ENERGY-ENERGY CORRELATION IN BACK-TO-BACK REGION AT $N^3LL + NNLO$ in QCD†

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ciao papo!

PLAN OF THE SEMINAR:

1. DEFINITION OF ENERGY-ENERGY CORRELATION (E.E.C.) AND BASIC PROPERTIES;

2. RESUMMATION OF EEC IN THE BACK-TO-BACK (2 JET) REGION;

3. COMPARISON WITH EXPE=RIMENTAL DATA FROM LEP1 + SLD AND INCLUSION OF NON-PERTURBATIVE EFFECTS;

4. CONCLUSIONS AND OUTLOOK.

1. DEFINITION OF EEC IN $e^+e^- \mapsto$ HADRONS

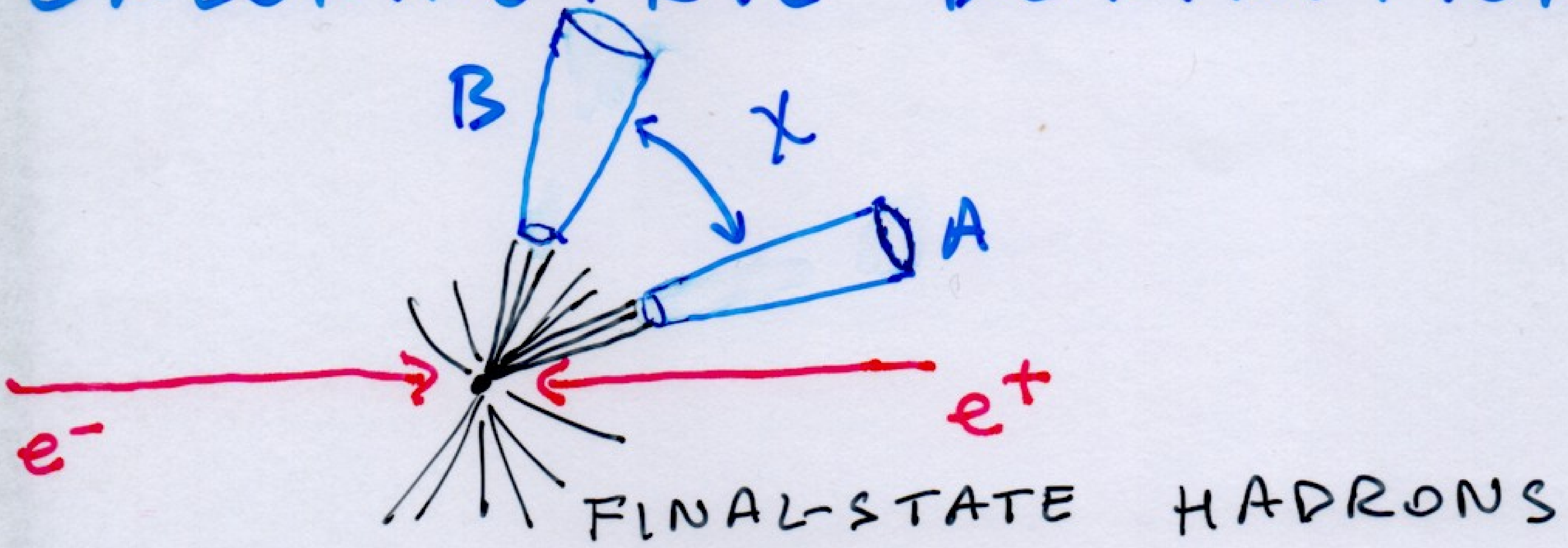
(BASHAM et al., 1978)

$$\frac{d\Sigma^{EEC}}{d\cos\chi} \equiv \sum_{N=2}^{\infty} \sum_{i,j}^{1,N} \frac{E_i E_j}{Q^2} \delta(\cos\vartheta_{ij} - \cos\chi) \times$$
$$\times\, d\sigma_N(e^+e^- \mapsto h_i h_j + X),$$

$$Q = \sum_{i=1}^{N} E_i = \text{HARD SCALE}$$

ϑ_{ij} = ANGLE BETWEEN $\vec{p}_i$ and $\vec{p}_j$

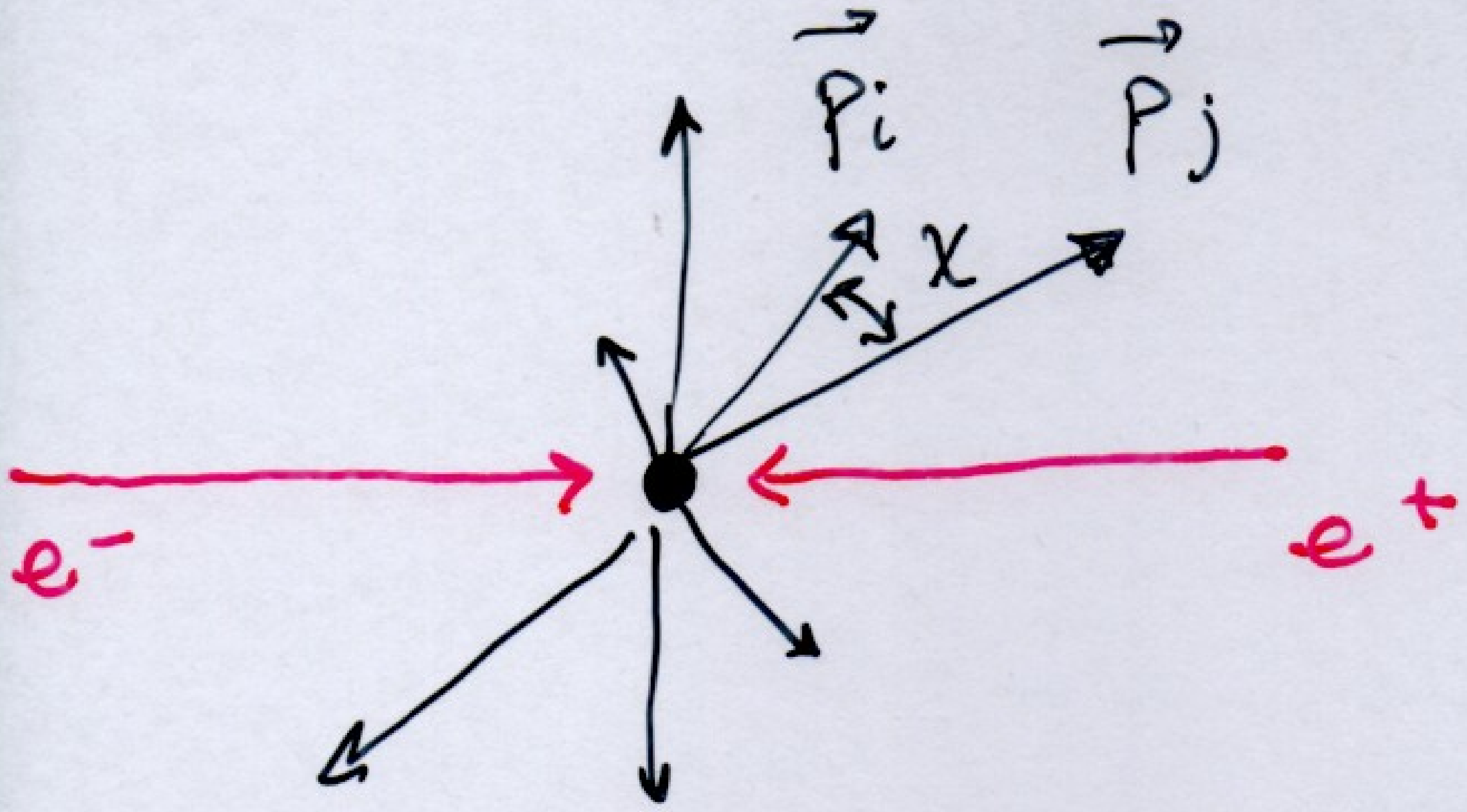
"CALORIMETRIC" DEFINITION:



χ = ANGULAR SEPARATION OF ENERGY DETECTORS A and B,

A, B = CALORIMETERS OF SMALL SOLID ANGLE.

(ii) "TRACK" DEFINITION:



1) CONSIDER A PAIR i, j OF HADRONS (PARTONS) IN A EVENT WITH ANGULAR SEPARATION χ AND GIVE TO THEIR PRODUCTION CROSS SECTION WEIGHT

$$\frac{E_i E_j}{Q^2} ;$$

2) SUM OVER ALL PAIRS OF HADRONS IN THE EVENT.

IDEA: STUDY ANGULAR CORRELATIONS OF HARD PARTICLE PAIRS.

EVALUATION OF EEC IN PERTURBATIVE QCD (pQCD)?

* EEC IS INFRARED (SOFT AND COLLINER) SAFE!

- A SOFT PARTICLE (GLUON), $E_i \mapsto 0$, DOES NOT CONTRIBUTE TO EEC BECAUSE OF ENERGY WEIGHT;

- A COLLINEAR SPLITTING,

$$E_i \mapsto z E_i + (1-z) E_i , \quad 0 < z < 1,$$

DOES NOT CHANGE THE EEC,

$$\frac{z E_i E_j}{Q^2} + \frac{(1-z) E_i E_j}{Q^2} = \frac{E_i E_j}{Q^2}$$

(ANGLES UNCHANGED, BY DEFINITION, IN COLLINEAR SPLITTING)

$g \mapsto gg$

COMMON UNITARY VARIABLE:

$$y \equiv \frac{1 + \cos \chi}{2}, \quad 0 \leq y \leq 1,$$

- $y \to 0^+ \iff \chi \to \pi^-$, BACK-TO-BACK OR 2-JET REGION;
- $y \to 1^- \iff \chi \to 0^+$, FORWARD (SMALL-ANGLE) REGION.

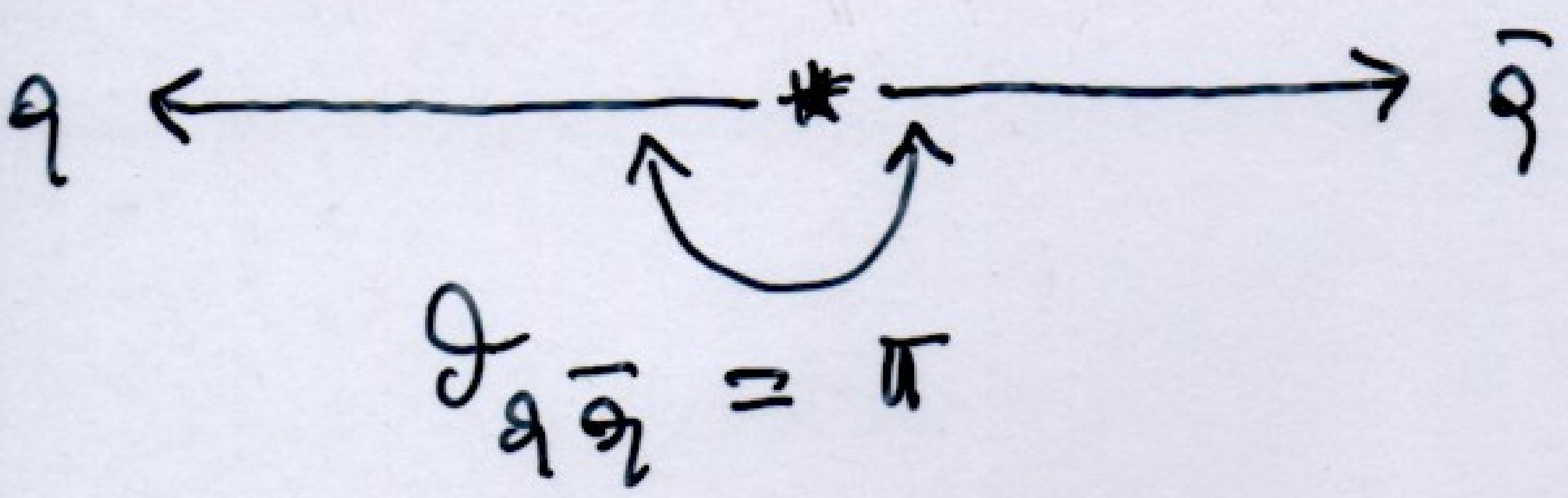
OTHER COMMON VARIABLE:

$$z \equiv 1 - y = \frac{1 - \cos \chi}{2}$$

(ENDPOINT REGIONS EXCHANGED)

i) EEC AT LOWEST ORDER???

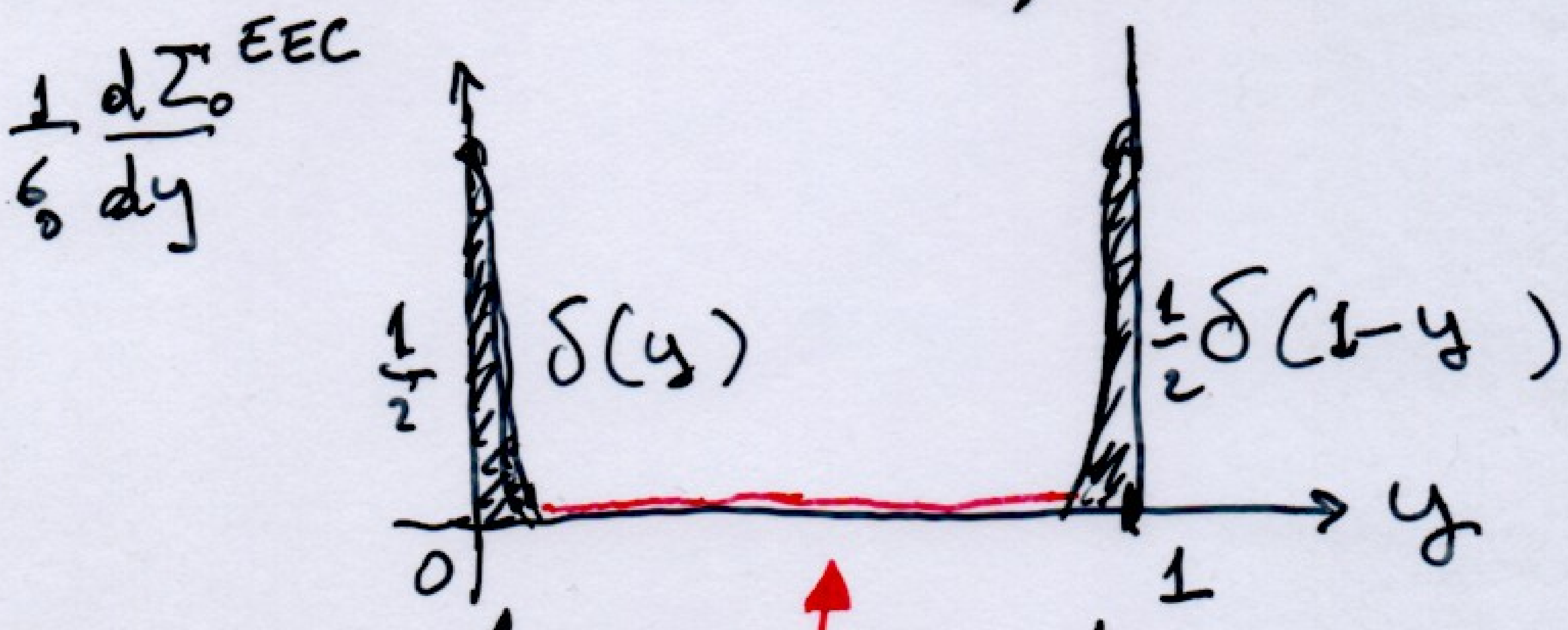
→ FINAL STATE:



(C.O.M. FRAME)

$$\frac{1}{\sigma_{tot}} \cdot \frac{d\Sigma^{EEC}}{dy} = \frac{1}{2}\,\delta(y) + \frac{1}{2}\,\delta(1-y) + O(\alpha_s)$$

2 PEAKS OF THE SAME STRENGHT AT THE ENDPOINTS,

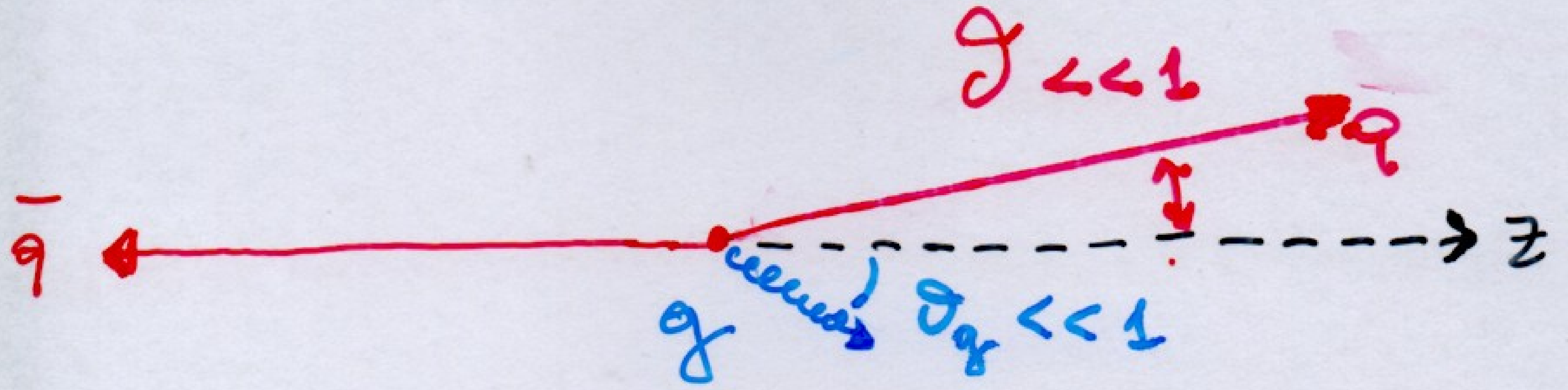


FROM BACK-TO-BACK $q\bar{q}$ PAIR

FROM SELF-CORRELATIONS

NO EVENTS AT ALL IN THE BULK $(0<y<1)$

ii) BACK-TO-BACK REGION TO $\mathcal{O}(\alpha_s)$?



SOFT, SMALL-ANGLE GLUON, EMITTED WITH LARGE PROBABILITY,

$$\sim \alpha_s \int \frac{dE_g}{E_g} \int \frac{d\vartheta_g}{\vartheta_g}$$

$\Downarrow$

A SMALL MOMENTUM UNBALANCE IS PRODUCED BECAUSE OF SOFT-GLUON EMISSION: QUARK AND ANTIQUARK ARE NO MORE EXACTLY BACK-TO-BACK.

$$\Rightarrow \quad \alpha_s \frac{\log(y)}{y} \quad \text{TERM}$$

$$\xrightarrow{+\text{ VIRTUAL}} \quad \alpha_s \left[\frac{\log(y)}{y}\right]_+ \quad \left[\begin{array}{l} \frac{\alpha_s}{2}\log^2(y) \\ \text{IN PARTIAL} \\ \text{INTEGR. RATE} \end{array}\right]$$

VERY LARGE FOR $0 < y \ll 1$, i.e. IN BACK-TO-BACK REGION

IN HIGHER ORDERS ?

$$\alpha_s^m \left[\frac{\log^k(y)}{y} \right]_+ ,$$

$$m = 1, 2, 3, \ldots \quad ; \quad 0 \le k \le 2m-1 .$$

$$[\ \alpha_s = \alpha_s(Q)\ ]$$

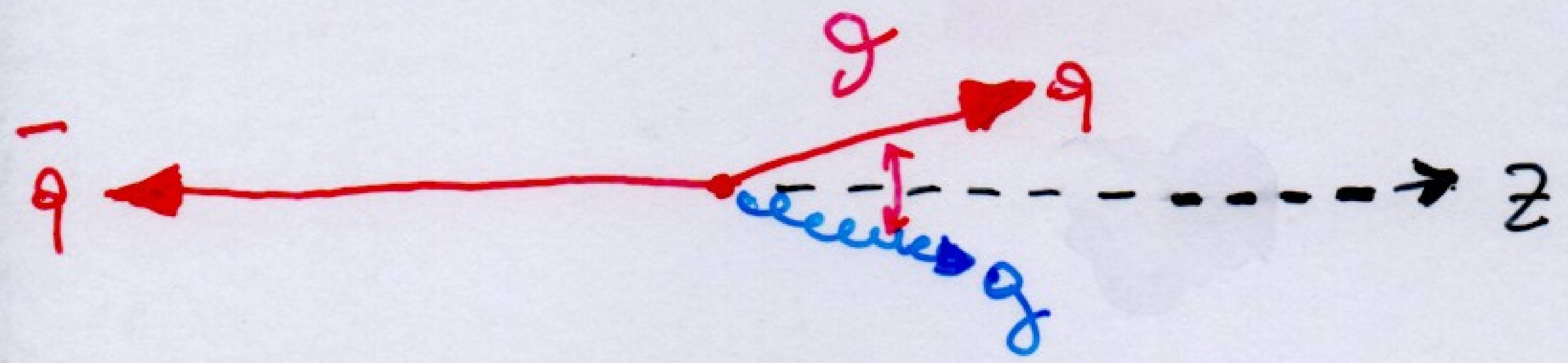
THESE TERMS NEED TO BE RESUMMED TO <u>ALL ORDERS</u> OF PERTURBATION THEORY TO HAVE A SENSIBLE RESULT !

USUAL, FIXED-ORDER PERTURBATION THEORY, FOR EXAMPLE NNLO,

$$\frac{1}{\sigma} \frac{d\Sigma}{dy} = \frac{1}{2} \left[\delta(y) + \delta(1-y) \right] + \frac{\alpha_s}{\pi} A(y) + \left(\frac{\alpha_s}{\pi} \right)^2 B(y) + \left(\frac{\alpha_s}{\pi} \right)^3 C(y) ,$$

<u>NOT GOOD</u> FOR BACK-TO-BACK REGION.

FORWARD REGION ALSO SINGULAR IN pQCD!



TO GIVE A SIZABLE CONTRIBUTION $\left(\frac{E_q E_g}{Q^2} \lesssim 1\right)$, THE QUARK AND THE GLUON HAVE TO BE BOTH HARD

$\Rightarrow$ COLLINEAR, $\int \frac{d\vartheta}{\vartheta}$, ENHANCEMENT, BUT NOT SOFT ENHANCEMENT, IN FORWARD REGION

$\Rightarrow$ NO SUDAKOV DOUBLE LOGS HERE.

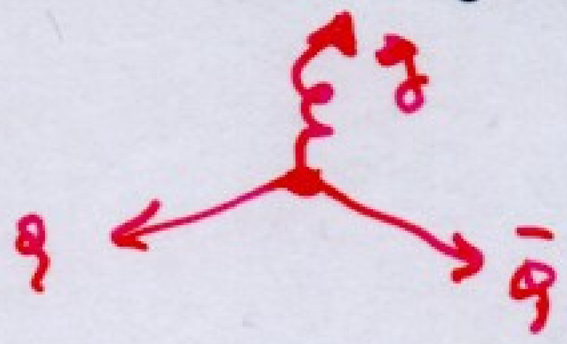
* AT MOST, ONLY ONE INFRARED LOGARITHM (OF HARD COLLINEAR NATURE) FOR EACH POWER OF α_S.

* PART 2

FIXED-ORDER EXPANSION, STATUS OF CALCULATIONS:

1) FUNCTION $\mathcal{A}(z)$, $O(\alpha_s)$, **LO**:

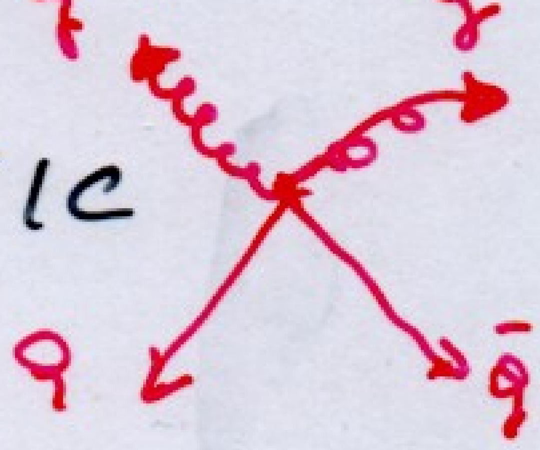
BASHAM, BROWN, ELLIS AND LOVE (1978) (AT MOST 3 HARD JETS);



2) $\mathcal{B}(z)$, $O(\alpha_s^2)$, **NLO**:

i) RICHARDS, STIRLING AND ELLIS (1982), NUMERICAL;

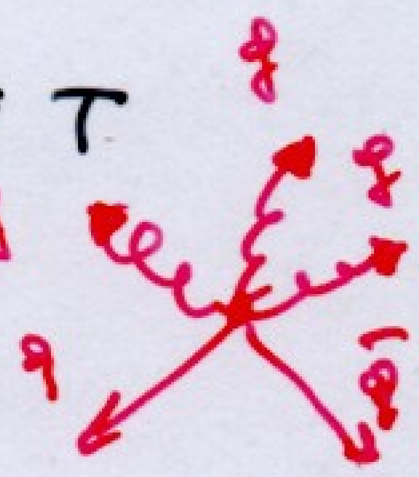
ii) DIXON ET AL. (2018), ANALYTIC (AT MOST 4 JETS);



3) $\mathcal{C}(z)$, $O(\alpha_s^3)$, **NNLO**:

a) DEL DUCA ET AL. (2016), NUMERICAL COMPUTATION OF THE ENTIRE SPECTRUM IN (FULL) QCD;

b) EBERT, MISTLBERGER AND VITA (2021), ANALYTIC COMPUTATION IN SCET IN THE BACK-TO-BACK REGION. (AT MOST 5 JETS).



[SOME CROSS-CHECKS MADE]

RESUMMATION IN BACK-TO-BACK REGION?

DIFFERENTIAL CROSS SECTION NATURALLY WRITTEN:

$$\frac{1}{\sigma}\frac{d\Sigma}{dy} = \frac{1}{\sigma}\frac{d\Sigma_{(RES)}}{dy} + \frac{1}{\sigma}\frac{d\Sigma_{(FIN)}}{dy},$$

WHERE:

$$\frac{1}{\sigma}\frac{d\Sigma_{(RES)}}{dy} = \frac{1}{2} H(\alpha_s) \int_0^\infty d\hat{b}\, \frac{\hat{b}}{2} J_0(\sqrt{y}\,\hat{b}) \times$$

$$\equiv S(Q,b) \left\{ \times e^{-\int_{b_0^2/b^2}^{\mu_Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q^2))\ln\left(\frac{Q^2}{q^2}\right) + B(\alpha_s(q^2))\right]} \right.,$$

WITH*: $\hat{b} \equiv bQ$; b = IMPACT PARAMETER;

$$A(\alpha_s) = \sum_{n=1}^{\infty} A_n \left(\frac{\alpha_s}{\pi}\right)^n = A_1 \frac{\alpha_s}{\pi} + A_2\left(\frac{\alpha_s}{\pi}\right)^2 + \dots$$

DOUBLE-LOG FUNCTION, RELATED TO THE EMISSION OF SOFT <u>AND</u> COLLINEAR GLUONS;

$$B(\alpha_s) = \sum_{n=1}^{\infty} B_n \left(\frac{\alpha_s}{\pi}\right)^n = B_1 \frac{\alpha_s}{\pi} + \dots$$

..... SOFT <u>OR</u> COLLINEAR GLUONS.

AFTER INTEGRATION OVER GLUON TRANSVERSE MOMENTUM SQUARED, q^2, THE SUDAKOV FORM FACTOR IS ORGANIZED AS A FUNCTION SERIES:

$$S(Q,b) = e^{L g_1(\lambda) + \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n g_{n+1}(\lambda)},$$

$L \equiv \log\left(\frac{\mu_Q^2 b^2}{b_0^2}\right)$, $\mu_Q \sim Q$, RESUMMATION SCALE

$\lambda \equiv \beta_0 \frac{\alpha_S(\mu_R)}{\pi} L$, $\left[b_0 = 2e^{-\gamma_E}, \text{ CONSTANT} \simeq 1.123 \right]$

LL: $g_1(\lambda) = \frac{A_1}{\beta_0} \frac{\lambda + \ln(1-\lambda)}{\lambda}$;

NLL: $+\, g_2(\lambda)$;

N^2LL: $+\, g_3(\lambda)$;

N^3LL: $+\, g_4(\lambda)$.

* NOTE THE SINGULARITY AT $\lambda = 1$, PRODUCED BY THE LANDAU POLE, BECOMING STRONGER IN HIGHER ORDERS. $\left[{}^{*}\log\left(\frac{Q^2}{q^2}\right) = \int_{q^2}^{Q^2} \frac{dE^2}{E^2} = \text{SOFT LOG.} \right]$

*ESSENTIAL THEORETICAL INGREDIENT TO OBTAIN FACTORIZATION OF SINGLE GLUON EMISSION AND (GENERALIZED) EXPONENTIATION:

TRASFORMATION TO IMPACT-PARAMETER (b-) SPACE:

$$\delta^{(2)}\Big(\underbrace{\vec{p}_\perp^{\,q}+\vec{p}_\perp^{\,\bar q}+\sum_{i=1}^{n}\vec{k}_\perp^{\,i}}\Big)=\int\frac{d^2b}{(2\pi)^2}\,e^{i\vec{b}\cdot\left(\vec{p}_\perp^{\,q}+\vec{p}_\perp^{\,\bar q}+\sum_{i=1}^{n}\vec{k}_\perp^{\,i}\right)}$$

SOFT-GLUON TRANSVERSE MOMENTA

PARISI AND PETRONZIO, 1981
CURCI AND GRECO, 1981

$$=\int\frac{d^2b}{(2\pi)^2}\,e^{i\vec{b}\cdot\vec{p}_\perp^{\,q}}\;e^{i\vec{b}\cdot\vec{p}_\perp^{\,\bar q}}\;\prod_{i=1}^{n}e^{i\vec{b}\cdot\vec{k}_\perp^{\,i}}$$

A FACTOR FOR EACH PARTON, TO BE ASSOCIATED TO ITS MATRIX ELEMENT.

NEXT:

$$H(\alpha_s) = 1 + \sum_{n=1}^{\infty} H_n \left(\frac{\alpha_s}{\pi}\right)^n = 1 + H_1 \frac{\alpha_s}{\pi} + \dots$$

SHORT-DISTANCE, PROCESS-DEPENDENT FACTOR (COEFFICIENT FUNCTION).

* GENERAL SCHEME FOR SHAPE-VARIABLE RESUMMATION:

CATANI, TRENTADUE, TURNOCK AND WEBBER (1993)

LOGARITHMIC EXPANSION:

i) LEADING-LOGARITHMIC (LL) APPROXIMATION:

A_1 (B_1) COLLINS AND SOPER, 1981

ii) NEXT-TO-LEADING LOGARITHMIC (NLL) APPROXIMATION:

$\underline{\underline{A_2}}, B_1, H_1$ KODAIRA AND TRENTADUE, 1981

iii) NNLL: A_3, B_2, H_2

DE FLORIAN AND GRAZZINI, 2001

- MOCH, VERMASEREN AND VOGT, 2004 $A_3^{(THR)}$
- BECKER AND NEUBERT, 2011 $\delta A_3 + A_3^{(THR)} \mapsto A_3$
- TULIPANT, KARDOS AND SOMOGYI, 2017 $N^2LL + N^2LO$
- U.G.A. AND G. FERRERA, 2024, H_2

iv) N^3LL : A_4, B_3, H_3

- VON MANTEUFFEL, PANZER, SCHABINGER, 2020 AND
- LI AND ZHU, 2017
- U.G.A. AND G. FERRERA, 2024

*IMPORTANT REMARK:

IN SCET, RESUMMATION TO N^3LL BY MOULT AND ZHU, 2018 AND EBERT, MISTLBERGER AND VITA, 2021 AND BEYOND BY DUHR, MISTLBERGER AND VITA, 2022: COMPATIBLE WITH QCD RESUMMATION.

* FINALLY (NOT TO FORGET!)

$$\frac{1}{6}\frac{d\Sigma_{(FIN)}}{dy}$$ = REMAINDER FUNCTION, PROCESS-DEPENDENT, "SMALL" IN THE BACK-TO-BACK REGION, BUT IMPORTANT IN THE "BULK" OF THE SPECTRUM.

THE REMAINDER FUNCTION HAS AN ORDINARY (TRUNCATED) PERTURBATIVE EXPANSION:

$$\frac{1}{6}\frac{d\Sigma_{(FIN)}}{dy} = \sum_{m=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^m \mathrm{Rem}^{(m)}(y)$$

THE $m=1$ (LO) AND $m=2$ (NLO) FUNCTIONS ARE KNOWN ANALITICALLY.

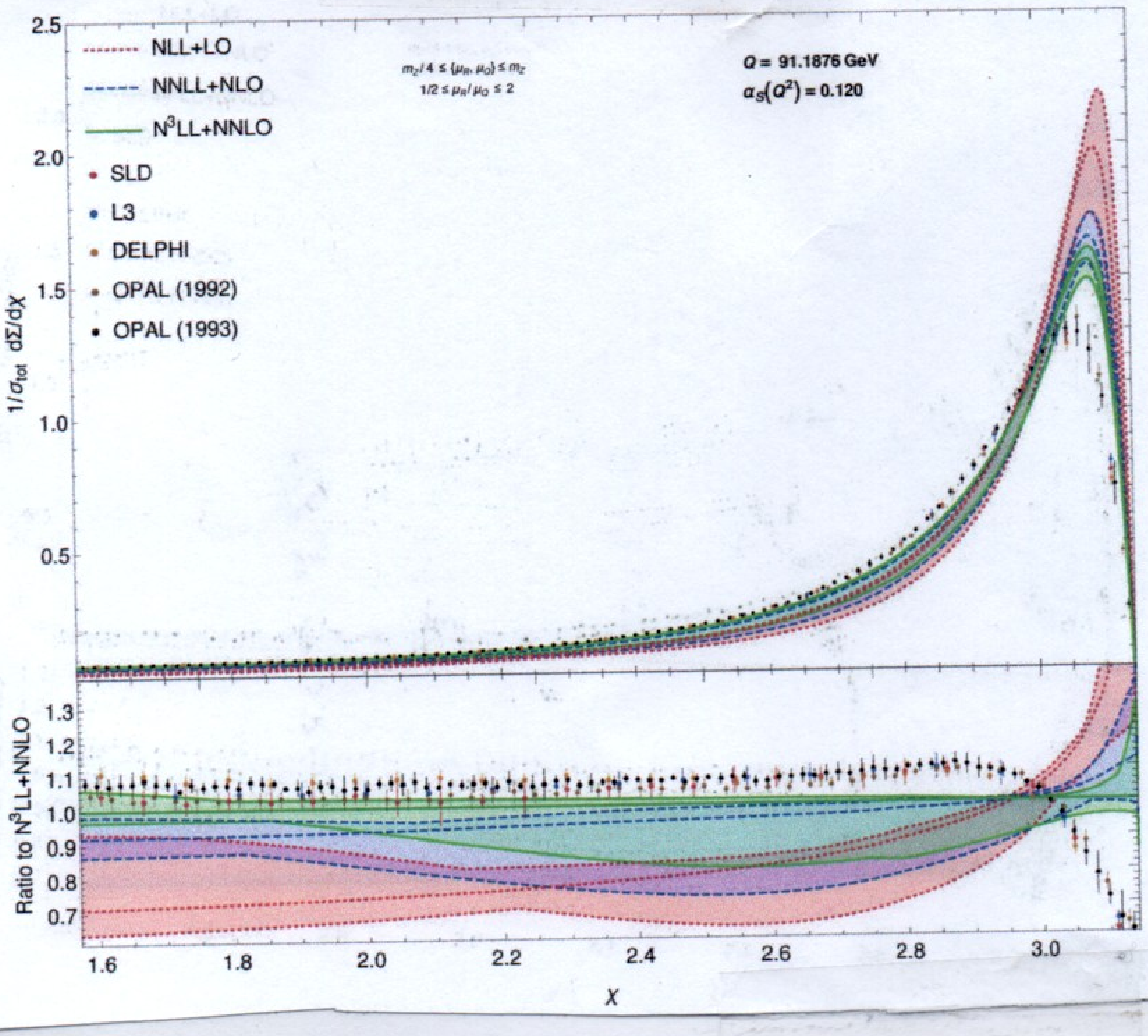
FOR $m=3$ (NNLO) THE NUMERICAL VALUES OF THE FUNCTION CAN BY FITTED BY:

$$\mathrm{Rem}^{(3)}(y) = \underset{\uparrow}{a_5} \ln^5 y + \underset{\uparrow}{a_4} \ln^4 y + \ldots + \underset{\uparrow}{a_1} \ln y + \underset{\uparrow}{a_0}$$

PARAMETERS TO FIT, U.G.A. AND G. FERRERA, 2024

PART 3:

COMPARISON WITH LEP1 DATA:

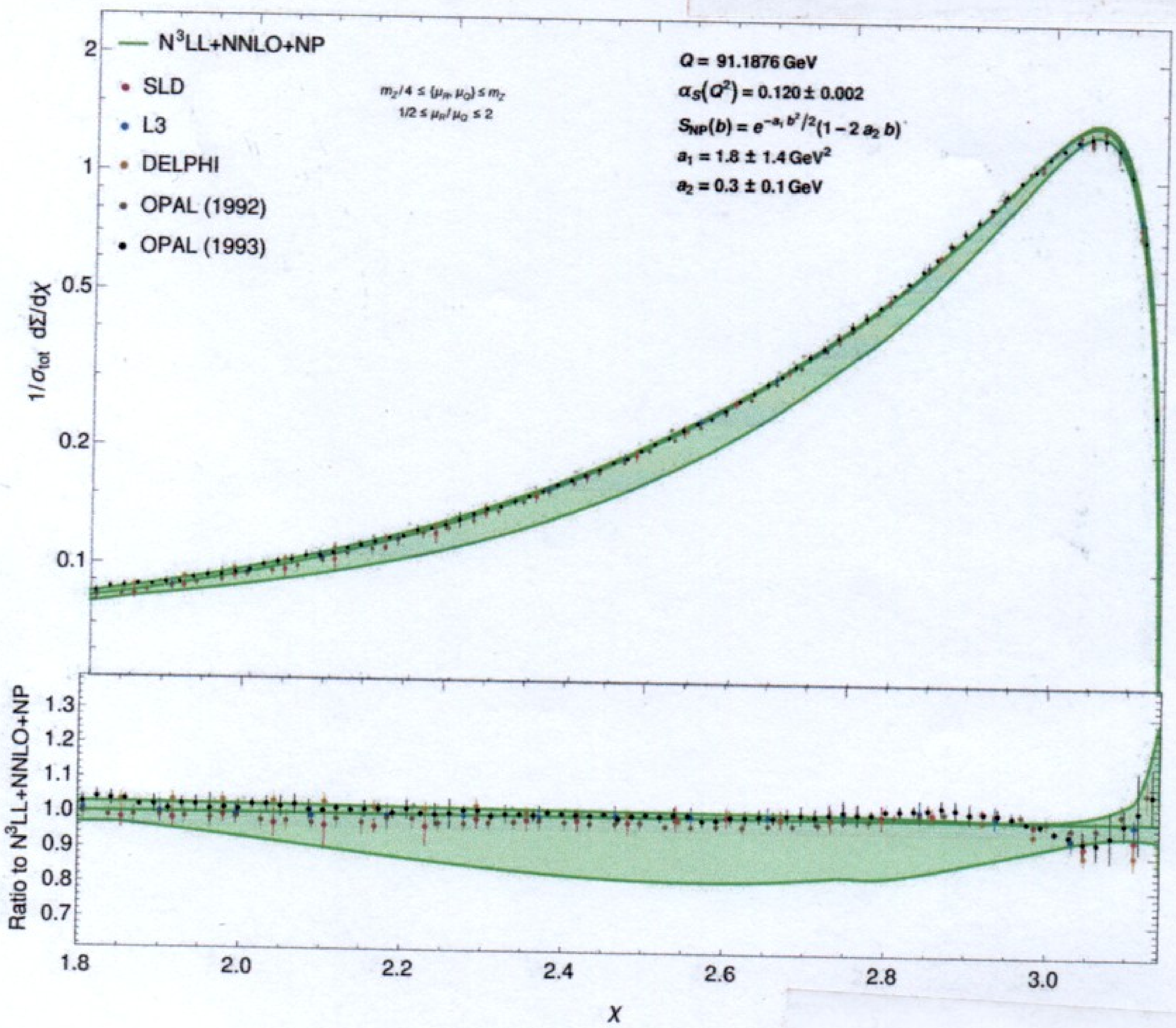


IN x SCALE, CORRELATION ANGLE χ.

BY INCRESING (PERTURBATIVE) ACCURACY, AGREEMENT WITH DATA IMPROVES; YET, AT BEST ACCURACY (GREEN LINE, N^3LL+N^2LO), EXPERIMENTAL BEHAVIOR IS FLATTER!

NEED TO ADD NON-PERTURBATIVE EFFECTS!

AGREEMENT FINALLY REACHED!



THE NON-PERTURBATIVE CORRECTION IS:

$$S_{NP}(b) = e^{-a_1 b^2/2}\left(1 - 2a_2 b\right)$$

$$\begin{cases} a_1 = 1.8 \pm 1.4\ \mathrm{GeV}^2 \\ a_2 = 0.3 \pm 0.1\ \mathrm{GeV} \end{cases}$$

COLLINS AND SOPER, 1981

DOKSHITZER, MARCHESINI AND WEBBER, 1999
(DMW DISPERSIVE MODEL)

PART 4:

CONCLUSIONS

1) EXPERIMENTAL DATA TAKEN OVER THIRTY YEARS AGO (LEP1 + SLD) STILL UNDER THEORETICAL ANALYSIS! A NEW ELECTRON MACHINE, PRODUCING HIGH-STATICS DATA ALSO OFF THE Z^0 PEAK COULD BE VERY USEFUL;

2) PERTURBATIVE COMPONENT OF EEC WELL UNDERSTOOD AND RATHER STABLE WITH RESPECT TO THE ORDER;

3) ALTERNATIVE NON-PERTURBATIVE MODELS TO THE DMW ONE, EXPLOITING UNIVERSALITY OF NON-PERTURBATIVE EFFECTS COULD BE INTERESTING.