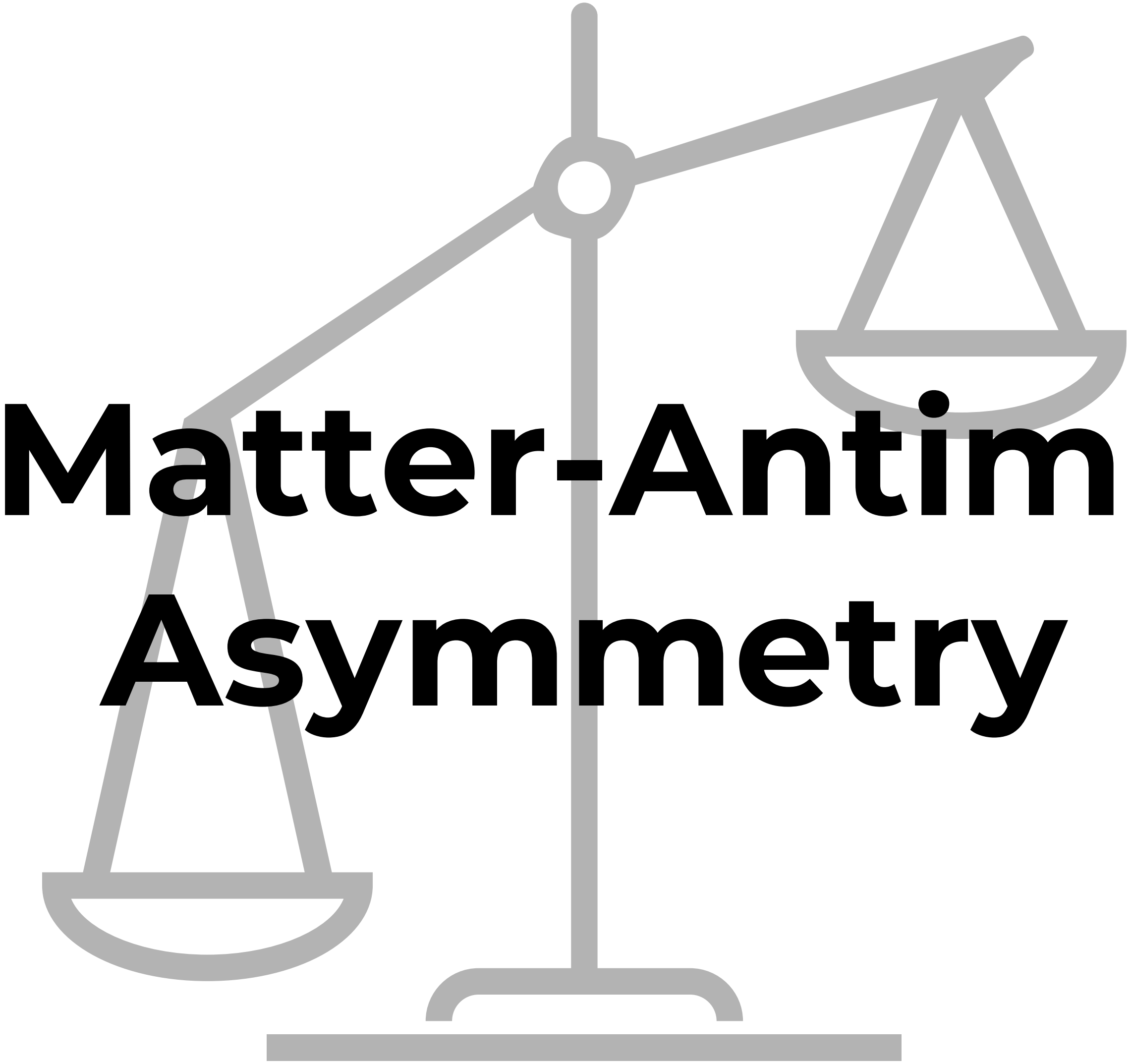
Leptogenesis

Speaker: **Alessandro Granelli**
Post-doc at University of
Bologna (Italy)

Theory and Phenomenology
of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA



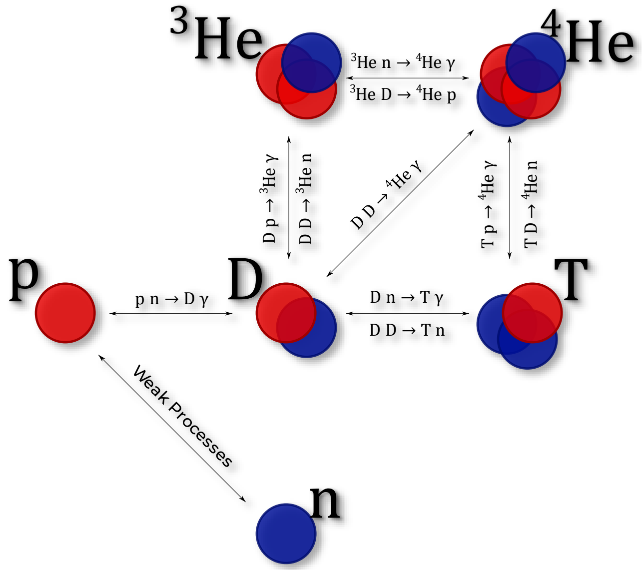
The Matter-Antimatter Asymmetry

The Baryon Asymmetry of the Universe

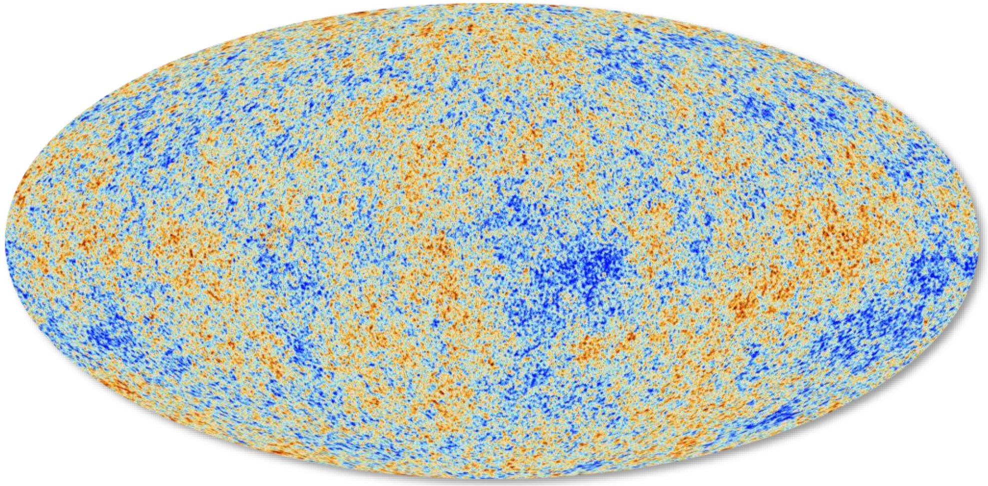
Observations from galactic to cosmological scales provide compelling evidence of a net **overabundance of matter** over antimatter, or, in terms of baryons, the **Baryon Asymmetry of the Universe (BAU)**.

The abundance of baryons in the present Unvierse is estimated from abundances of light primordial elements produced during BBN and from anisotropies in the CMB spectrum.

Big Bang Nucleosynthesis (BBN)



Cosmic Microwave Background (CMB)



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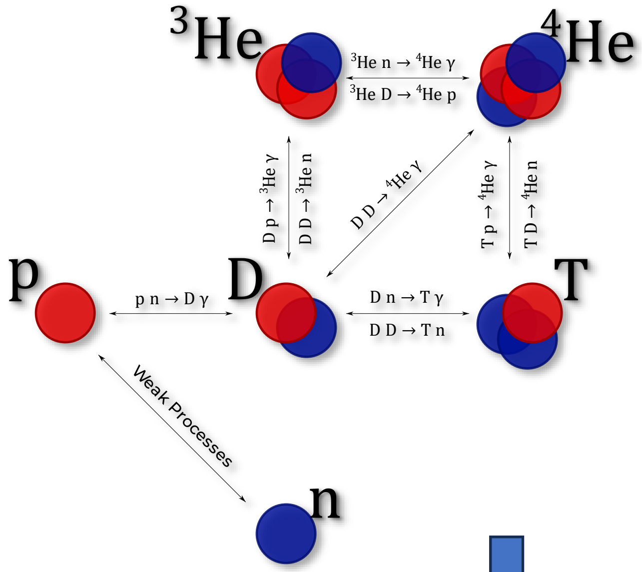

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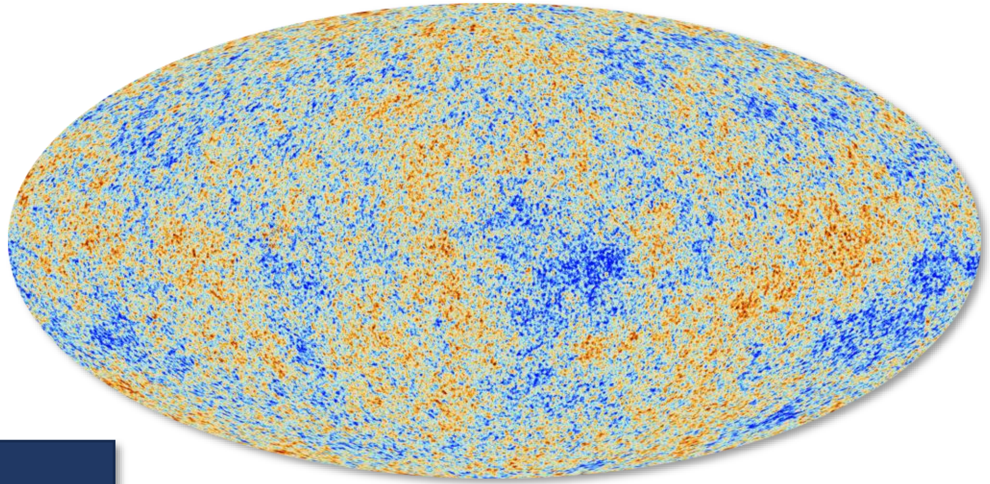

Baryon-to-photon ratio

$$\eta_B = \frac{(n_B - n_{\bar{B}})}{n_\gamma} \simeq 6.1 \times 10^{-10}$$

$\sim 2 \times 10^9 + 1$ baryons every 2×10^9 of antibaryons!

The Lepton Asymmetry of the Universe

- **Charged lepton-to-photon ratio**: $\eta_\ell = \frac{(n_\ell - n_{\bar{\ell}})}{n_\gamma}, \ell = e, \mu, \tau, \eta_e + \eta_\mu + \eta_\tau = \eta_e \cong \eta_B$

  Charge neutrality of the Universe imposes $\eta_e/\eta_B < 1 + 10^{-26}$

  C. Caprini and P. G. Ferreira, hep-ph/0310066

Pics from M. Escudero, A. Ibarra and V. Maura, arXiv:2208.03201

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- **Neutrino-to-photon ratio**: $\eta_{\nu_\ell} = \frac{(n_{\nu_\ell} - n_{\overline{\nu_\ell}})}{n_\gamma}, \ell = e, \mu, \tau;$ η_{ν_e} mostly affects helium abundance at BBN, while $|\eta_{\nu_e} + \eta_{\nu_\mu} + \eta_{\nu_\tau}|$ alters N_{eff}. Mild constraints from observations: $\boxed{|\eta_{\nu_e} + \eta_{\nu_\mu} + \eta_{\nu_\tau}| < O(0.1).}$

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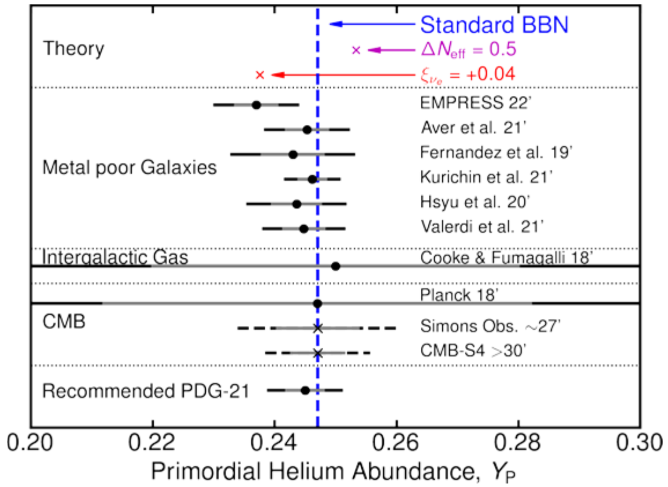
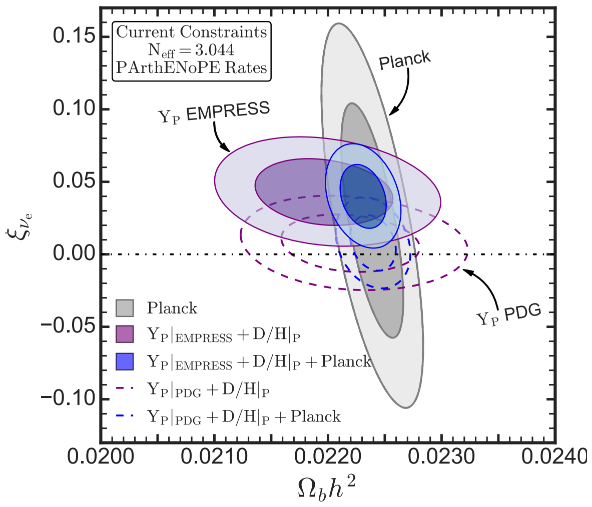
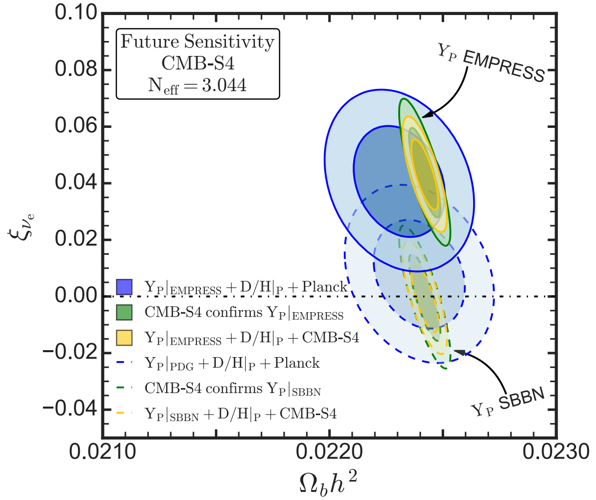
  Anomaly in the **EMPRESS** sruvey on 4-helium abundance suggests $\eta_{\nu_e} = 0.043 \pm 0.015$, but need more data!

EMPRESS collab. 2203.09617





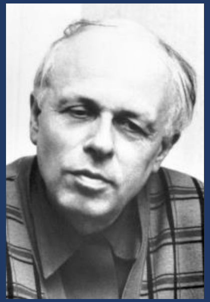


Pics from M. Escudero, A. Ibarra and V. Maura, arXiv:2208.03201

Sakharov's conditions

Three necessary **Sakharov's conditions** for a dynamical generation of a baryon (B) or lepton (L) asymmetry

☐ **B (L) violation (BNV or LNV)**.



A. D. Sakharov (1967)

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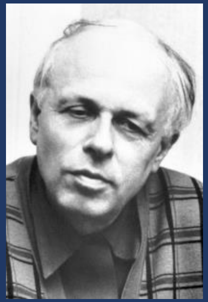
- ☐ **C and CP violation (CPV)**



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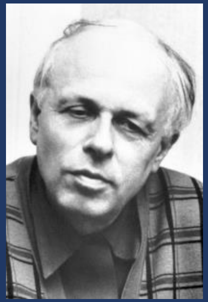
☐ **Out-of-equilibrium dynamics (OoE)**



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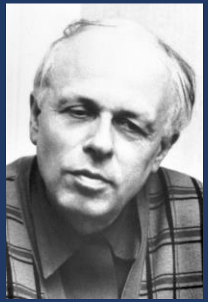
Non-perturbative SM sphalerons are (B + L)-violating, that conserve B − L.

☒ **C and CP violation (CPV)**

C is maximally violated in the SM, while CP is broken by quark mixing: this CP-violation is too small!

☒ **Out-of-equilibrium dynamics (OoE)**

The electroweak phase transition in the EU is not enough first order.



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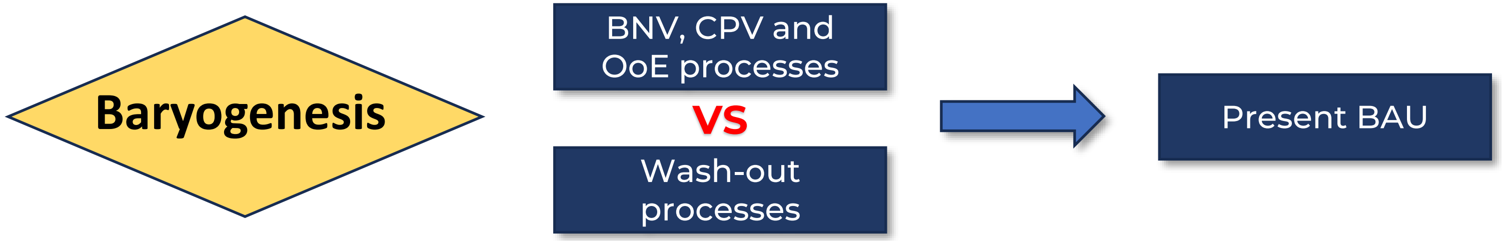
Physics beyond the SM is needed!

Baryogenesis and Leptogenesis



Baryogenesis

BNV, CPV and OoE processes

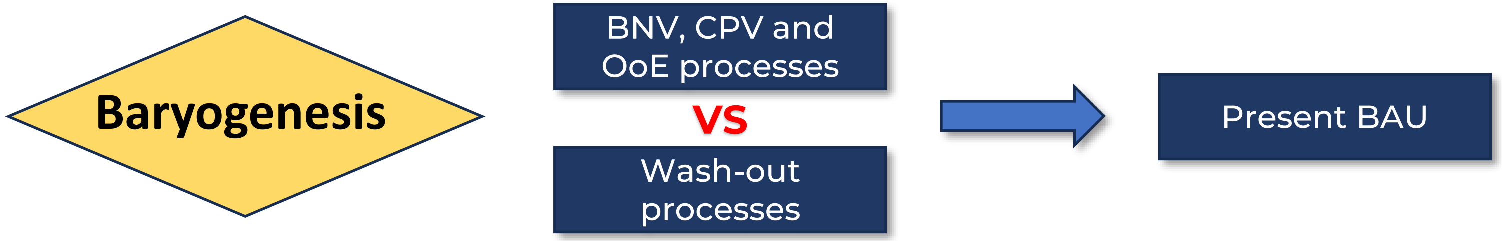
VS

Wash-out processes

Present BAU

Recent Review: D. Bodeker, W. Buchmuller, 2009.07294

Baryogenesis and Leptogenesis



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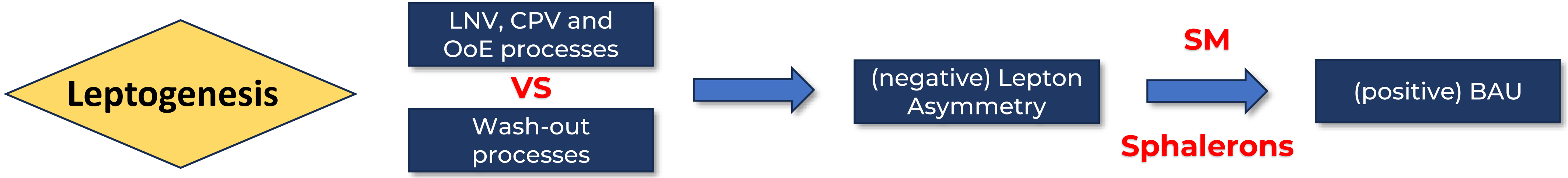


Leptogenesis

LNV, CPV and OoE processes

VS

Wash-out processes

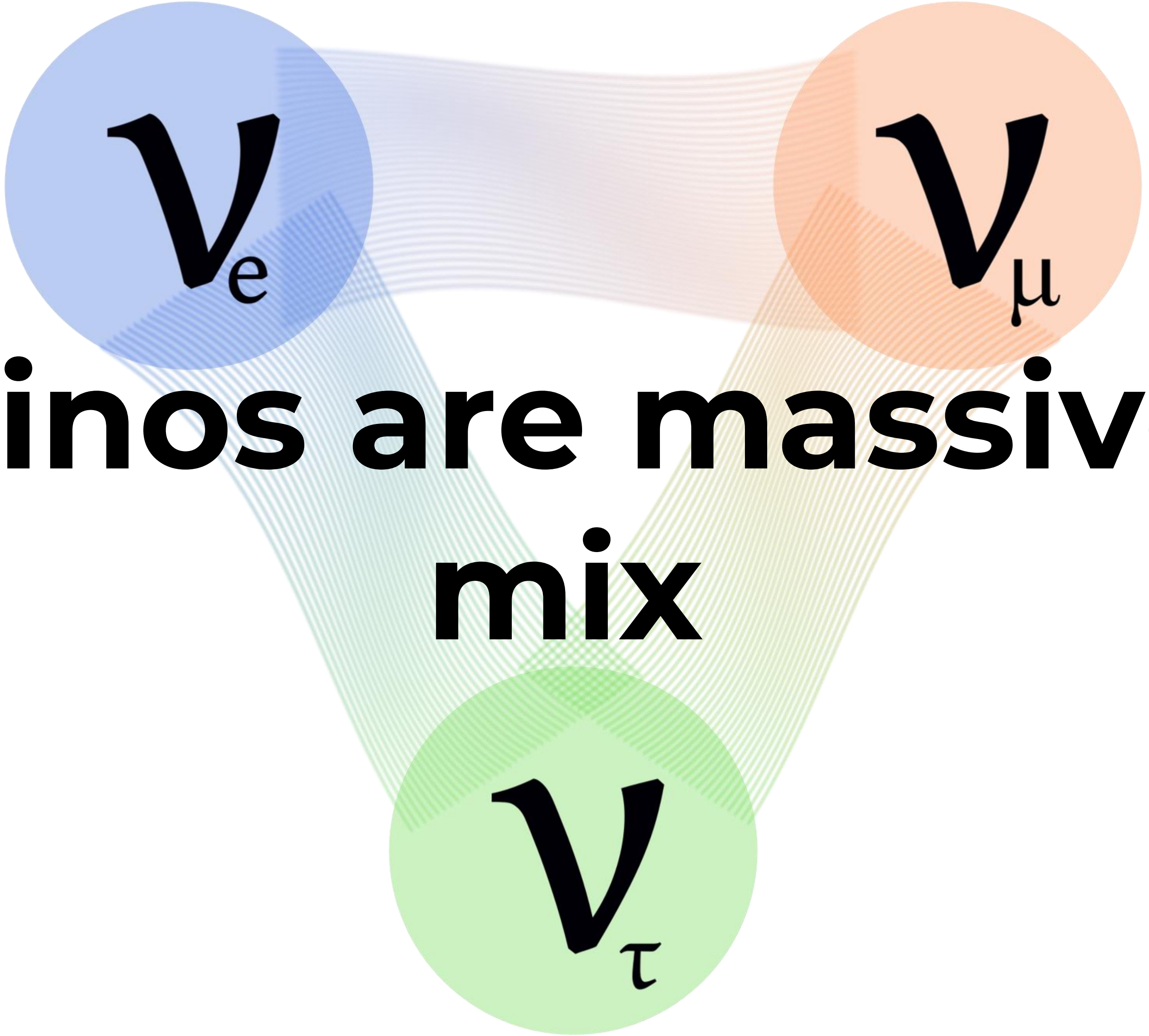
(negative) Lepton Asymmetry

SM

Sphalerons

(positive) BAU

First proposed by Fukugita & Yanagida (1986)



Neutrinos are massive and mix

Neutrino masses and mixing

Neutrinos have non-zero masses and mix: $\nu_{\alpha L}(x) = \sum_{a=1}^{3} U_{\alpha a} \nu_{aL}(x)$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{i\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{\frac{i\alpha_{31}}{2}} \end{pmatrix}$$

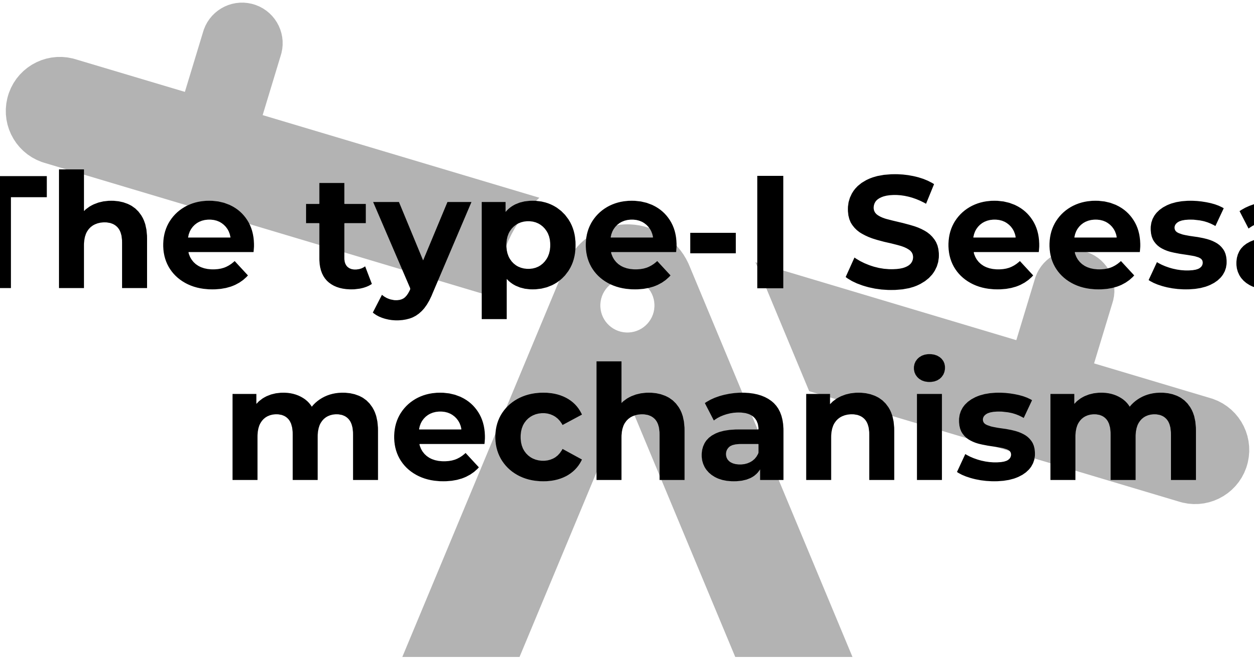
Summary of neutrinos observations:

- **Normal Ordering (NO):** $m_1 < m_2 < m_3$
- **Inverted Ordering (IO):** $m_3 < m_1 < m_2$

- **Normal Hierarchical (NH):** $0 \simeq m_1 < m_2 < m_3$
- **Inverted Hierarchical (IH):** $0 \simeq m_3 < m_1 < m_2$
- **Quasi Degenerate:** $m_1 \simeq m_2 \simeq m_3$

Ordering	θ_{12} (°)	θ_{13} (°)	θ_{23} (°, 3σ)	δ (°, 3σ)	Δm_{21}^2 ($10^{-5}\mathrm{eV}^2$)	$\Delta m_{31(32)}^2$ ($10^{-3}\mathrm{eV}^2$)
NO	33.67	8.58	39.9 – 51.1	139 – 350	7.41	2.505
IO	33.67	8.57	39.9 – 51.4	195 – 342	7.41	-2.487

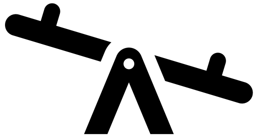
I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou (2020), NuFIT 5.3 (2024), www.nu-fit.org



The type-I Seesaw mechanism

Type-I seesaw mechanism

Seesaw lagrangian



Yukawa and mass terms

$$\mathcal{L}_{\mathrm{Y,M}}(x) = -\left(Y_{\alpha j}\,\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + h.c.\right) - \frac{1}{2}\, M_j\, \overline{N_j}(x)\, \boxed{N_j(x)}$$

Right-handed neutrinos/sterile neutrinos/ heavy Majorana neutrinos

Type-I seesaw mechanism

Seesaw lagrangian

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Right-handed neutrinos/sterile neutrinos/ heavy Majorana neutrinos

Electroweak Symmetry Breaking

Neutrino mass generation

Neutrino mass matrix

$$m_\nu \simeq -(v^2/2) Y \widehat{M}^{-1} Y^T$$

Neutrino mixing

$$\nu_{\alpha L} \simeq U_{\alpha a}\nu_{aL} + \boxed{\Theta_{\alpha j}} N^c_{jR}$$

$$\Theta_{\alpha j} \simeq \left(v/\sqrt{2}\right) Y_{\alpha j}/M_j$$

Mixing angle/Coupling

Type-I seesaw mechanism

Seesaw lagrangian

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Mixing angle/Coupling

Model Parameters

Casas-Ibarra Parameterisation

$$Y = \pm i\left(\sqrt{2}/v\right) U \sqrt{\widehat{m}}\, O^T \sqrt{\widehat{M}}$$

Casas-Ibarra matrix

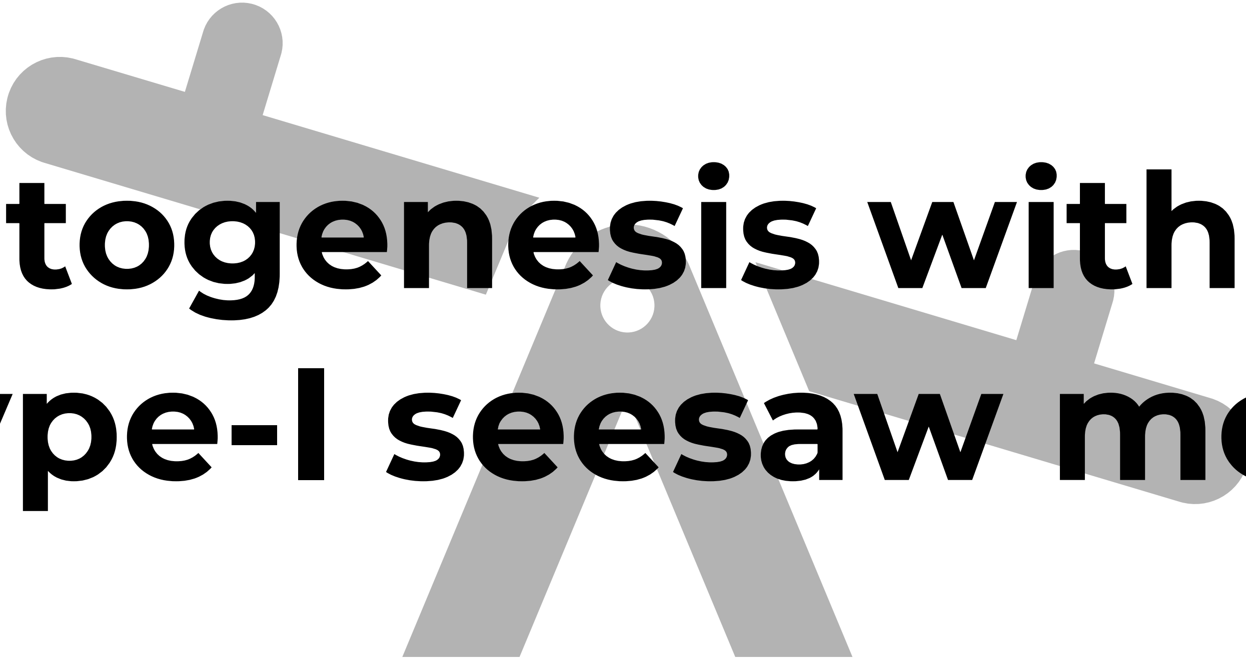
$$\boldsymbol{OO^T = 1}$$

E.g., with 2 heavy Majorana Ns

$$O^{(\mathrm{NH})} = \begin{pmatrix} 0 & \cos\theta & \varphi\sin\theta \\ 0 & -\sin\theta & \varphi\cos\theta \end{pmatrix}$$

$$O^{(\mathrm{IH})} = \begin{pmatrix} \cos\theta & \varphi\sin\theta & 0 \\ -\sin\theta & \varphi\cos\theta & 0 \end{pmatrix}$$

$$\theta = \omega + i\,\xi$$
$$\varphi = \pm 1$$

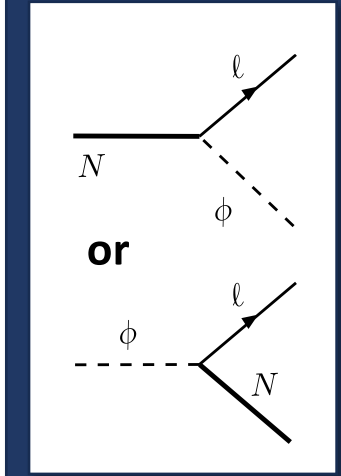


Leptogenesis within the type-I seesaw model

Leptogenesis within the type-I seesaw mechanism

Lepton Number violating processes via Yukawa coupling

Decays



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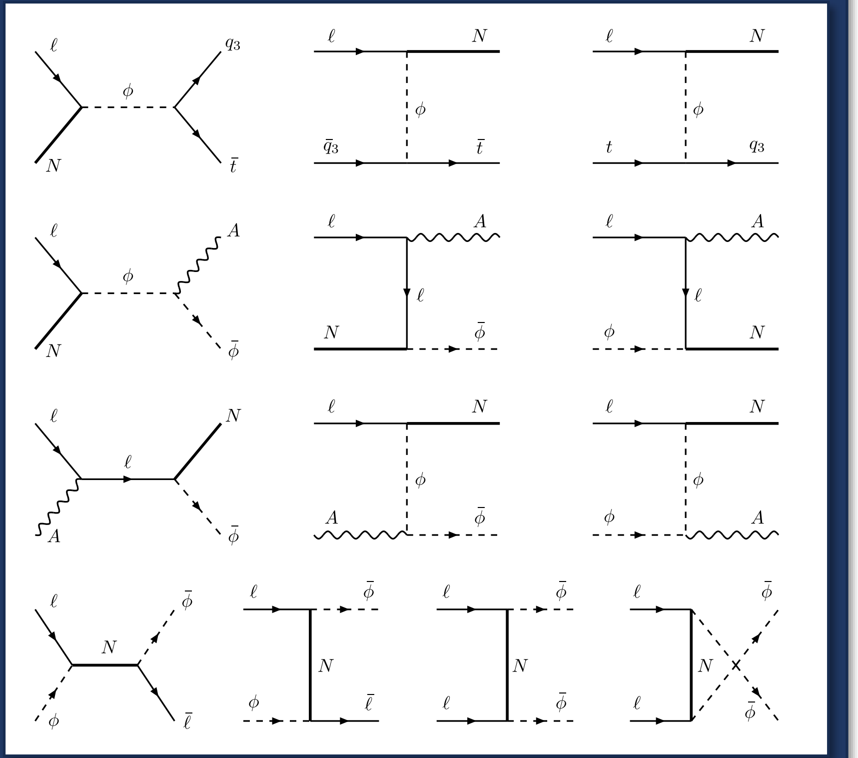
Scatterings

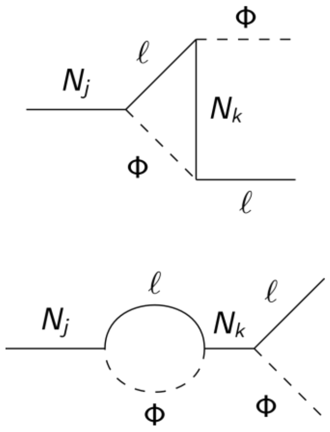


G. F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia hep-ph/0310123
S. Davidson, E. Nardi, Y. Nir arXiv:0802.2962

+

CP-violation at loop-level $O(Y^6)$

$$\epsilon^{\mathrm{CP}} = \frac{\Gamma(N \to \mathrm{l}\ldots) - \Gamma(N \to \bar{\mathrm{l}}\ \ldots)}{\Gamma(N \to \text{anything})}$$



L. Covi, E. Roulet, F. Vissani
hep-ph/9605319,
W. Buchmuller, M. Plumacher
hep-ph/9710460,
A. Pilaftsis hep-ph/9702393,
...

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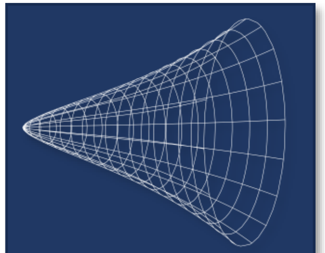
Expansion of the Universe



L A U → B A U

Sphaleron freeze-out

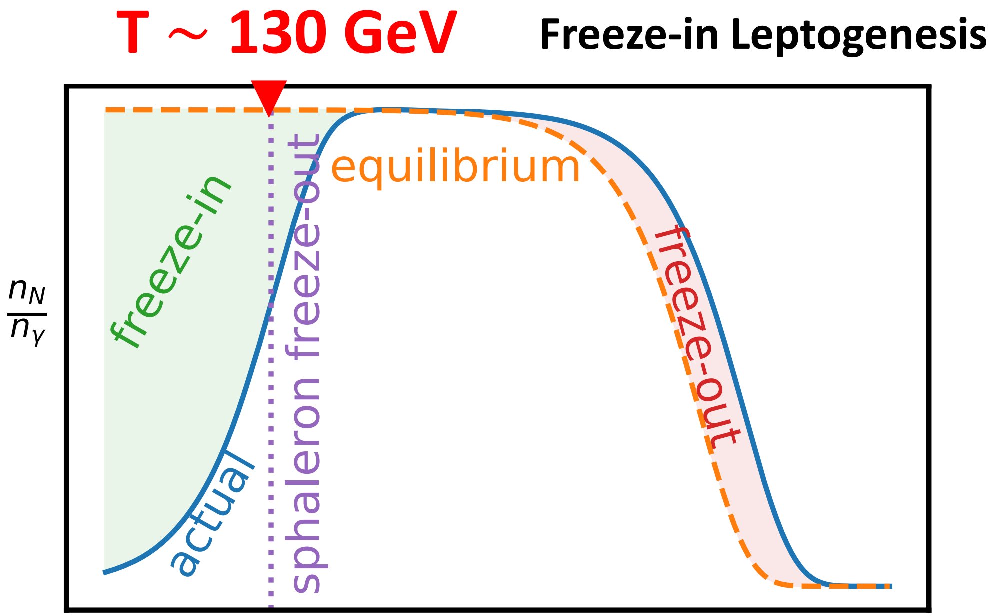
T ~ 130 GeV

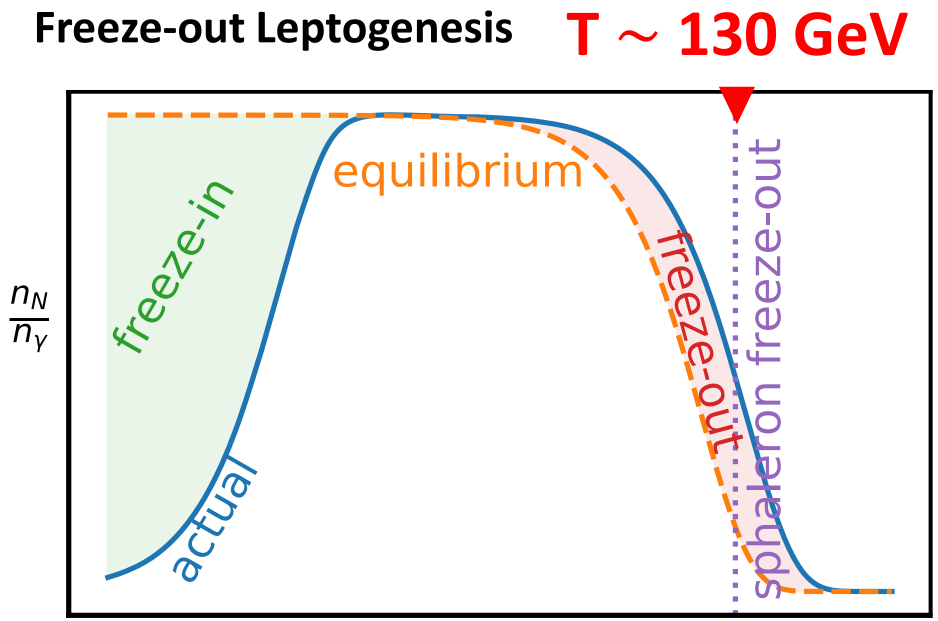
Leptogenesis within the type-I seesaw mechanism

Heavy neutrinos at the beginning can either have

- **Thermal Initial Abundance** (TIA);
- **Vanishing Initial Abundance** (VIA).

BAU generation after sphaleorn decouple either during production (**freeze-in**) or departure from equilibrium (**freeze-out**)





J. Klarić, M. Shaposhnikov, I. Timiryasov, PRL.127.111802 and PRD.104.055010
A. G., K. Moffat, S. T. Petcov, arXiv:2009.03166

CP-violation from heavy neutrino oscillations

Leptogenesis via heavy neutrino oscillations

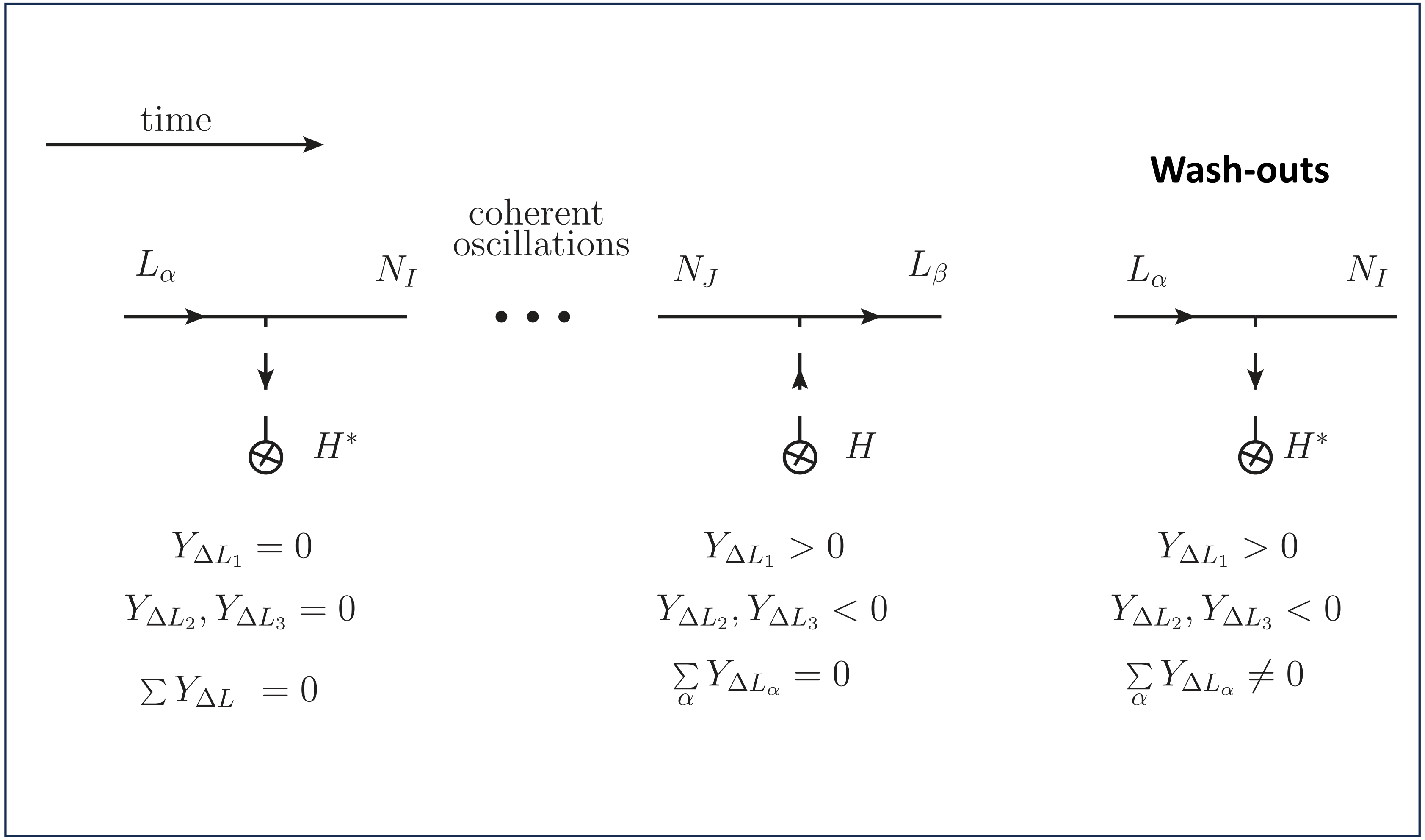


Fig. from B. Shuve, I. Yavin arXiv:1401.2459

- **In the relativistic regime $T \gg M$**, the Majorana nature of relativistic RHNs is not manifest. The first two steps conserve total lepton number. A total lepton asymmetry is generated at $O(Y^6)$ via **oscillations + flavour effects and wash-outs**. This is the Akhmedov-Rubakov-Smirnov (**ARS**) mechanisms for LG proposed in 1998, relevant for M < 100 GeV

- When **non-relativistic effects** are relevant, the Majorana nature of RHNs is important and oscillations alone give total lepton asymmetry at $O(Y^4 M^2/T^2)$, relevant for M ~100 GeV or higher.

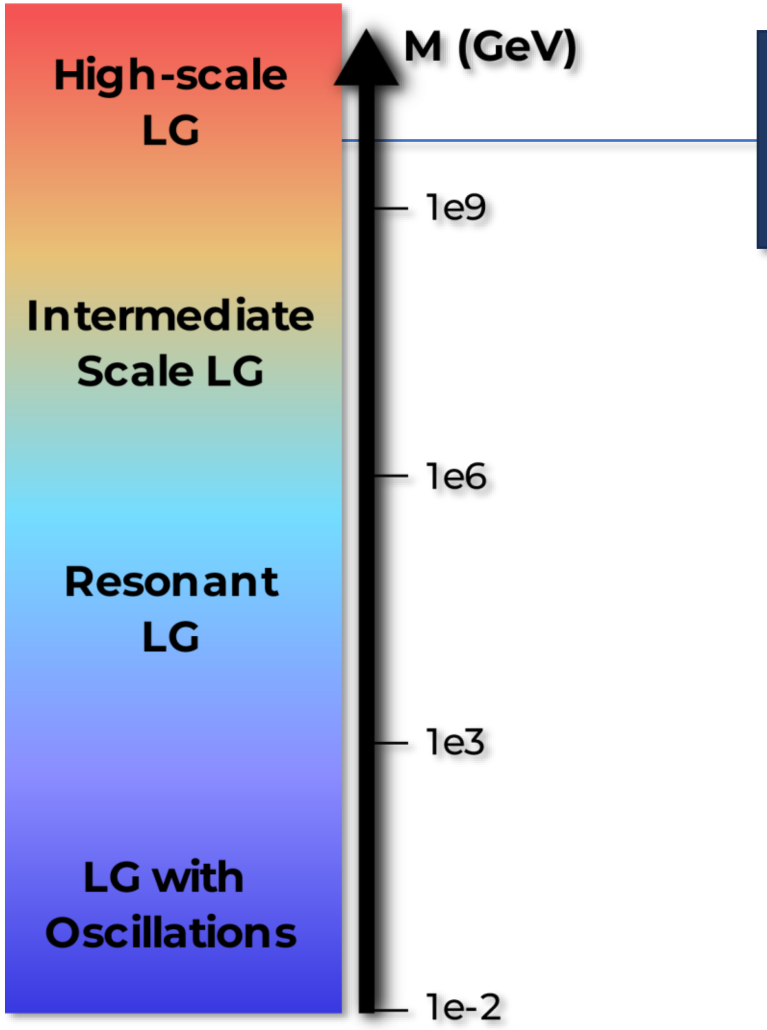
Leptogenesis scales

Fukugita & Yanagida (1986)

Racker, Rius & Pena (2012)

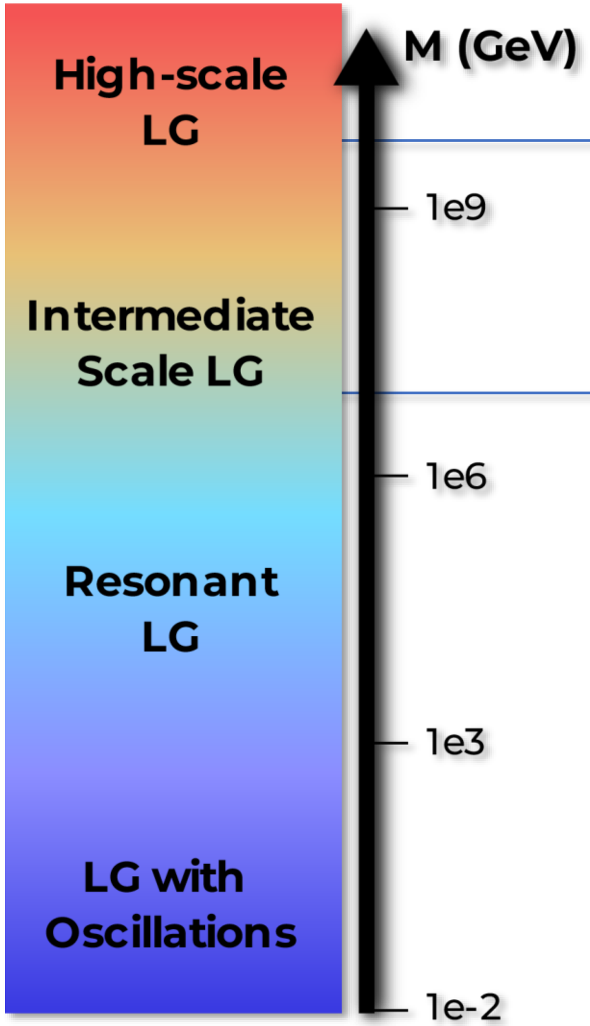
Pilaftsis & Underwood (2003)

Ahmedov, Rubakov & Smirnov (1998)



M1<<M2<<M3
Davidson-Ibarra bound M1>1e9 GeV

Leptogenesis scales



Fukugita & Yanagida (1986) — **High-scale LG**

Racker, Rius & Pena (2012) — **Intermediate Scale LG**

Pilaftsis & Underwood (2003) — **Resonant LG**

Ahmedov, Rubakov & Smirnov (1998) — **LG with Oscillations**

M1<<M2<<M3
Davidson-Ibarra bound M1>1e9 GeV

Flavour effects,
Mild hierarchy: M2<3M1, M3<3M2

Take into account that charged lepton Yukawa interactions activate at different epochs for each flavour.

Leptogenesis scales

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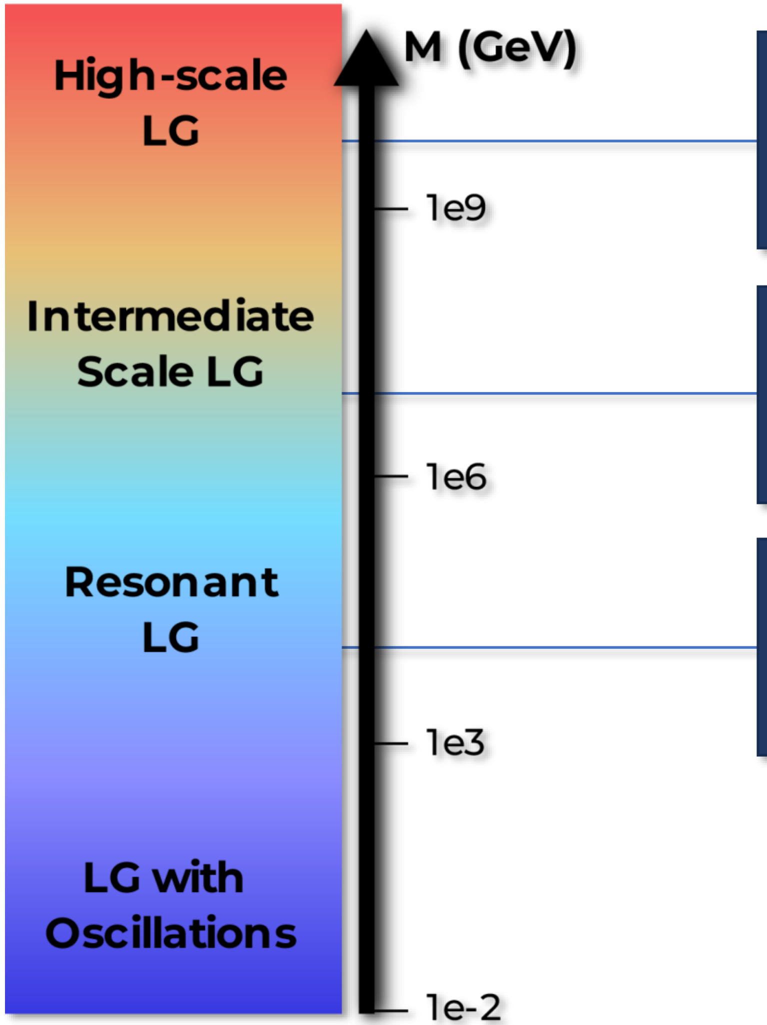


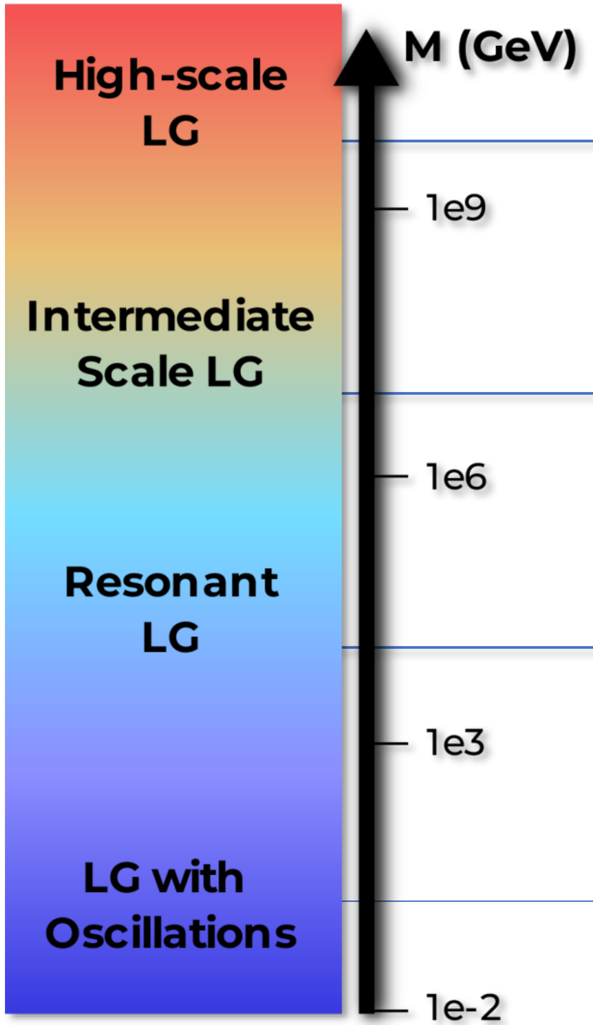
M1<<M2<<M3 Davidson-Ibarra bound M1>1e9 GeV

Flavour effects, Mild hierarchy: M2<3M1, M3<3M2

Quasi-degenerate Ns Non-relativistic regime (T<<M)

Resonance effects in the CP-violation parameter appears when two RHNs are quasi-degenerate

Leptogenesis scales



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Resonant LG

Quasi-degenerate Ns
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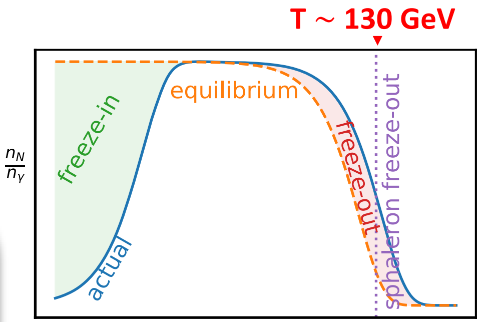
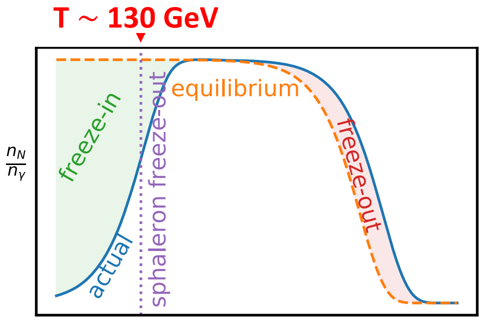
Ahmedov, Rubakov & Smirnov (1998)

LG with Oscillations

Quasi-degenerate Ns
Relativistic (T>>M)
Thermal effects





J. Klarić, M. Shaposhnikov, I. Timiryasov, PRL.127.111802 and PRD.104.055010

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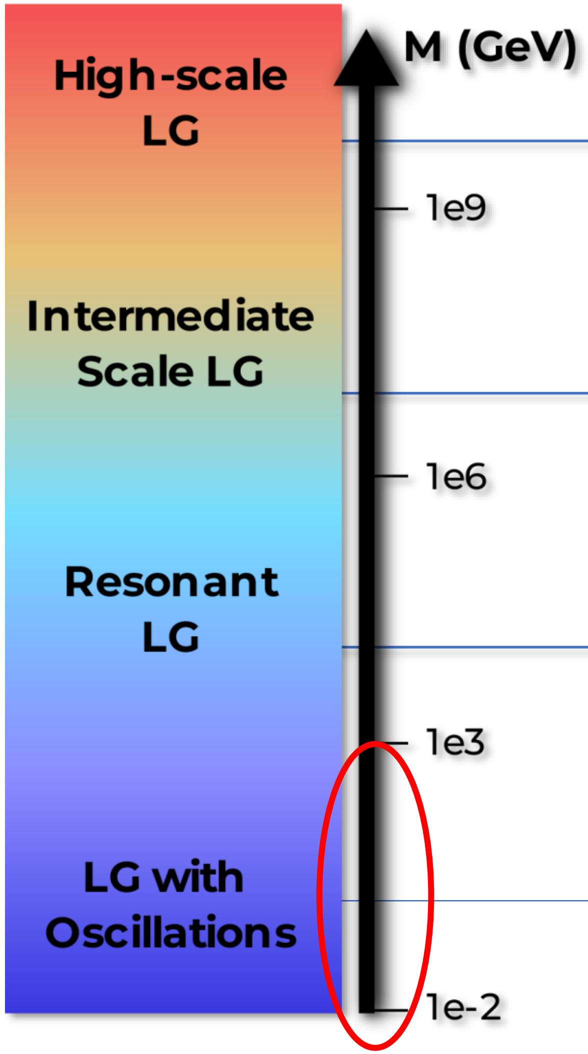


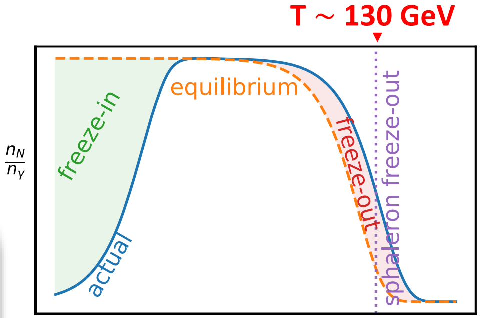
Accessible energies!

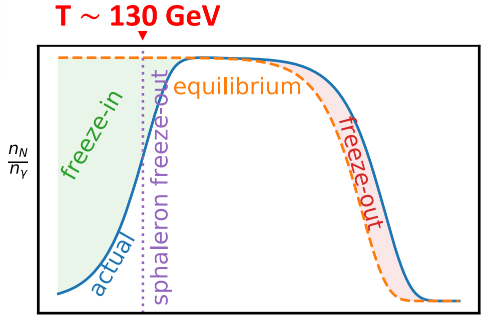
M1<<M2<<M3
Davidson-Ibarra bound M1>1e9 GeV

Flavour effects,
Mild hierarchy:
M2<3M1, M3<3M2

Quasi-degenerate Ns
Non-relativistic regime (T<<M)

Quasi-degenerate Ns
Relativistic (T>>M)
Thermal effects

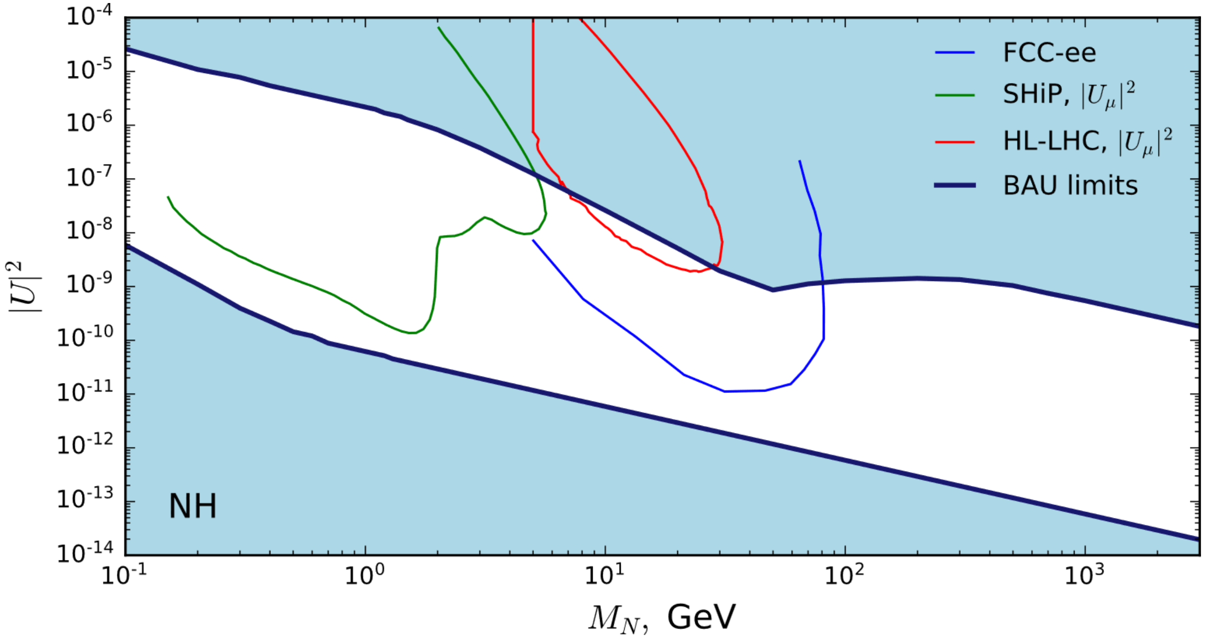


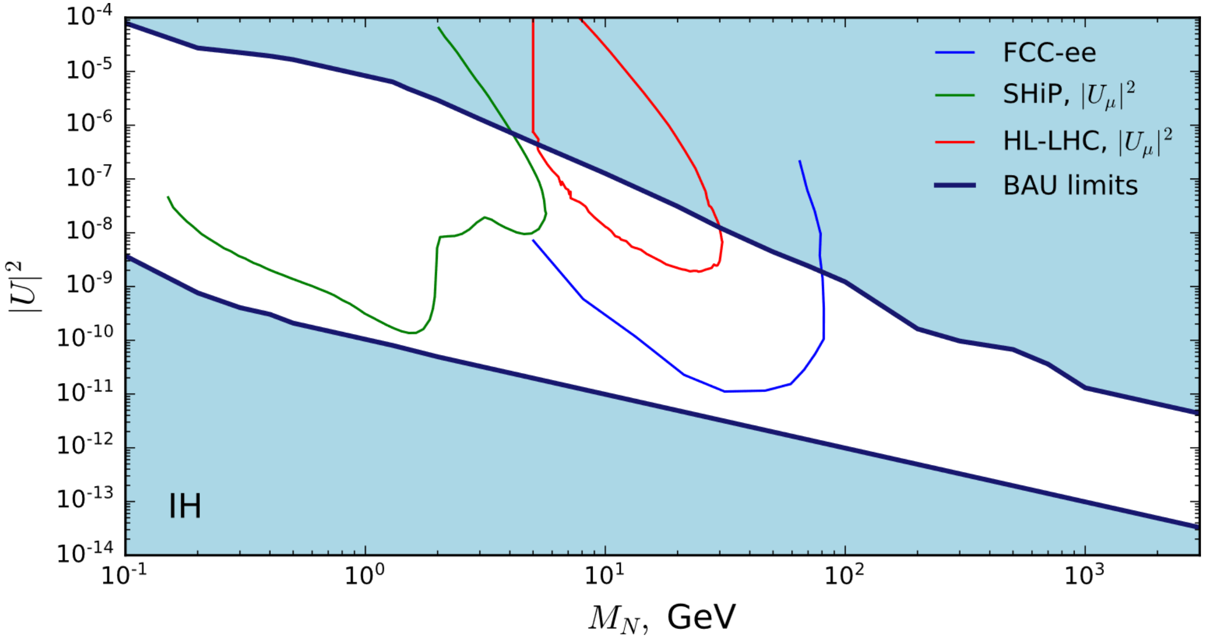


J. Klarić, M. Shaposhnikov, I. Timiryasov, PRL.127.111802 and PRD.104.055010

Parameter Space of low-scale LG with 2RHNs

- The parameter space of low-scale leptogenesis can be tested in the future, including at future collider experiments!





Parameter Space of low-scale LG with 3RHNs

LG with 3 RHNs is even more promising!

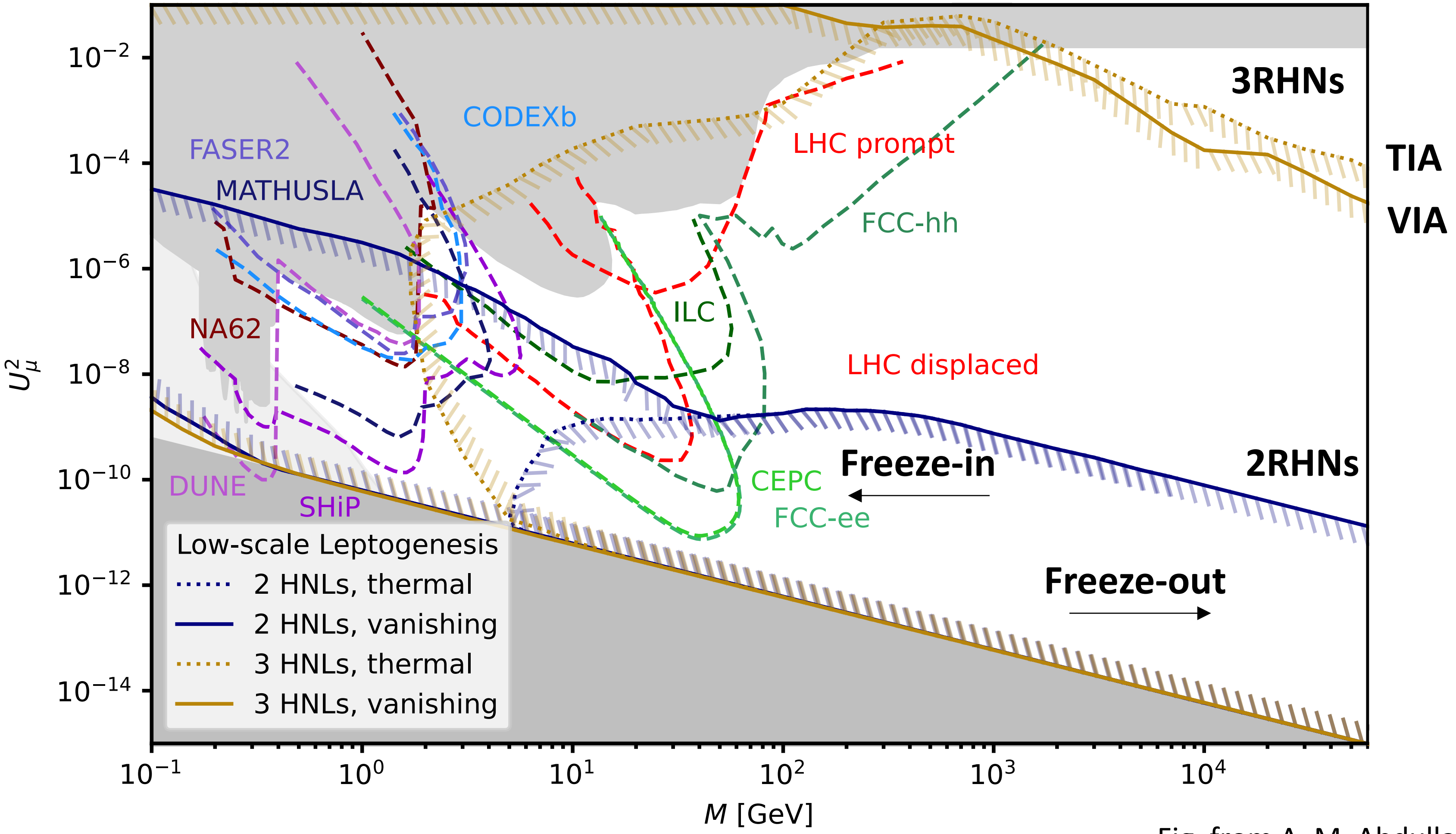


Fig. from A. M. Abdullahi et al., arXiv:2203.08039

Parameter Space of low-scale LG

- Experiments looking at **charged lepton flavour violating processes** involving muons, such as MEG II on the μ→eγ decay, Mu3e on μ→eee decay, Mu2e and COMET (PRISM/PRIME) on μ−e conversion in Al (Ti) will be able to probe the LG parameter space.

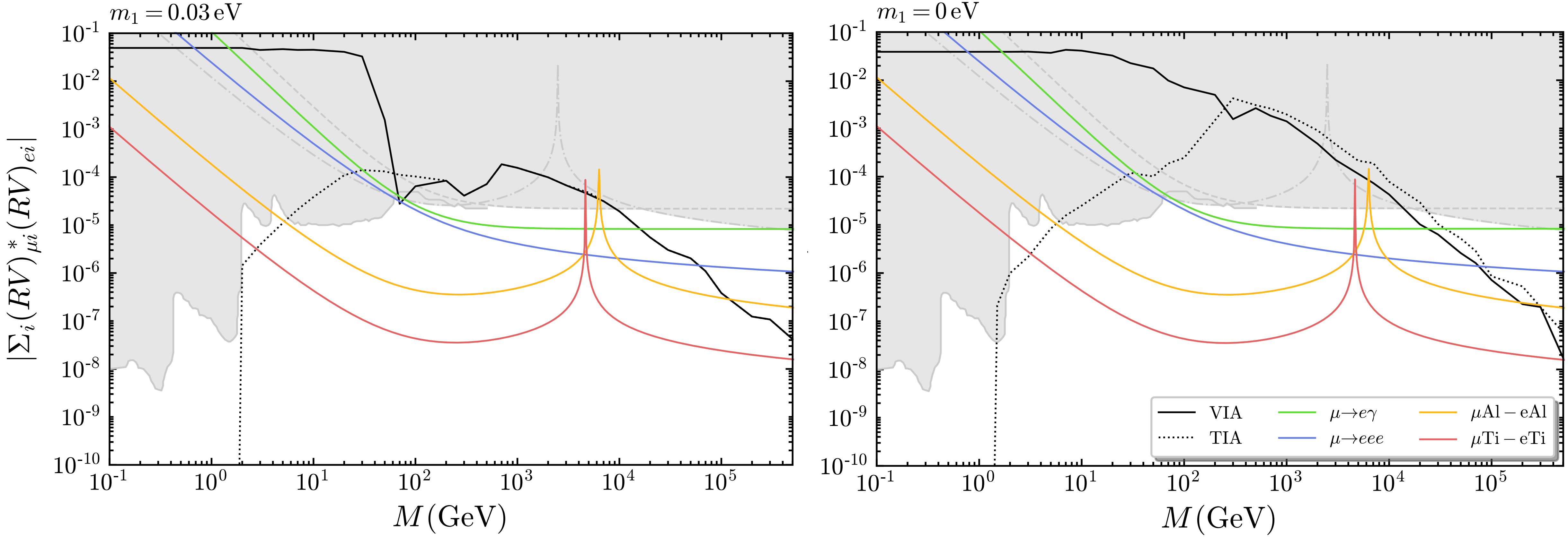


Fig. from A.G., J. Klarić, S. T. Petcov, *Phys. Lett. B, 837 (2023) 137643* [2206.04342]

LG with low-energy CP-violation

CP-violation in the Seesaw model

Casas-Ibarra Parameterisation

$$Y = \pm i(\sqrt{2}/v)U\sqrt{\hat{m}}O^T\sqrt{\hat{M}}$$

CP-violating phases in U and O!

CP-parities, $\pm i$

CP-conservation implies : $Y_{\alpha j} = -iY^*_{\alpha j}\eta^{NCP}_j$ $\Longrightarrow$ $U^*_{\alpha a} = -iU_{\alpha a}\eta^{\upsilon CP}_a$ and $O_{aj} = -iO^*_{aj}\eta^{NCP}_j\eta^{\upsilon CP}_a$

- **CP-conserving Casas-Ibarra:** $O_{aj} = \pm O^*_{aj}$ $\Longrightarrow$
- **Low-energy CP-violation**: Dirac δ, Majorana α_{21}, α_{31}.

Casas-Ibarra angle real or purely imaginary:
Real $\xi = 0, \omega \neq 0$
Imaginary $\omega = 0, \xi \neq 0$

CPV in LG in connection with that on neutrino oscillations and 0νββ-decay

CP-conserving values:
$\boldsymbol{\delta = k\pi}$,
$\boldsymbol{\alpha_{21} = k_2\pi}$,
$\boldsymbol{\alpha_{31} = k_3\pi}$,
$\boldsymbol{k, k_1, k_2 = 0, 1, 2}$ …

S. Pascoli, S. T. Petcov, A. Riotto hep-ph/0611338

Models with generalised CP-symmetry at high-energy:
e.g., P. Chen, G.-J. Ding, S. F. King arXiv:1402.03873

Large couplings are allowed!

$$\text{E.g., } O^{(\mathrm{NH})} = \begin{pmatrix} 0 & \cosh\xi & \pm i\sinh\xi \\ 0 & -i\sinh\xi & \pm\cosh\xi \end{pmatrix}$$

Dirac CP-violation

The **Dirac phase** can be the **unique source of CP-violation** in the neutrino sector.
The Dirac phase alone can be responsible for the generation of the present **matter-antimatter asymmetry.**

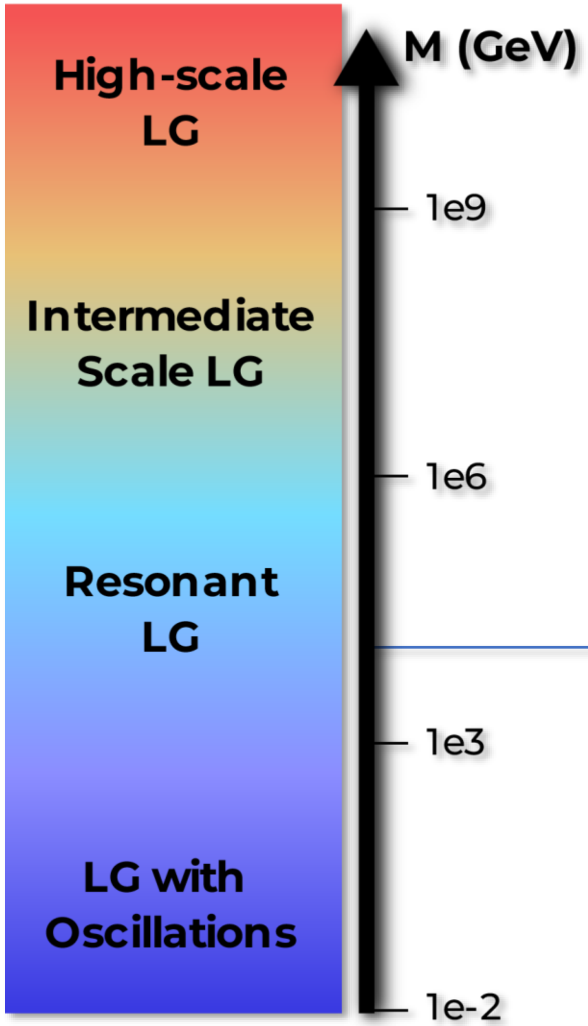
Leptogenesis scales

Fukugita &
Yanagida
(1986)

Racker, Rius &
Pena (2012)

Pilaftsis &
Underwood
(2003)

Ahmedov,
Rubakov &
Smirnov (1998)



High-scale LG with low-energy CP-violation

S. Pascoli, S. T. Petcov, A. Riotto (2007),
S. Blanchet. P. Di Bari (2007),
…,
K. Moffat, S. Pascoli, S. T Petcov, J. Turner (2018),
A.G., K. Moffat, S. T. Petcov (2022)

Remarks

- LG can work down to the 1e6 GeV scale with low-energy CP-violation (1809.08251)
- In the NO case, there is a one-to-one correspondence between the sign of the BAU (> 0) and that of $\sin\delta$ (2107.02079)

Low-scale LG with low-energy CP-violation

2 RHNs: A.G., S. Pascoli, S. T. Petcov (2023),
3 RHNs: A.G., J. Klaric, S. T. Petcov (in preparation)

Viable LG with 2 RHNs and Dirac CPV

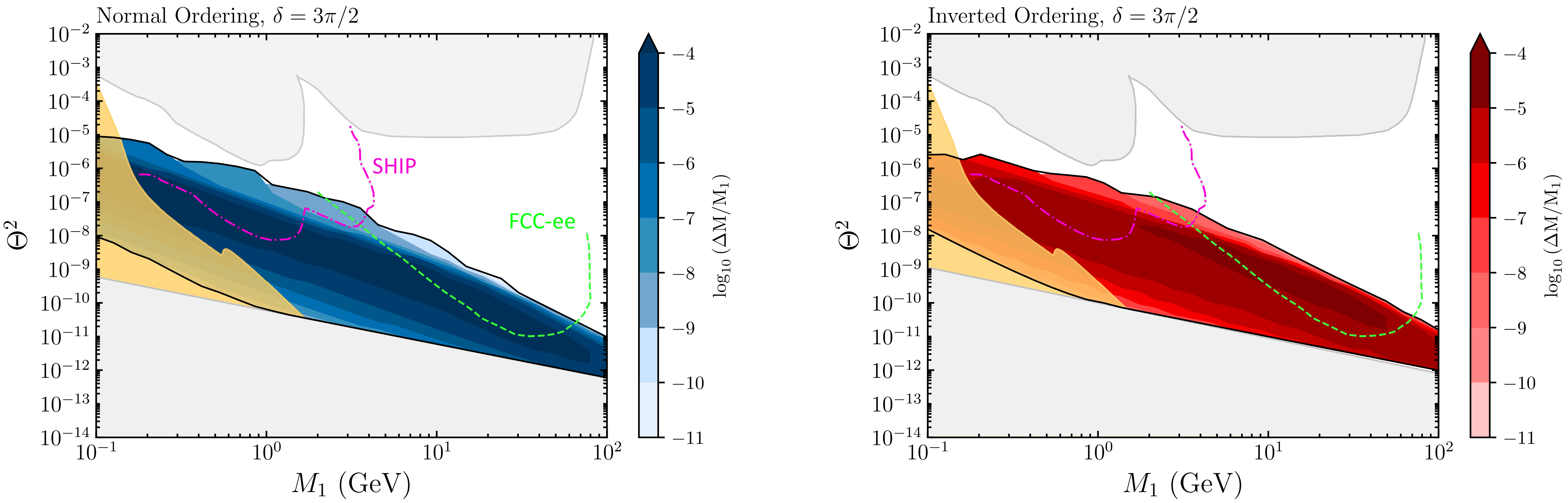
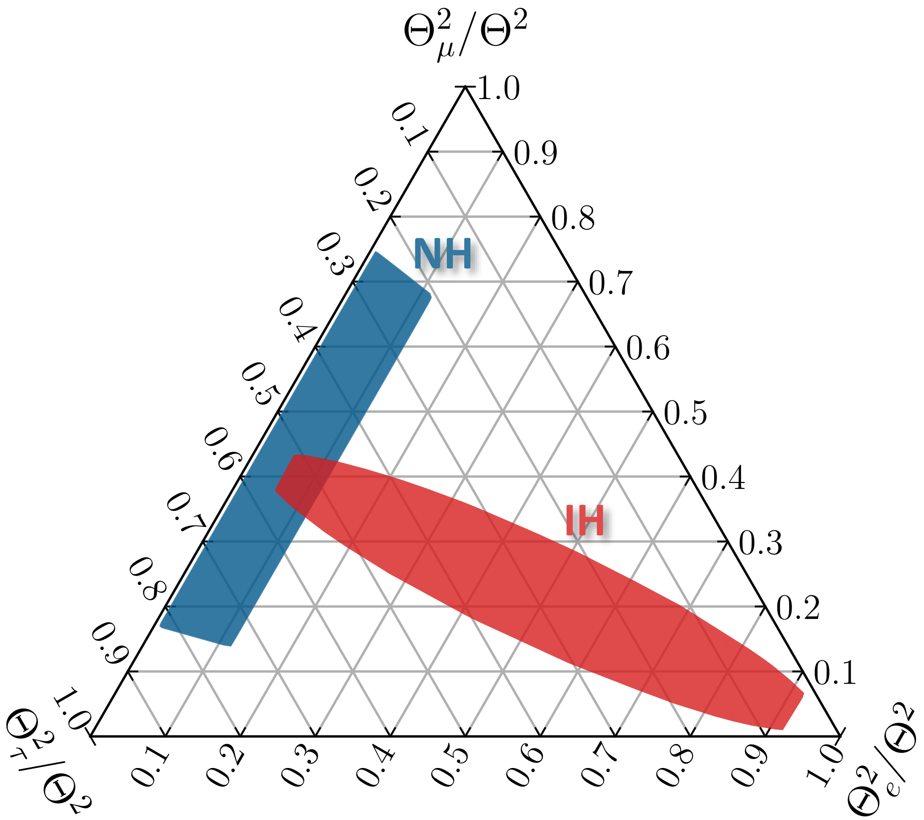


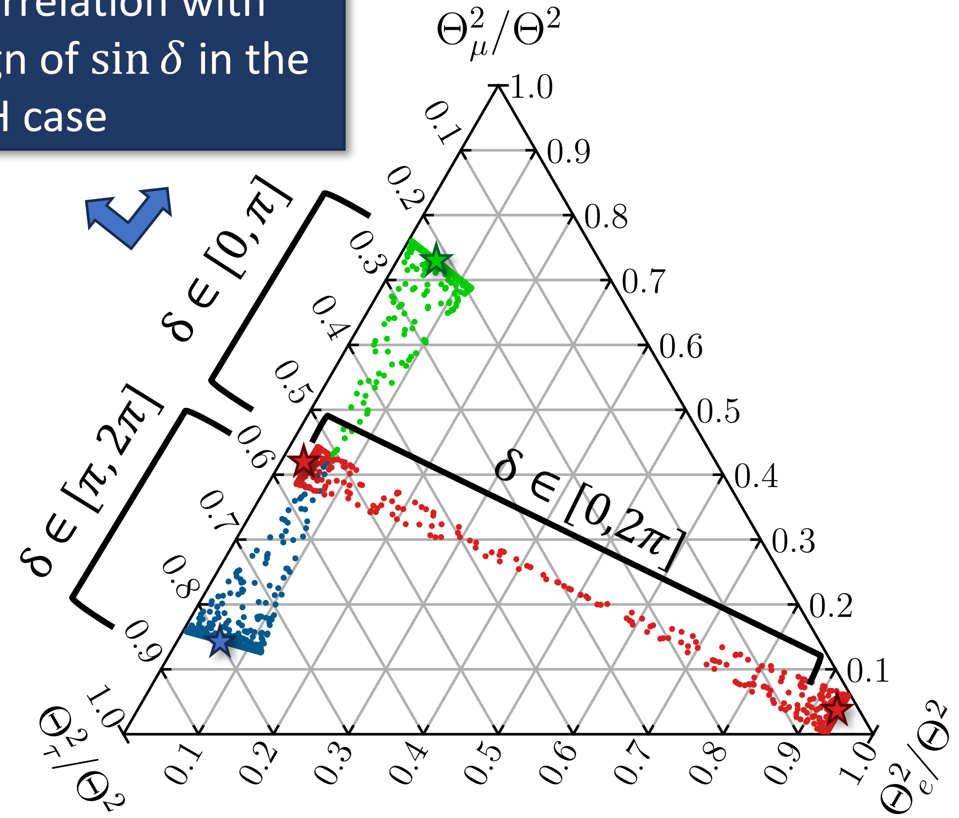
Fig. from A. G., S. Pascoli, S. T. Petcov, *Low-Scale LG with Low-Energy Dirac CPV,* arXiv:2307.07476.

Flavour ratios compatible with viable LG



LG with low- or high-energy CP-violation

Correlation with sign of $\sin\delta$ in the NH case



Low-energy Dirac CP-violation

★ Large mixings $\xi > 1$, Θ^2 in the accessible region

A. G., S. Pascoli, S. T. Petcov arXiv:2307.07476.

Summary and conclusions

Final remarks and take-home messages:

- The parameter space of **low-scale LG via oscillations** with **two/three quasi degenerate heavy Majorana neutrinos** can be **probed by future collider searches,** e.g. at **SHiP, HL-LHC and FCC,** looking for heavy neutral leptons in the mass range **[100 MeV, 100 GeV]**.

- Experiments on **charged lepton flavour violation processes** involving muons can **probe** the parameter space of **low-scale LG via oscillations** with **three quasi degenerate heavy Majorana neutrinos**.

- The **Dirac CP-violating phase** can **alone** provide the requisite **CP-violation** necessary **for successful LG. a future measurements of the CP-violation in neutrino oscillations would be in favour of low-scale LG.**

- The **complementarity** between different experiments is the **key** to reveal hints of leptogenesis ad low-scales!

Thanks for your attention!

Back-up

Density Matrix Equations for LG via oscillations

$$
\begin{aligned}
Hx\frac{dr_N}{dx} = & -i\left[\langle\mathcal{H}\rangle, r_N\right] - Hx\frac{r_N}{N_N^{\rm eq}}\frac{dN_N^{\rm eq}}{dx} - \frac{\langle\gamma_N^{(0)}\rangle}{2}\left\{Y^\dagger Y, r_N - 1\right\} + \langle\gamma_N^{(1)}\rangle Y^\dagger \mu Y - \frac{\langle\gamma_N^{(2)}\rangle}{2}\left\{Y^\dagger \mu Y, r_N\right\} + \\
& - \frac{\langle S_N^{(0)}\rangle}{2T^2}\left\{MY^{\rm T}Y^* M, r_N - 1\right\} - \frac{\langle S_N^{(1)}\rangle}{T^2} MY^{\rm T}\mu Y^* M + \frac{\langle S_N^{(2)}\rangle}{2T^2}\left\{MY^{\rm T}\mu Y^* M, r_N\right\}, \\
\kappa Hx\frac{d\mu_{\Delta_\alpha}}{dx} = & -\frac{\langle\gamma_N^{(0)}\rangle}{2}\left(Y r_N Y^\dagger - Y^* r_{\overline{N}} Y^{\rm T}\right)_{\alpha\alpha} + \langle\gamma_N^{(1)}\rangle\left(YY^\dagger\right)_{\alpha\alpha}\mu_\alpha - \frac{\langle\gamma_N^{(2)}\rangle}{2}\left(Y r_N Y^\dagger + Y^* r_{\overline{N}} Y^{\rm T}\right)_{\alpha\alpha}\mu_\alpha + \\
& + \frac{\langle S_N^{(0)}\rangle}{2T^2}\left(Y^* M r_N M Y^{\rm T} - Y M r_{\overline{N}} M Y^\dagger\right)_{\alpha\alpha} + \frac{\langle S_N^{(1)}\rangle}{T^2}\left(Y M^2 Y^\dagger\right)_{\alpha\alpha}\mu_\alpha + \\
& - \frac{\langle S_N^{(2)}\rangle}{2T^2}\left(Y M r_{\overline{N}} M Y^\dagger + Y^* M r_N M Y^{\rm T}\right)_{\alpha\alpha}\mu_\alpha\,, \\
Hx\frac{dr_{\overline{N}}}{dx} = & \; r_N \to r_{\overline{N}},\ \mu \to -\mu,\ Y \to Y^*
\end{aligned}
$$

Computationally very demanding!

Thermal averaged rates

J. Ghiglieri, M. Laine arXiv:1703.06087 and 1711.08469
http://www.laine.itp.unibe.ch/leptogenesis/

Freely available codes!

`Python`: A. G., C. Leslie, Y. F. Perez-Gonzalez, H. Schulz, B. Shuve, J. Turner, R. Walker, `ULYSSESv2`, arXiv:2301.05722
`C++`: P. Hernández, J. López-Pávon, N. Rius and S. Sandner, `amiqs`, arXiv:2207.01651