The q_T and $\Delta\Phi_{t\bar{t}}$ spectra in top-antitop hadroproduction at LHC at NNLL+NNLO

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[2210.09272, 2407.03501, and work in progress]

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qTRes@Drell-Yan and Higgs production

1. Leading power factorization [Collins:1984kg, Catani:2000vq, Bozzi:2005wk, Bozzi:2007pn, Ebert:2016gcn, Monni:2016ktx, Bizon:2017rah, GarciaEchevarria:2011rb, Becher:2011dz, Chiu:2011qc, Chiu:2012ir, Li:2016axz, Li:2016ctv, Becher:2010tm]

$$\tilde{\sigma}_{\mathrm{DY}}(\vec{b}_{\mathrm{T}}) = \int \mathrm{d}^2\vec{q}_{\mathrm{T},\ell\bar{\ell}}\, \exp\left(-\mathrm{i}\,\vec{b}_{\mathrm{T}}\cdot\vec{q}_{\mathrm{T},\ell\bar{\ell}}\right)\, \frac{\mathrm{d}\sigma_{\mathrm{DY}}}{\mathrm{d}\vec{q}_{\mathrm{T},\ell\bar{\ell}}} = \sum_{m=2,n=1} c^{\mathrm{DY}}_{m,n}\,\alpha_s^m\, L_{\mathrm{T}}^n + \sum_{m=2} \alpha_s^m\, d^{\mathrm{DY}}_m\,.$$

Here $L_{\mathrm{T}} = \ln(\vec{b}_{\mathrm{T}}^2 Q^2)$.

2. Fourier transformation

$$L_{\mathrm{T}}^m \to \frac{\ln^{m-1}(q_{\mathrm{T},\ell\bar{\ell}})}{q_{\mathrm{T},\ell\bar{\ell}}} + \frac{\ln^{m-2}(q_{\mathrm{T},\ell\bar{\ell}})}{q_{\mathrm{T},\ell\bar{\ell}}} + \frac{\ln^{m-3}(q_{\mathrm{T},\ell\bar{\ell}})}{q_{\mathrm{T},\ell\bar{\ell}}} + \ldots$$

Exponentiating L_{T} → Resumming $\ln^m(q_{\mathrm{T},\ell\bar{\ell}})/q_{\mathrm{T},\ell\bar{\ell}}$.

qTRes@Drell-Yan and Higgs production

- Methodologies
  dQCD: [Catani:1988vd,Davies:1984hs,Collins&Soper&Sterman1984,Catani:2000vq,Bozzi:2005wk...]
  Momentum space: [Monni:2016ktx,Bizon:2017rah,Bizon:2019zgf,Bizon:2018foh,Ebert:2016gcn...]
  SCET: [Becher:2010tm,Chiu:2011qc,Chiu:2012ir,Li:2016axz...].
  ...
- Latest logarithmic accuracy at leading power:
  N3LL' [Re:2021con,Camarda:2021ict,Ju:2021lah...]
  N4LL [Camarda:2023dqn,Neumann:2022lft,Moos:2023yfa,Piloneta:2024aac...]
  ...
- Recent developments at next-to-leading power:
  Leptonic tensor: [Camarda:2021jsw,Ebert:2020dfc...]
  Hadronic tensor:
  [Ebert:2019zkb,Oleari:2020wvt,Cieri:2019tfv,Inglis-Whalen:2020rpi,Inglis-Whalen:2021bea,Ferrera:2023vsw...]

qTRes@The process $pp \to t\bar{t} + X$

1. Leading power factorization [Zhu:2012ts,Li:2013mia,Catani:2014qha,Catani:2017tuc,Catani:2018mei]

$$\tilde{\sigma}_{t\bar{t}}(\vec{b}_{\mathrm{T}}) = \sum_{m=2,n=1} \alpha_s^m \left[c_{m,n}^{t\bar{t}} + \tilde{c}_{m,n}^{t\bar{t}}(\phi_b) \right] L_{\mathrm{T}}^n + \sum_{m=2} \alpha_s^m \left[d_m^{t\bar{t}} + \tilde{d}_m^{t\bar{t}}(\phi_b) \right],$$

where $L_{\mathrm{T}} = \ln(\vec{b}_{\mathrm{T}}^2 Q^2)$. Both L_{T} and ϕ_b are associated with the asymptotic terms

2. Fourier transformation [Catani:2014qha,Catani:2017tuc]

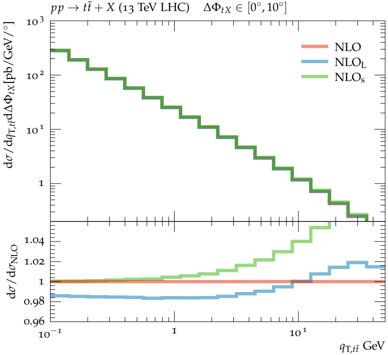
$$L_{\mathrm{T}}^m \to \frac{\ln^{m-1}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \frac{\ln^{m-2}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \frac{\ln^{m-3}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \ldots$$

$$\tilde{d}_m^{t\bar{t}}(\phi_b) \to \frac{1}{q_{\mathrm{T},t\bar{t}}}$$

$$\tilde{c}_{m,n}^{t\bar{t}}(\phi_b) L_{\mathrm{T}}^m \to \frac{\ln^{m}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \frac{\ln^{m-1}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \frac{\ln^{m-2}(q_{\mathrm{T},t\bar{t}})}{q_{\mathrm{T},t\bar{t}}} + \ldots$$

qTRes@The process $pp \to t\bar{t} + X$

The spectrum of q_{T} in slice of $\Delta\Phi_{tX}$



1) $\Delta\Phi_{tX}$: the azimuthal separation between t and the emitted partons
2) NLO$_{\mathrm{L}}$: including only L_{T}
3) NLO$_{\mathrm{s}}$: comprising both L_{T} and ϕ_b dependent terms
4) NLO: full theory from SHERPA

The discrepancies between NLO$_{\mathrm{s}}$ and NLO$_{\mathrm{L}}$ reflect the influences from the ϕ_b dependences.

By comparison to the QCD outputs, the L_{T} contribution fails to cover the leading asymptotic behaviour.

Improving the fixed-order ingredients for $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}\vec{q}_{\mathrm{T},t\bar{t}}$

[Nadolsky:2007ba,Catani:2010pd,Catani:2014qha,Catani:2017tuc]

1. $\tilde{c}^{t\bar{t}}_{m,n}(\phi_b)$ and $\tilde{d}^{t\bar{t}}_{m}(\phi_b) \to \mathcal{O}(L_\mathrm{T})$
2. Matching accuracy N^{k}LL'+N^{k}LO
3. Azimuthal harmonics @ NLL$'$ [Catani:2017tuc]

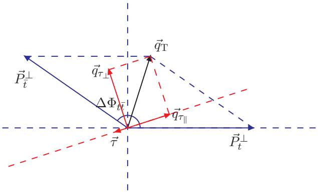
Observables insensitive to azimuthal asymmetric divergences

[Zhu:2012ts,Li:2013mia,Catani:2018mei]

1. Azimuthally averaged spectra $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}q_{\mathrm{T},t\bar{t}}$
   [Zhu:2012ts,Li:2013mia,Catani:2018mei,Bonciani:2015sha,Catani:2019iny,Catani:2019hip,Catani:2021cbl]

$$\int \mathrm{d}\phi_b \tilde{\sigma}_{t\bar{t}}(\vec{b}_\mathrm{T}) \to \sum_{m=2,n=1} \alpha_s^m \left[c^{t\bar{t}}_{m,n}\right] L^n_\mathrm{T} + \sum_{m=2} \alpha_s^m \left[d^{t\bar{t}}_{m}\right]$$

2. Projection of $\vec{q}_\mathrm{T}$ onto a reference vector $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}q_\tau$
   [Ju:2022wia]



A Generic circumstance:
$\vec{\tau}$-2D unit reference vector: $q_\tau = |\vec{q}_{t\bar{t},\mathrm{T}} \cdot \vec{\tau}|$

$$\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}q_\tau}$$

B Numeric implementations:

$$q_\tau = q_{\mathrm{T,out}}\,, \qquad \text{if } \vec{\tau} = \pm\,\vec{n} \times \frac{\vec{P}_t^{\perp}}{|\vec{P}_t^{\perp}|}\,;$$

$$q_\tau = q_{\mathrm{T,in}}\,, \qquad \text{if } \vec{\tau} = \pm\,\frac{\vec{P}_t^{\perp}}{|\vec{P}_t^{\perp}|}\,,$$

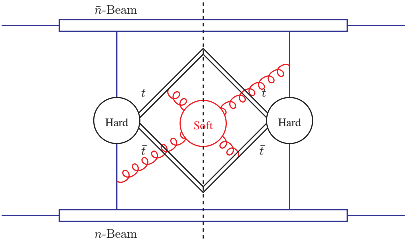
B Azimuthal separation between top and antitop quarks:

$$\Delta\Phi_{t\bar{t}} \equiv \cos^{-1}\left[\frac{\vec{P}_t^{\perp} \cdot \vec{P}_{\bar{t}}^{\perp}}{|\vec{P}_t^{\perp}||\vec{P}_{\bar{t}}^{\perp}|}\right] \sim \pi - \frac{q_{\mathrm{T,out}}}{|\vec{P}_t^{\perp}|} + \mathcal{O}(\lambda_\tau^2)\,.$$

Factorization in SCET$_{\mathrm{II}}$+HQET

$$\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}q_\tau} \sim \sum_\kappa \int \mathrm{d}b_\tau \cos(q_\tau b_\tau) \mathcal{B}_n^{[\kappa]}(\eta_n, b_\tau, \mu, \nu)\, \mathcal{B}_{\bar{n}}^{[\kappa]}(\eta_{\bar{n}}, b_\tau, \mu, \nu) \sum_{\alpha,\beta} \Big\{ \mathcal{H}_{\alpha\beta}^{[\kappa]}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu) \times \mathcal{S}_{[\kappa]}^{\alpha\beta}(b_\tau \vec{\tau}, \mu, \nu) \Big\}\ ,$$

where $b_\tau \sim 1/q_\tau \sim \mathcal{O}(\lambda_\tau^{-1})$



1) Hard function $\mathcal{H}_{\alpha\beta}^{[\kappa]}$: Recola@NLO [Actis:2012qn,Actis:2016mpe].
2) Beam function $\mathcal{B}_n^{[\kappa]}$: [Luo:2020epw,Luo:2019szz,Luo:2019bmw,Gutierrez-Reyes:2019rug,Catani:2022sgr];
3) Soft Function: [Zhu:2012ts,Li:2013mia,Angeles-Martinez:2018mqh,Catani:2014qha,Catani:2021cbl];
Azimuthally resolved NLO in exp rapidity regulator → This work;

Azimuthally resolved soft function with the exponential rapidity regulator

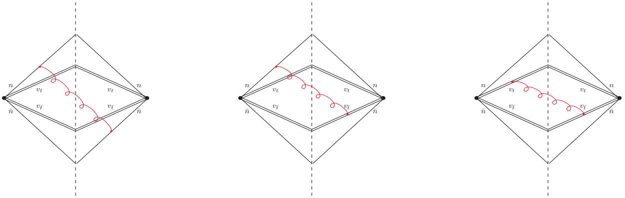
Exponential regulator[Li:2016axz] to curb the rap div

$$\mathcal{S}^{\alpha\beta}_{[\kappa]}(\vec{b}_{\rm T},\mu,\nu) \equiv \sum_{m=0} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^m \mathcal{S}^{\alpha\beta,(m)}_{[\kappa]}(\vec{b}_{\rm T},\mu,\nu)\,, \quad (1)$$

where

$$\mathcal{S}^{\alpha\beta,(0)}_{[\kappa]}(\vec{b}_{\rm T},\mu,\nu) = \delta_{\alpha\beta}\,,$$
$$\mathcal{S}^{\alpha\beta,(1)}_{[\kappa]}(\vec{b}_{\rm T},\mu,\nu) = \sum_{a,b} \langle c^{\alpha}_{\kappa}|\,\mathbf{T}_a\cdot\mathbf{T}_b\,|c^{\beta}_{\kappa}\rangle\, \mathcal{I}_{ab}(\vec{b}_{\rm T},\mu,\nu)$$

Mellin-Barnes (MB) transformation[Smirnov:1999gc, Tausk:1999vh]&MBtools[Czakon:2005rk, Ochman:2015fho, Czakon:Hepforge]

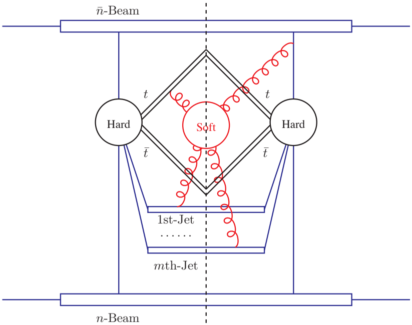


(a) Light-light correlation (b) Light-heavy correlation (c) heavy-heavy correlation

Power suppression for the energetic transverse recoil

$$\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}q_\tau} \sim \sum_{\kappa} \int \mathrm{d}q_{\tau_\perp}\, \mathrm{d}y_k \mathcal{B}_n(\eta'_n, 0, \mu, \nu)\, \mathcal{B}_{\bar{n}}(\eta'_{\bar{n}}, 0, \mu, \nu)\, \mathcal{J}^{[g]}_{n_\mathrm{J}}(\vec{k}_\mathrm{T}, y_k) \times \sum_{\alpha,\beta} \left\{ \mathcal{H}^{[q_n^i \bar{q}_{\bar{n}}^j]}_{\alpha\beta,[g_{n_\mathrm{J}}]}(\vec{P}_t, \vec{k}_\mathrm{T}, Y_{t\bar{t}}, y_k, \mu)\, \mathcal{S}^{\alpha\beta,[g_{n_\mathrm{J}}]}_{[q_n \bar{q}_{\bar{n}}]}(\vec{0}, \mu, \nu) \right\} + \ldots, \qquad (2)$$

where $q_{\tau_\perp} \sim k_{\tau_\perp} \sim \mathcal{O}(1) \gg q_{\tau_\parallel} \sim \mathcal{O}(\lambda_\tau)$



$\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}q_\tau$ is free of azimuthally asymmetric divergence !!

Proof:

$$\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}M_{t\bar{t}}^2\,\mathrm{d}^2\vec{P}_t^\perp\,\mathrm{d}Y_{t\bar{t}}\,\mathrm{d}q_\tau} \sim \sum_{m,n}\int_{-\infty}^{\infty}\mathrm{d}b_{\tau_\parallel}\;\cos(b_{\tau_\parallel}q_\tau)\,L_{\mathrm{M}}^n\Big\{s_{m,n}(\beta_{t\bar{t}},x_t,Y_{t\bar{t}}) + a_{m,n}(\mathrm{sign}[b_{\tau_\parallel}],\beta_{t\bar{t}},x_t,Y_{t\bar{t}})\Big\}$$
$$= \sum_{m,n}\Big\{2\,s_{m,n}(\beta_{t\bar{t}},x_t,Y_{t\bar{t}}) + a_{m,n}(+1,\beta_{t\bar{t}},x_t,Y_{t\bar{t}}) + a_{m,n}(-1,\beta_{t\bar{t}},x_t,Y_{t\bar{t}})\Big\}\,\mathcal{F}_\tau^{(n)}(q_\tau,M_{t\bar{t}})\,,$$

where the function $\mathcal{F}_\tau^{(n)}$ is defined as,

$$\mathcal{F}_\tau^{(n)}(q_\tau,M_{t\bar{t}}) = \int_0^\infty \mathrm{d}b_{\tau_\parallel}\;\cos(b_{\tau_\parallel}q_\tau)\,L_{\mathrm{M}}^n\,.$$

and

$$\mathcal{F}_\tau^{(0)}(q_\tau,M_{t\bar{t}}) = 0\,,\quad \mathcal{F}_\tau^{(1)}(q_\tau,M_{t\bar{t}}) = -\frac{\pi}{q_\tau}\,,\quad \mathcal{F}_\tau^{(2)}(q_\tau,M_{t\bar{t}}) = -\frac{2\pi}{q_\tau}\ln\left[\frac{M_{t\bar{t}}^2}{4q_\tau^2}\right]\,,\ldots$$

Logarithmic exponentiations with RGE and RaGE

[Chiu:2011qc,Chiu:2012ir,Li:2016axz,Li:2016ctv]

A Renormalisation group equations (RGE) $\rightarrow$
exponentiate the visuality-div-associated logs

B Rapidity renormalisation group equations (RaGE) $\rightarrow$
exponentiate the rapidity-div-associated logs

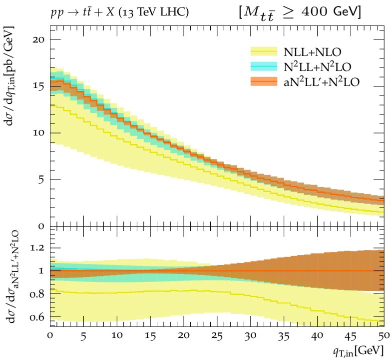
Master formula for resummed spectra

$$\frac{\mathrm{d}\sigma_{t\bar{t}}^{\mathrm{res}}}{\mathrm{d}M_{t\bar{t}}^2\,\mathrm{d}^2\vec{P}_t^{\perp}\,\mathrm{d}Y_{t\bar{t}}\,\mathrm{d}q_\tau} \sim \sum_{\mathrm{sign}[x_t],\kappa} \int_{-\infty}^{\infty} \mathrm{d}b_{\tau_\parallel}\;\cos\left(b_{\tau_\parallel} q_\tau\right)\;\widetilde{\Sigma}_{t\bar{t}}^{\mathrm{res},[\kappa]}(b_{\tau_\parallel}\vec{\tau}, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}})$$

where

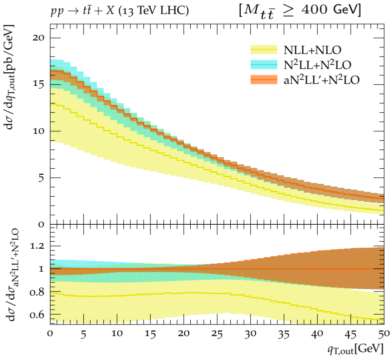
$$\begin{aligned}\widetilde{\Sigma}_{t\bar{t}}^{\mathrm{res},[\kappa]}(\vec{b}_\mathrm{T}, M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}) \sim &\sum_{\{\alpha,\beta,h,h'\}} \mathcal{D}_{[\kappa]}^{\mathrm{res}}(b_\mathrm{T}, M_{t\bar{t}}, \mu_h, \mu_b, \mu_s, \nu_b, \nu_s)\\ &\times\Bigg\{\mathcal{S}_{[\kappa]}^{\alpha_1\beta_1}(\vec{b}_\mathrm{T}, \mu_s, \nu_s)\otimes\mathcal{B}^{[\kappa]}_{n,h_n' h_n}(\eta_n, \vec{b}_\mathrm{T}, \mu_b, \nu_b)\otimes\mathcal{B}^{[\kappa]}_{\bar{n},h_{\bar{n}}' h_{\bar{n}}}(\eta_{\bar{n}}, \vec{b}_\mathrm{T}, \mu_b, \nu_b)\\ &\otimes\left[\mathcal{V}^{[\kappa]}_{\alpha_1\alpha_2}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h)\right]^*\otimes\mathcal{V}^{[\kappa]}_{\beta_1\beta_2}(\beta_{t\bar{t}}, x_t, \mu_s, \mu_h)\\ &\otimes\mathcal{H}^{[\kappa]}_{\alpha_2\beta_2;h_{\bar{n}}' h_{\bar{n}};h_n' h_n}(M_{t\bar{t}}, \beta_{t\bar{t}}, x_t, \mu_h)\Bigg\}.\end{aligned}$$

$$\vec{\tau} = \pm \frac{\vec{P}_t^{\perp}}{|\vec{P}_t^{\perp}|}, \quad q_\tau = q_{\mathrm{T,in}}$$



- PDG Inputs and LHAPDFs (`NNPDF31_nnlo_as_0118`)
- FO:
  - Event-Gen: `SHERPA+Recola`
  - Analysis: `Rivet`
- Res: `Cuba+Recola+Diag`
- Asymptotic regime
  - The central values are close to each other.
  - With the accuracy growing, the uncertainties decrease considerably.
  - The error bands of higher accuracy are contained by those with lower accuracy.

$$\vec{\tau} = \pm \vec{n} \times \frac{\vec{P}_t^{\perp}}{|\vec{P}_t^{\perp}|}\,, \quad q_\tau \;=\; q_{\mathrm{T,out}}$$



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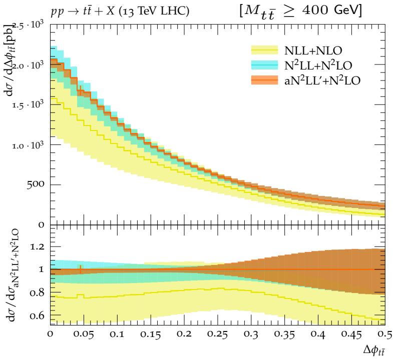
Resummation in the slice $M_{t\bar{t}} \geq 400$ GeV

$$\Delta\phi_{t\bar{t}} = \pi - \Delta\Phi_{t\bar{t}} \sim \frac{q_{\mathrm{T,out}}}{|\vec{P}_t^{\perp}|} + \mathcal{O}(\lambda_\tau^2)\,.$$



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Prescriptions and extrapolation

Single differential distribution:

$$\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}\Delta\phi_{t\bar{t}}} \sim \int \mathrm{d}M_{t\bar{t}}^2\,\mathrm{d}^2\vec{P}_t^\perp\,\mathrm{d}Y_{t\bar{t}}\,\frac{\mathrm{d}\sigma_{t\bar{t}}^{\mathrm{SCET+HQET}}}{\mathrm{d}M_{t\bar{t}}^2\,\mathrm{d}^2\vec{P}_t^\perp\,\mathrm{d}Y_{t\bar{t}}\,\mathrm{d}\Delta\phi_{t\bar{t}}}$$

where

$$\frac{\mathrm{d}\sigma_{t\bar{t}}^{\mathrm{SCET+HQET}}}{\mathrm{d}M_{t\bar{t}}^2\,\mathrm{d}^2\vec{P}_t^\perp\,\mathrm{d}Y_{t\bar{t}}\Delta\phi_{t\bar{t}}} \sim \mathcal{B}_n \otimes \mathcal{B}_{\bar{n}} \otimes \mathbf{Tr}\Big[\mathcal{V}^\dagger\,\mathcal{S}\,\mathcal{V}\,\mathcal{H}\Big]$$

In the threshold limit $\beta_{t\bar{t}} \equiv \sqrt{1-4m_t^2/M_{t\bar{t}}^2} \to 0$

$$\mathrm{NLL}: \mathcal{B}_n \sim \mathcal{B}_{\bar{n}} \sim \mathcal{V} \sim \mathcal{H} \sim S \sim \mathcal{O}(1)$$

$$\mathrm{NNLL}: \mathcal{B}_n \sim \mathcal{B}_{\bar{n}} \sim S \sim \mathcal{O}(1)\,, \quad \underbrace{\mathcal{V} \sim \mathcal{O}(\beta_{t\bar{t}}^{-2})\,, \quad \mathcal{H} \sim \mathcal{O}(\beta_{t\bar{t}}^{-1})}_{\text{Coulomb Interaction}}$$

......

An idea resolution for single differential distribution $\mathrm{d}\sigma^{\mathrm{res}}_{t\bar{t}}/\mathrm{d}\Delta\phi_{t\bar{t}}$:

$$\frac{\mathrm{d}\sigma^{\mathrm{res}}_{t\bar{t}}}{\mathrm{d}\Delta\phi_{t\bar{t}}} \sim \int \mathrm{d}M^2_{t\bar{t}} \left\{ \frac{\mathrm{d}\sigma^{\mathrm{SCET+pNRQCD}}_{t\bar{t}}}{\mathrm{d}M^2_{t\bar{t}}\,\mathrm{d}\Delta\phi_{t\bar{t}}} + \left\langle \frac{\mathrm{d}\sigma^{\mathrm{SCET+HQET}}_{t\bar{t}}}{\mathrm{d}M^2_{t\bar{t}}\,\mathrm{d}\Delta\phi_{t\bar{t}}} \right\rangle + \left\langle \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{t\bar{t}}}{\mathrm{d}M^2_{t\bar{t}}\,\mathrm{d}\Delta\phi_{t\bar{t}}} \right\rangle \right\}$$

where

- $\sigma^{\mathrm{SCET+pNRQCD}}_{t\bar{t}}$ refers to the combined resummation of Coulomb/soft/collinear correction.
- $\langle\sigma^{\mathrm{SCET+HQET}}_{t\bar{t}}\rangle$ compensates power corrections in $\beta_{t\bar{t}}$ for the non-relativistic limit.
- $\langle\sigma^{\mathrm{FO}}_{t\bar{t}}\rangle$ makes up power corrections in small q_τ expansion

Prescriptions and extrapolation

Challenges on $\mathrm{d}\sigma_{t\bar{t}}^{\mathrm{SCET+pNRQCD}}/\mathrm{d}q_\tau$ at NNLL:

- Mixed contributions: $\alpha_s^2\, L \sim \underbrace{(\alpha_s/\beta_{t\bar{t}})}_{\text{Coulomb vertex}} \otimes \underbrace{(\alpha_s \beta_{t\bar{t}})}_{\text{NLP@NLO}}$
- NLP vertices from SCET
  [Bauer:2001yt,Bauer:2001ct,Bauer:2000yr,Bauer:2000ew,Bauer:2002nz]
  [Beneke:2002ph,Beneke:2002ni,Bauer:2002aj,Lange:2003pk,Beneke:2003pa]
- NLP vertices from pNRQCD
  [Pineda:1997bj, Brambilla:1999x,Beneke:1999zr, Beneke:1999qg, Kniehl:2002br]
- NLP corrections from zero-bin subtraction
  Colorless-particles: [Inglis-Whalen:2021bea,Ferrera:2023vsw]
  Colorful-particles: ???

Prescriptions and extrapolation

The single differential distribution $\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}\Delta\phi_{t\bar{t}}}$ at NNLL:

$$\frac{\mathrm{d}^3\sigma^{\mathrm{res}}_{t\bar{t}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}\Delta\phi_{t\bar{t}}} \xrightarrow{\beta_{t\bar{t}}\to 0} \underbrace{\beta^3_{t\bar{t}}}_{\mathrm{kin}} \otimes \underbrace{\Big\{\overbrace{\mathcal{O}(\beta^{-1}_{t\bar{t}})}^{\mathrm{N^2LL}} + \dots\Big\}}_{\mathcal{H}^{[\kappa]}_{\alpha,\{h\}}} \otimes \underbrace{\Big\{\overbrace{\mathcal{O}(\beta^{-4}_{t\bar{t}})}^{\mathrm{N^2LL}} + \dots\Big\}}_{(\mathbf{V}^{[\kappa]}_h)^\dagger \mathbf{V}^{[\kappa]}_h} \otimes \dots$$

$$\sim \underbrace{\mathcal{O}(\beta^{-2}_{t\bar{t}})}_{\mathrm{N^2LL}} + \dots .$$

Prescriptions for NNLL resummation

- D-prescriptions: Further decomposition of $\mathbf{V}$ and postponing threshold enhanced term to N^{3}LL and beyond. [Kulesza:2017ukk,Alioli:2021ggd,Zhu:2012ts,Li:2013mia,Ahrens:2010zv]

$$\left.\frac{\mathrm{d}^3\sigma^{\mathrm{res}}_{t\bar{t}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}\Delta\phi_{t\bar{t}}}\right|_{\mathrm{NNLL_D}} \xrightarrow{\beta_{t\bar{t}}\to 0} \mathcal{O}(\beta^2_{t\bar{t}})\,,$$

The single differential distribution $\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}\Delta\phi_{t\bar{t}}}$ at NNLL:

$$\frac{\mathrm{d}^3\sigma_{t\bar{t}}^{\mathrm{res}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}\Delta\phi_{t\bar{t}}} \xrightarrow{\beta_{t\bar{t}}\to 0} \underbrace{\beta_{t\bar{t}}^3}_{\mathrm{kin}} \otimes \underbrace{\left\{ \overbrace{\mathcal{O}(\beta_{t\bar{t}}^{-1})}^{\mathrm{N^2LL}} + \ldots \right\}}_{\mathcal{H}_{\alpha,\{h\}}^{[\kappa]}} \otimes \underbrace{\left\{ \overbrace{\mathcal{O}(\beta_{t\bar{t}}^{-4})}^{\mathrm{N^2LL}} + \ldots \right\}}_{(\mathbf{V}_h^{[\kappa]})^\dagger \mathbf{V}_h^{[\kappa]}} \otimes \ldots$$

$$\sim \underbrace{\mathcal{O}(\beta_{t\bar{t}}^{-2})}_{\mathrm{N^2LL}} + \ldots .$$

Prescriptions for NNLL resummation

- D-prescription: Further decomposition of $\mathbf{V}$ and postponing threshold enhanced term to N^{3}LL and beyond. [Kulesza:2017ukk,Alioli:2021ggd,Zhu:2012ts,Li:2013mia,Ahrens:2010zv]
- R-prescription: Re-exponentiating threshold enhanced term for $\mathbf{V}$ in the limit $\beta_{t\bar{t}}$ and then matching onto $\mathbf{V}$ in HQET →This work

$$\left.\frac{\mathrm{d}^3\sigma_{t\bar{t}}^{\mathrm{res}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}\Delta\phi_{t\bar{t}}}\right|_{\mathrm{NNLL_R}} \xrightarrow{\beta_{t\bar{t}}\to 0} \mathcal{O}(\beta_{t\bar{t}}) ,$$

Prescriptions and extrapolation

Straightforward generalization to single differential distribution $\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}q_\mathrm{T}}$ at NNLL:

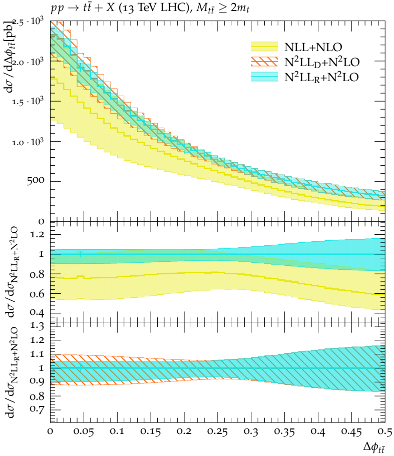
- D-prescription: Further decomposition of $\mathbf{V}$ and postponing threshold enhanced term to N^{3}LL and beyond. [Kulesza:2017ukk,Alioli:2021ggd,Zhu:2012ts,Li:2013mia,Ahrens:2010zv]

$$\left.\frac{\mathrm{d}^3\sigma_{t\bar{t}}^{\mathrm{res}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}q_\mathrm{T}}\right|_{\mathrm{NNLL_D}} \xrightarrow{\beta_{t\bar{t}}\to 0} \mathcal{O}(\beta_{t\bar{t}})\,,$$

- R-prescription: Re-exponentiating threshold enhanced term for $\mathbf{V}$ in the limit $\beta_{t\bar{t}}$ and then matching onto $\mathbf{V}$ in HQET $\rightarrow$This work

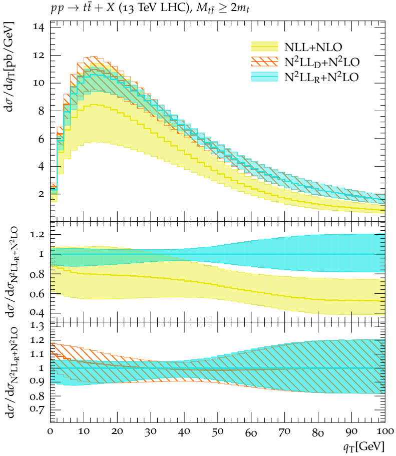
$$\left.\frac{\mathrm{d}^3\sigma_{t\bar{t}}^{\mathrm{res}}}{\mathrm{d}\beta_{t\bar{t}}\mathrm{d}Y_{t\bar{t}}\mathrm{d}q_\mathrm{T}}\right|_{\mathrm{NNLL_R}} \xrightarrow{\beta_{t\bar{t}}\to 0} \mathcal{O}(\beta_{t\bar{t}}^0)\,,$$

Resummation over the full phase space



- PDG Inputs and LHAPDFs (`NNPDF31_nnlo_as_0118`)
- FO:
  - Event-Gen: `SHERPA`+`Recola`
  - Analysis: `Rivet`
- Res: `Cuba`+`Recola`+`Diag`
- Negligible ambiguity in the low $\Delta\phi_{t\bar{t}}$ domain

Resummation over the full phase space



- PDG Inputs and LHAPDFs (`NNPDF31_nnlo_as_0118`)
- FO:
  - Event-Gen: `SHERPA`+`Recola`
  - Analysis: `Rivet`
- Res: `Cuba`+`Recola`+`Diag`
- $\sim 10\%$ ambiguity in the low q_T domain

Conclusion

Upshot

- We propose the projected transverse momentum for the top-antitop hadroproduction, offering alternative choices to circumvent the azimuthal asymmetric divergences.
- In the context of SCET$_{\rm II}$+HQET, we derive the leading power factorization formula of the q_τ spectra and in turn evaluate $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}\Delta\phi_{t\bar{t}}$ for the region $M_{t\bar{t}} \geq 400$ GeV.
- With the help of D- and R- prescriptions, we extrapolate the SCET$_{\rm II}$+HQET based resummation to the whole phase space, thereby evaluating the single differential distributions $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}\Delta\phi_{t\bar{t}}$ and $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}q_{\rm T}$.
- Comparing the results from D- and R-prescriptions, we deliver a quantitative assessment on the theoretical ambiguity due to the threshold divergences, finding that $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}\Delta\phi_{t\bar{t}}$ presents much less scheme-dependences than $\mathrm{d}\sigma_{t\bar{t}}/\mathrm{d}q_{\rm T}$.

Thank you for your attention!