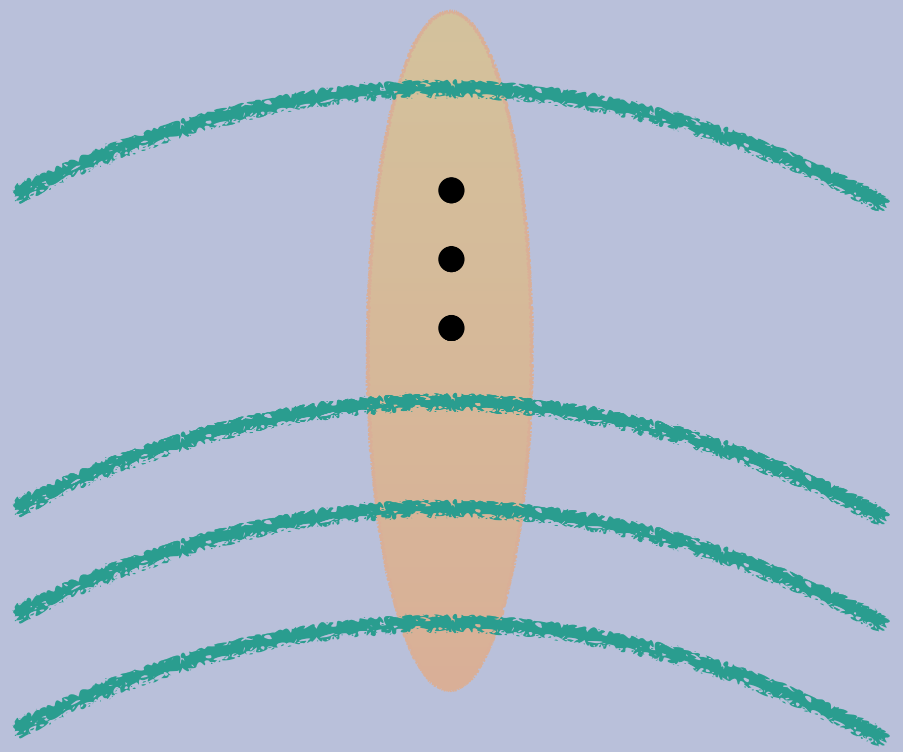


$= |g\rangle \quad = |0\rangle$



$= |r\rangle \quad = |1\rangle$

By applying a specific pulse we can create a superposition



$$= \frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|1\rangle$$

Each state with probability
of being populated of:

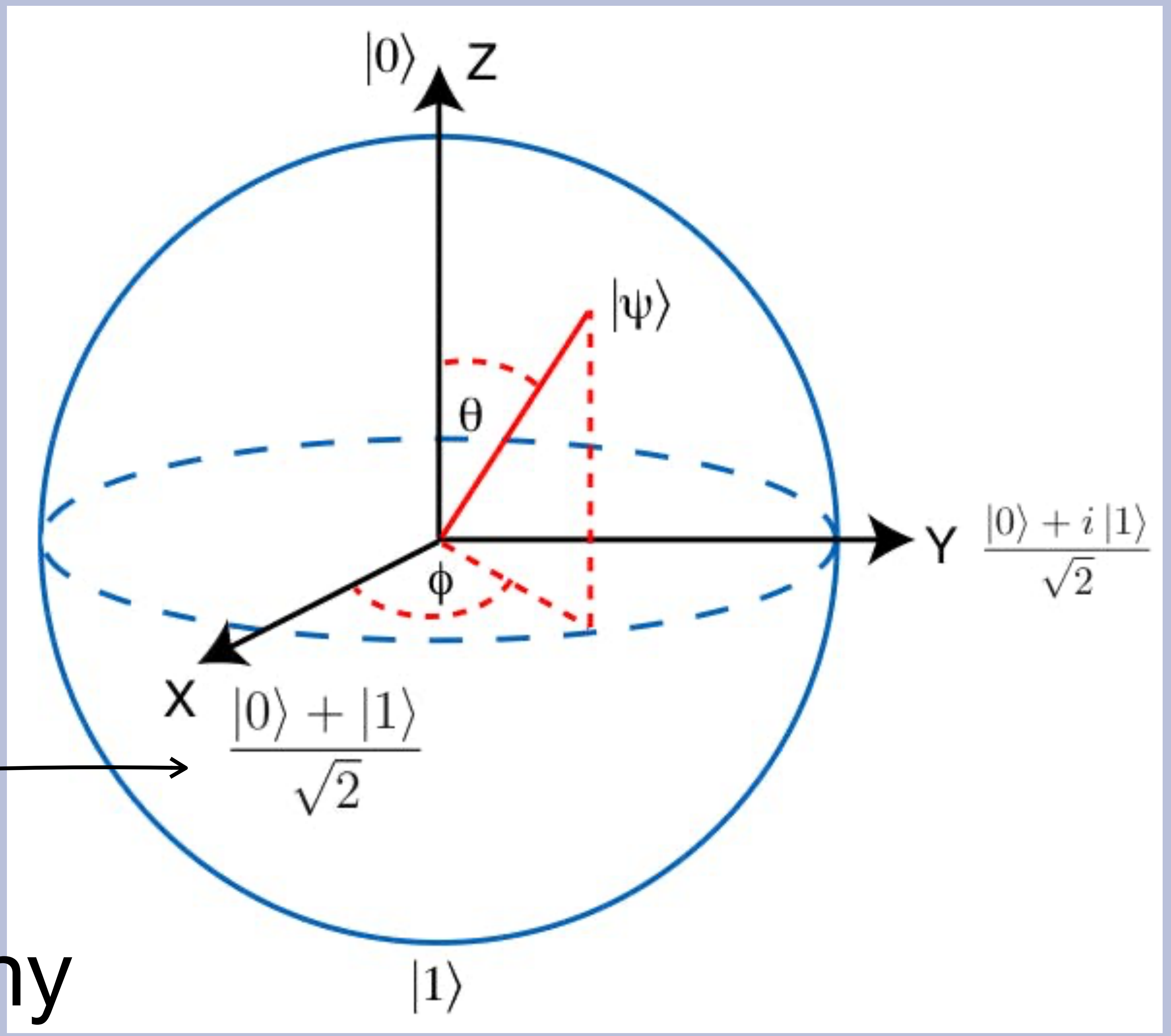
$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

And we can do much more

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\Psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$



This is one of infinitely many possible superposition states

This is a qubit

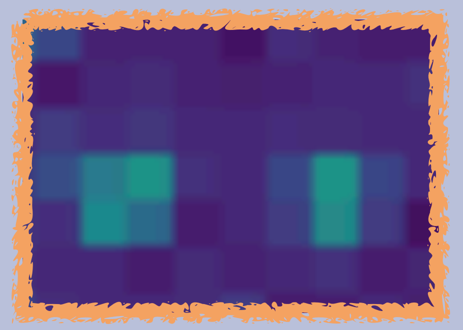
Rotating the qubit is like applying (quantum) gates

$$\boxed{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

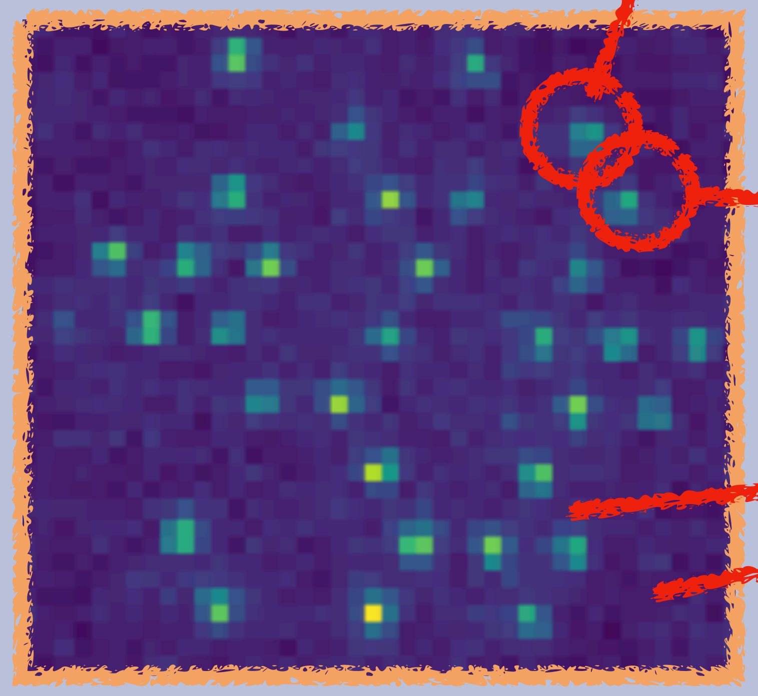
$$\boxed{R_z} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Example: $H|\Psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

And we can add more qubits as well…

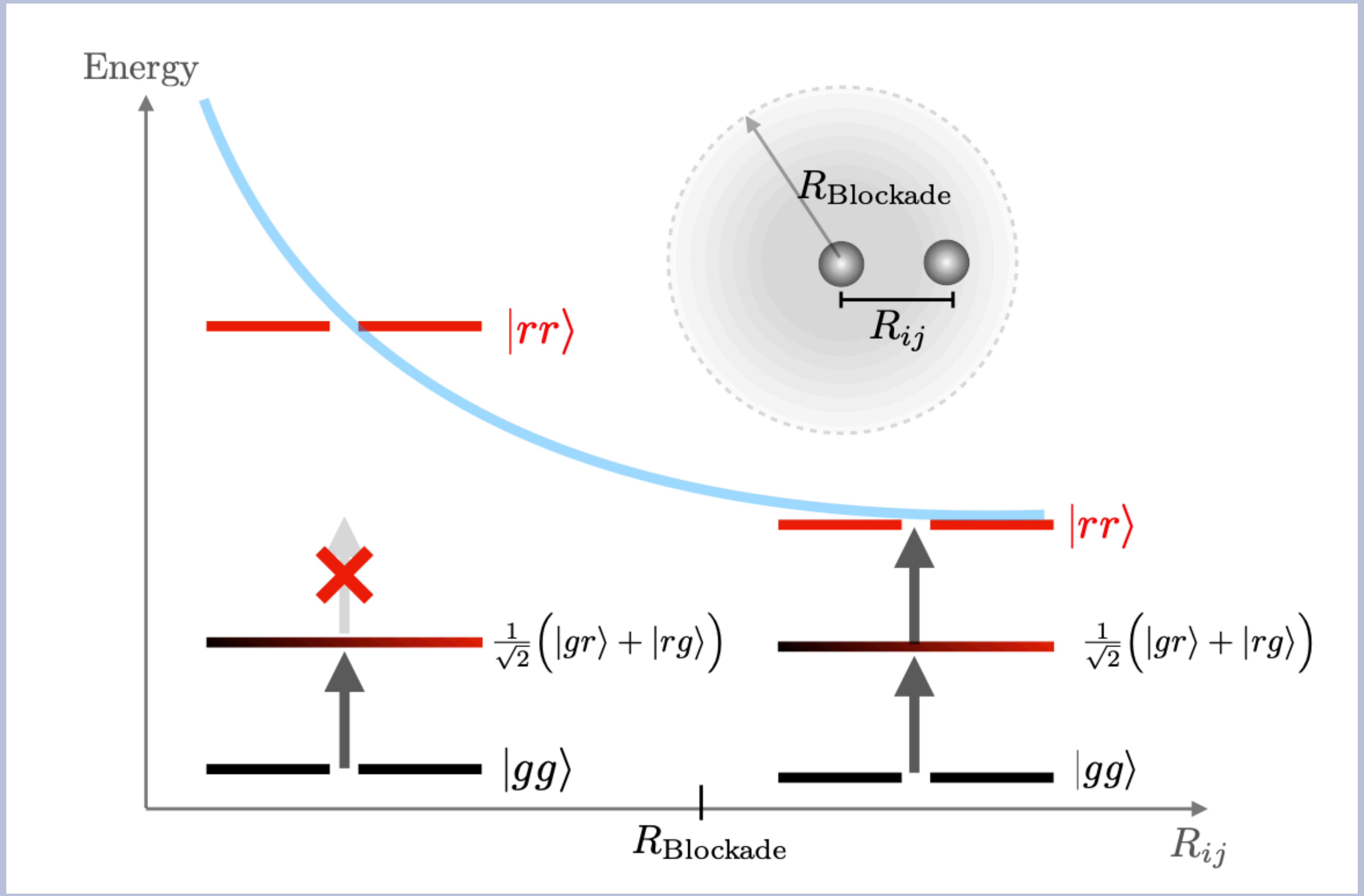


$$\begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \otimes \begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix} = \begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix} = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$



$$|\Psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \otimes \begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix} \otimes \begin{pmatrix}\alpha_3\\ \beta_3\end{pmatrix} \otimes \cdots \otimes \begin{pmatrix}\alpha_N\\ \beta_N\end{pmatrix} = \begin{pmatrix}\alpha_1\alpha_2\alpha_3\ldots\alpha_N\\ \alpha_1\alpha_2\alpha_3\ldots\beta_N\\ \vdots\\ \beta_1\beta_2\beta_3\ldots\beta_N\end{pmatrix} = 2^N$$

With 2 qubits we can also create entanglement…



… exploiting the Rydberg blockade.

When two atoms are close enough, electrons cannot be excited to |r> at the same time

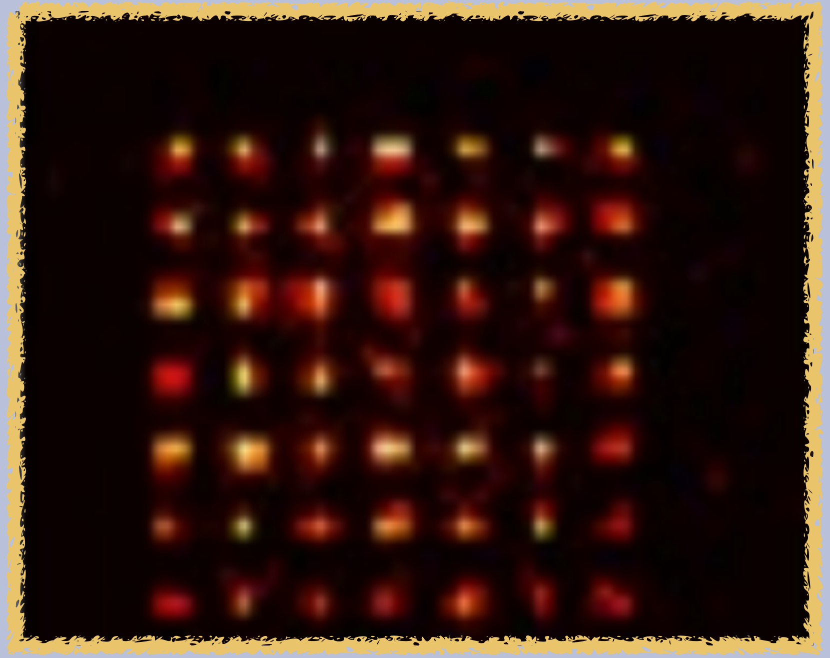
The result:

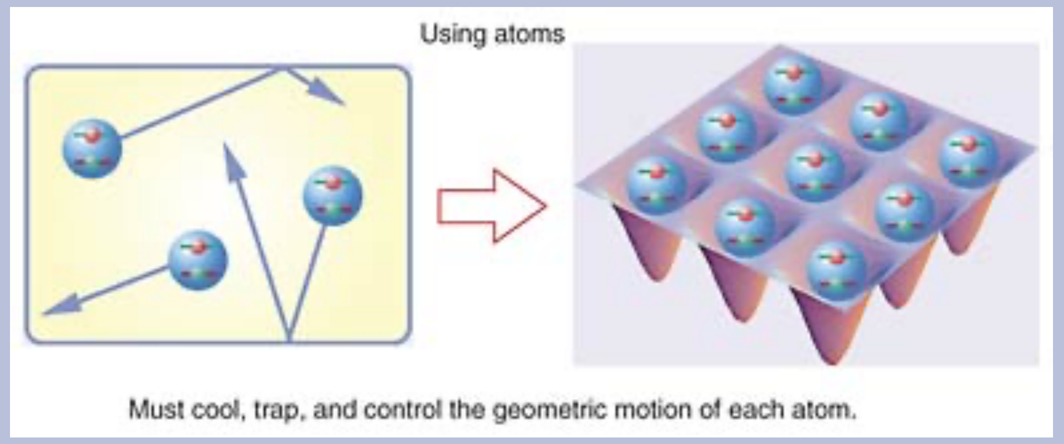
Far apart: $$|gg> \to \frac{1}{2}(|gg\rangle + |gr\rangle + |rg\rangle + |rr\rangle) = \frac{1}{\sqrt{2}}(|g\rangle + |r\rangle) \otimes \frac{1}{\sqrt{2}}(|g\rangle + |r\rangle)$$

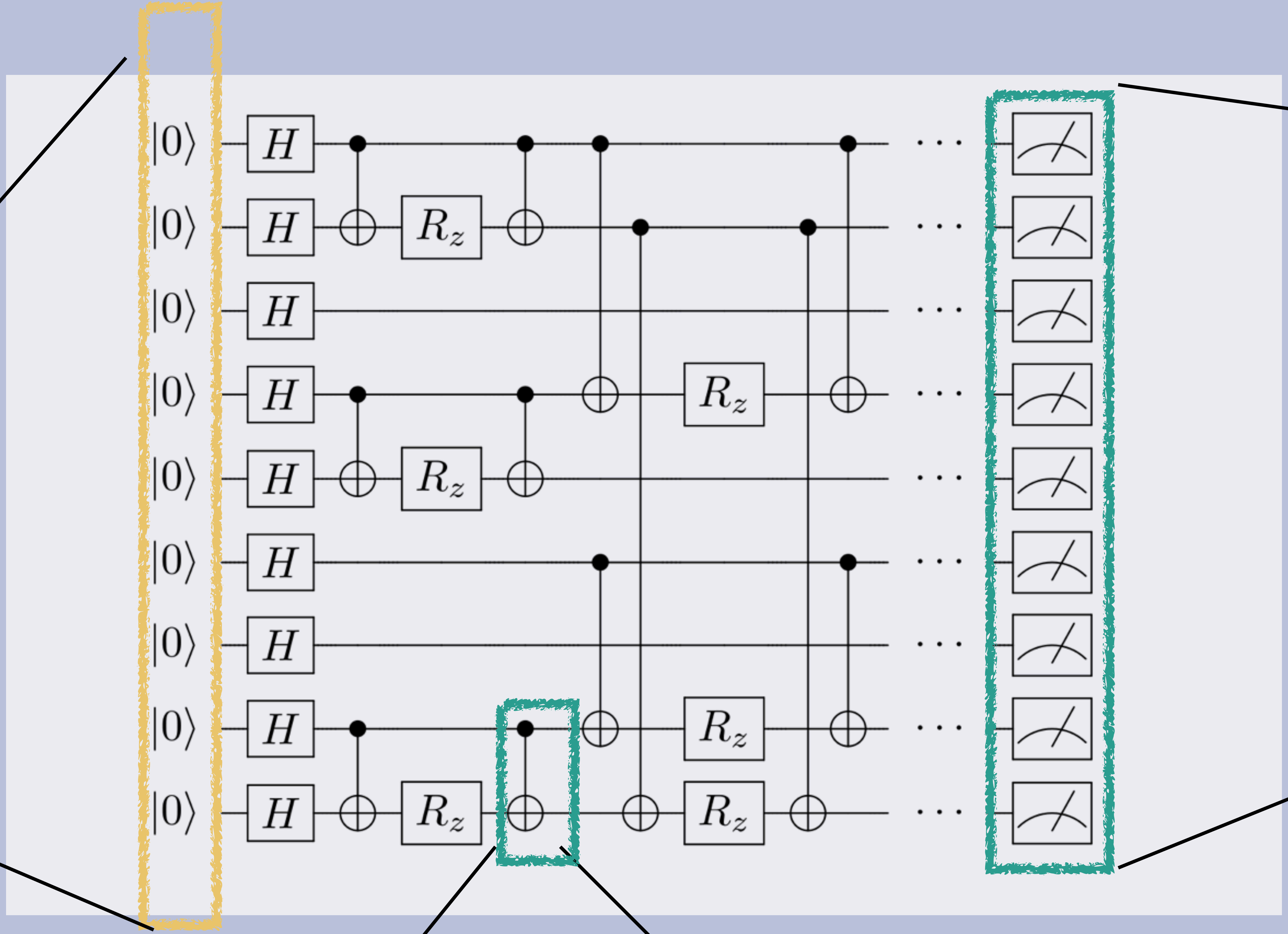
Closer than R $$|gg> \to \frac{1}{2}(|gg\rangle + |gr\rangle + |rg\rangle - |rr\rangle)$$ Two electrons are entangled!

Rotating a qubit + entanglement = quantum computation

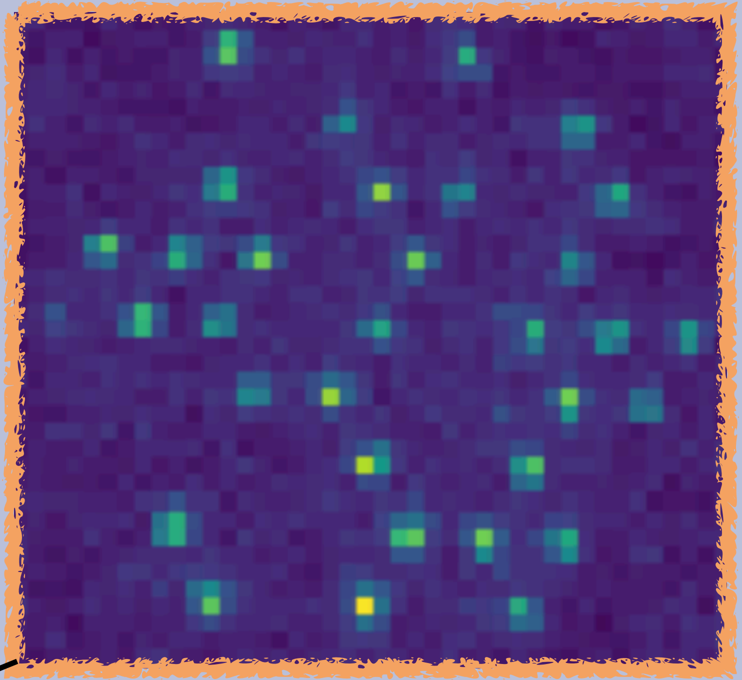
Prepare a state



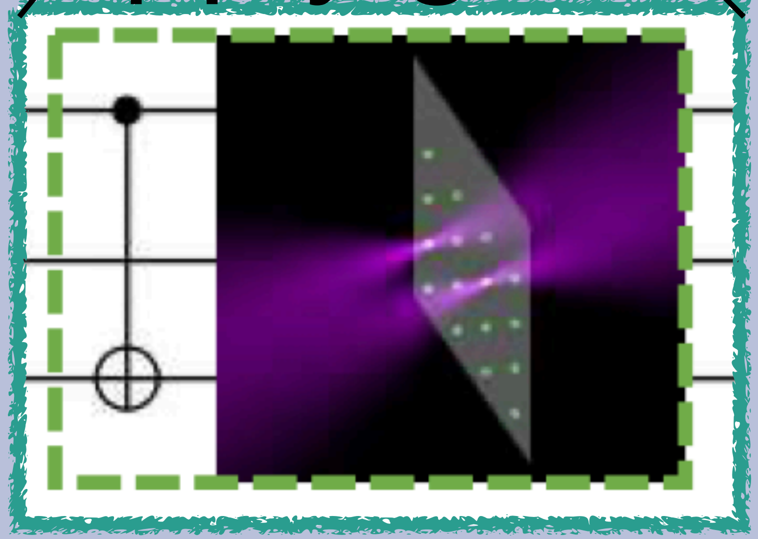




Measure

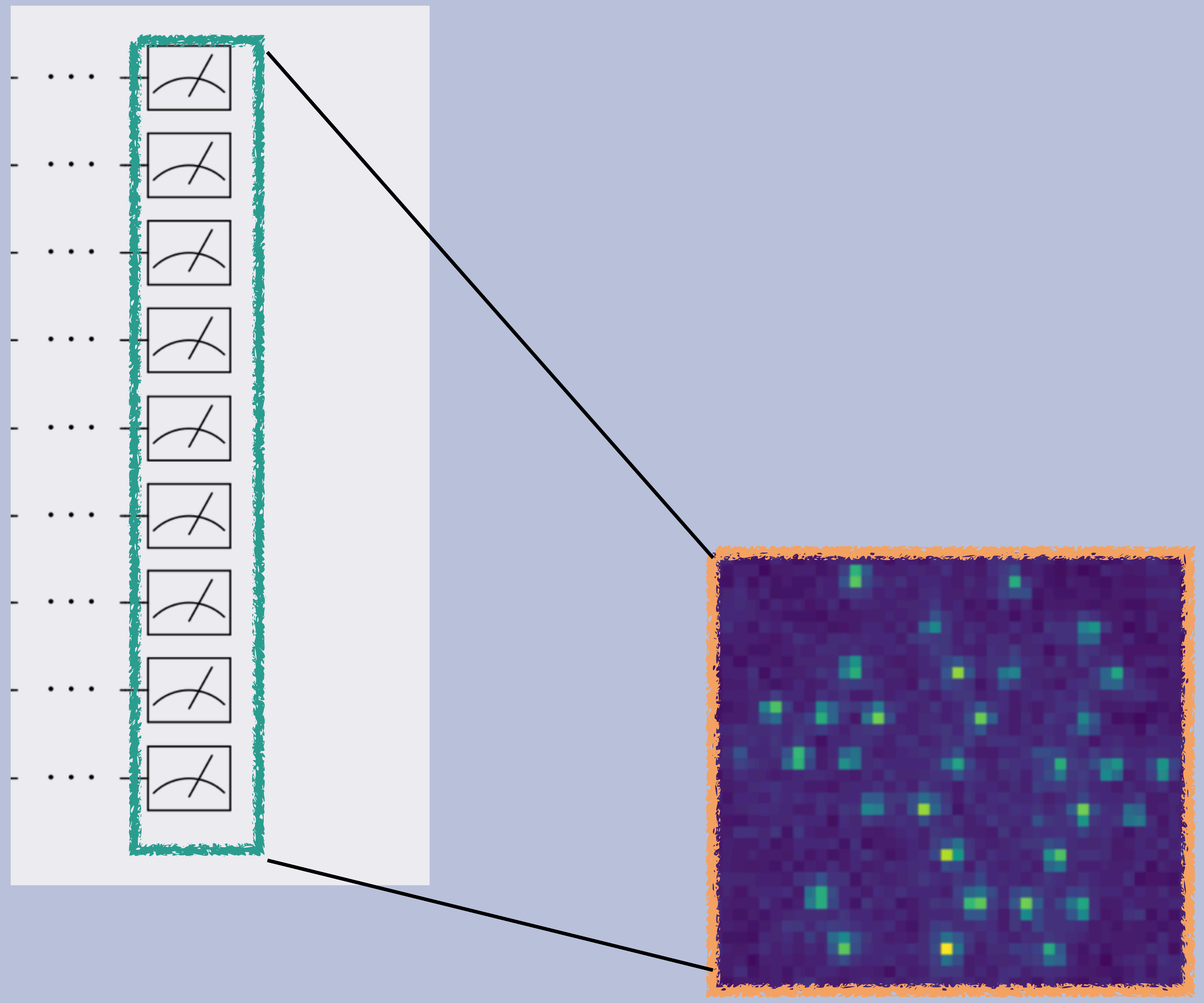


Apply gates



This is called
DIGITAL MODE

The measurement problem is central in every type of QC because measuring means destroying a state.



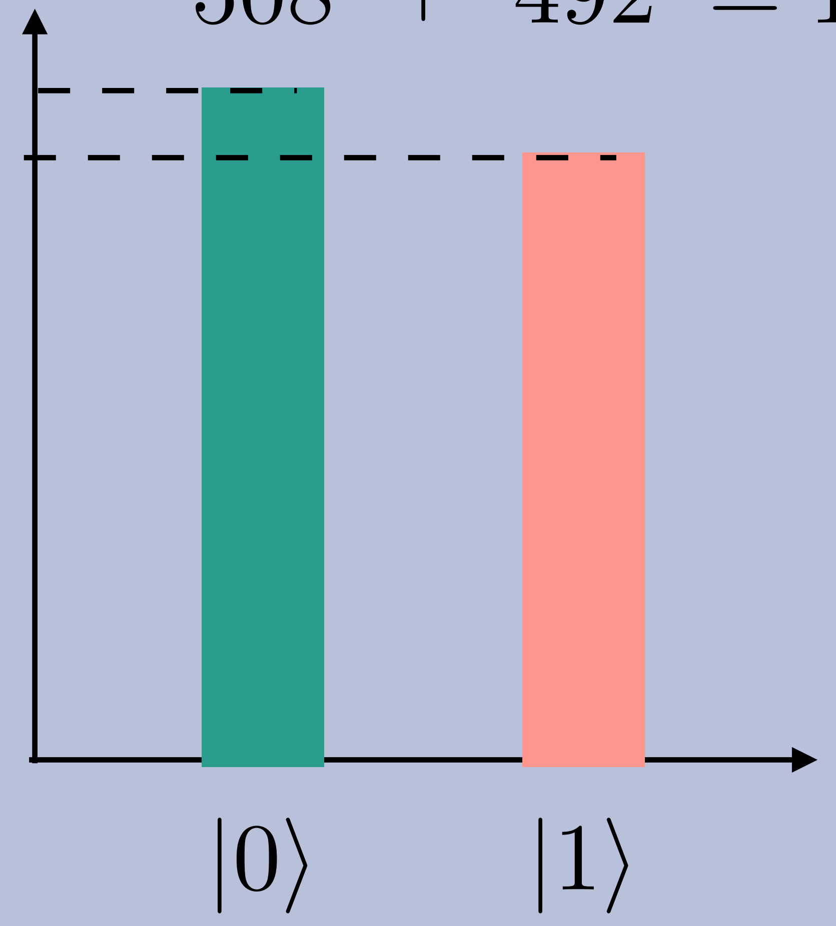
A camera flashes the qubits and erases the ones in |r>

Example

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} = p_0 = p_1$$

$$508 + 492 = 1000$$



DIGITAL MODE

What can we do with digital?

Any algorithm, all of them.

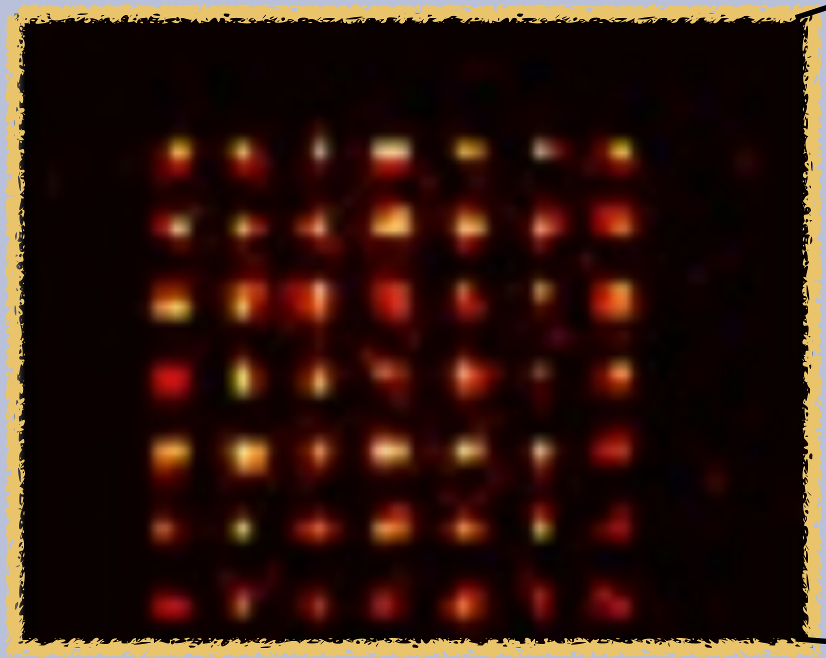
Ex: Shor, quantum neural networks …

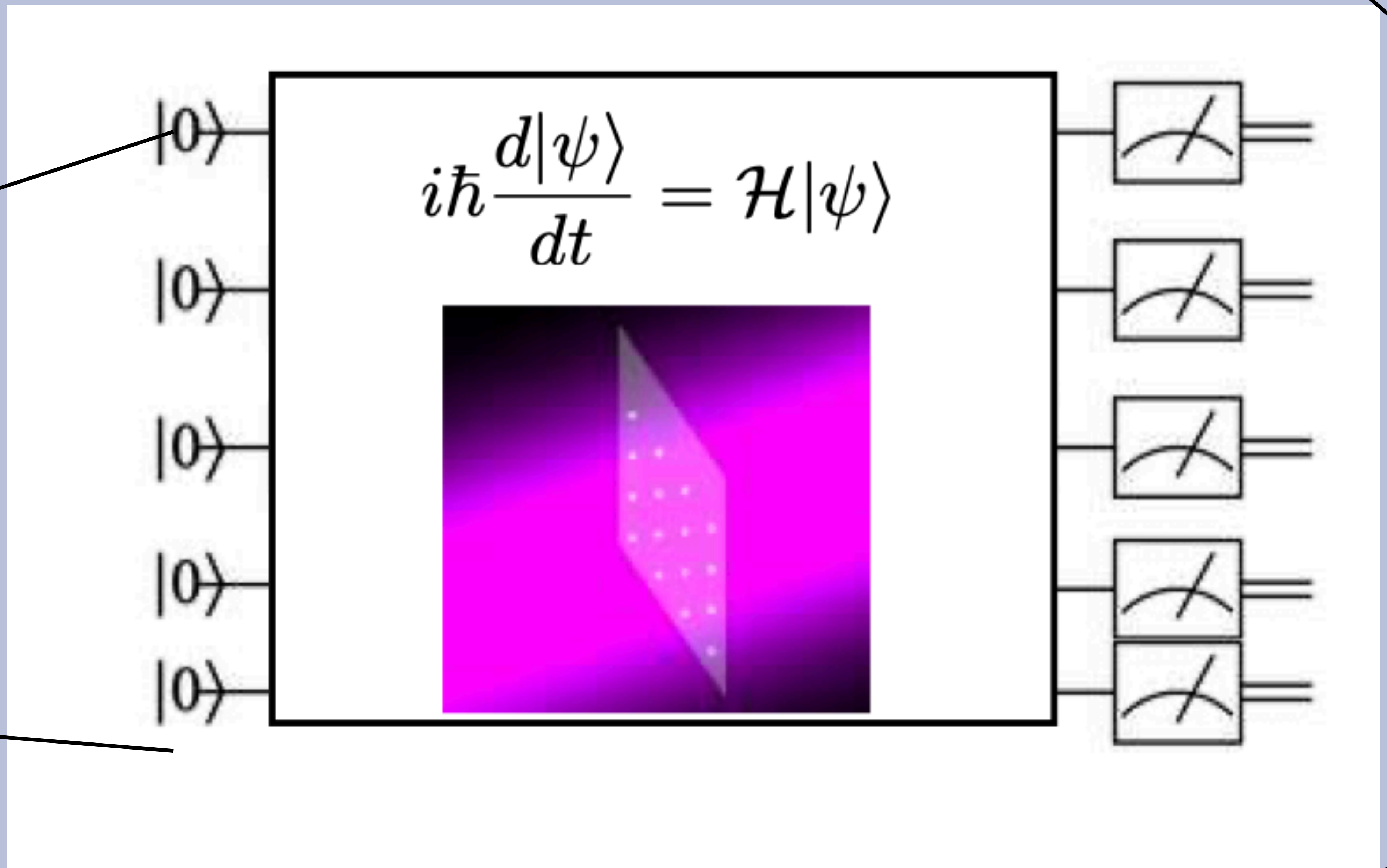
Why isn’t everything digital?

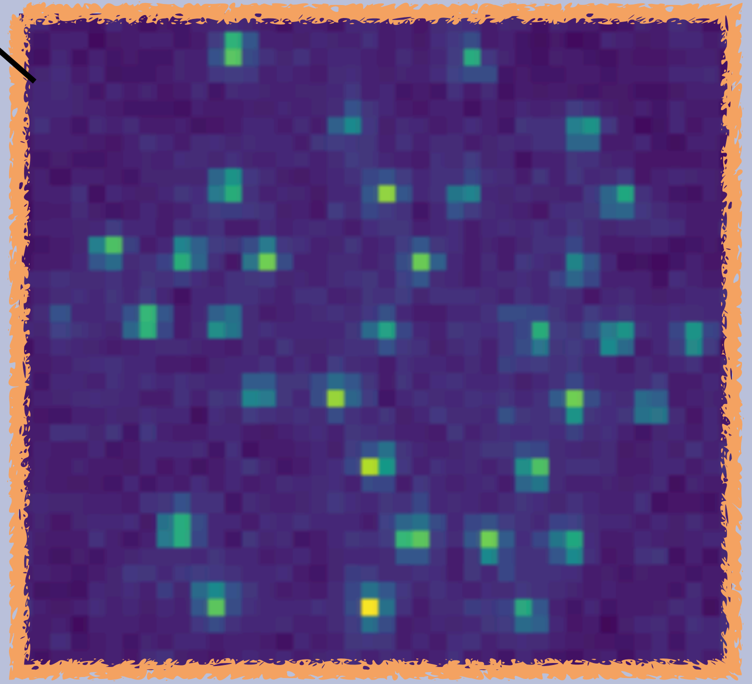
It’s difficult :(

An “easier” approach: Analog Quantum Computation

Prepare a state







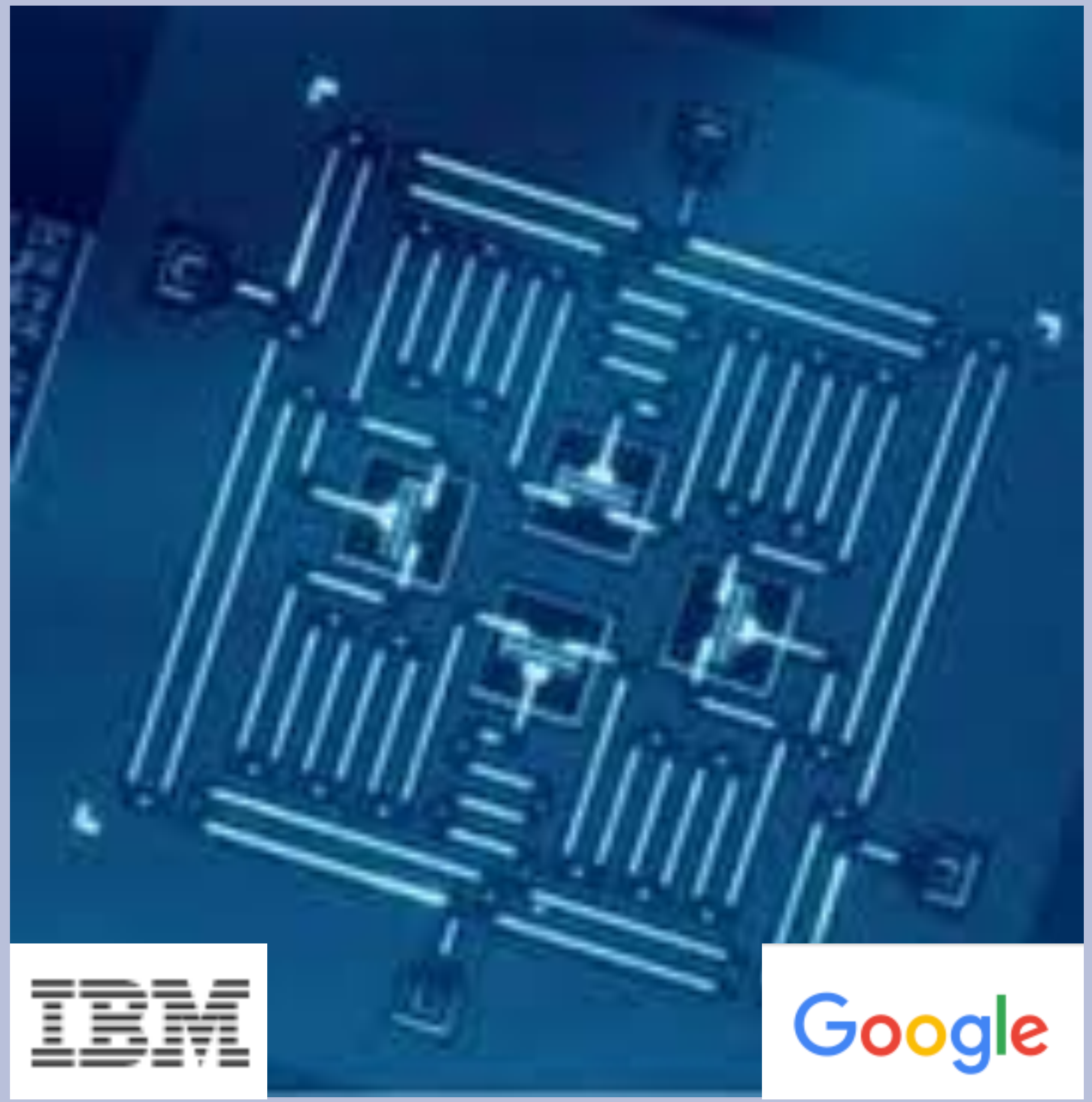
What can we do?

Almost every algorithm.

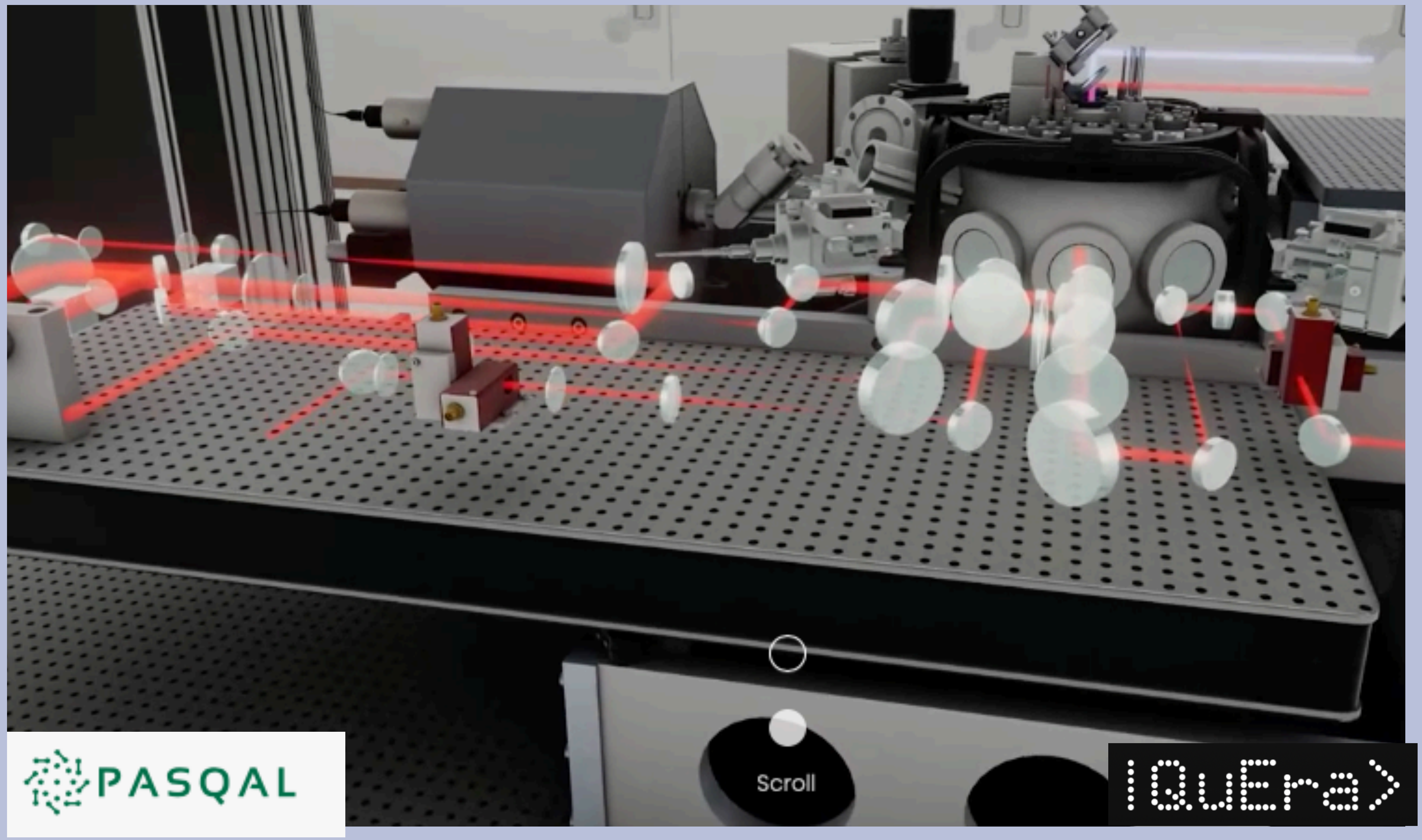
Are there other ways to create a QC?

Yes, a few people are working on it

analog QC,
superconducting
technology



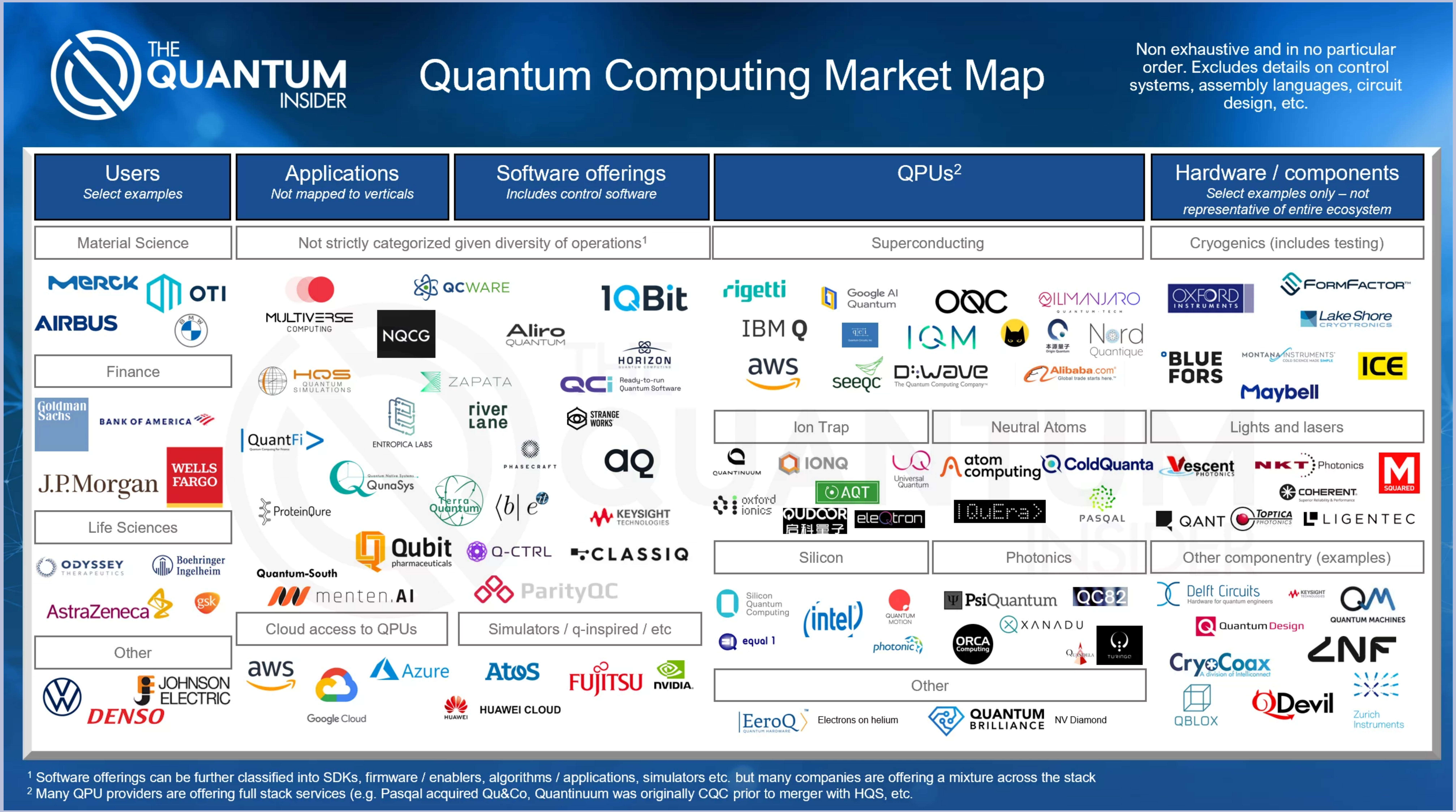
digital QC,
superconducting
technology



analogue/digital QC,
Rydberg atom
technology

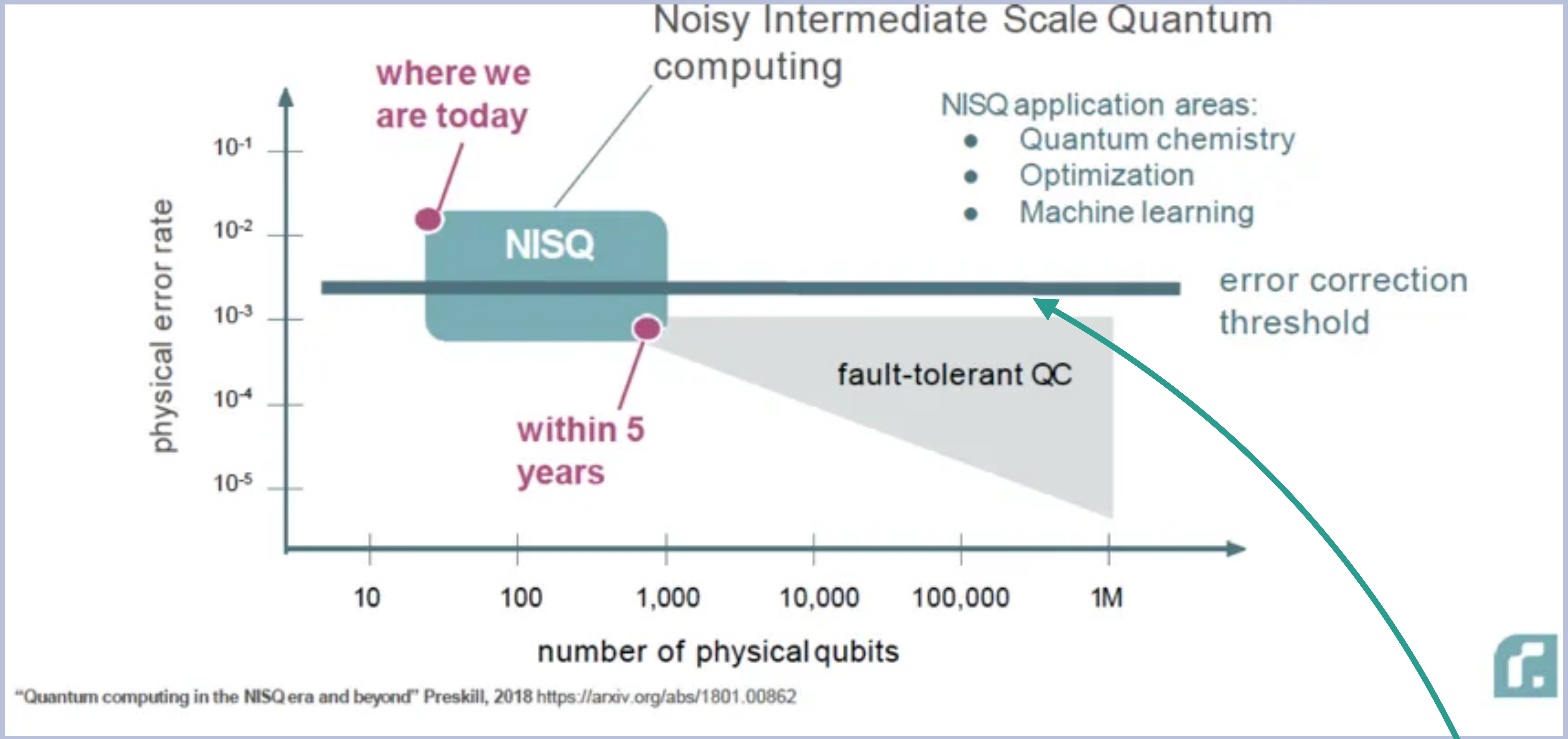
Are there other ways to create a QC?

Probably more than a few



Is any technology better than other?

Two main ways to tell: no. of qubits and noise



Each company/university/research center has a roadmap (path) inside this graph over time

Lets run an algorithm

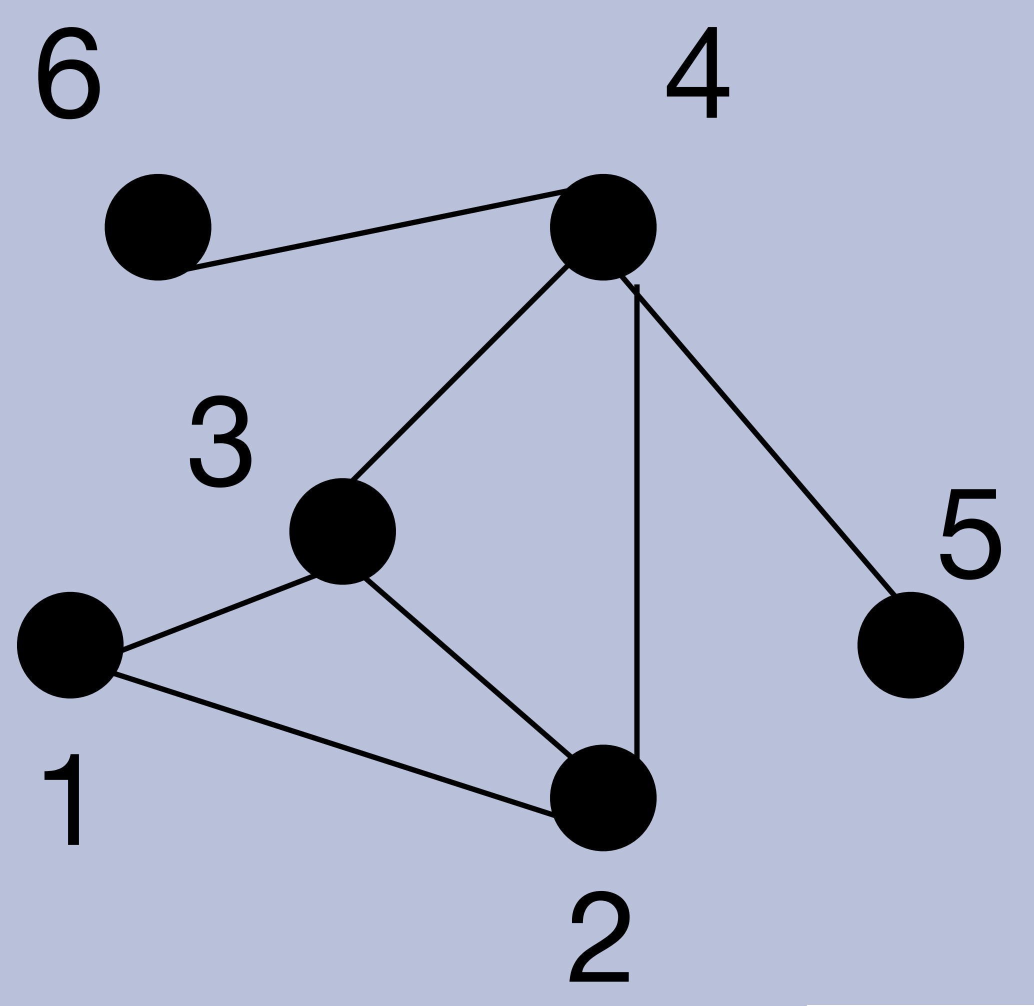
Pick an architecture: neutral atoms

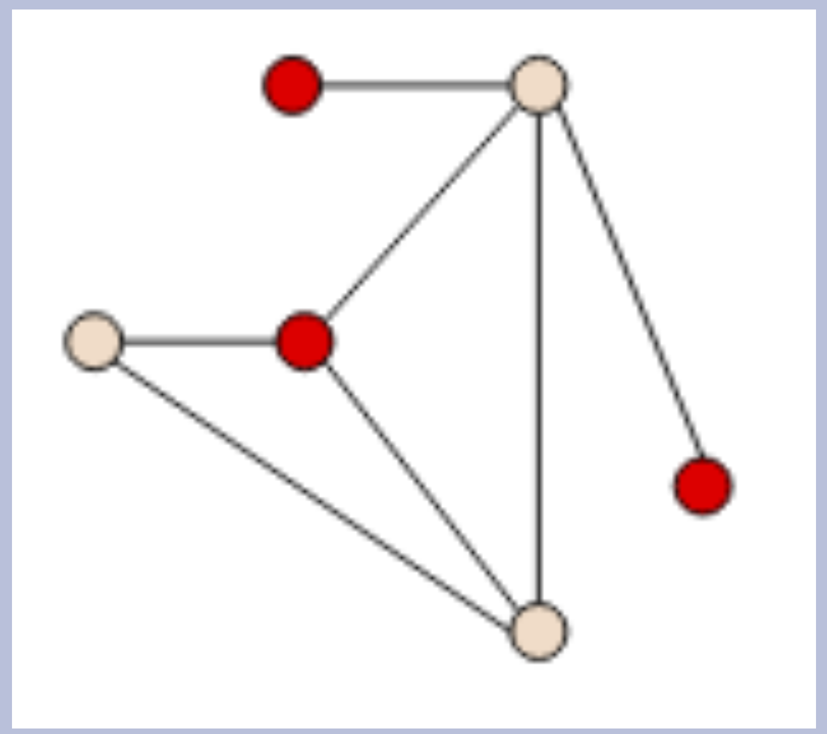
Pick a method: analog, no choice

Pick a classical problem: combinatorial

Pick an algorithm: variational quantum circuit

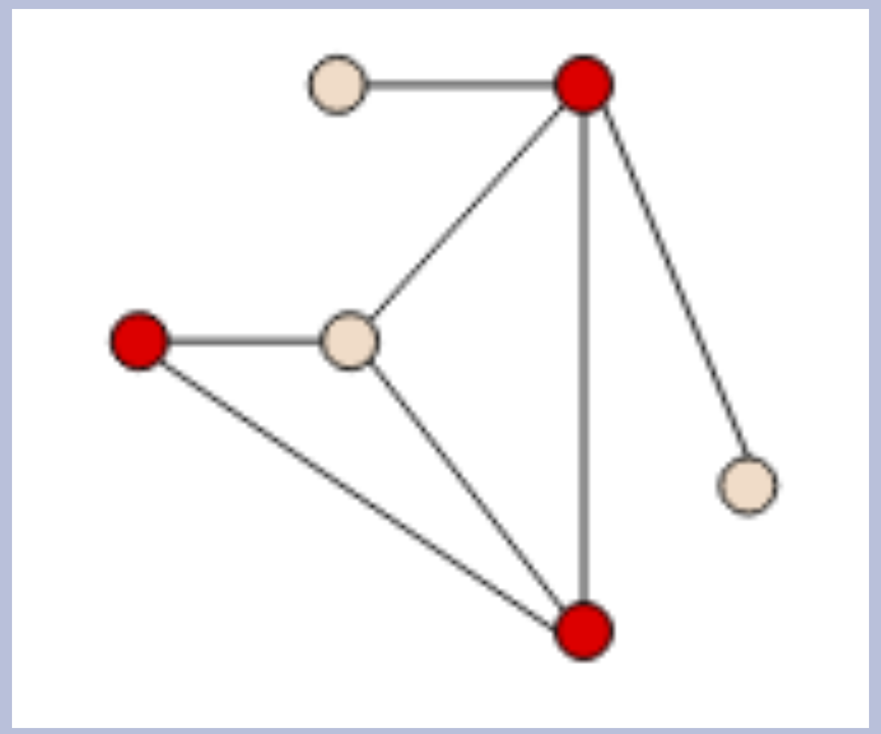
THE MIS problem





001011

Is a solution



110100

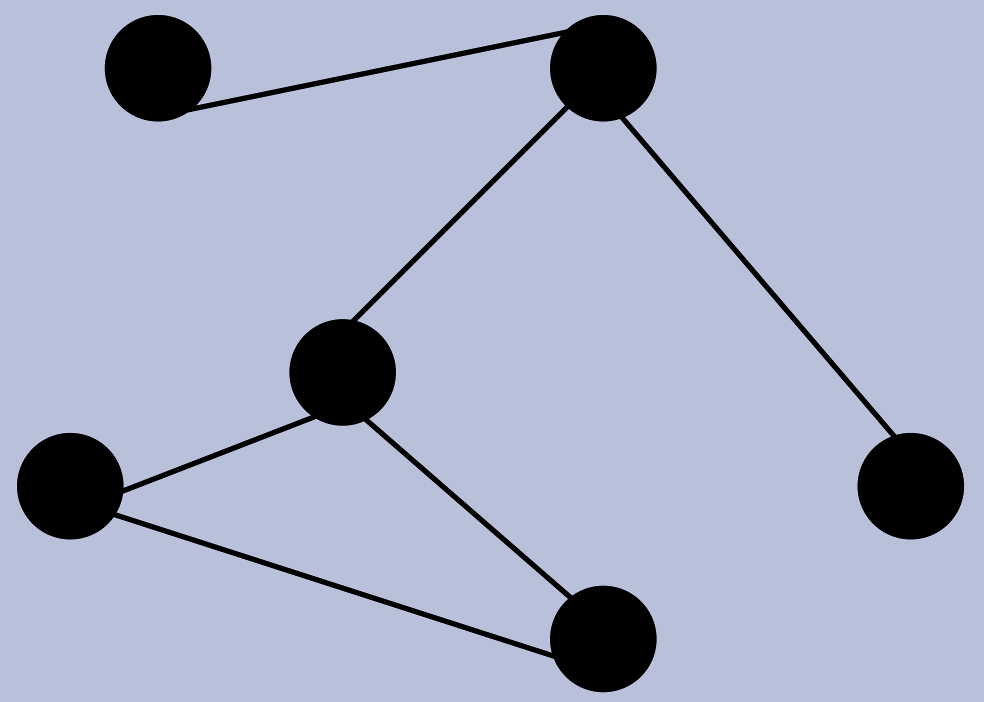
It’s not a solution

$$E = -\sum_i x_i + \sum_{i,j} x_i x_j$$

$E = -3$

$E = -3 + 2 = -1$

Why MIS?



First: Ubiquitous

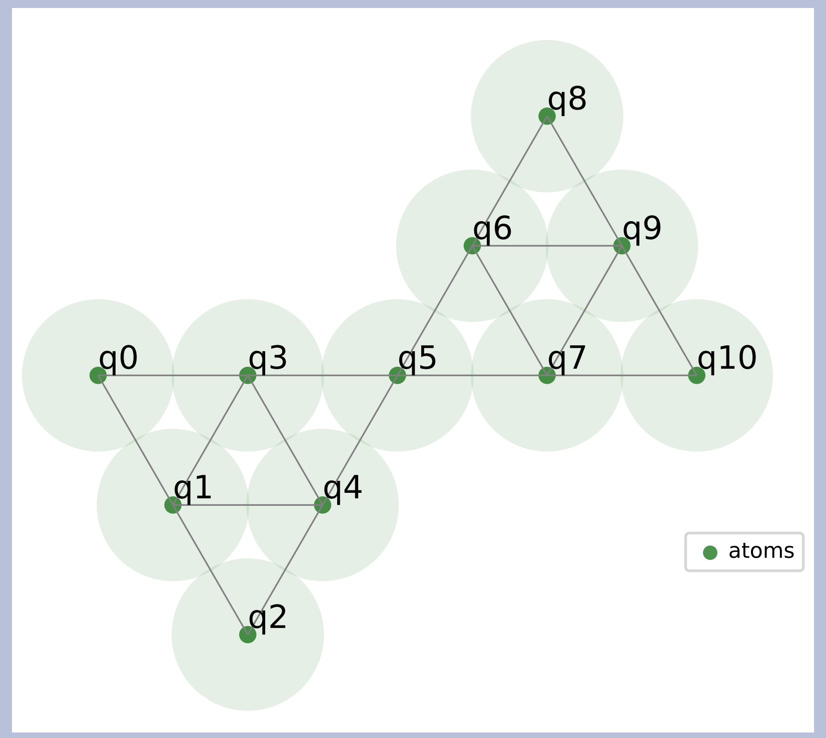
Second: Ising encoding

$$E = -\sum_{i\in G} x_i + \sum_{(i,j)\in E} x_i x_j \quad \longrightarrow \quad H = \sum_{i\in G} Z_i + \sum_{(i,j)\in E} (1 - Z_i)(1 - Z_j)$$

$x_i \in \{0,1\}$

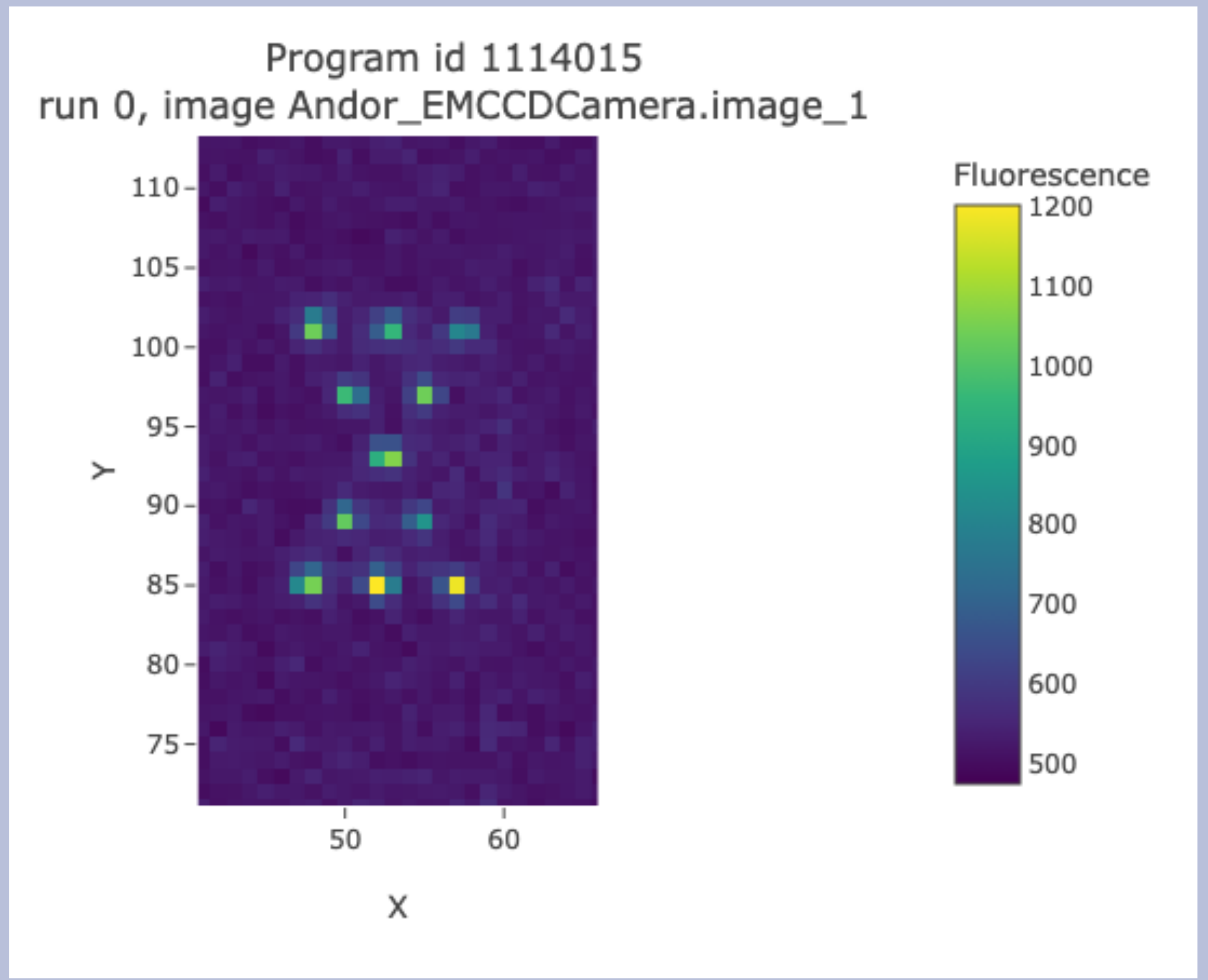
$$Z_i|1\rangle = -1|1\rangle$$
$$Z_i|0\rangle = +1|0\rangle$$

Third: spatial dependency

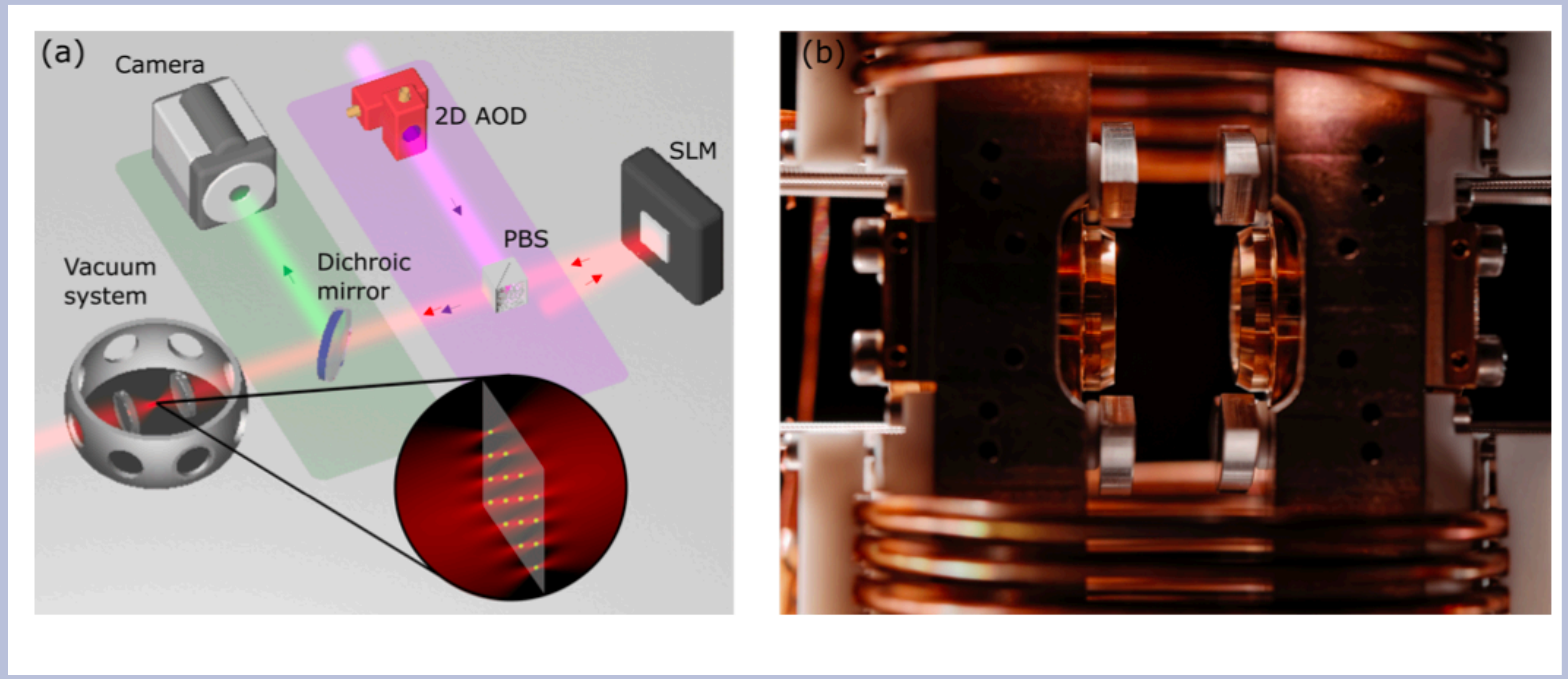


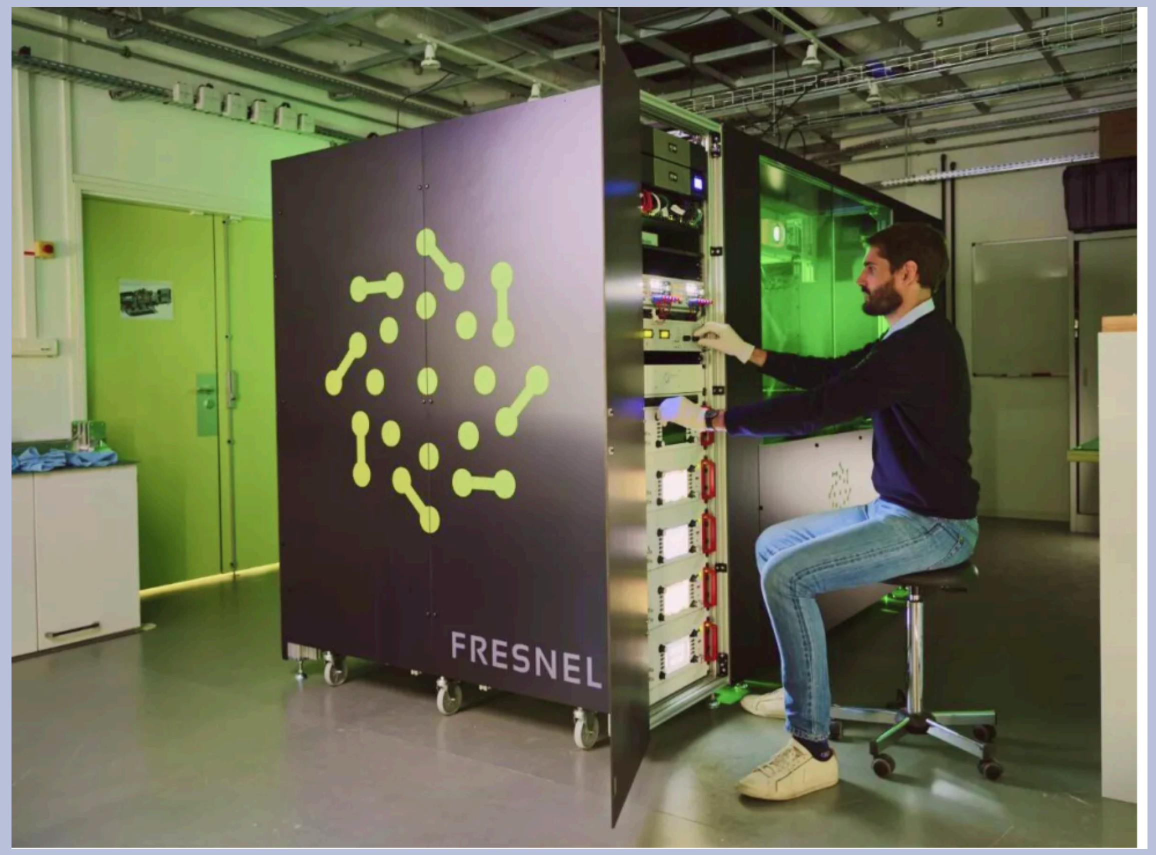
Equivalence between graph problem

and atoms positions

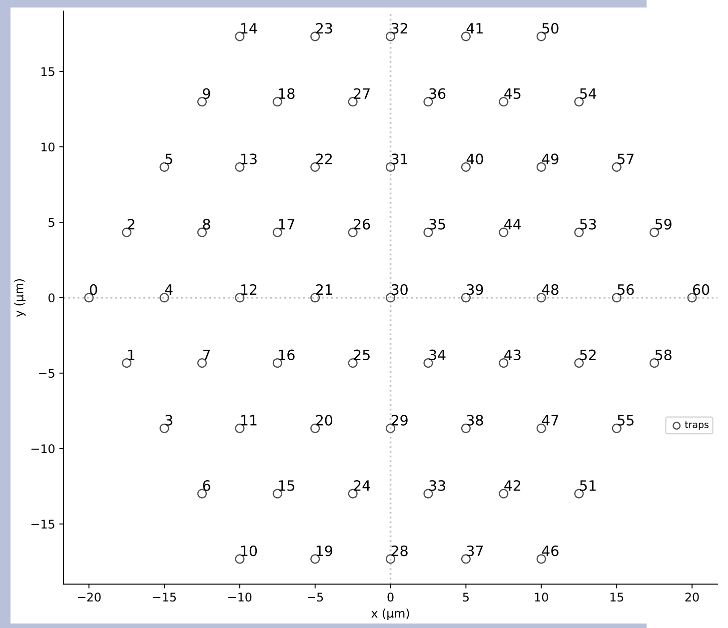


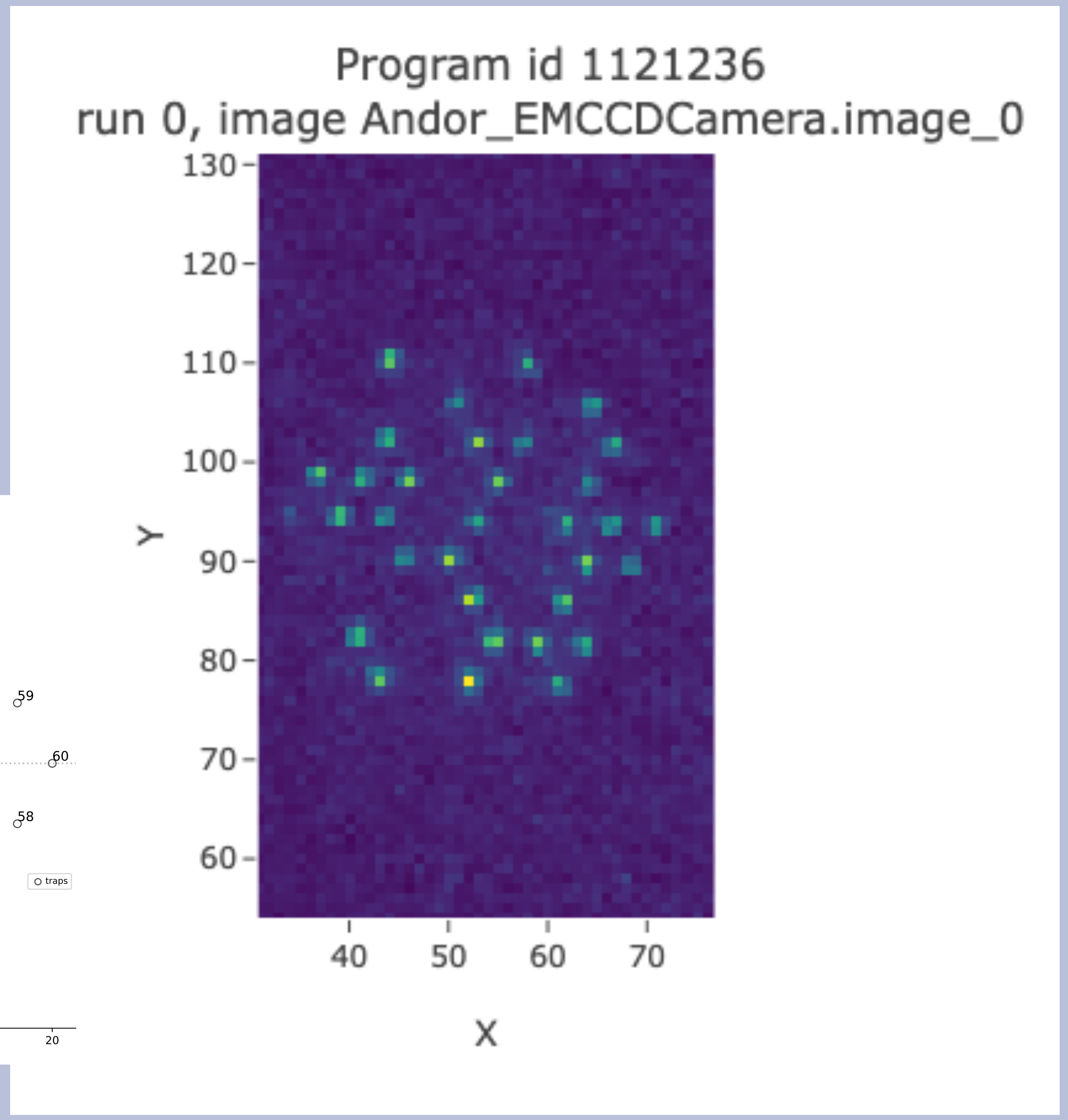
HOW TO RUN AN ANALOG ALGORITHM: 1. Load the graph



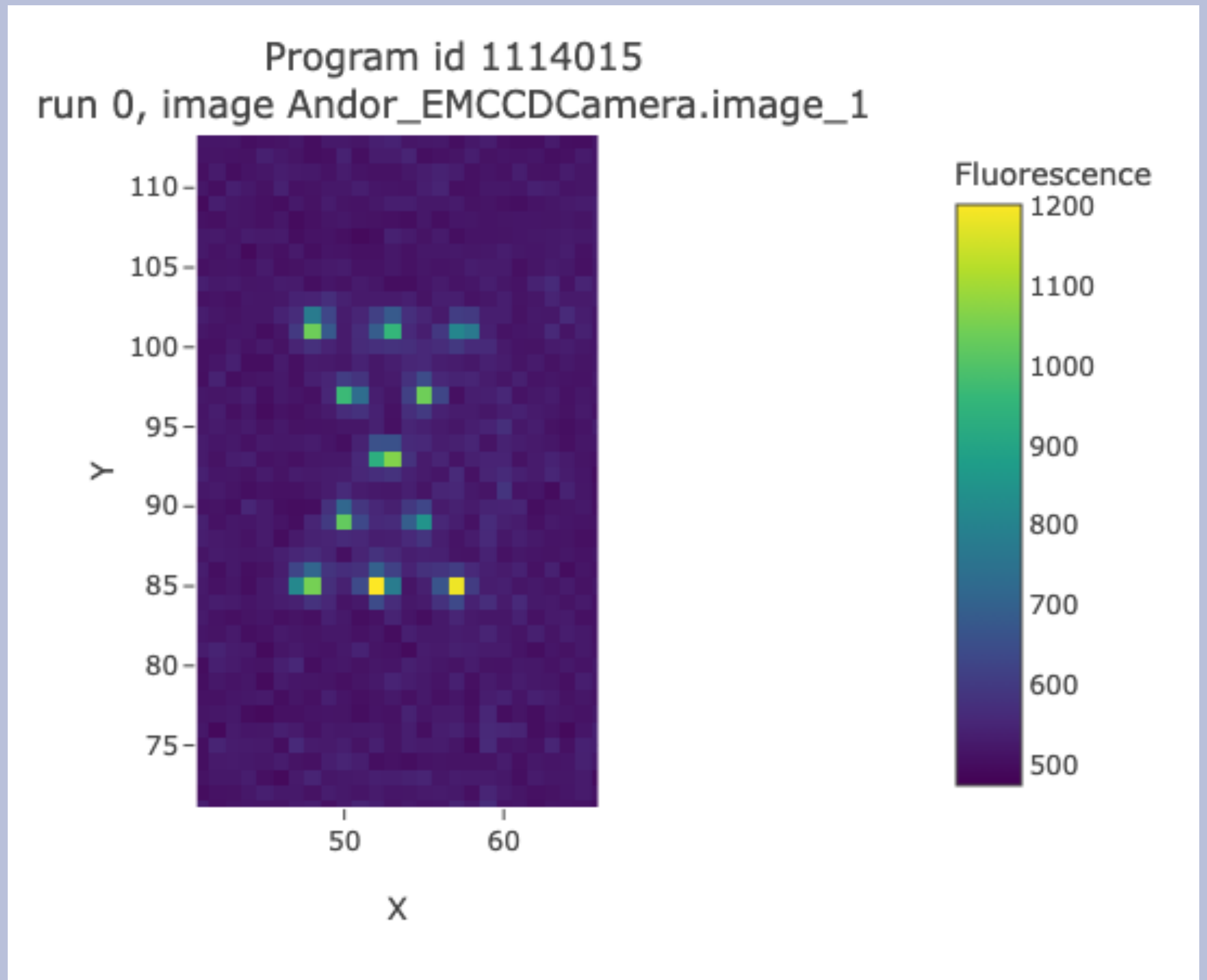


First:
load randomly

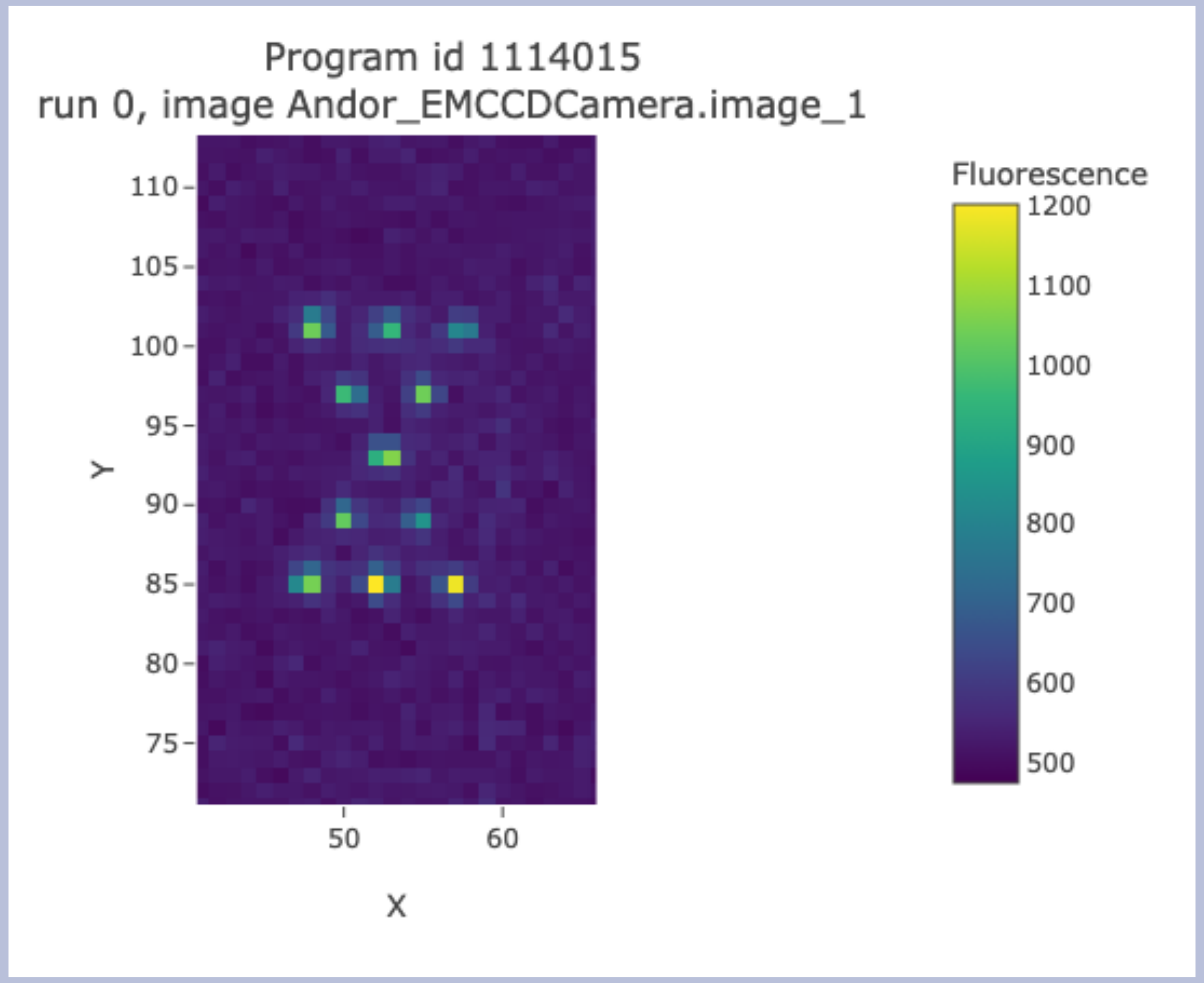




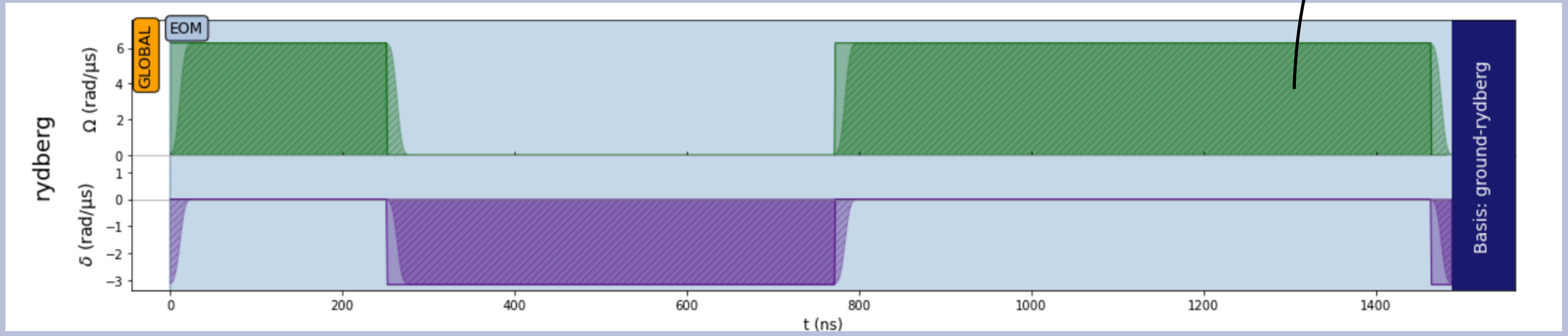
Then, readjust



HOW TO RUN AN ANALOG ALGORITHM: 2. Apply pulses for random times



This durations are initially random



Rotates qubits from |0> to |1>

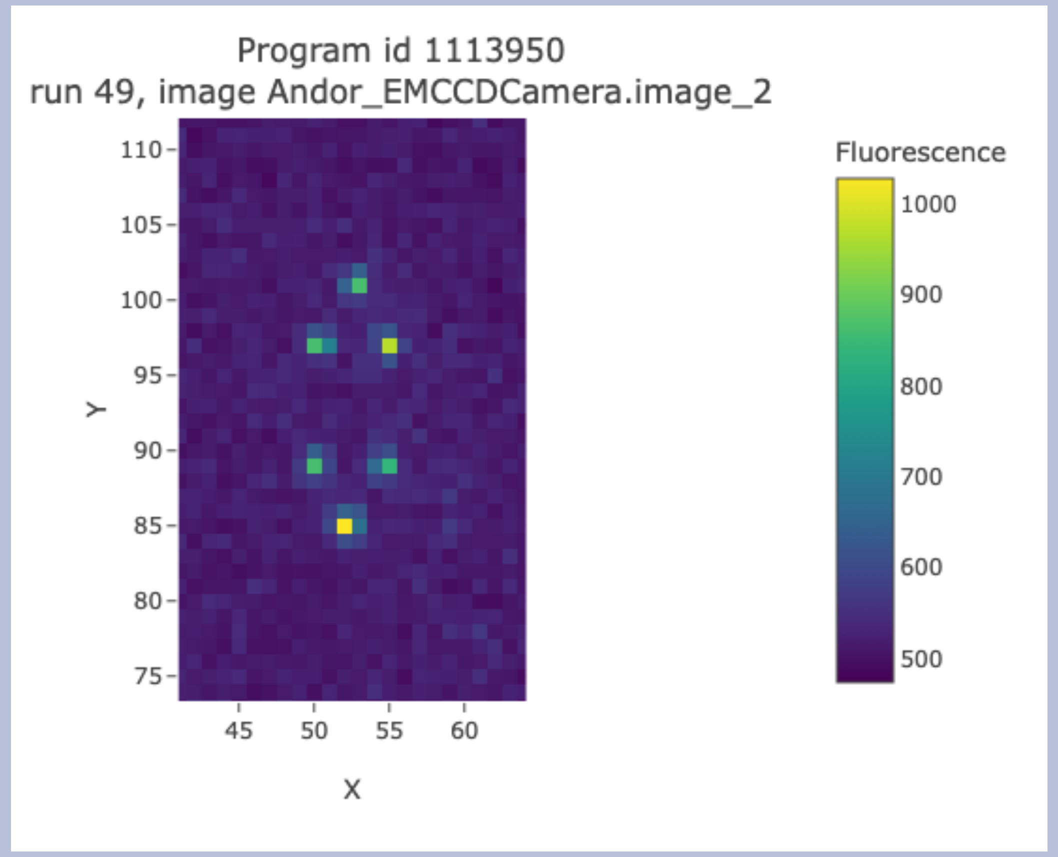
Applies phase according to the energy of the problem

$$H_M = \Omega(t) \sum_i \sigma_i^x$$

$$H_C = -\delta(t) \sum_i n_i + U_{i,j} \sum_{(i,j)\in E} n_i n_j$$

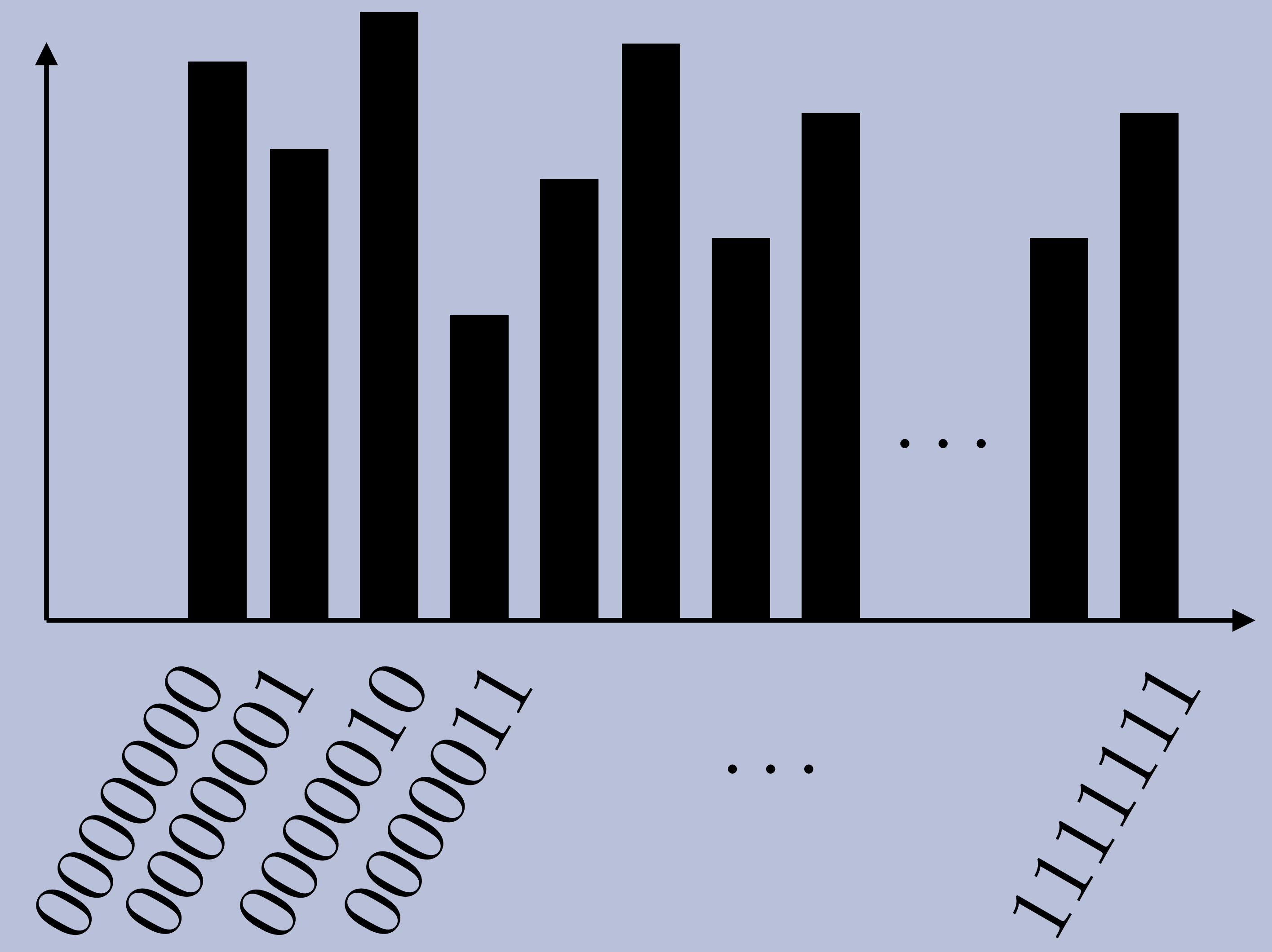
HOW TO RUN AN ANALOG ALGORITHM: 3. Measure and repeat

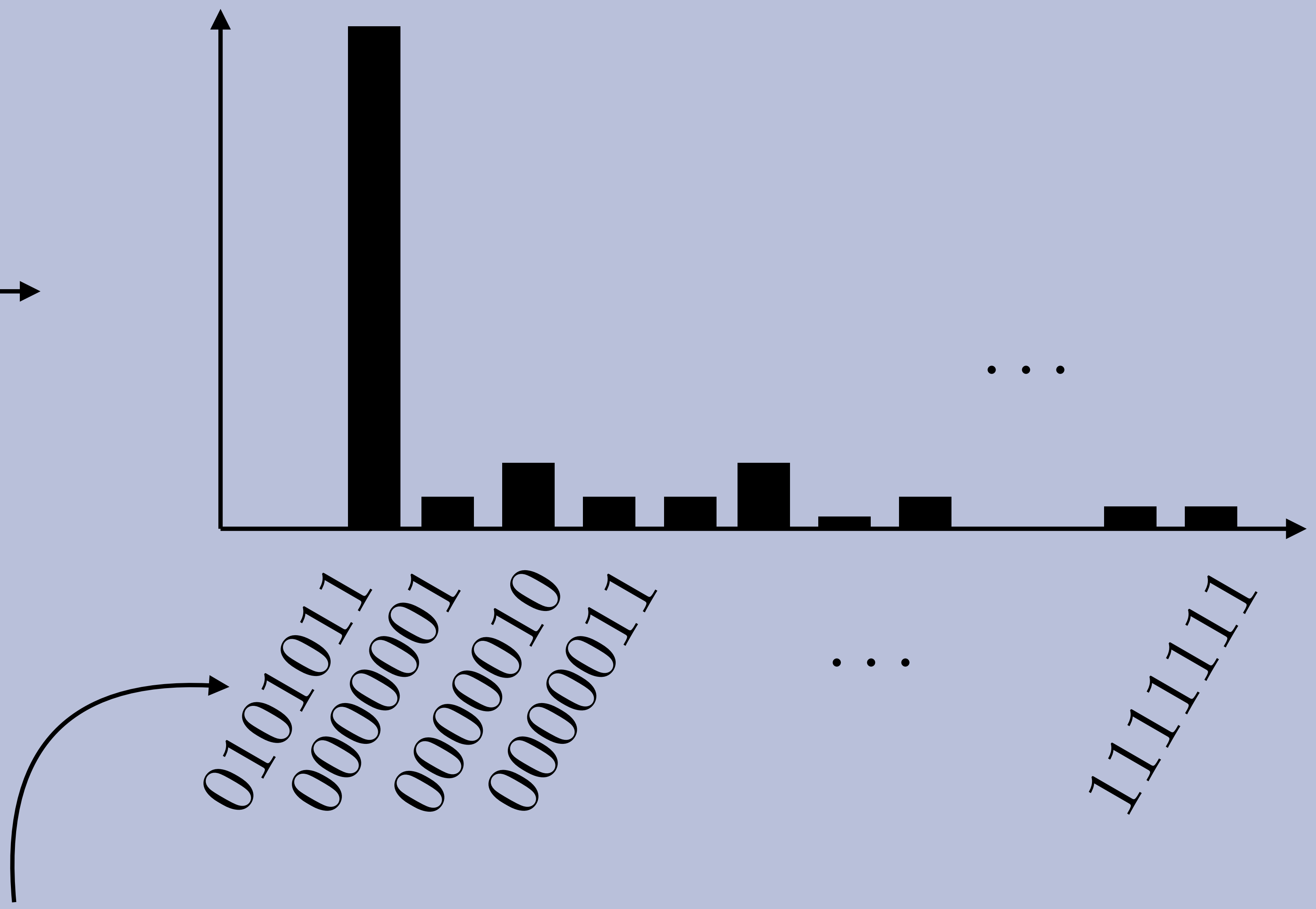
We measure, obtain a bitstring, repeat N times



An example of a measured solution

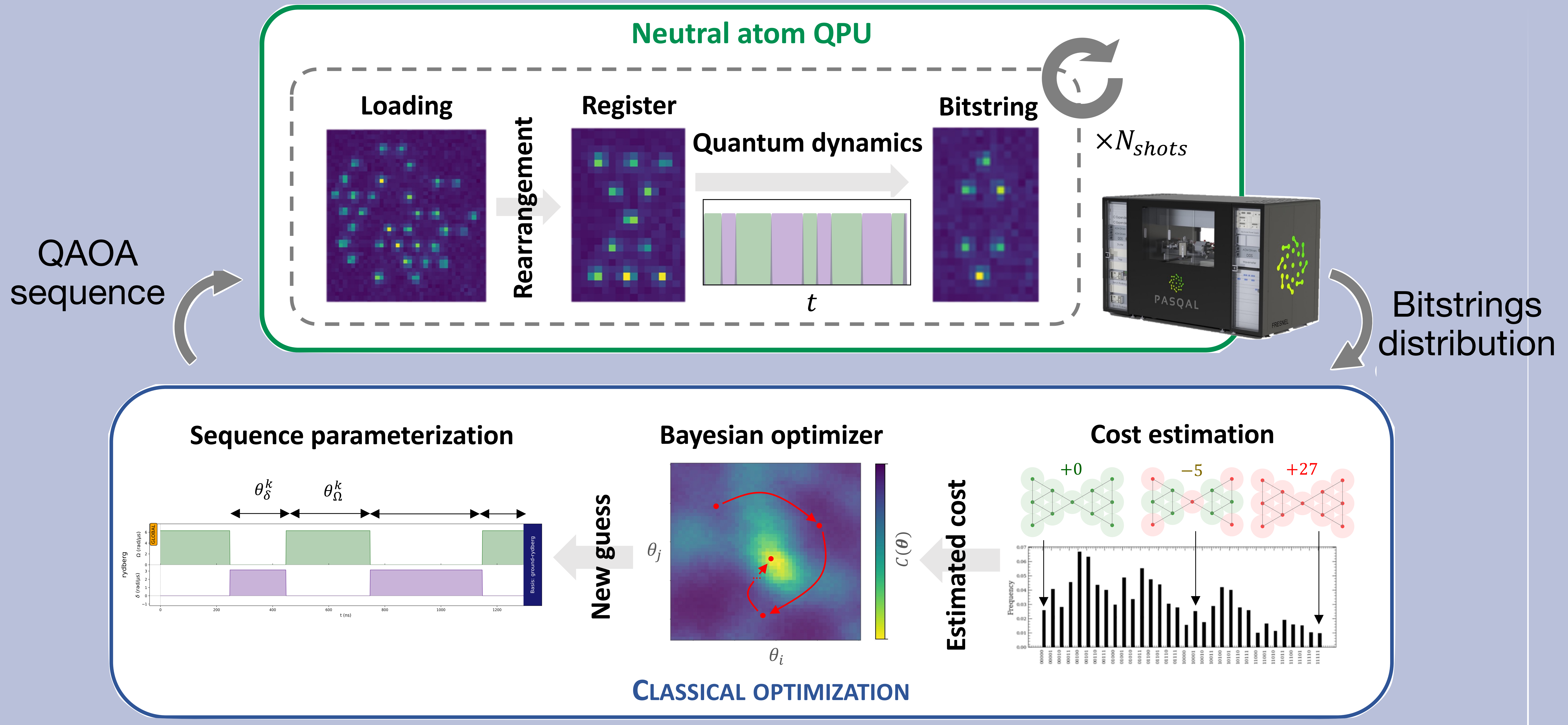
How do we go from





Solution

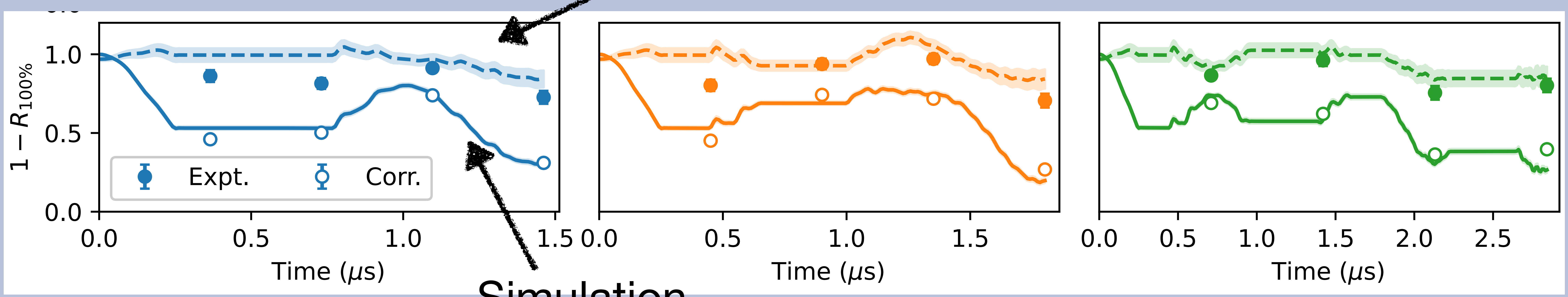
How do we solve it: QAOA



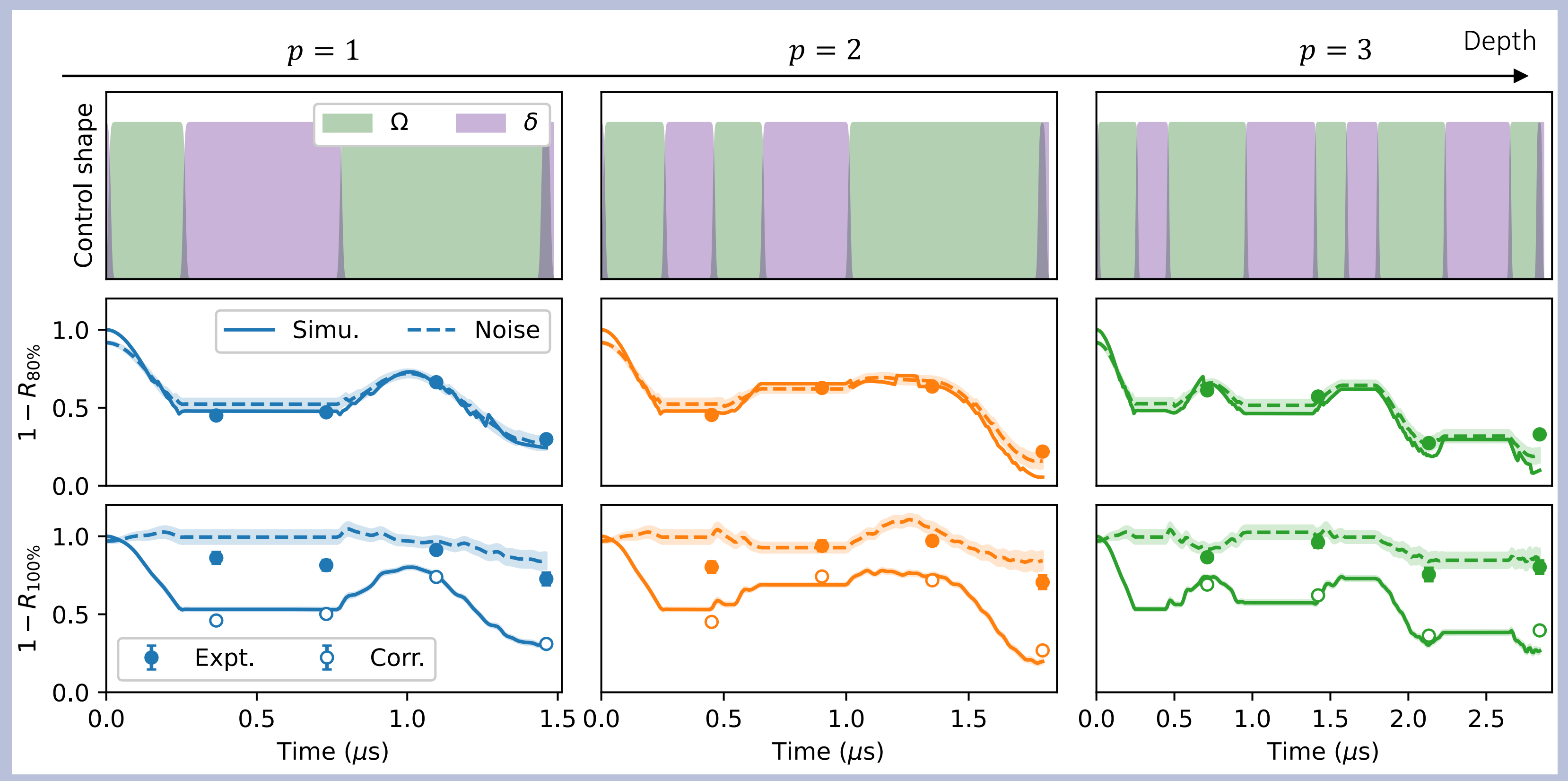
How do we solve it: QAOA

As a results

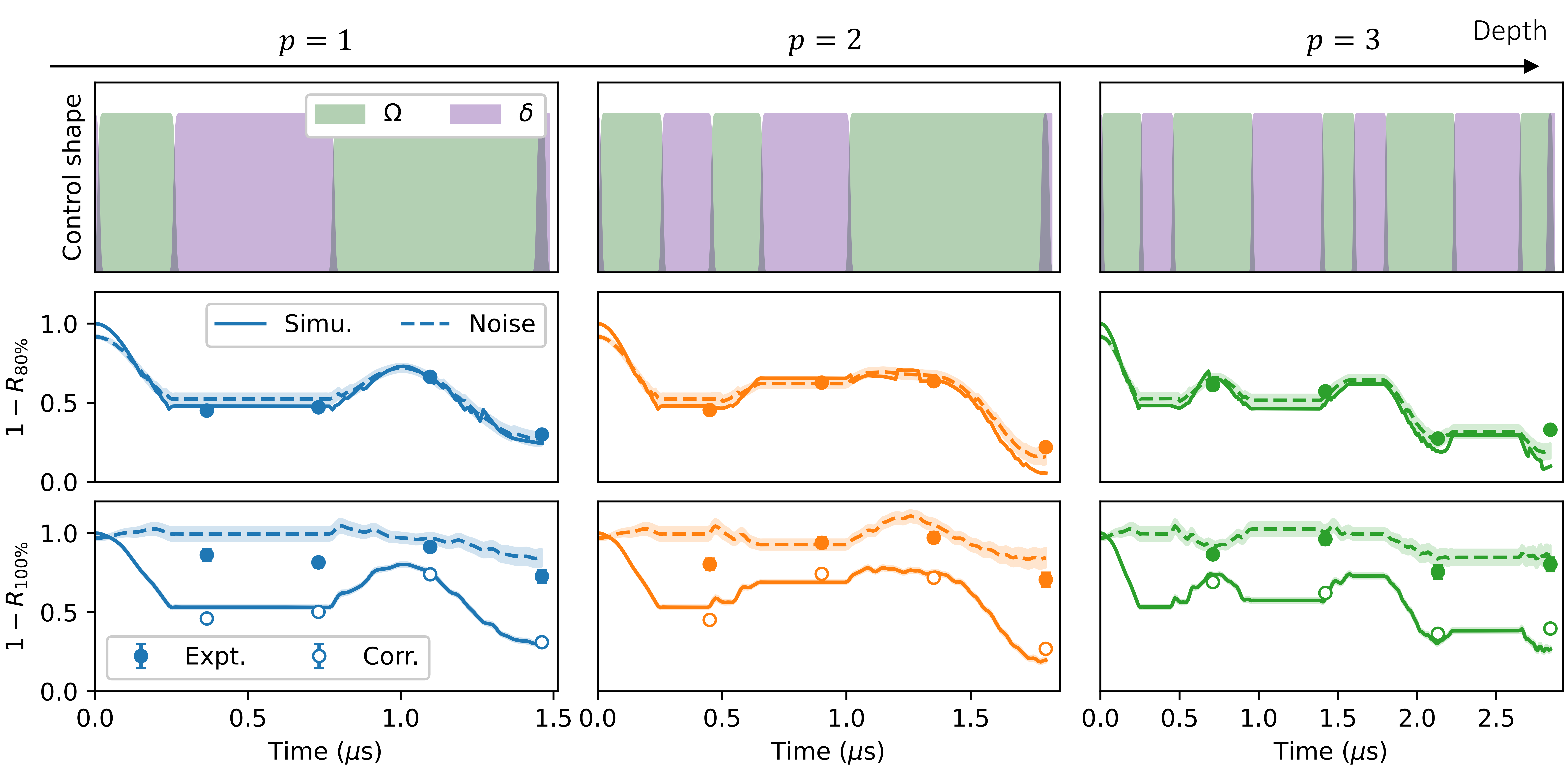
Reality



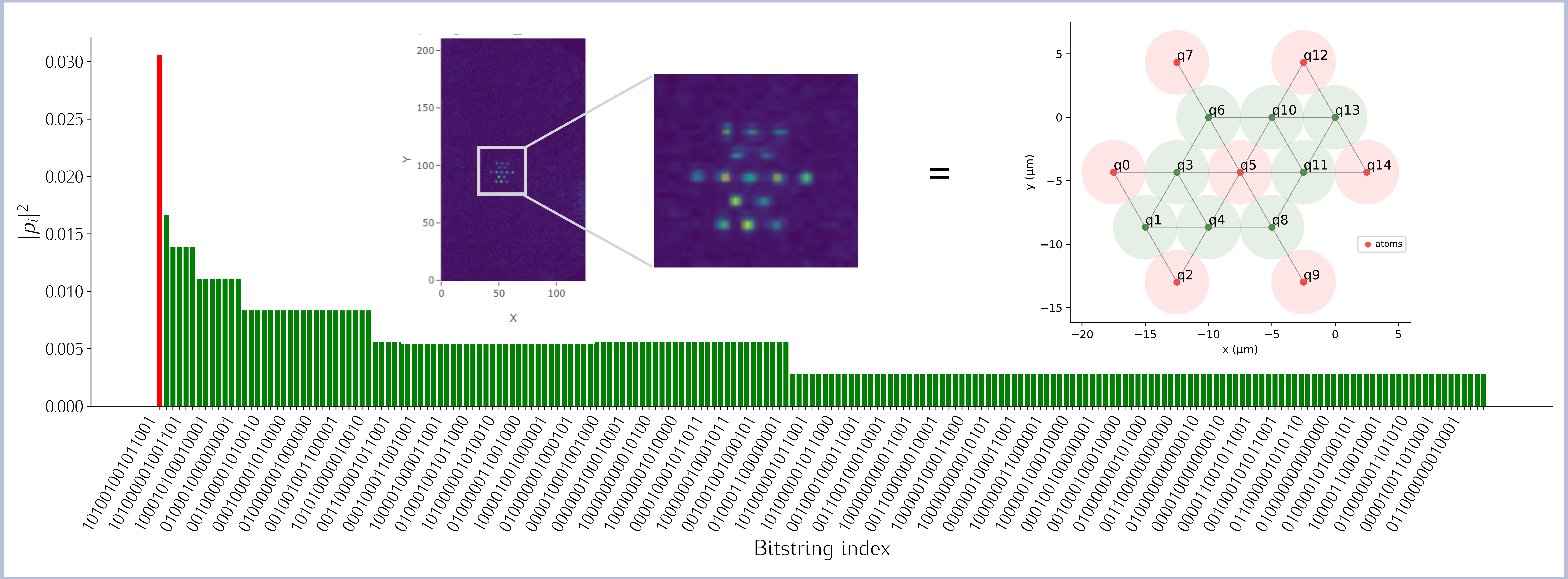
Simulation



So nothing works?

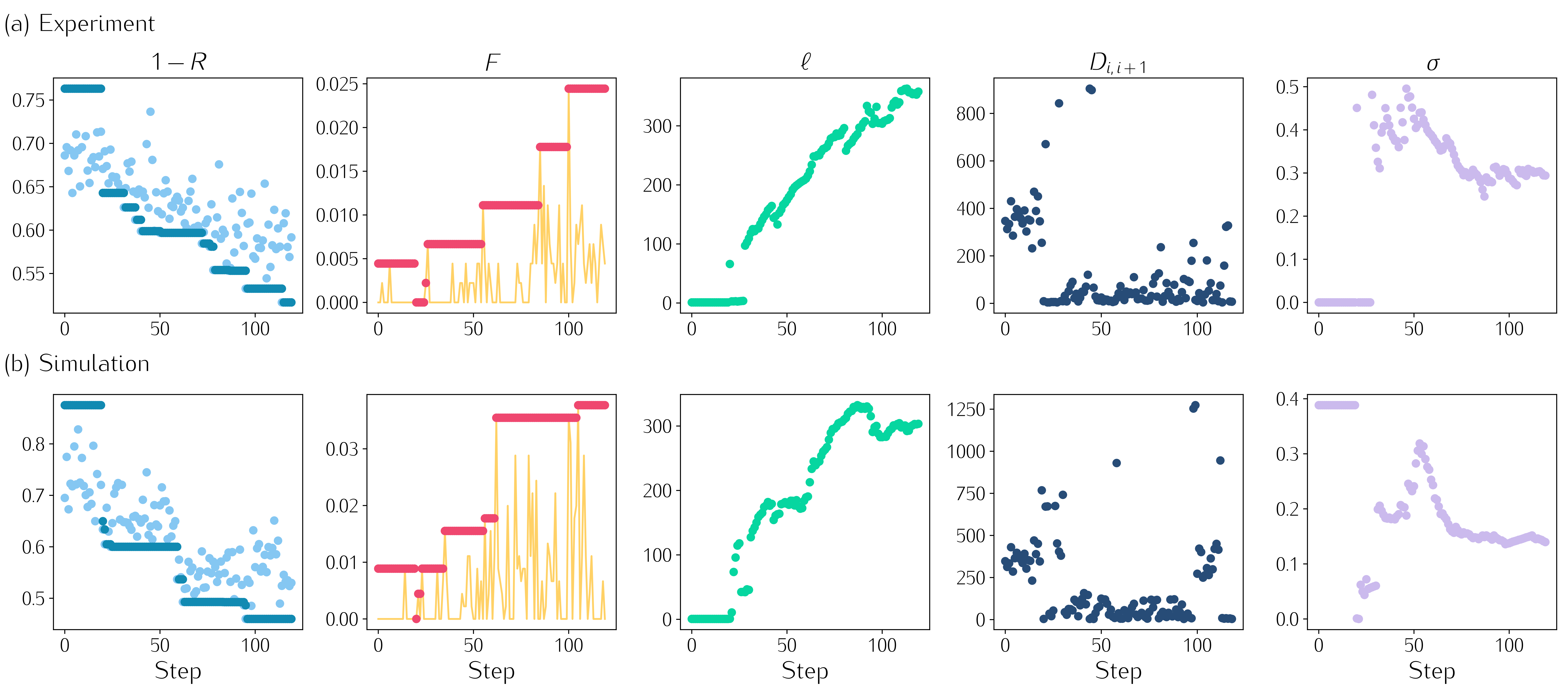


Conclusions

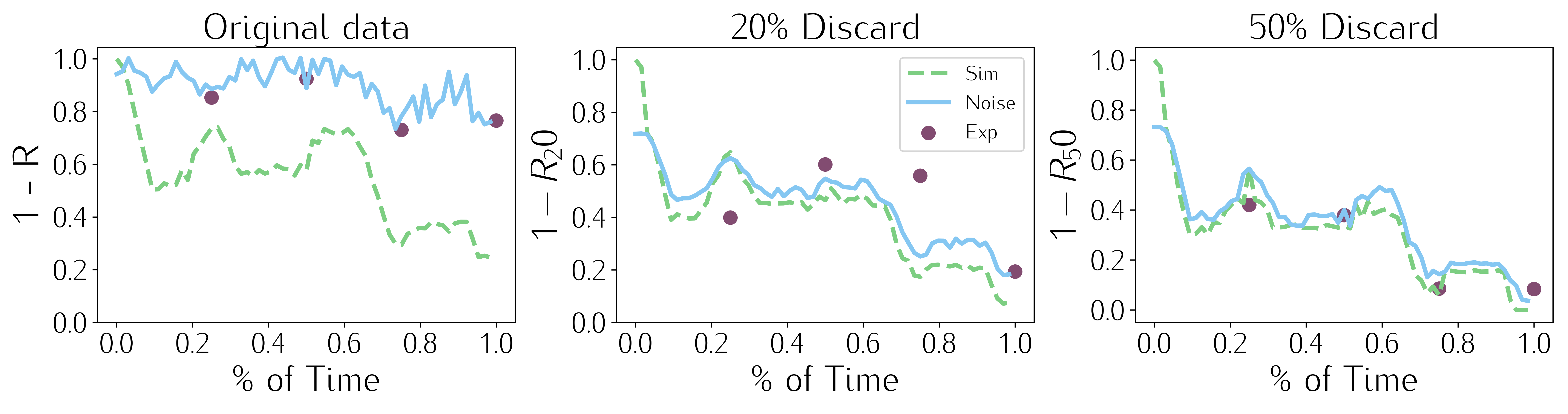


Be careful but be hopeful

Thank you



(a) Experiment
$1-R$
F
ℓ
$D_{i,i+1}$
σ
(b) Simulation
Step
Step
Step
Step
Step



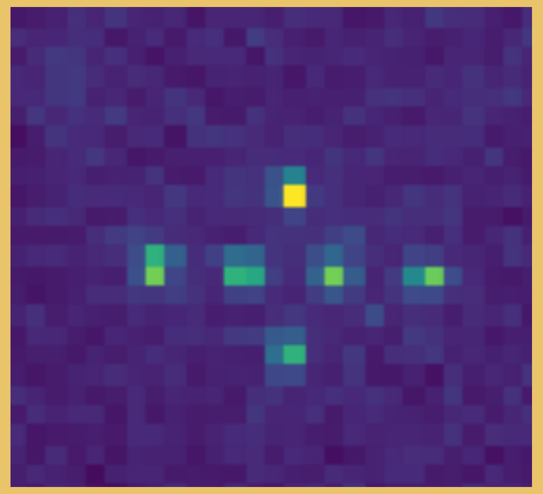
Original data
20% Discard
50% Discard
1 - R
$1 - R_{20}$
$1 - R_{50}$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
% of Time
Sim
Noise
Exp

BENCHMARK CORRECTING FOR SPAM: ONE WAY

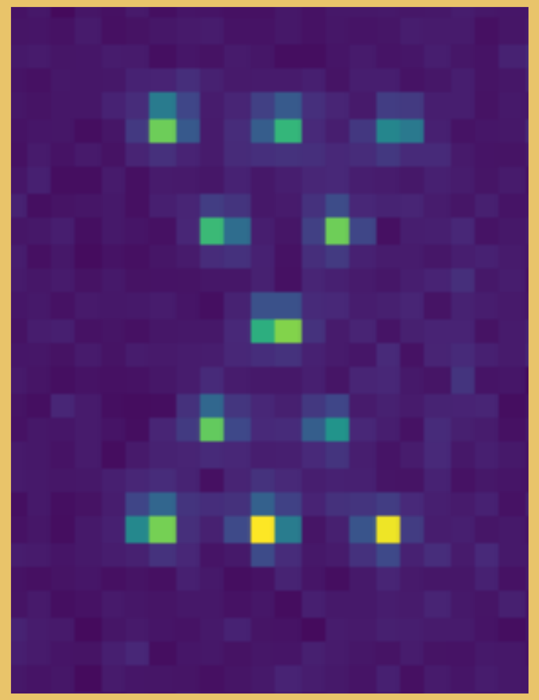
CREATE A TRANSFER MATRIX
WITH BINOMIAL PROBABILITIES

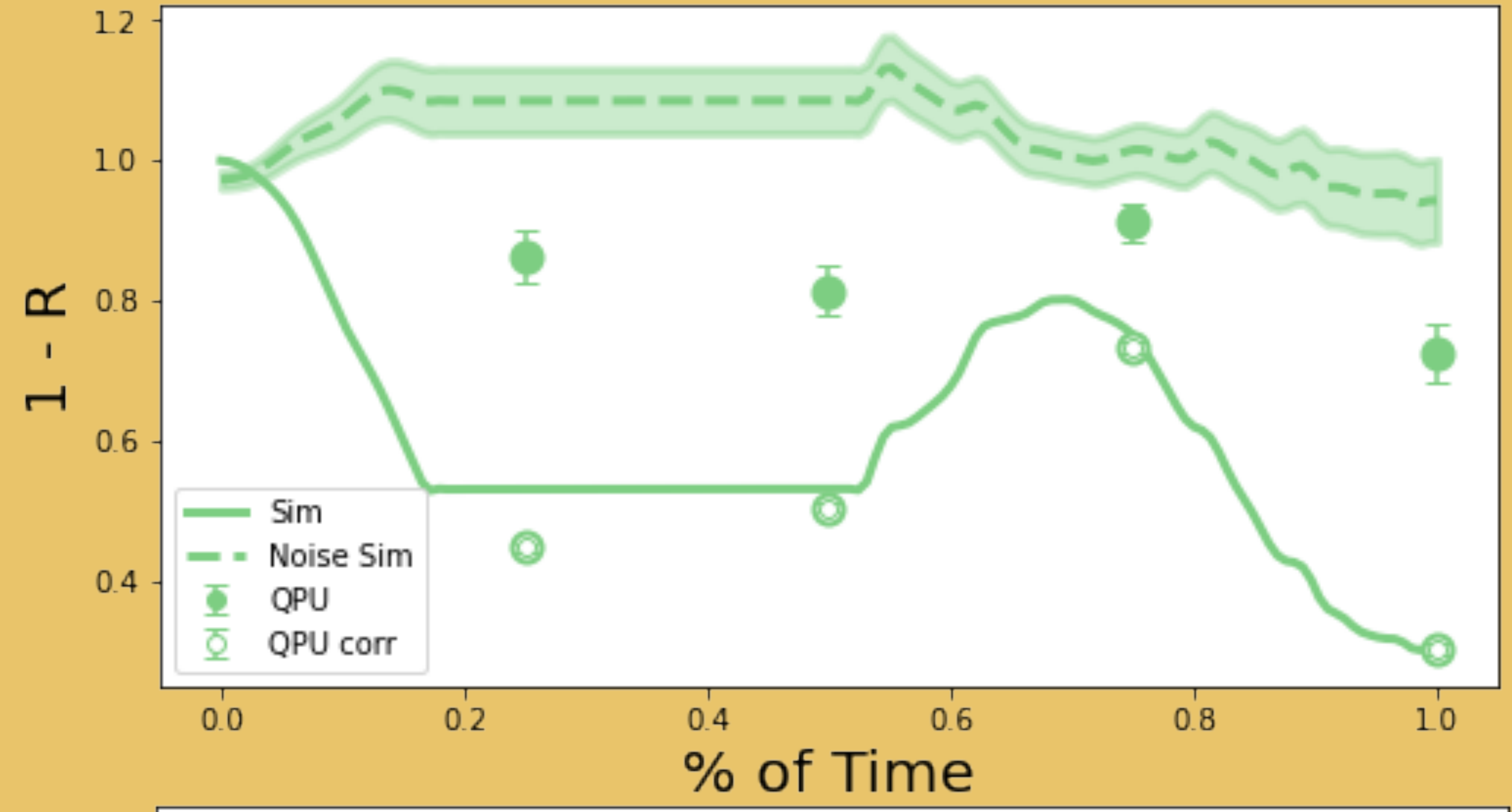
$$\left(\; p_{i\to j} \; \right)$$

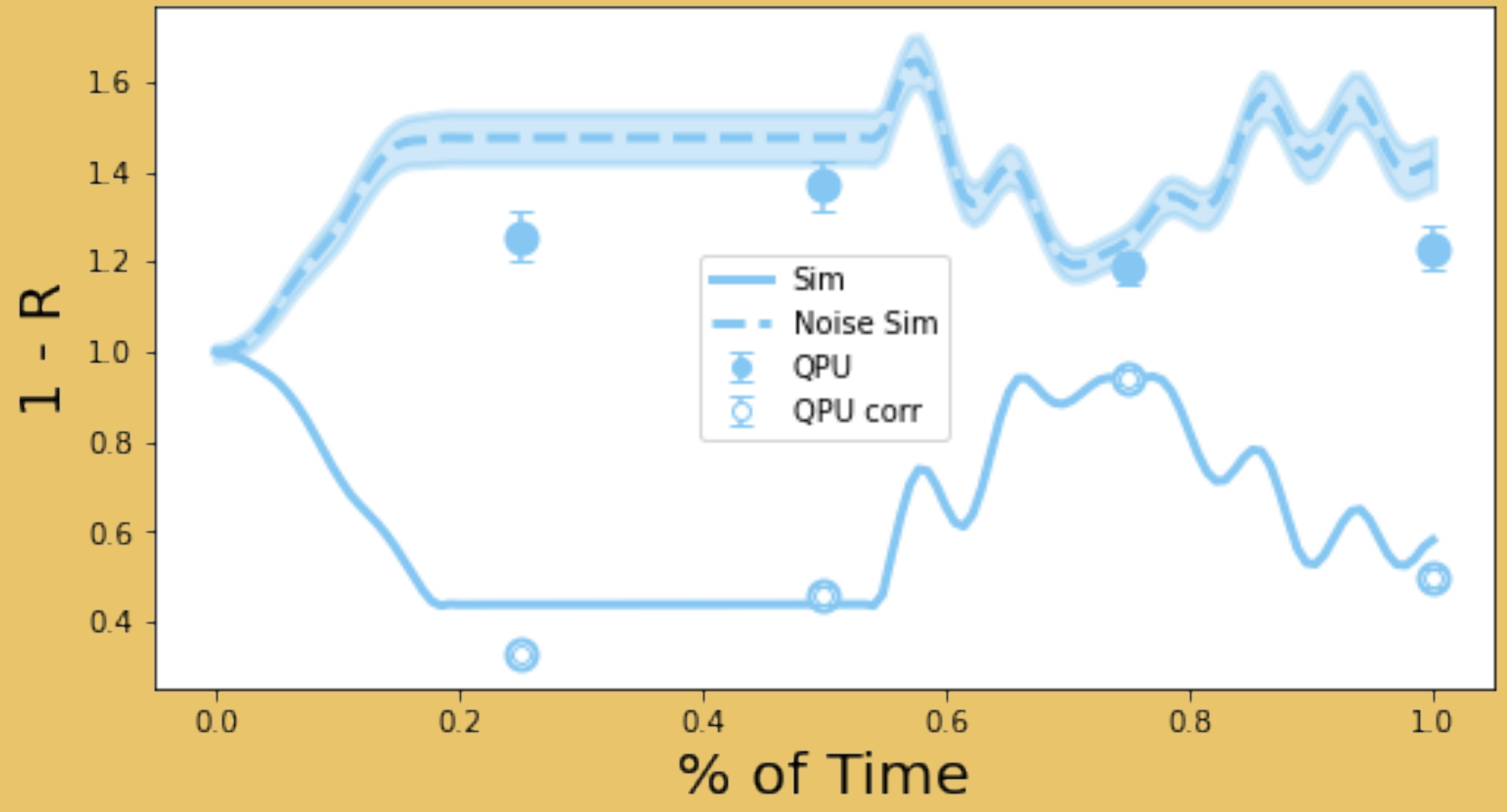
N=6



N=11

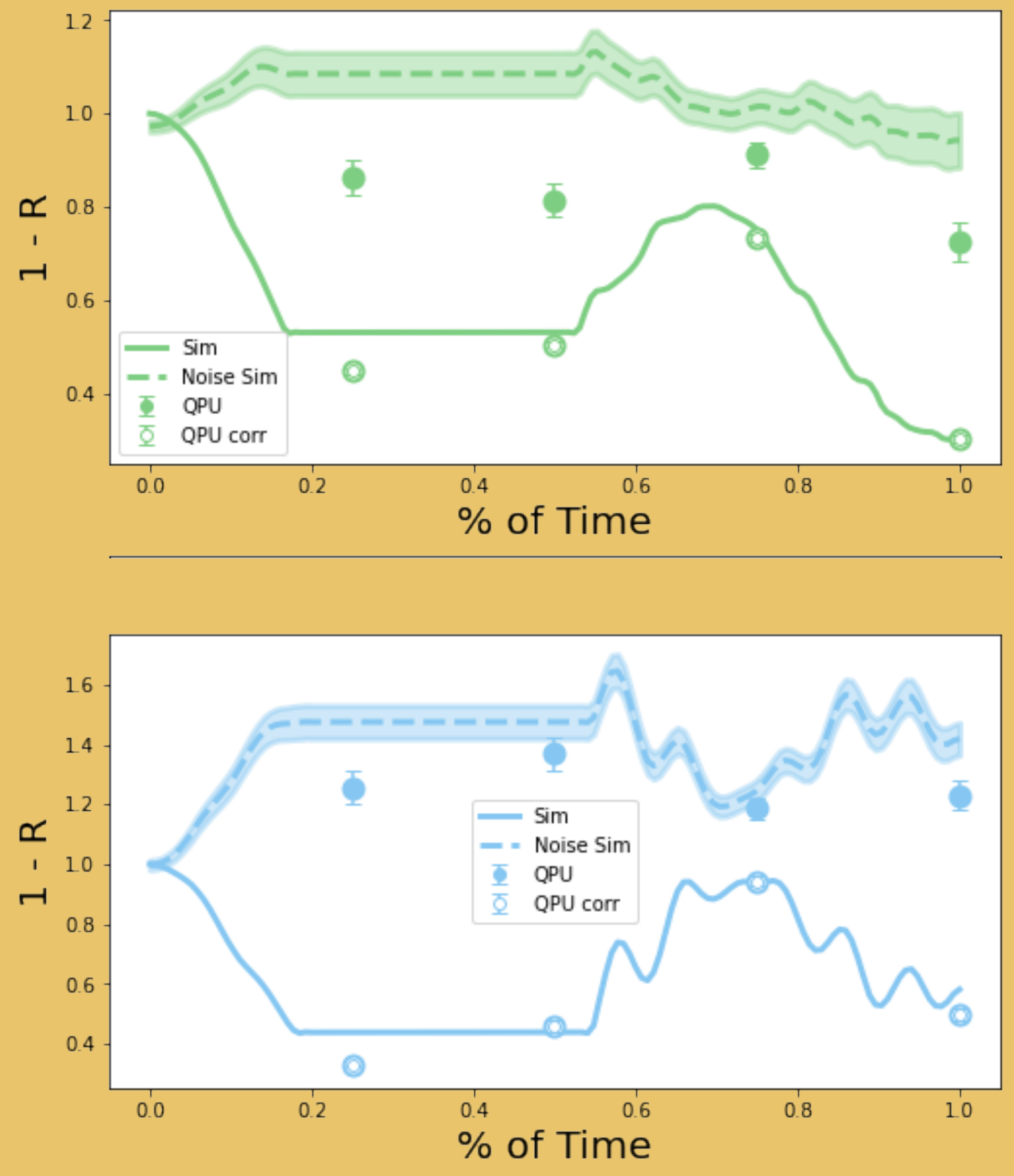




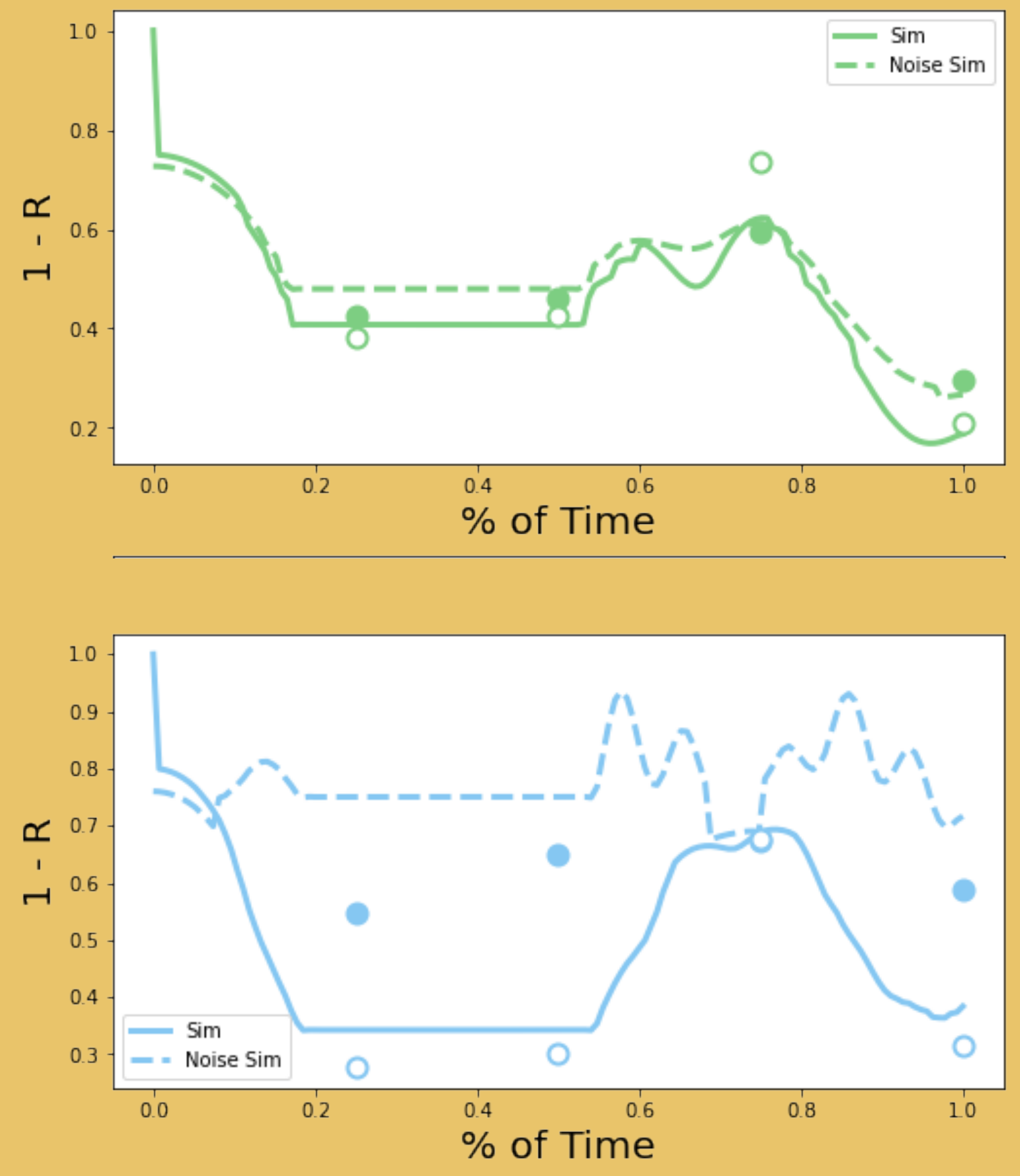


BENCHMARK Another way: discard bitstrings

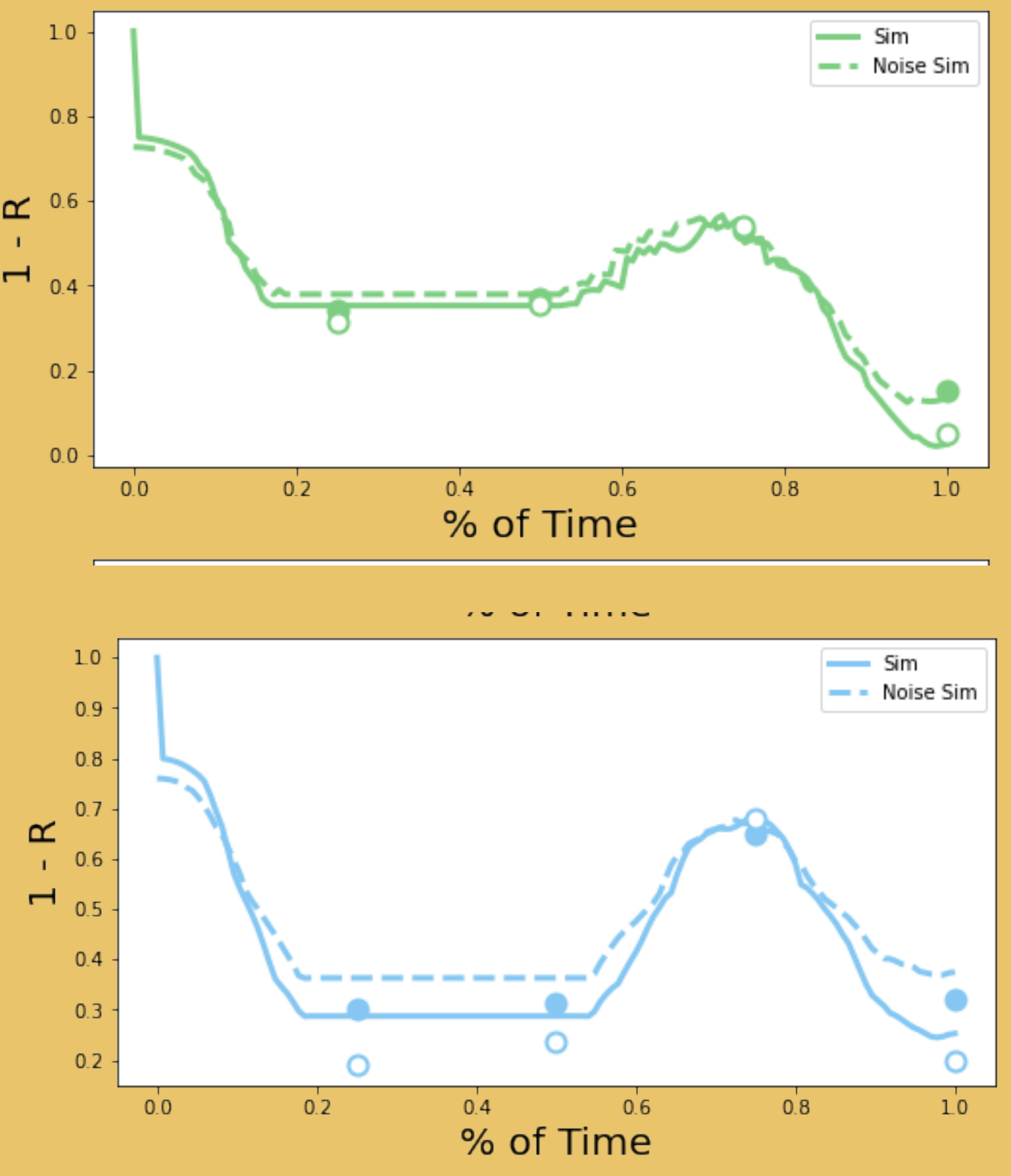
ORIGINAL DATA



DISCARD 20%

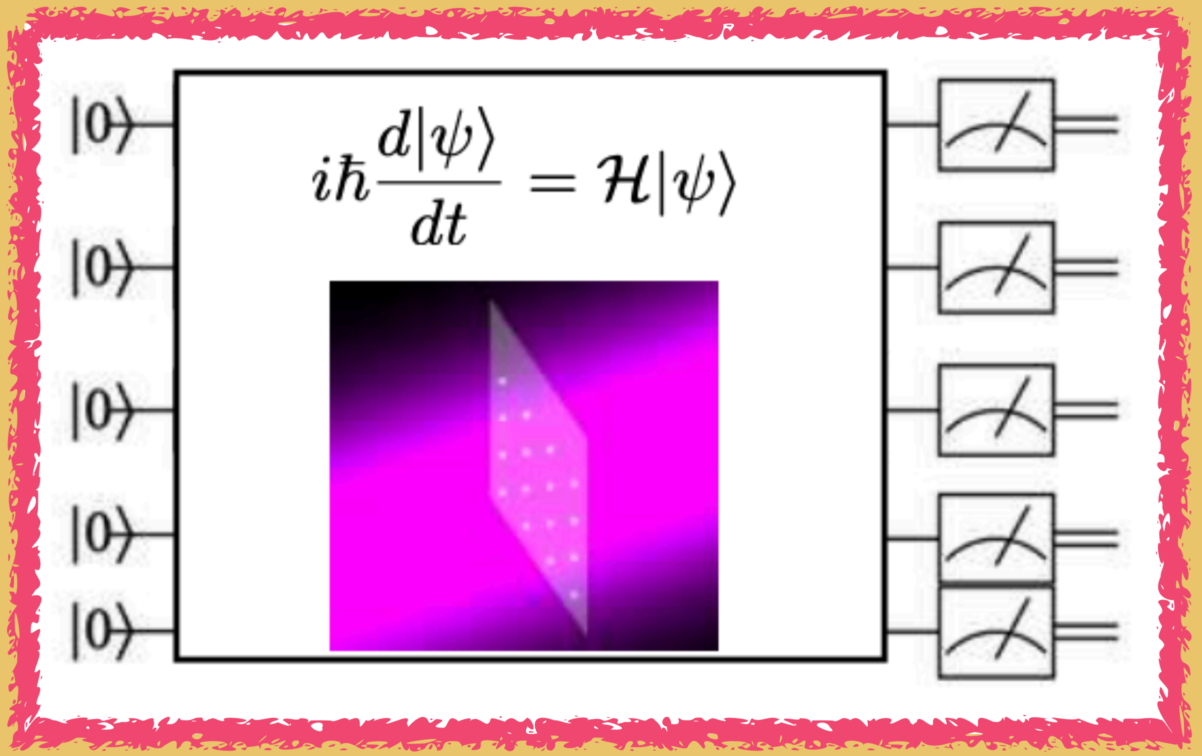


DISCARD 50%



Bayesian Optimization to optimize QAOA

[L Heneriet et al, Quantum (2020)]



Parameters

$(\theta_1, \theta_2, \cdots)$

Output

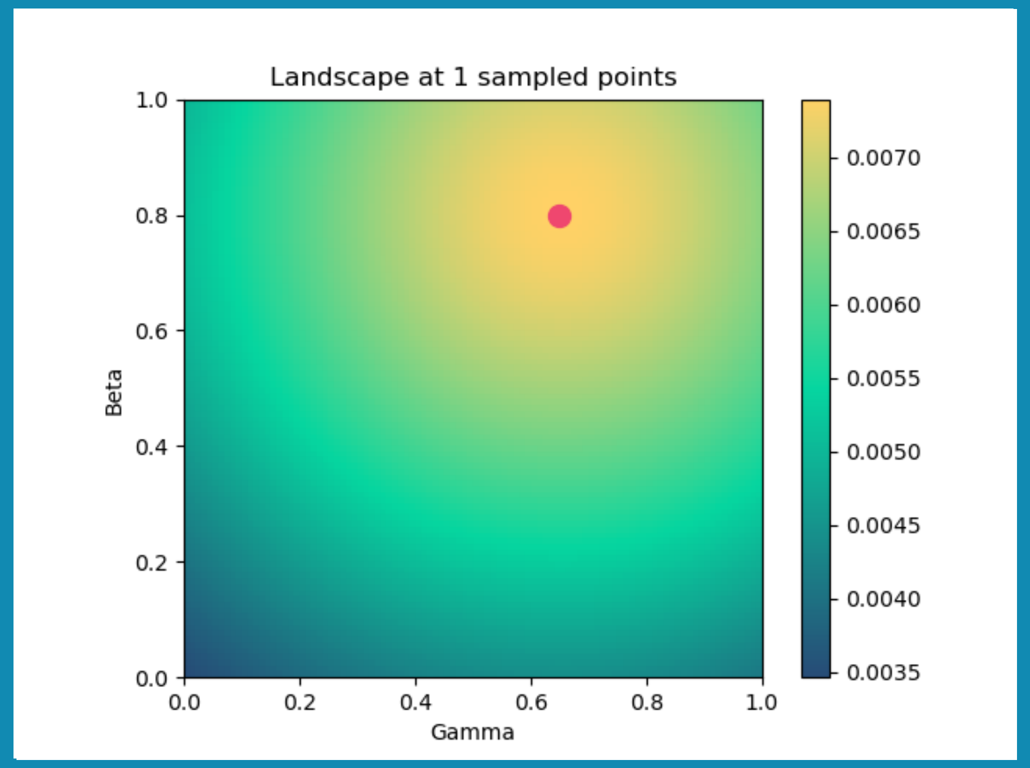
$E(\theta_1, \theta_2, \cdots)$

Acquisition Function

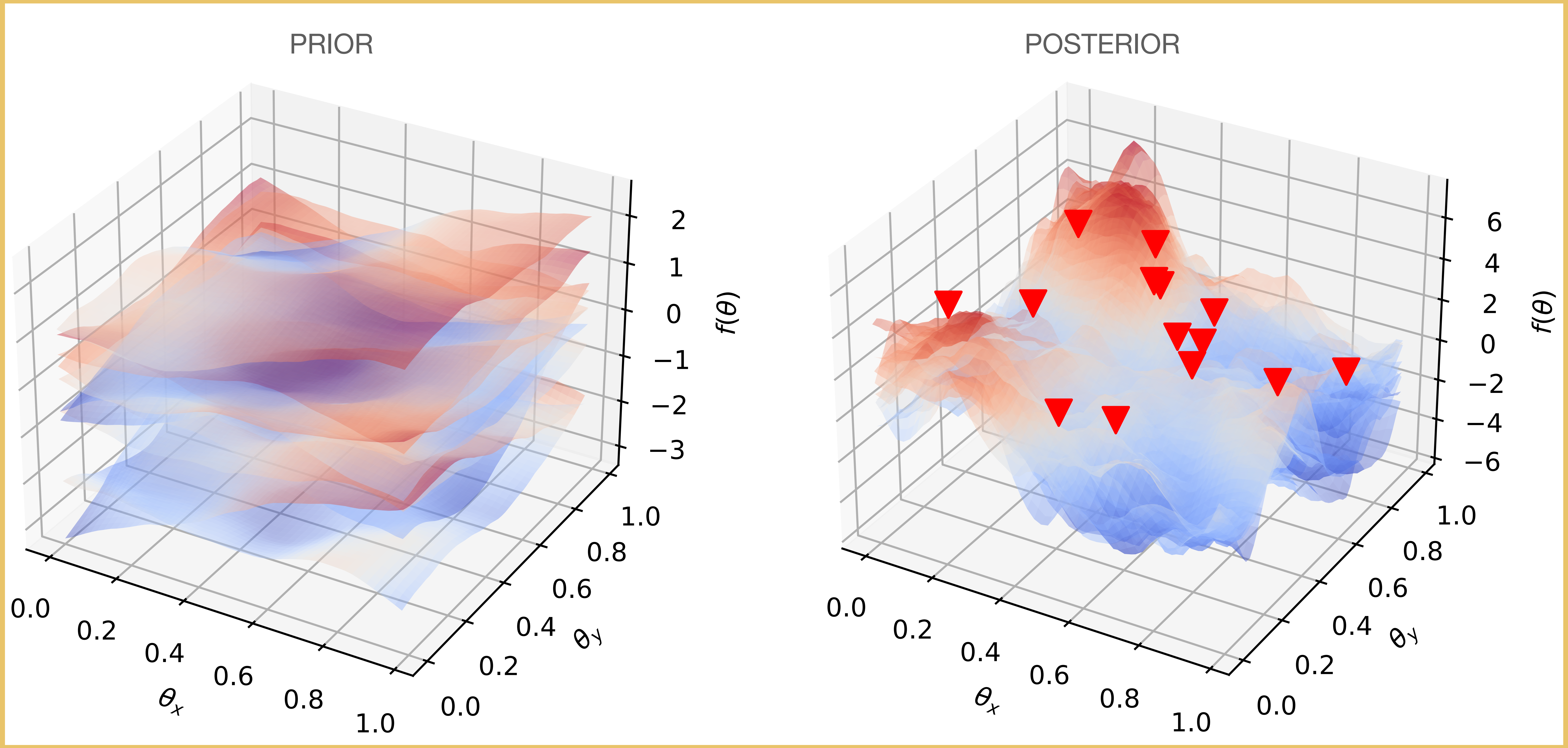
$$\mathbb{EI}(\vec{\theta}) = \sigma[\theta^\star \Phi(\theta^\star) + \phi(\theta^\star)]$$

$$\vec{\theta} = \max_{\vec{\theta}} \mathbb{EI}(\vec{\theta})$$

Gaussian Process



What is a Gaussian Process



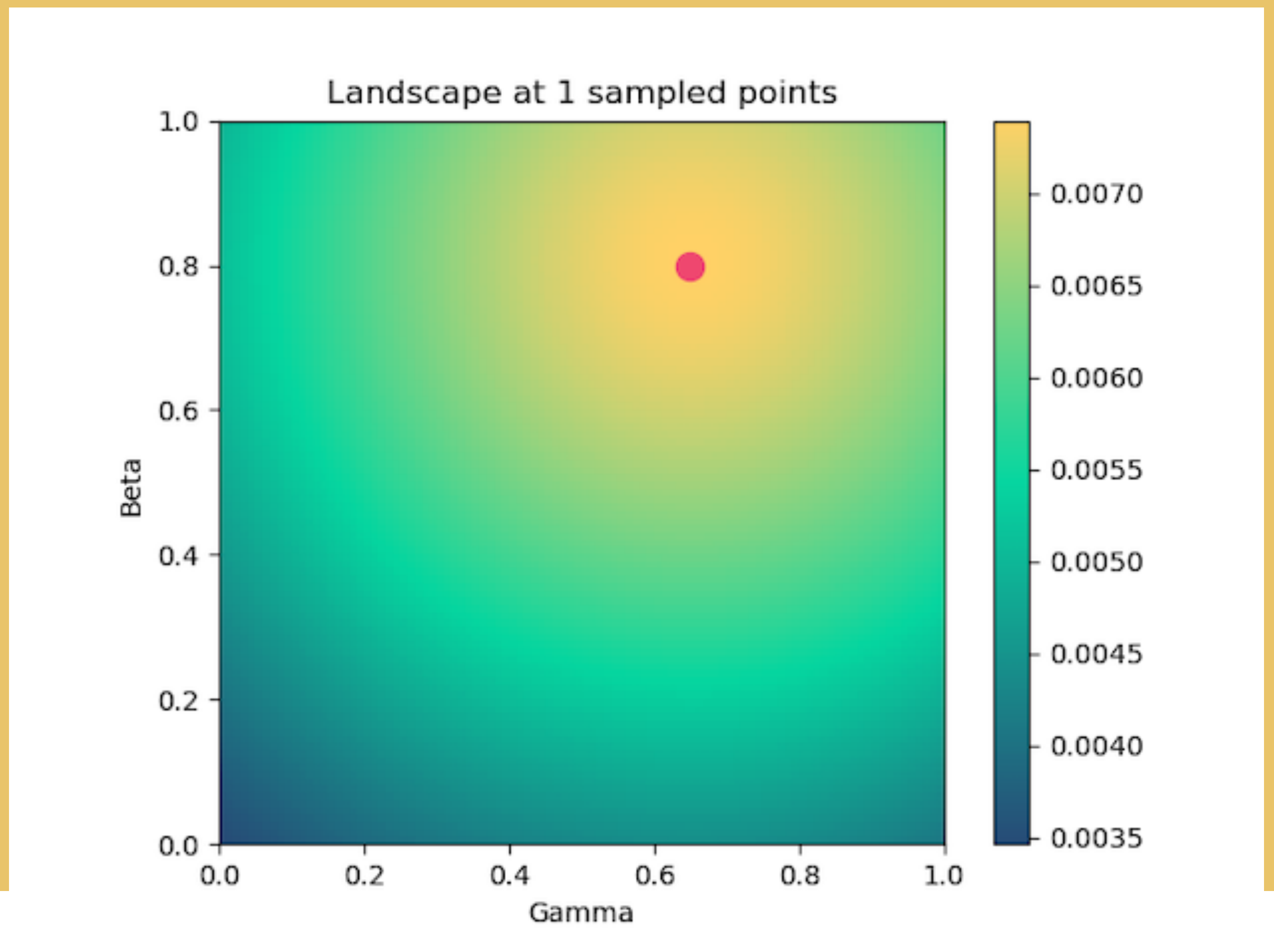
Kernel

$$k(\theta_i, \theta_j) \propto \sigma^2 e^{\frac{\sqrt{3}||\theta_i - \theta_j||}{\ell}} + \sigma_N^2$$

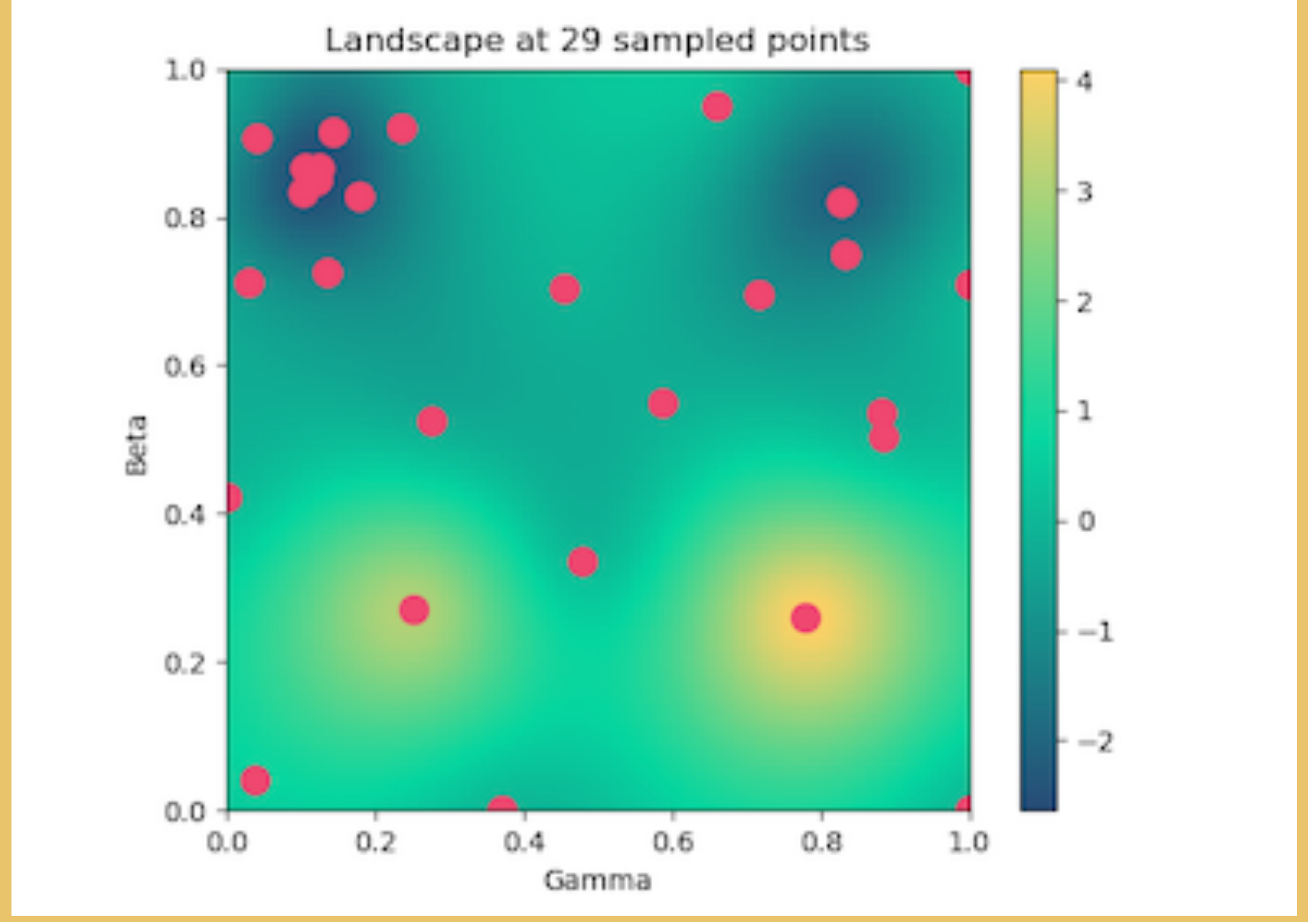
Hyperparameters
Selected with optimization

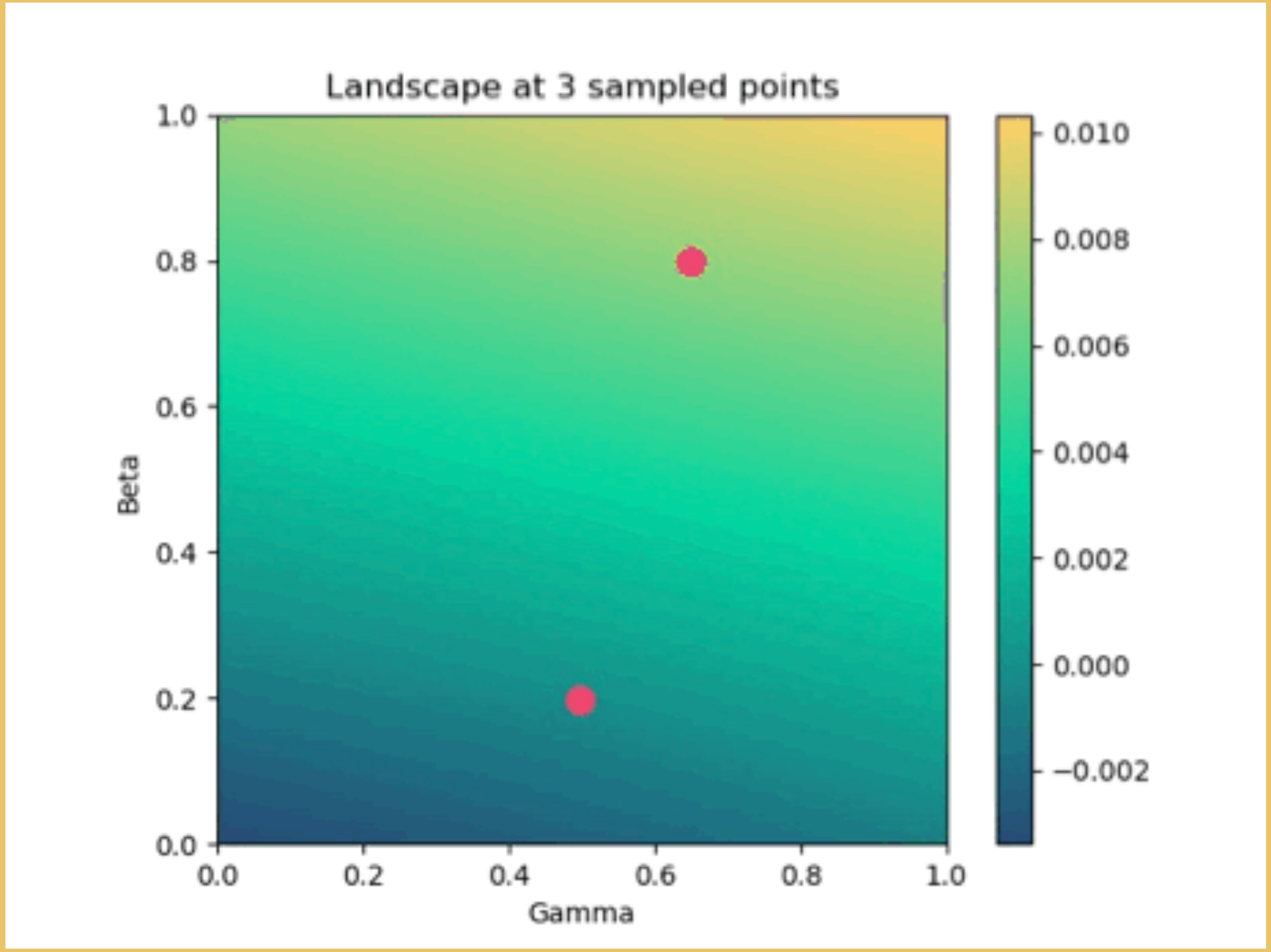
What is a Gaussian Process

After sampling
1 point

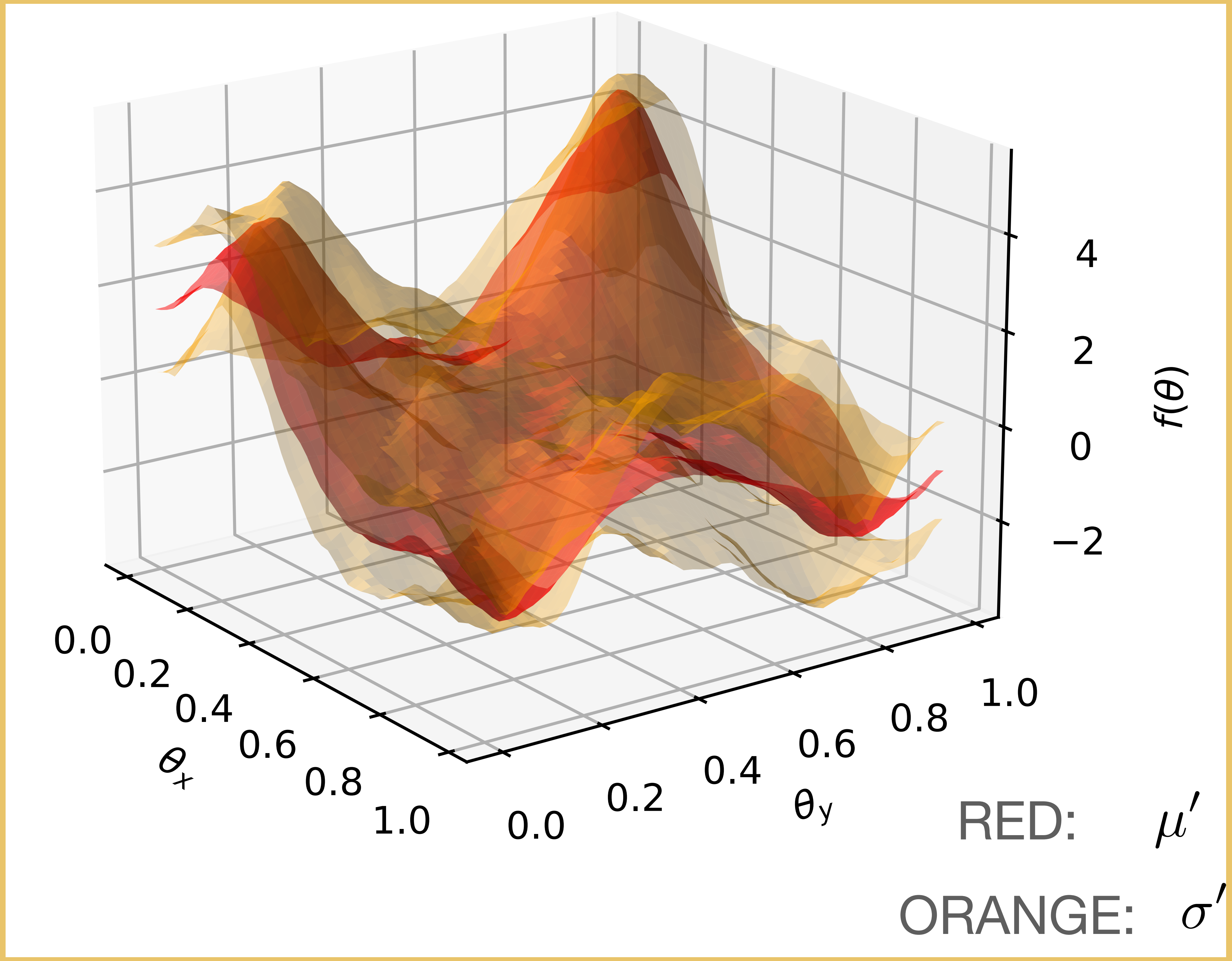


After sampling
30 points





What is an Acquisition Function



$$\mathrm{EI}(\theta) = \Phi(z)(\theta_{BEST} - \mu') + \phi(z)\sigma'$$

CURRENT BEST

p

POSTERIOR MEAN

POSTERIOR VARIANCE

AT EACH BAYESIAN OPT STEP:

$$\vec{\theta} = \max_{\vec{\theta}} \mathbb{EI}(\vec{\theta})$$