μTRISTAN:

$\mu^+ e^-$ and $\mu^+ \mu^+$ collider

Yu Hamada (DESY)

Based on:

- 2201.06664, **YH**, R. Kitano, R. Matsudo, H. Takaura, and M. Yoshida
- 2210.11083, **YH**, R. Kitano, R. Matsudo, and H. Takaura
- 2406.04500, L. Chen, **YH**, and S. Iguro
- 2408.01068, **YH**, R. Kitano, R. Matsudo, S. Okawa, R. Takai, H. Takaura, and L. Treuer

19th Sep 2024, LFC24@SISSA

Plan of talk

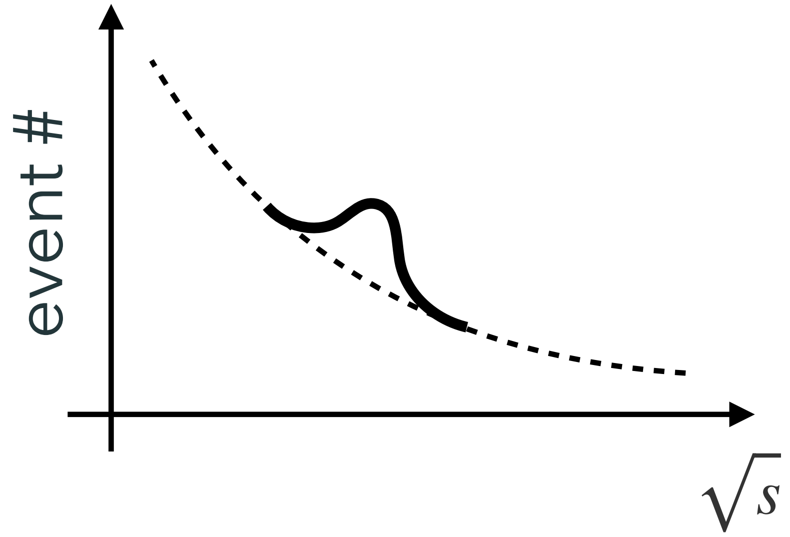
- Introduction: What is μTRISTAN?
- Higgs physics at μTRISTAN
- New physics search at μTRISTAN
- Summary

Introduction: What is μTRISTAN?

Task of future collider

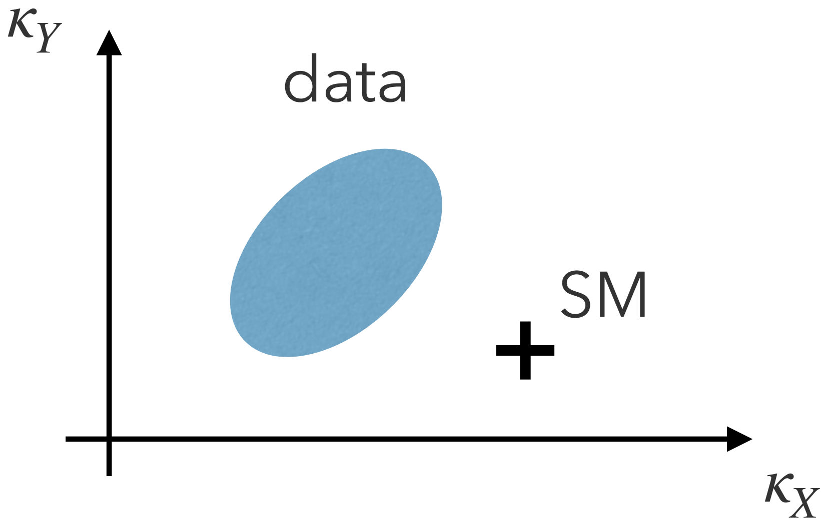
1. **Go to higher energy**

   production of new particle
   → direct search



2. **Precision measurement**

   probe deviation from SM
   → indirect search

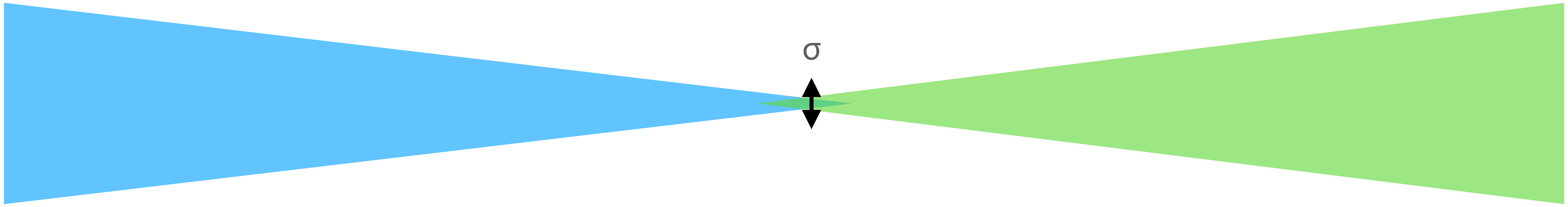


- **Muon colliders are very nice for both!**

  precision measurements ← lepton collider

  high energy e.g. O(10) TeV ← less synchrotron rad. than $e^{\pm}$

Luminosity



# of particles

How frequently collisions occur

$$\mathscr{L} = \frac{N_{\text{beam1}}\, N_{\text{beam2}}}{4\pi\,\sigma_x\,\sigma_y} f_{\text{rep}}$$

beam size

We need many particles and narrow beam!

Current difficulties of $\mu^+\mu^-$ collider

- **MAP** (Muon Accelerator Program)

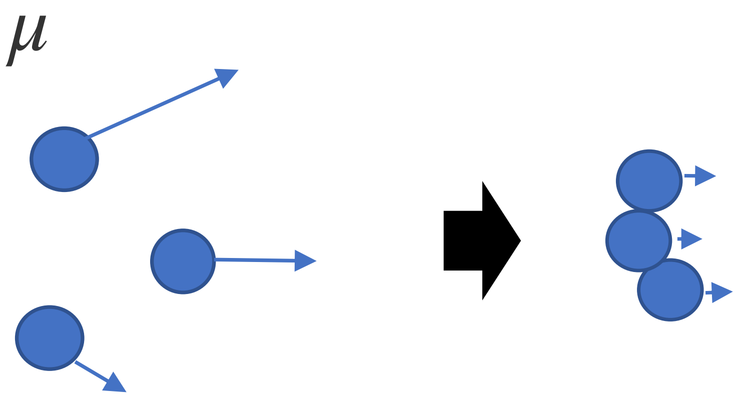
  $\mu^\pm$ produced from $\pi^\pm$ decay

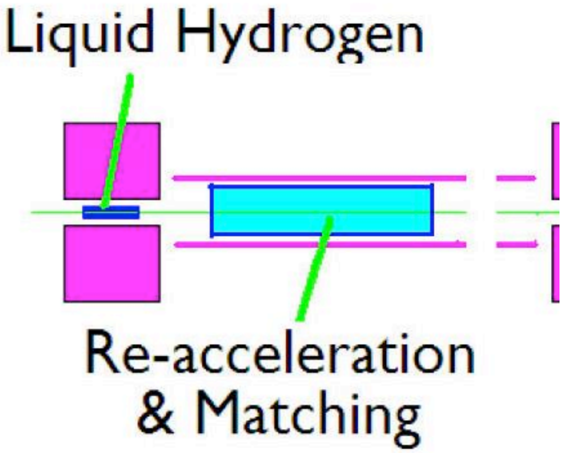
  → **too hot, randomly distributed**

  → cooling with ionized material

  principle works (MICE), but not yet enough

  [1907.08562]





[from slide by Daniel Schulte]

- **LEMMA** (Low Emittance Muon Accelerator)

  $e^+e^- \to \mu^+\mu^-$ at threshold

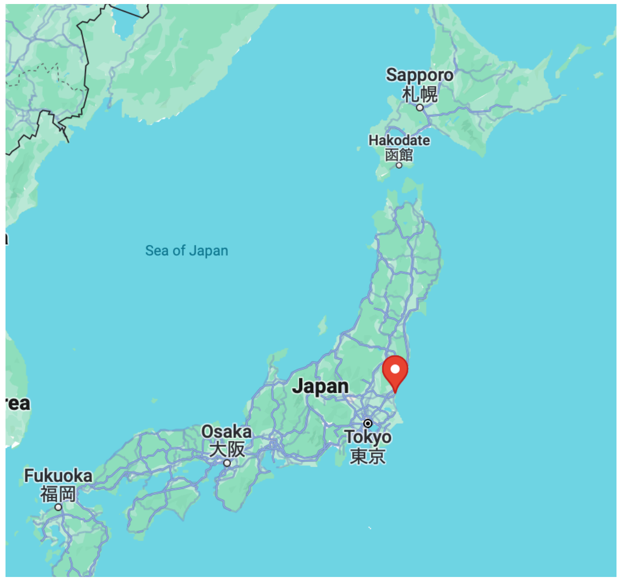
  → almost at rest i.e. **already cold, but less amount**

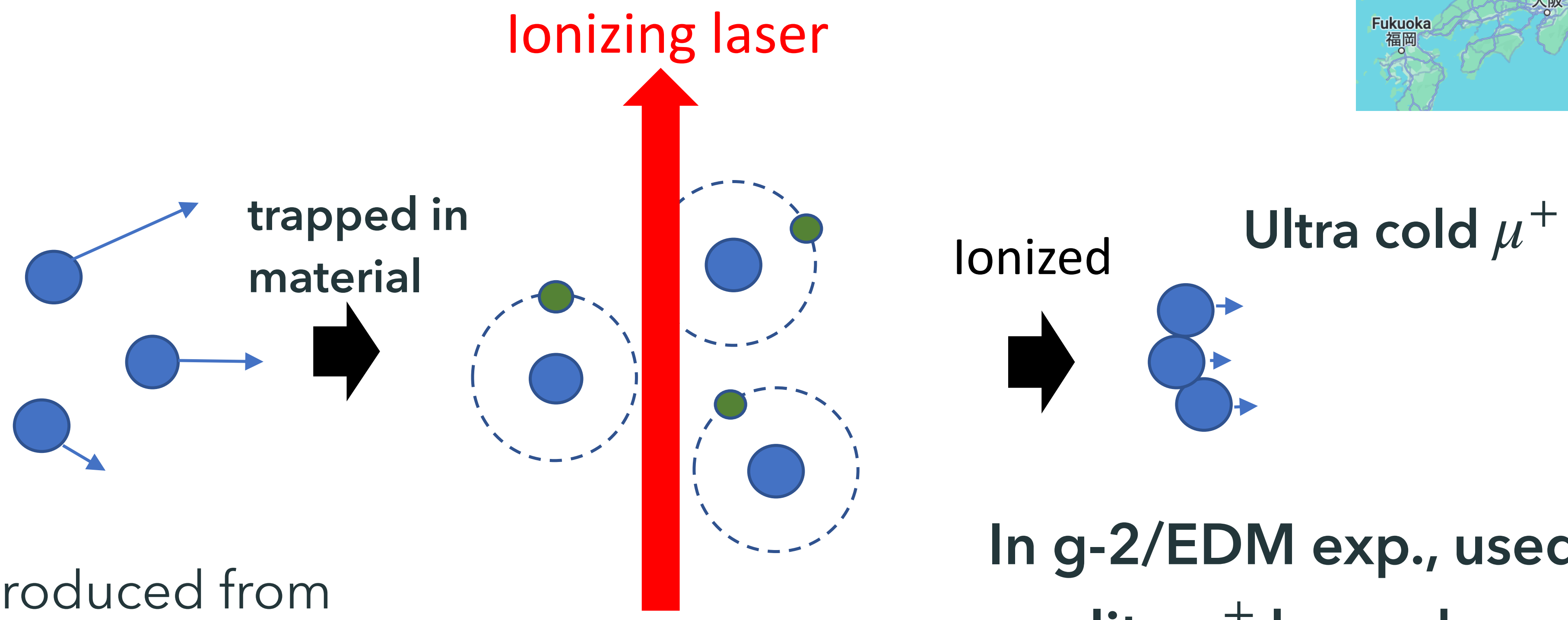
→ μ cooling w/ large amount is very difficult!

Technology for μ^+ cooling exists!

- J-PARC is planning μ g-2/EDM experiment.

The key technology is cooling of μ^+, which is available today!





μ^+ produced from π^+ decay

μ^+e^- bound state (muonium)

In g-2/EDM exp., used as a high-quality μ^+ beam by accelerating to 300 MeV.

fig from slide by Takaura

Technology for μ^+ cooling exists!

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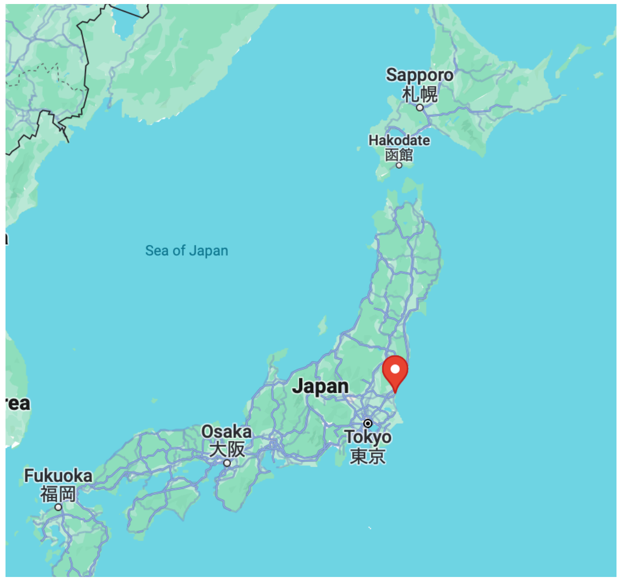
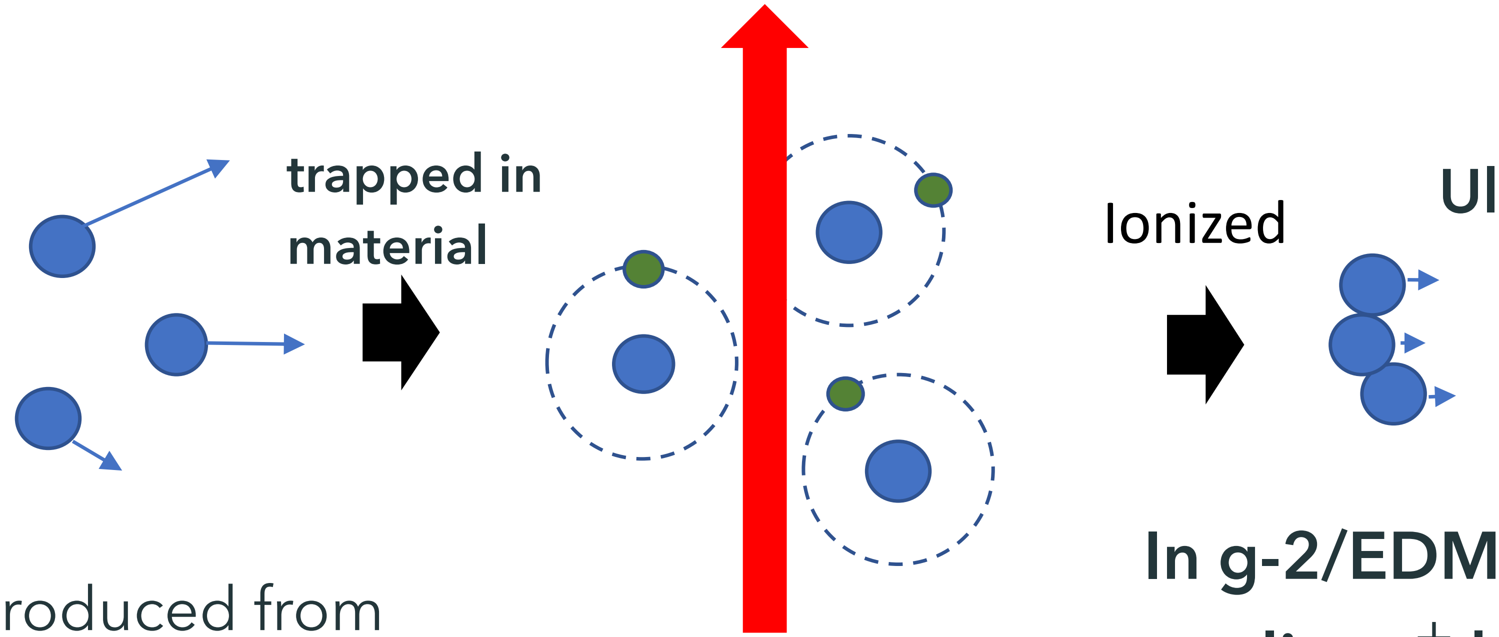




Ionizing laser

trapped in material

Ionized

Ultra cold μ^+

μ^+ produced from π^+ decay

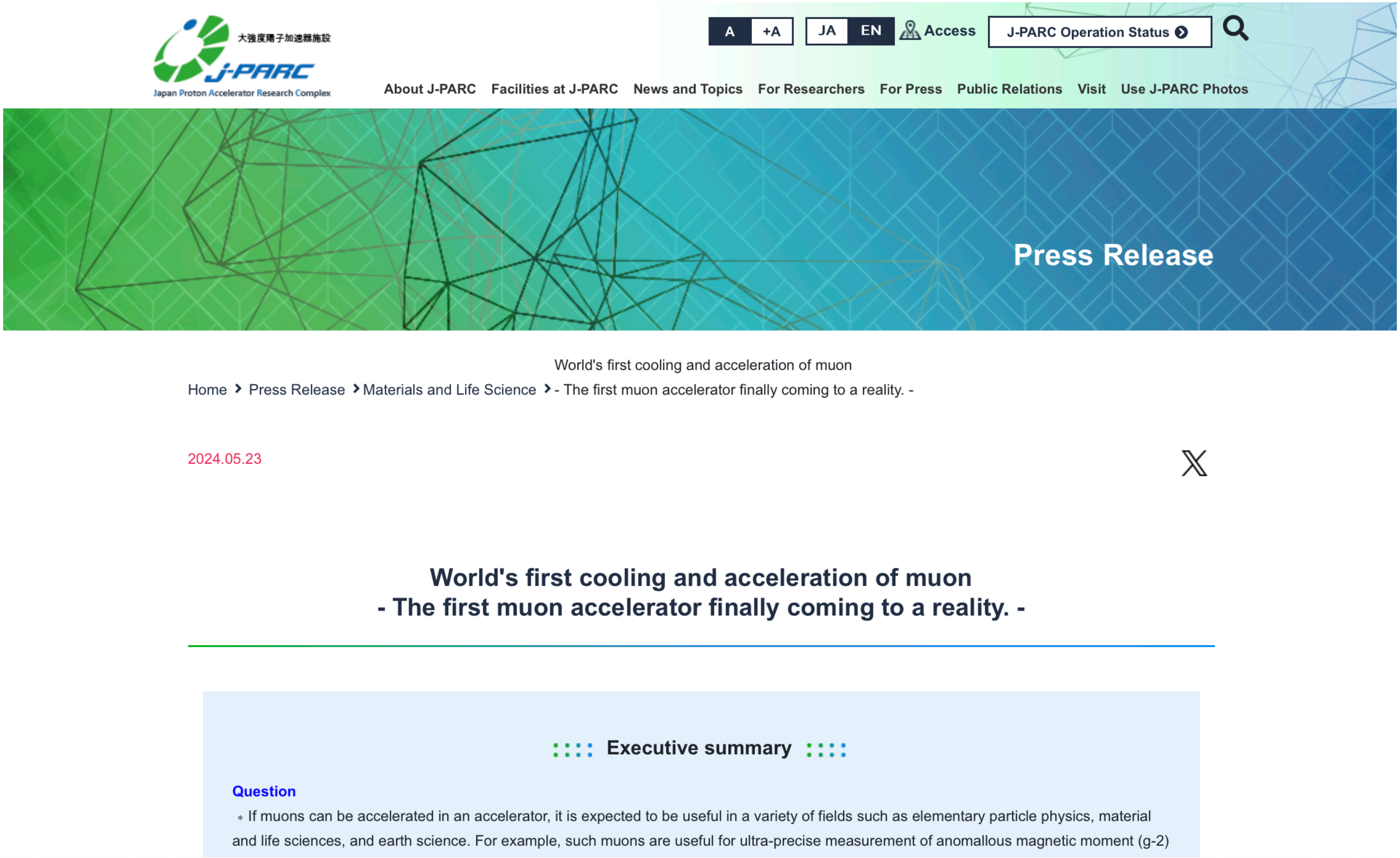
μ^+e^- bound state (muonium)

In g-2/EDM exp., used as a high-quality μ^+ beam by accelerating to 300 MeV.

Our proposal: accelerate it to TeV!

fig from slide by Takaura

Technology for μ^+ cooling exists!



World's first cooling and acceleration of muon

Home › Press Release › Materials and Life Science › - The first muon accelerator finally coming to a reality. -

2024.05.23

World's first cooling and acceleration of muon
- The first muon accelerator finally coming to a reality. -

Executive summary

Question

⁎ If muons can be accelerated in an accelerator, it is expected to be useful in a variety of fields such as elementary particle physics, material and life sciences, and earth science. For example, such muons are useful for ultra-precise measurement of anomallous magnetic moment (g-2)

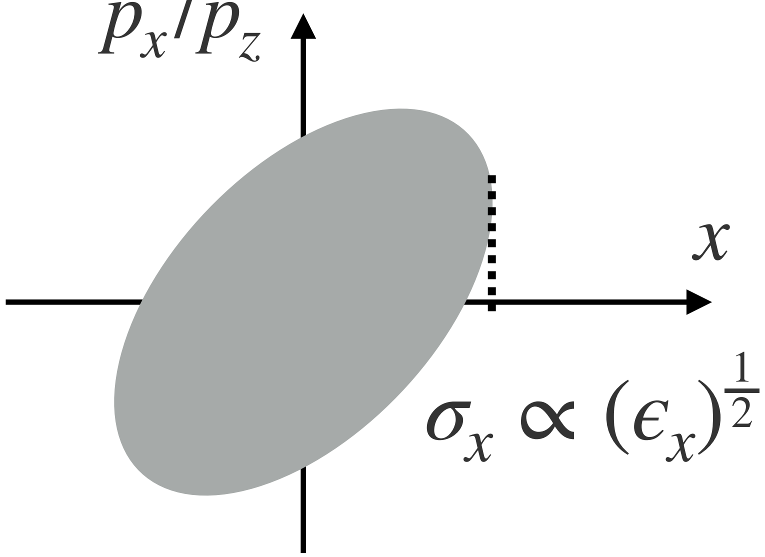
https://www.j-parc.jp/c/en/press-release/2024/05/23001341.html

ready to be accelerated

Comparison of beam emittance

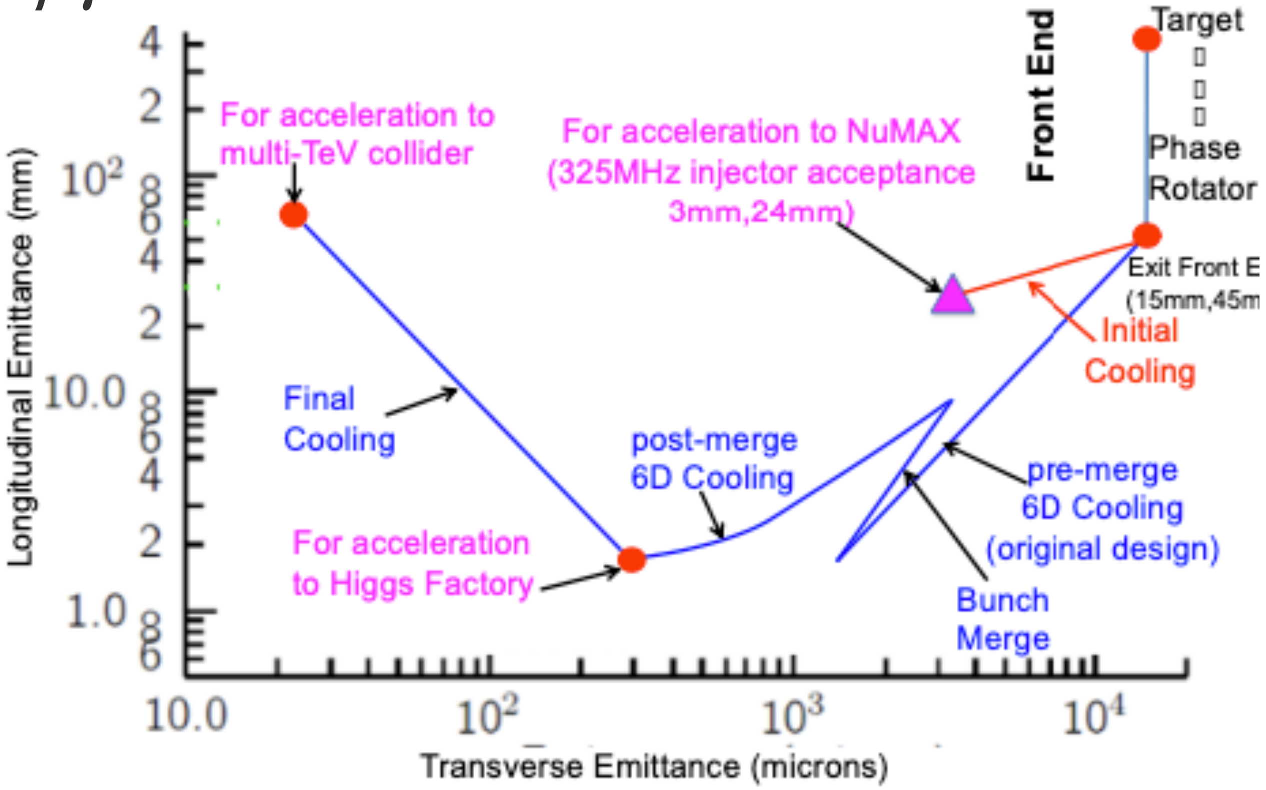
- Beam size is determined by emittance $\epsilon_{x,y}$

  emittance = area of distribution in phase space
  → **reflects quality of beam**



[J-PARC EDM/g-2, 1901.03047]

For ultra-cold μ^+: $\epsilon_x \, , \epsilon_y = \dfrac{4\,\mu\mathrm{m}}{\beta\gamma}$



planned emittance in MAP

Comparison of beam emittance

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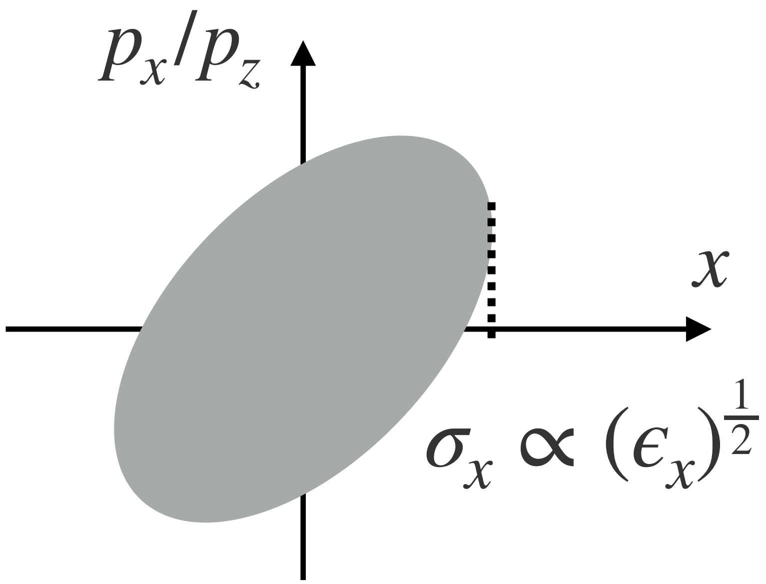
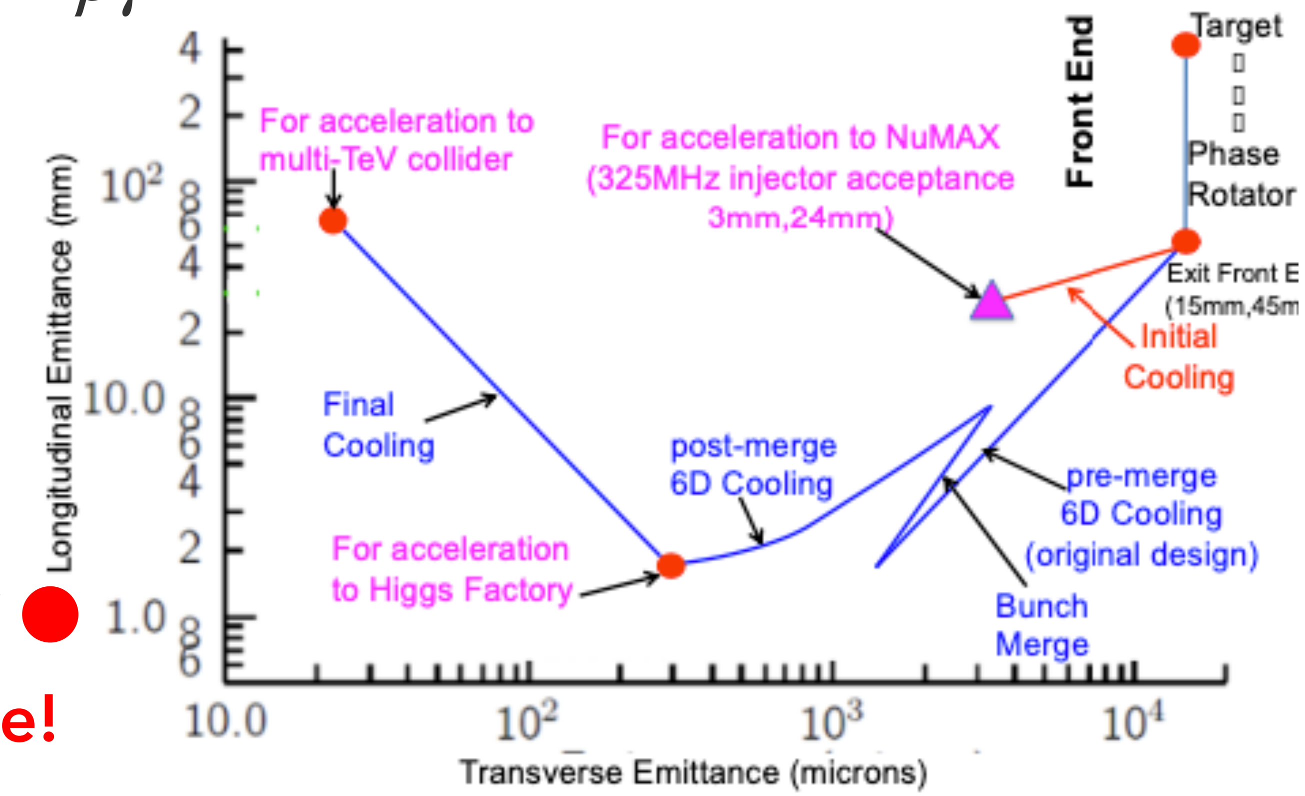


$\sigma_x \propto (\epsilon_x)^{\frac{1}{2}}$

[J-PARC EDM/g-2, 1901.03047]

For ultra-cold μ^+: $\epsilon_x\,, \epsilon_y = \frac{4\ \mu\mathrm{m}}{\beta\gamma}$



ultra-cold μ^+ is here!

planned emittance in MAP

Proposal of new experiment: μTRISTAN!

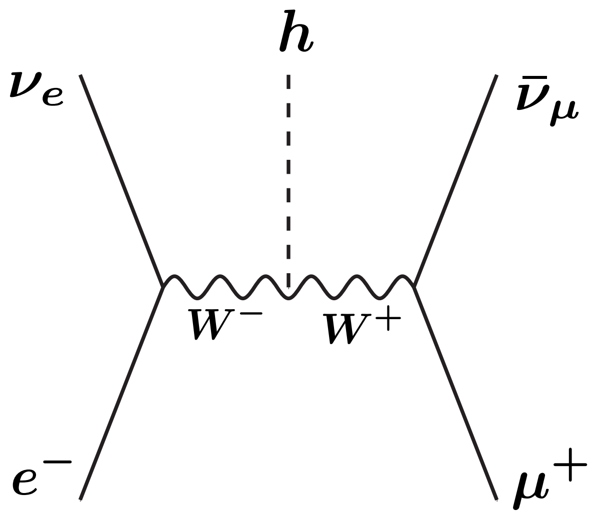
high-quality μ^+ beam accelerated to O(1) TeV

- $\mu^+ e^-$ collider

$$E_{\mu^+} = 1\,\mathrm{TeV}, \quad E_{e^-} = 30\,\mathrm{GeV}$$ (TRISTAN energy)

$$\rightarrow \sqrt{s} = 346\,\mathrm{GeV} \qquad \mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Higgs factory/EW precision



- $\mu^+\mu^+$ collider (instead of $\mu^+\mu^-$)

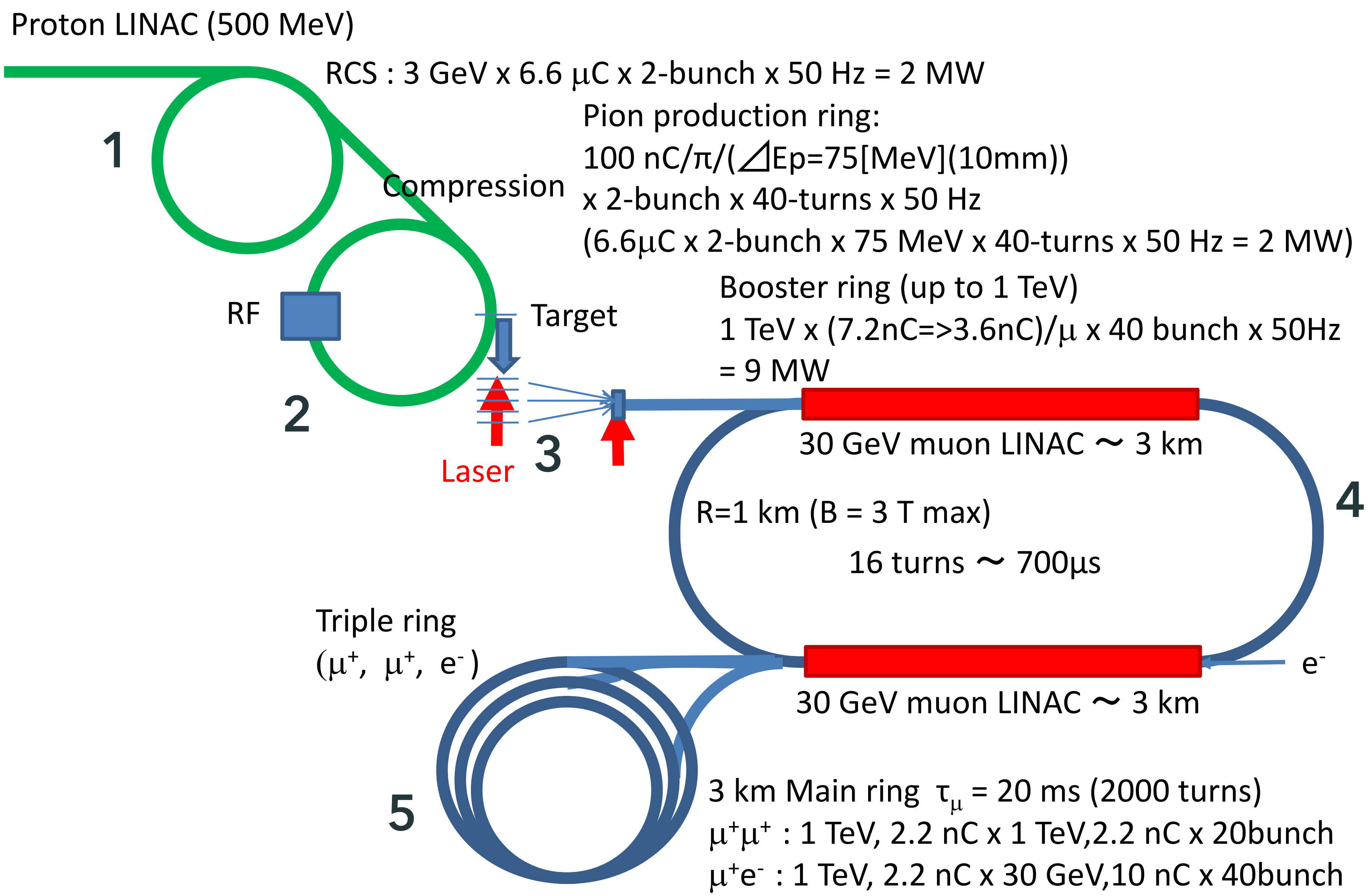
$$E_{\mu^+} = 1\,\mathrm{TeV}, \quad E_{\mu^+} = 1\,\mathrm{TeV}$$

$$\rightarrow \sqrt{s} = 2\,\mathrm{TeV} \qquad \mathscr{L}_{\mu^+\mu^+} = 5.7 \times 10^{32}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

New physics search/Higgs factory(!?)

Design of μTRISTAN

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]



Proton LINAC (500 MeV)

RCS : 3 GeV x 6.6 μC x 2-bunch x 50 Hz = 2 MW

Pion production ring:
100 nC/π/(⊿Ep=75[MeV](10mm))
x 2-bunch x 40-turns x 50 Hz
(6.6μC x 2-bunch x 75 MeV x 40-turns x 50 Hz = 2 MW)

Booster ring (up to 1 TeV)
1 TeV x (7.2nC=>3.6nC)/μ x 40 bunch x 50Hz
= 9 MW

30 GeV muon LINAC ∼ 3 km

R=1 km (B = 3 T max)

16 turns ∼ 700μs

30 GeV muon LINAC ∼ 3 km

Triple ring
(μ^+, μ^+, e^-)

3 km Main ring τ_μ = 20 ms (2000 turns)
$\mu^+\mu^+$: 1 TeV, 2.2 nC x 1 TeV,2.2 nC x 20bunch
μ^+e^- : 1 TeV, 2.2 nC x 30 GeV,10 nC x 40bunch

Plan of talk

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- Higgs physics at μTRISTAN
- New physics search at μTRISTAN
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Higgs physics at μTRISTAN

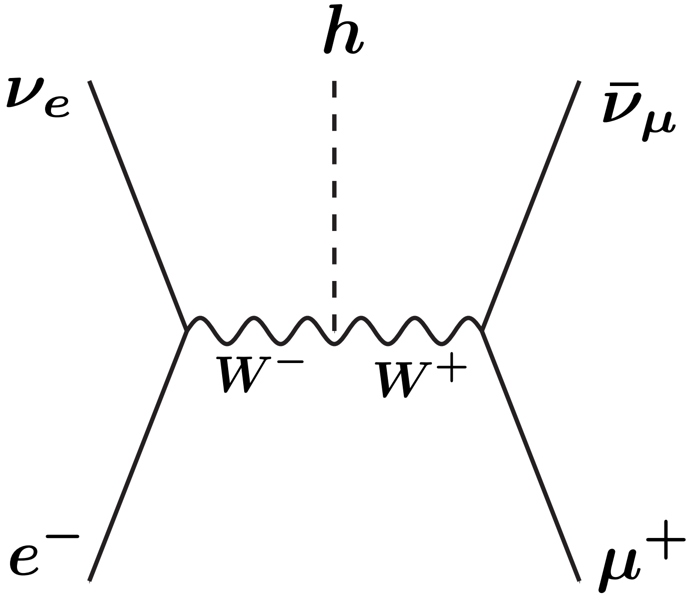
Higgs production

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

Due to the large luminosity, the $\mu^+ e^-$ collider is more suitable.

$$(E_{\mu^+}, E_{e^-}) = (1\,\mathrm{TeV}, 30\,\mathrm{GeV}) \qquad \sqrt{s} = 346\,\mathrm{GeV}$$

$$(P_{\mu^+}, P_{e^-}) = (0.8, -0.7)$$



Main process: **W-boson fusion $\sigma_{\mathrm{WBF}} \simeq 91\,\mathrm{fb}$**

- **luminosity** $\mathscr{L}_{\mu^+ e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- $\int dt\, \mathscr{L}_{\mu^+ e^-} = 1.0\,\mathrm{ab}^{-1}$ w/ ten-year running

$$\longrightarrow \quad N(\mathrm{Higgs}) = 9.5 \times 10^4 \times \frac{(\text{integrated luminosity})}{1.0\,\mathrm{ab}^{-1}}$$

Higgs precision measurement is possible!

Higgs coupling measurement

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

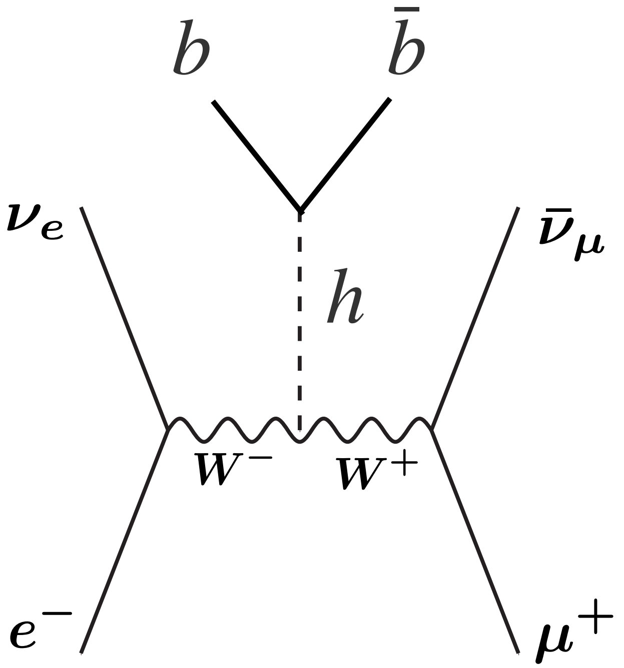
- Higgs mainly decays into $b\bar{b}$ (Br. = 58.2 % in SM)
- All couplings are parameterized by κ's (κ-scheme)

$$g_{hWW} = \kappa_W \, g_{hWW}^{\mathrm{SM}}$$

$$g_{hb\bar{b}} = \kappa_b \, g_{hb\bar{b}}^{\mathrm{SM}} \qquad \longrightarrow \qquad \sigma = \frac{\kappa_W^2 \kappa_b^2}{\kappa_H^2} \sigma_{\mathrm{SM}}$$

$$\Gamma_{H,tot} = \kappa_H^2 \, \Gamma_{H,tot}^{\mathrm{SM}}$$

$$\kappa_\bullet = 1 + \Delta\kappa_\bullet$$



κ's can be measured with the statistical uncertainty:

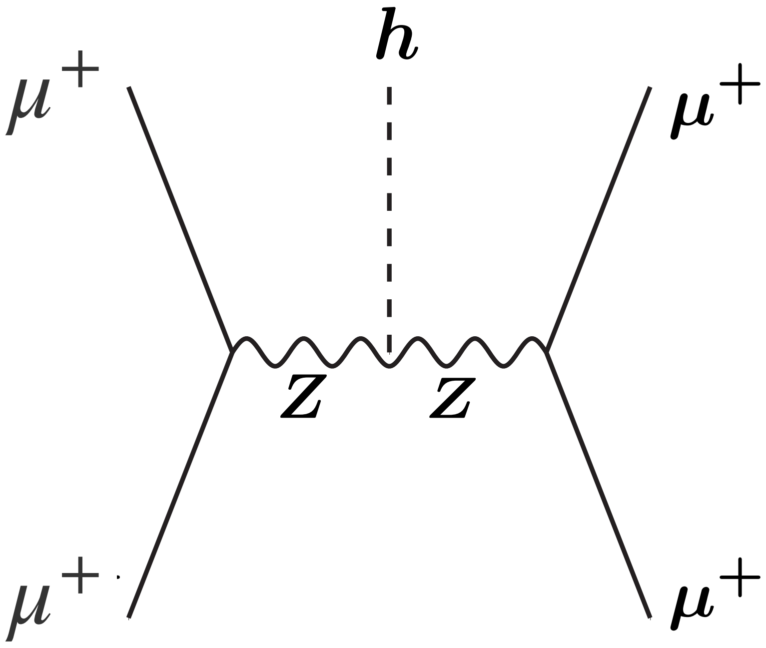
$$\left| \Delta\kappa_W + \Delta\kappa_b - \Delta\kappa_H \right| \lesssim 3.1 \times 10^{-3} \times \left(\frac{\text{integrated luminosity}}{1.0\,\mathrm{ab}^{-1}} \right)^{-1/2} \left(\frac{\text{efficiency}}{0.5} \right)^{-1/2}$$

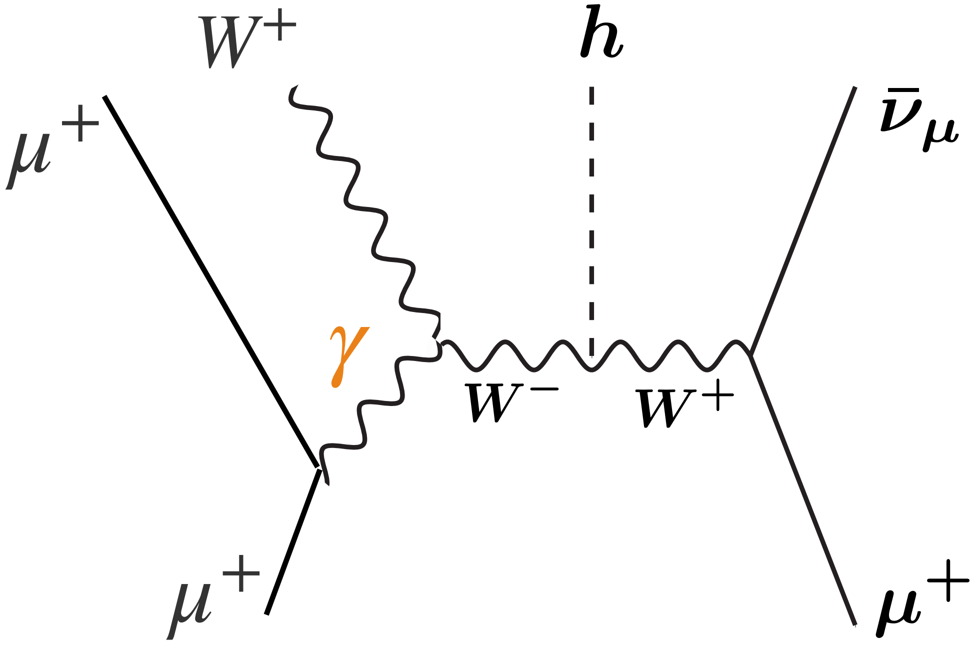
O(0.1)% measurement!

Higgs production @ $\mu^+\mu^+$?

[2408.01068, **YH**, Kitano, Matsudo, Okawa, Takai, Takaura, and Treuer]

- **Naively, only ZBF is possible at $\mu^+\mu^+$ collider → suppressed?**
- **However, γ-emitted WBF can be significant at high energy!!**



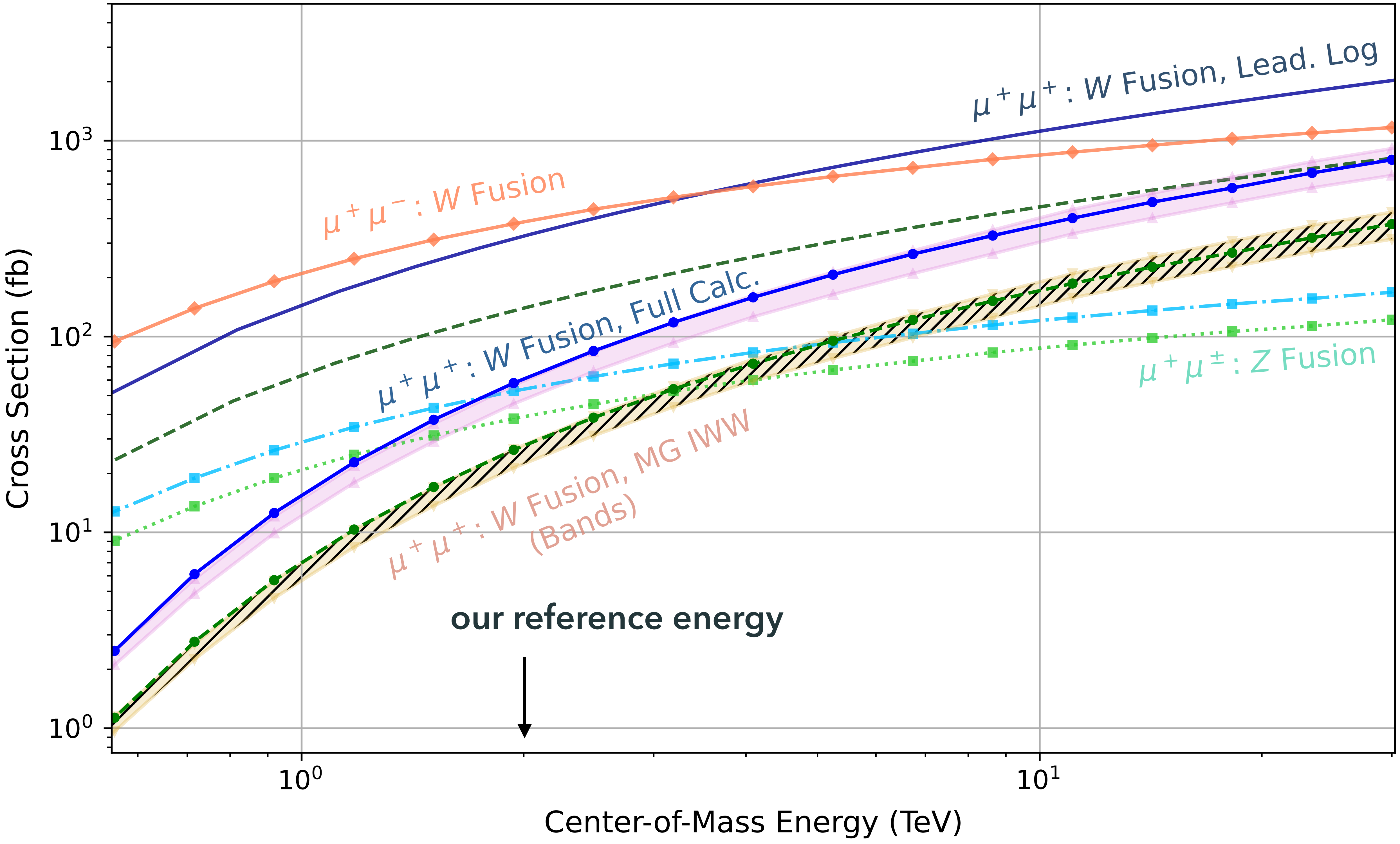


$$\sigma_{\mathrm{ZBF}} \propto \alpha^3 \log\frac{s}{m_h^2}$$

$$\sigma_{\gamma\mathrm{WBF}} \propto \alpha^4 \log\frac{s}{m_\mu^2}\left(\log\frac{s}{s_{\mathrm{min}}}\right)^2$$

Higgs production @ $\mu^+\mu^+$?

[2408.01068, **YH**, Kitano, Matsudo, Okawa, Takai, Takaura, and Treuer]



our reference energy

Plan of talk

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New physics search at μTRISTAN

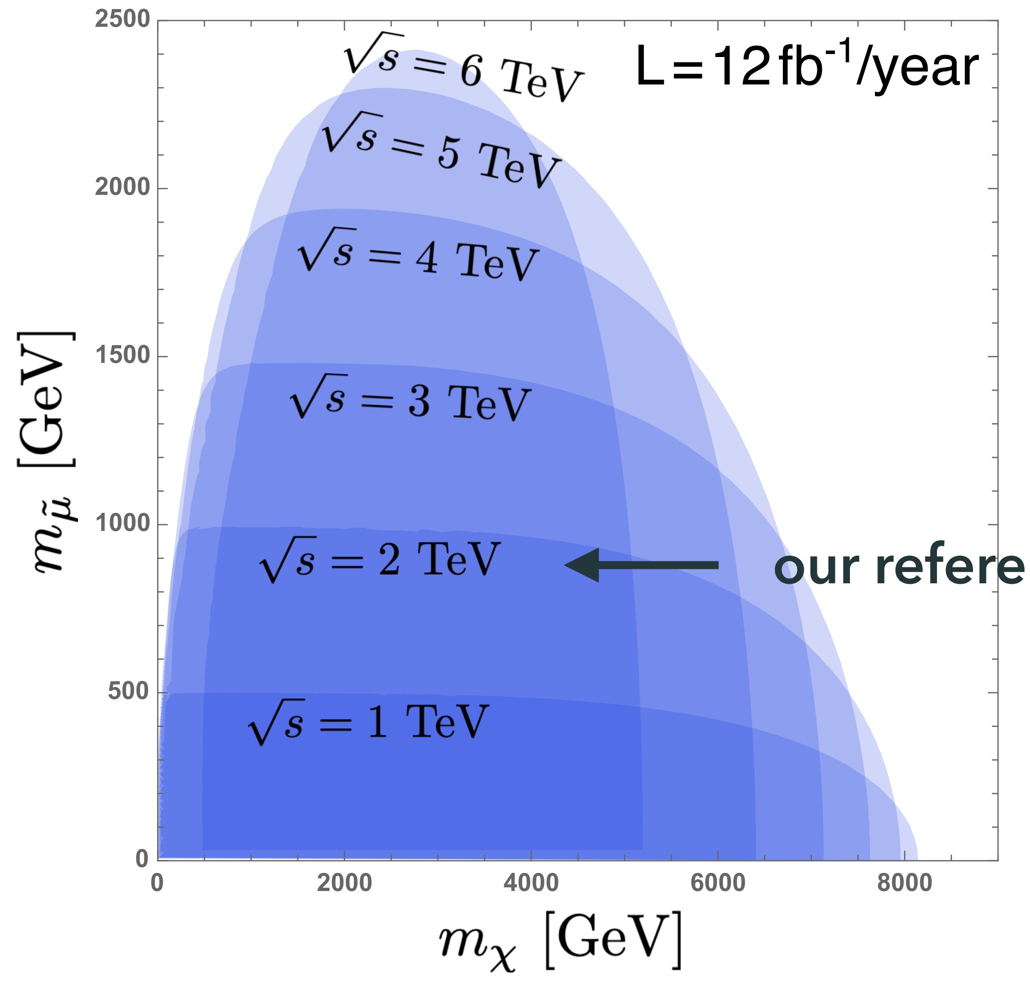
Slepton production at μTRISTAN

[**YH**, Kitano, Matsudo, Takaura, Yoshida, 2201.06664]

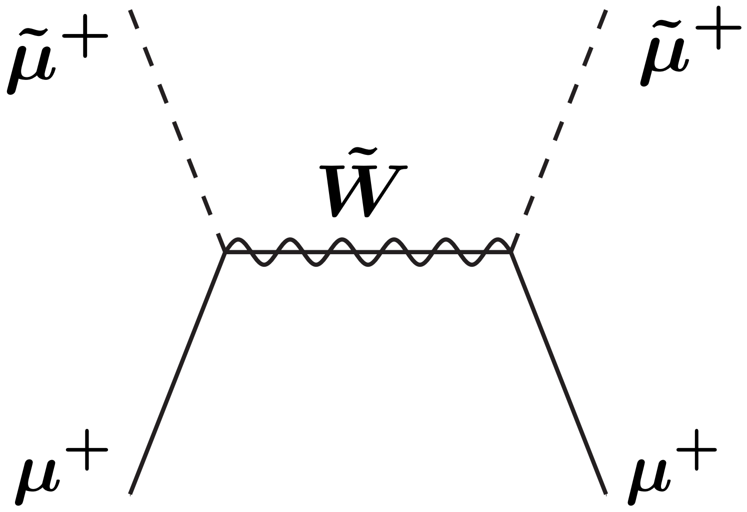
- For simplicity, only Wino $\tilde{W}$ exchange (no other neutralinos)

- Mass parameter region where # of events exceeds 100

(We do not consider decay of sleptons)



our reference energy



$\mu^+\mu^+$ **collider**

$m_{\tilde{\mu}} \lesssim 1\,\mathrm{TeV}$ can be explored

Indirect new physics search at μTRISTAN

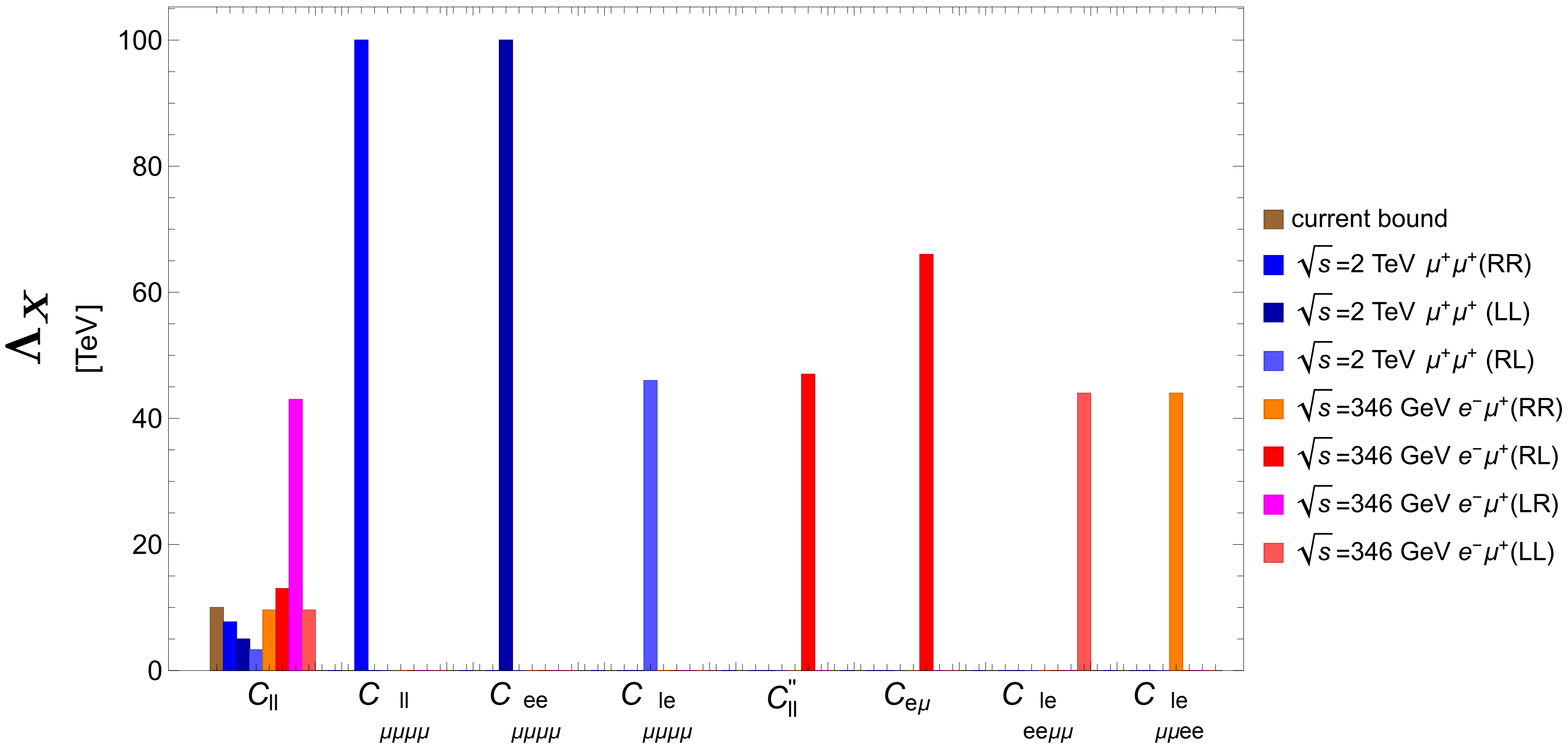
[**YH**, Kitano, Matsudo, Takaura, 2210.11083]

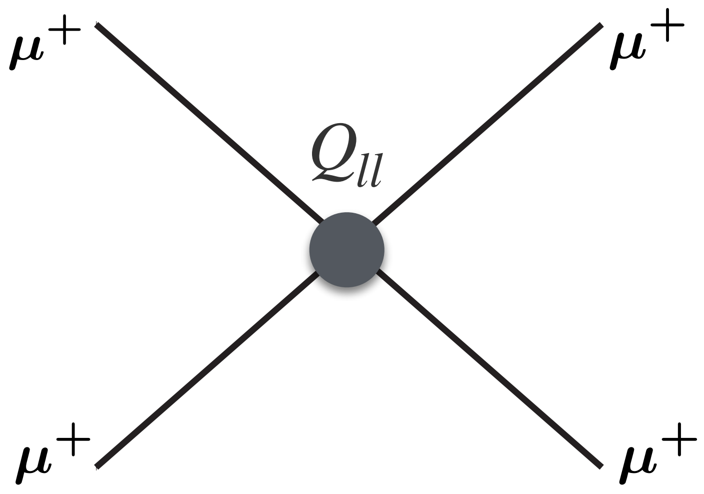
Constrain SMEFT dim-6 operators via Møller scattering

$$Q_{ll} = \frac{1}{\Lambda^2}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) \qquad Q_{HD} = \frac{1}{\Lambda^2}(H^\dagger D_\mu H)^* (H^\dagger D^\mu H) \qquad \text{etc.}$$

$$C_X = \frac{1}{\Lambda_X^2}$$

Current bound from 2204.05260 by Bagnaschi et al.





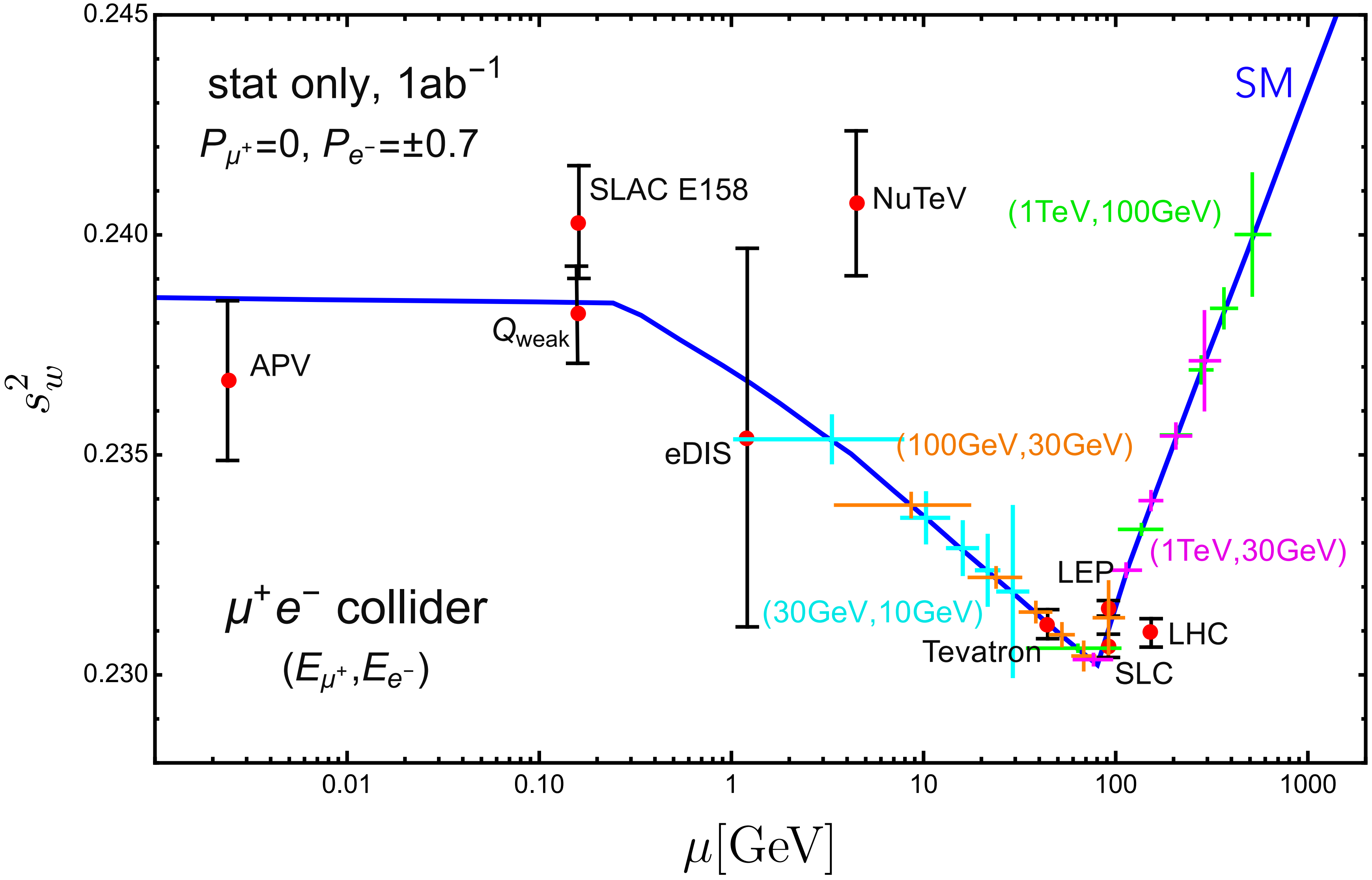
with 95% C.L.

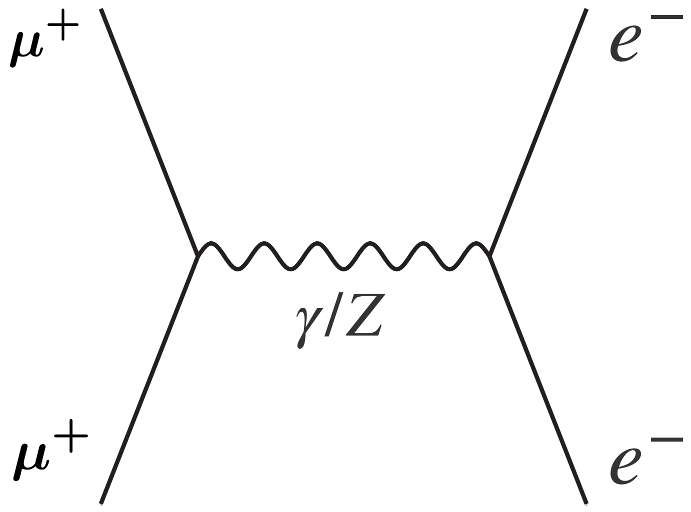
We can detect $\Lambda \lesssim 100\,\mathrm{TeV}$ at most!

Weak mixing angle at μTRISTAN

[Chen, **YH**, Iguro, 2406.04500]

Møller-like scattering btw $\mu^+ e^-$





scan over wide range with precision!

Summary

- **We proposed a new collider μTRISTAN using ultra-cold μ^+, which is already available!**

$\mu^+ e^-$ collider $\mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ $\sqrt{s} = 346\,\mathrm{GeV}$

- Higgs factory → coupling measurement w/ O(0.1) % precision
- Weak mixing angle measurement

$\mu^+ \mu^+$ collider $\mathscr{L}_{\mu^+\mu^+} = 5.7 \times 10^{32}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ $\sqrt{s} = 2\,\mathrm{TeV}$

- direct NP search like slepton
- indirect NP search based on SMEFT → $\Lambda \simeq 100\,\mathrm{TeV}$ at most
- Higgs factory at higher energy (γ-emit WBF)

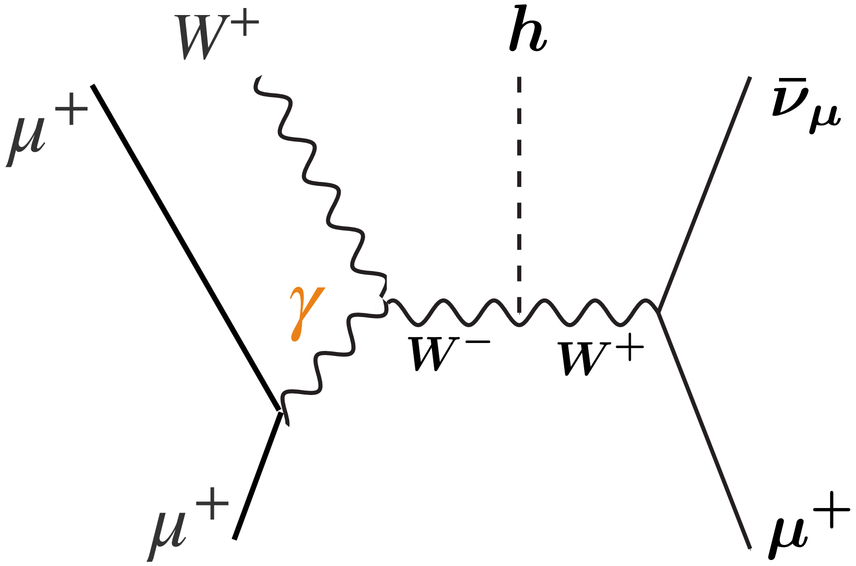
Backup

Higgs production @ $\mu^+\mu^+$?

[2408.01068, **YH**, Kitano, Matsudo, Okawa, Takai, Takaura, and Treuer]

$$f_{\gamma/\mu}(x,\mu_f^2) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\log\frac{\mu_f^2}{m_\mu^2}$$

$$f_{W_{\mathrm{L}}^+/\mu_R^+}(x) = \frac{\alpha}{2\pi\sin^2\theta_{\mathrm{W}}}\frac{1-x}{x}$$

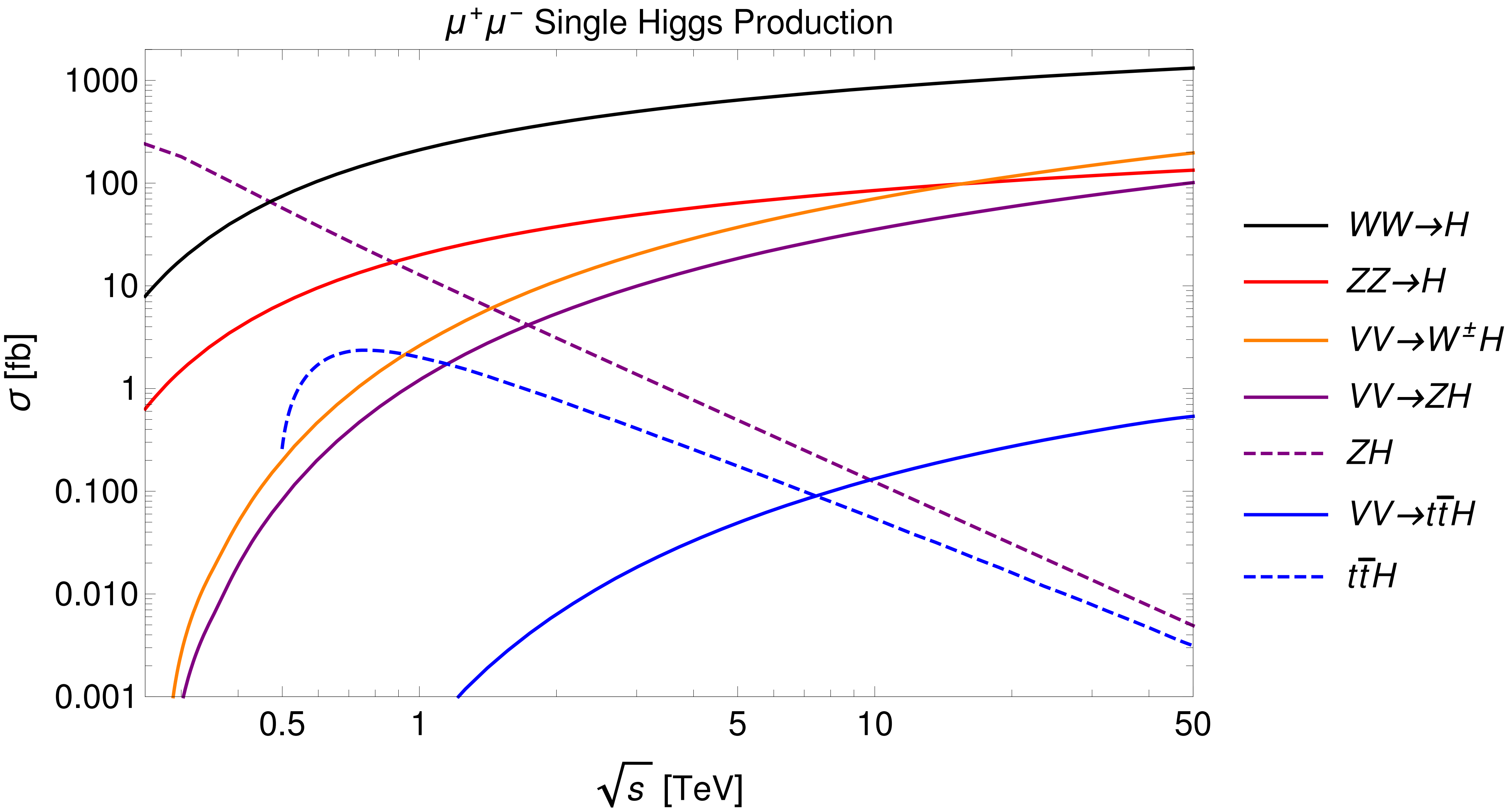


$$\sigma_{\gamma W}(s) = \frac{\pi\alpha^2}{m_W^2\sin^2\theta_{\mathrm{W}}} + \mathcal{O}\left(\frac{1}{s}\right) \qquad \because \sigma_{\gamma W} \sim \left(\frac{s}{t-m_{\mathrm{W}}^2}\frac{\partial}{m_{\mathrm{W}}}\right)^2 \times \frac{1}{s}$$

$$\sigma_\gamma(s) \simeq 2\int_{xys>s_{\min}} \mathrm{d}x\mathrm{d}y\, f_{\gamma/\mu^+}(x,\mu_f^2)\, f_{W_{\mathrm{L}}^+/\mu^+}(y)\,\sigma_{\gamma W}(xys)$$

$$\mu_f^2 = xys$$

Higgs production cross section



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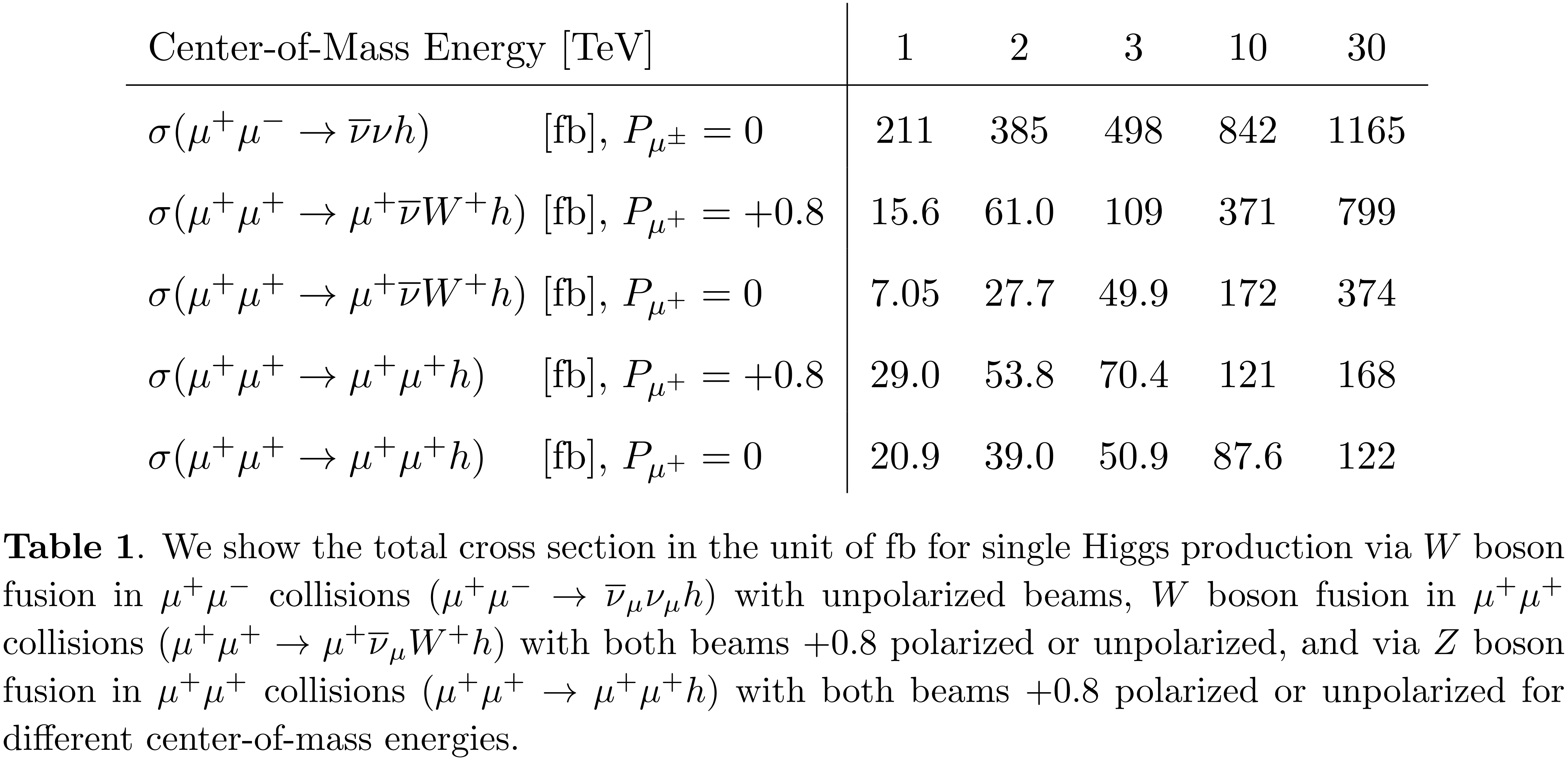


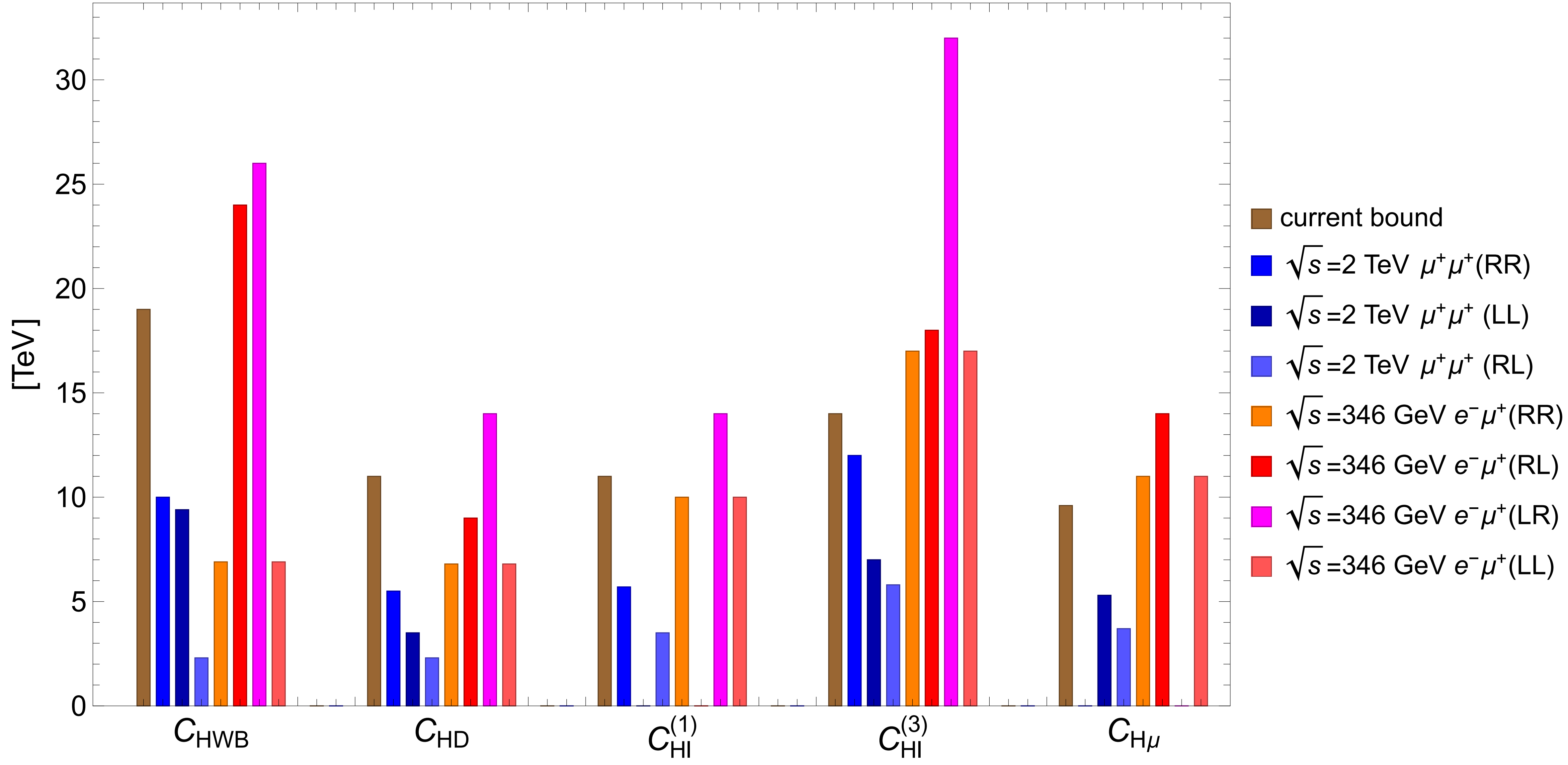
Center-of-Mass Energy [TeV]		1	2	3	10	30
$\sigma(\mu^+\mu^- \to \bar{\nu}\nu h)$	[fb], $P_{\mu^\pm} = 0$	211	385	498	842	1165
$\sigma(\mu^+\mu^+ \to \mu^+\bar{\nu}W^+h)$	[fb], $P_{\mu^+} = +0.8$	15.6	61.0	109	371	799
$\sigma(\mu^+\mu^+ \to \mu^+\bar{\nu}W^+h)$	[fb], $P_{\mu^+} = 0$	7.05	27.7	49.9	172	374
$\sigma(\mu^+\mu^+ \to \mu^+\mu^+h)$	[fb], $P_{\mu^+} = +0.8$	29.0	53.8	70.4	121	168
$\sigma(\mu^+\mu^+ \to \mu^+\mu^+h)$	[fb], $P_{\mu^+} = 0$	20.9	39.0	50.9	87.6	122

Table 1. We show the total cross section in the unit of fb for single Higgs production via W boson fusion in $\mu^+\mu^-$ collisions ($\mu^+\mu^- \to \bar{\nu}_\mu\nu_\mu h$) with unpolarized beams, W boson fusion in $\mu^+\mu^+$ collisions ($\mu^+\mu^+ \to \mu^+\bar{\nu}_\mu W^+h$) with both beams +0.8 polarized or unpolarized, and via Z boson fusion in $\mu^+\mu^+$ collisions ($\mu^+\mu^+ \to \mu^+\mu^+h$) with both beams +0.8 polarized or unpolarized for different center-of-mass energies.

Indirect new physics search at μTRISTAN

$$C_X = \frac{1}{\Lambda_X^2}$$

Current bound from 2204.05260 by Bagnaschi et al.



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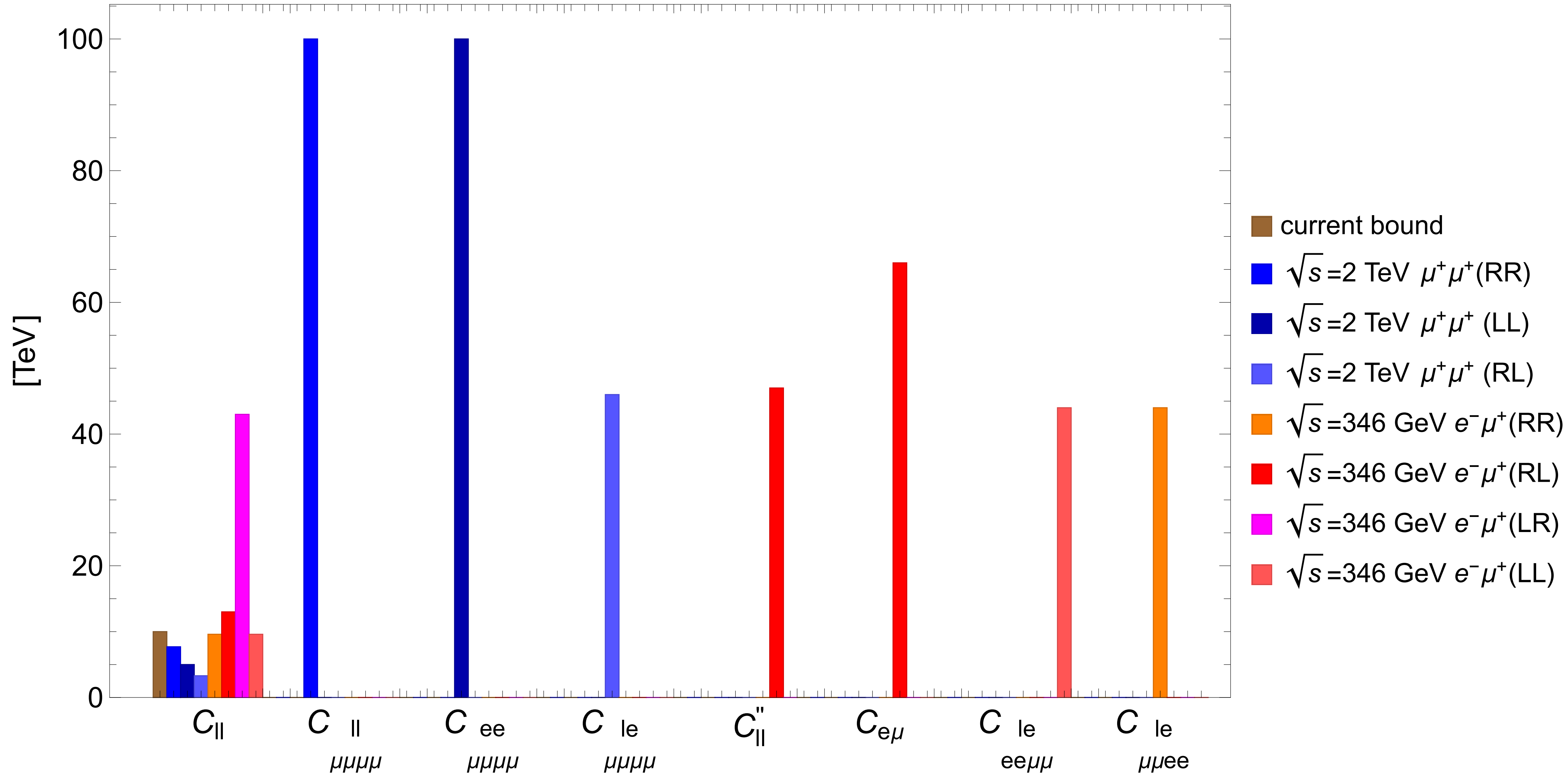


with 95% C.L.

We can detect $\Lambda \lesssim 100\,\mathrm{TeV}$ at most!

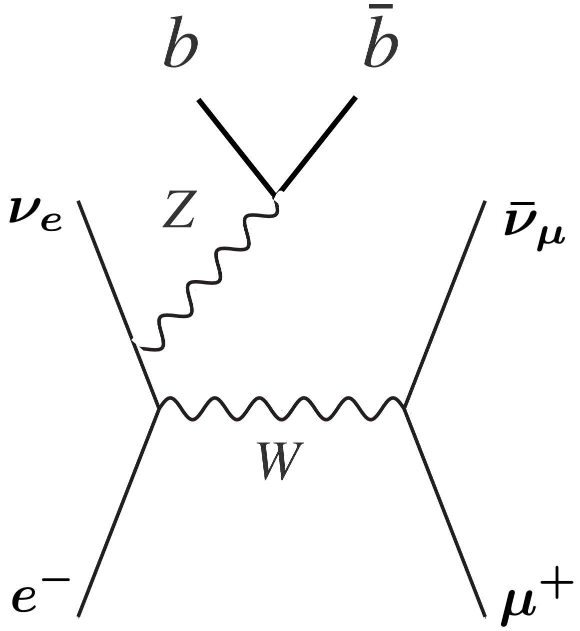
Efficiency

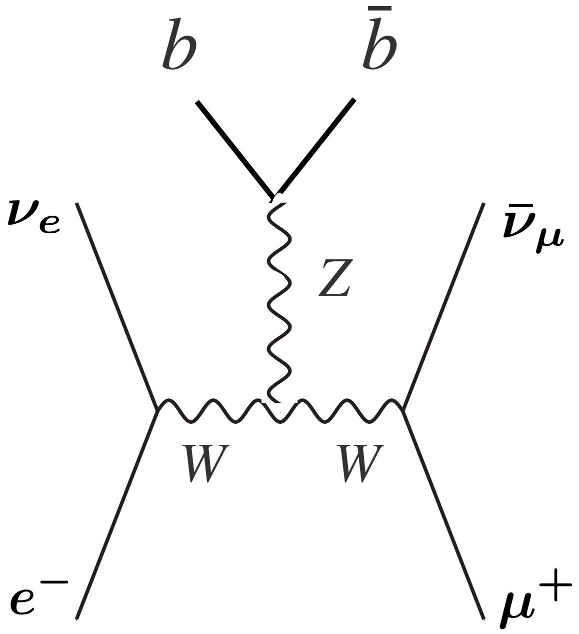
in progress with experimentalists

The efficiency to detect the events is important and under studied.

- **Background events**

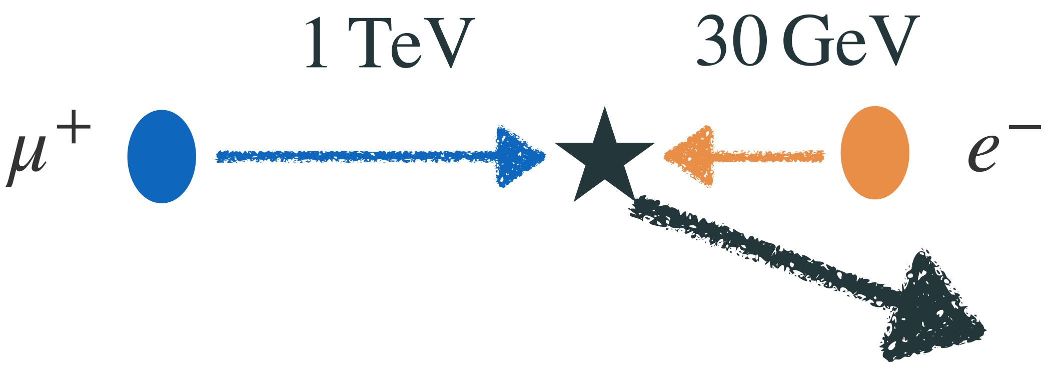
  By reconstructing the invariant mass from b-jets, we would suppress the BG events.





- **Coverage of detector in a small angle region**

  The produced particle is strongly boosted.



Efficiency

in progress with experimentalists

from slide by Toshiaki Kaji

Benchmark Detector : HL-LHC

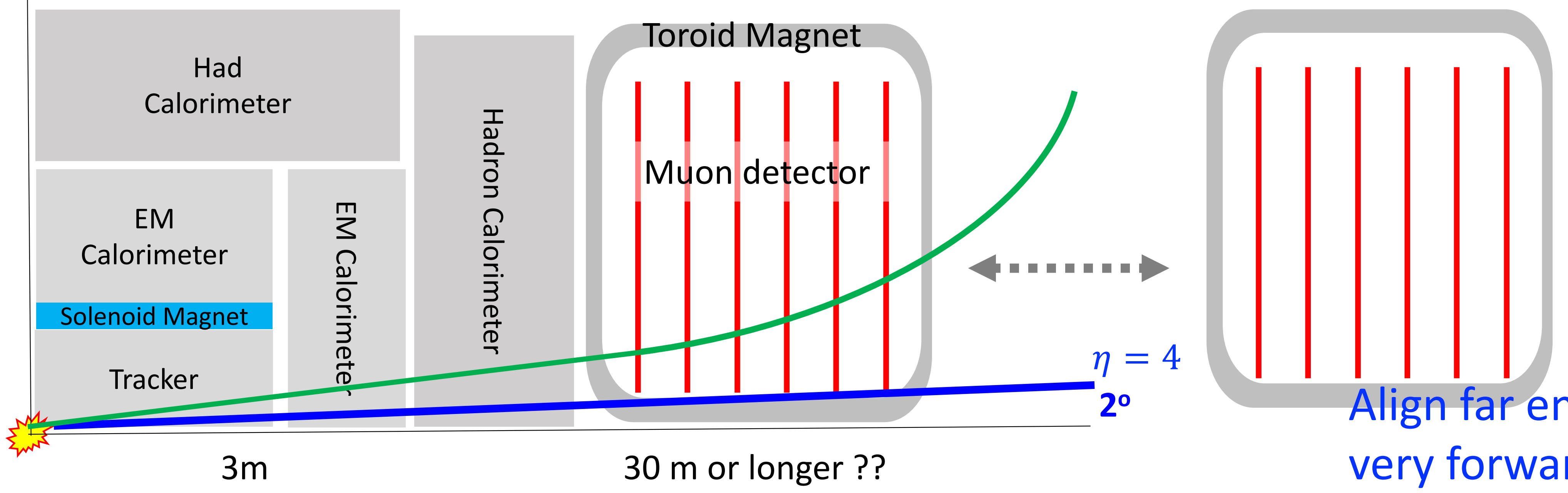
Coverage of HL-LHC @ Delphes

- Muon |η| < 4.0
- Electron |η| < 4.0
- B-jet |η| < 4.0, pT > 25 GeV

Delphes card for HL-LHC is used for this study.

https://github.com/delphes/delphes/blob/3.5.0/cards/delphes_card_HLLHC.tcl

Tentative rough schematic for muon collider detector



Forward muons should be detected by combination of forward magnet and muon detector.

Align far enough to detect very forward muons

Efficiency

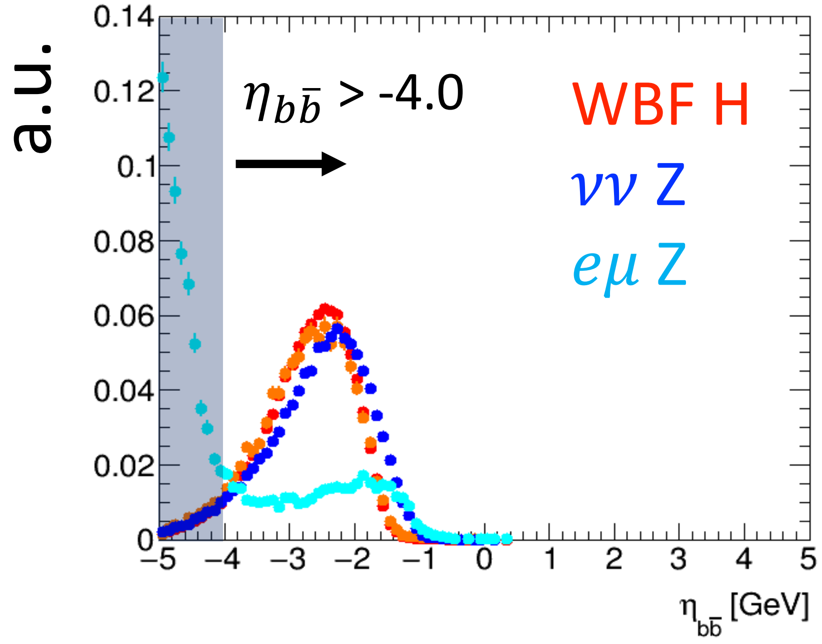
in progress with experimentalists

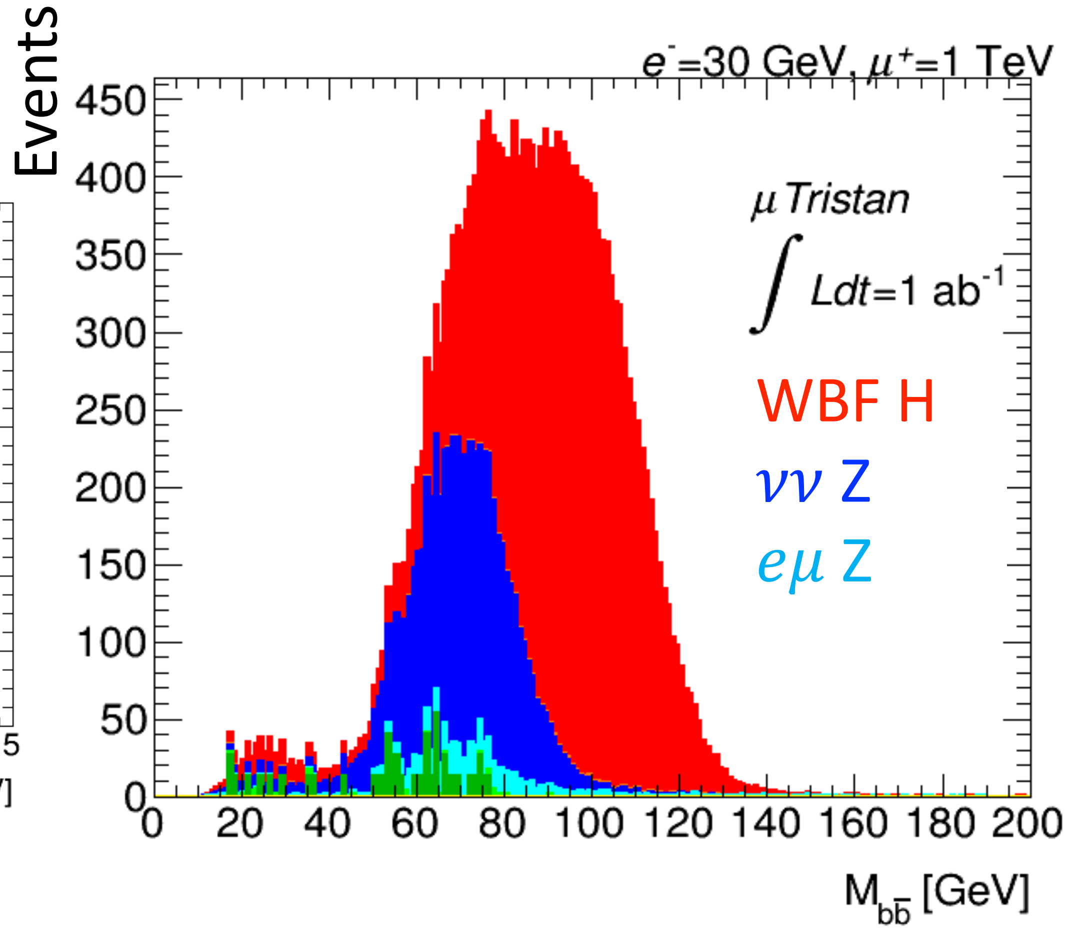
from slide by Toshiaki Kaji

WBF Higgs Measurements

Requirements

- No muon
- No electron
- Exact 2 b-jets
- $\eta_{b\bar{b}}$ > -4.0





Radiative Z background can be removed by lepton veto and $\eta_{b\bar{b}}$ cut.

Efficiency : ~ 23%

→ 12k events @ 1 ab^{-1}

→ $\Delta(\kappa_W + \kappa_b - \kappa_H)_{\mathrm{stat}} =$ 0.5% @ 1 ab^{-1}

Trilinear coupling in higher energy case

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{e^-} = 50\,\mathrm{GeV}$$

$$\sqrt{s} = 775\,\mathrm{GeV} \qquad (P_{\mu^+}, P_{e^-}) = (0.8, -0.7)$$

- WBF process $\sigma_{\mathrm{WBF}} \simeq 472\,\mathrm{fb}$

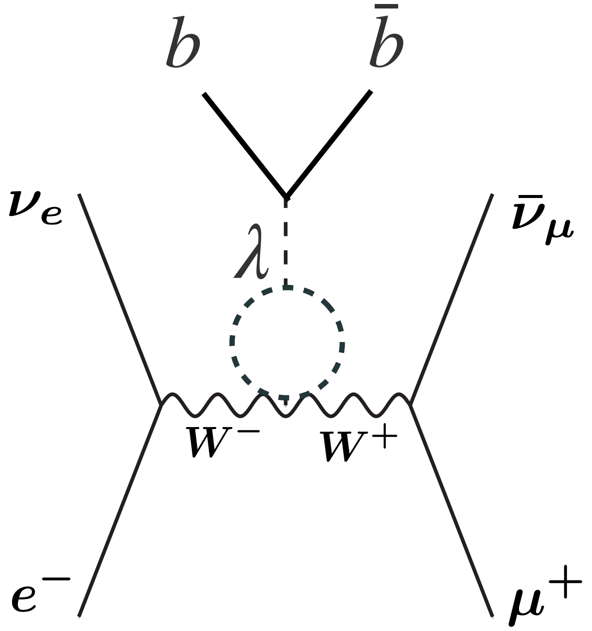
can probe Higgs trilinear coupling via 1-loop

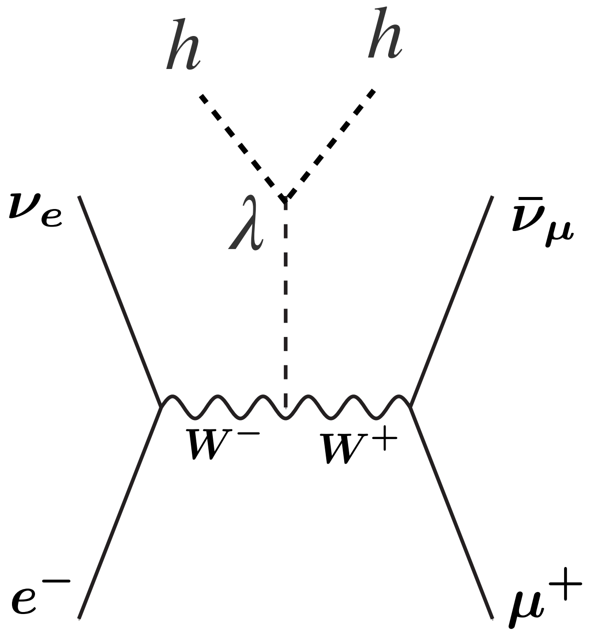
[Di Vita+, 1711.03978]

$$\left|\Delta\kappa_W + \Delta\kappa_b - \Delta\kappa_H + 0.006\Delta\kappa_\lambda\right|$$

$$\lesssim 1.3 \times 10^{-3} \times \left(\frac{\text{integrated luminosity}}{1.0\,\mathrm{ab}^{-1}}\right)^{-1/2} \left(\frac{\text{efficiency}}{0.5}\right)^{-1/2}$$

$$\longrightarrow \underline{\left|\Delta\kappa_\lambda\right| \lesssim 20\,\%}$$ (if other Δκ's are zero)





- also probed by Higgs pair production

$$N(\text{di Higgs}) \simeq 89 \times \left(\frac{\text{integrated luminosity}}{1.0\,\mathrm{ab}^{-1}}\right)^{-1/2} \left(\frac{\text{efficiency}}{0.5}\right)^{-1/2} \longrightarrow \left|\Delta\kappa_\lambda\right| \lesssim 100\,\%$$

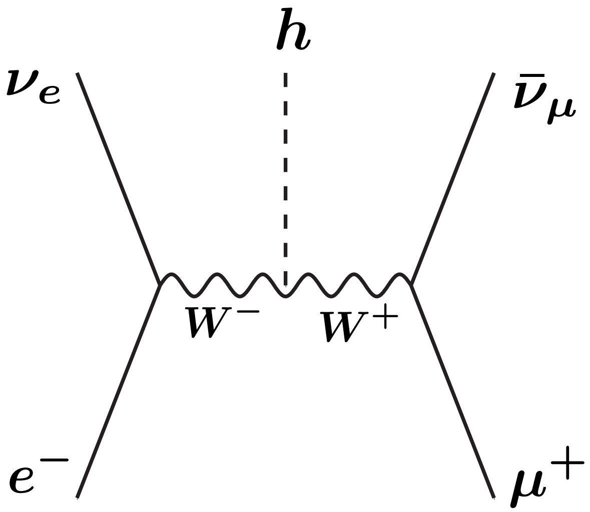
Proposal of new experiment: μTRISTAN!

If a larger 6 (or 9) km ring is available (Tevatron size), we can explore higher energy:

- $\mu^+ e^-$ collider

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{e^-} = 50\,\mathrm{GeV}$$

$$\to \sqrt{s} = 775\,\mathrm{GeV}$$ **Higgs factory**



- $\mu^+ \mu^+$ collider (instead of $\mu^+ \mu^-$)

$$E_{\mu^+} = 3\,\mathrm{TeV}, \quad E_{\mu^+} = 3\,\mathrm{TeV}$$

$$\to \sqrt{s} = 6\,\mathrm{TeV}$$ **New physics search**

Comment on polarization

Polarization is important to enhance cross sections

- **Electron beam polarization:** $P_{e^-} = \pm 0.7$ same polarization as superKEKB
- **Muon beam polarization:** $P_{\mu^+} = \pm 0.8$

eg., $P = 0.4$ means 40 % is RH while 60% is unpolarized.

muon production sequence:

$\pi^+ \to \mu^+ + \nu_\mu$ ⟶ Muonium ($\mu^+ e^-$) formation in target silica aerogel ⟶ Ultra-cold muon

ionized by laser

$P_{\mu^+} = +1.0$

initial muonium states: $|+\rangle_{\mu^+} |+\rangle_{e^-}$ & $|+\rangle_{\mu^+} |-\rangle_{e^-}$

oscillate $|-\rangle_{\mu^+} |+\rangle_{e^-}$

By applying magnetic field of 0.3 T, the polarization can be maintained.

[CDR for muon g-2/EDM exp. at J-PARC]

Thus $P_{\mu^+} = \pm 0.8$ would be a reasonable estimation.

SMEFT operator

$$Q_{HWB} = H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu}$$ $\longleftarrow$ S parameter

$$Q_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$$ $\longleftarrow$ T parameter

$$Q^{(1)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$$

$$Q^{(3)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}{}^I_\mu H)(\bar{L}\tau^I \gamma^\mu L)$$

$$Q_{H\mu} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\mu}\gamma^\mu P_+ \mu)$$

$$Q_{\substack{ll\\prst}} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

$$Q_{\substack{le\\prst}} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

$$Q_{\substack{ee\\prst}} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$

$$C_{\substack{\ell\ell\\ \mu\mu\mu\mu}}, \quad C''_{\ell\ell} \equiv \frac{1}{2}\left(C_{\substack{\ell\ell\\ ee\mu\mu}} + C_{\substack{\ell\ell\\ \mu\mu ee}}\right), \quad C_{\substack{\ell e\\ \mu\mu\mu\mu}}, \quad C_{\substack{\ell e\\ ee\mu\mu}}, \quad C_{\substack{\ell e\\ \mu\mu ee}}, \quad C_{\substack{ee\\ \mu\mu\mu\mu}},$$

$$C_{e\mu} \equiv \frac{1}{4}\left(C_{\substack{ee\\ \mu\mu ee}} + C_{\substack{ee\\ ee\mu\mu}} + C_{\substack{ee\\ \mu ee\mu}} + C_{\substack{ee\\ e\mu\mu e}}\right).$$

$$C_{\substack{\ell\ell\\ e\mu\mu e}} = C_{\substack{\ell\ell\\ \mu ee\mu}} \equiv C_{\ell\ell}.$$

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}}\, N_{\text{beam2}}}{4\pi\, \sigma_x\, \sigma_y} f_{\text{rep}}$$

- Collision frequency (per detector)

$$f_{\text{rep}}^{(\mu^+e^-)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 40 = 4\,\text{MHz}$$

speed of beam

circumference of ring

# of bunches

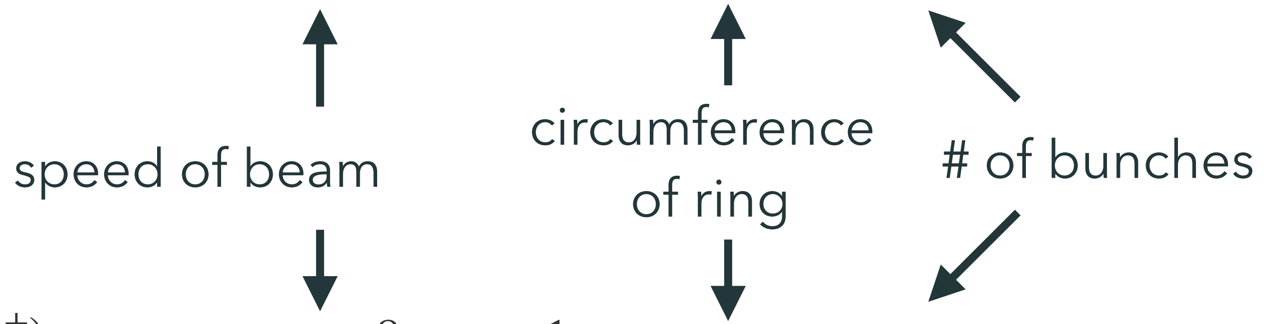
$$f_{\text{rep}}^{(\mu^+\mu^+)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 20 = 2\,\text{MHz}$$

Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}} N_{\text{beam2}}}{4\pi \, \sigma_x \, \sigma_y} f_{\text{rep}}$$

- Collision frequency (per detector)

$$f_{\text{rep}}^{(\mu^+ e^-)} = 3.0 \times 10^8 \, \text{m s}^{-1} / (3 \, \text{km}) \times 40 = 4 \, \text{MHz}$$



speed of beam

circumference of ring

# of bunches

$$f_{\text{rep}}^{(\mu^+ \mu^+)} = 3.0 \times 10^8 \, \text{m s}^{-1} / (3 \, \text{km}) \times 20 = 2 \, \text{MHz}$$

$$\beta\gamma \sim 10^4$$

- Beam size is determined by emittance

emittance: $\epsilon_x \, , \epsilon_y = \frac{4 \, \text{mm mrad}}{\beta\gamma}$

$$\sigma_i = \sqrt{\epsilon_i \beta_i}$$

beta function

$\beta_x = 30 \, \text{mm}$ $\sigma_x = 3.6 \, \mu\text{m}$

$\beta_y = 7 \, \text{mm}$ $\sigma_y = 1.7 \, \mu\text{m}$

Luminosity

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- Collision frequency (per detector)

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Thanks to ultra-cold muon, low emittance is realized!

$$\sigma_i = \sqrt{\epsilon_i \beta_i}$$

beta function

$\beta_x = 30\,\mathrm{mm}$ $\quad\sigma_x = 3.6\,\mu\mathrm{m}$

$\beta_y = 7\,\mathrm{mm}$ $\quad\sigma_y = 1.7\,\mu\mathrm{m}$

Luminosity

$$\mathscr{L} = \frac{N_{\mathrm{beam1}} N_{\mathrm{beam2}}}{4\pi \sigma_x \sigma_y} f_{\mathrm{rep}}$$

- Collision frequency (per detector)

$$f_{\mathrm{rep}}^{(\mu^+e^-)} = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}/(3\,\mathrm{km}) \times 40 = 4\,\mathrm{MHz}$$

speed of beam

circumference of ring

# of bunches

$$f_{\mathrm{rep}}^{(\mu^+\mu^+)} = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}/(3\,\mathrm{km}) \times 20 = 2\,\mathrm{MHz}$$

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Thanks to ultra-cold muon, low emittance is realized!

beta function

$\beta_x = 30\,\mathrm{mm}$ $\sigma_x = 3.6\,\mu\mathrm{m}$

$\beta_y = 7\,\mathrm{mm}$ $\sigma_y = 1.7\,\mu\mathrm{m}$

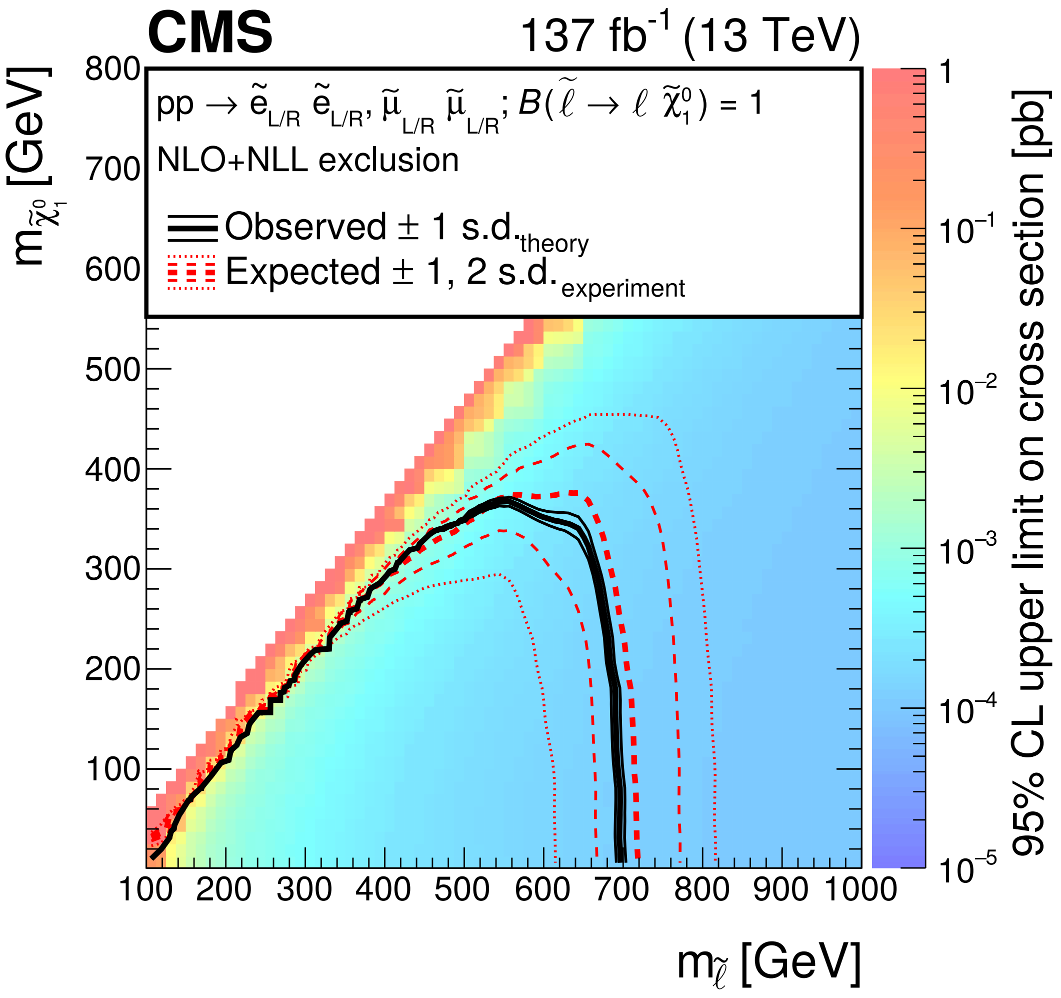
- # of beam particles

$N_{e^-} = 10\,\mathrm{nC}$ per bunch

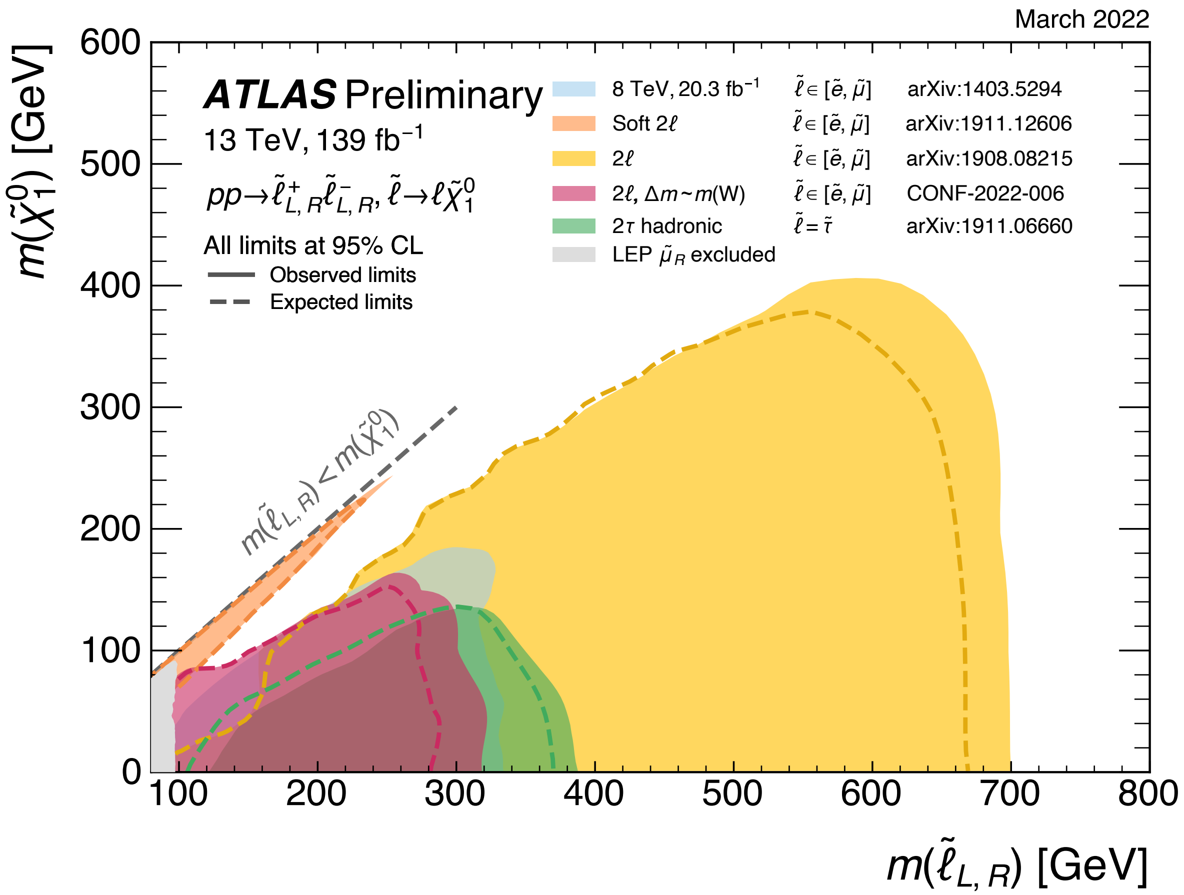
$N_{\mu^+} = 3.6\,\mathrm{nC} \rightarrow 1.3\,\mathrm{nC}$ per bunch due to decay

Current status of SUSY search

[2012.08600]



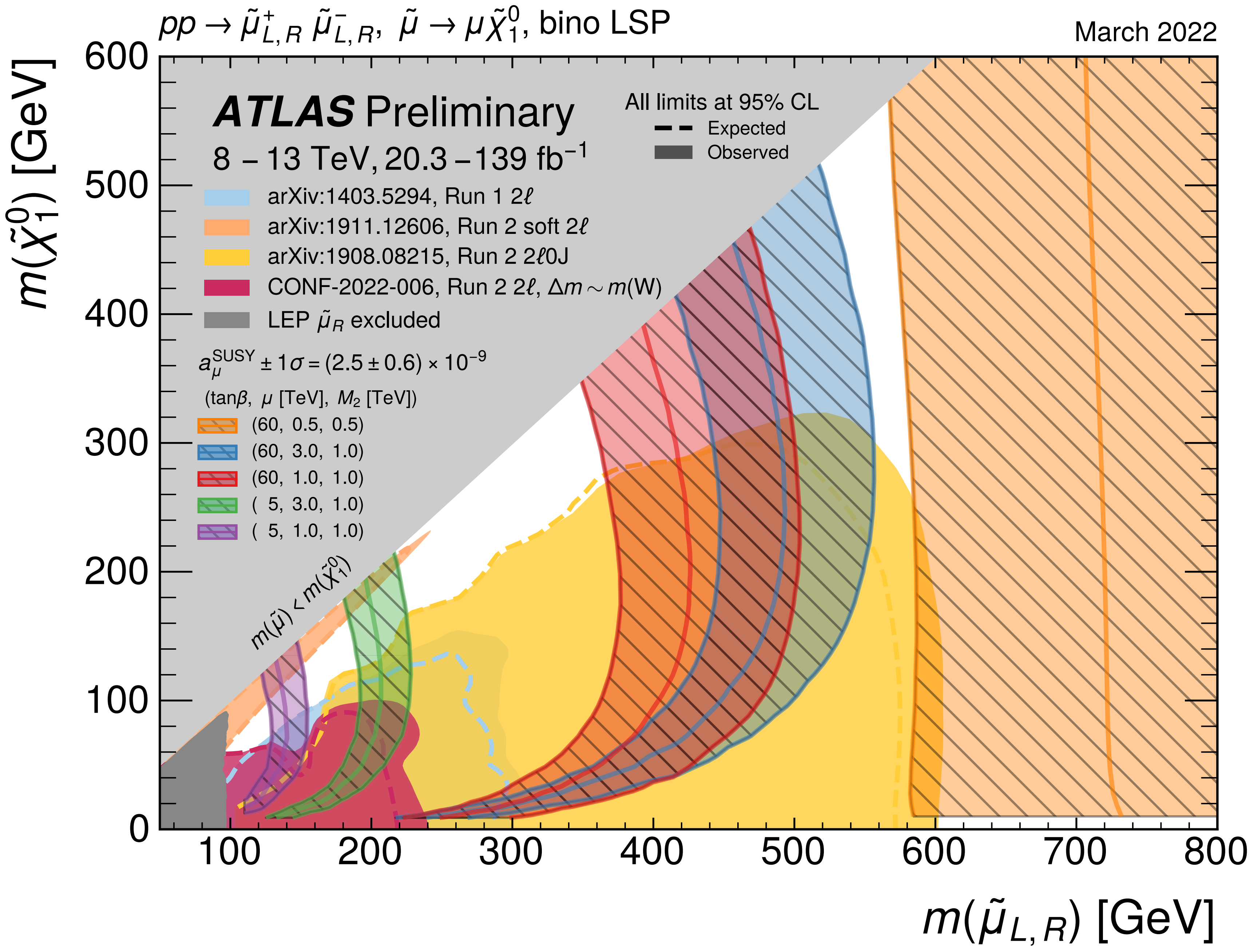
[ATL-PHYS-PUB-2022-013]



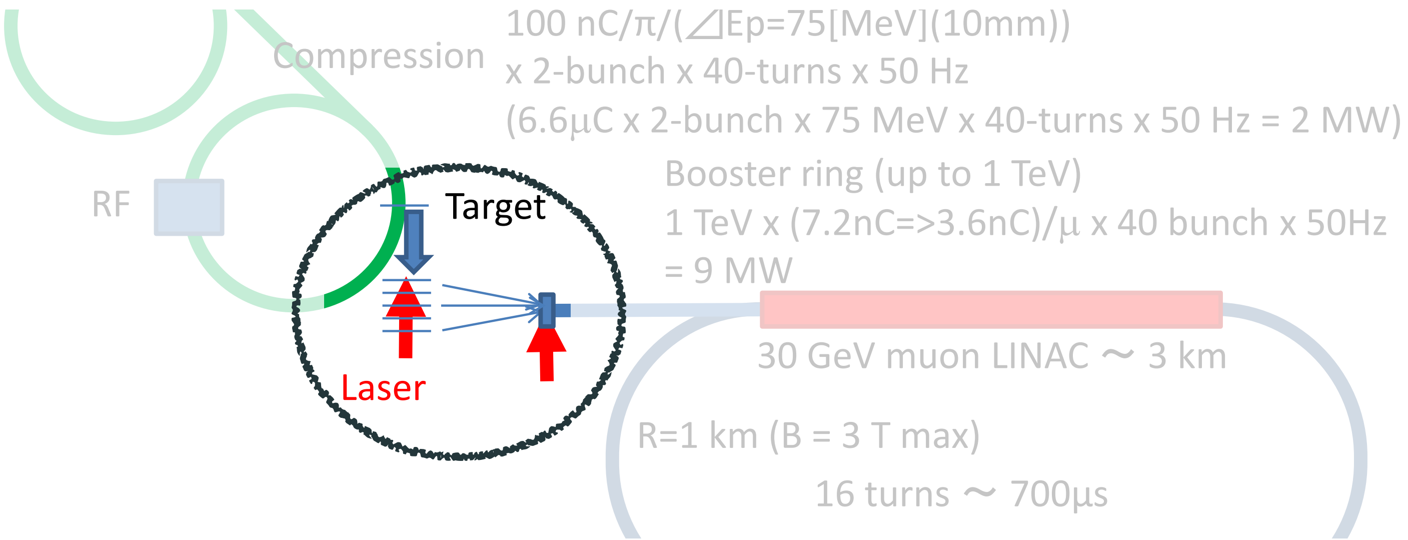
Typically, slepton w/ mass $m_{\tilde{l}} \lesssim 700\,\mathrm{GeV}$ is excluded.

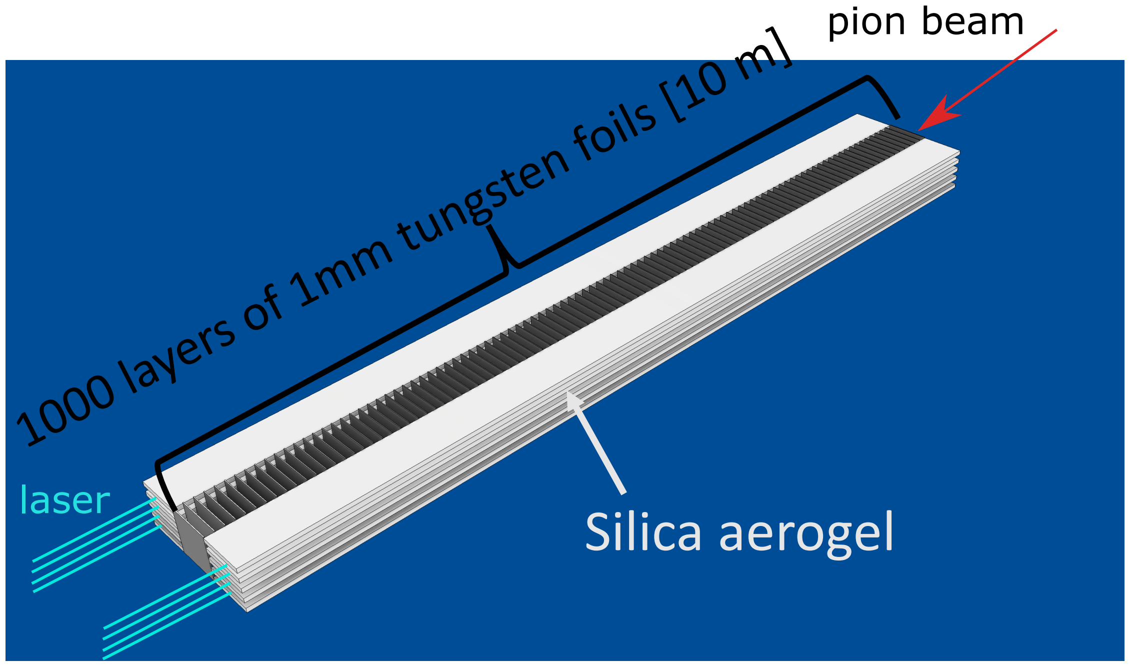
SUSY search and muon g-2

[ATL-PHYS-PUB-2022-013]



Ultra-cold muons





1. Pions are stopped at tungsten foils and decay into muons.

   pion transportation to the target: 50%

2. Muons are transported into the aerogel target and form muoniums.

   muonium formation: 52 %

3. Neutral muoniums become thermalized w/ $E_K \sim 25\,\mathrm{meV}$ and thermally diffused from the target. → ionized by laser

   muonium emission: 60 % & decay loss: 60 %

   laser ionization: 73%

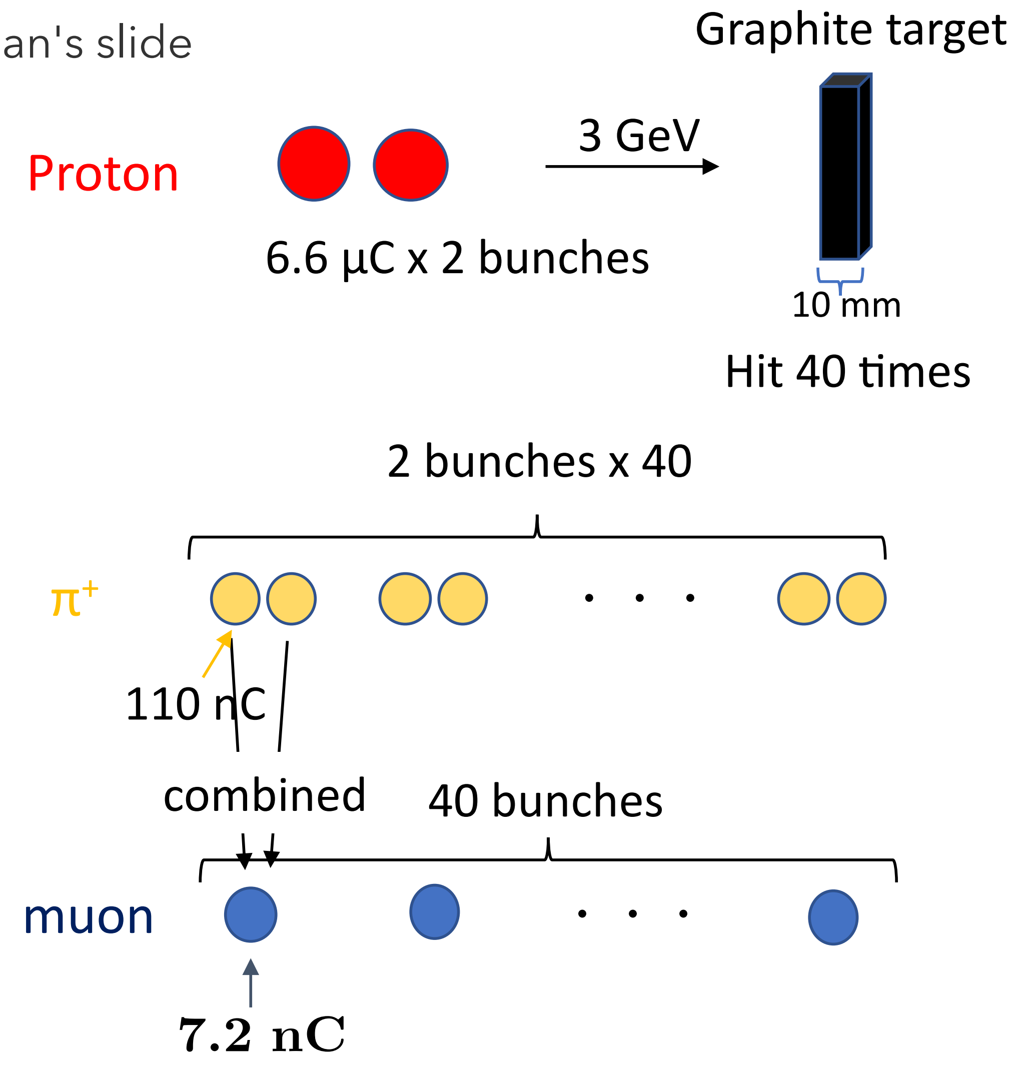
repeat step 2 and 3 twice: **1st target size ~ 10m, 2nd target ~ O(1) cm**

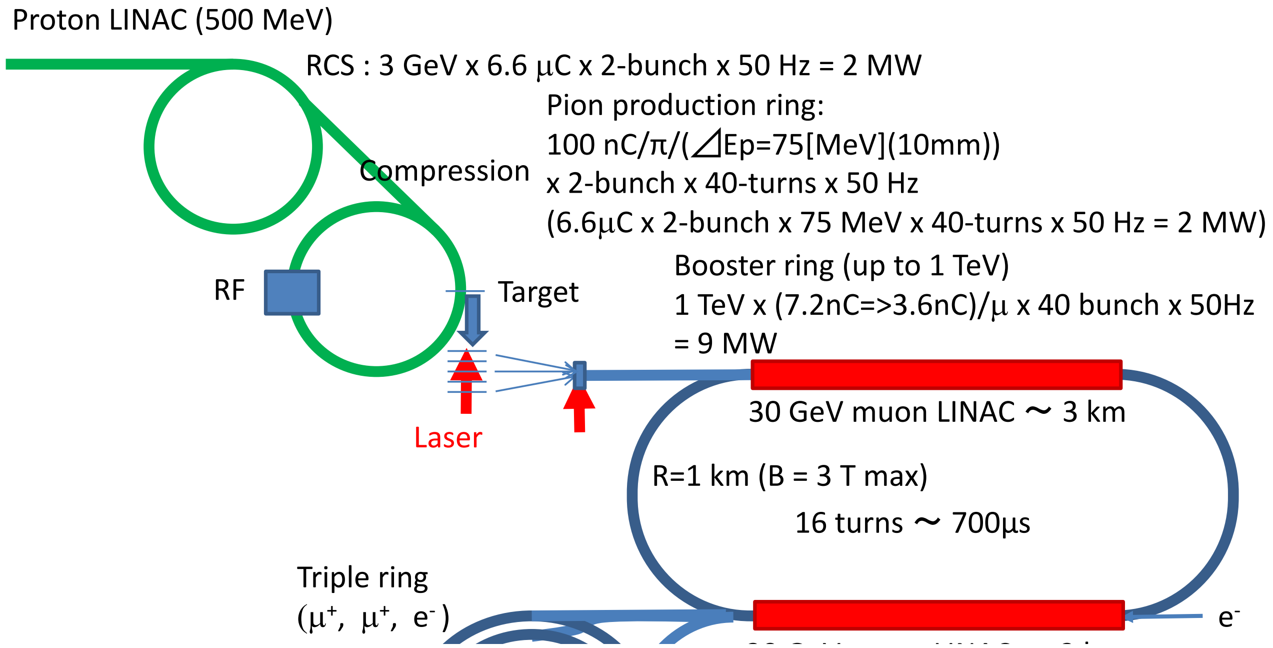
50% at second time because of a thin target

$$\therefore N_{\mu^+}/N_{\pi^+} \simeq 0.5 \times 0.52 \times 0.73 \times 0.6 \times 0.6 \times 0.5 \simeq 3.4\,\%$$

Estimation of N_{μ^+}

Fig taken from
Takaura-san's slide


Graphite target
Proton
3 GeV
6.6 μC x 2 bunches
10 mm
Hit 40 times
2 bunches x 40
π^+
110 nC
combined
40 bunches
muon
7.2 nC


Proton LINAC (500 MeV)
RCS : 3 GeV x 6.6 μC x 2-bunch x 50 Hz = 2 MW
Pion production ring:
100 nC/π/(⊿Ep=75[MeV](10mm))
x 2-bunch x 40-turns x 50 Hz
(6.6μC x 2-bunch x 75 MeV x 40-turns x 50 Hz = 2 MW)
Compression
RF
Target
Laser
Booster ring (up to 1 TeV)
1 TeV x (7.2nC=>3.6nC)/μ x 40 bunch x 50Hz
= 9 MW
30 GeV muon LINAC ~ 3 km
R=1 km (B = 3 T max)
16 turns ~ 700μs
Triple ring
(μ^+, μ^+, e^-)
e^-

These operations are repeated at every 20 ms.

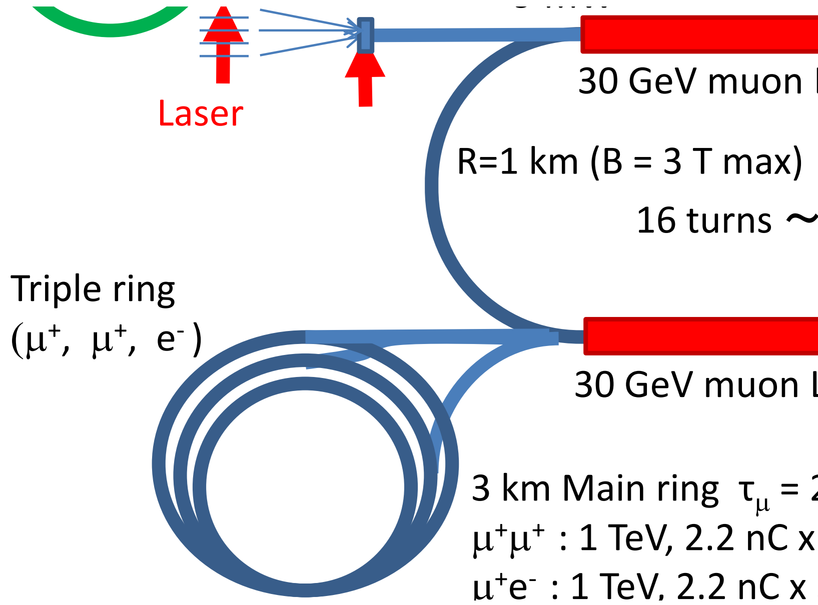
$$\text{Initial \# of ultra-cold muons} = 7.2\,\mathrm{nC} \times 40\,\mathrm{bunch}/(20\,\mathrm{ms}) = 9.0 \times 10^{13}/\mathrm{s}$$

acceleration

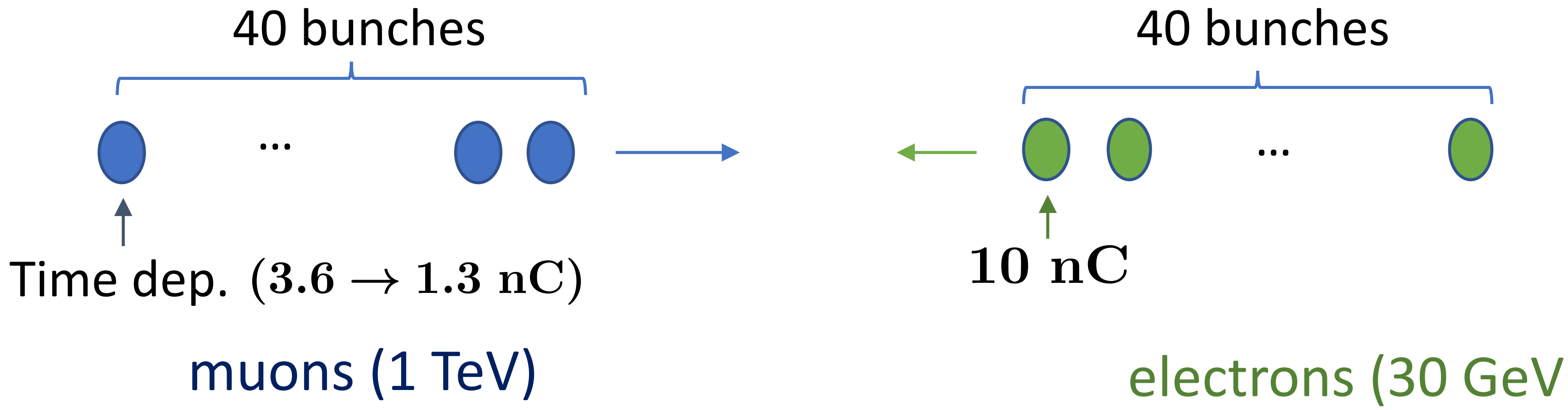
$$\longrightarrow \quad \text{\# of ultra-cold muons} = 3.6\,\mathrm{nC} \times 40\,\mathrm{bunch}/(20\,\mathrm{ms}) = 4.5 \times 10^{13}/\mathrm{s}$$

Beam in main ring

Fig taken from
Takaura-san's slide



- μ^+e^- collider



40 bunches

…

Time dep. $(3.6 \to 1.3\ \mathrm{nC})$

muons (1 TeV)

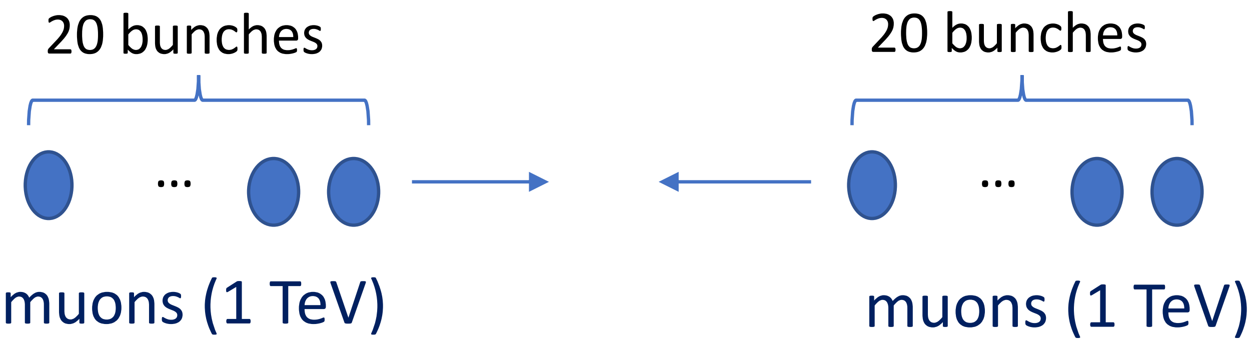
40 bunches

…

$10\ \mathrm{nC}$

electrons (30 GeV)

- $\mu^+\mu^+$ collider



20 bunches

…

muons (1 TeV)

20 bunches

…

muons (1 TeV)

Luminosity

$$\mathscr{L} = \frac{N_{\mathrm{beam1}} N_{\mathrm{beam2}}}{4\pi\,\sigma_x\,\sigma_y} f_{\mathrm{rep}}$$

- Collision frequency (per detector)

$$f_{\mathrm{rep}}^{(\mu^+e^-)} = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}/(3\,\mathrm{km}) \times 40 = 4\,\mathrm{MHz}$$

speed of beam

circumference of ring

# of bunches

$$f_{\mathrm{rep}}^{(\mu^+\mu^+)} = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}/(3\,\mathrm{km}) \times 20 = 2\,\mathrm{MHz}$$

$$\beta\gamma \sim 10^4$$

- Beam size

emittance: $\epsilon_x, \epsilon_y = \frac{4\,\mathrm{mm\,mrad}}{\beta\gamma}$

$$\sigma_i = \sqrt{\epsilon_i \beta_i}$$

beta function

$\beta_x = 30\,\mathrm{mm}$ $\sigma_x = 3.6\,\mu\mathrm{m}$

$\beta_y = 7\,\mathrm{mm}$ $\sigma_y = 1.7\,\mu\mathrm{m}$

Thanks to ultra-cold muon, low emittance is realized!

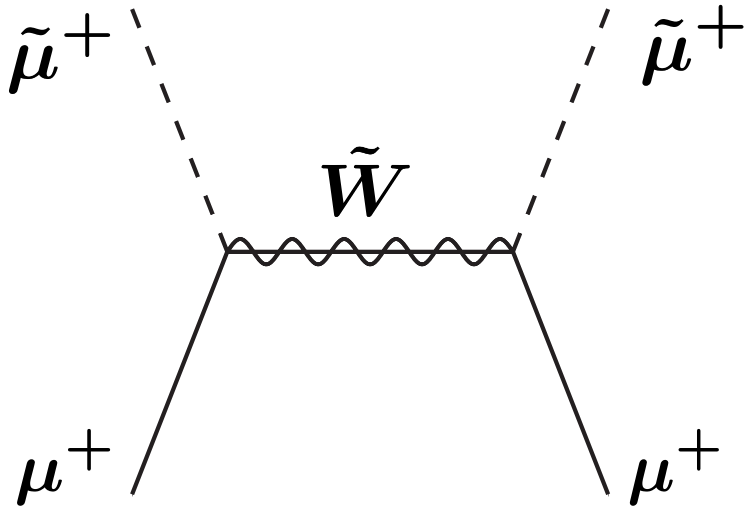
Without cooling, the normalized emittance is $\sim 10^3\pi\,\mathrm{mm\,mrad}$

Cross section

$$d\sigma = \frac{d\cos\theta}{32\pi}\frac{\beta}{s}|M_{\mathrm{RR}}|^2\frac{(1+P_{\mu 1})(1+P_{\mu 2})}{4}, \quad 0 \le \cos\theta \le 1,$$

$$M_{\mathrm{RR}} = -\frac{g_2^2}{2}\cdot\frac{4\sqrt{x}_A(1+2x_A-2x_3)}{(1+2x_A-2x_3)^2-\beta^2\cos^2\theta},$$

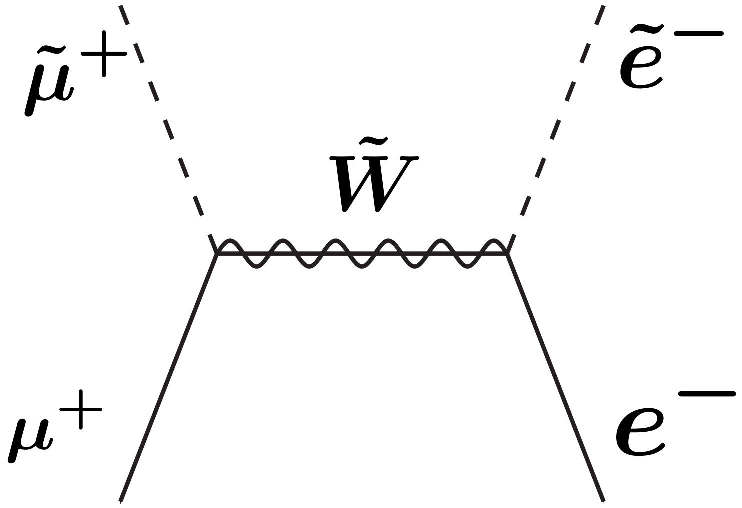
$$x_A = \frac{m_\chi^2}{s}, \quad x_3 = \frac{m_{\tilde{\mu}}^2}{s}, \quad \beta = \sqrt{1-4x_3}.$$



$$\sigma = \frac{g_2^4}{64\pi}\frac{1}{s}\left[\frac{\beta x_A}{x_A+(x_A-x_3)^2}+\frac{2x_A}{1+2x_A-2x_3}\log\frac{1+2x_A-2x_3+\beta}{1+2x_A-2x_3-\beta}\right]$$
$$\times\,\frac{(1+P_{\mu 1})(1+P_{\mu 2})}{4}.$$

Cross section

$$d\sigma = \frac{d\cos\theta}{32\pi}\frac{(1+x_3-x_4)\beta}{s}|M_{\mathrm{LR}}|^2\frac{(1-P_{e^-})(1+P_{\mu^+})}{4}, \quad -1 \le \cos\theta \le 1,$$



$$M_{\mathrm{LR}} = -\frac{g_2^2}{2}\cdot\frac{(1+x_3-x_4)\beta\sin\theta}{1+2x_A-x_3-x_4-(1+x_3-x_4)\beta\cos\theta},$$

$$x_A = \frac{m_\chi^2}{s}, \quad x_3 = \frac{m_{\tilde{e}}^2}{s}, \quad x_4 = \frac{m_{\tilde{\mu}}^2}{s}, \quad \beta = \frac{\sqrt{1-2x_3-2x_4+(x_3-x_4)^2}}{1+x_3-x_4}.$$

Comparison with ILC

- **μTRISTAN: μ^+e^- collider**

$$\sqrt{s} = 346\,\mathrm{GeV} \qquad \mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\mathrm{cm^{-2}s^{-1}}$$

main ring: 3km circumference

booster ring: 2km LINAC x2 + R=1km arc

construction cost: 5000 billion yen? (with large uncertainty)

(1 dollar ~130 yen)

- **ILC: e^+e^- collider**

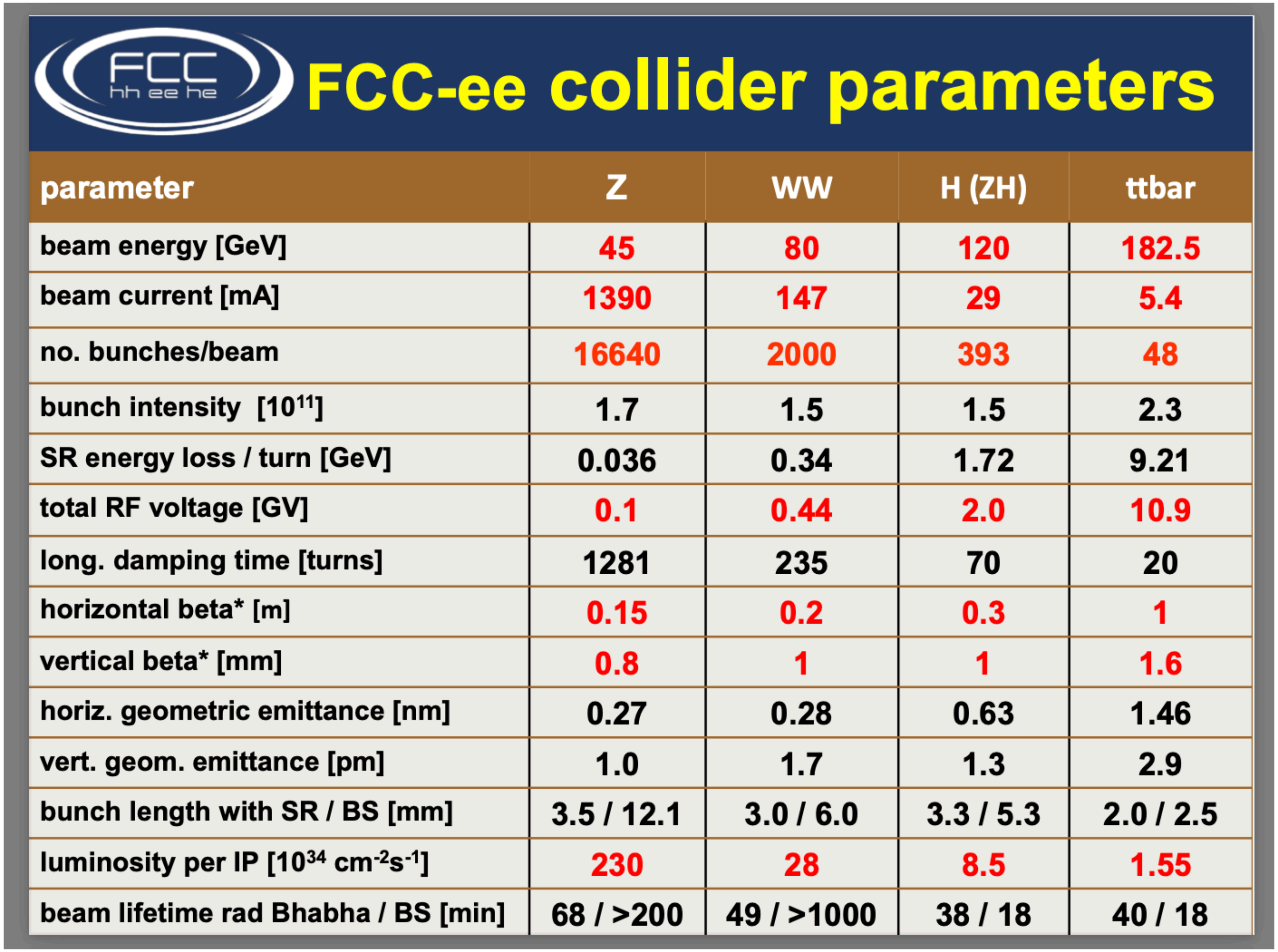
$$\sqrt{s} = 250\,\mathrm{GeV} \qquad \mathscr{L} = 1.35 \times 10^{34}\,\mathrm{cm^{-2}s^{-1}}$$

length: 20km

construction cost: 7300-8000 billion yen

FCC-ee

[slide by Frank Zimmermann]

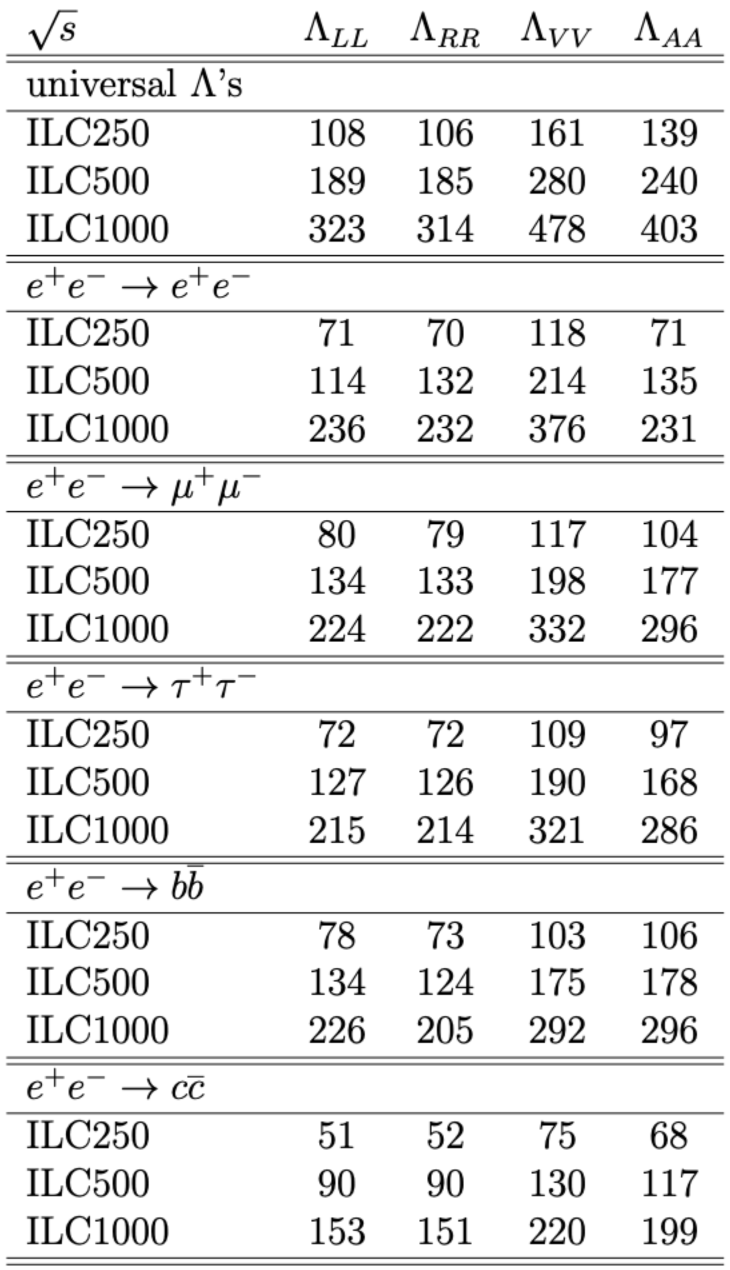


FCC-ee collider parameters

parameter	Z	WW	H (ZH)	ttbar
beam energy [GeV]	45	80	120	182.5
beam current [mA]	1390	147	29	5.4
no. bunches/beam	16640	2000	393	48
bunch intensity [10^{11}]	1.7	1.5	1.5	2.3
SR energy loss / turn [GeV]	0.036	0.34	1.72	9.21
total RF voltage [GV]	0.1	0.44	2.0	10.9
long. damping time [turns]	1281	235	70	20
horizontal beta* [m]	0.15	0.2	0.3	1
vertical beta* [mm]	0.8	1	1	1.6
horiz. geometric emittance [nm]	0.27	0.28	0.63	1.46
vert. geom. emittance [pm]	1.0	1.7	1.3	2.9
bunch length with SR / BS [mm]	3.5 / 12.1	3.0 / 6.0	3.3 / 5.3	2.0 / 2.5
luminosity per IP [10^{34} cm^{-2}s^{-1}]	230	28	8.5	1.55
beam lifetime rad Bhabha / BS [min]	68 / >200	49 / >1000	38 / 18	40 / 18

Four fermi measurement at ILC

[1908.11299]



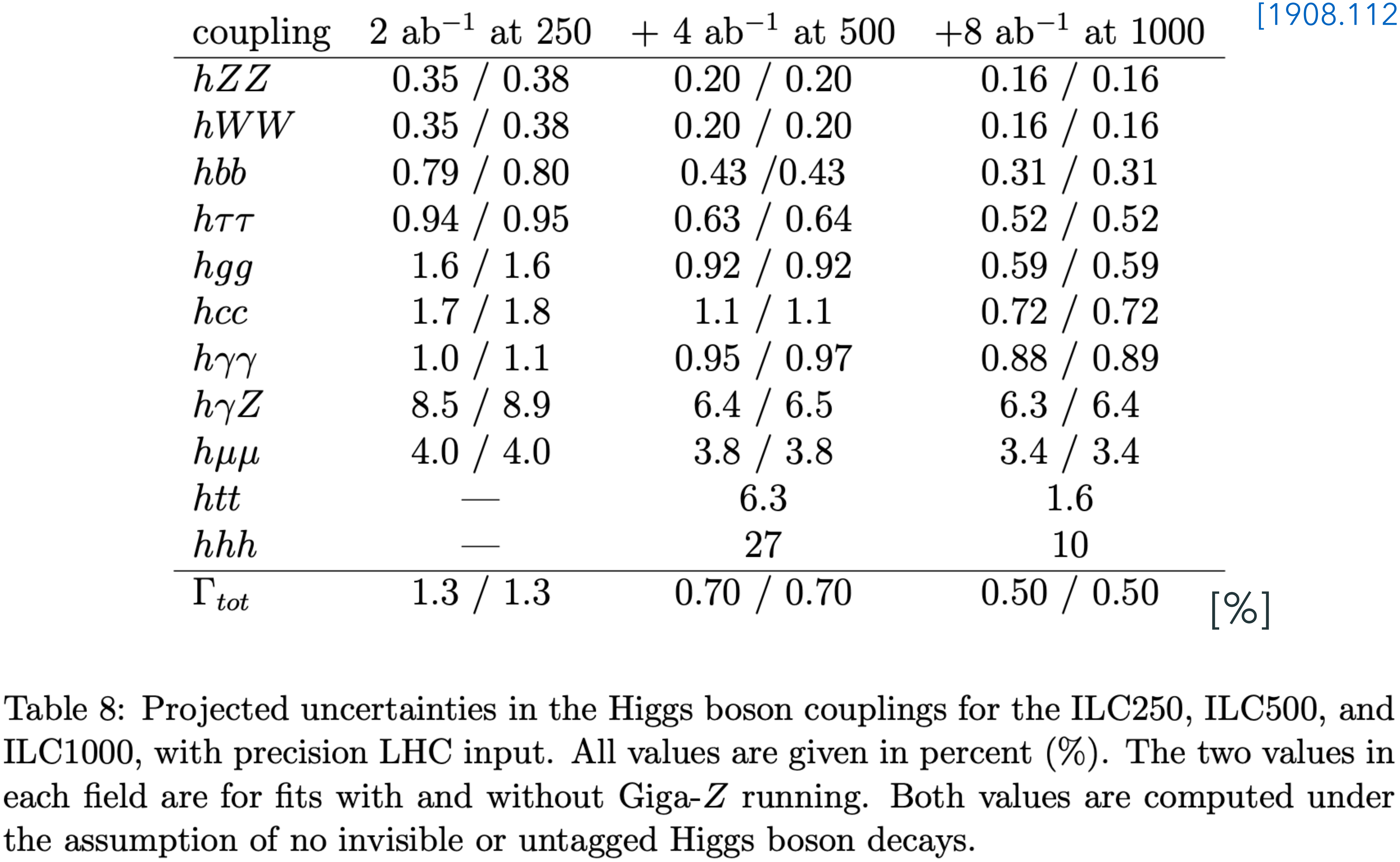
$\sqrt{s}$	Λ_{LL}	Λ_{RR}	Λ_{VV}	Λ_{AA}
universal Λ's				
ILC250	108	106	161	139
ILC500	189	185	280	240
ILC1000	323	314	478	403
$e^+e^- \to e^+e^-$				
ILC250	71	70	118	71
ILC500	114	132	214	135
ILC1000	236	232	376	231
$e^+e^- \to \mu^+\mu^-$				
ILC250	80	79	117	104
ILC500	134	133	198	177
ILC1000	224	222	332	296
$e^+e^- \to \tau^+\tau^-$				
ILC250	72	72	109	97
ILC500	127	126	190	168
ILC1000	215	214	321	286
$e^+e^- \to b\bar{b}$				
ILC250	78	73	103	106
ILC500	134	124	175	178
ILC1000	226	205	292	296
$e^+e^- \to c\bar{c}$				
ILC250	51	52	75	68
ILC500	90	90	130	117
ILC1000	153	151	220	199

[TeV]

Note: They use a different convention: $\frac{4\pi}{\Lambda^2}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$

Higgs coupling measurement at ILC

[1908.11299]



coupling	2 ab^{-1} at 250	+ 4 ab^{-1} at 500	+8 ab^{-1} at 1000
hZZ	0.35 / 0.38	0.20 / 0.20	0.16 / 0.16
hWW	0.35 / 0.38	0.20 / 0.20	0.16 / 0.16
hbb	0.79 / 0.80	0.43 /0.43	0.31 / 0.31
$h\tau\tau$	0.94 / 0.95	0.63 / 0.64	0.52 / 0.52
hgg	1.6 / 1.6	0.92 / 0.92	0.59 / 0.59
hcc	1.7 / 1.8	1.1 / 1.1	0.72 / 0.72
$h\gamma\gamma$	1.0 / 1.1	0.95 / 0.97	0.88 / 0.89
$h\gamma Z$	8.5 / 8.9	6.4 / 6.5	6.3 / 6.4
$h\mu\mu$	4.0 / 4.0	3.8 / 3.8	3.4 / 3.4
htt	—	6.3	1.6
hhh	—	27	10
Γ_{tot}	1.3 / 1.3	0.70 / 0.70	0.50 / 0.50

[%]

Table 8: Projected uncertainties in the Higgs boson couplings for the ILC250, ILC500, and ILC1000, with precision LHC input. All values are given in percent (%). The two values in each field are for fits with and without Giga-Z running. Both values are computed under the assumption of no invisible or untagged Higgs boson decays.

Magnet

Dipole magnet with the magnetic field of 10T

⟶ Main ring 3km for $(E_{\mu^+}, E_{e^-}) = (1\,\mathrm{TeV},\ 30\,\mathrm{GeV})$

9km for $(E_{\mu^+}, E_{e^-}) = (3\,\mathrm{TeV},\ 50\,\mathrm{GeV})$

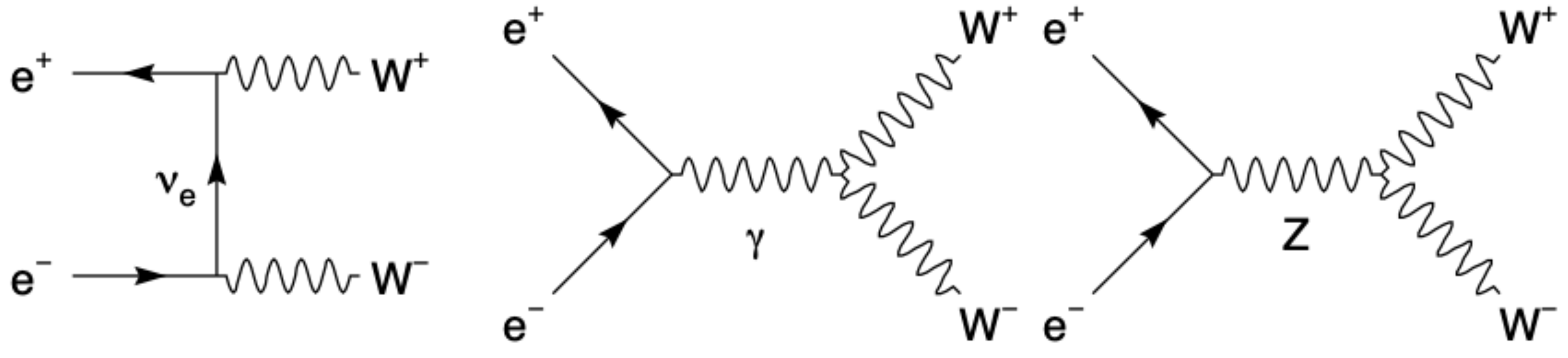
cf. High-luminosity LHC: 11T

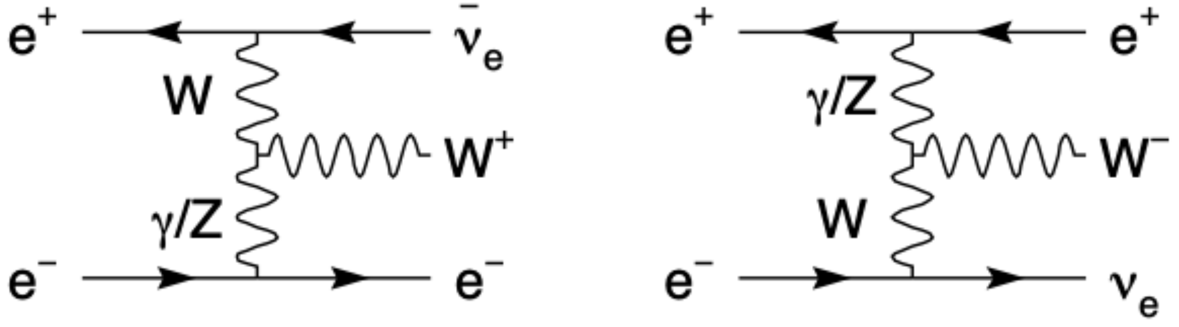
If dipole magnet with the magnetic field of 16T is possible,

⟶ Main ring 6km for $(E_{\mu^+}, E_{e^-}) = (3\,\mathrm{TeV},\ 50\,\mathrm{GeV})$

W boson mass

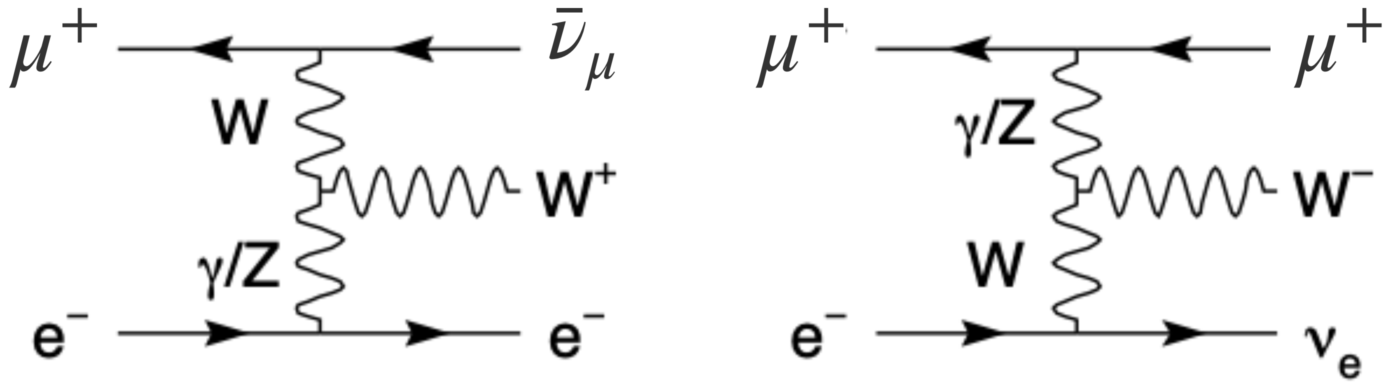
- ILC (and LEP):





W boson mass

- μTRISTAN



& hadronic decay $W \to q\bar{q}$

In ILC 250 study, $\Delta M_W \simeq 3.7\,\mathrm{MeV}$, which is dominated by systematic uncertainty (particularly hodronization). [1310.6708]

We expect a similar precision at μTRISTAN.

Cf.) CDF II result: $M_W = 80{,}433.5 \pm 9.4\,\mathrm{MeV}$

Design of μTRISTAN

Proton acceleration (proton LINAC & RCS) ⟶ Pion production (pion production ring)

$p(3\,\mathrm{GeV})$

$p(3\,\mathrm{GeV}) + C \rightarrow \pi^+ + X$

⟶ Ultra-cold muon production (explained later)

$\pi^+ \rightarrow \mu^+ + \nu_\mu$ ⟶ Muonium ($\mu^+ e^-$) formation in target silica aerogel ⟶ Ultra-cold muon

ionized by laser

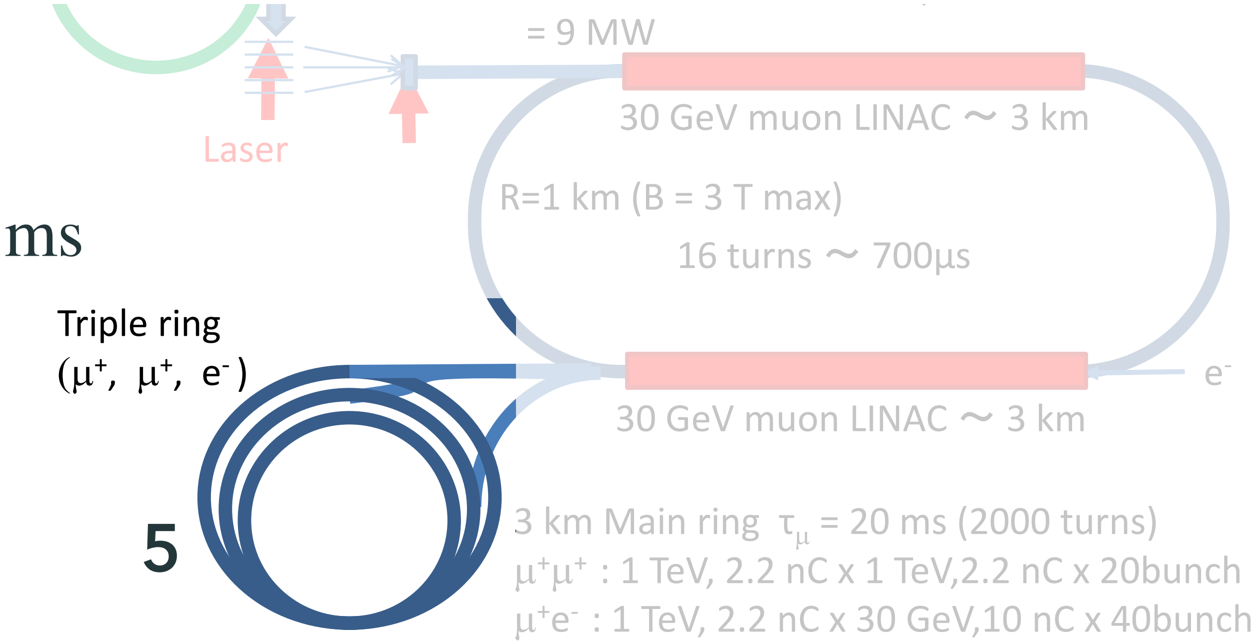
⟶ Muon acceleration (booster ring) to 1TeV & electron acceleration (LINAC part)

⟶ Collide (main ring) : $[\mu^+\,(1\,\mathrm{TeV}),\ e^-\,(30\,\mathrm{GeV})]$ or $[\mu^+\,(1\,\mathrm{TeV}),\ \mu^+\,(1\,\mathrm{TeV})]$

$\tau_{\mu(1\mathrm{TeV})} = 20\,\mathrm{ms}$

New beam injected every $20\,\mathrm{ms}$

Ring circumference 3 km



Luminosity

$$\mathscr{L} = \frac{N_{\text{beam1}}\, N_{\text{beam2}}}{4\pi\, \sigma_x\, \sigma_y} f_{\text{rep}}$$

$$f_{\text{rep}}^{(\mu^+e^-)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 40 = 4\,\text{MHz}$$

$$f_{\text{rep}}^{(\mu^+\mu^+)} = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}/(3\,\text{km}) \times 20 = 2\,\text{MHz}$$

$$\sigma_x = 3.6\,\mu\text{m} \qquad \sigma_y = 1.7\,\mu\text{m}$$

$\text{f} = 10^{-15}, \text{a} = 10^{-18}, \text{p} = 10^{-12}$

$N_{e^-} = 10\,\text{nC}$ per bunch $\qquad N_{\mu^+} = 3.6\,\text{nC} \to 1.3\,\text{nC}$ per bunch due to decay

Our estimate:

(10 years running w/ 70 % duty factor)

$$\mathscr{L}_{\mu^+e^-} = 4.6 \times 10^{33}\,\text{cm}^{-2}\text{s}^{-1} \quad \Rightarrow \quad \int dt\, \mathscr{L}_{\mu^+e^-} \simeq 1.0\,\text{ab}^{-1}$$

$$\mathscr{L}_{\mu^+\mu^+} = 5.7 \times 10^{32}\,\text{cm}^{-2}\text{s}^{-1} \quad \Rightarrow \quad \int dt\, \mathscr{L}_{\mu^+\mu^+} \simeq 130\,\text{fb}^{-1}$$