Quantum Tops: Quantum Information meets High-Energy Physics

LFC24 Workshop, SISSA, Trieste, Italy

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Overview

- The Standard Model is a Quantum Field Theory:
  - Special Relativity.
  - Quantum Mechanics.
- Recently, it was shown that fundamental properties of Quantum Mechanics can be tested via processes of the Standard Model.
- An opportunity to study concepts of Quantum Information at High-Energy colliders, like the LHC.
- In this talk, I will focus on $t\bar{t}$.
- Three main parts are in the talk:
  1. Theory: Basic concepts.
  2. Phenomenology: Implementation for $t\bar{t}$ in hadron colliders.
  3. Experiment: Recent measurements.

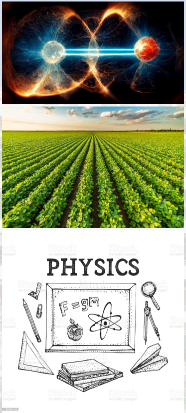

Figure: Quantum + Field + Theory.

First part: Theory

Basic concepts.

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- **Quantum Tomography**: reconstruction of the quantum state from measurement of a set of expectation values.

Quantum Tomography: One Qubit

- Qubit: quantum system with two states (e.g., spin-1/2 particle).
- Most general density matrix for a qubit:

$$\rho = \frac{I_2 + \sum_i B_i \sigma^i \otimes I_2}{2}$$

- Only 3 parameters $B_i \to$ Quantum tomography is the measurement of spin polarization **B**:

$$B_i = \langle \sigma^i \rangle = \mathrm{tr}(\sigma^i \rho)$$

Quantum Tomography: Two Qubits

- Most general density matrix for 2 qubits:

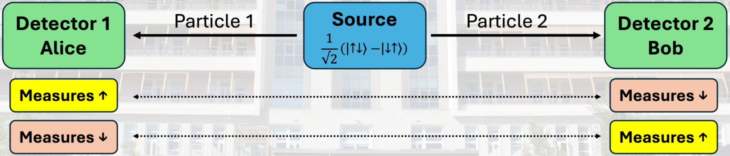
$$\rho = \frac{I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- 15 parameters $B_i^\pm, C_{ij}$ $\rightarrow$ Quantum tomography=Measurement of individual spin polarizations $\mathbf{B}^\pm$ and spin correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \rangle \,, \;\; B_i^- = \langle \bar{\sigma}^i \rangle \,, \;\; C_{ij} = \langle \sigma^i \bar{\sigma}^j \rangle$$


What is Quantum Entanglement?

- Quantum state of one particle cannot be described independently from another particle.
- $\Rightarrow$ **Correlations** of observed physical properties of both systems.
- $\Rightarrow$ **Measurement** performed on one system seems to be influencing other systems entangled with it.



- Observed in photons, atoms, superconductors, mesons, analog Hawking radiation, nitrogen-vacancy centers in diamond and even macroscopic diamond. **Recently it has been observed in $t\bar{t}$ pairs.**

Quantum Entanglement Definition

- Two different systems A and B: $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$.
- Separable: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$.
- $\rho_n^{a,b}$ are quantum states in A, B, $\sum_n p_n = 1,\ p_n \geq 0$
- Classically correlated state in $\mathcal{H}$ $\rightarrow$ can be written in this form.

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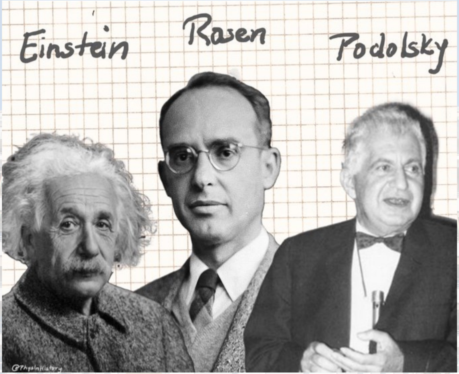
- Classically correlated state in $\mathcal{H} \rightarrow$ can be written in this form.
- Non-separable state is called **entangled** and hence, it is a non-classical state.

Separable

Non-Separable

- For two qubits:
  - Separability $\Longleftrightarrow$ Classical probability distribution.
  - **Entanglement** $\Longleftrightarrow$ No classical probability distribution description.

EPR Paradox



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

EPR Paradox

- Entanglement: "spooky action at a distance" (A. Einstein).

- Assuming two particles with spacial distance.
- When a measurement is done on one of the particles, the other one "knows" about it immediately.
- Information travel faster than light?
- Contradicts the theory of relativity.
- **Conclusion:** the theory of Quantum Mechanics is incomplete.

Hidden Variables

- By EPR, each particle "carries" variables that know the state before the measurement.
  $\Rightarrow$ There are some hidden variables that are missing in order to have a full theory.

Hidden Variables

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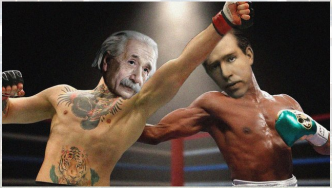
  ⇒ There are some hidden variables that are missing in order to have a full theory.
- The Copenhagen Interpretation: superposition of states until a measurement was done.
- Bohr Vs. Einstein.

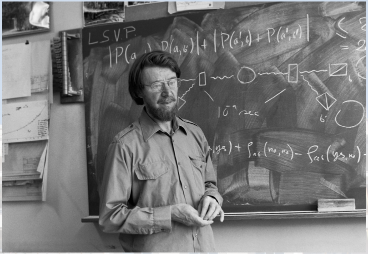
"God does not play at dice with the universe".



"Quit telling God what to do!"

- Who is right?

Bell Inequality



ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

- If local hidden variables hold, they should satisfy some inequality.
- $C(x, y)$ are the correlations between different measurements at different detectors.
- The parameters a,b,c are different directions for the measurement.
- Original form: $1 + C(b, c) \geq |C(a, b) - C(a, c)|$.

The Nobel Prize in Physics 2022

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science". (link)

Figure: Alain Aspect, John F. Clauser and Anton Zeilinger.

How does all this related to High-Energy Physics?

Second part: Phenomenology

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Implementation for $t\bar{t}$ in hadron colliders.

Top Quark

- **Top-quark:**
  - The most massive particle in the Standard Model.
  - Lifetime: $\sim 10^{-25}$ s.
- **General:**
  - Hadronisation: $\sim 10^{-24}$ s.
  - Spin-decorrelation: $\sim 10^{-21}$ s.
- Spin information $\rightarrow$ decay products.
- Spin-correlations between top-quark pairs can be measured.
- Considering di-leptonic decays.

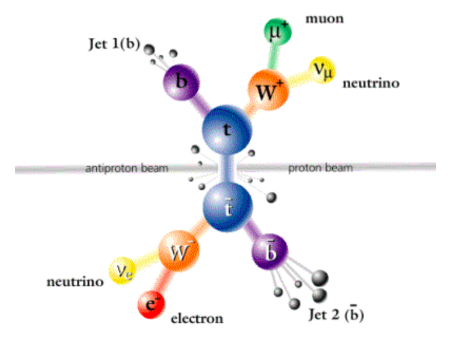
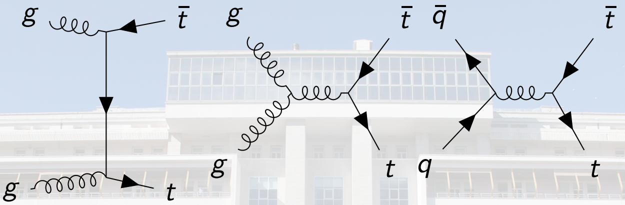


Figure: Di-leptonic decay of a $t\bar{t}$ pair.

Spin-Correlations between Top-Quark Pairs

- Studied extensively theoretically.
- Measured by the D0, CDF, ATLAS and CMS collaborations.
- No link between spin-correlations of top-quarks and concepts of Quantum Information until recently.
- Spin-Correlations can be a classical property.
  For example, **Spin-Correlations $\neq$ Quantum Entanglement!**
  However, Quantum Entanglement $\subset$ Spin-Correlations.

Leading-order Analytical Calculation



- Analytical calculation at leading-order. The system is defined by:
  - $\hat{k}$: the direction of the top with respect to the beam axis.
  - The invariant mass $M_{t\bar{t}}$, $\beta = \sqrt{1 - \frac{4 \cdot m_t^2}{M_{t\bar{t}}^2}}$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

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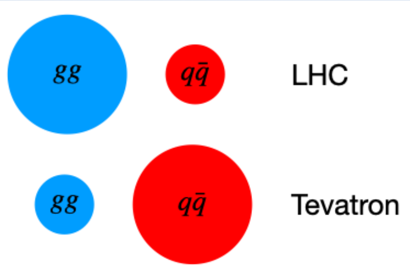

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Experimental Observables

Quantum Entanglement:

- **Concurrence** $\mathcal{C}[\rho]$: quantitative measurement of entanglement.
- $0 \leq \mathcal{C}[\rho] \leq 1$, $\mathcal{C}[\rho] \neq 0$ iff the state is entangled.
- Here, $\mathcal{C}[\rho] = \max(\Delta, 0)$; $\Delta = \frac{-C_{nn}+|C_{kk}+C_{rr}|-1}{2}$.

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Non-Separable

Bell Non-locality:

- A violation of the CHSH inequality: $|\mathbf{a}_1^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 + \mathbf{b}_2)\,| > 2$.
  - $\mathbf{C}$ - spin correlation matrix.
  - $\mathbf{a}_1, \mathbf{a}_2$ $(\mathbf{b}_1, \mathbf{b}_2)$ - axes in which we measure the spin of the top (antitop).
- Maximization: $2\sqrt{\mu_1 + \mu_2} \leq 2\sqrt{2}$ where $0 \leq \mu_i \leq 1$ are the eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$.

Entanglement and Bell Non-locality Before Integration

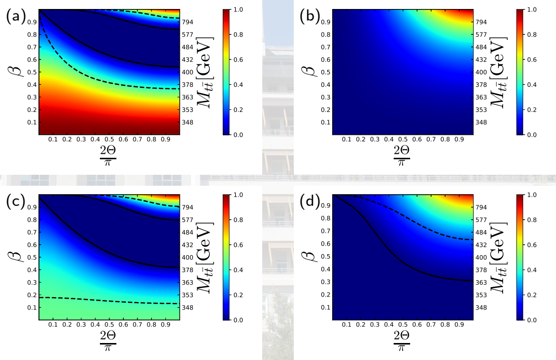
a) $gg \to t\bar{t}$ Concurrence.

b) $q\bar{q} \to t\bar{t}$ Concurrence.

c) Full LHC $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

d) Full Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

- Solid line: entanglement limit; Dashed line: Bell non-locality limit.
- Figures are from YA, de Nova, Quantum (2022).



- We have identified two regions of strong quantum correlations:
  - Close to the production threshold of $\sim 2 \cdot m_t$.
  - At high $M_{t\bar{t}}$ and high top-p_T.

Entanglement Observable

- Plots are shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- Single observable:
  $\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$,
  $D = \frac{\mathrm{tr}[\mathbf{C}]}{3} = -3\cdot\langle\cos\varphi\rangle$, φ is the angle between the leptons measured in the parent top/antitop rest frame, and **C** is the spin correlation matrix.
- $D < -\frac{1}{3} \Rightarrow$ entanglement.
- Can be achieved by measuring D close to threshold at the LHC.
- Theory framework:
  - YA, de Nova, EPJP (2021).
  - Severi, Boschi, Maltoni, Sioli, EPJC (2022).
  - YA, de Nova, Quantum (2022).

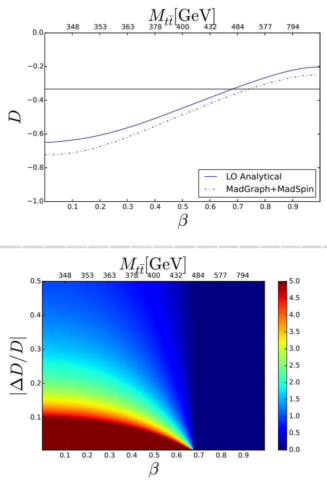


Figure: Up: the value of D; bottom: statistical deviation from the null hypothesis ($D = -1/3$).

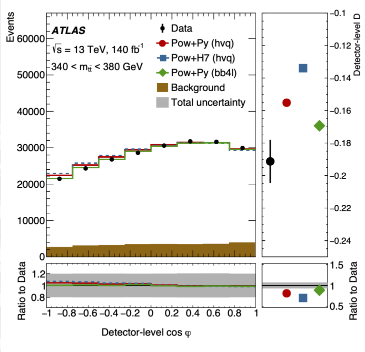
Third part: Experiment

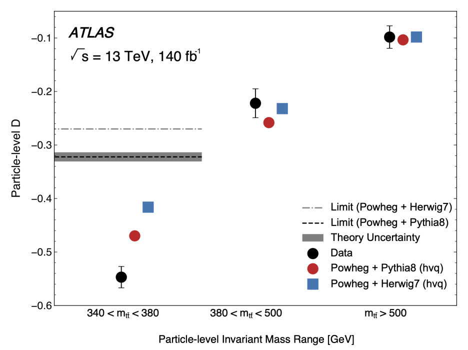
Recent measurements.

Recent Measurements

- So far, three related measurements were done by ATLAS and CMS.
- Entanglement in $t\bar{t}$ pairs close to the production threshold:
  - **ATLAS: 2311.07288 (Accepted to Nature). The paper will be published tomorrow here: ATLAS, Nature (2024).**
  - **CMS: 2406.03976 (Submitted to Reports on Progress in Physics).**
- Entanglement in $t\bar{t}$ pairs with boosted tops:
  - **CMS: CMS-PAS-TOP-23-007.**
- Many more are at work.

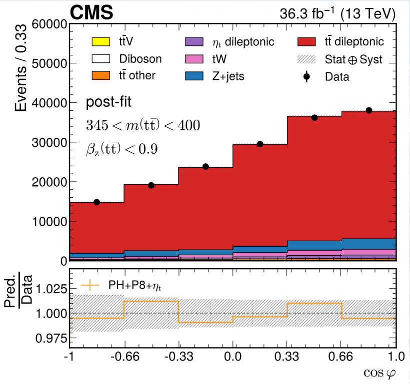
Threshold Region - ATLAS

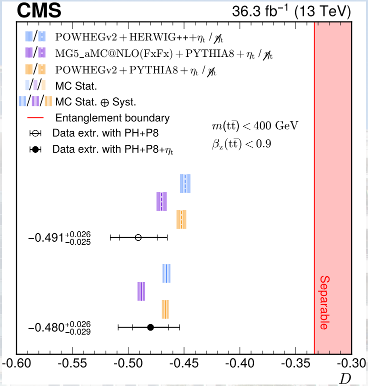




- No clear preference of a specific MC prediction.
- The limit of $D = -1/3$ is folded from parton to particle level.
- **Entanglement is observed (expected) with well more than** 5σ**.**
  Observed: $D = -0.547 \pm 0.002$ [stat.] ± 0.021 [syst.]
  Expected: $D = -0.470 \pm 0.002$ [stat.] ± 0.018 [syst.]

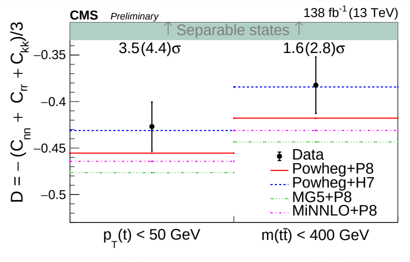
Threshold Region - CMS

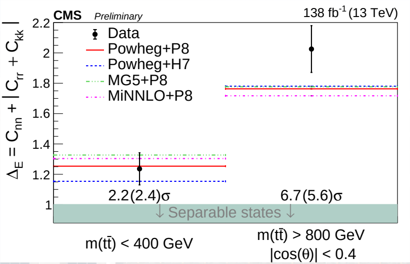




- Data seem to prefer Toponium.
- The limit of $D = -1/3$ is shown at parton-level.
- **Entanglement is observed (expected) with** 5.1σ **(**4.7σ**).**
  Observed: $D = -0.480^{+0.016}_{-0.017}$ [stat.]$^{+0.020}_{-0.023}$ [syst.]
  Expected: $D = -0.467^{+0.016}_{-0.017}$ [stat.]$^{+0.021}_{-0.024}$ [syst.]

Boosted Region - CMS





- Different final state: $t\bar{t} \to \ell^{\pm} + jets$.
- The limits of separability are shown at parton-level.
- **Entanglement is observed (expected) with 6.7σ (5.6σ).**
  Observed: $\Delta_E = -2.03 \pm 0.15$.
- Sensitivity at the threshold region is lower.

What is Next?

- **Test more quantum information concepts with $t\bar{t}$.**
  YA, de Nova, PRL (2023).
- Use other $t\bar{t}$ decay mechanisms and / or in other parts of phase-space.
  Fabbrichesi, Floreanini, Panizzo, PRL (2021).
  Dong, Gonçalves, Kong, Navarro, PRD (2024).
- Test quantum information concepts with other systems.
  Barr, PLB (2022).
  Aguilar-Saavedra, PRD (2023).
  YA, Kats, de Nova, Soffer, Uzan, 2406.04402.
- Use quantum information techniques to search for new physics.
  Aoude, Madge, Maltoni, Mantani, PRD (2022).
  Severi, Vryonidou, JHEP (2023).
  Fabbrichesi, Floreanini, Gabrielli, EPJC (2023).
  Maltoni, Severi, Tentori, Vryonidou, JHEP (2024).
- **Use quantum information techniques to better understand high-energy physics processes.**
  Aguilar-Saavedra, 2407.20330.

Quantum Information Hierarchy

- Complete picture of quantum correlations in top-quark pairs. YA, de Nova, PRL (2023).
- Quantum Discord:
  - The most basic form of quantum correlations.
  - Asymmetric between different subsystems, natural test of *CP*.
- Quantum Steering:
  - Measurement of how one subsystem can be used to "steer" the other one.
  - A non-local feature that lies between entanglement and Bell non-locality.

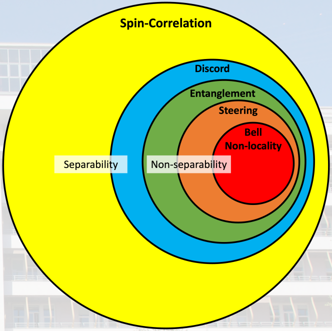
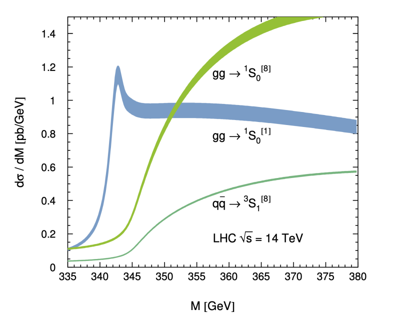


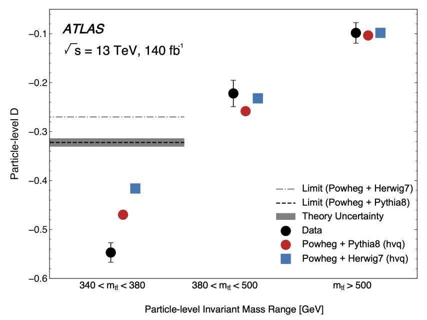
Figure: Schematic description of the relation between the different concepts discussed in the talk.

These measurements are difficult to make in conventional labs, and are naturally accessible at the LHC due to the large statistics.

Toponium?

- Left: invariant mass distribution close to threshold including all partonic production channels. Figure is from Eur.Phys.J.C 60 (2009) 375-386.
- Right: the recent ATLAS result.
- Toponium: higher cross-section next to threshold, more spin-singlet (maximally entangled). Not included in MC generators.





Summary

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- It constitutes as a proof of concept that quantum information measurements can be done in high-energy colliders.
- This is a new and exciting way to analyze collider data.
- This line of research is rapidly evolving, with many new ideas and dedicated workshops:
  - Oxford (March 2023): link, Oxford (October 2024): link.
  - GGI (November 2023): link.
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  - GGI (November 2023): link.
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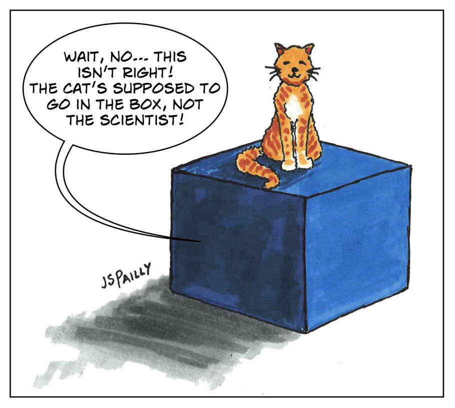
- There is a lot to do, especially on the experimental side.

"Theory will only take you so far", from the movie Oppenheimer.

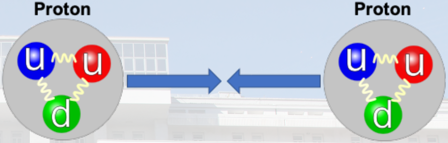
Thank You


WAIT, NO... THIS ISN'T RIGHT! THE CAT'S SUPPOSED TO GO IN THE BOX, NOT THE SCIENTIST!
JSPAILLY

Backup Slides

Backup

Collisions at the LHC



- At the LHC, protons are being collided at high energies.
- The proton is a complex creature!
- Proton: quarks and gluons (partons).
- Parton distribution function (PDF): the density of each parton in the proton.

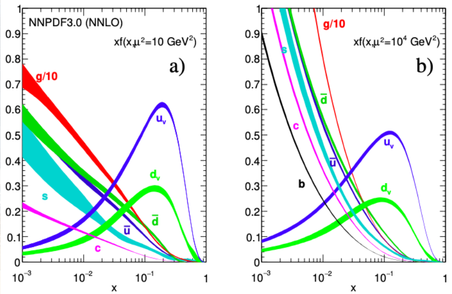


Figure: Parton density at the proton.
Figure is from JHEP 2015, 40 (2015).

Loopholes in a Collider Experiment

- Loopholes: experimental tests of Bell inequality may not fulfill all hypotheses of the theorem.
- Collider experiment:
  - Free-will loophole: spin measurement directions should be free, independent from hidden-variables.
  - Detection loophole: only a subset of events is selected for the measurement, which can be biased.
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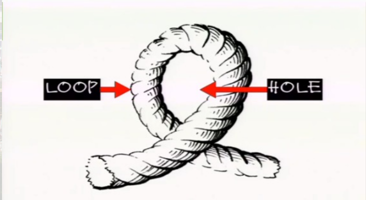
- Collider experiments were not designed to test Bell Inequality!
  $\Rightarrow$ Can only detect a *weak* violation of CHSH (Bell) Inequality.



- **Bell-Inequality** $\subset$ **Quantum Entanglement**.

Recent Related Measurement

- Recently, D was measured inclusively, i.e. with no selection on $M_{t\bar{t}}$, by the CMS collaboration.
- Results:
  $D = -0.237 \pm 0.011 > -1/3$;
  $\Delta D/D = 4.6\%$.
- No evidence of quantum entanglement.
  $\Rightarrow$ **We need a dedicated analysis!**

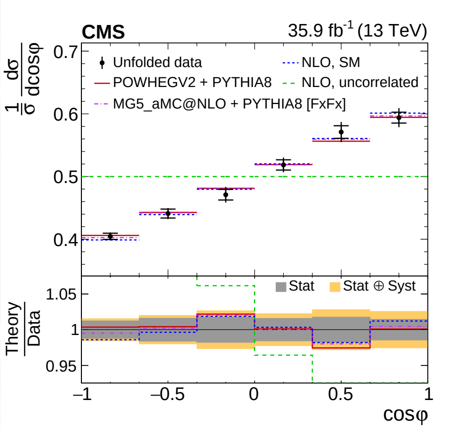


Figure: Distribution of $\cos\varphi$. Figure is from Phys. Rev. D 100, 072002.

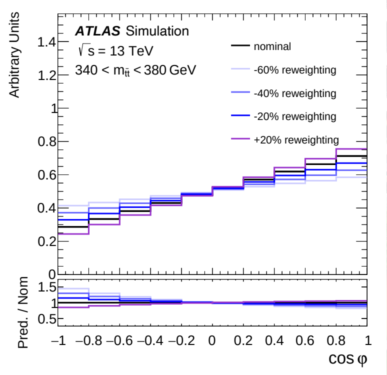
Reweighting Method

- To test the alternative hypotheses we must change D.
- Inherent in particle generators.
- Alternative approach: each event is reweighted (at parton-level) taking into account $m_{t\bar{t}}$ to preserve linearity in $\cos\varphi$.

- $D(m_{t\bar{t}})$ is calculated for each modeling systematic.
- The reweighting is done for all systematic uncertainties.

$$w = \frac{1 - D(m_{t\bar{t}}) \cdot \chi \cdot \cos\varphi}{1 - D(m_{t\bar{t}}) \cdot \cos\varphi}$$

$$\chi = 0.4, 0.6, 0.8, 1.2.$$



Systematic Uncertainties

- Three categories:
  - Signal ($t\bar{t}$) modeling.
  - Background modeling.
  - Detector uncertainties.

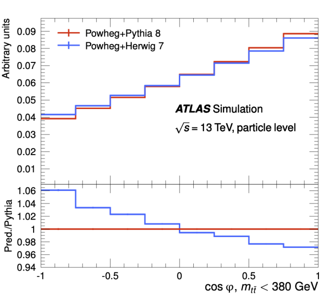
Systematic source	$\Delta D_{\text{expected}}(D = -0.470)$	ΔD (%)
Signal Modelling	0.015	3.2
Electron	0.002	0.4
Muon	0.001	0.1
Jets	0.004	0.8
b-tagging	0.002	0.4
Pileup	< 0.001	< 0.1
E_T^{miss}	0.002	0.4
Backgrounds	0.009	1.8
Stat.	0.002	0.4
Syst.	0.018	3.9
Total	0.018	3.9

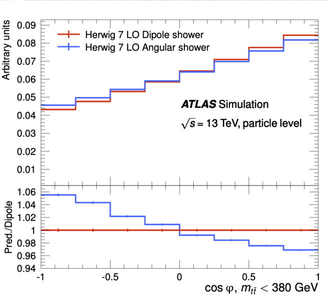
Table: Systematic uncertainties for the **expected** D.

- Signal ($t\bar{t}$) modeling breakdown:
  - Top decay (MadSpin): 1.6%
  - PDF (PDF4LHC): 1.2%
  - Recoil To Top: 1.1%
  - FSR: 1.1%
  - Scales (μ_R, μ_F): 1.1%
  - NNLO Reweighting: 1.1
  - pThard1 (`pThard = 1`): 0.8%
  - m_t (172.5 ± 0.5 GeV): 0.7%
  - ISR: 0.2%
  - Parton Shower (Herwig): 0.2%
  - h_{damp}: 0.1%
- Background modeling is dominated by $Z \to \tau^+\tau^-$ uncertainty.
- For each systematic, we extract a curve. The difference w.r.t. the nominal curve is the uncertainty.

Parton Shower Modeling

- Large difference between PowhegBox+Pythia 8.230 PowhegBox+Herwig 7.21, especially in the SR.
- A reason for an extensive scrutiny, to understand the difference.
- Comparison at particle-level.
- Main origin: the ordering of the shower.
- Observed both at detector and particle-level.
  → **Parton-level analysis: huge uncertainty.**
  → **Particle-level analysis: small uncertainty.**



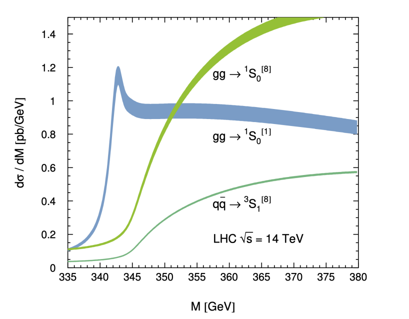


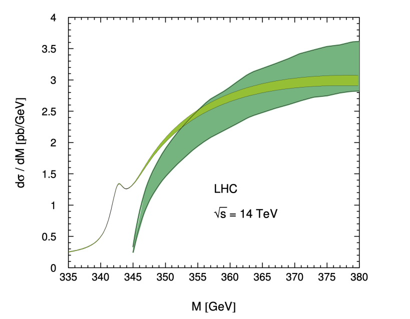
Systematic Uncertainties

Systematic source	$\Delta D_{\text{particle}}(D = -0.470)$	ΔD (%)	$\Delta D_{\text{observed}}(D = -0.547)$	ΔD (%)
Signal Modelling	0.017	3.2	0.015	3.2
Electron	0.002	0.4	0.002	0.4
Muon	0.001	0.1	0.001	0.1
Jets	0.004	0.7	0.004	0.8
b-tagging	0.002	0.4	0.002	0.4
Pileup	< 0.001	< 0.1	< 0.001	< 0.1
E_T^{miss}	0.002	0.3	0.002	0.4
Backgrounds	0.010	1.8	0.009	1.8
Stat.	0.002	0.3	0.002	0.4
Syst.	0.021	3.8	0.018	3.9
Total	0.021	3.8	0.018	3.9

Differential Cross-Section - $m_{t\bar{t}}$ Dependence

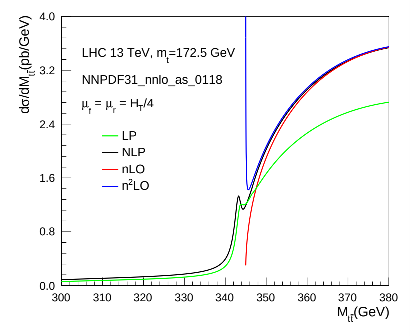
- Left: Invariant mass distribution close to threshold including all partonic production channels.
- Right: Comparison of threshold re-summed results with fixed order QCD predictions.
- Figures are from Eur.Phys.J.C 60 (2009) 375-386.





Differential Cross-Section - $m_{t\bar{t}}$ Dependence

- The comparison between the NLP resummed result and the LP resummed, nLO and nnLO ones.
  - NLP: next-to-leading power.
  - LP: leading power.
  - nLO: next-to-leading order.
  - nnLO: next-to-next-to-leading order.
- Figure is from JHEP 06 (2020) 158.



Critical Values After Integration

- We focus on *pp* interactions.
- Clear motivation to restrict to selected regions of phase space.
- Plot is shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- We focus on the region close to threshold. For high p_T see:
  - Fabbrichesi, Floreanini, Panizzo, PRL (2021).
  - Severi, Boschi, Maltoni, Sioli, EPJC (2022).

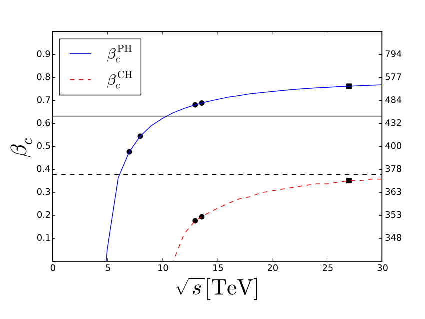


Figure: Critical values below which entanglement and CHSH violation can be observed, for different COM values.

Top Reconstruction

- Three methods:
    - 85%: Ellipse Method. Calculates two ellipses for p_T^ν and finds the intersections.
    - 5%: Neutrino Weighting.
    - 10%: Rudimentary pairing.
- The solution with the smallest $m_{t\bar{t}}$ is taken.

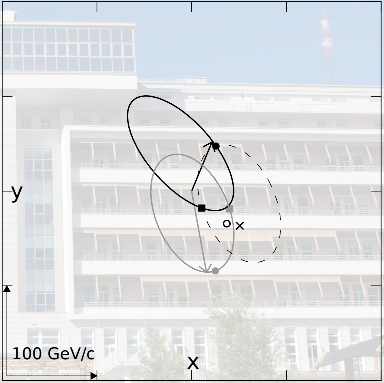


Figure: Constrain on neutrino momenta. Figure is from Nucl.Instrum.Meth.A 736 (2014) 169-178.

Quantum Discord

- Classically: $I(A,B) = H(A) + H(B) - H(A,B) = H(A) - H(A|B)$, $H(X)$ is the Shannon entropy.
- QM "discord": $\mathcal{D}(A,B) \equiv H(B) - H(A,B) + H(A|B) \neq 0$.
- The condition for discord in a two-qubit system is:
  $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}}) \neq 0$.

with $S(\rho) = -\mathrm{Tr}\,\rho \log_2 \rho$
the Von Neumann entropy.

- Can be asymmetric:
  $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$.
  $\rightarrow$ A test for CP-violation.

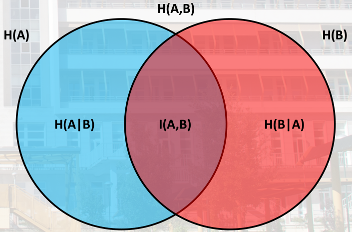


Figure: Schematic description of two subsystems with mutual information.

Steering

- Measurement of how Alice can "steer" the quantum state of Bob.
- Original conception of Schrödinger for the EPR paradox, only well-defined in 2007 (Wiseman, Jones, Doherty, PRL (2007)).

Steering

- Measurement of how Alice can "steer" the quantum state of Bob.
- Original conception of Schrödinger for the EPR paradox, only well-defined in 2007 (Wiseman, Jones, Doherty, PRL (2007)).
- Alice performs a spin measurement x and obtains the result $a = \pm$.
- Bob's resulting state is the corresponding conditional states $\rho(a|x)$.
- Bob has to believe that Alice can influence his state, unless local hidden state holds.
- Can be asymmetric.
  $\rightarrow$ A test for CP-violation.

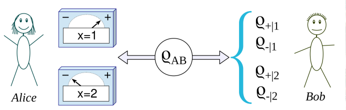
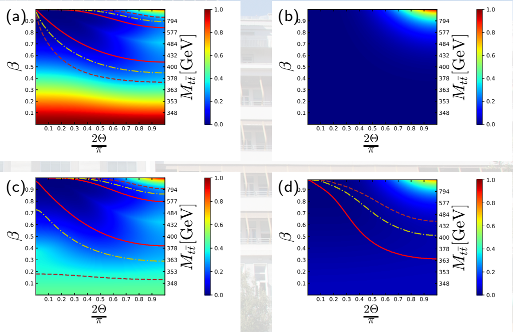


Figure: Schematic description of the steering phenomenon: Figure is from Uola, Costa, Nguyen, Gühne, Rev. Mod. Phys. (2020).

Discord and Steering Before Integration

a) $gg \to t\bar{t}$ Discord.
b) $q\bar{q} \to t\bar{t}$ Discord.
c) Full LHC $\rho(M_{t\bar{t}}, \hat{k})$ Discord.
d) Full Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Discord.

- Solid red, dashed-dotted yellow, and dashed brown lines are the critical boundaries of separability, steerability, and Bell locality, respectively.



Full picture of quantum correlations in $t\bar{t}$.

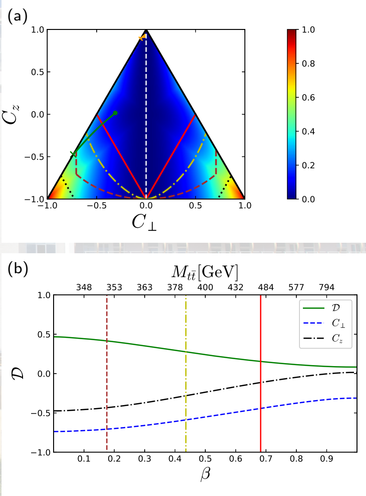
Discord and Steering After Integration

- Integration only for $[2m_t, M_{t\bar{t}}]$.

a) Discord for $C_\perp, C_z$ (symmetry around the beam axis).

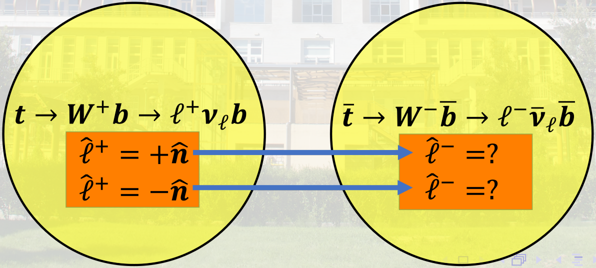
- Green: LHC trajectory; Orange: Tevatron trajectory.
- Cross: $\beta = 0$; Circle: $\beta = 1$.
- Quantum discord: $C_\perp \neq 0$. Solid red, dashed-dotted yellow, dashed brown, and dotted black lines are the critical boundaries of separability, steerability, Bell locality, and NAQC, respectively.

b) Detailed trajectory of green line in the upper panel.



Experimental Measurement - Discord

- The tomography is required for $\rho_{A,B}$, ρ, $\rho_{\hat{\mathbf{n}}}$, $\rho_{-\hat{\mathbf{n}}}$:
  $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}}) \neq 0.$
  $\rightarrow$ Can be done by measuring the differential cross-sections.
- One-qubit tomography of $\rho_{\hat{\mathbf{n}}}$ from conditional Bloch vectors $\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}$:
  $$p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-}{(4\pi)^2}$$
  $$p(\hat{\ell}_\pm | \hat{\ell}_\mp = \mp \hat{\mathbf{n}}) = \frac{p(\hat{\ell}_\pm, \hat{\ell}_\mp = \mp\hat{\mathbf{n}})}{p(\hat{\ell}_\mp = \mp\hat{\mathbf{n}})} = \frac{1 \pm \mathbf{B}^{\pm}_{\hat{\mathbf{n}}} \cdot \hat{\ell}_\pm}{4\pi}.$$
- Actual discord is evaluated from minimization over $\hat{\mathbf{n}}$.
  $\rightarrow$ Measuring discord according to its very definition.



Experimental Measurement - Steering

- Steering ellipsoid: the set of states to which Bob can steer Alice.
  - Forms an ellipsoid $\mathcal{E}_A$ in Alice's Bloch sphere, containing her Bloch vector $\mathbf{a}$.
  - Fundamental object in Quantum Information.
  - Contains most of the information about system's quantumness.
- Measurement of $\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}$ enables the reconstruction of $t, \bar{t}$ steering ellipsoids.
- Highly-challenging measurements in conventional setups.
  $\rightarrow$ Natural implementation in colliders.

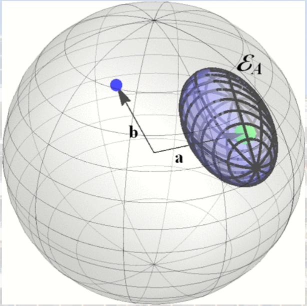
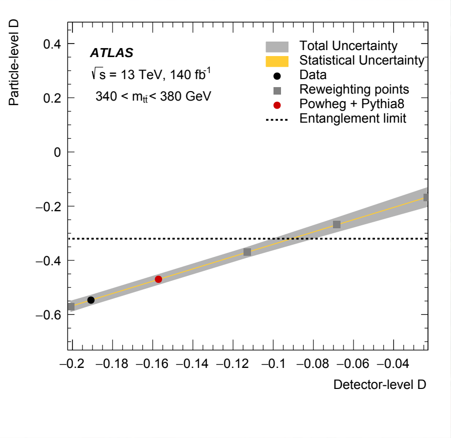


Figure: Ellipsoid representation of a two-qubit state. Figure is from Jevtic, Pusey, Jennings, Rudolph, PRL (2014).

Results



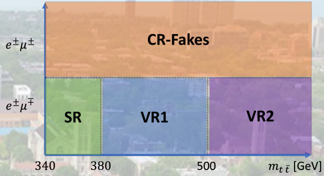
Systematic source	$\Delta D_{\text{observed}}(D = -0.547)$	ΔD (%)
Signal Modelling	0.017	3.2
Electron	0.002	0.4
Muon	0.001	0.1
Jets	0.004	0.7
b-tagging	0.002	0.4
Pileup	< 0.001	< 0.1
E_T^{miss}	0.002	0.3
Backgrounds	0.010	1.8
Stat.	0.002	0.3
Syst.	0.021	3.8
Total	0.021	3.8

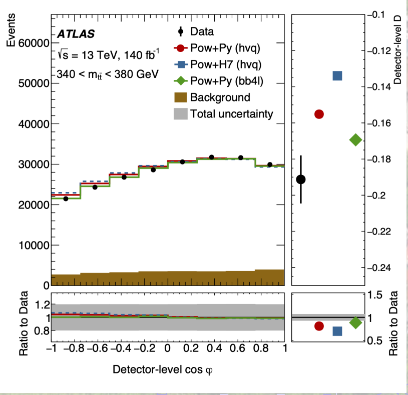
Table: Systematic uncertainties for the **observed** D.

- The calibration curve for the SR and the uncertainties for the observed values are presented.

Analysis Strategy

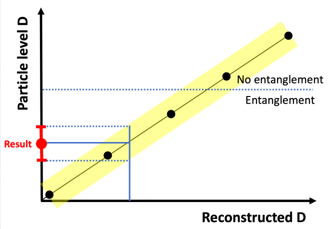
- Analysis selection:
  - $1\mu, 1e$ with opposite charges.
  - Single lepton triggers.
  - Lepton $p_T > 25$–28 GeV.
  - $N_b \geq 1$ (85% b-tag efficiency).
- Backgrounds:
  - tW.
  - $t\bar{t} + X(X = H, W, Z)$.
  - $VV(V = W, Z)$.
  - $Z \to \tau^+\tau^-$.
  - Fakes.
- Regions are categorized by $m_{t\bar{t}}$. The $t\bar{t}$ purity is 90% across the signal region (SR) and the validation regions (VR1, VR2).
- **Particle level fiducial regions are defined with similar selections.**





Calibrating the Observable

- Measure the particle level value of D using a calibration curve.
- The curve is built from alternative sets of **reconstructed** D and **particle level** D, with variations of the parton level D value: -60%, -40%, -20%, SM, +20%



- A first order polynomial is used to interpolate between the points.
- The data are corrected to the particle level value of D.
- One curve for each systematic. The difference w.r.t. the nominal curve is the uncertainty.

Particle level *D* **Reconstructed *D*** **Corrected *D***