Lepton Flavour Violation
What do we know?   What can we learn?

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1. what do we know?
   - only upper bounds   *But neutrinos have mass*
   - what do we(theorists) want to know? *A New Physics Lagrangian!*
     more practical: the effective Lagrangian
   - Interpreting exptal bounds with the $\mathcal{L}_{eff}$

2. what we can learn?
   - exptal sensitivities to come
   - what could we learn from data about effective Lagrangian ?
   - (??how does one get from $\mathcal{L}_{eff}$ to the New Physics Lagrangian?)

Lepton Flavour Violation ... would be evidence for New Physics!

recent reviews:
Paradisi, Feldmann
Hirsch,...

Nonetheless, no model predictions in this talk!

... data should tell us what the theory is ...(d'après moi)

What do we know?

(LFV $\equiv$ flavour changing point interaction of charged leptons
$\equiv$ FCNC in charged leptons)

1. we know $m_\nu \neq 0 \Rightarrow$ *Beyond the Standard Model in the leptons!* 

   NB: the relation of lepton flavour to BSM, vs BSM to quark flavour, is *different* :

   - In **leptons**, put BSM to reproduce flavour data.
   - In **quarks**: SM predicts (most) observed CPV and FCNC. Put BSM to address hierarchy problem; quark flavour physics is an obstacle it must get around... $\Rightarrow$ require that BSM flvour predictions are patterned on SM. $\Rightarrow$ MFV.

2. But $\mathcal{A}(\mathrm{LFV}) \propto m_\nu^2/m_W^2 \sim 20^{-24}, \Rightarrow$ observable LFV requires dynamics other than m_ν

   entertainment for theorists: obtain log GIM in leptons...

What do we know? (experimentally)

some processes	current sensitivities
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$
$\frac{\sigma(\mu + Au \to e + Au)}{\sigma(\mu \text{ capture})}$	$< 7 \times 10^{-13}$
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$
$BR(\tau \to e\phi)$	$< 3.1 \times 10^{-8}$
$BR(\tau \to \ell + X_{m \lesssim m_\pi})$	$< 2.7 - 5 \times 10^{-3}$
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$
$BR(K^+ \to \pi^+ \bar{\nu}\nu)$	$= 1.7 \pm 1.1 \times 10^{-10}$
$BR(\overline{K^+} \to \pi^+ X_{m \sim 0})$	$< 5.9 \times 10^{-11}$
$BR(B^+ \to K^+ \tau\bar{\mu})$	$< 7.7 \times 10^{-5}$

What a theorist might want to know

- The symmetries which define the New Physics Lagrangian

  ?how to measure a symmetry?

- pragmatic: new particles, masses and interactions of that Lagrangian

  1. New particles are heavy (SUSY, GUTs, etc)
  2. New particles interact weakly (axion, majoron, ...) Interesting. Not covered here. Explore more?

  ?how to measure masses and interactions of particles that can't produce?

- mais on n'a pas accès à ces choses — what to do?

What a theorist might want to know

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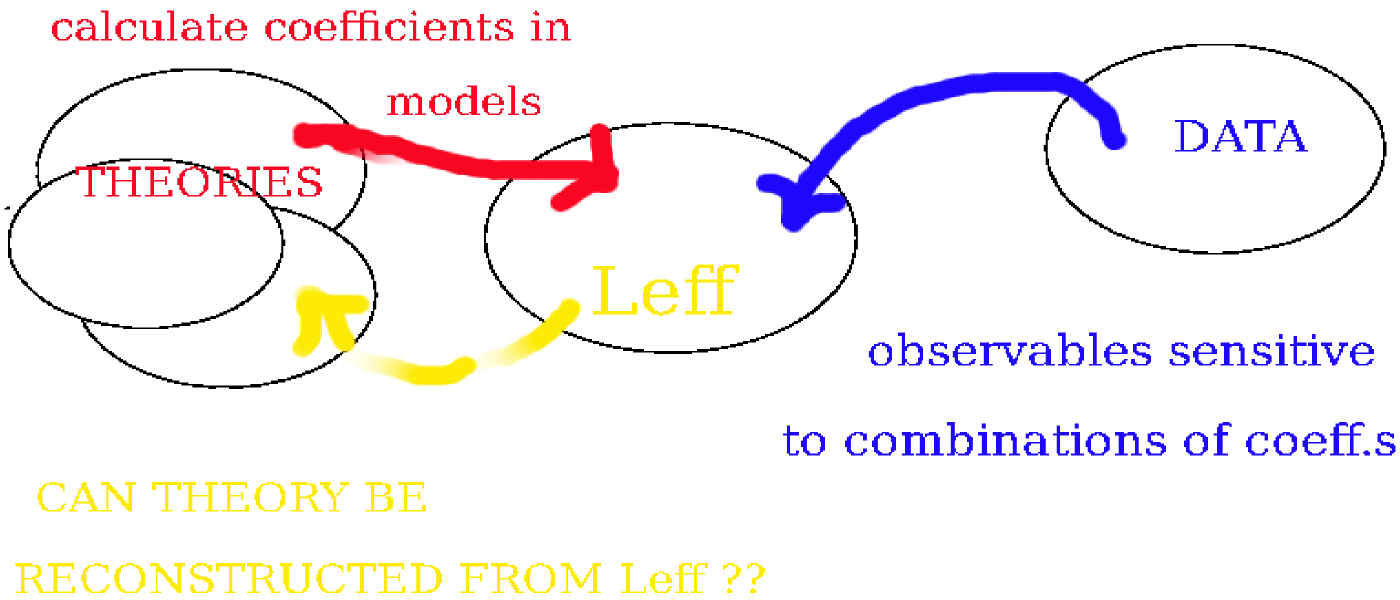
- mais on n'a pas accès à ces choses — what to do?
  At SM scales, footprints of heavy NP are encoded an "effective Lagrangian"; can use $\mathcal{L}_{eff}$ as a bridge between data and theories...



(Organising and interpreting) what we know: the effective Lagrangian

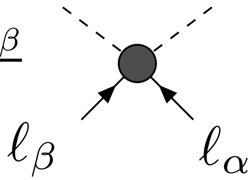
Suppose that new particles are above (fuzzy) mass scale Λ. Describe the interactions they induce among SM particles, at energies $\ll \Lambda$, with an "effective Lagrangian":

$$\Delta \mathcal{L}_{eff}^{LFV} = \sum_{d \geq 5} \sum_{n} \frac{C^n}{\Lambda^{d-4}} O_n(H, \{\psi\}, A_\mu, ...) + h.c.$$

The operators $\{O_n\}$ describe the legs of the LFV diagrams (including Higgs vevs)
The (dimless) coefficients C^n contain coupling constants, $1/16\pi^2$, ...

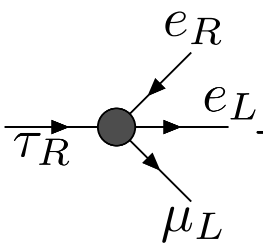
More friendly $\mathcal{L}_{eff}$

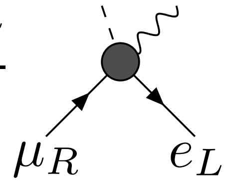
coefficients $\frac{C^{(n)}}{\Lambda^{d-4}}$ $\approx$ couplings constants, operators $\Leftrightarrow$ diagrams

$\Delta\mathcal{L}_{eff} = \frac{[m_\nu]_{\alpha\beta}}{v_2^2}$ 

$+$ h.c.

ang. dist, in $\tau \to 3\ell$: KitanoOkada

$+ \ldots + \frac{C^{e\tau\mu e}}{16\pi^2\Lambda^2}$ 

$+ \frac{e m_\mu C^{e\mu}}{16\pi^2\Lambda^2}$ 

$+ \ldots +$ h.c.

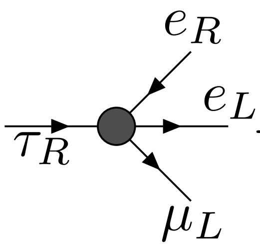
$$+ [\nu \text{ mag mos and other dim 7}] + h.c$$

$$+ [NSI \text{ and other dim 8}] + \ldots$$

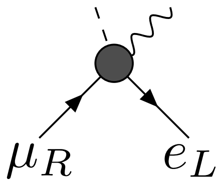
More friendly $\mathcal{L}_{eff}$

coefficients $\frac{C^{(n)}}{\Lambda^{d-4}}$ ≈ couplings constants, operators ⇔ diagrams

$$\Delta\mathcal{L}_{eff} = ... + \frac{C^{e\tau\mu e}}{16\pi^2\Lambda^2}$$



$$+ \quad \frac{e m_\mu C^{e\mu}}{16\pi^2\Lambda^2}$$



$+ ... +$ h.c.

$$+ [\nu \text{ mag mos and other dim } 7] + h.c$$

$$+ [NSI \text{ and other dim } 8] + ...$$

An first interpretation of current bounds using $\mathcal{L}_{eff}$:
For a given process with $BR <$ ..., can obtain a lower bound on Λ:
find lowest dimension operator/diagram corresponding to a process, set $C \simeq 1$, compute rate,...
(dimension 6, $\propto 1/(16\pi^2\Lambda^2)$), for most LFV processes)

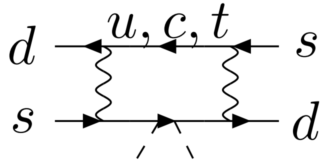
Interpreting what we know: bounds assuming dimension 6 operators

process	bound	scale, dim 6, loop
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	48 TeV
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	174 TeV (tree)
		14 TeV
$\frac{\sigma(\mu+Ti \to e+Ti)}{\sigma(\mu Ti \to \nu Ti')}$	$< 4.3 \times 10^{-13}$	40 TeV
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$	2.8 TeV
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	0.8 TeV
$BR(\tau \to e\pi)$	$< 8.1 \times 10^{-8}$	0.5 TeV
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	25 TeV$(V \pm A)$
		140 TeV$(S \pm P)$
$BR(B^+ \to K^+\tau\bar{\mu})$	$< 7.7 \times 10^{-5}$	0.3 TeV

smaller Λs than ... G Isidori@NA62 workshop...

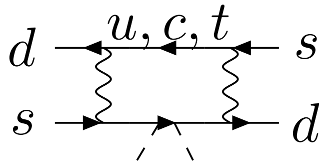
? μ searches sensitive to higher scale than τ ??
LFV in kaons vs Bs?

SM FCNC are at dimension 8 (GIM)—what if its true in BSM too?



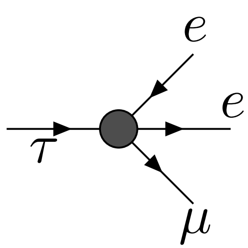
Giudice,Goudelis,Lebedev,Park
Babu

SM FCNC are at dimension 8 (GIM)—what if its true in BSM too?

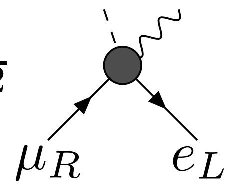


set dimension six coefficients $\frac{C^{(6)}}{\Lambda^2} = 0$, consider operators/diagrams with two additional Higgs legs (vevs)

$$\Delta\mathcal{L}_{eff} = ... + \frac{\mathbf{0}}{16\pi^2\Lambda^2}$$



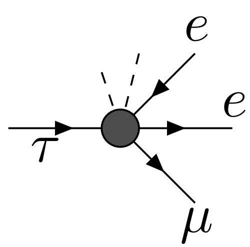
$$+ \quad \frac{\mathbf{0}}{16\pi^2\Lambda^2}$$



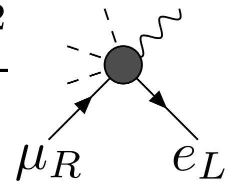
$$+ ... + \text{h.c.}$$

$$+[\nu \text{ mag mos and other dim 7}] + h.c.$$

$$+[NSI + ...] + \frac{C^{(8)}_{e\tau\mu e} v^2}{16\pi^2\Lambda_8^4}$$



$$+ \quad \frac{e m_\mu}{16\pi^2}\frac{C^{(8)}_{e\mu} v^2}{\Lambda_8^4}$$



$$+ ... + \text{h.c.}$$

For a given process with $BR < ...$, can obtain a lower bound on Λ at dimension 8: set $C^8 \simeq 1$, compute rate,...

Interpreting what we know: bounds at dimension 6 and 8

process	bound	scale (dim 6, loop)	scale (dim 8, loop)
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	48 TeV	2.9 TeV
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	170 TeV (tree)	5.5 TeV (tree)
		14 TeV	1.5 TeV
$\frac{\sigma(\mu+Ti\to e+Ti)}{\sigma(\mu \text{ capture})}$	$< 4.3 \times 10^{-12}$	40 TeV	2.6 TeV
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$	2.8 TeV	0.7 TeV
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	9 TeV (tree)	1 TeV (tree)
$BR(\tau \to e\pi)$	$< 8.1 \times 10^{-8}$	0.5 TeV	0.3 TeV
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	25 TeV$(V \pm A)$	2.1 TeV$(V \pm A)$
		140 TeV$(S \pm P)$	5 TeV$(S \pm P)$

New particles could be accessible to colliders. Such mass determination very useful for raising coupling$\leftrightarrow$ mass degeneracy of $\mathcal{L}_{eff}$ coefficients.

But flavoured couplings we know are not 1?

Lets suppose

1. a mass scale for new particles $\sim$ TeV
2. tree diagrams (no factors of $1/(16\pi^2)$)
3. flavoured fermion couplings $\propto$ SM fermion masses:

$$\lambda_{ij} \simeq \sqrt{\frac{m_i m_j}{v^2}} \quad , \quad i,j \text{ any SM fermion}$$

Cheng Sher
extra dim ...

estimate rates assuming no additional (eg chiral) suppression factors...
(except when estimate is to big)

Current bounds vs naive expectations

process	bound	expectation
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	$\sim 6.5 \times 10^{-8}$, 2.2×10^{-14}
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	$\sim 1.3 \times 10^{-23}$
$\frac{\sigma(\mu + Ti \to e + Ti)}{\sigma(\mu \text{ capture})}$	$< 4.3 \times 10^{-12}$	$\sim 2.5 \times 10^{-19}$
$BR(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$	$\sim 8 \times 10^{-7}$, 8×10^{-11}
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	$\lesssim 3 \times 10^{-16}$
$BR(\tau \to \mu\pi)$	$< 8.0 \times 10^{-8}$	$\sim 10^{-17}$
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	$\sim 1 \times 10^{-12}$
$BR(K^+ \to \pi^+\bar{\nu}\nu)$	$= 1.7 \pm 1.1 \times 10^{-10}$	$\sim 2 \times 10^{-10}$ (ν_τ)
$BR(B^+ \to K^+\tau\bar{\mu})$	$< 7.7 \times 10^{-5}$	$\sim 3 \times 10^{-10}$

1. tree level
2. a mass scale for new particles $\sim$ TeV
3. flavoured couplings $\propto$ SM masses:

$$\lambda_{ij} \simeq \sqrt{\frac{m_i m_j}{v^2}} \quad , \quad i, j \text{ any SM fermion}$$

Summary: what we know

Neutrinos have mass $\Leftrightarrow$ there is New Physics dedicated to Lepton Flavour!

NB: different relation between BSM and lepton flavour, vs BSM and quark flavour!

But, no flavour-changing processes observed among charged leptons (yet).

• current bounds allow, in loops, most new flavoured particles with masses $\gtrsim$ few $\rightarrow$ 10 TeV, with $\mathcal{O}(1)$ couplings

•new flavoured particles with masses $\sim$ TeV and hierarchical couplings can contribute at tree

• different classes of BSM scenarios (in loops, at dimension 6 or 8, with hierarchical couplings), can most readily be found in various processes (μ decays, τ decays, K decays ,...)

$$\Rightarrow \textit{look everywhere!}$$

What can LFV tell us about New Physics?

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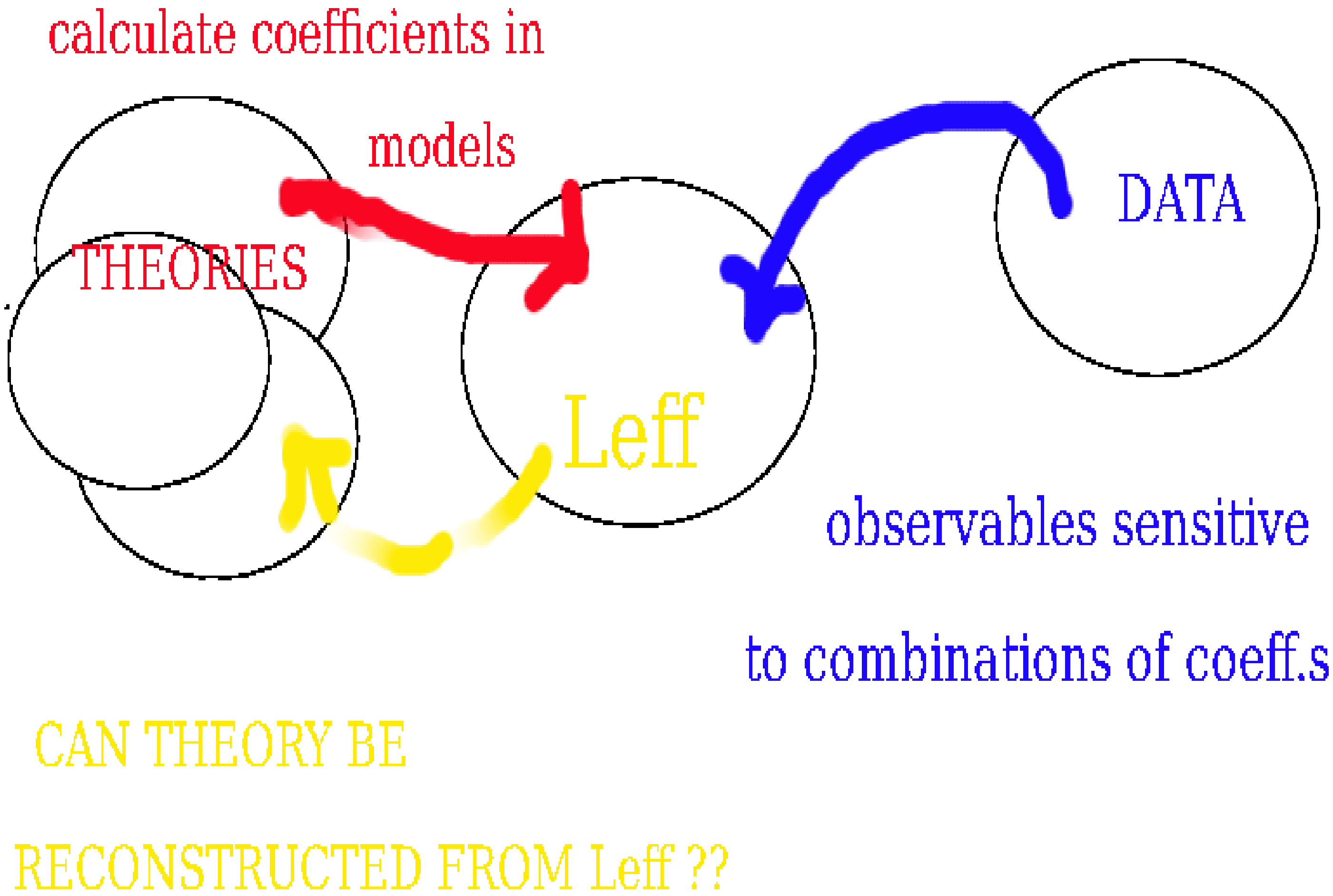


What can we learn — future exptal sensitivities

some processes	current sensitivities	future sensitivity
$BR(\mu \to e\gamma)$	$< 2.4 \times 10^{-12}$	$\sim 10^{-13}(10^{-14}?)$ (MEG)
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	
$\frac{\sigma(\mu+Au\to e+Au)}{\sigma(\mu\ \mathrm{capture})}$	$< 7 \times 10^{-13}$	$10^{-16} - 10^{-18}$(J-PARC)
$BR(\tau \to \ell\gamma)$	$< 3.3, 4.4 \times 10^{-8}$	few $\times 10^{-9}$ (S-B fact)
$BR(\tau \to 3\ell)$	$< 1.5 - 2.7 \times 10^{-8}$	$\lesssim 10^{-9}$ (S-B fact)
$BR(\tau \to e\phi)$	$< 3.1 \times 10^{-8}$	$\lesssim 10^{-9}$ (S-B fact)
$BR(\tau \to \ell + X_{m \lesssim m_\pi})$	$< 2.7 - 5 \times 10^{-3}$	?
$BR(\overline{K_L^0} \to \mu\bar{e})$	$< 4.7 \times 10^{-12}$	
$BR(K^+ \to \pi^+\bar{\nu}\nu)$	$= 1.7 \pm 1.1 \times 10^{-10}$	100 evts (NA62)

NA62 will have K^+s, can do $BR(K^+ \to \pi^+\mu^+e^-) \sim 10^{-12}$, but for LFV, 'tis not better than $K \to \mu^+e^-$??

What can we learn about coefficients of $\mathcal{L}_{eff}$? ??

two examples

1. *combining different observables allows to identify the operator in* $\mathcal{L}_{eff}$.
   e.g. : $\mu - e$ conversion, $\mu \to e\gamma$, *and* $K_L \to \mu^{\pm}e^{\mp}$

2. *measuring the same process for different flavours, tells about flavour structure of the operqtor coefficient,*
   e.g. :$\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$

What can we learn about coefficients of $\mathcal{L}_{eff}$? ?? And about theories ??

two examples

1. *combining different observables allows to identify the operator in $\mathcal{L}_{eff}$.*
   (This can tell about properties of New Particles , such as spin, colour)
   $\mu - e$ conversion, $\mu \to e\gamma$, *and* $K_L \to \mu^{\pm} e^{\mp}$

2. *measuring the same process for different flavours, tells about flavour structure of the operator coefficient.*
   And (?) therefore of NP couplings?
   $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$

...but we are a far from reconstructing the New Physics Lagrangian ...

What can we learn: $\mu - e$ conversion, $\mu \to e\gamma$, and $K_L \to \mu^\pm e^\mp$?

A μ^- stops in matter, gets bound to a nucleus (in $1s$).

$$\text{In the SM}: \mu + (A,Z) \to \left\{ \begin{array}{c} \nu_\mu + (A, Z-1) \\ e + \bar{\nu_e} + \nu_\mu + (A,Z) \end{array} \right.$$

In BSM ; $\mu + (A,Z) \to e + (A,Z)$ due to dipole and (various) 4-fermion operators:

$$\frac{e_{em} C^{\mu e} m_\mu}{16\pi^2 \Lambda^2} \overline{\mu} \sigma^{\alpha\beta} e F_{\alpha\beta} + \sum_\Gamma \left\{ \frac{C_\Gamma^{e\mu dd}}{\Lambda^2} (\bar{\mu}\Gamma e)(\bar{d}\Gamma d) + \frac{C_\Gamma^{e\mu dd}}{\Lambda^2} (\bar{\mu}\Gamma e)(\bar{u}\Gamma u) \right\} + h.c.$$

(off-shell photon is included in the 4-fermion operator).

Look for single e^- with $E \simeq m_\mu - E_{bind}$. From SINDRUM II @PSI:

$$\frac{\Gamma(\mu Au \to eAu)}{\Gamma(\mu Au \to \nu Au')} < 7 \times 10^{-13} \qquad \to \qquad ?\ \ 10^{-18} (PRISM/PRIME, \mu 2e?)$$

$\mu \to e\gamma$ and $\mu N \to eN$: relative importance of 4-f. and dipole ops:

1. if see $\mu \to e\gamma$

   - dipole contribution to $\mu N \to eN$ predicts lower bound:

$$\frac{BR(\mu N \to eN)}{BR(\mu \to e\gamma)} \simeq \frac{B(A,Z)}{428} \sim \frac{\alpha}{3} \qquad B : 1.1 \to 1.8$$

Czarnecki Marciano Melnikov

(Dipole dominates in (low $\tan\beta$) SUSY models, due to cancellations in penguin/box contributions to 4-f. ops)

$\Rightarrow$ if $BR(\mu N \to eN) \gg \alpha BR(\mu \to e\gamma)/3 \quad \Rightarrow$ 4-f dominates.

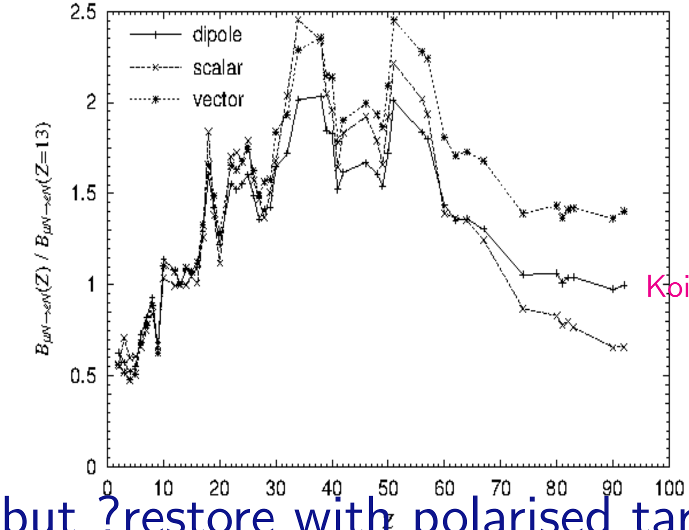
Which ones? no relation between 4-f and dipole coeffs in Little Higgs + T, Blanke etal 0703117 and leptoquarks/ R_pV SUSY

2. if see $\mu N \to eN$ but not $\mu \to e\gamma$?

Which operator: $\mu - e$ conversion, and $K_L \to \mu^\pm e^\mp$?

1. measure with different nuclei? The various operators have different parities, dependance on A, Z. So different operators give different dependance of BR on Z :

   plot for BR measured at $Z = 13(Al)$



Koike,Kitano,Okada

2. μ polarisation? $\simeq$ lost in cascade to $1s$, but ?restore with polarised target? Kuno Nagamine, Yamazaki

   $\vec{p}_e \cdot \vec{s}_\mu = \pm$, distinguishes operators giving e_L or e_R.

3. $\Lambda^{(6)}_{K \to \mu e} > 140$ TeV, $\Lambda^{(6)}_{\mu - e \text{ conversion}} > 40 \to 1200$

   information about how LFV interacts with quarks ? (only to singlets ? if to doublets, then via penguins??)

$\tau \to \ell\gamma$ and $\mu \to e\gamma$: flavour structure of the dipole coefficient

Only one operator (two chiralities):

$$\frac{em_\alpha}{16\pi^2\Lambda^2}[C_L]_{\alpha\beta}\overline{e_R}_\beta \sigma^{\mu\nu} e_{L\alpha} F_{\mu\nu} + \frac{em_\alpha}{16\pi^2\Lambda^2}[C_R]_{\alpha\beta}\overline{e}_\beta \sigma^{\mu\nu} e_{R\alpha} F_{\mu\nu}$$

lets assume chirality flip on external leg (for simplicity):

- Suppose *see* a $\tau \to \ell\gamma$ decay(!). Lets suppose observe $\tau \to e\gamma$.
  - not ridiculous (many models can predict this)
  - to learn something, have to see something
  - interesting scenario for learning about flavour structure: two pieces of info (can "test" hierarchical structure!)

- Suppose coefficient $em_\tau C/16\pi^2\Lambda^2$ dominated by largest eigenvalue
  (like $[\mathbf{Y}_u^\dagger \mathbf{Y}_u]_{bs} \simeq V_{tb}^* y_t^2 V_{ts}$)

  $\Rightarrow$ 3 parameters $(\Lambda, |V_{3e}|, |V_{3\mu}|)$ to parametrise $\mu \to e\gamma, \tau \to e\gamma, \tau \to \mu\gamma$.

Then... the hierarchy predicts that not see $\tau \to \mu\gamma$ ...

1. "sufficiently heavy" BSM induces $\tau \to e\gamma$:

$$\widetilde{BR}(\tau \to e\gamma) \simeq 10^{-8} \left(\frac{500GeV}{\Lambda}\right)^4 \frac{|V_{3e}|^2}{10^{-4}} \gtrsim 10^{-8}$$

2. $\widetilde{BR}(\mu \to e\gamma) \lesssim 10^{-12}$ imposes an "approximate zero" (irrespective if is seen or not)

$$\frac{\widetilde{BR}(\mu \to e\gamma)}{\widetilde{BR}(\tau \to e\gamma)} \simeq |V_{3\mu}|^2 \lesssim 10^{-4}$$

Want to argue that:

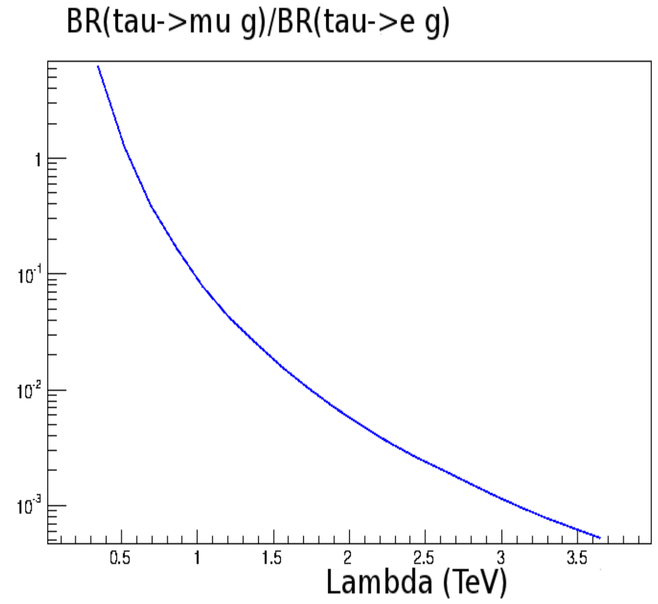
$1 \Rightarrow$ a large mixing angle V_{3e},

(for suff large Λ)

$2 \Rightarrow$ a small mixing angle $V_{3\mu}$,

so $\tau \to \mu_L\gamma (\propto |V_{3\mu}|^2)$ suppressed below S-B sensibilities

but caveats...



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LOTS TO DO (besides ski)

There is New Physics dedicated to Lepton Flavour. Things we could do:

1. measure LFV (its there....just... what rates?)

2. (...invent models so beautiful they must be true...and calculate their predictions)

3. ?maybe theorists could think about reconstructing “the” theory from $\mathcal{L}_{eff}$?