Phenomenological Review of Lepton Flavour Violation

S. Davidson[1]

[1] IPNL, Université de Lyon, Université Lyon 1, CNRS/IN2P3, 4 rue E. Fermi 69622 Villeur-
     banne cedex, France

Summary. — This is a review of current bounds on Lepton Flavour Violation, and
some discussion of what could be learned about New Physics from an observation
of LFV. There are no model predictions (see for instance reference [2].

For the purposes of this review, a lepton is a Standard Model fermion without strong
interactions, such as the electron or its neutrino. Lepton flavour, or generation, is a
quantum number distinguishing the three copies e, μ, and τ of a massive electrically
charged lepton plus its neutrino. Finally, Lepton Flavour Violation (LFV), is a flavour
changing point interaction of charged leptons. By this definition, LFV is equivalent to
a Flavour-Changing Neutral Current (FCNC) contact interaction among the charged
leptons, such as $\tau \to \mu\gamma$. Neutrino oscillations do not qualify.

The relation of LFV to New Physics, is fundamentally different from the relation
between quark flavour and New Physics(NP). In the Standard Model, neutrinos are
massless, and lepton flavour is conserved. So the observation of LFV is a signal of
Beyond-the-Standard-Model (BSM)[1] Physics. But we know that there is BSM in the
lepton sector, because neutrinos oscillate and therefore have mass. So LFV happens, due
to the New Physics responsable for neutrino masses — but the rate is unknown. This
situation can be constrasted with the quark sector, where the SM predicts FCNC, and
most observations are in such good agreement with the SM, that quark flavour bounds
are perceived as a hurdle for New Physics models, introduced to address some other issue.

The amplitudes for LFV induced by the neutrino masses, treated as Dirac masses,
are $\propto m_\nu^2/m_W^2 \sim 20^{-24}$. So observable LFV requires dynamics other than m_ν. A variety
of models fit oscillation data and current LFV bounds, but give different predictions for
LFV rates. This wide diversity can be parametrised via the Effective Lagrangian.

The scale(s) of the New Physics in the lepton sector are unknown. I assume here that
the New Particles are heavier than the Higgs vev $v = 175$ GeV, so that the only "light"
fields in the Effective Lagrangian are the known SM fields.
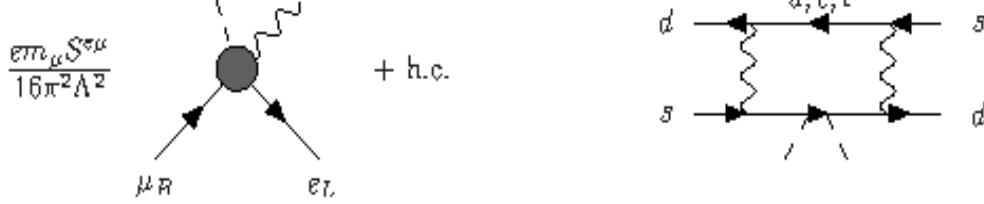
---

[1] I use BSM and NP interchangeably

Fig. 1. – On the left, the diagram and "effective coupling" corresponding to the dipole operator of eq. (2). Notice that the normalisation of the coefficient assumes that the chirality flip is due to the heaviest lepton Yukawa coupling, and that the NP contributes via a loop. Only the combination $S^{\alpha\beta}/\Lambda^2$ is measurable, but it is intuitive to separate it into the dimensionless $S^{\alpha\beta}$ which contains New Physics couplings, and the New Physics mass scale Λ. On the right, an GIM-suppressed FCNC diagram in the SM. Since two quark mass insertions are required, the diagram has two Higgs legs and is of dimension eight.

1. – Current Bounds and Where to look?

Experimentally, we know that LFV rates are below current sensitivities(for refernces, see for instance [1]). A selection of bounds is presented in the second colomn of table I. An interesting question is therefore "where is most promising place to look?"

New particles can have escaped detection to date because they are heavy (*e.g.* SUSY, etc), or because they interact weakly (like axions, majorons, or sterile neutrinos). Here, I only consider heavy New Particles. At SM scales, footprints of heavy NP are encoded an the "effective Lagrangian" $\mathcal{L}_{eff} = \mathcal{L}_{SM} + \Delta\mathcal{L}_{eff}^{LFV} + \Delta\mathcal{L}_{eff}^{other}$. It has the SM particle content, SM gauge symmetries, and all (mass) dimension> 4 operators are allowed. If the new particles masses are of order a (fuzzy) mass scale Λ, the interactions they induce among SM particles can be described, at energies $\ll \Lambda$, via:

$$(1) \qquad \Delta\mathcal{L}_{eff}^{LFV} = \sum_{d\geq 5}\sum_n \frac{C^n}{\Lambda^{d-4}}O_n(H,\{\psi\},A_\mu,...) + h.c.$$

where the operators $\{O_n\}$ are built with SM fields, respect SM gauge symmetries, and, more intuitively, describe the legs of LFV diagrams (including Higgs vevs). See figure 1. From the New Physics perspective, the (dimensionless) coefficients C^n contain SM and NP coupling constants and loop factors; it can be convenient to factor out the SM coupling constants and $1/(16\pi^2)$, so that C appears to be a product of New Physics couplings. For instance the dipole operator, which describes $g-2$ and $\ell_\alpha \to \ell_\beta\gamma$, in this review is normalised:

$$(2) \qquad \frac{em_\alpha}{16\pi^2\Lambda^2}[S_L]_{\alpha\beta}\overline{e_{R\beta}}\sigma^{\mu\nu}e_{L\alpha}F_{\mu\nu} + \frac{em_\alpha}{16\pi^2\Lambda^2}[S_R]_{\alpha\beta}\overline{e}_\beta\sigma^{\mu\nu}e_{R\alpha}F_{\mu\nu}$$

$\mathcal{L}_{eff}$ can provide a useful bridge between data and theories. From data, the operator coefficients can be constrained. From a theory, the operator coefficients can be calculated. From the perspective that data should identify the correct theory, it is interesting to ask

process	bound	scale (dim 6, loop)	scale (dim 8, loop)
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	48 TeV	2.9 TeV
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	170 TeV (tree)	5.5 TeV (tree)
		14 TeV	1.5 TeV
$\frac{\sigma(\mu+Ti \to e+Ti)}{\sigma(\mu\ \text{capture})}$	$< 4.3 \times 10^{-12}$	40 TeV	2.6 TeV
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$	2.8 TeV	0.7 TeV
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	9 TeV (tree)	1 TeV (tree)
$BR(\tau \to e\pi)$	$< 8.1 \times 10^{-8}$	0.5 TeV	0.3 TeV
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	25 TeV$(V \pm A)$	2.1 TeV$(V \pm A)$
		140 TeV$(S \pm P)$	5 TeV$(S \pm P)$
$BR(B \to e^{\pm}\mu^{\mp})$	$< 6.4 \times 10^{-8}$	3 TeV $(S \pm P)$	

TABLE I. – *A selection of LFV processes and current bounds. The third colomn gives the mass scale of New Particles which could induce the process at dimension six via a loop with couplings of $\mathcal{O}(1)$. For such scenarios, μ searches are sensitive to higher scales than τ searches.Similarly, LFV is more likely to be found in Ks than in Bs. The last colomn gives the mass scale of New Particles which induce the process via a loop with two extra Higgs legs (saturated by Higgs vevs) and couplings of $\mathcal{O}(1)$. All channels are promising to search for such New Physics scenarios. The New particles in such scenarios could be accessible to the LHC.*

to what degree "the" theory can be reconstructed from the coefficients of $\mathcal{L}_{eff}$. However, we make no progress on this question here.

A lower bound on the mass scale Λ of perturbative New Physics can be obtained from the experimental bounds as follows. First, find the lowest dimension operator/diagram corresponding to a process(usually dimension 6 for LFV), set the New Couplings to 1 (on the assuption that perturbative couplings are ≤ 1), and compute the rate. Notice that the bound obtained will depend on what loop or SM coupling factors are scaled out of C in eq. (1). In table I, the New Phyics is assumed to contribute via loop diagrams, as if New Particles had a conserved quantum number, so $C/(\Lambda^2)$ was taken to be $1/(16\pi^2\Lambda^2)$.

In the SM, quark FCNC are suppressed by the quadratic GIM mechanism. The additional m_q^2/m_W^2 factor can be interpreted as placing SM FCNC at dimension eight, with 4 fermion legs and two Higgs legs (see figure 1 on the right). From a phenomenological bottom-up perspective, one can ask if this might also occur in New Physics scenarios [3].

Bounds on the scale of New Physics that contributes to LFV at one loop via dimension eight operators, can be obtained following a similar recipe to the dimension 6 bounds. The coefficients $\frac{C^{(6)}}{\Lambda^2}$ of the dimension six operators contributing to a process are set to 0, and replaced by the coefficients $\frac{C^{(8)}v^2}{16\pi^2\Lambda^4}$ of the dimension eight operators/diagrams which have similar fermion legs and two additional Higgs legs (vevs). The lower bounds on Λ at dimension 8, given in table I, are obtained by setting $C^8 \simeq 1$.

An objection to the bounds of table I is that the flavoured couplings we know in the SM are not 1. Bounds that take into account a possible hierarchy in flavoured New Physics couplings can be obtained by following the Cheng-Sher ansätz [4], which is that flavoured fermion couplings are $\propto$ SM fermion masses

$$(3) \qquad\qquad \lambda_{ij} \simeq \sqrt{\frac{m_i m_j}{v^2}} \quad , \quad i, j \text{ any SM fermion.}$$

process	bound	expectation
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	$\sim 2 \times 10^{-14}$ (avec mass insertion)
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	$\sim 10^{-17}$ (long distance loop)
$BR(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$	$\sim 8 \times 10^{-11}$ (avec mass insertion)
$BR(\tau \to 3\ell)$	$< 2.1 \times 10^{-8}$	$\sim\ 0^{-14}$ (long distance loop)
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	$\sim 5 \times 10^{-15}$ $(S \pm P)$
		$\sim 10^{-17}$ $(V \pm A)$
$BR(B \to \tau^{\pm}e^{\mp})$	$< 2.8 \times 10^{-5}$	$\sim 4 \times 10^{-15}$ $(S \pm P)$
$BR(B_s \to \tau^{\pm}\mu^{\mp})$		$\sim 10^{-11}$ $(S \pm P)$
$BR(B \to e^{\pm}\mu^{\mp})$	$< 6.4 \times 10^{-8}$	$\sim 4 \times 10^{-16}$ $(S \pm P)$
$BR(B \to K^0\mu^{\pm}e^{\mp})$	$< 2.7 \times 10^{-7}$	$\sim 10^{-15}$ $(V \pm A)$
$BR(B^+ \to K^+\tau\bar{\mu})$	$< 7.7 \times 10^{-5}$	$\sim 10^{-11}$

TABLE II. – *Expected Branching Ratios due to tree level TeV-scale New Particles with hierarchical couplings, as in eq. (3). In meson decays, the chiral structure of the matrix element is indicated. The "long-distance loop" estimates correspond to an a dipole operator, where the off-shell photon decays to a charged lepton pair.*

Such patterns arise, for instance, in Randall-Sundrum extra-dimensional models. To obtain the rate estimates given in table II (see also [5]), I assume that new particles with masses $\sim$ TeV and couplings like eq. (3) contribute via tree diagrams (when possible) to the various processes. The $\ell_\alpha \to \ell_\beta\gamma$ branching ratios are estimated with a $1/(16\pi^2)$ loop factor, and chirality flip due to a Higgs insertion on an external leg, as in figure 1. Without this factor, the prediction exceeds the current upper bounds.

In **summary**, neutrino masses imply that there *is* New Physics dedicated to Lepton Flavour. However, no flavour-changing processes have yet been observed among charged leptons. Current bounds are consistent with various patterns of New Physics. Most new flavoured particles with masses $\gtrsim$ few $\to$ 10 TeV, and $\mathcal{O}(1)$ couplings are allowed if they contribute to LFV via loops. New flavoured particles with masses $\sim$ TeV and hierarchical couplings can contribute at tree level. Most importantly, the three classes of BSM scenarios considered here (in loops at dimension 6 or 8, with hierarchical couplings), can most readily be found in different processes (μ decays, τ decays, K decays ,...). This means that improving the sensitiveity of all LFV modes is interesting, because there is no model independent "golden mode" which is the "best place" to look.

2. – What can we learn ?

Some anticipated sensitivities to various LFV processes [2] are listed in table III. In this section, we suppose that some LFV is observed, and discuss an example of what such data could tell us about New Physics. An early discussion in this perspective is [6]. There are two steps to learning about NP: first, determining the coefficients of the effective Lagrangian, then, in principle, it would be interesting to "reconstruct" the New Physics Lagrangian from the Effective Lagrangian.

---

[2] NA62 will have K^+s, and could explore $BR(K^+ \to \pi^+\mu^+e^-) \sim 10^{-12}$. However, for LFV, its not clear this is more sensitive that the current bounds from $K \to \mu^+e^-$

some processes	current sensitivities	future sensitivity
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	$\sim 10^{-13}(10^{-14}$ (MEG)
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	
$\frac{\sigma(\mu+Au \to e+Au)}{\sigma(\mu \text{ capture})}$	$< 7 \times 10^{-13}$	$10^{-16} - 10^{-18}$(J-PARC)
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$	few $\times 10^{-9}$ (S-B fact)
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	$\lesssim 10^{-9}$ (S-B fact)
$BR(\tau \to e\phi)$	$< 3.1 \times 10^{-8}$	$\lesssim 10^{-9}$ (S-B fact)
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	
$BR(K^+ \to \pi^+ \bar{\nu}\nu)$	$= 1.7 \pm 1.1 \times 10^{-10}$	100 evts (NA62)

TABLE III. – *Future sensitivities of various experiments to LFV processes.*

One way to learn about New Physics is to combine various observables. In many processes, such as $\tau \to 3\ell$ or $\mu - e$ conversion, there are several operators of the same dimension which can contribute to the rate, so experimental observables depend on combinations of operator coefficients. Interesting studies [7] have shown that these coefficients could be disentangled with additional observables, such as angular correlations in $\tau \to 3\ell$, or nucleus-dependance in $\mu - e$ conversion. Knowing the various coefficients in the Effective Lagrangian can give some information on the properties of New mediating Particles, such as their colour or spin.

Measuring the same process for different flavours (*e.g.* :$\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$) tells about the flavour structure of the Effective Lagrangian coefficient, and, possibly also of the New couplings. Consider $\tau \to \ell\gamma$ and $\mu \to e\gamma$, which constrain the flavour structure of the dipole coefficient. Only one operator contributes, although it is convenient to separate it in two according to fermion chirality (as in eqn (2)), rather than write the operator $+h.c.$. For simplicity, I assume chirality flip on an external leg.

Recall that $BR(\mu \to e\gamma) \leq 10^{-12}$. And suppose we see $BR(\tau \to e\gamma) \sim 10^{-8}$ at a Super-B factory. This is an interesting scenario for learning about flavour structure, because we have two pieces of information: the $\tau \to e\gamma$ rate, and the "approximate zero" from $\mu \to e\gamma$. However, S_L and S_R combine to an arbitrary complex three by three matrix, which cannot be reconstructed from two observations.

So I make one more assumption, which is common in hierarchical flavour physics: suppose that the dipole coefficient $em_\alpha S_{\alpha\beta}/16\pi^2\Lambda^2$ is dominated by its largest eigenvalue (this is like taking $[\mathbf{Y}_u^\dagger \mathbf{Y}_u]_{bs} \simeq V_{tb}^* y_t^2 V_{ts}$). Then there are three parameters, Λ, $|V_{3e}|$, and $|V_{3\mu}|$, to parametrise $\mu \to e\gamma, \tau \to e\gamma, \tau \to \mu\gamma$. If one allows that the LHC can give a lower bound on Λ, an upper bound on the remaining rate $\tau \to \mu\gamma$ can be predicted. This bound is shown in figure 2. It arises because V_{3e} must be large, if "sufficiently heavy" NP induces $\tau \to e\gamma$:

$$\widetilde{BR}(\tau \to e\gamma) \simeq 10^{-8} \left(\frac{500GeV}{\Lambda}\right)^4 \frac{|V_{3e}|^2}{10^{-4}} \gtrsim 10^{-8}$$

Then $\widetilde{BR}(\mu \to e\gamma) \propto |V_{3\mu}V_{3e}^*|^2 \lesssim 10^{-12}$ imposes that $|V_{3\mu}|$ is "approximately zero" (assuming $|V_{3e}^*|$ is large). This argument is relevant for the experimental scenario where the LHC puts a lower bound on the mass of LFV mediators, and a Super-B factory
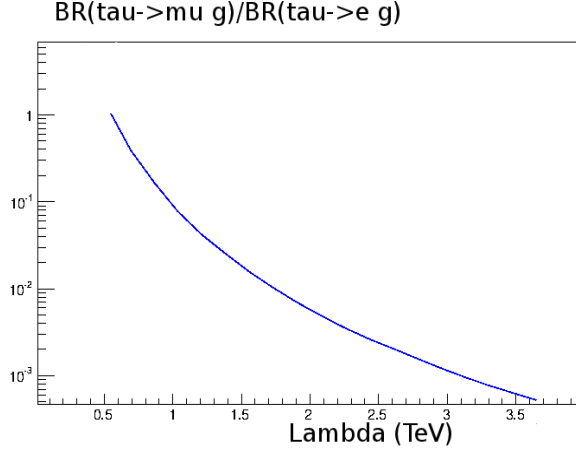
BR(tau->mu g)/BR(tau->e g)

Fig. 2. – the hierarchy predicts $BR(\tau \to \mu\gamma)$ below anticipated Super B fact sensitivities...

sees a $\tau \to \ell\gamma$ decay. Then the argument says that : if the New Physics couplings are hierarchical, then only one of $\tau \to \mu\gamma$ or $\tau \to e\gamma$ should be seen. Notice that this upper bound arises irrespective of whether $\mu \to e\gamma$ is observed or not. See [8] for caveats to this argument.

In **summary**, it is the authors opinion that it is interesting to explore how much of the fundamental New Physics Lagrangian can be reconstructed from coefficients of the Effective Lagrangian. I described here a simple example (with some hidden assumptions) where measuring one rare τ decay allows to learn whether the New couplings are hierarchical. This example also illustrates that discovering an LFV process in τs is arguably more interesting than discovering it in μs, because combining a τ detection at $BR \sim 10^{-8}$ with a μ bound at $BR \lesssim 10^{-12}$ gives information about both the New Physics flavour structure and scale.

$$* * *$$

I thank the organisers for an invitation to La Thuile the year of first LHC results.

REFERENCES

[1] K. Nakamura et al. (Particle Data Group), J. Phys. **G 37** (2010) 075021
[2] T. Feldmann, "Lepton Flavour Violation Theory," PoS BEAUTY **2011** (2011) 017 [arXiv:1105.2139 [hep-ph]]. P. Paradisi, "LFV: where are we?," PoS HQL **2010** (2010) 052. M. Hirsch, "Charged lepton flavour violation," Nucl. Phys. Proc. Suppl. **217** (2011) 318. A. Hoecker, "Charged-Lepton Flavour Physics," arXiv:1201.5093 [hep-ph].
[3] A. Goudelis, O. Lebedev and J. -h. Park, "Higgs-induced lepton flavor violation," Phys. Lett. B **707** (2012) 369 [arXiv:1111.1715 [hep-ph]]. G. F. Giudice and O. Lebedev, "Higgs-dependent Yukawa couplings," Phys. Lett. B **665** (2008) 79 [arXiv:0804.1753 [hep-ph]]. K. S. Babu and S. Nandi, "Natural fermion mass hierarchy and new signals for the Higgs boson," Phys. Rev. D **62** (2000) 033002 [hep-ph/9907213].
[4] T. P. Cheng and M. Sher, "Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets," Phys. Rev. D **35** (1987) 3484.
[5] M. Carpentier and S. Davidson, "Constraints on two-lepton, two quark operators," Eur. Phys. J. C **70** (2010) 1071 [arXiv:1008.0280 [hep-ph]].

[6] A. Brignole and A. Rossi, "Anatomy and phenomenology of mu-tau lepton flavor violation in the MSSM," Nucl. Phys. B **701** (2004) 3 [hep-ph/0404211].

[7] V. Cirigliano, R. Kitano, Y. Okada and P. Tuzon, "On the model discriminating power of mu —¿ e conversion in nuclei," Phys. Rev. D **80** (2009) 013002 [arXiv:0904.0957 [hep-ph]]. R. Kitano, M. Koike and Y. Okada, "Detailed calculation of lepton flavor violating muon electron conversion rate for various nuclei," Phys. Rev. D **66** (2002) 096002 [Erratum-ibid. D **76** (2007) 059902] [hep-ph/0203110]. R. Kitano and Y. Okada, "P odd and T odd asymmetries in lepton flavor violating tau decays," Phys. Rev. D **63** (2001) 113003 [hep-ph/0012040].

[8] S. Davidson, "Learning about flavour structure from tau to ell gamma and mu to e gamma?," Eur. Phys. J. C **72** (2012) 1897 [arXiv:1112.2956 [hep-ph]].