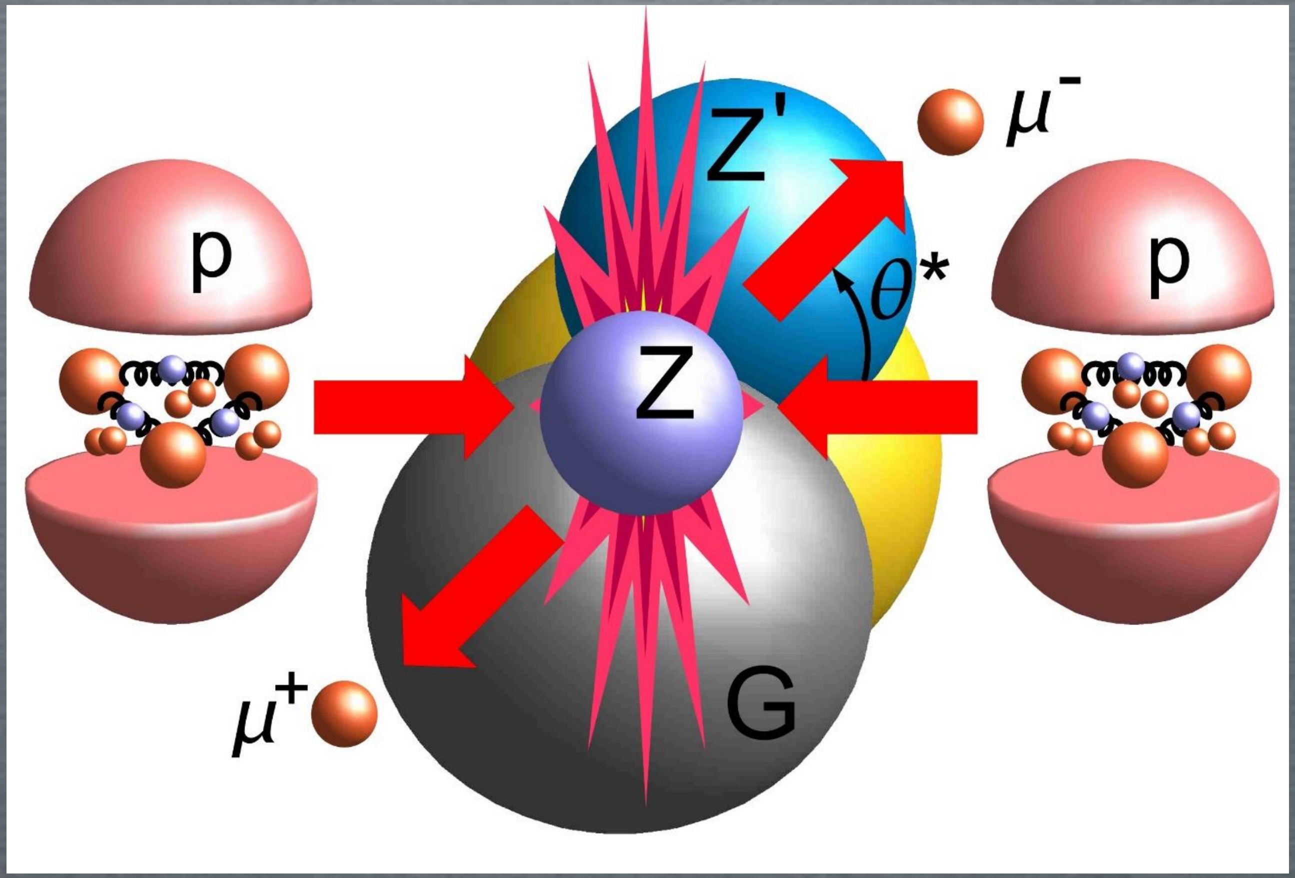
Precision QCD status and perspectives



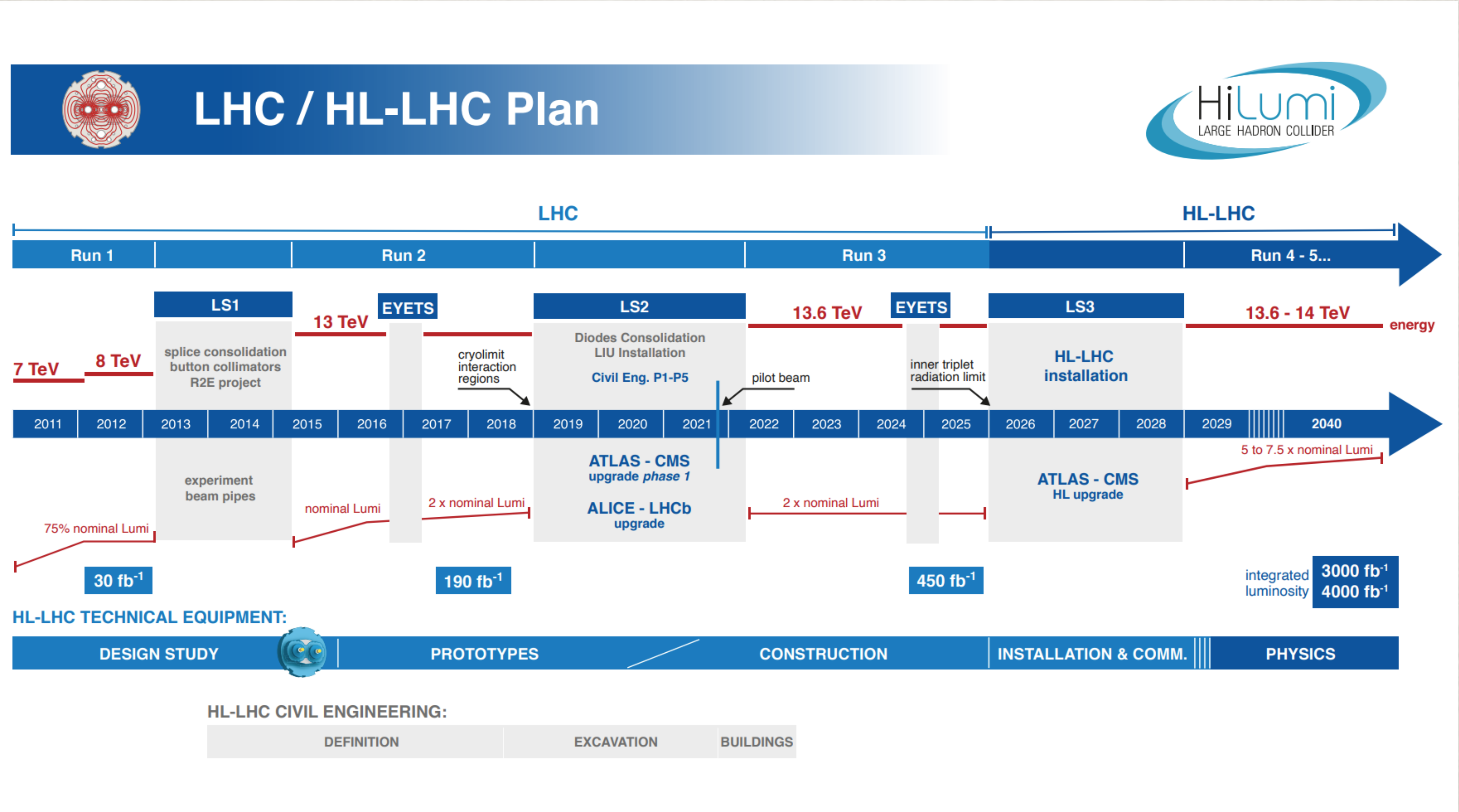
Andrea Banfi

University of Sussex

LCF24 - 17 september 2024 – SISSA Trieste

High luminosity LHC

In a few years, the LHC will enter its high-luminosity phase



At the moment, HL-LHC is the only high-energy collider on the horizon.
Nevertheless, the issues raised here apply to future colliders too

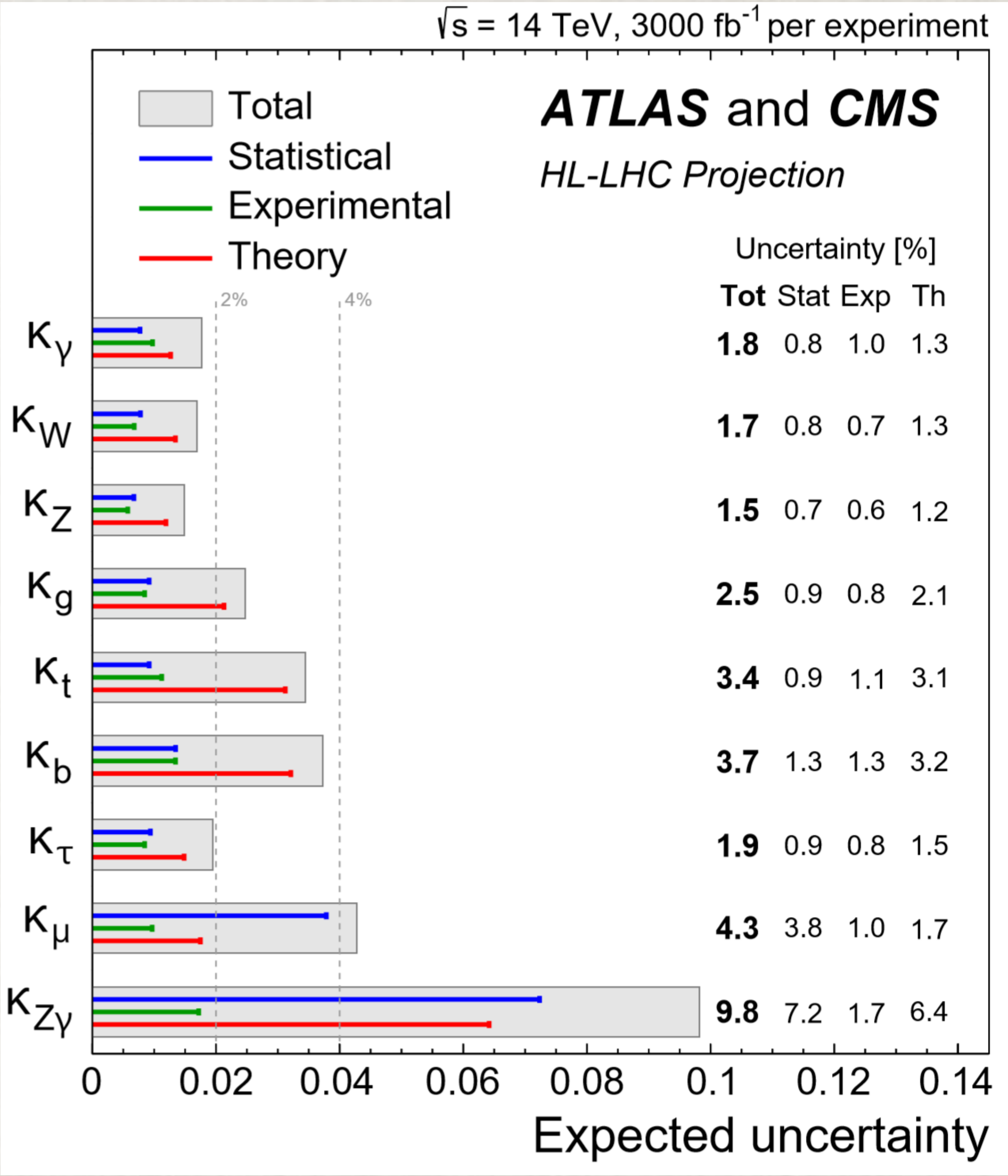
Discovery through precision

Even in the absence of striking new physics signals, we can still grasp the scale of new physics by looking at deviations from SM expectations

After HL-LHC, uncertainties will be largely dominated by theory

Precision is the key: can theory accuracy be pushed at the % level?

Higgs couplings after HL-LHC



[HL-LHC W2 report 1902.00134]

Outline

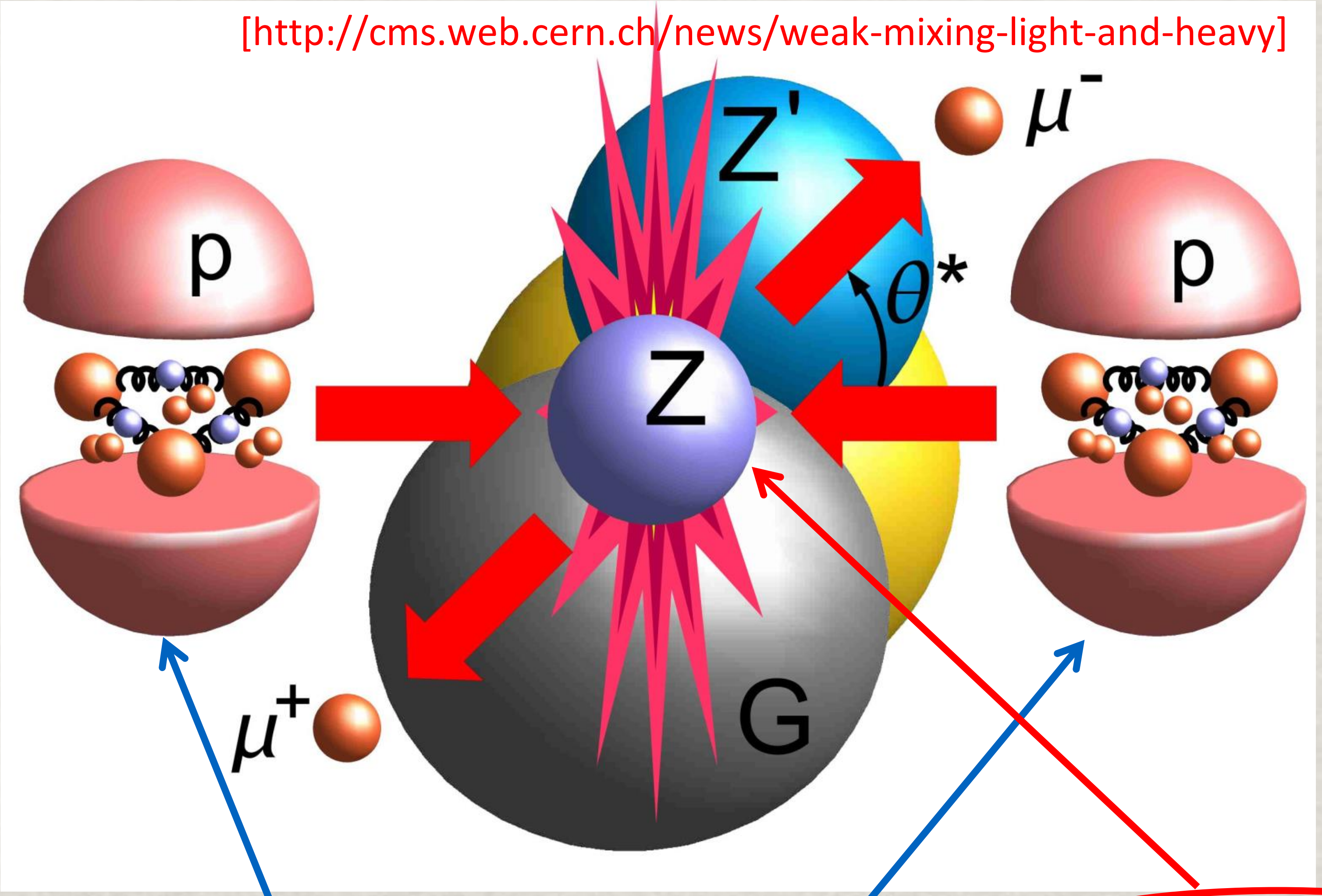
I will present some recent theoretical progress in precision QCD in its three main areas, using explicit examples as showcases of recent advances

- Fixed-order calculations
  - 2→3 processes @ NNLO: tree-jet production and $t\bar{t}H$
  - Mass effects in Higgs cross-sections
  - Heavy-flavour production
- Resummations
  - Global and non-global observables: where are we?
  - Transverse momentum distributions
- Parton-shower event generators
  - NNLO matching
  - The quest towards NNLL accuracy

Fixed-order calculations

Precision through PT QCD

Cross sections are computed in QCD via a factorisation formula



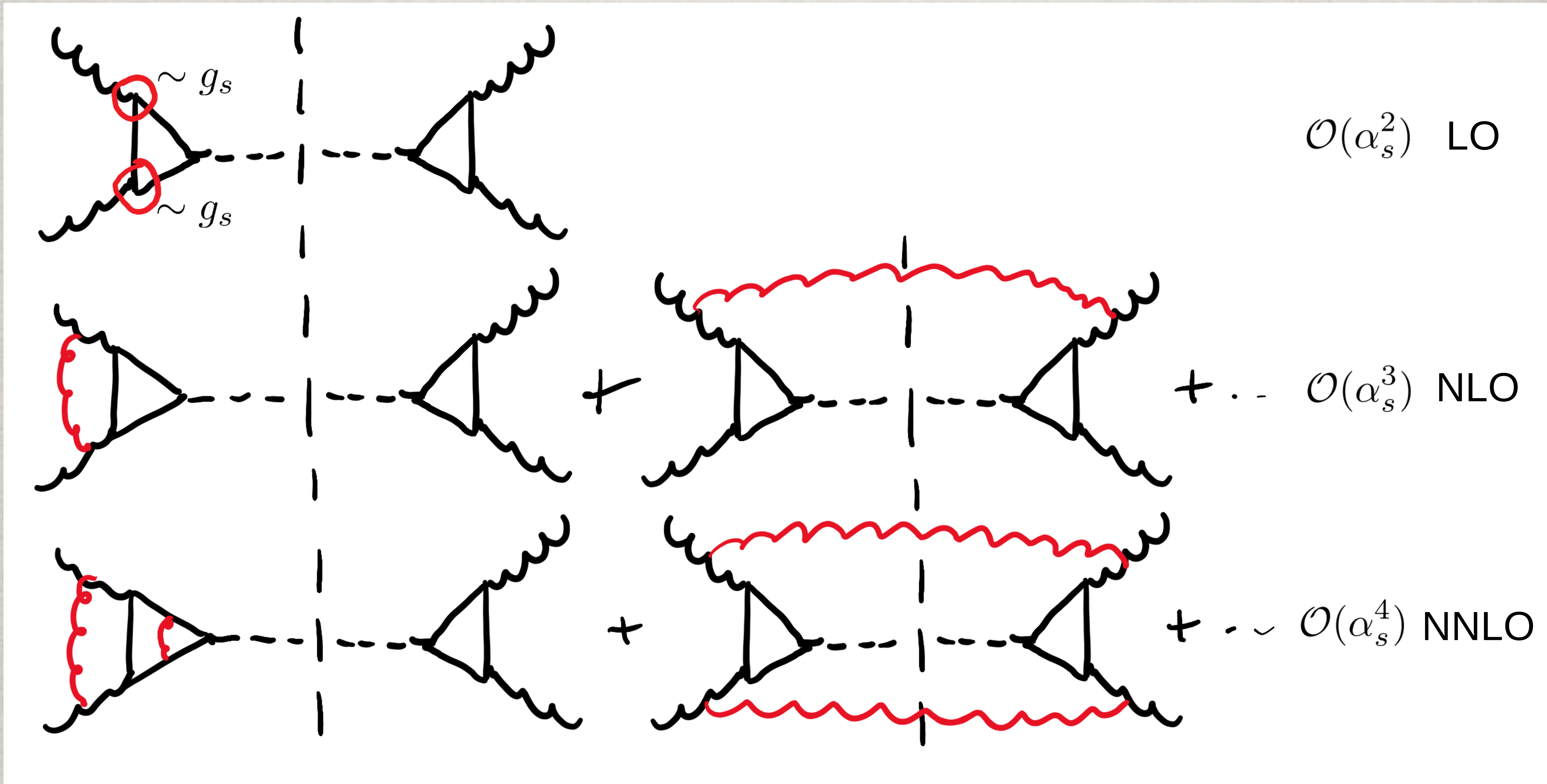
$$d\sigma_{pp\to X} \simeq \sum_{a,b} \int dx_a\, dx_b\, f_{a/p}(x_a, \mu_F)\, f_{b/p}(x_b, \mu_F) \times d\hat{\sigma}_{ab\to X}(\alpha_s(\mu_R), \mu_R, \mu_F)$$

PDFs: extracted from data evolution is perturbative

Partonic cross section: expansion in the QCD coupling

Perturbative QCD

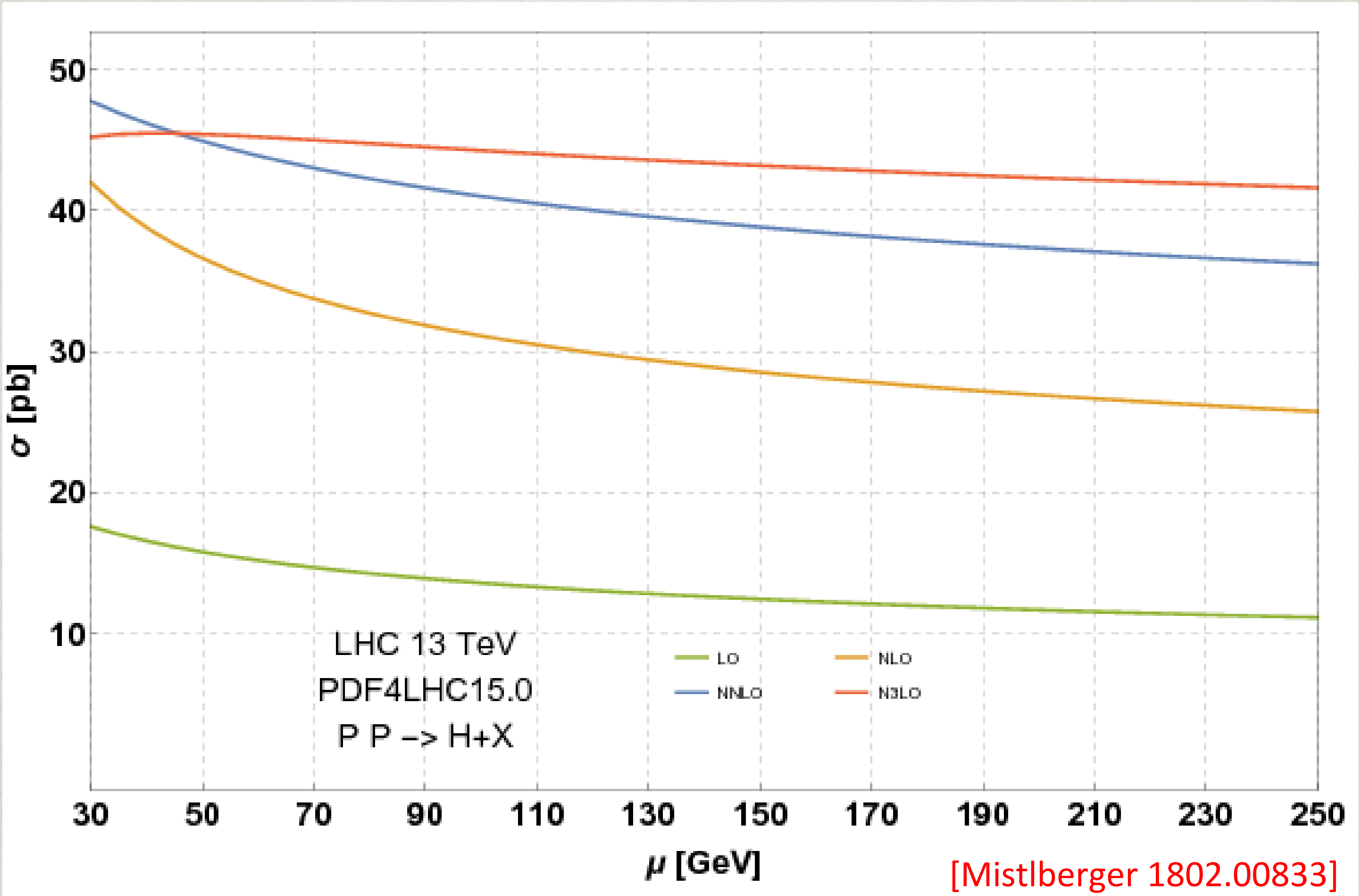
We need to square real and virtual amplitudes according to the desired order



Obtaining numerically stable results requires ensuring the cancellation of infrared and collinear (IRC) singularities between real and virtual contributions

QCD is strong interactions

QCD cross sections converge slowly ⇒need to go to high perturbative order to have satisfactory theoretical control



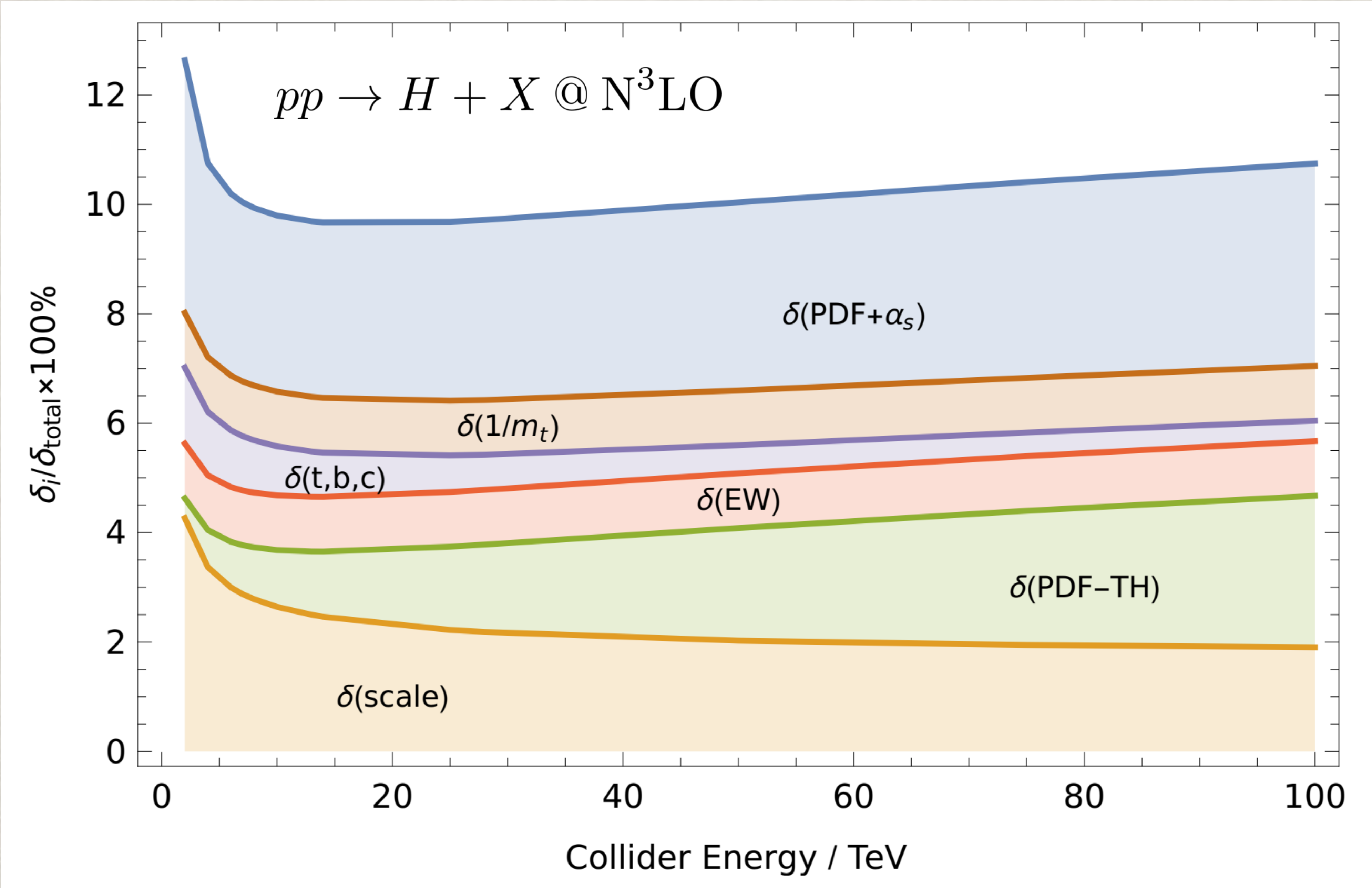
[Mistlberger 1802.00833]

Inclusive observables (e.g. Higgs total cross-sections in the heavy-top limit) can be computed at a high level of accuracy, at least at N^3LO

[Fully differential Higgs cross-section: Chen Gehrmann Glover Huss Mistlberger Pelloni 2102.07607]

QCD IS STRONG INTERACTIONS

Pushing theoretical uncertainties below 10% is challenging and requires control over many effects, including PDFs and EW corrections



[Dulat Lazopoulos Mistlberger 1802.00827]

THE FIXED-ORDER WISH LIST

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO^{(t)}_{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.) $\mathrm{NNLO^{(b,c)}_{QCD}}$
$pp \to H + j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$
$pp \to H + 2j$	$\mathrm{NLO_{HTL} \otimes LO_{QCD}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ $\mathrm{NNLO^{(VBF)}_{QCD}}$
$pp \to H + 3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to VH$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{NLO^{(t,b)}_{gg\to HZ}}$	
$pp \to VH + j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$	$\mathrm{NLO_{EW}}$
$pp \to HH + 2j$	$\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	
$pp \to HHH$	$\mathrm{NNLO_{HTL}}$	
$pp \to H + t\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO_{QCD}}$ (off-diag.)	$\mathrm{NNLO_{QCD}}$
$pp \to H + t/\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to 2\,\text{jets}$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + NLO_{EW}}$
$pp \to 3\,\text{jets}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{+\,NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
$pp \to V + b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to VV' + 1j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to VV' + 2j$	$\mathrm{NLO_{QCD}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^+W^+ + 2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to W^+W^- + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to W^+Z + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to ZZ + 2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to VV'V''$	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^\pm W^+ W^-$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to \gamma\gamma$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma\gamma + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{+\,NLO_{QCD}}$ (gg channel)	
$pp \to \gamma\gamma\gamma$	$\mathrm{NNLO_{QCD}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$

[Huss Huston Jones Pellen 2207.02122, update on 2406.00708]

THE FIXED-ORDER WISH LIST

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO^{(t)}_{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.) $\mathrm{NNLO^{(b,c)}_{QCD}}$
$pp \to H+j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$
$pp \to H+2j$	$\mathrm{NLO_{HTL} \otimes LO_{QCD}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ $\mathrm{NNLO^{(VBF)}}$
$pp \to H+3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	NL(
$pp \to VH$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{NLO^{(t,b)}_{gg\to HZ}}$	
$pp \to VH+j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	NNI
$pp \to HH$	$\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$	NL(
$pp \to HH+2j$	$\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	
$pp \to HHH$	$\mathrm{NNLO_{HTL}}$	
$pp \to H+t\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO_{QCD}}$ (off-diag.)	NNI
$pp \to H+t/\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	NNI
$pp \to 2$ jets	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to 3$ jets	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{+NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V+j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	hadronic decays
$pp \to V+2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
		$\mathrm{NNLO_{QCD} + NLO_{EW}}$
		$\mathrm{NNLO_{QCD}}$
		Full $\mathrm{NLO_{QCD} + NLO_{EW}}$
		$\mathrm{NLO_{QCD} + NLO_{EW}}$
		$\mathrm{N^3LO_{QCD}}$
		$\mathrm{N^3LO_{QCD}}$
		$\mathrm{NNLO_{QCD} + NLO_{EW}}$

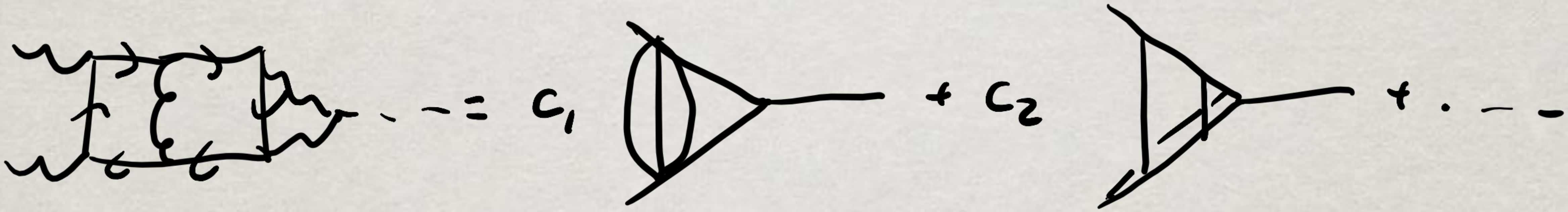
process	known	desired
$pp \to t\bar{t}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell effects) $\mathrm{NNLO_{QCD}}$ (w/ decays)	$\mathrm{N^3LO_{QCD}}$
$pp \to t\bar{t}+j$	$\mathrm{NLO_{QCD}}$ (off-shell effects) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/ decays)
$pp \to t\bar{t}+2j$	$\mathrm{NLO_{QCD}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/ decays)
$pp \to t\bar{t}+V'$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/ decays)
$pp \to t\bar{t}+\gamma$	$\mathrm{NLO_{QCD}}$ (off-shell effects)	
$pp \to t\bar{t}+Z$	$\mathrm{NLO_{QCD}}$ (off-shell effects)	
$pp \to t\bar{t}+W$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell effects)	
$pp \to t/\bar{t}$	$\mathrm{NNLO_{QCD}}$*(w/ decays) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/ decays)
$pp \to tZj$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/ decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays)
$pp \to t\bar{t}t\bar{t}$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell effects) $\mathrm{NNLO_{QCD}}$

[Huss Huston Jones Pellen 2207.02122, update on 2406.00708]

NNLO is the state of the art, and for many processes, $\mathrm{N^3LO}$ is needed

NNLO ingredients: virtual

- We need to know all relevant amplitudes up to two loops (and higher for N^{3}LO and beyond) in $D = 4 - 2\epsilon$ dimensions. This requires
- Decomposition of the amplitude in terms of master integrals



NNLO ingredients: virtual

- We need to know all relevant amplitudes up to two loops (and higher for N^{3}LO and beyond) in $D = 4 - 2\epsilon$ dimensions. This requires
- Decomposition of the amplitude in terms of basis integrals
- Calculation of the basis integrals
  - All two-loop integrals for massless processes up to 2→3 are known; they generally involve multiple polylogarithms

    [Agarwal Buccioni Devoto Gambuti von Manteuffel Tancredi 2311.09870]
  - Integrals with massive particles in the loops involve elliptic functions and are currently a subject of intense studies from a mathematical point of view

    [SAGEX review: Blümlein Schneider 2203.13015]
  - All loop integrals can be expressed as integrals over Feynman parameters with deformed contours in the complex plane $\Rightarrow$ numerical evaluation

    [pySecDec: Heinrich Jahn Jones Kerner Langer Magerya Pöldrau Schlenk Villa 2108.10807]
    [FIESTA: Smirnov Shapurov Visotsky 2110.11660]

NNLO ingredients: real

- Real emission matrix elements are also IR divergent. The divergences need to be subtracted before integrating them over the phase space
- There are various methods to achieve the subtraction. These are automated at NLO, but no general formula at NNLO

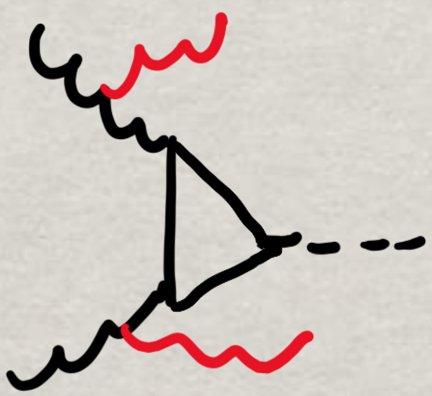
$$\int \left[d\sigma^{R}_{n+1} J_{n+1} - d\sigma^{\text{sub}}_{n+1} J_n \right] + \int \left[d\sigma^{V}_{n} - d\sigma^{\text{sub}}_{n+1} \right] J_n$$

numerical cancellation of singularities in 4 dimensions

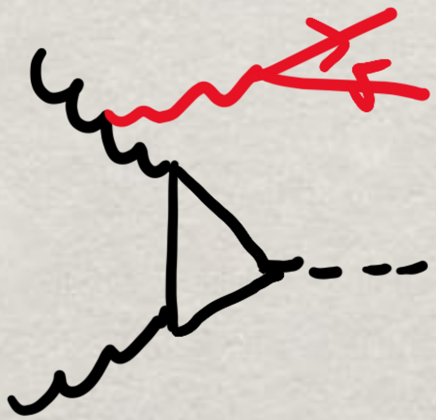
analytic cancellation of singularities in D dimensions

measurement function

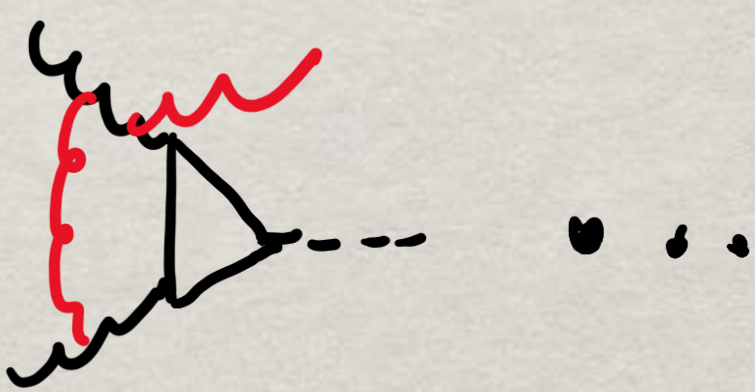
- At NNLO, real IR singularities are more involved and need to be systematically subtracted with the appropriate counterterms



double soft



triple collinear



soft-virtual

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- There are various methods to achieve the subtraction. These are automated at NLO, but no general formula at NNLO

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measurement function

numerical cancellation of singularities in 4 dimensions

analytic cancellation of singularities in D dimensions

- At NNLO, real IR singularities are more involved, and need to be systematically subtracted with the appropriate counterterms

- The singular structure of gauge theories at NNLO is known: the bottleneck is the analytic cancellation of singularities between virtual corrections and counterterms

NNLO ingredients: real

- Real emission matrix elements are also IR divergent. The divergences need to be subtracted before integrating them over the phase space

- There are various methods to achieve the subtraction. These are automated at NLO, but no general formula at NNLO

$$\int \left[d\sigma_{n+1}^{R} J_{n+1} - d\sigma_{n+1}^{\mathrm{sub}} J_n\right] + \int \left[d\sigma_n^{V} - d\sigma_{n+1}^{\mathrm{sub}}\right] J_n$$

- Phase space slicing: introduce a resolution variable v_{cut}, as small as possible

$$d\sigma^{\mathrm{sub}} = d\sigma_{n+1}^{R}\Theta(v_{\mathrm{cut}} - V_{n+1}) \implies \begin{array}{l} \int \left[d\sigma_{n+1}^{R} J_{n+1} - d\sigma_{n+1}^{\mathrm{sub}} J_n\right] = \int^{v_{\mathrm{cut}}} d\sigma_{n+1}^{R} J_{n+1} \\ \int \left[d\sigma_n^{V} - d\sigma_{n+1}^{\mathrm{sub}}\right] = \Sigma_n(v_{\mathrm{cut}}) \end{array}$$

- The function $\Sigma_n(v_{\mathrm{cut}})$ needs to be computed as accurately as possible, and at least including all non-vanishing contributions for $v_{\mathrm{cut}} \to 0$

NNLO ingredients: real

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- There are various methods to achieve the subtraction. These are automated at NLO, but no general formula at NNLO

$$\int \left[d\sigma^{R}_{n+1} J_{n+1} - d\sigma^{\rm sub}_{n+1} J_n \right] + \int \left[d\sigma^{V}_{n} - d\sigma^{\rm sub}_{n+1} \right] J_n$$

- Phase space slicing: introduce a resolution variable $v_{\rm cut}$, as small as possible
- Only two resolution variables are known at NNLO
  - the transverse momentum of a colour singlet or a top-antitop pair: allows to deal with initial-state radiation only [Catani Grazzini hep-ph/0703012]
  - N-jettiness (related to the invariant mass of the hard jets): enables final-state radiation, known up to V+jet [Gaunt Stahlhofen Tackmann Walsh 1505.04794]

NNLO ingredients: real

- Real emission matrix elements are also IR divergent. The divergences need to be subtracted before integrating them over the phase space
- There are various methods to achieve the subtraction. These are automated at NLO, but no general formula at NNLO

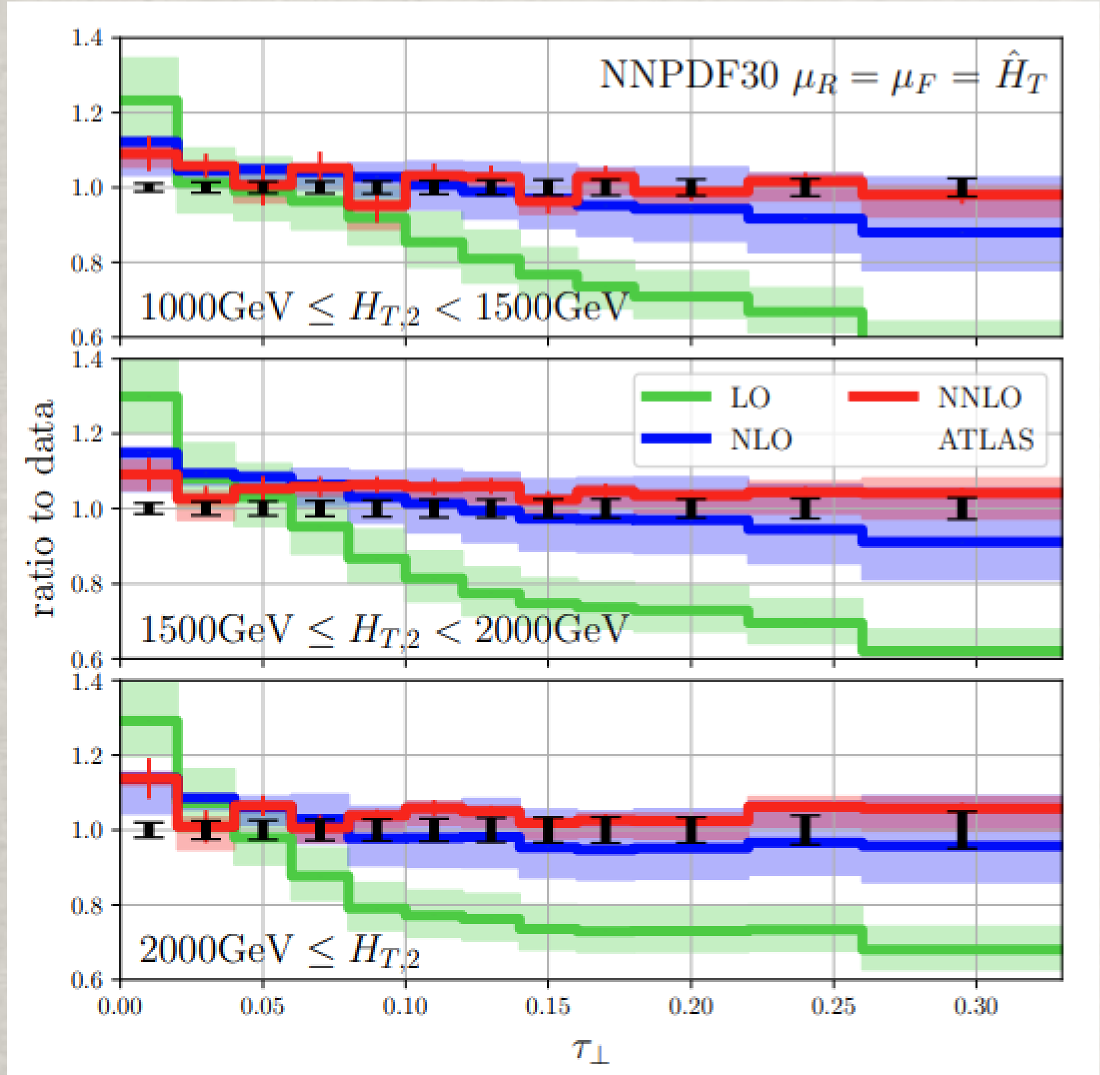
$$\int \left[d\sigma^R_{n+1} J_{n+1} - d\sigma^{\rm sub}_{n+1} J_n\right] + \int \left[d\sigma^V_n - d\sigma^{\rm sub}_{n+1}\right] J_n$$

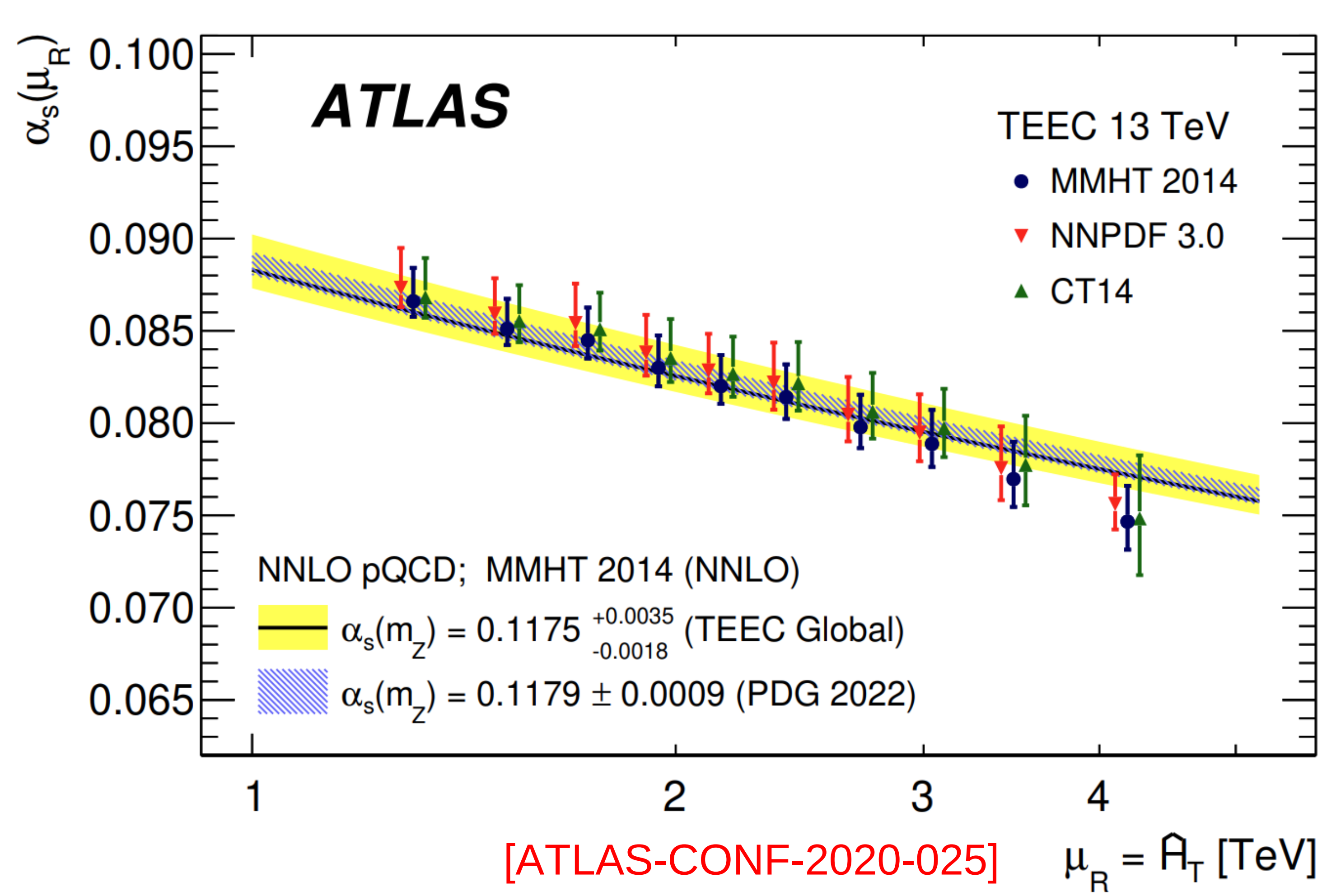
- Subtraction: $d\sigma^{\rm sub}$ reproduces the singularities of QCD amplitudes, but its integral over the full phase space must be computed analytically in D dimensions

[Antennae: Gehrmann-de Ridder Gehrmann hep-ph/050511]
[CoLoRFul: Del Duca Durh Somogyi Tramontano Trócsánzi Tulipánt 1606.03453]
[Sector-improved: Czakon 1005.0274]
[Nested soft-collinear: Caola Melnikov Röntsch 1702.01352]
[Projection to Born: Cacciari Dreyer Karlberg Salam Zanderighi 1506.02660]
[Local analytic sector: Magnea Maina Pelliccioli Signorile-Signorile Torrielli Uccirati 1806.09570]

Three-jet production

- Important 2→3 process for the measurement of the QCD running coupling
- Phenomenological predictions are based quasi full-colour amplitudes and subtraction with improved sector decomposition [Czakon Mitov Poncelet 2106.05331]





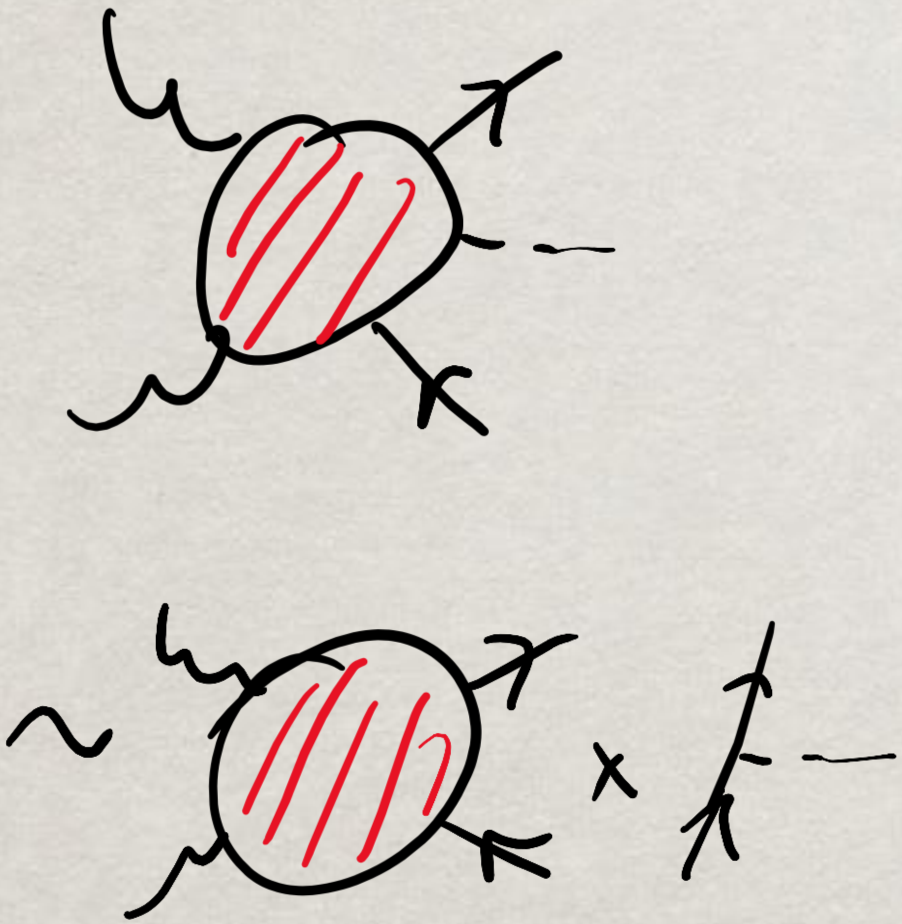
[ATLAS-CONF-2020-025]

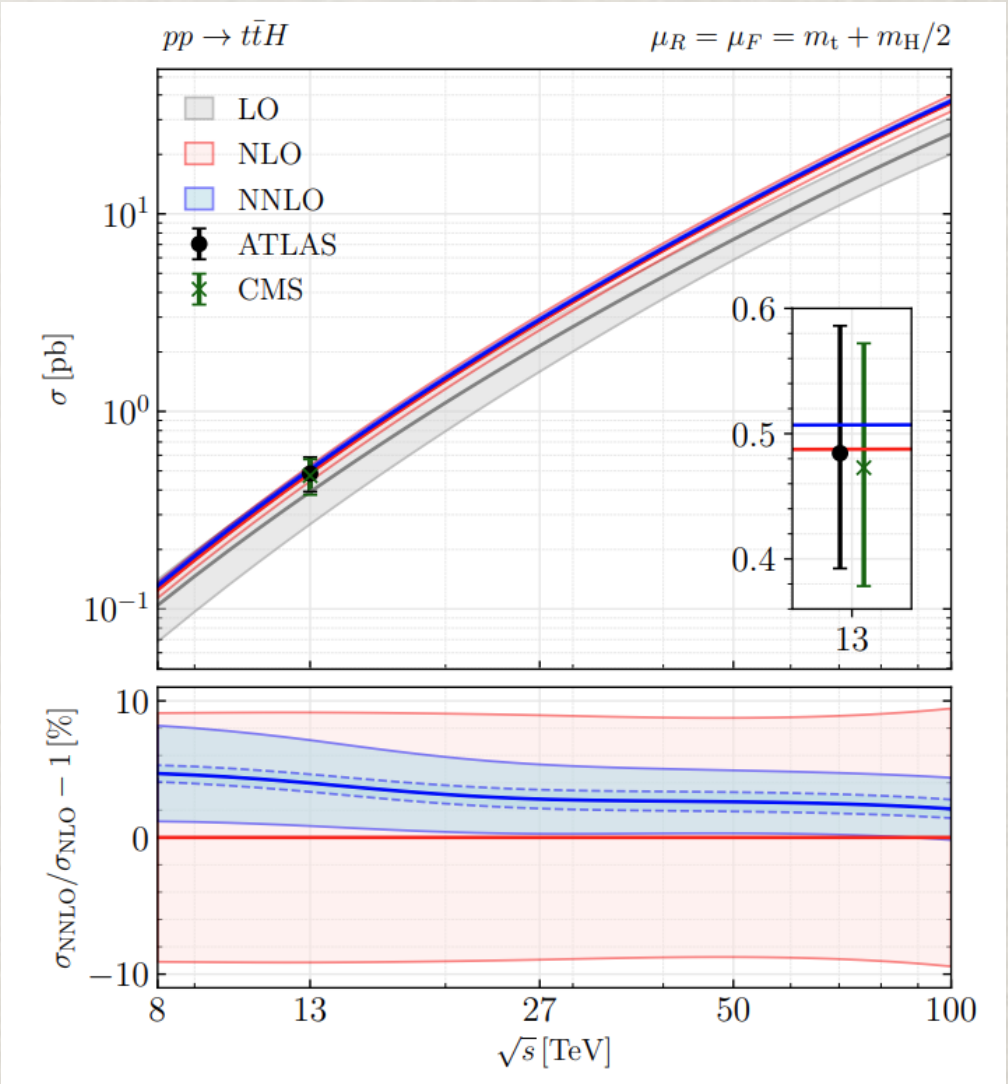
The full massless five-parton scattering amplitudes at two loops have become available

[Agarwal Buccioni Devoto Gambuti von Manteuffel Tancredi 2311.09870]

Higgs in association with tops

Important 2→3 process for direct access to the top Yukawa





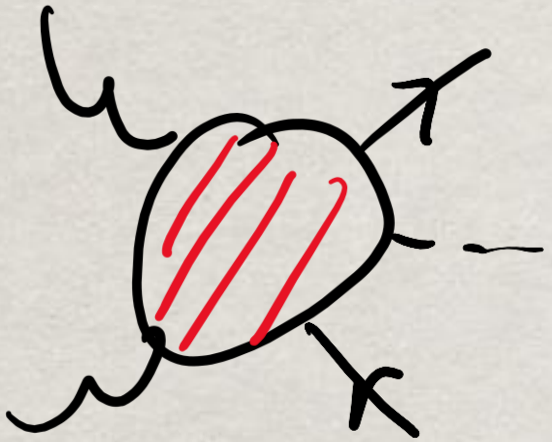
- Approximate NNLO exist treating the Higgs as soft

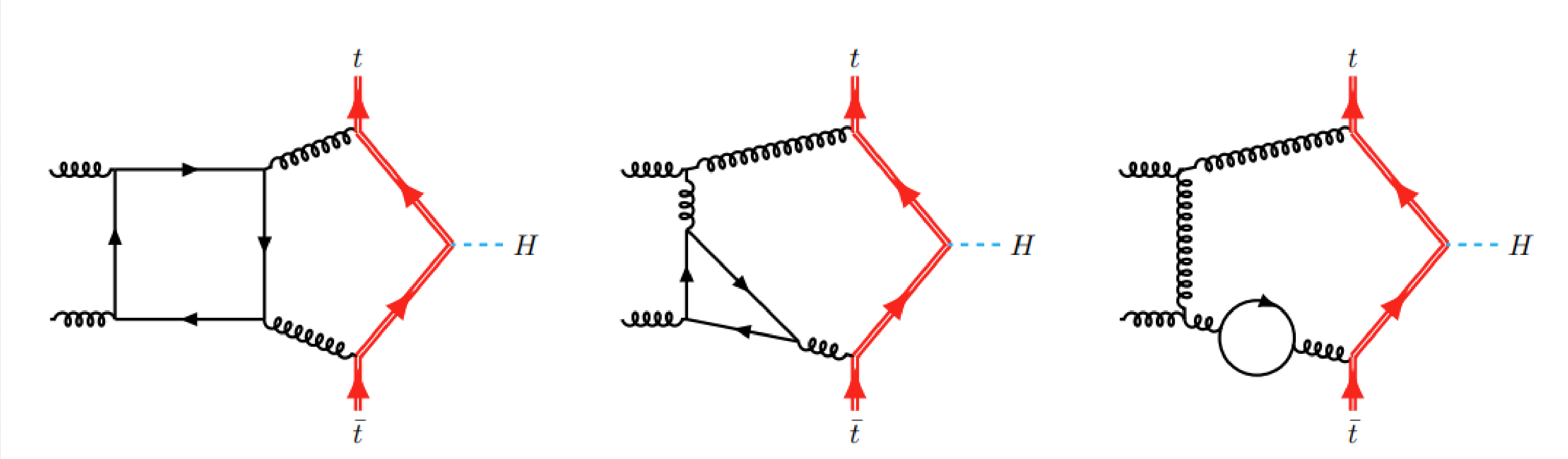
[Catani Devoto Grazzini Kallweit Mazzitelli Savoini 2210.07846]

- IR singularities eliminated using q_T subtraction

Higgs in association with tops

Important 2→3 process for direct access to the top Yukawa





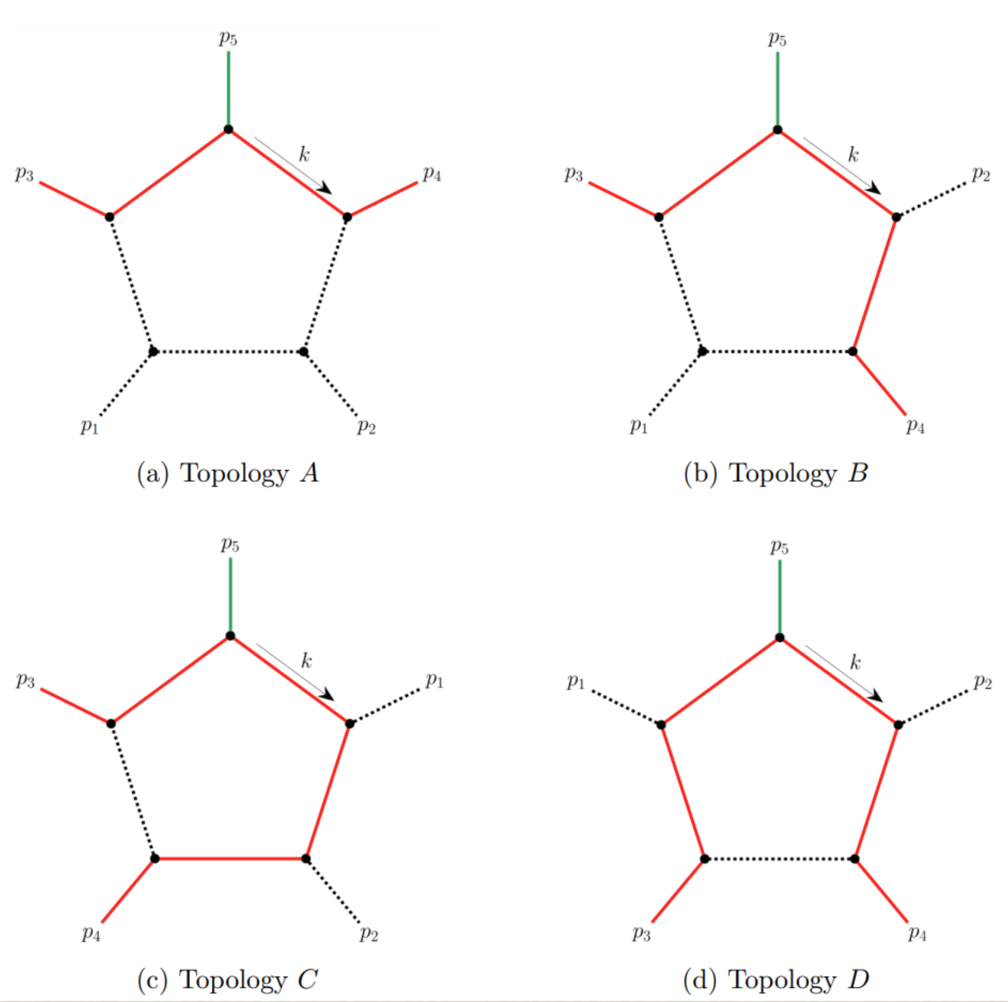
Recent steps towards full NNLO

- Two-loop integrals proportional to the number of light flavours

  [Cordero Figuereido Kraus Page Reina 2312.08131]

- One-loop corrections to $gg \to t\bar{t}H$ up to order ϵ^2, as needed for NNLO

  [Buccioni Kreer Liu Tancredi 2312.10015]

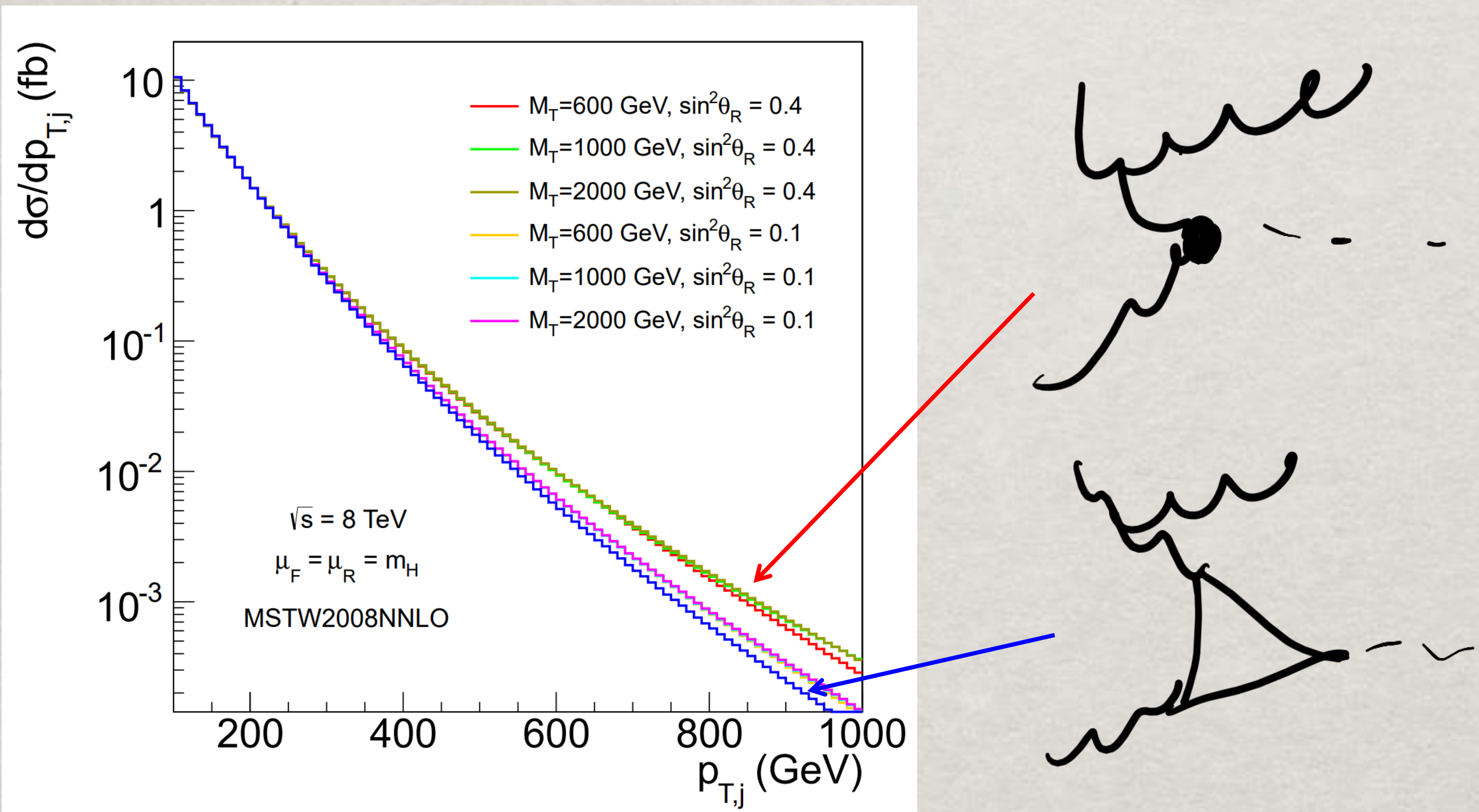


Mass effects in Higgs production

- The tail of the p_T distribution of a Higgs or the recoiling jet is sensitive to higher-dimensional operators, and/or heavy top partners (e.g. stops)

[Azatov Paul 1309.5273]
[Grojean Salvioni Schlaffer Weiler 1312.3317]
[AB Martin Sanz 1308.4771]

- The top is light in this regime, so exact mass effects are crucial



$$\kappa_g \frac{\alpha_s}{12\pi} \frac{h}{v} G^a_{\mu\nu} G^{a,\mu\nu}$$

$$\kappa_t m_t \frac{h}{v} \bar{t} t$$

Mass effects in Higgs production

- The tail of the p_T distribution of a Higgs or the recoiling jet is sensitive to higher-dimensional operators, and/or heavy top partners (e.g. stops)

[Azatov Paul 1309.5273]
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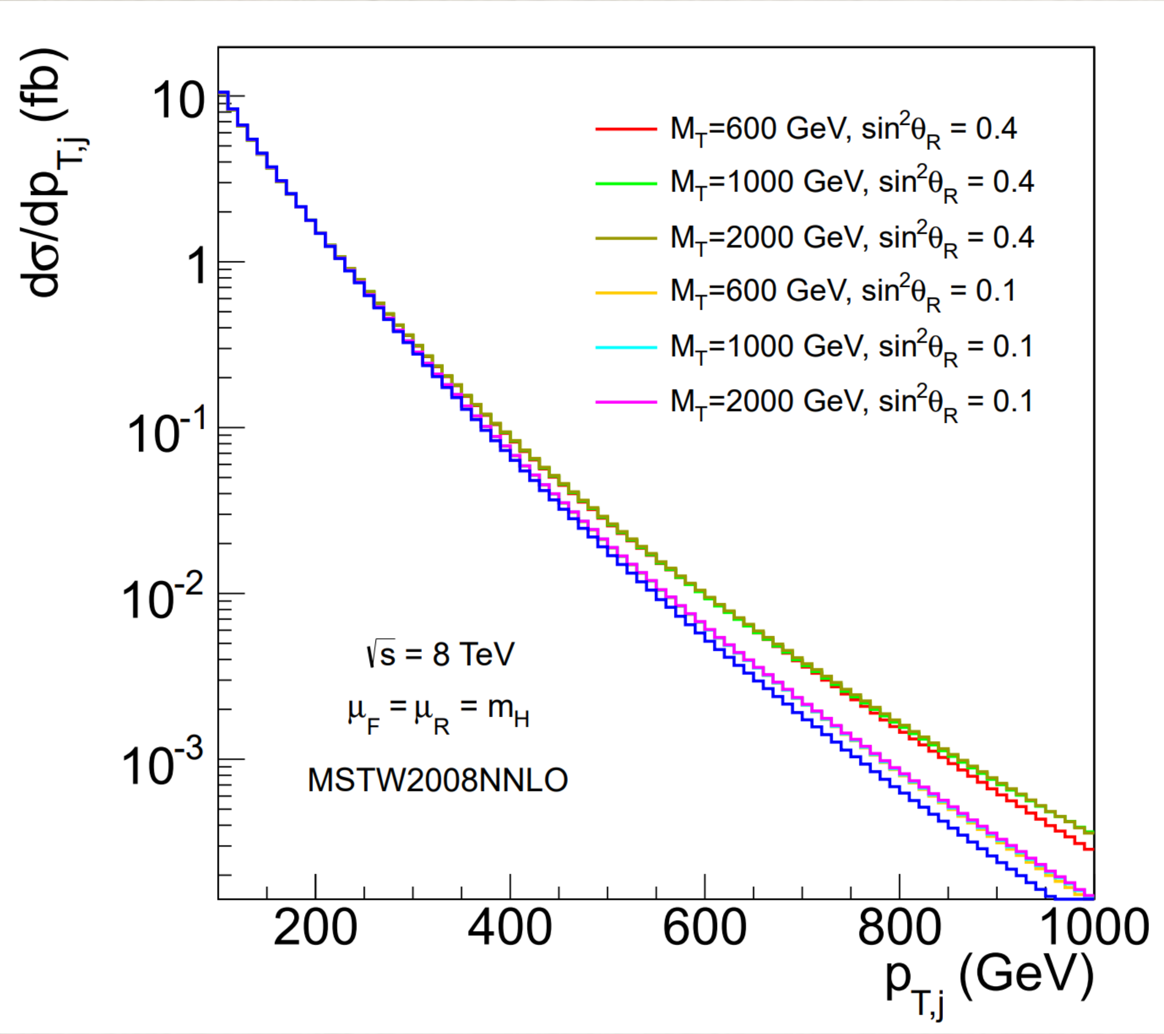
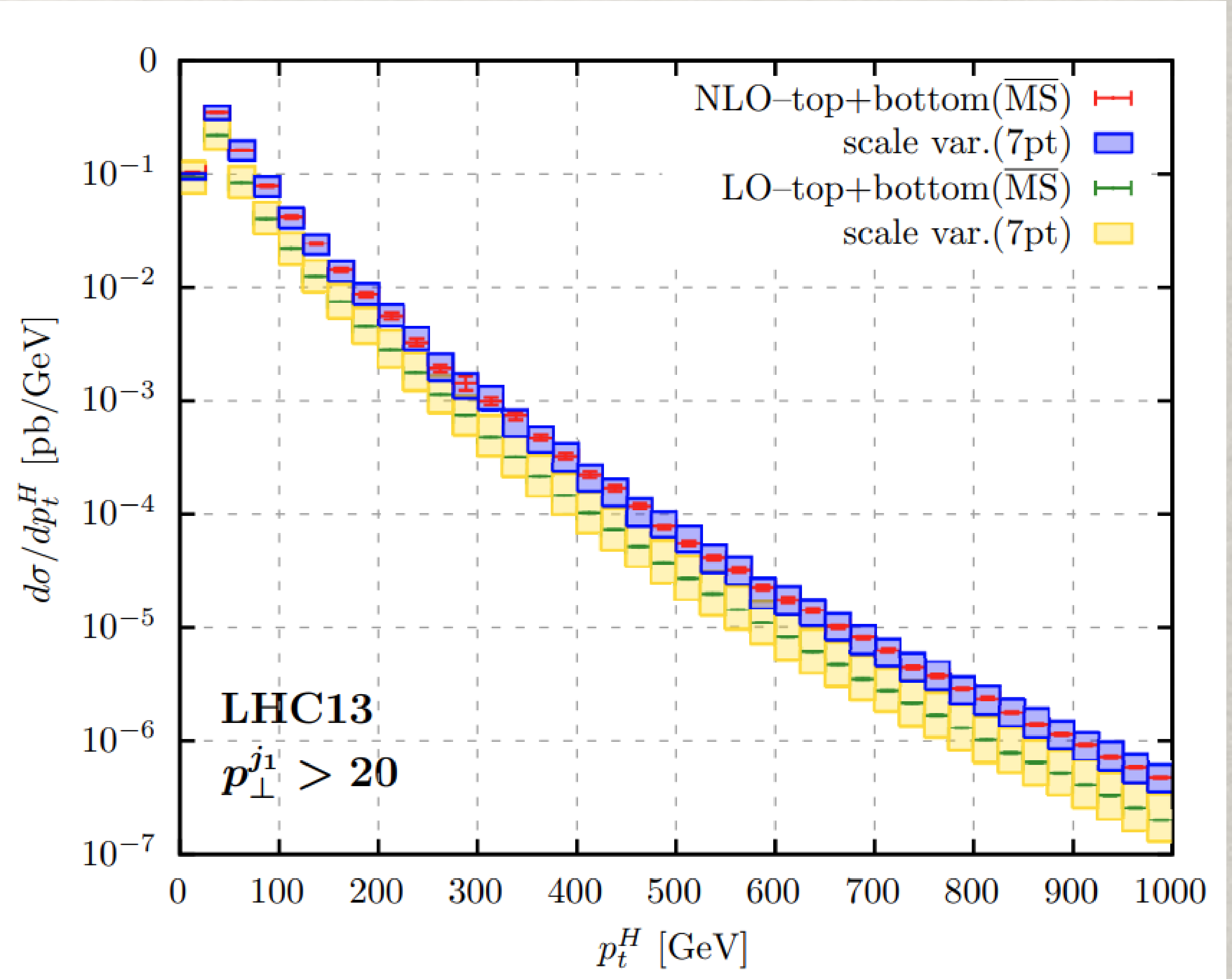
[AB Martin Sanz 1308.4771]

- The top is light in this regime, so exact mass effects are crucial



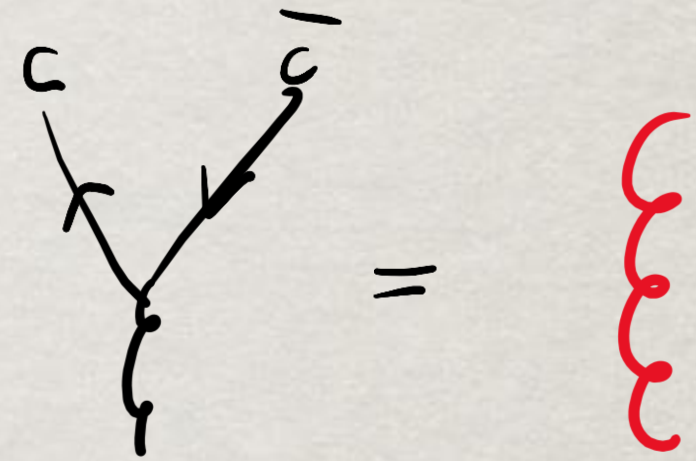


NLO (two-loop) corrections have finally been computed using differential equations: their contribution is very large

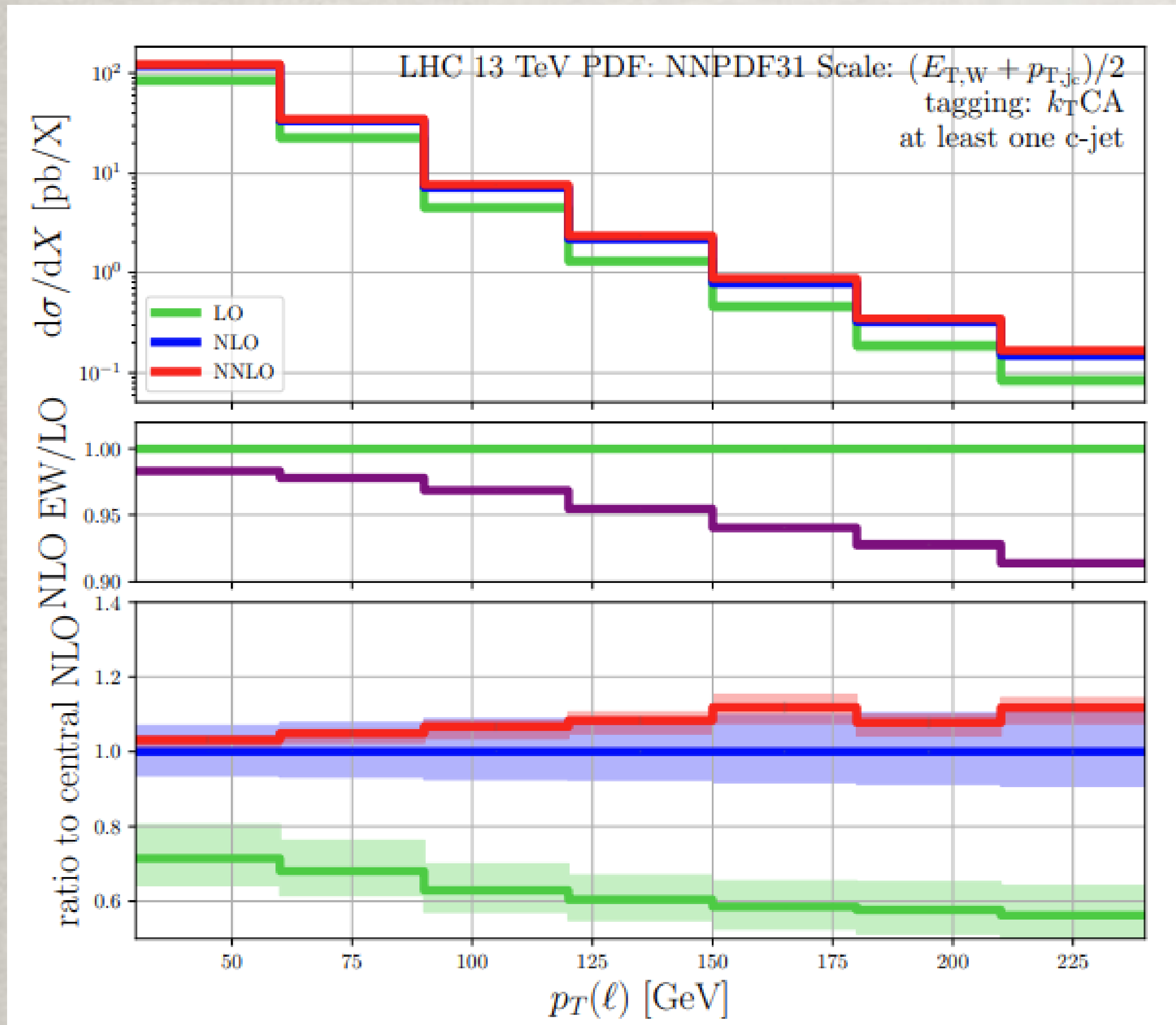
[Bonciani Del Duca Frellesvig Hidding Hirschli 2206.10490]

Heavy-flavoured jets

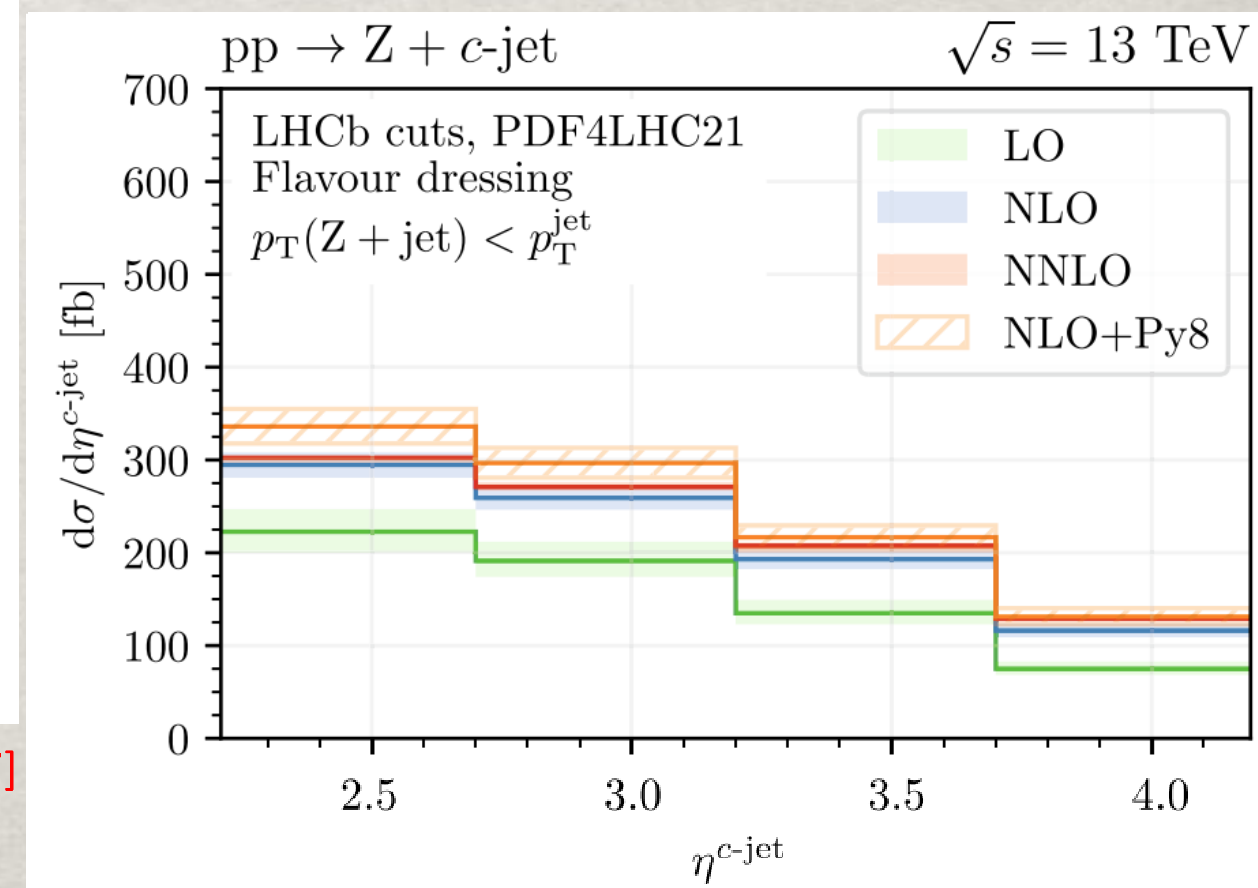
New NNLO predictions for vector boson and a flavoured jet use IRC safe jet-flavour algorithms $\Rightarrow$ heavy quarks can be considered massless



[Gauld et al 2302.12844]

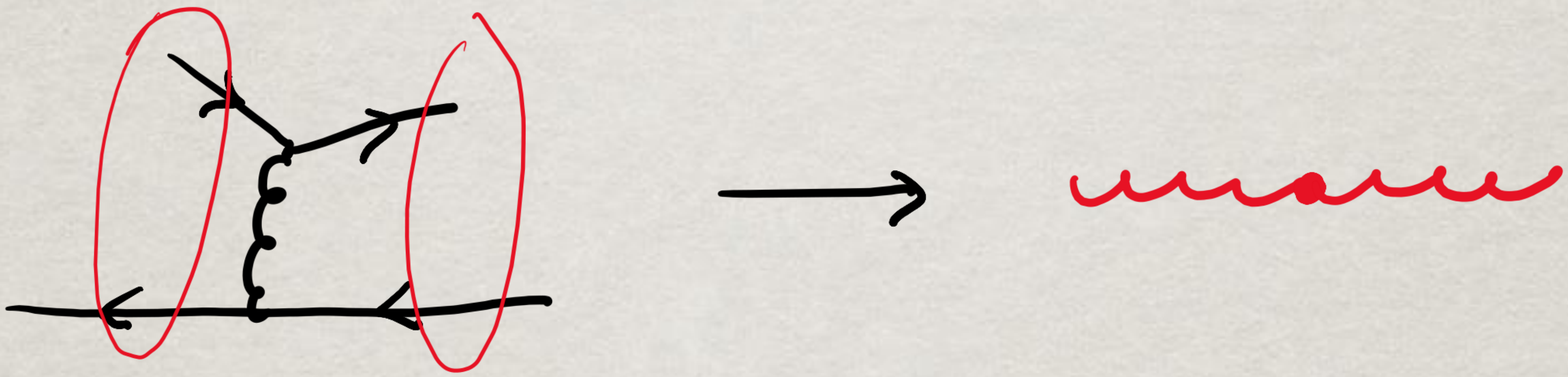


[Czakon Mitov Pellen Poncelet 2212.00467]



Heavy-flavoured jets

- The main theoretical problem of heavy-flavoured jets is to prevent the clustering of large-angle quarks originating from a single gluon splitting into different jets



- Some IRC safe flavour-jet algorithms (including the very first one) modify the jet resolution, however this does not lead to anti-k_t jets [AB Salam Zanderighi hep-ph/0601139] [Czakon Mitov Pellen Poncelet 2212.00467]

- New solutions to the problem based on neutralising the flavour of the quarks that would need to be recombined: this leads to jet with anti-k_t kinematics and the correct flavour assignment [Gauld Huss Stagnitto 2208.11138] [Caola Grabarczyk Hutt Salam Scyboz Thaler 2306.07314]

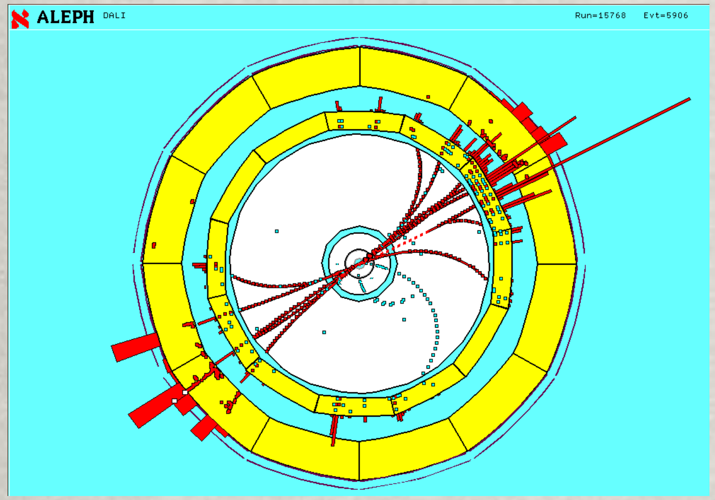
- These new jet algorithms have not been implemented yet in current experiments

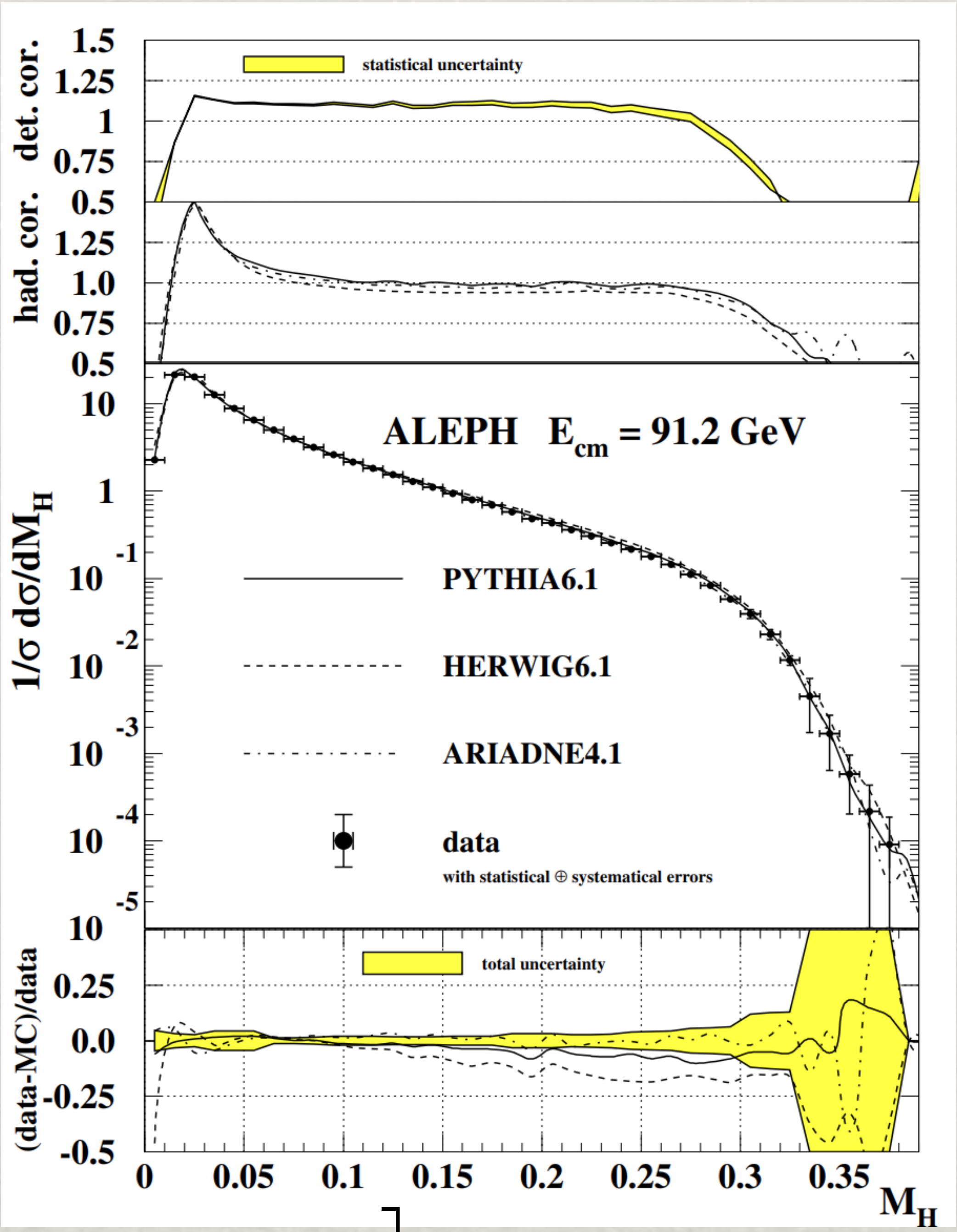
Resummations

Why do we resum?

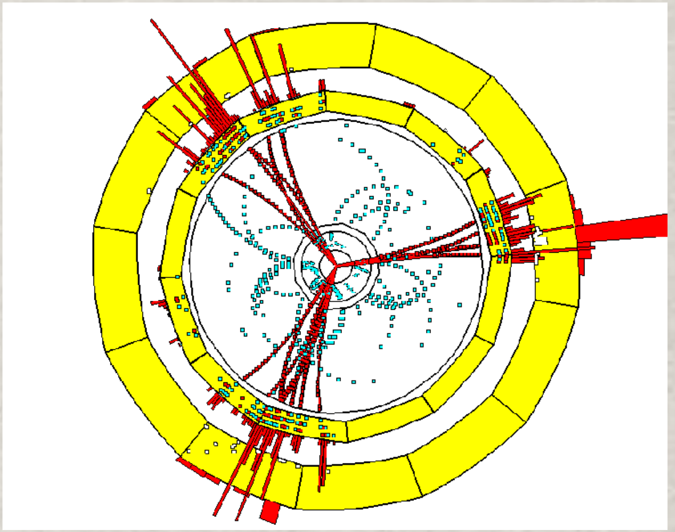
For many jet observables, the bulk of events lies in the region $\alpha_s L \sim 1$, so large logaritms L of the observable's value compensate the smallness of α_s

resummation





fixed order



$$\sim \exp\left[\underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \dots\right]$$

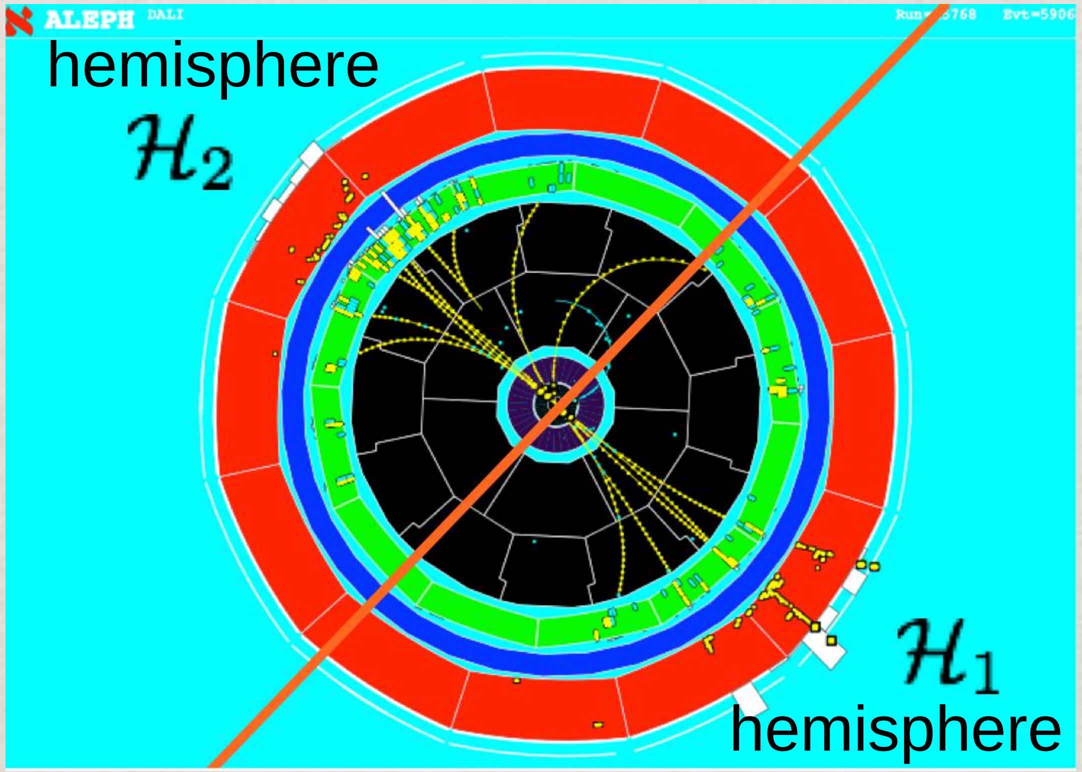
$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \dots$$

Global vs non-global

Global observables are those whose rate does not restrict emissions in a selected phase-space region

global

$$\rho_H = \max\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$



non-global

$$\rho_L = \min\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$

Non-global observables are most common in hadron collisions, where the definition of jet observables often imposes restrictions on hadron phase space (e.g. only measure hadrons inside the central tracker region)

Status of global resummation

The majority of global event shapes and jet resolution parameters can be resummed at very high logarithmic accuracy

- NLL resummation is a solved problem, and can be performed automatically with the CAESAR program [AB Salam Zanderighi hep-ph/0407286]
- Some observables (e.g. thrust, broadening) enjoy factorisation theorems in SCET, which enables N^3LL resummation [Becher Schwartz 0803.0342] [Becher Bell 1210.0580] [Hoang Kolodubrez Mateu Stewart 1501.04111]
- General NNLL resummation of factorisable observables in SCET with the semi-numerical program SoftServe [Bell Rahn Talbert 2004.08396]
- General semi-numerical NNLL resummation of event shapes and jet rates in e^+e^- annihilation with the ARES method [AB Monni McAslan Zanderighi 1412.2126] [AB Monni McAslan Zanderighi 1607.03111] [AB El-Menoufi Monni 1807.11487] [Arpino AB El-Menoufi 1912.09341]

Status of non-global resummation

- Non-global observables give rise to characteristic non-Abelian non-global logarithms (NGL) which can be resummed in two equivalent ways
  - non-linear equations
  - an infinite number of factorisation conditions

[Dasgupta Salam hep-ph/0104277]
[AB Marchesini Smye hep-ph/0206076]
[Becher Neubert 1605.02737]

- NL NGLs have been resummed in two equivalent ways
  - By constructing the NL generalisation of BMS equation

    [AB Dreyer Monni 2104.06416 , 2111.02413]
    [GNOLE https://github.com/non-global/gnole]
  - By computing the anomalous dimensions of the hard function at two loops

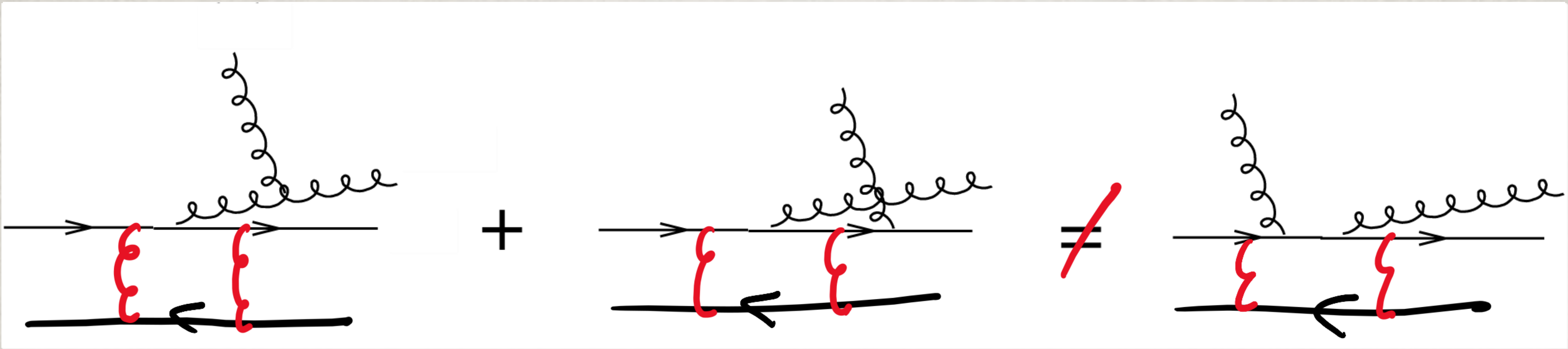
    [Becher Rau Xu 2112.02108]
- Resummation of NGLs has been extended to hadron

  [Becher Schalch Xu 2307.02283]
- Non-global observables in the presence of jet clustering algorithms give rise to additional Abelian "clustering" logarithms

  [AB Dasgupta hep-ph/0508159]
  [Becher Haag 2309.17355]

Coherence violating logarithms

Colour can be transferred between two initial-state hard partons



At the amplitude level, splitting functions acquire extra dependence on the momentum of all hard partons [Catani De Florian Rodrigo 1112.4405]

- In non-global observables, this leads to super-leading logarithms $(\alpha_s L^2)(\alpha_s L)^n$

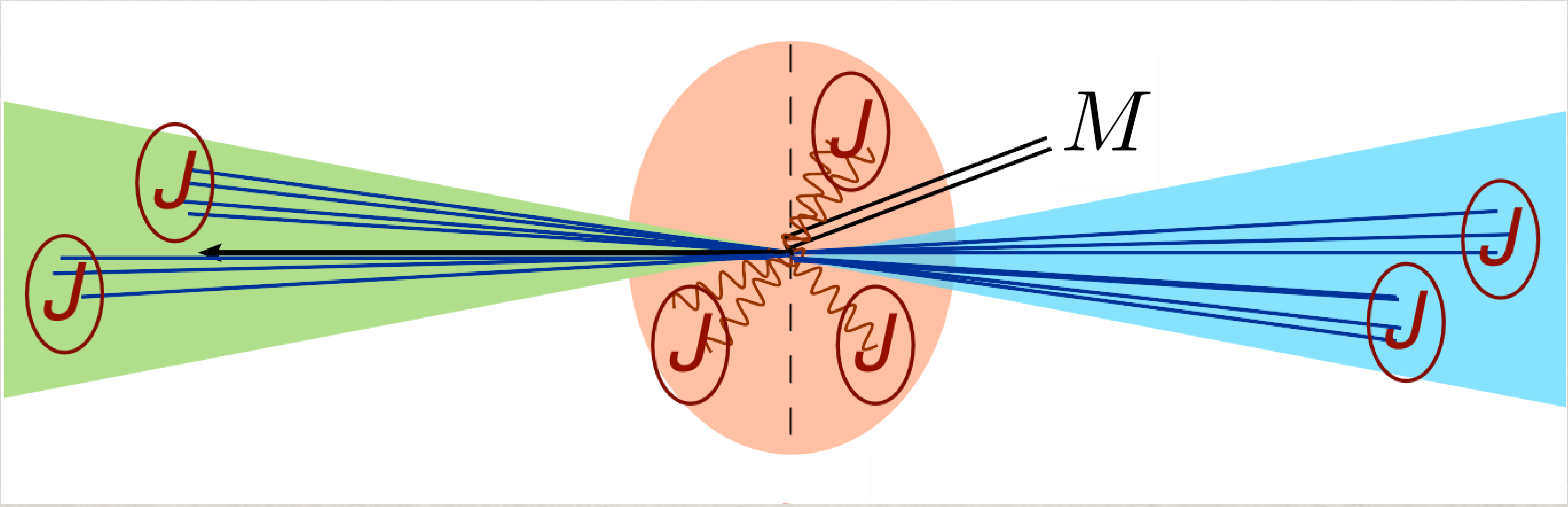
  [Forshaw Kyrieleis Seymour hep-ph/0604094]
  [Becher Neubert Shao Stillger 2307.06359]

- Do coherence-violating logarithms occur in global observables as well?

  [Forshaw Holguin 2109.03665]

Refactorisation

Both in QCD and Soft Collinear Effective Theory (SCET) it is possible to write "refactorisation" theorems: the resummation consists of the convolution of hard, jet and soft functions, that can be computed independently

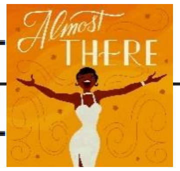


- All resummation functions are obtained via evolution equations involving anomalous dimensions and initial conditions that can be computed using loop techniques
- Involved observable-dependent coefficients can be computed numerically

Transverse momentum distribution

Impressive progress in the calculation of anomalous dimension and coefficient functions allows resummation up to approximate N^{4}LL accuracy

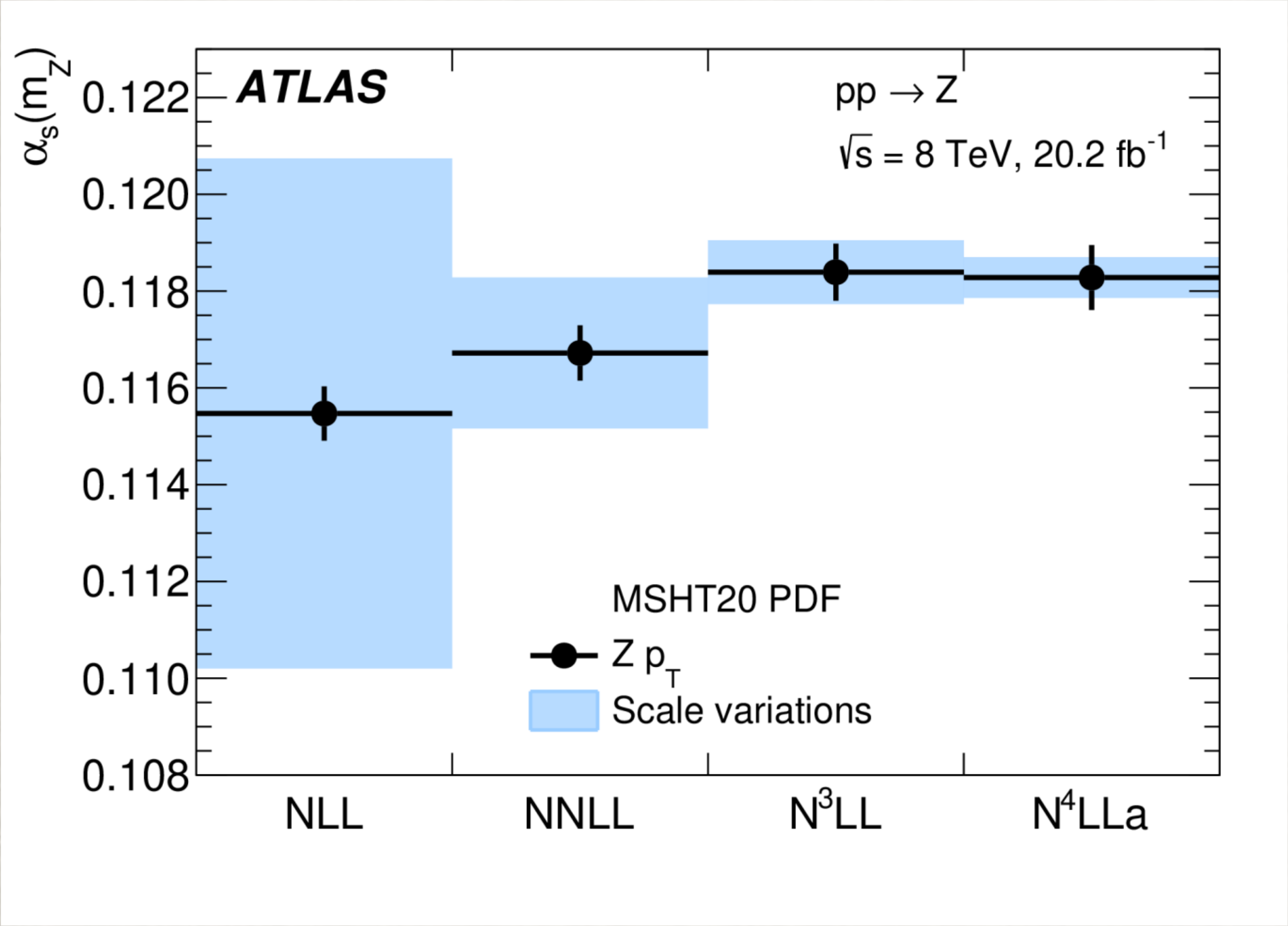
$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J,\nu)\otimes\mathcal{H}(M,\mu_H)\otimes\mathcal{S}(\mu_S,\nu)\otimes\mathcal{B}(\mu_J,\nu)$$

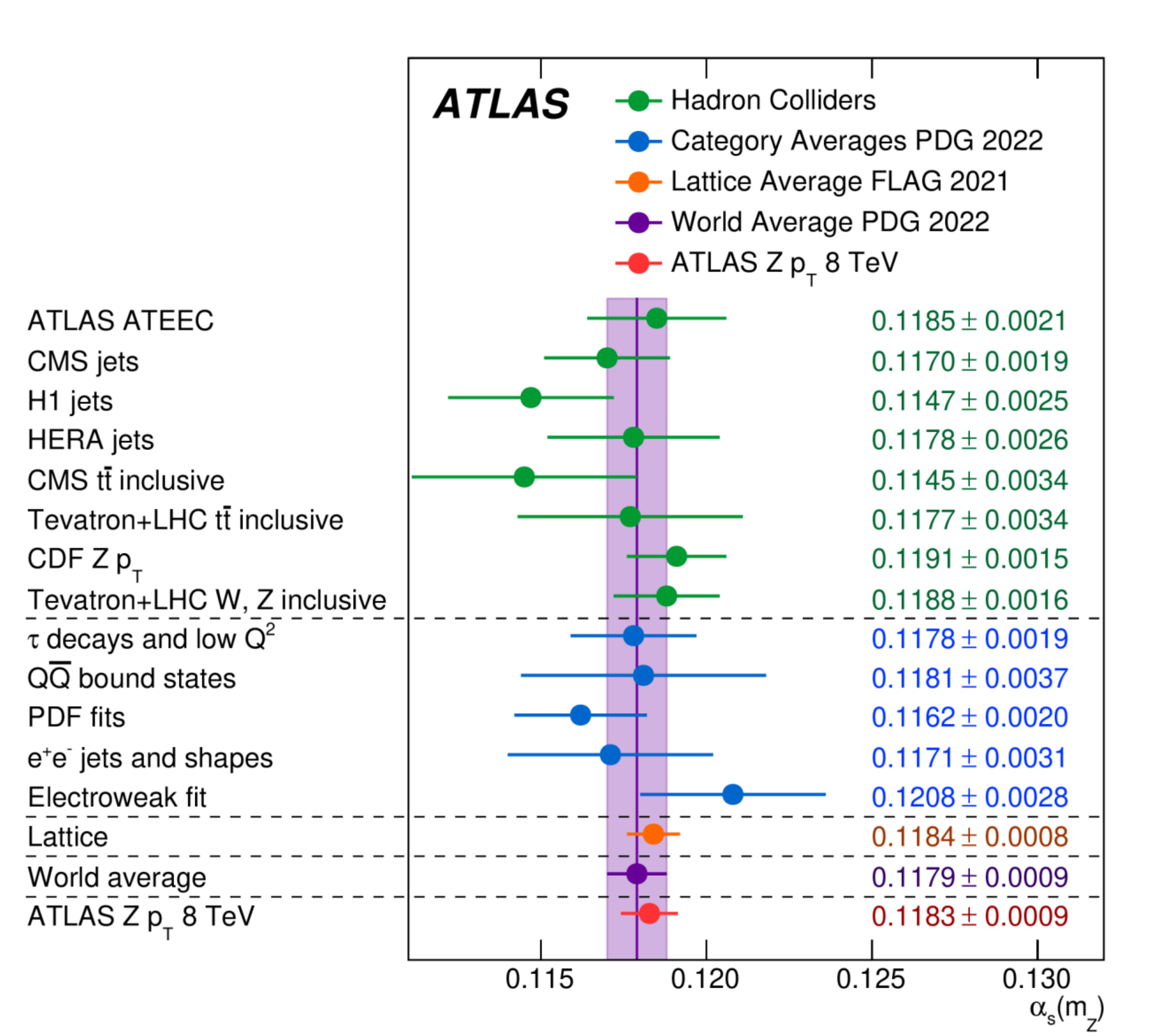
Accuracy	$\mathcal{H},\mathcal{B}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level ✓	1-loop ✓	–	–	1-loop ✓
NLL	Tree level ✓	2-loop ✓	1-loop ✓	1-loop ✓	2-loop ✓
NLL′	1-loop ✓	2-loop ✓	1-loop ✓	1-loop ✓	2-loop ✓
NNLL	1-loop ✓	3-loop ✓	2-loop ✓	2-loop ✓	3-loop ✓
NNLL′	2-loop ✓	3-loop ✓	2-loop ✓	2-loop ✓	3-loop ✓
N^{3}LL	2-loop ✓	4-loop ✓	3-loop ✓	3-loop ✓	4-loop ✓
N^{3}LL′	3-loop ✓	4-loop ✓	3-loop ✓	3-loop ✓	4-loop ✓
N^{4}LL	3-loop ✓	5- 	4-loop ✓	4-loop ✓	5-loop ✓
N^{4}LL′	4-loop ✓	5-	4-loop ✓	4-loop ✓	5-loop ✓

3-loop beam function [Ebert Mistberger Vita 2205.02242]

4-loop hard function [Lee von Manteuffel Schabinger Smirnov Smirnov 2202.04660]

MEASURING THE STRONG COUPLING





[ATLAS 2309.12986]

- Predictions obtained with DYturbo

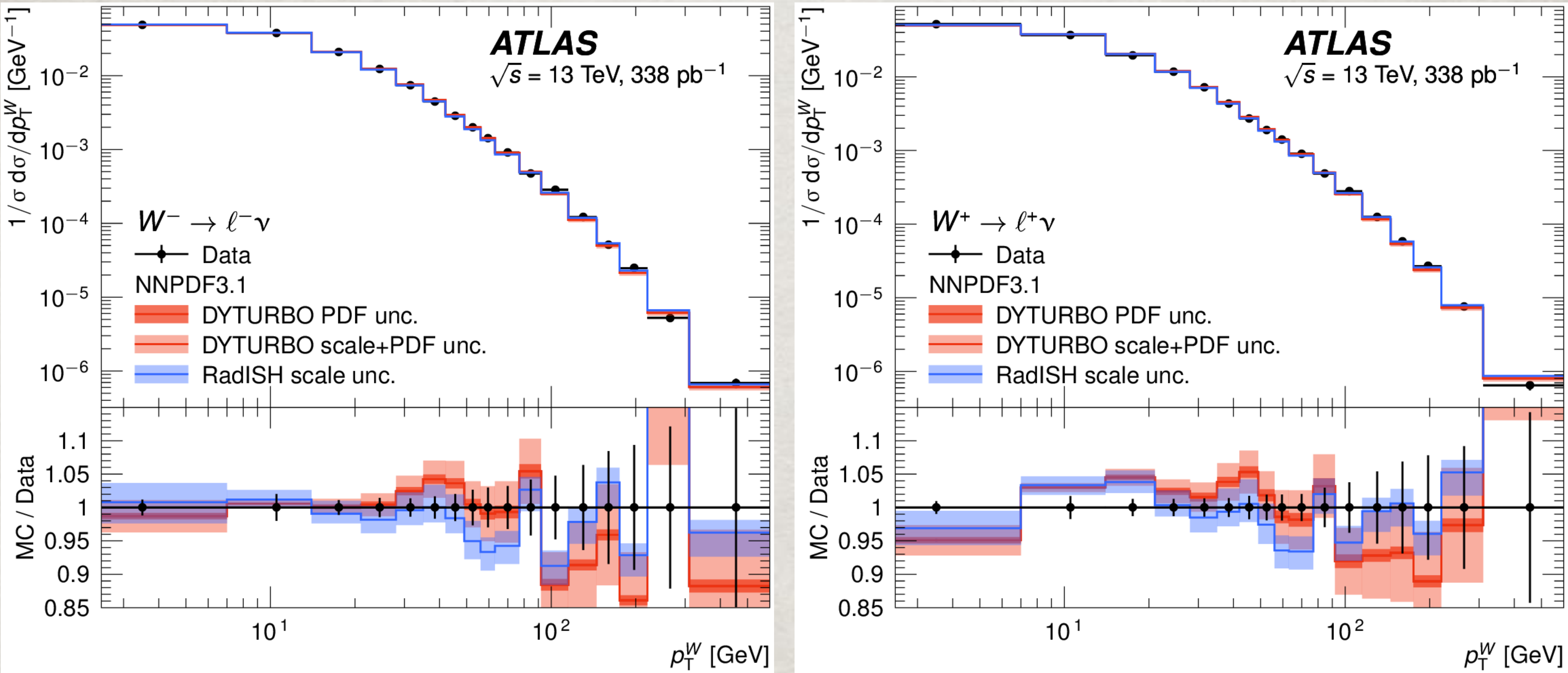
  [Camarda Cieri Ferrera 2303.12781]

- Two-parameter non-perturbative model

  [Collins Rogers 1412.3820]

Very precise determination of α_s

New: W^+ and W^- p_T distributions

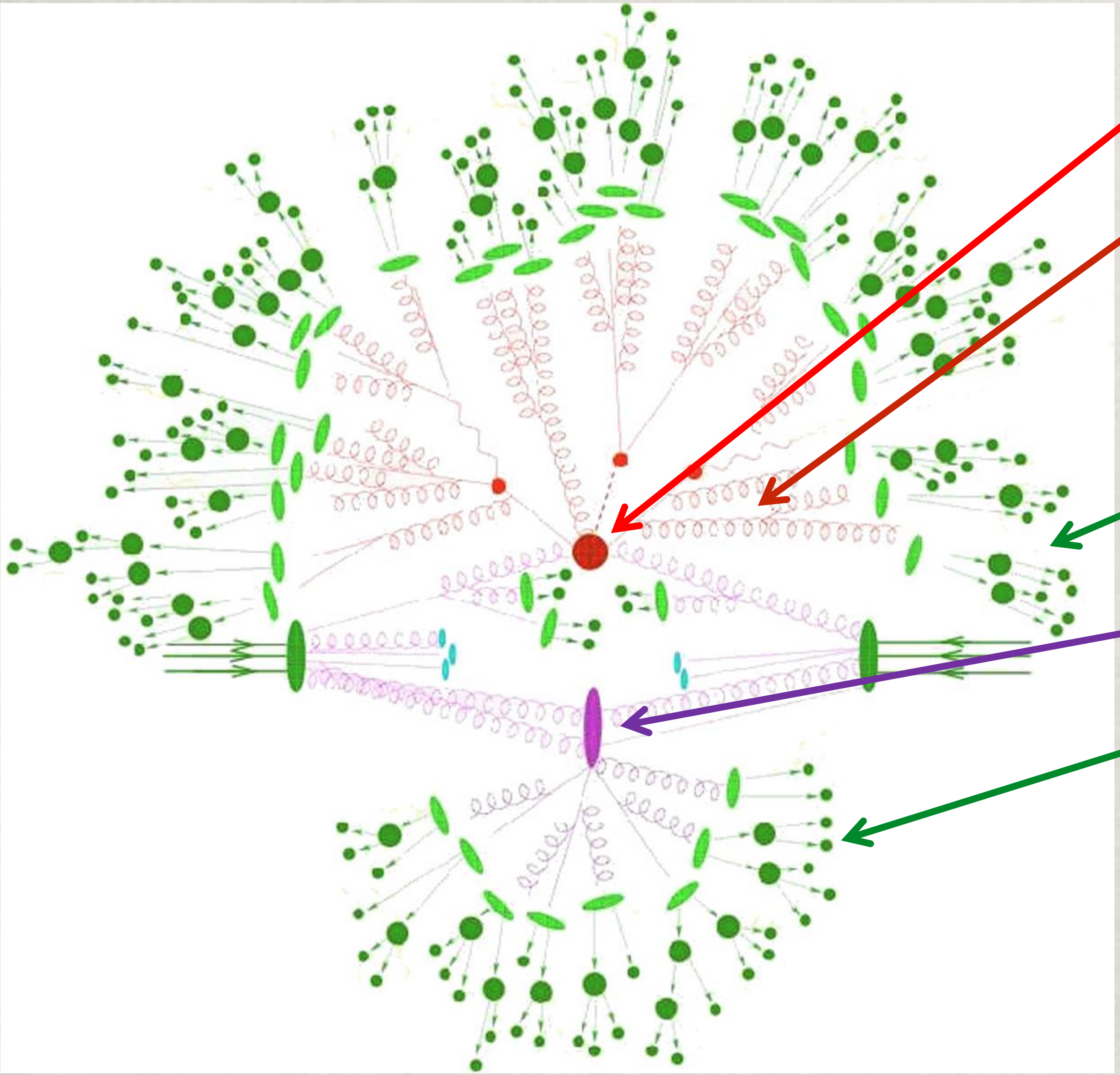


- W transverse momentum reconstructed via hadronic recoil, important for precise determinations of the W mass [ATLAS 2404.06204]
- Very good agreement with QCD resummed predictions (N^{3}LL+NNLO) in all regions of the spectrum [Camarda Cieri Ferrera 2303.12781] [Chen Gehrmann Glover Huss Monni Re Rottoli Torrielli 2203.01565]

Parton-shower event generators

Realistic final states

Below is a fully exclusive event produced in $t\bar{t}H$



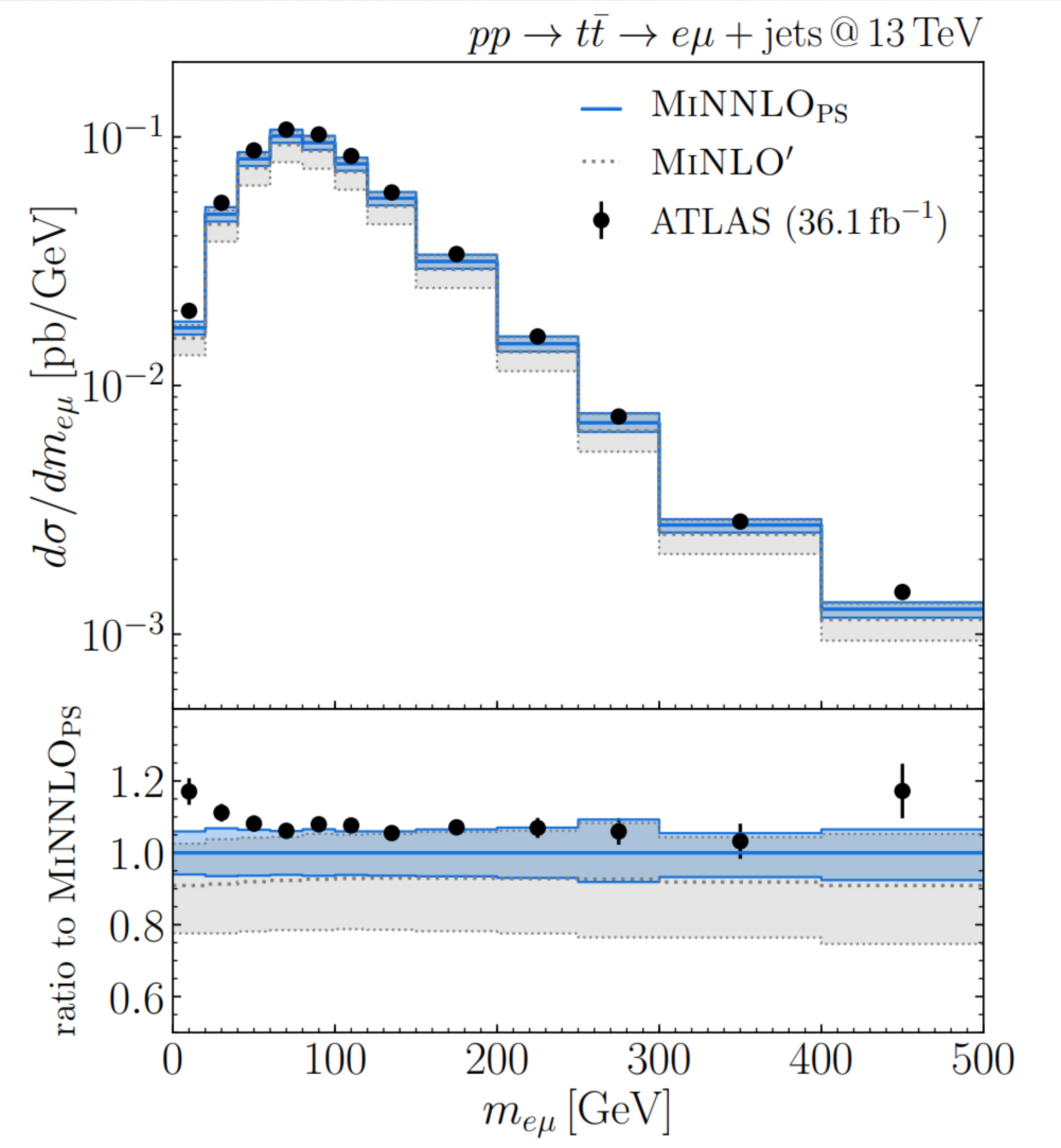
- Hard event: fixed-order
- Multiple soft and collinear emissions: resummation
- Hadronisation
- Underlying event
- Interface with detectors

parton-shower event generators

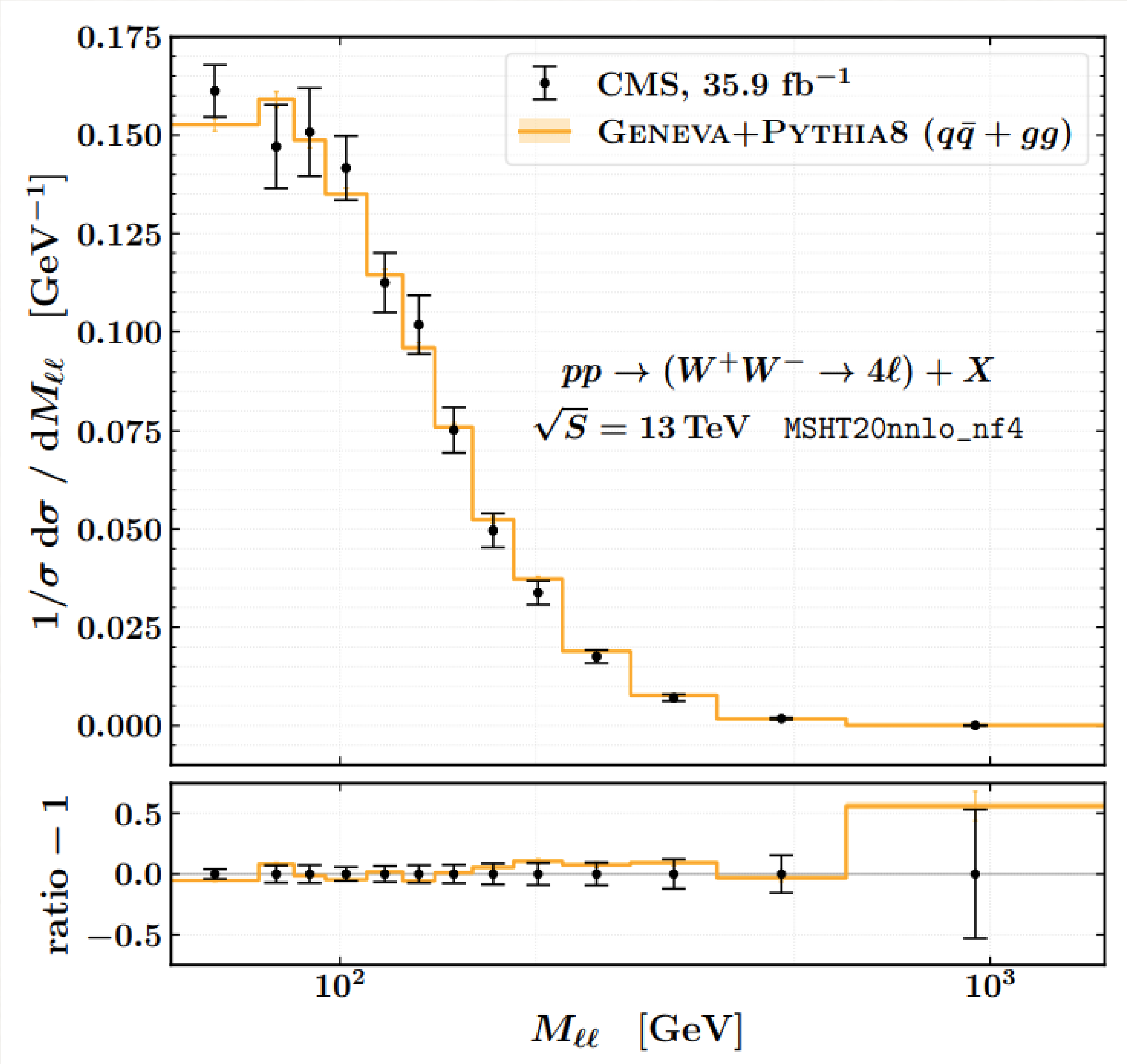
Can we improve parton-shower event generators so that they have the same accuracy as state-of-the-art fixed-order calculations and resummations?

Matching to NNLO

MiNNLO and GENEVA are two competing approaches that are able to achieve NNLO accuracy for a number of processes



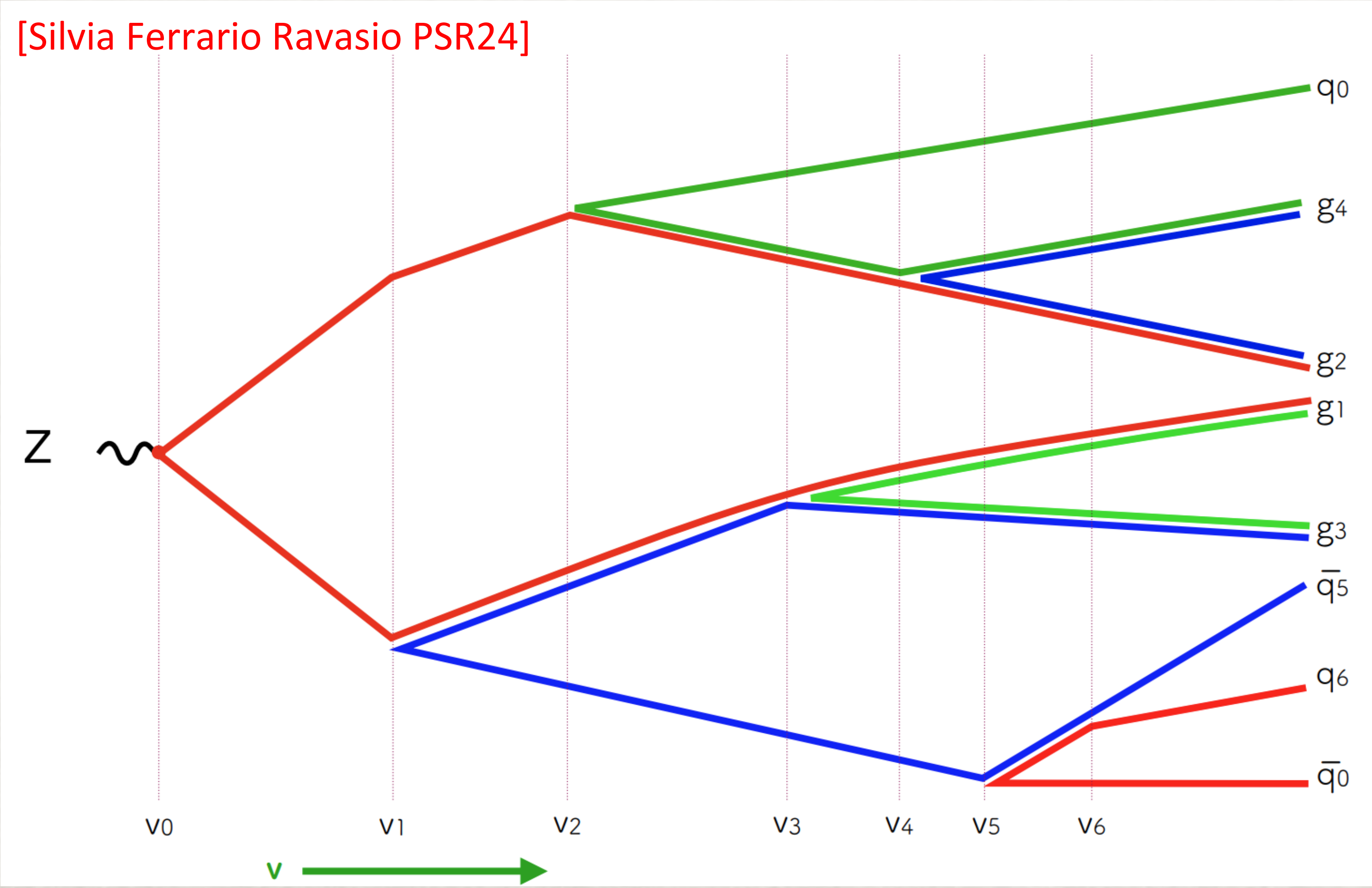
[Mazzitelli et al 2012.14267]



[Gavardi Lim Alioli Tackmann 2308.11577]

As for phase-space slicing, both methods require the NNLL resummation and the two-loop constant for a selected resolution variable

Toward NNLL accuracy



Main ingredients of a parton-shower generator

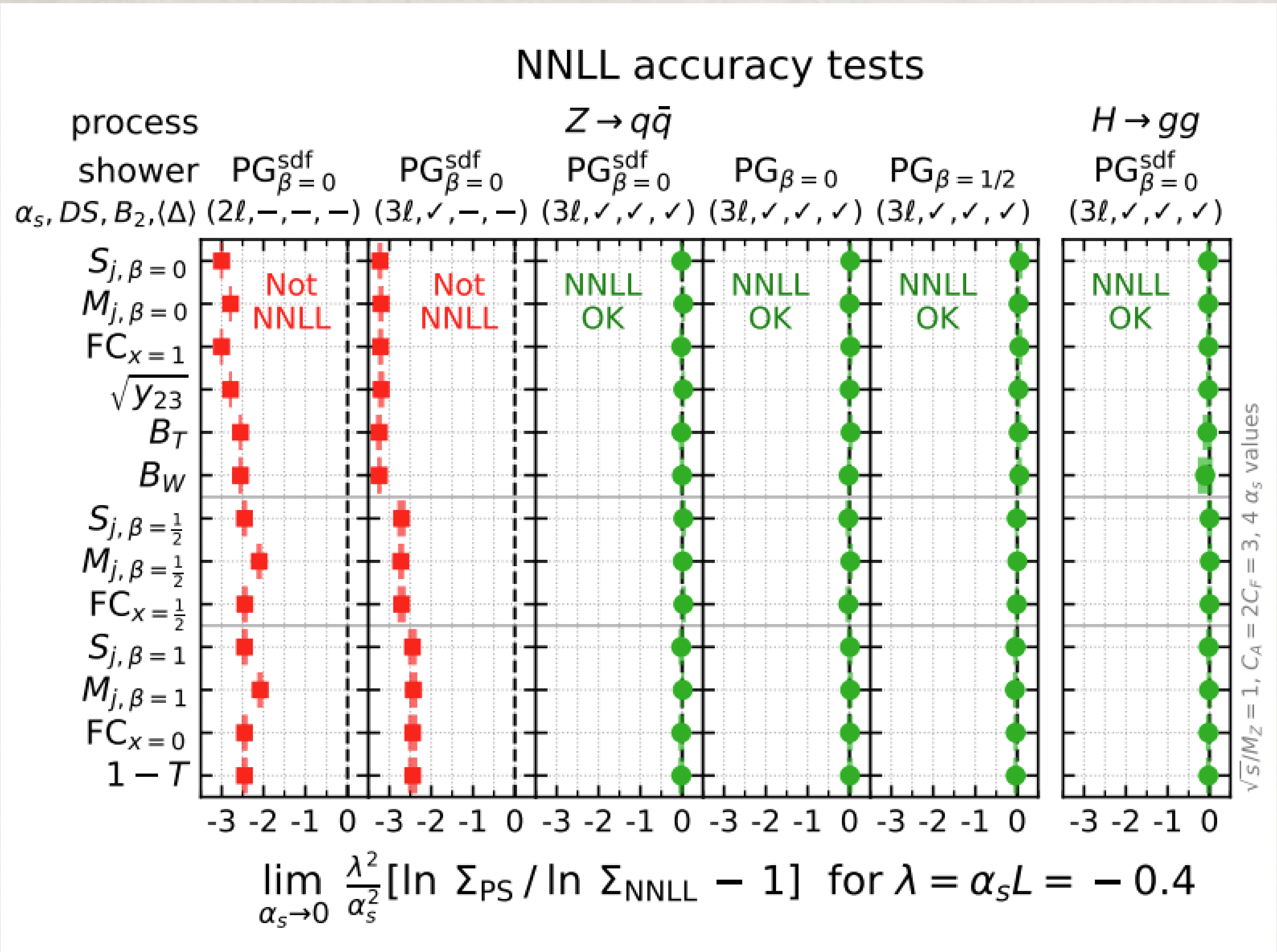
- algorithm: single parton or dipole branching
- splitting probability
- evolution variable v, e.g. transverse momentum
- prescription to accommodate recoil after each splitting

- A careful choices of these ingredients makes it possible to achieve NLL accuracy in e^+e^- annihilation, DIS and hadron-hadron collisions
  [Dasgupta Dreyer Hamilton Monni Salam Soyez 2002.11114]
  [van Beekveld Ferrario-Ravasio Salam Soto-Ontoso Soyez Verheyen 2205.02237]
  [van Beekveld Ferrario-Ravasio 2305.08645]

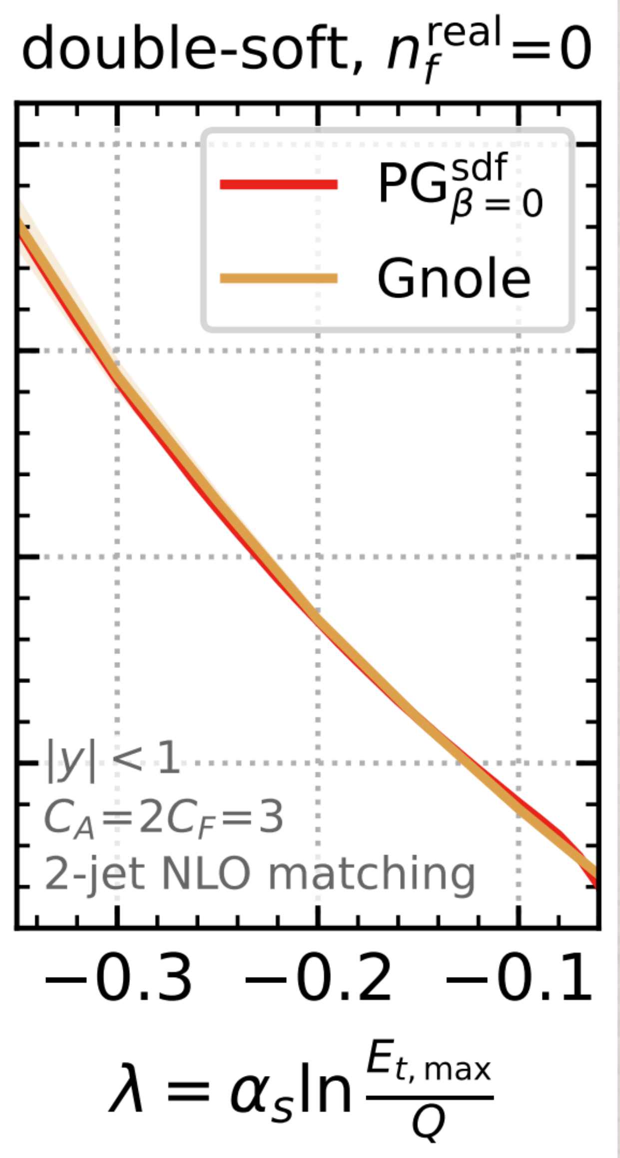
- In e^+e^- annihilation, it is possible to improve the splitting probability to achieve NNLL accuracy [van Beekveld Dasgupta El-Menoufi Ferrario-Ravasio et al 2406.02661]

Log accuracy of parton shower

Testing the logarithmic accuracy of parton-shower event generators require the calculation of a number of observables at the required logarithmic accuracy



[van Beekveld Dasgupta El-Menoufi Ferrario-Ravasio et al 2406.02661]



[Ferrario-Ravasio et al 2307.11142]

NNLL resummations for suitable observables needed in multi-jet DIS and hadron-hadron collisions

Concluding remarks

Impressive progress towards reaching the % accuracy goal in precision calculations for many relevant processes at present and future colliders

- NNLO is the state of the art for many fixed-order calculations, and N^3LO accuracy is available for a number of inclusive cross-sections
- Resummations have reached NNLL accuracy, and for inclusive observables approximate N^4LL
- Parton-shower event generators are moving from models to precision tools: in selected cases the accuracy is NNLO and NNLL

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Broad research quests encompassing different areas

- Two-loop integrals with internal masses: these are important for both QCD and EW corrections
- Resummation of jet observables in DIS and hadron-hadron collisions needed for both fixed-order and parton-shower event generators

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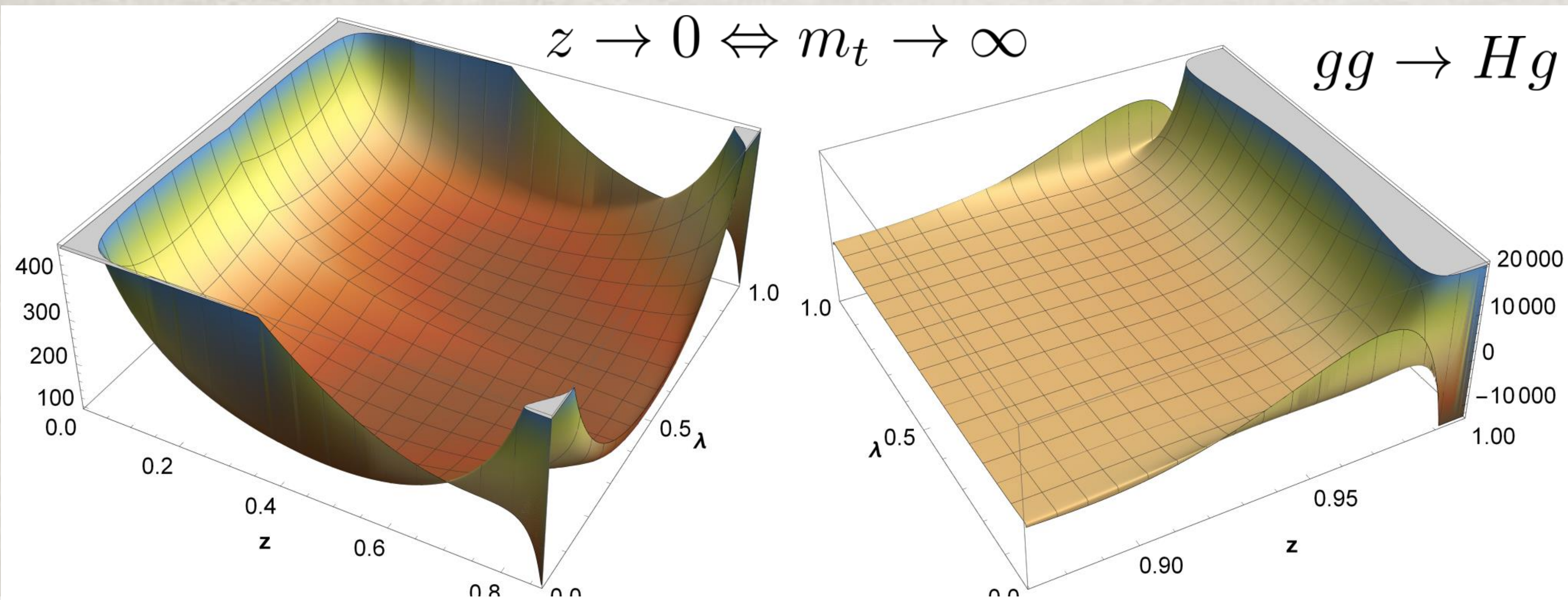
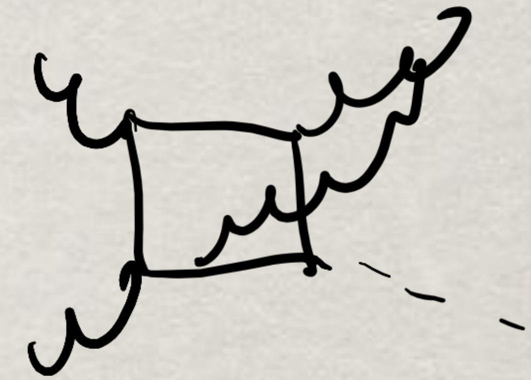
Thank you for your attention

Extra

Mass effects in Higgs production

- Three-loop amplitudes for $gg \to H$ and two-loop amplitudes H+1parton are computed by solving numerically a system of differential equations taking the heavy-top limit as an initial condition [Czakon Harlander Klappert Niggetiedt 2105.04436]





channel	$\sigma^{\text{NNLO}}_{\text{HEFT}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\text{NNLO}}_{\text{exact}} - \sigma^{\text{NNLO}}_{\text{HEFT}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma^{\text{NNLO}}_{\text{exact}}/\sigma^{\text{NNLO}}_{\text{HEFT}} - 1)$ [%]
		$\sqrt{s} = 8$ TeV		
gg	$7.39 + 8.58 + 3.88$	$+0.0353$	$+0.0879 \pm 0.0005$	$+0.62$
qg	$0.55 + 0.26$	-0.1397	-0.0153 ± 0.0002	-19
qq	$0.01 + 0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39 + 9.14 + 4.18$	-0.0873	$+0.0535 \pm 0.0006$	-0.16
		$\sqrt{s} = 13$ TeV		
gg	$16.30 + 19.64 + 8.76$	$+0.0345$	$+0.2431 \pm 0.0020$	$+0.62$
qg	$1.49 + 0.84$	-0.3696	-0.0408 ± 0.0005	-18
qq	$0.02 + 0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30 + 21.15 + 9.70$	-0.3029	$+0.1522 \pm 0.0021$	-0.32

- Absolute value of the correction of the order of 2%
- Saturates the estimated uncertainty
- Actual contribution smaller due to cancellations

Can we automate?

It is possible to derive general formulae for the resummation of a large class of observable in an arbitrary process

[AB Salam Zanderighi hep-ph/0407286]

[AB McAslan Monni Zanderighi 1412.2126]

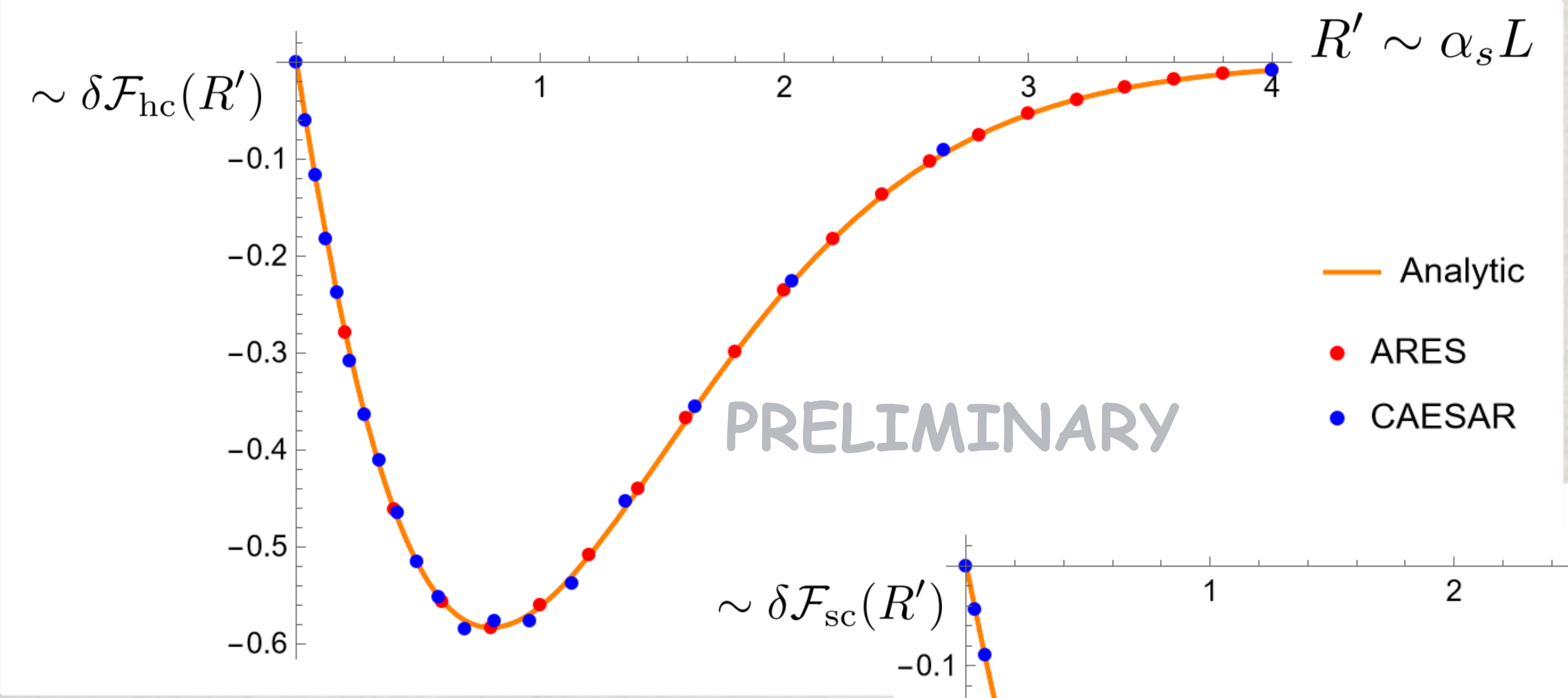
- CAESAR performs automatically NLL resummations given only the observable definition in terms of momenta
- Requires careful numerical extrapolations to be extended beyond NLL

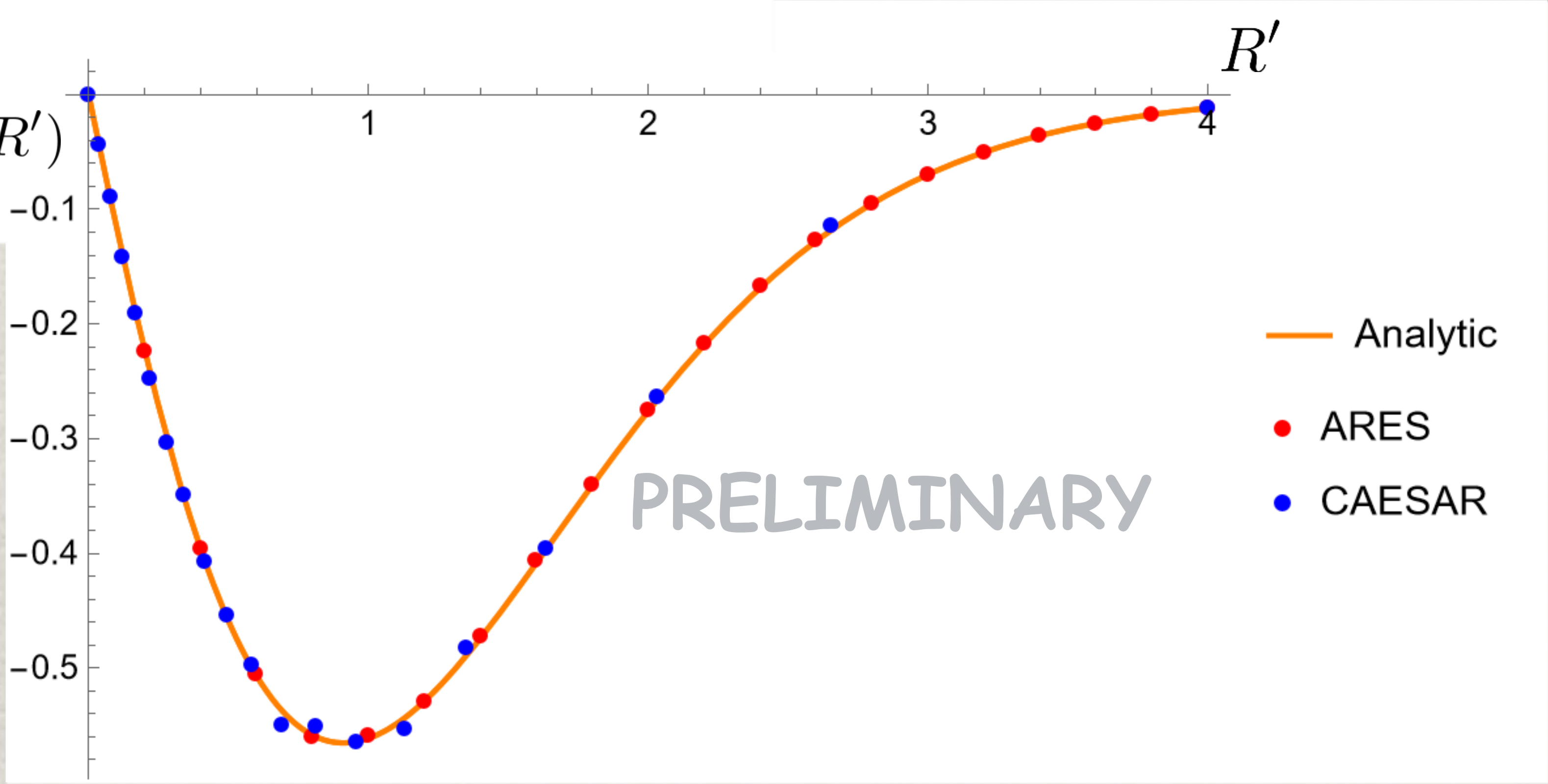
- ARES uses analytically determined soft and collinear limits of each observable
- Currently NNLL, but can be in principle extended to an arbitrary logarithmic accuracy (see e.g. RadISH)

[Monni Re Torrielli 1604.02191]

First-ever NNLL with CAESAR

Soft and collinear emissions are correctly implemented in CAESAR $\Rightarrow$ one can compute NNLL corrections arising from soft and collinear emissions "out of the box"





Example: additive observables, e.g. thrust

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \sum_{i=1}^{n} V(\{\tilde{p}\}, k_i)$$