EW corrections at future colliders

Istituto Nazionale di Fisica Nucleare
SEZIONE DI BOLOGNA

Davide Pagani

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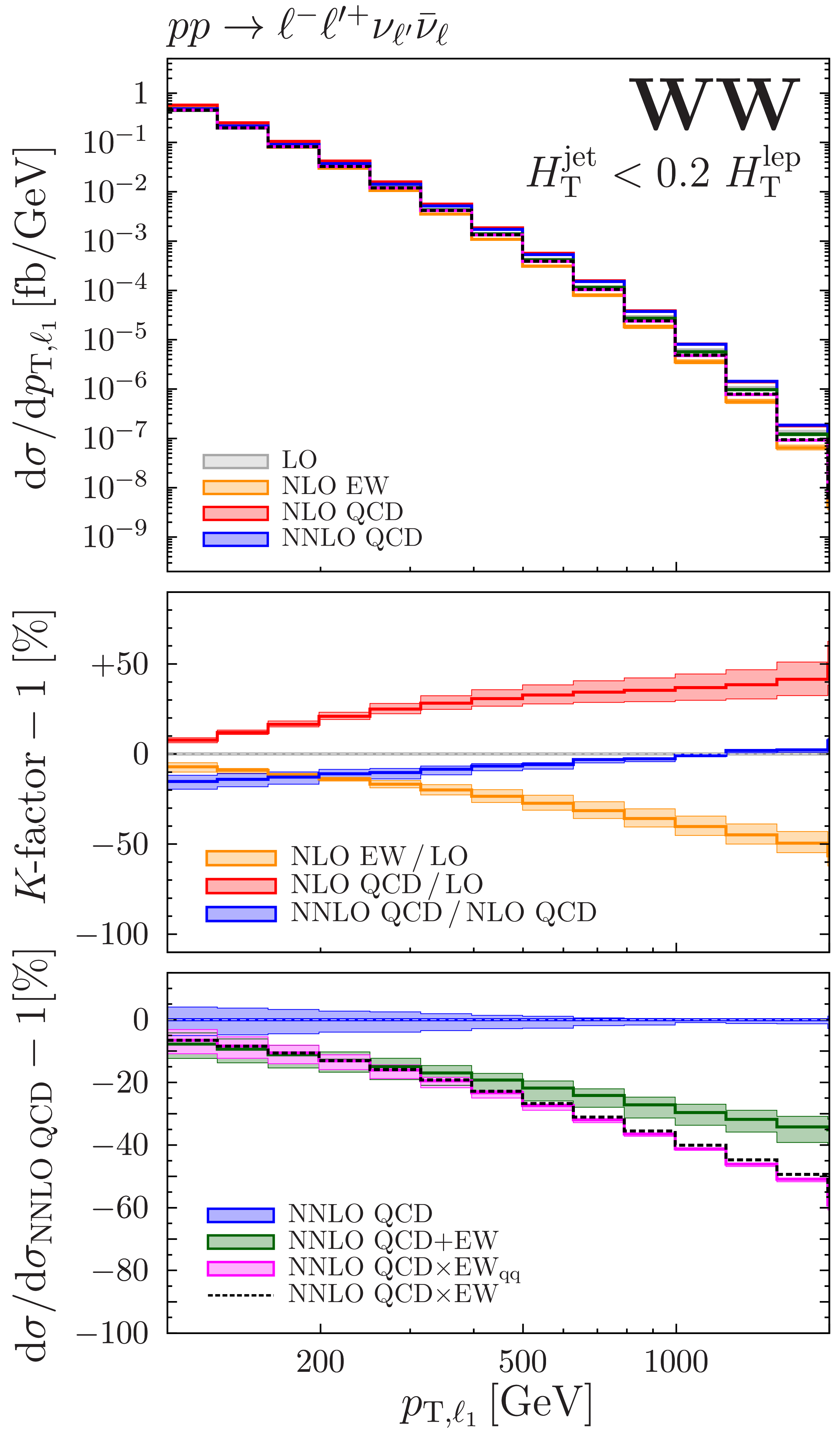
EW corrections are already relevant at the LHC: an example



Grazzini, Kallweit, Lindert, Pozzorini, Wiesemann '19

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm BSM} = {} & \delta g^Z_{uL}\left[Z^\mu \bar u_L \gamma_\mu u_L + \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar u_L\gamma_\mu d_L + \text{h.c.}) + \dots\right] + \delta g^Z_{uR}\left[Z^\mu \bar u_R\gamma_\mu u_R\right] \\
& + \delta g^Z_{dL}\left[Z^\mu \bar d_L \gamma_\mu d_L - \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar u_L\gamma_\mu d_L + \text{h.c.}) + \dots\right] + \delta g^Z_{dR}\left[Z^\mu \bar d_R\gamma_\mu d_R\right] \\
& + ig\cos\theta_W \delta g_1^Z\left[Z^\mu(W^{+\nu}W^-_{\mu\nu} - \text{h.c.}) + Z^{\mu\nu}W^+_\mu W^-_\nu + \dots\right] \\
& + ie\delta\kappa_\gamma\left[(A_{\mu\nu} - \tan\theta_W Z_{\mu\nu})W^{+\mu}W^{-\nu} + \dots\right],
\end{aligned}
$$

Banerjee, Reichelt, Spannowsky '24

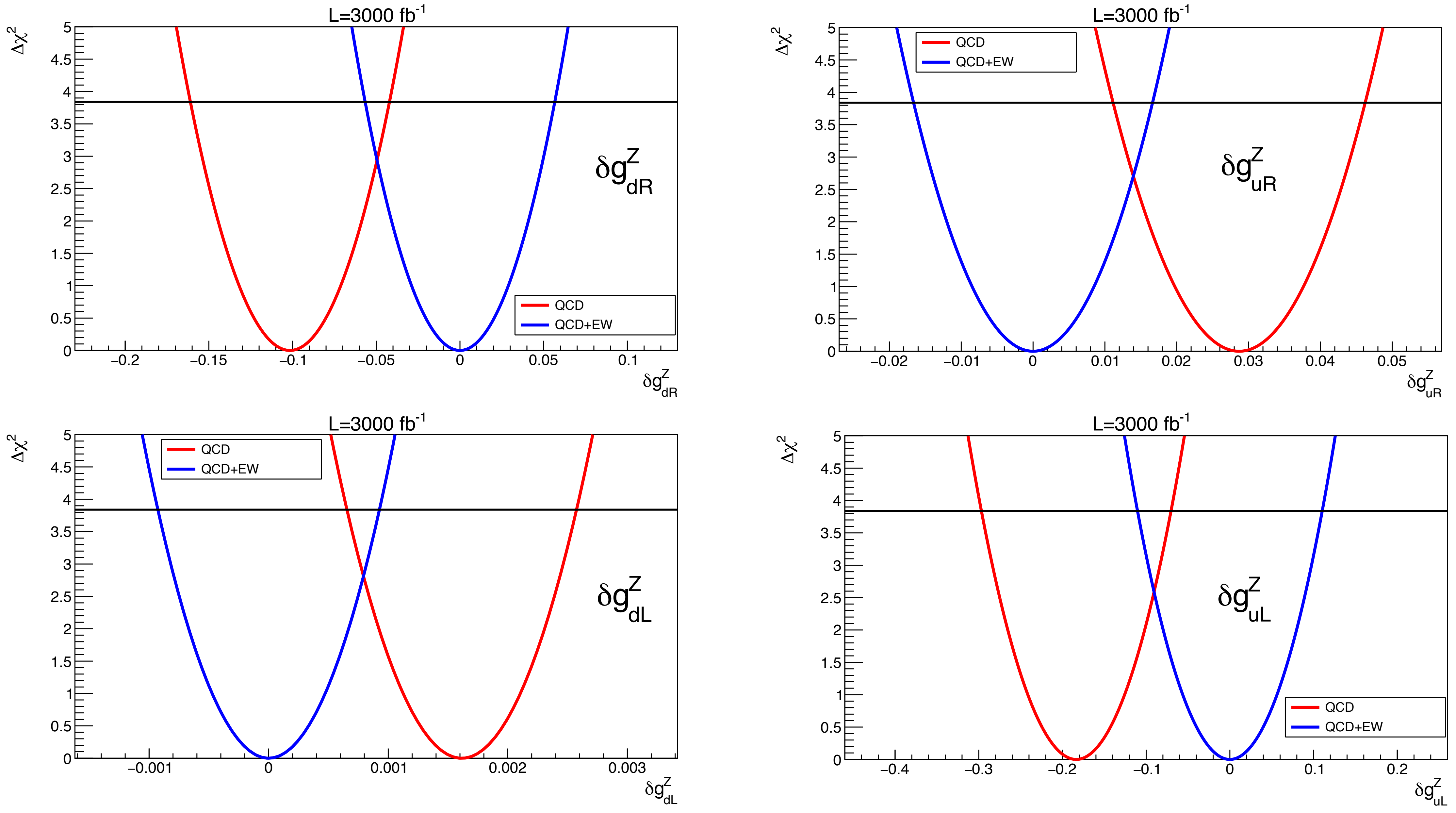


Figure 3: Comparison of the one-dimensional bounds at 95% C.L. between theory assumptions of SM@(NLO QCD) + SMEFT@LO and SM@(NLO QCD + approximate NLO EW) + SMEFT@LO. The expectation assumption is the same, SM@(NLO QCD + approximate NLO EW) for both theory assumptions. The other parameters have not been marginalised over and we consider $\Delta\chi^2 = 3.84$ for a one-parameter χ^2 fit to present our allowed regions.

Future Colliders vs. EW corrections

(1) Higher Precision

EW corrections become even more relevant. NLO EW is not sufficient and higher orders are necessary.

(2) Higher Energy

EW corrections become larger (Sudakov).

EW corrections become relevant both for signal/bkg or BSM/SM, not only for precision studies.

For Precision: see **(1)**.

Future Colliders vs. EW corrections

(1) Higher Precision

EW corrections become even more relevant. NLO EW is not sufficient and higher orders are necessary.

I will focus more on the high-energy case, which gives to EW corrections a new and different role in pheno studies.

EW is the new **QCD**

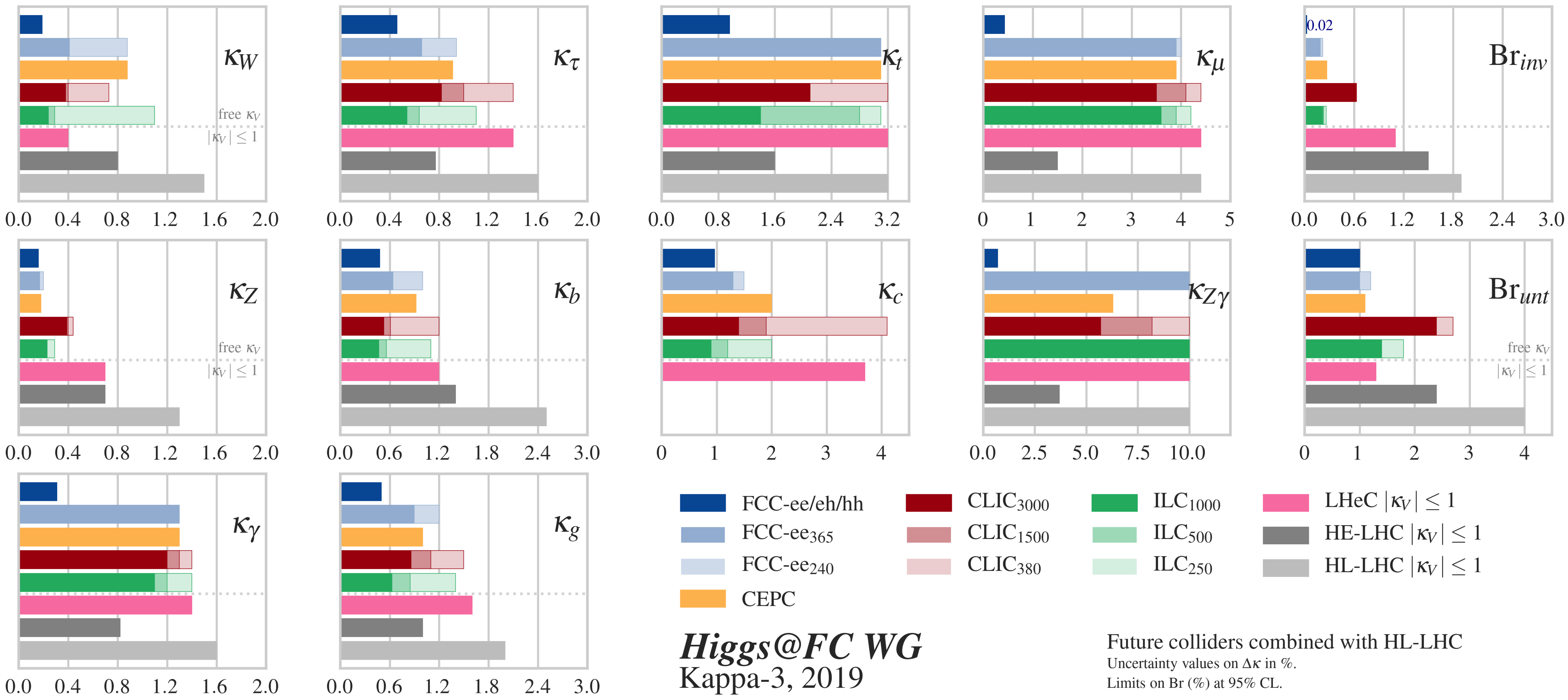
(2) Higher Energy

EW corrections become larger (Sudakov).

EW corrections become relevant both for signal/bkg or BSM/SM, not only for precision studies.

For Precision: see **(1)**.

Precision: the Higgs case

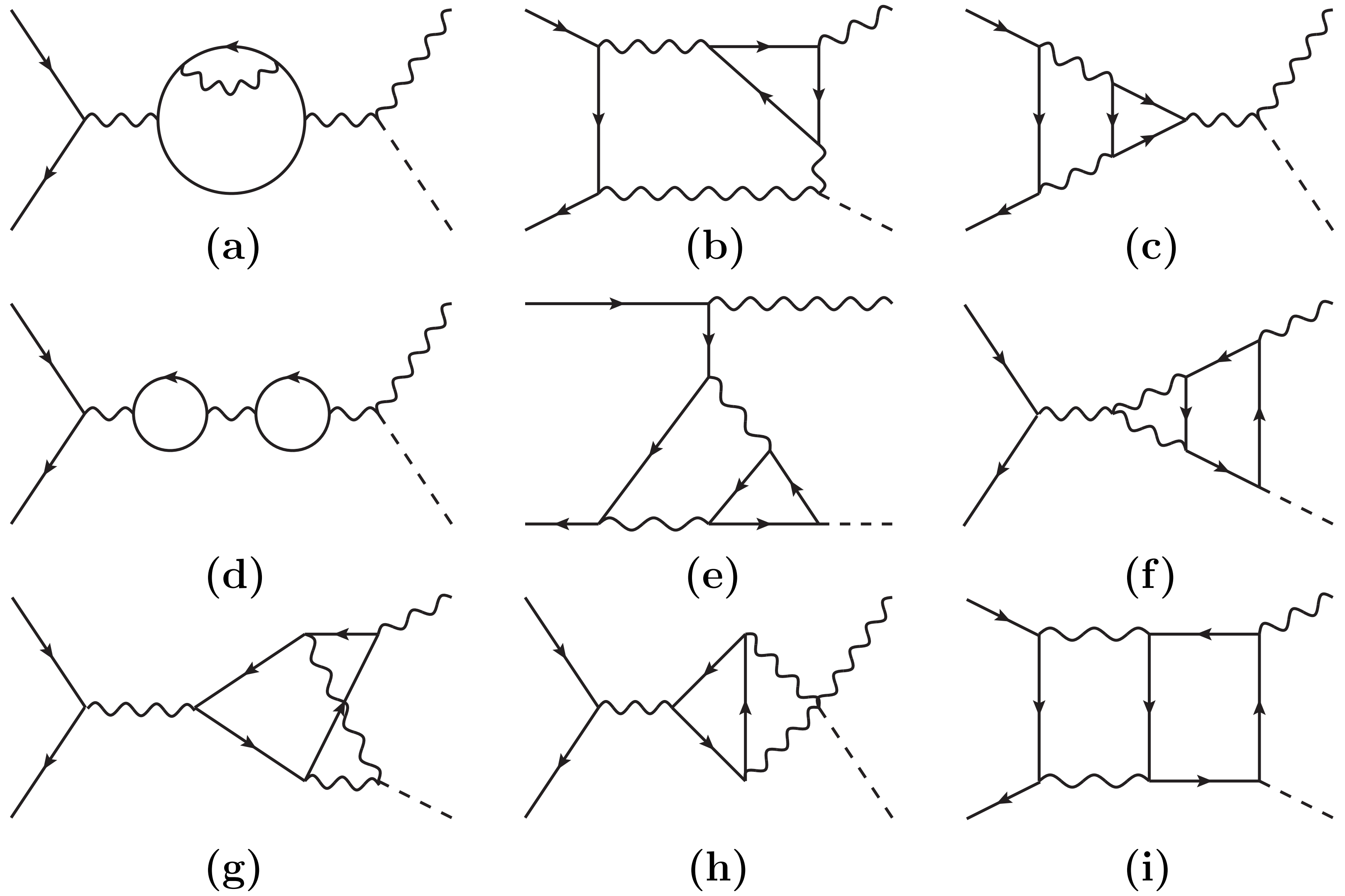


Sub-percent precision is expected and so NLO EW is not enough.

$e^+e^- \to ZH$

Contributions from fermion loops at NNLO

Freitas, Song ‘22
Freitas, Song, Xie ‘24



		$\alpha(0)$ scheme		G_μ scheme
σ^{LO} [fb]		222.96		239.18
σ^{NLO} [fb]	+3.1%	229.89	-2.9%	232.08
σ^{NNLO} [fb]	+0.7%	231.55	+0.3%	232.74
$\mathcal{O}(\alpha^2_{N_f=2})$		1.88		0.73
$\mathcal{O}(\alpha^2_{N_f=1})$		-0.23		-0.07

$\sqrt{S} = 240$ GeV

Small corrections, but necessary for matching the experimental precision.

The choice of the renormalisation scheme is relevant.

This is only one of the effects: e.g. PDFs and ISR are not taken into account.

NLO EW corrections at high energies

NLO EW corrections for energies of the order of few TeVs are as large as (or even more than) NLO QCD corrections at the LHC. Origin: **EW Sudakov logarithms.**

EW corrections should be considered not only for precision physics, since they give $\mathcal{O}(10-100\%)$ effects. This includes also BSM scenarios.

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-Z	$3.330(2)\cdot 10^{1}$	$2.568(8)\cdot 10^{1}$	$-22.9(2)$
W^+W^-H	$1.1253(5)\cdot 10^{0}$	$0.895(2)\cdot 10^{0}$	$-20.5(2)$
ZZZ	$3.598(2)\cdot 10^{-1}$	$2.68(1)\cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4)\cdot 10^{-2}$	$6.60(3)\cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1)\cdot 10^{-2}$	$2.451(5)\cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6)\cdot 10^{-8}$	$0.86(7)\cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1)\cdot 10^{0}$	$0.993(6)\cdot 10^{0}$	$-33.1(4)$
W^+W^-ZZ	$1.209(1)\cdot 10^{0}$	$0.699(7)\cdot 10^{0}$	$-42.2(6)$
W^+W^-HZ	$8.754(8)\cdot 10^{-2}$	$6.05(4)\cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1)\cdot 10^{-2}$	$0.655(5)\cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2)\cdot 10^{-3}$	$1.799(7)\cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2)\cdot 10^{-3}$	$1.766(6)\cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7)\cdot 10^{-4}$	$6.24(2)\cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1)\cdot 10^{-4}$	$1.165(4)\cdot 10^{-4}$	$-25.7(2)$

3 TeV Muon Collider

WHIZARD

Bredt, Kilian, Reuter, Steinemeier '22

What are EW Sudakov logarithms?

QCD: virtual and real terms are separately IR divergent ($1/\epsilon$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon-mass λ. So $1/\epsilon$ poles $\rightarrow \log(Q^2/\lambda^2)$, where Q is a generic scale.

EW: with weak interactions $\lambda \rightarrow m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$-\alpha^k \log^n(s/m_W^2)$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

Future Colliders: are EW Sudakov logarithms a good and robust approximation for EW corrections at high energies?

Currently: exact NLO EW automated for SM but not for BSM.

Since EW corrections are expected to be relevant also for BSM, can we safely use the high-energy Sudakov approximation?

MadGraph5_aMC@NLO: EW corrections for FC

NLO EW hadron colliders: *Frederix, Frixione, Hirshi, DP, Shao, Zaro '18*

NLO EW e^+e^- colliders: *Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22*

One-loop EW Sudakov alone: *DP, Zaro '21*

one-loop EW virtual corrections $\mathcal{O}(\alpha)$

=

α **[Sudakov Logs $\mathcal{O}(-\log^k(s/m_W^2), k = 1,2)$ +**

constant term $\mathcal{O}(1)$ +

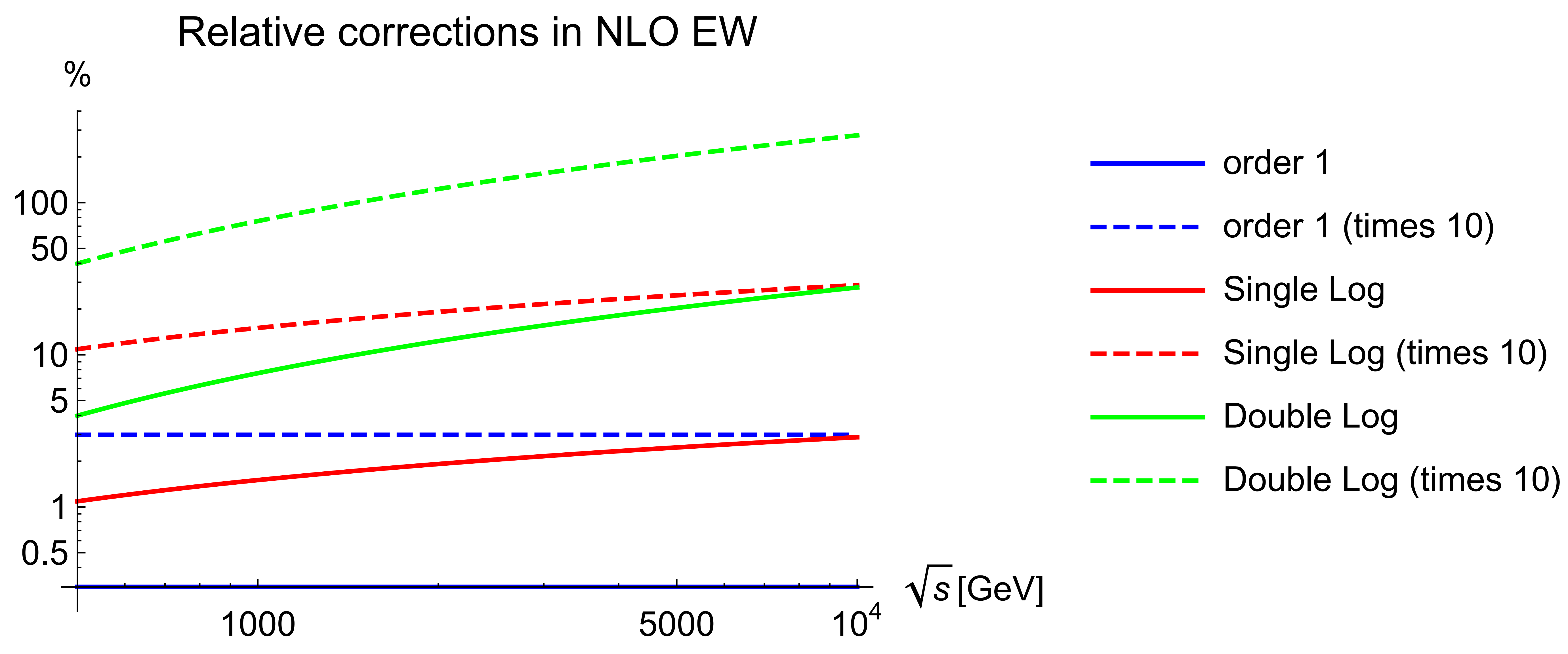
mass-suppressed terms $\mathcal{O}(m_W^2/s)$]

Having separately exact NLO EW and EW Sudakov logarithms is possible to study the goodness of the high-energy approximation(s). **SM as a test case!**

How large are expected to be the EW Sudakov?

$$\mathcal{O}(1) \to \frac{\alpha}{4\pi s_w^2} \sim 0.3\,\%\,,\quad \textbf{Single Log} \to \frac{\alpha}{4\pi s_w^2}\log(s/m_W^2),$$

$$\textbf{Double Log} \to \frac{\alpha}{4\pi s_w^2}\log^2(s/m_W^2)$$



Taking into account only DL, and not SL, is not safe for partonic energies up to 10 TeV.

Just a representative example of a process

The estimate done via the variation of a factor of 10 is actually conservative.

$$\delta^{\mathrm{RR,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} \boxed{= -2.58\,L(s)}\boxed{-\,5.15\left(\log\frac{t}{u}\right)l(s) + 0.29\,l_{\mathrm{Z}} + 7.73\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},}$$

$$\delta^{\mathrm{RL,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} \boxed{= -4.96\,L(s)}\boxed{-\,2.58\left(\log\frac{t}{u}\right)l(s) + 0.37\,l_{\mathrm{Z}} + 14.9\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},}$$

Denner Pozzorini '01

NLO EW: some open questions/issues

Resummation?
When is it necessary to resum EW (Sudakov) corrections?

BSM?
What features of NLO EW corrections are universal and can be extended to the BSM case?

Heavy Boson Radiation (HBR)?
What should one do with Z,W radiation? Experimental set-up may impact the calculation result.

PDFs or VBF with matrix elements?
If PDFs involve weak effects, weak counter terms in NLO EW corrections should be included. Resum logs or keep power corrections? Both?

Master formula (Denner&Pozzorini)

Denner Pozzorini '01

Born amplitude: $\mathcal{M}_0^{i_1\dots i_n}(p_1,\dots,p_n)$

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\dots i_n}(p_1,\dots,p_n) = \mathcal{M}_0^{i'_1\dots i'_n}(p_1,\dots,p_n)\delta_{i'_1 i_1\dots i'_n i_n}$

other tree-level amplitudes — the logs

eikonal approximation of soft EW boson exchange

$$\delta = \delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

Leading Soft-Collinear — Subleading Soft-Collinear — Collinear — Parameter renormalis.

It depends only on s and it is the only term involving double logarithms.

The only one involving ratios of s with other invariants and also angular dependences.

In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.

The logs inside the δ^i have always the form:

$$L(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log^2\frac{|r_{kl}|}{M^2}$$

$$l(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log\frac{|r_{kl}|}{M^2}$$

$$M = M_W, M_Z, m_f, \lambda, \dots$$

$$r_{kl} \equiv (p_k + p_l)^2$$

ASSUMPTIONS:

$$r_{kl} \equiv (p_k + p_l)^2 \simeq 2p_k p_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$$

the high-energy limit

$$r_{kl}/r_{k'l'} \simeq 1$$

All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.

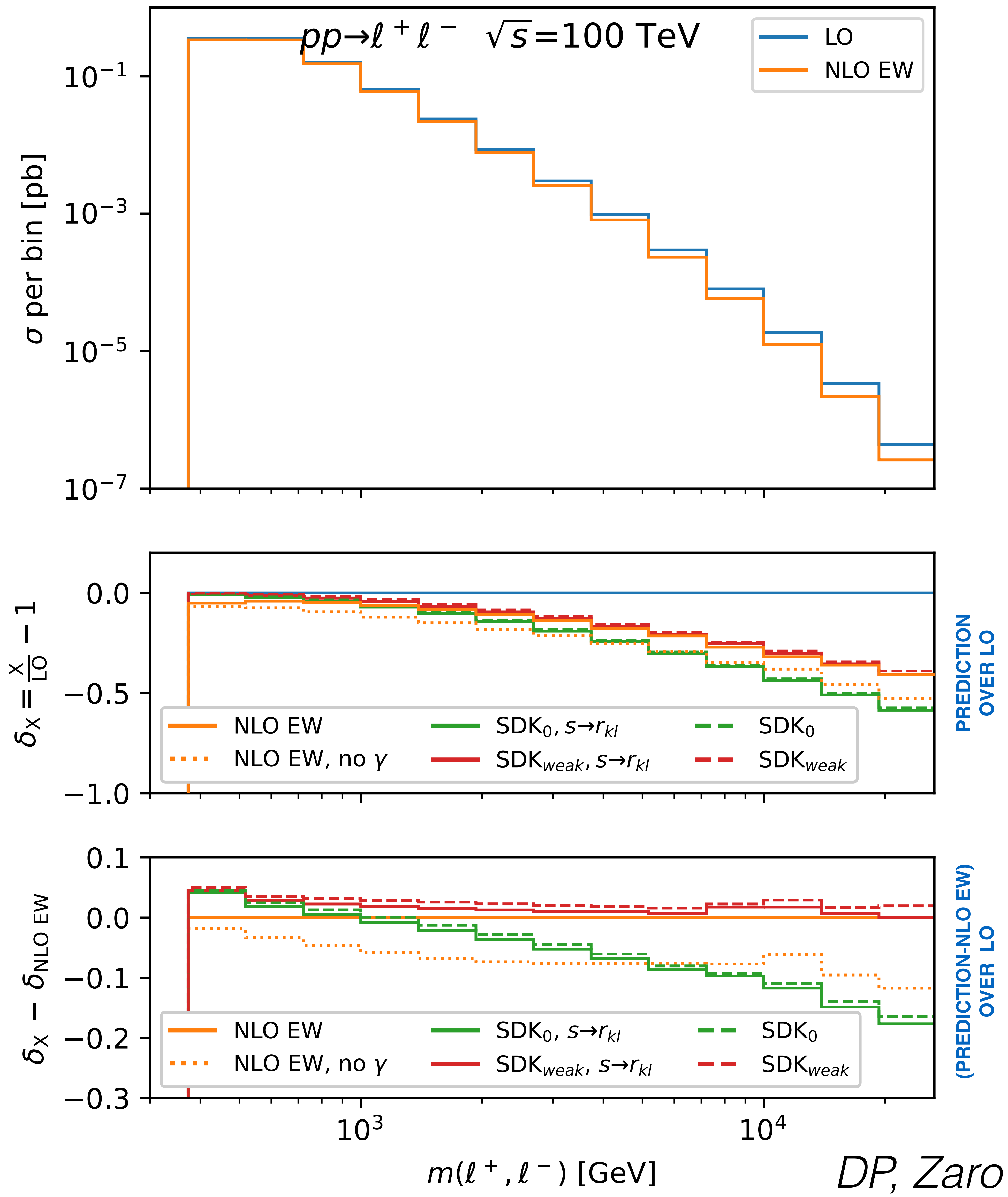
Our revisitation:

DP, Zaro '21

Logs among invariants: Logs like $\log(t/s)$ taken into account.

$\mathrm{SDK}_{\mathrm{Weak}}$ scheme: A purely Weak (no QED) scheme for improving approximation of IR-finite physical observables. Different to the more common SDK_0 scheme that has been used in the literature.

e^+e^- production at 100 TeV **FCC-hh**



DP, Zaro '21

$p_T(\ell^\pm) > 200$ GeV, $|\eta(\ell^\pm)| < 2.5$, $m(\ell^+,\ell^-) > 400$ GeV, $\Delta R(\ell^+,\ell^-) > 0.5$.

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = $\mathrm{SDK_0}$, **Red** = $\mathrm{SDK_{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction: Red-solid line

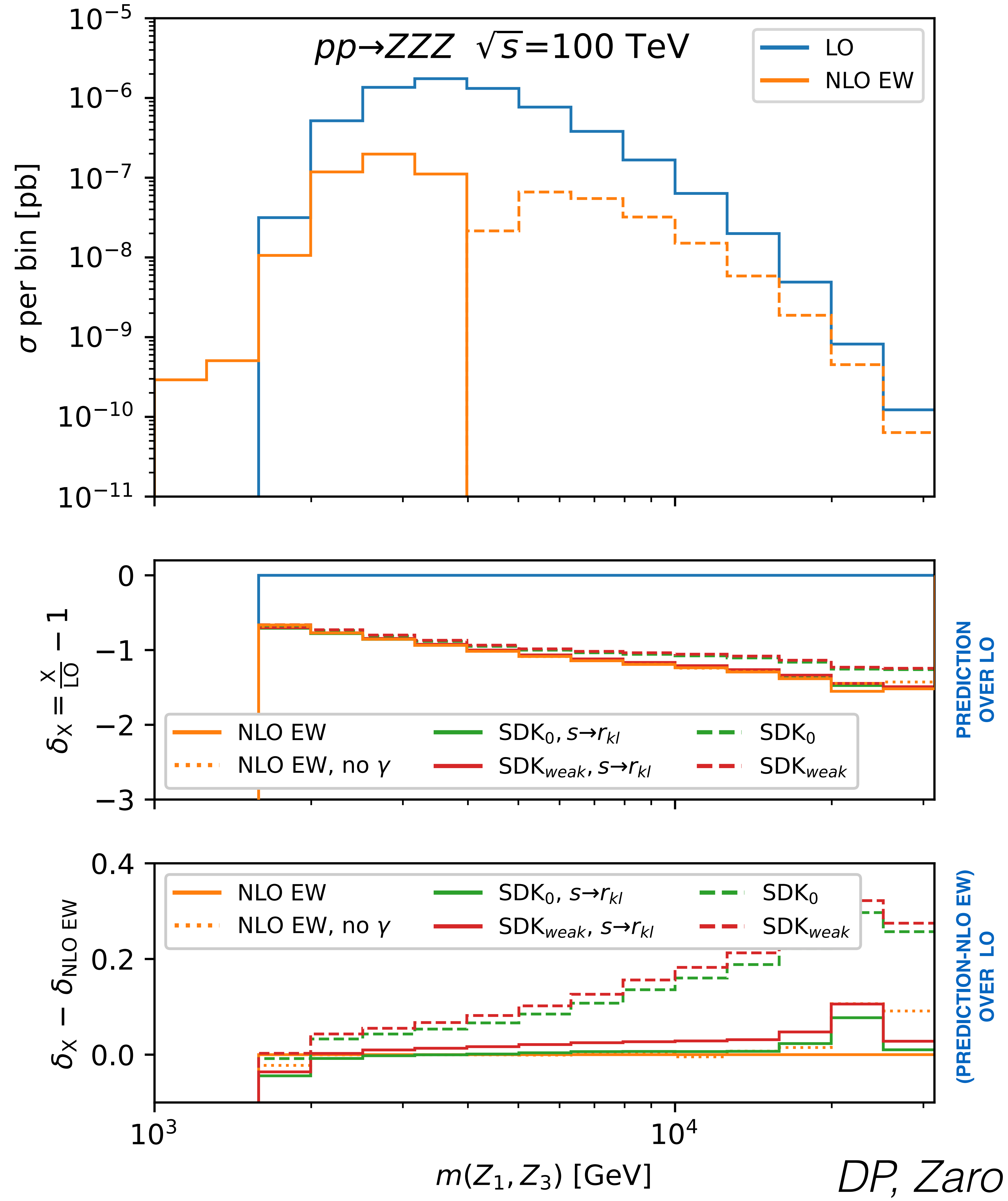
Only the $\mathrm{SDK_{weak}}$ approach correctly captures the NLO EW prediction.

Solid and dashed very similar.

Photon PDF cannot be ignored.

Larger invariant -> larger correction

ZZZ production at 100 TeV **FCC-hh**



DP, Zaro '21

$p_T(Z_i) > 1\ \text{TeV}, \quad |\eta(Z_i)| < 2.5, \quad m(Z_i, Z_j) > 1\ \text{TeV}, \quad \Delta R(Z_i, Z_j) > 0.5.$

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction:
Red-solid line

$\mathrm{SDK}_{\mathrm{weak}}$ and SDK_0 not so relevant for neutral final state).

Only the solid lines, having more angular information, correctly capture NLO EW.

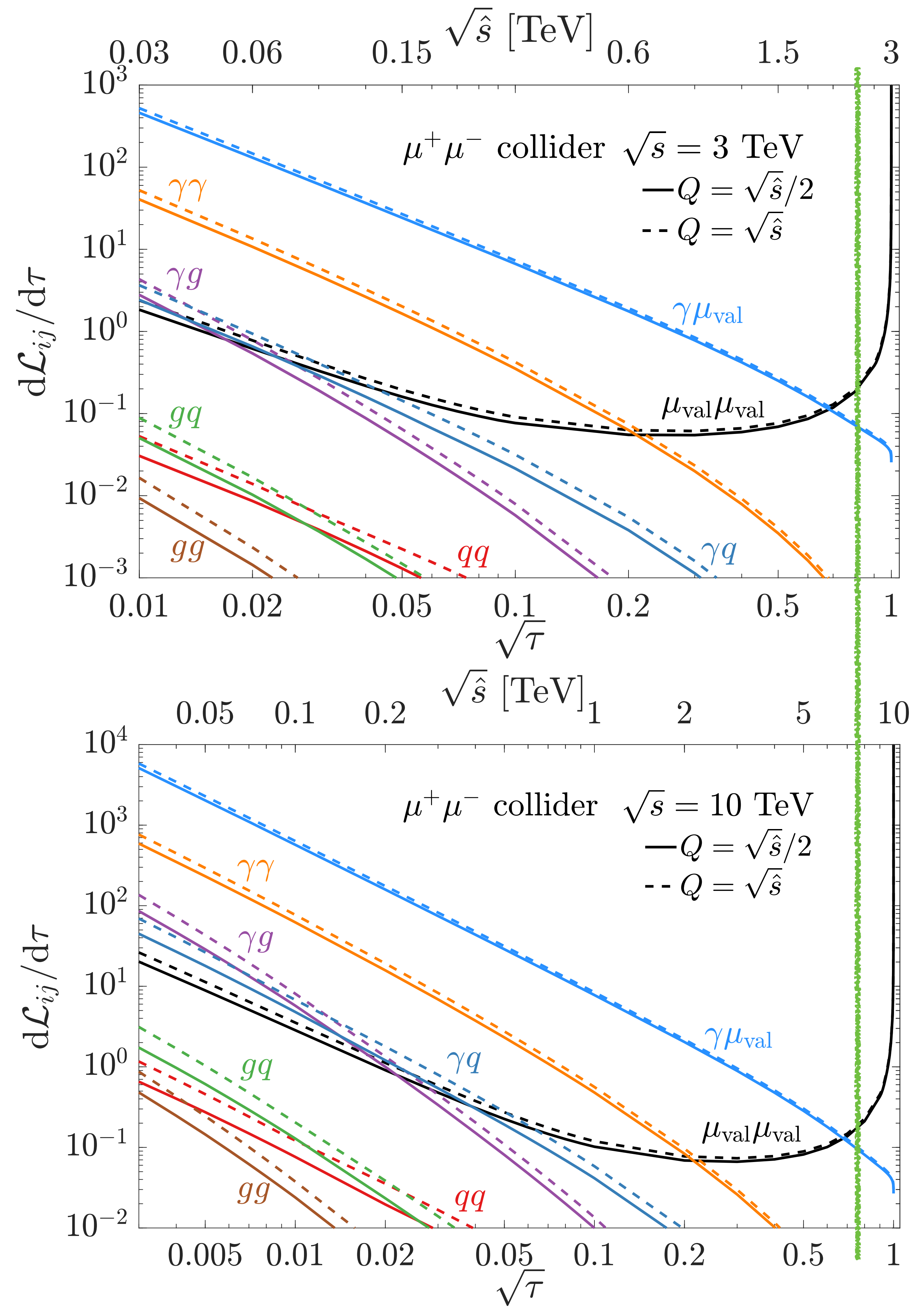
One cannot forget terms as $\log^2[m^2(Z_2, Z_3)/s]$

Larger invariant -> larger correction

The muon collider case

Ma, DP, Zaro **TODAY**

$\mu^+\mu^- \longrightarrow F$, where F is a generic final state involving W, Z, t, H.
Thus we select direct production, with no VBF contributions.



We require $m(F) > 0.8\sqrt{S}$, so that neither VBF nor PDFs other than μ are relevant.

We apply further experimentally motivated cuts for each X, Y particle in F:

$$p_T(X) > 100 \text{ GeV},\ |\eta(X)| < 2.44,\ \Delta R(X, Y) > 0.4$$

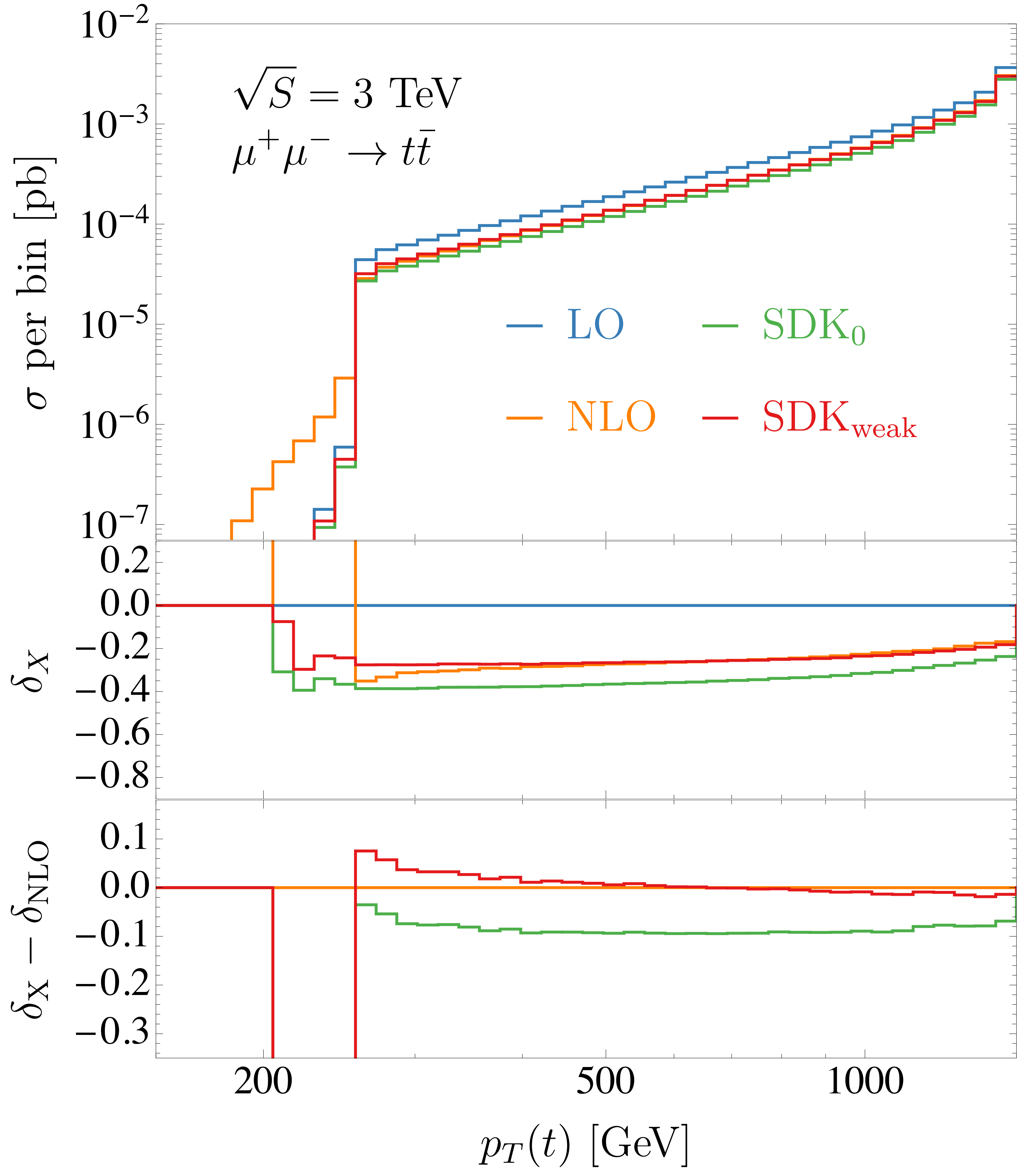
And we recombine photons with charged (also massive) particles.

Han, Ma, Xie '20, '21

The μ PDF in the μ is peaked at Bjorken-x=1, therefore: Collider $S \simeq$ partonic s

$t\bar{t}$

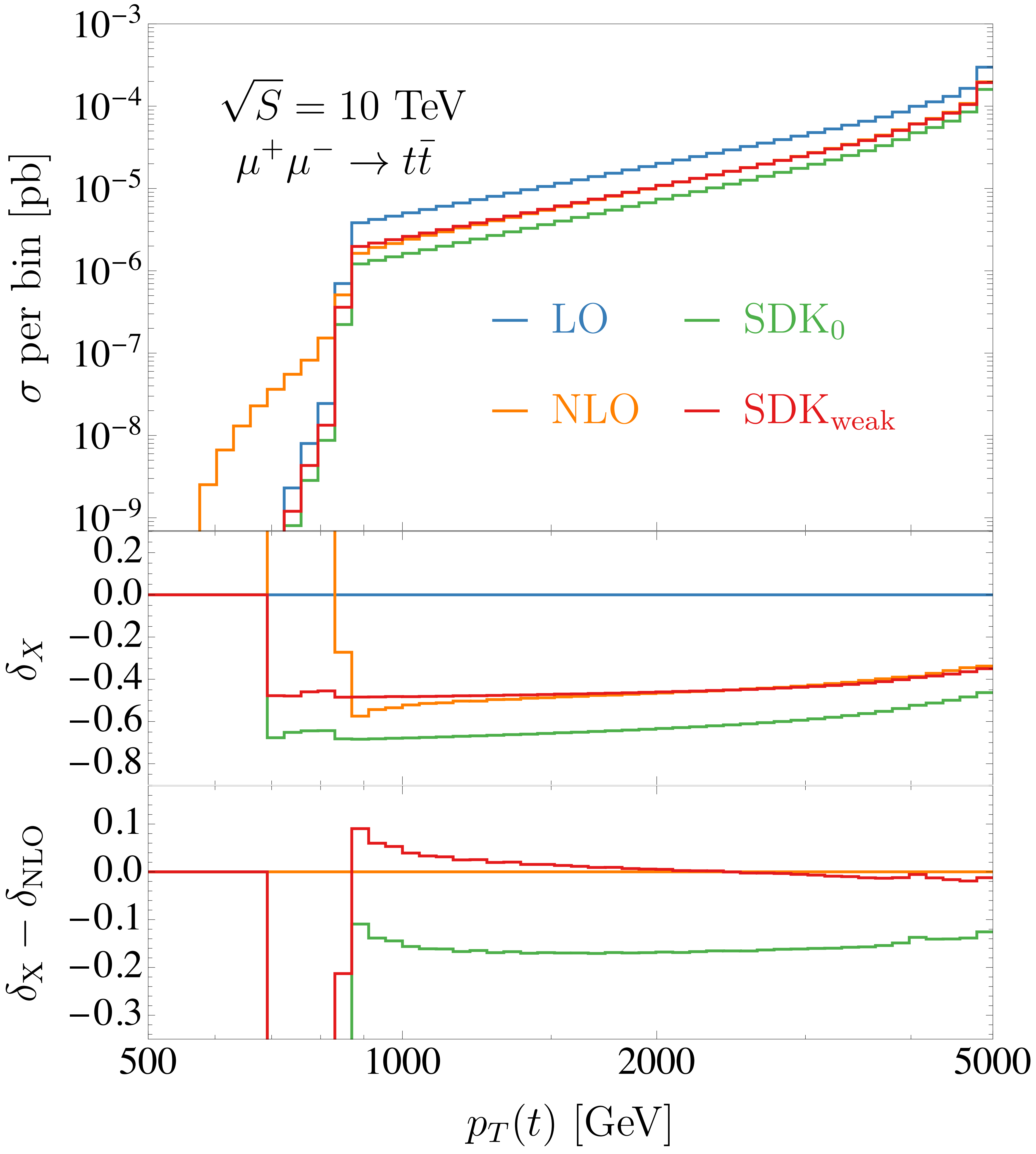
Ma, DP, Zaro **TODAY**



For smaller p_T , larger corrections.

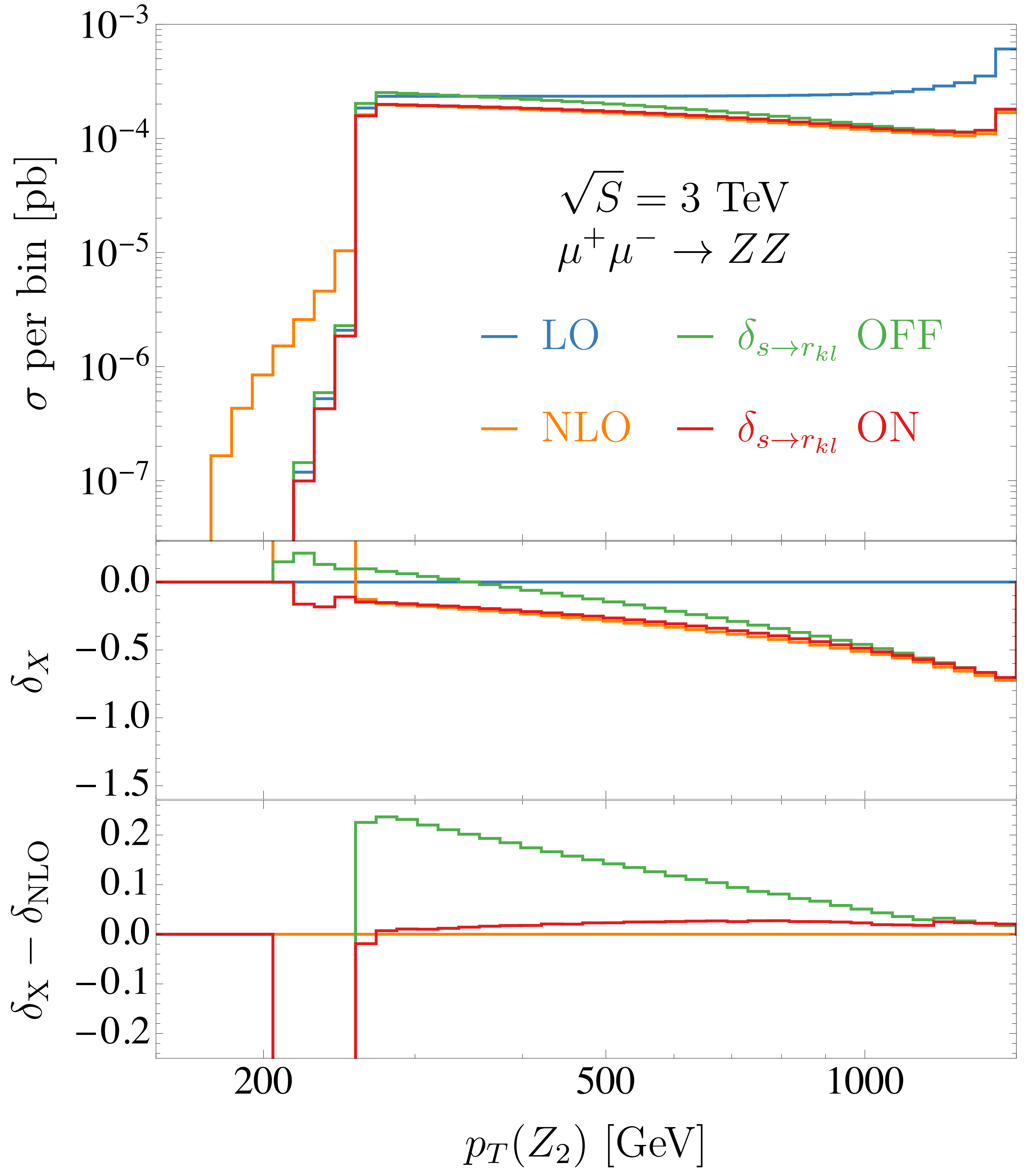
Sudakov (in the SDK$_\mathrm{weak}$ scheme) **capture NLO EW corrections** up to the % level.

If double logs are written in the form $\log^2(s/m_W^2)$, the shapes observed here are all arising from **single logs**.



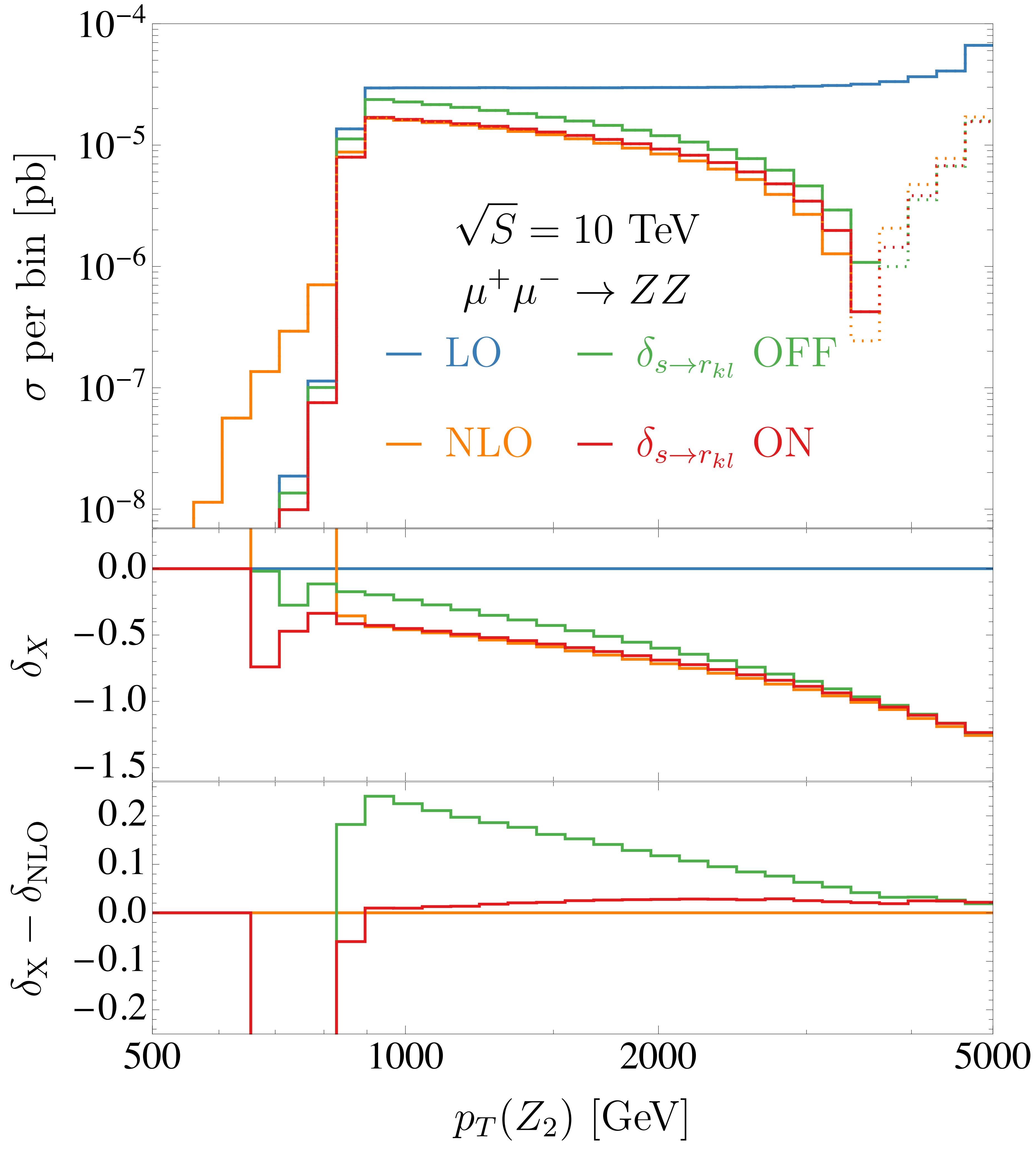
ZZ

Ma, DP, Zaro **TODAY**



Sudakov logs **capture NLO EW corrections** up to the % level, but only if all the logs of the form $\log(t/s)$ are taken into account.

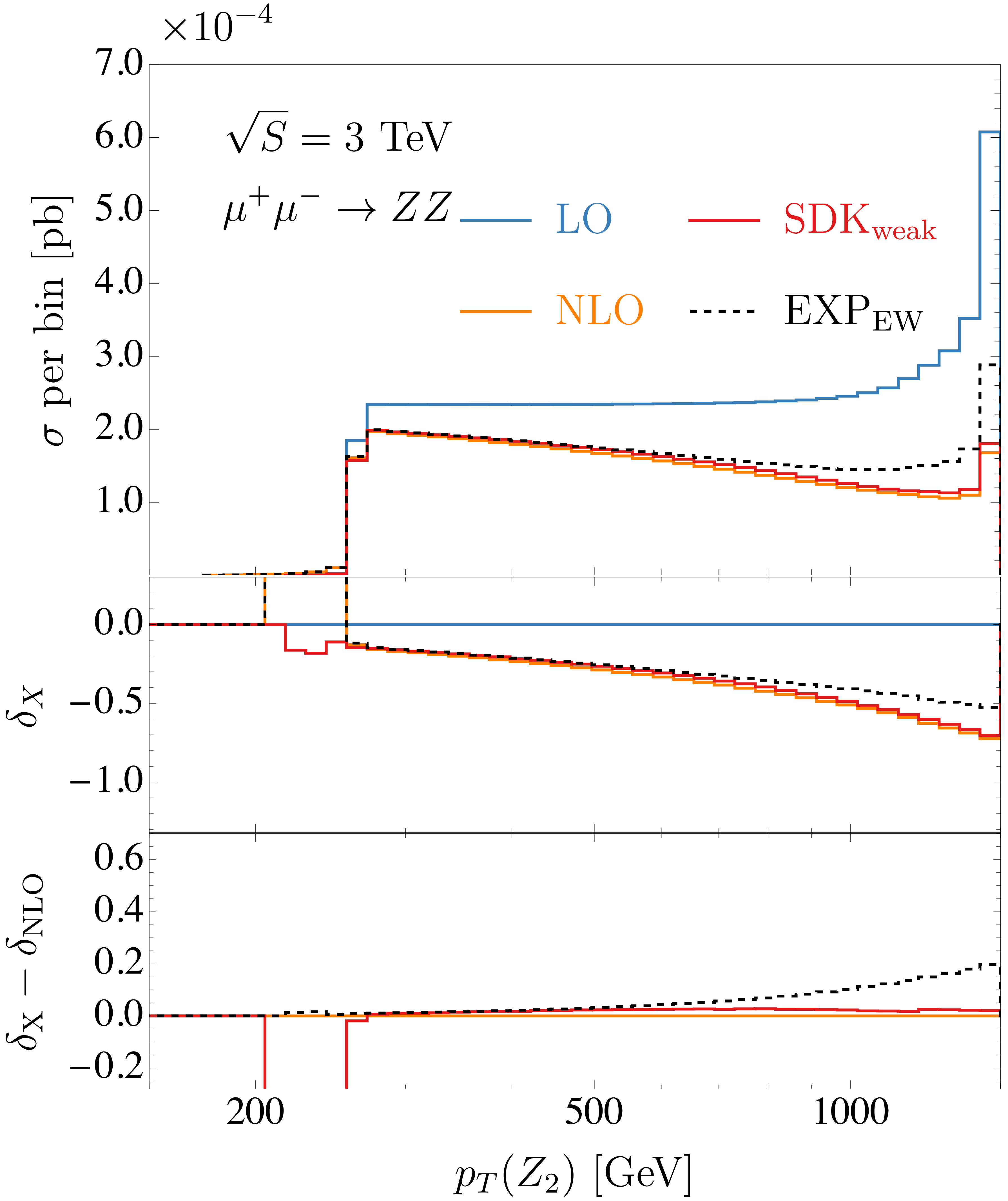
Green: logs of the form $\log^2(t/s)$ or $\log(t/s)$ ignored.



ZZ

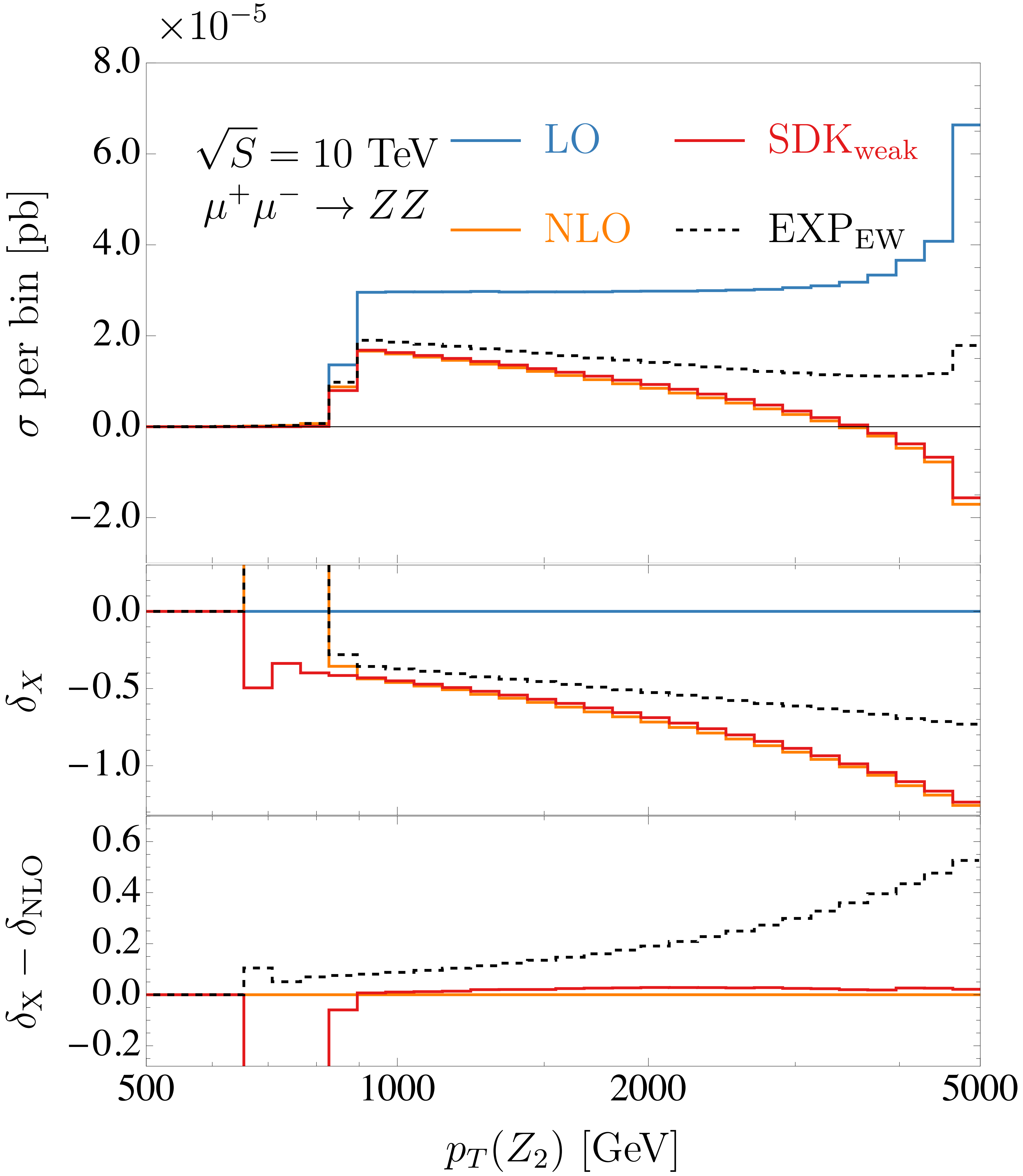
Ma, DP, Zaro **TODAY**

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



Exponentiation as an approximation of proper resummation.

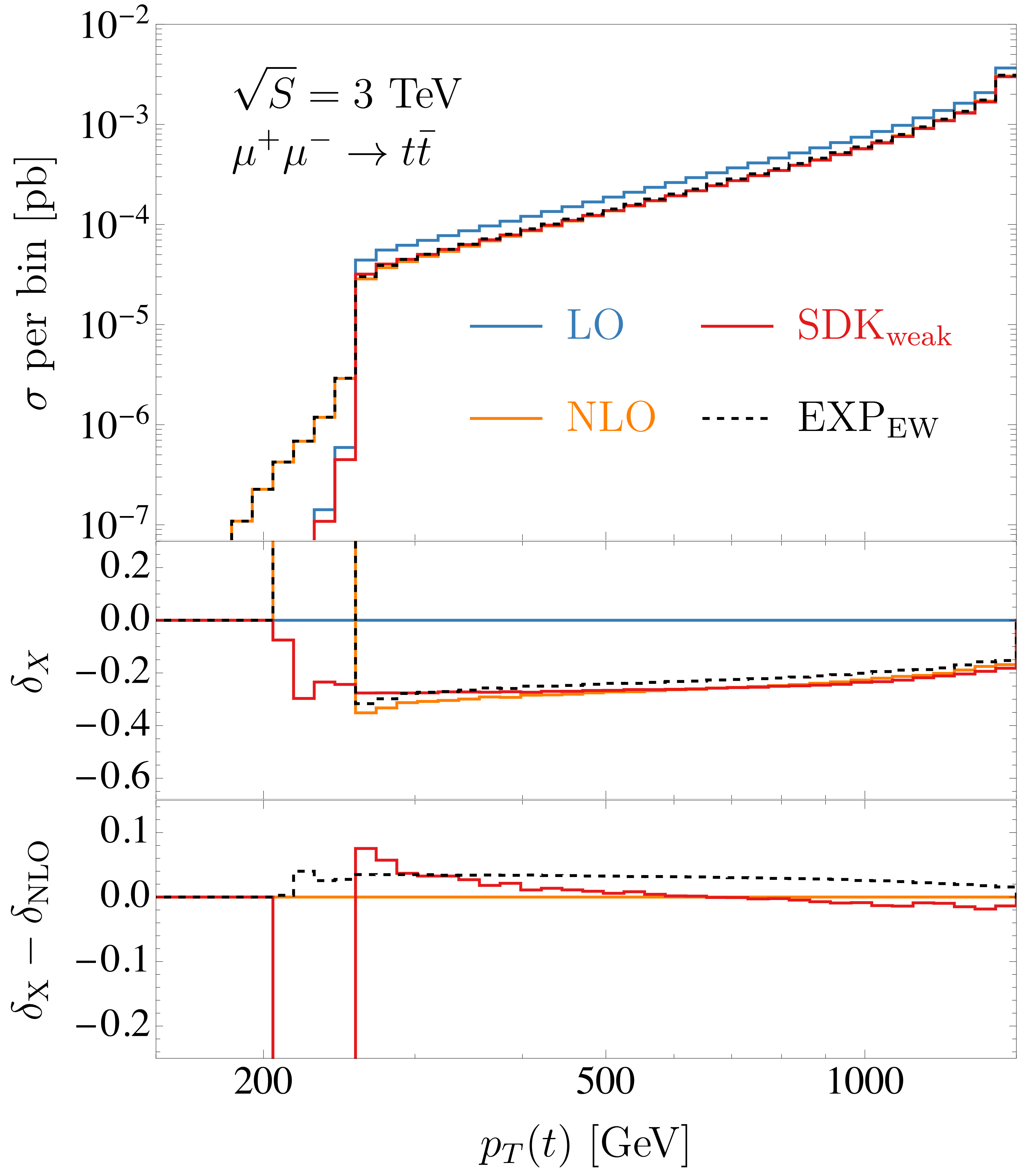
At 10 TeV resummation is unavoidable for sensible predictions, and it is necessary for precision at 3 TeV.



$t\bar{t}$

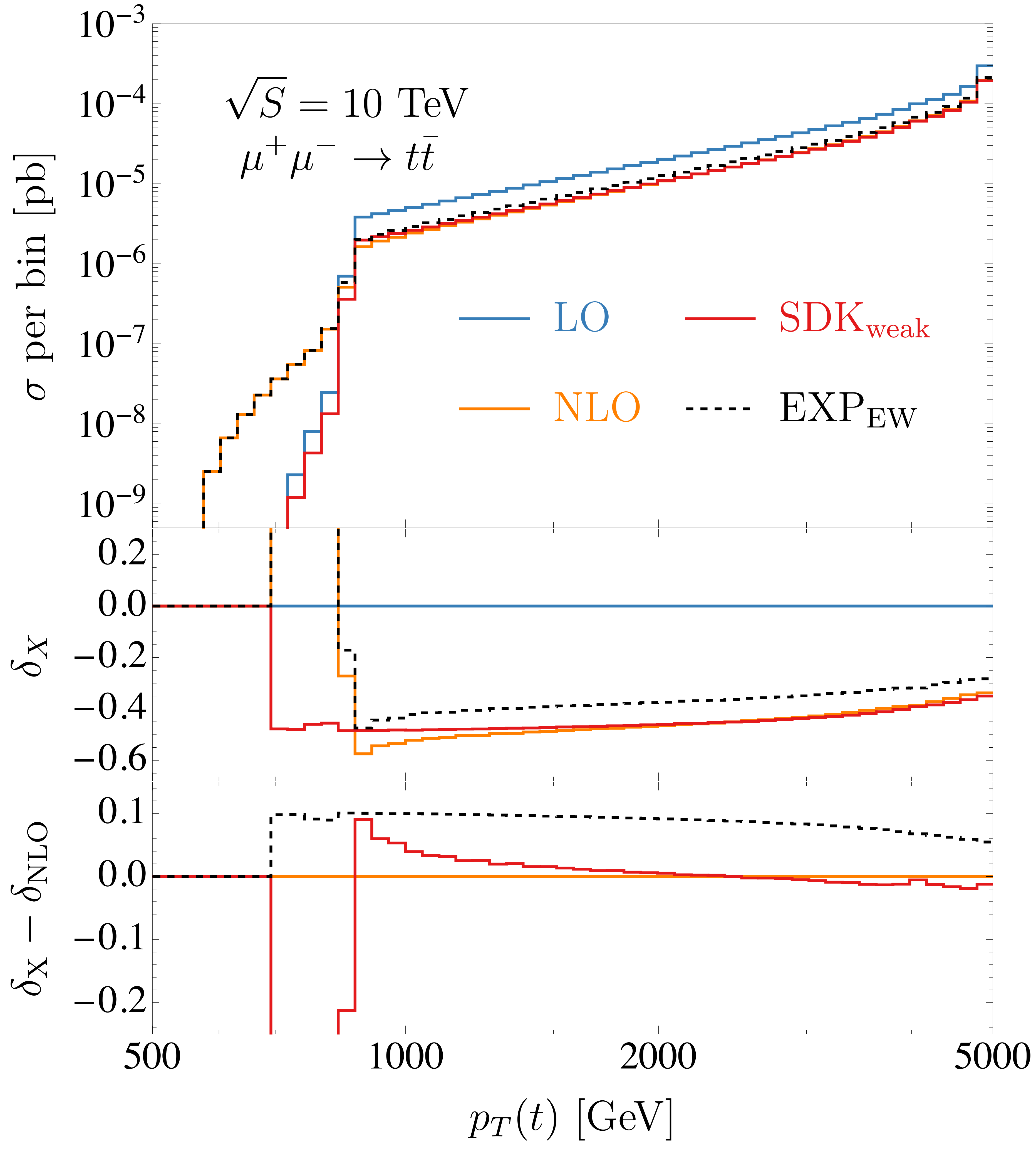
Ma, DP, Zaro **TODAY**

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



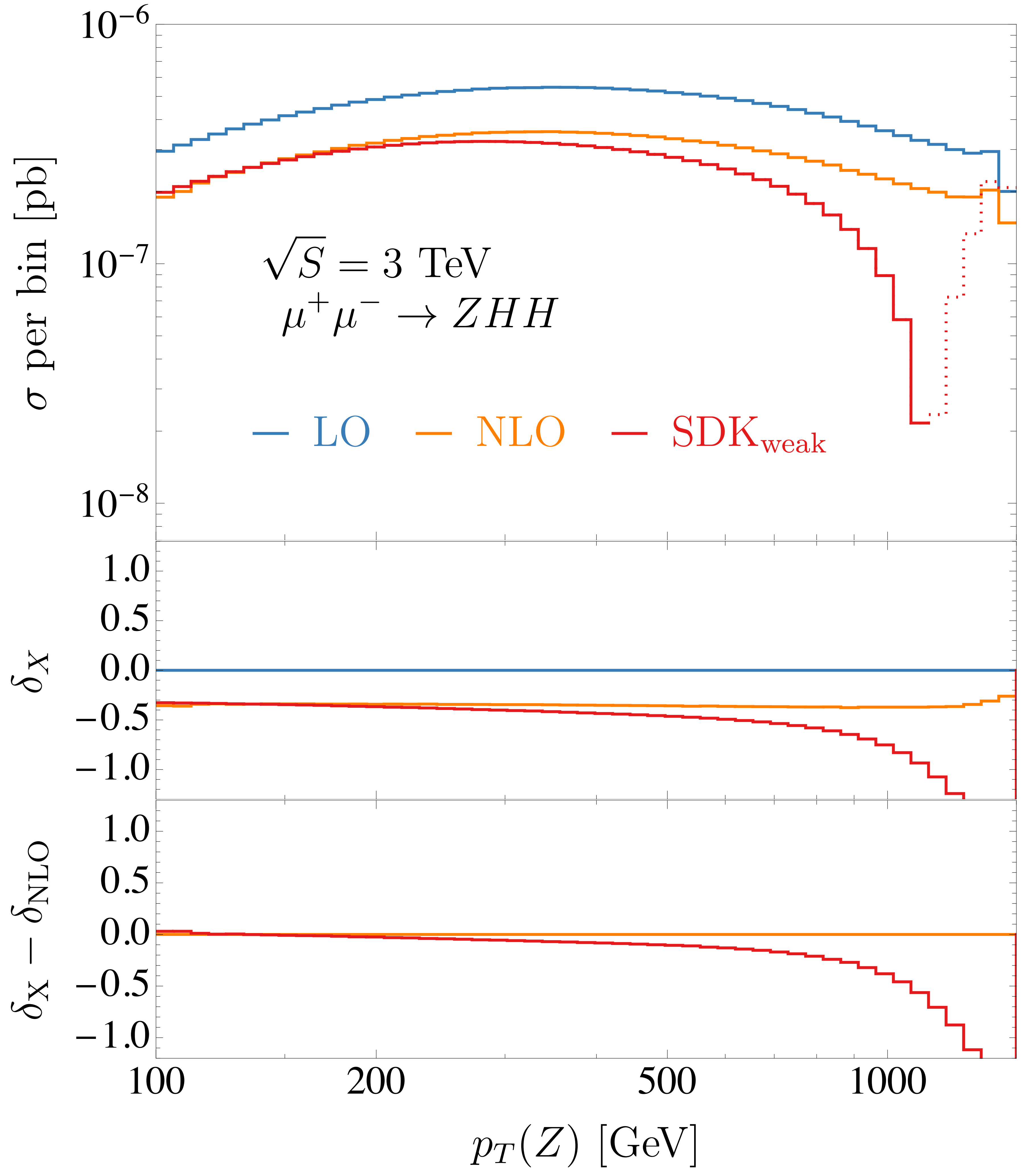
Exponentiation as an approximation of proper resummation.

Unlike ZZ, for $t\bar{t}$ **also at 10 TeV resummation is** necessary only for precision.



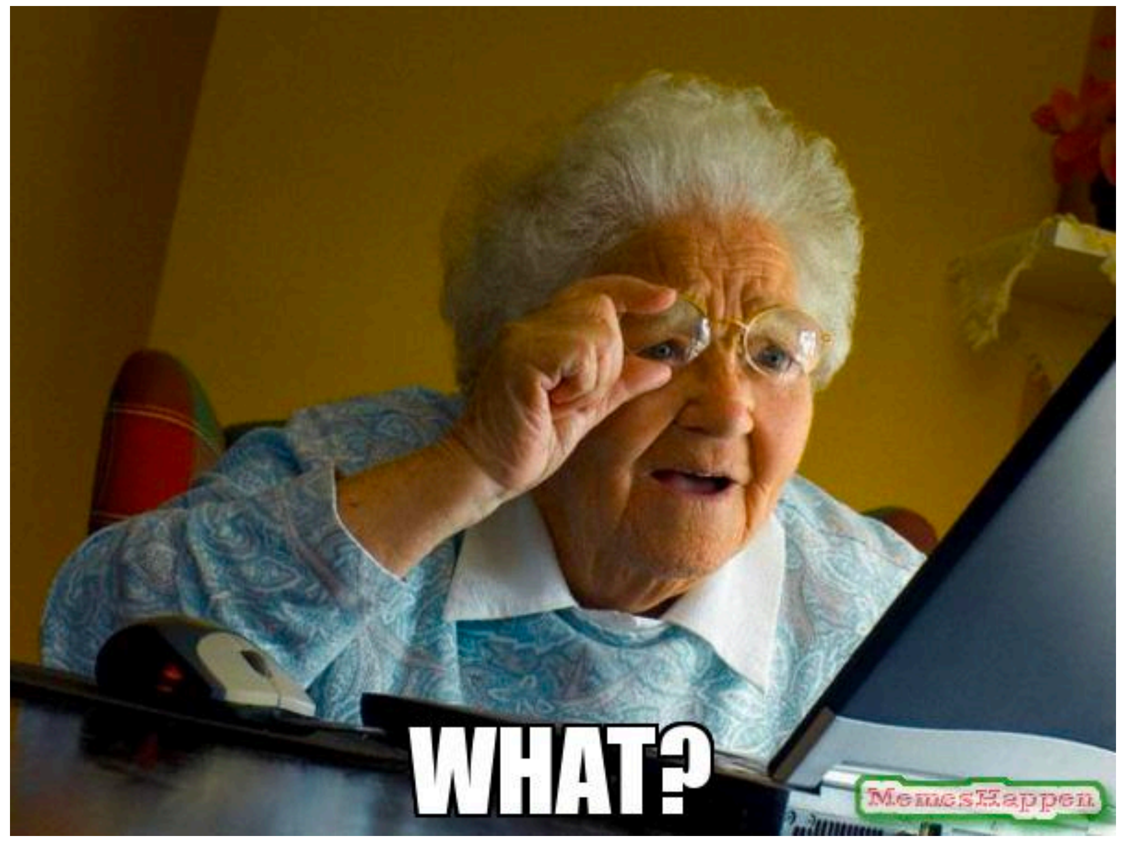
Sudakov may completely fail: ZHH

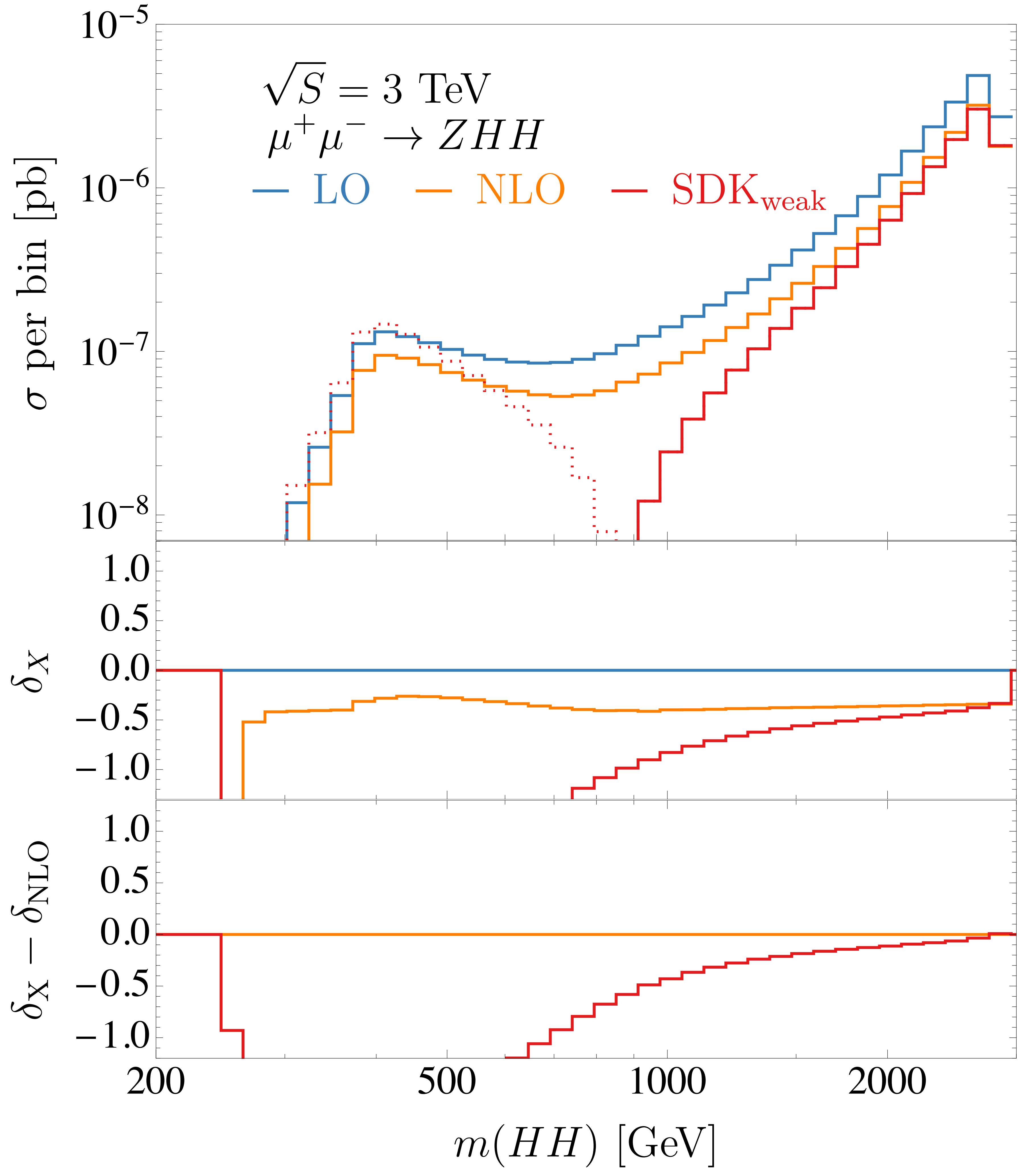
Ma, DP, Zaro **TODAY**



NLO EW corrections are flat.

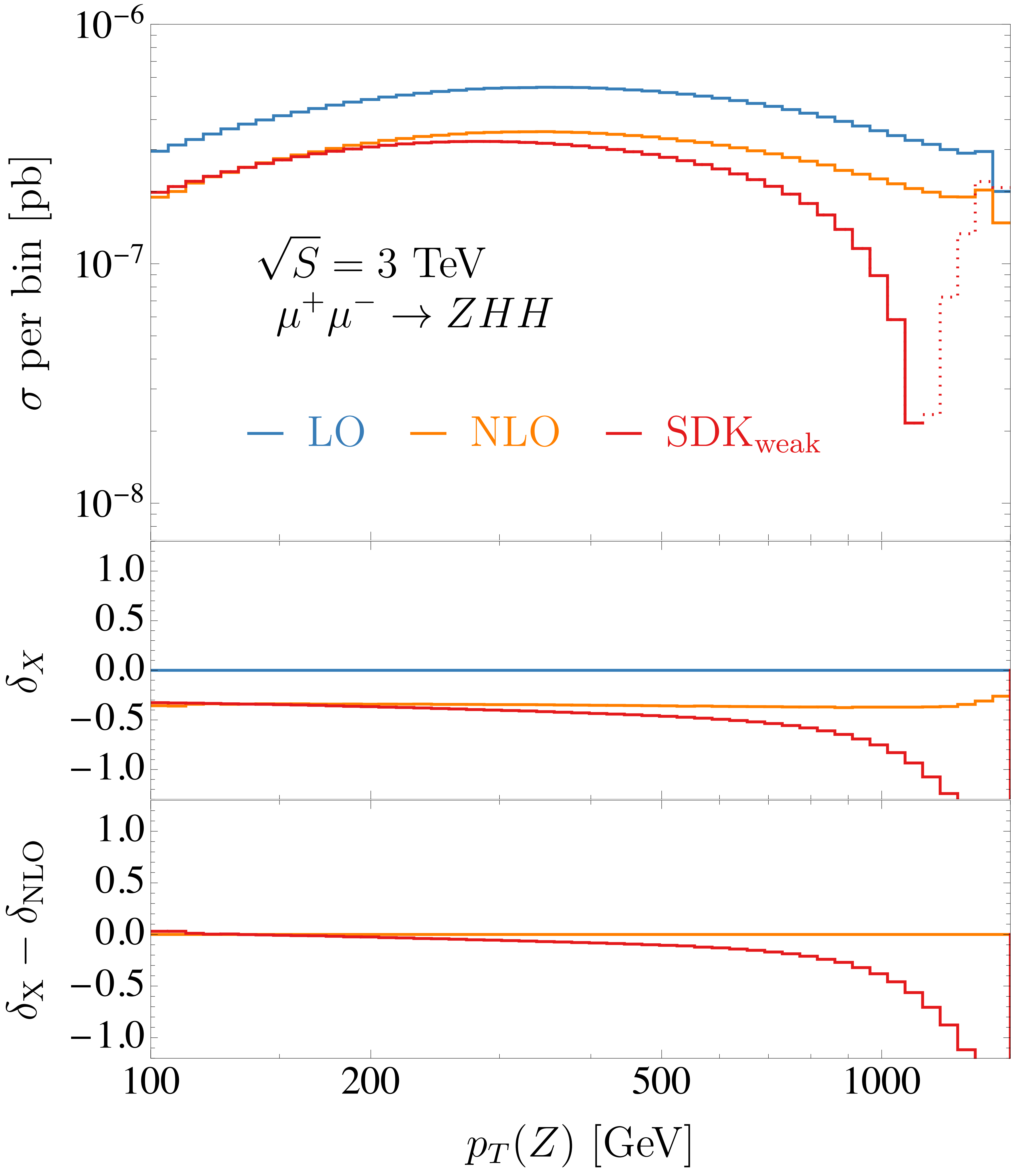
Sudakov logarithms work **very well at low pt and very bad at high pt.**





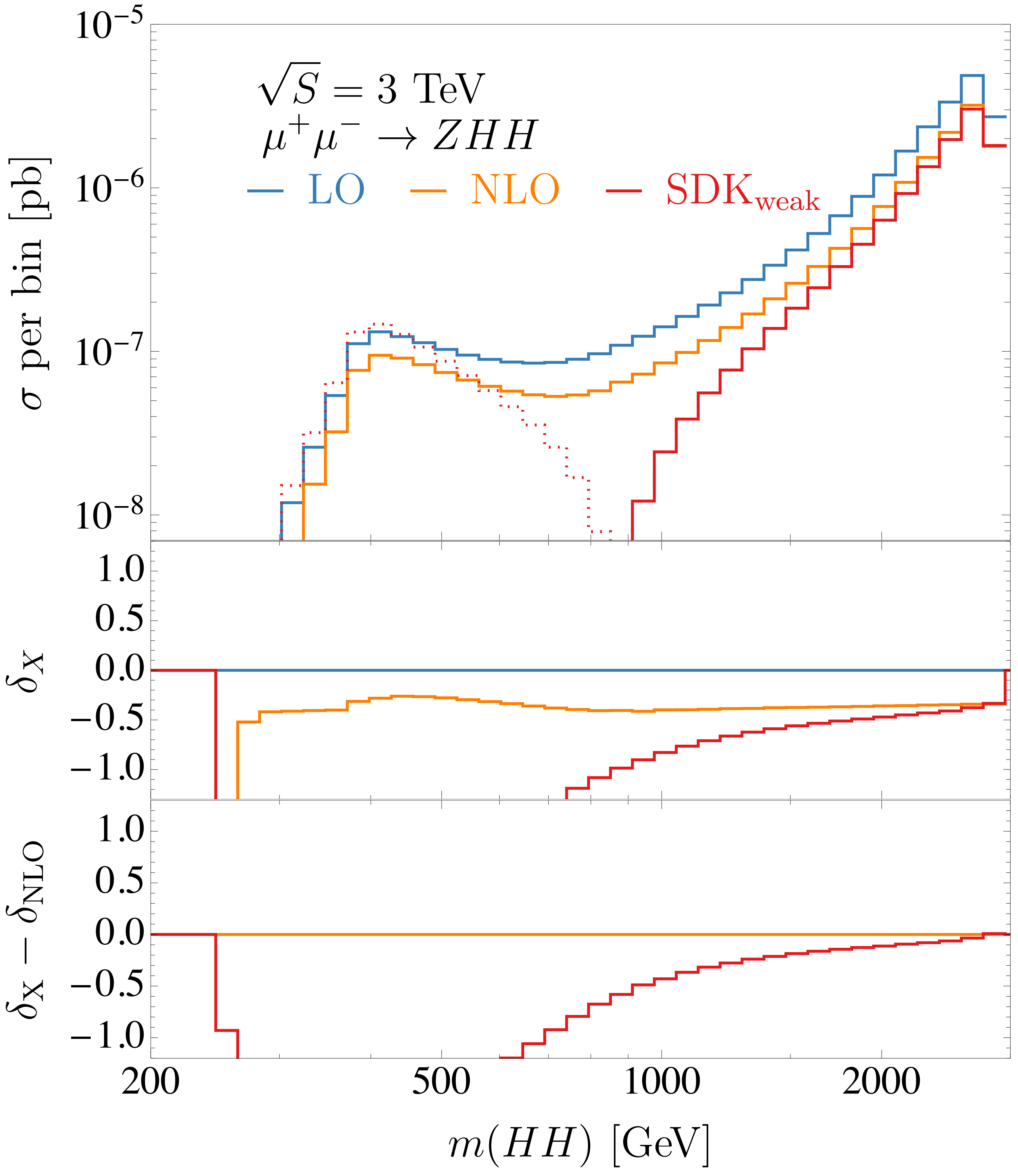
Sudakov may completely fail: ZHH

Ma, DP, Zaro **TODAY**



For High pt of the Z boson, the two Higgs can have very small ΔR and so small $m(HH)$, recoiling against the Z.

In that configuration, formally **mass suppressed** terms $\sim \dfrac{v}{m(H_1H_2)}$ can become numerically sizeable, and the **DP algorithm fails**.



What about extra radiation of Z (and H)?

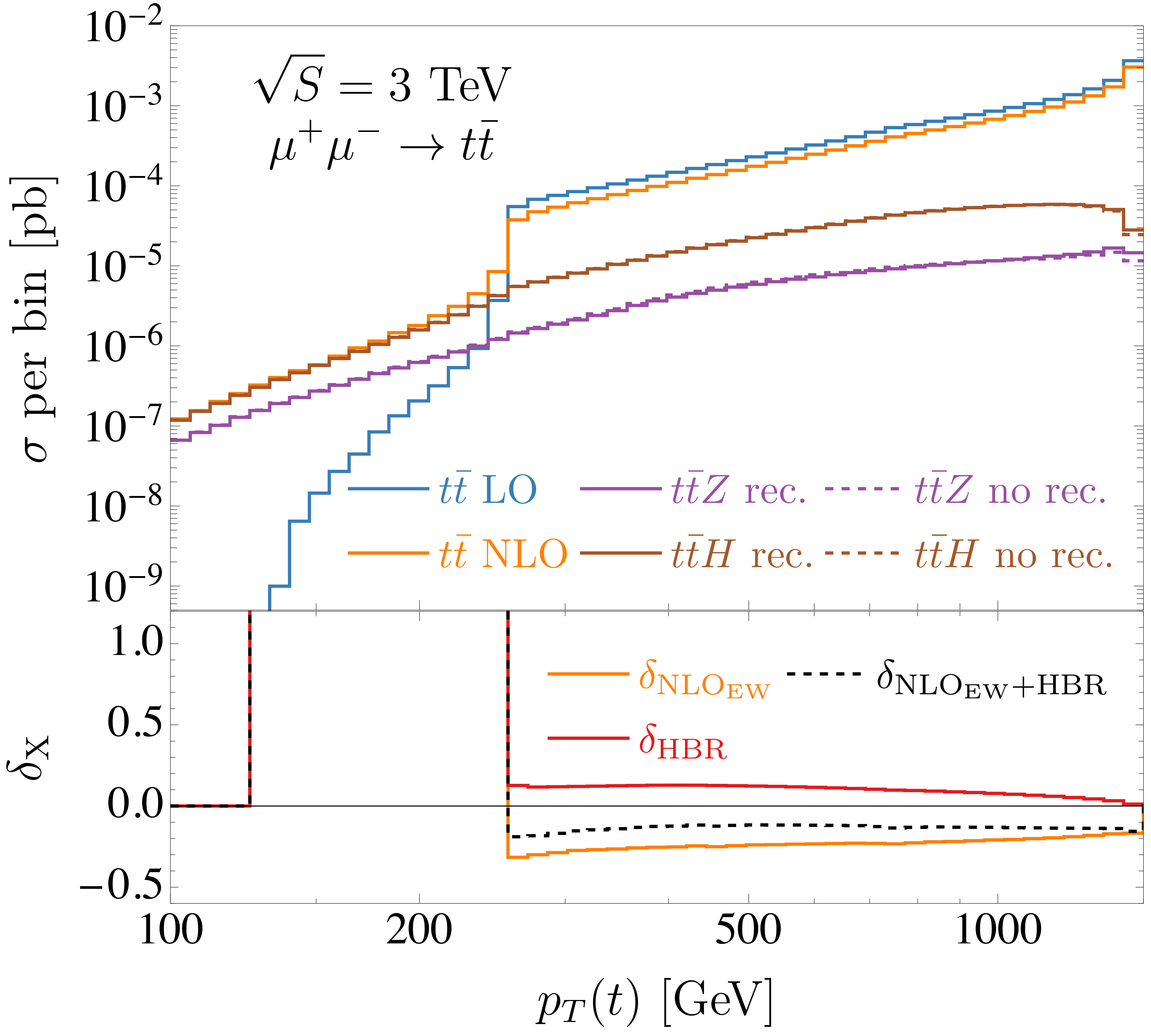
We know that unlike QCD in virtual+real there is not the exact cancellation of logarithms.

But a cancellation is still present, how much large?

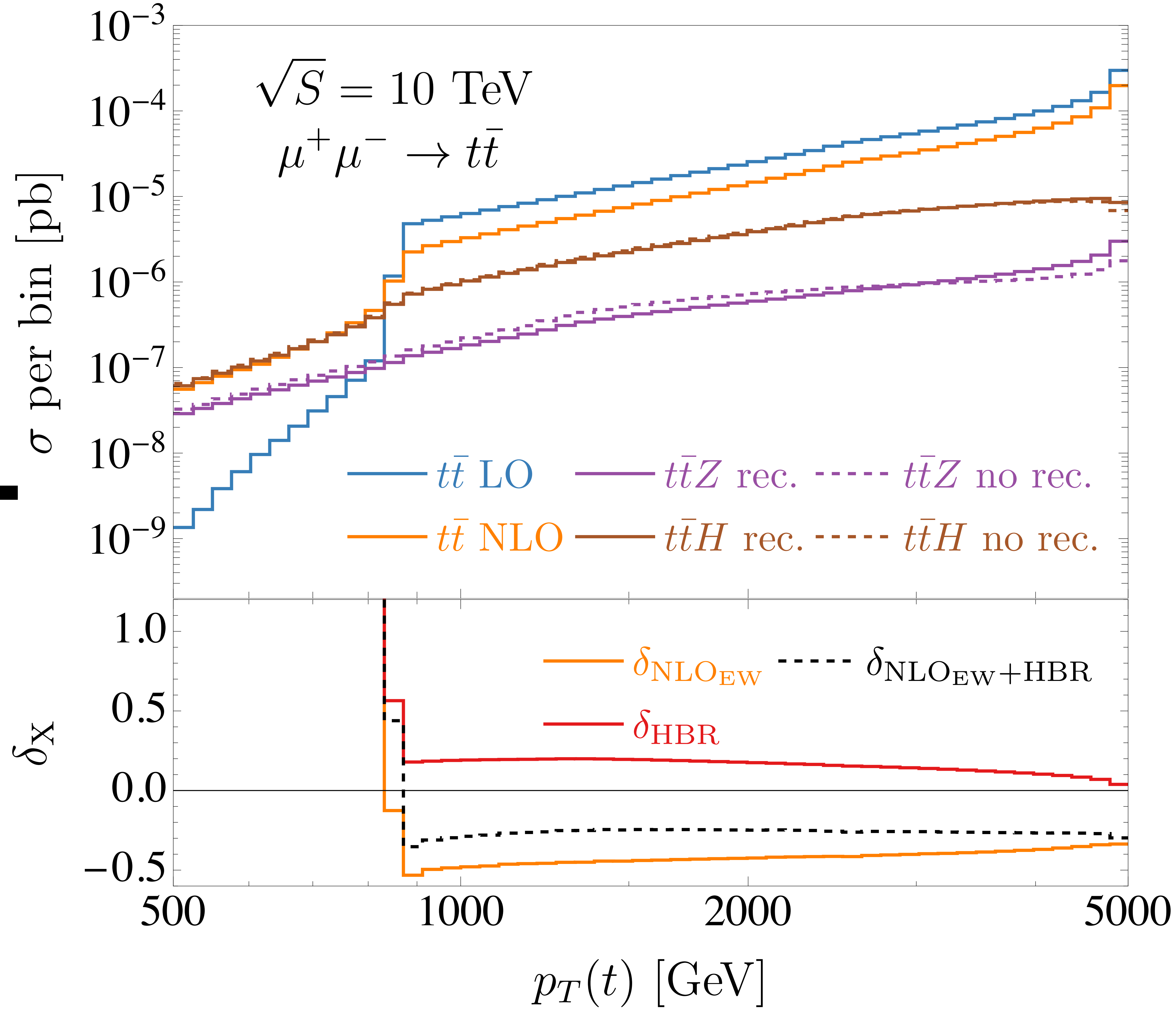
Is it really Heavy-Boson-Radiation (HBR) leading to $\mathcal{O}(1)$ corrections?

EW is the new **QCD**,
but it is not exactly as the QCD!

Very small effects from Z and H radiation, especially in the bulk: $p_T(t) \simeq \sqrt{S}/2$



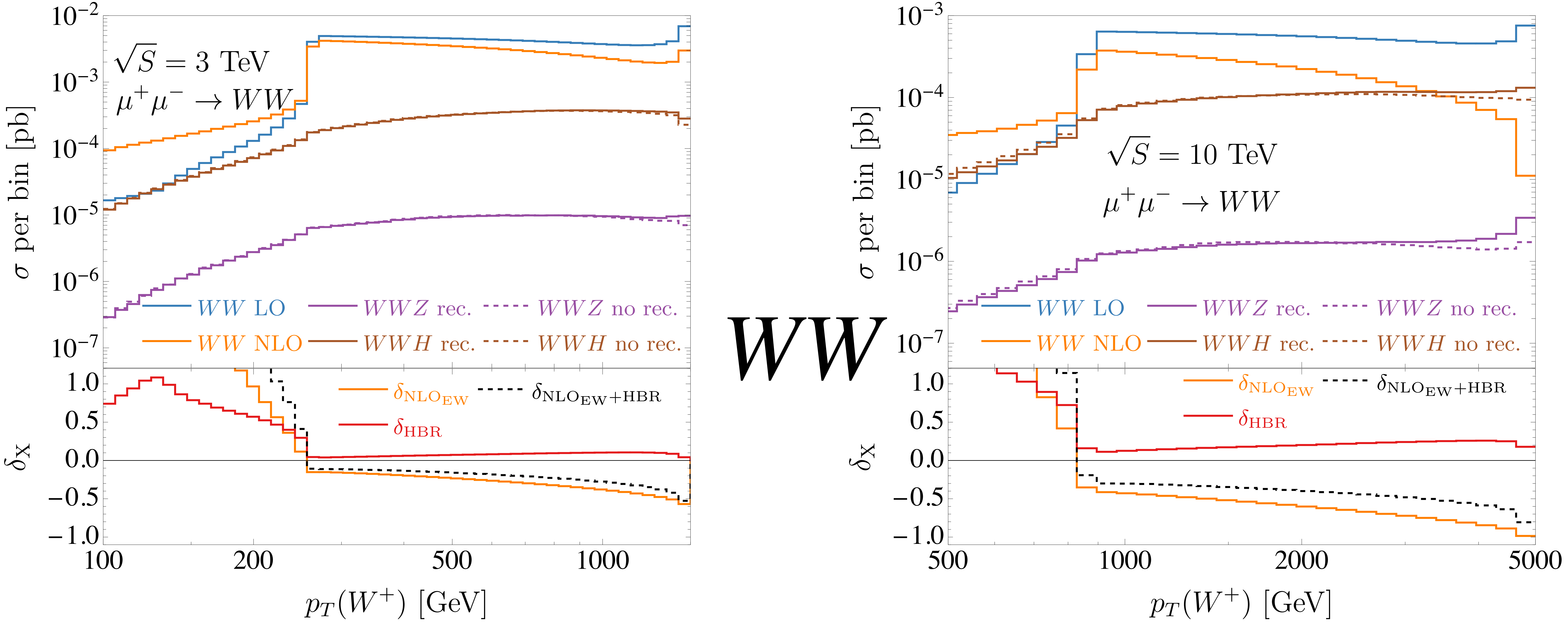
$t\bar{t}$



Notice that in order to allow more phase space we required just $m(F) > 0.5\sqrt{S}$.
Still HBR << NLO EW in absolute value.

Ma, DP, Zaro **TODAY**

Very small effects from Z and H radiation, especially in the bulk: $p_T(W) \simeq \sqrt{S}/2$



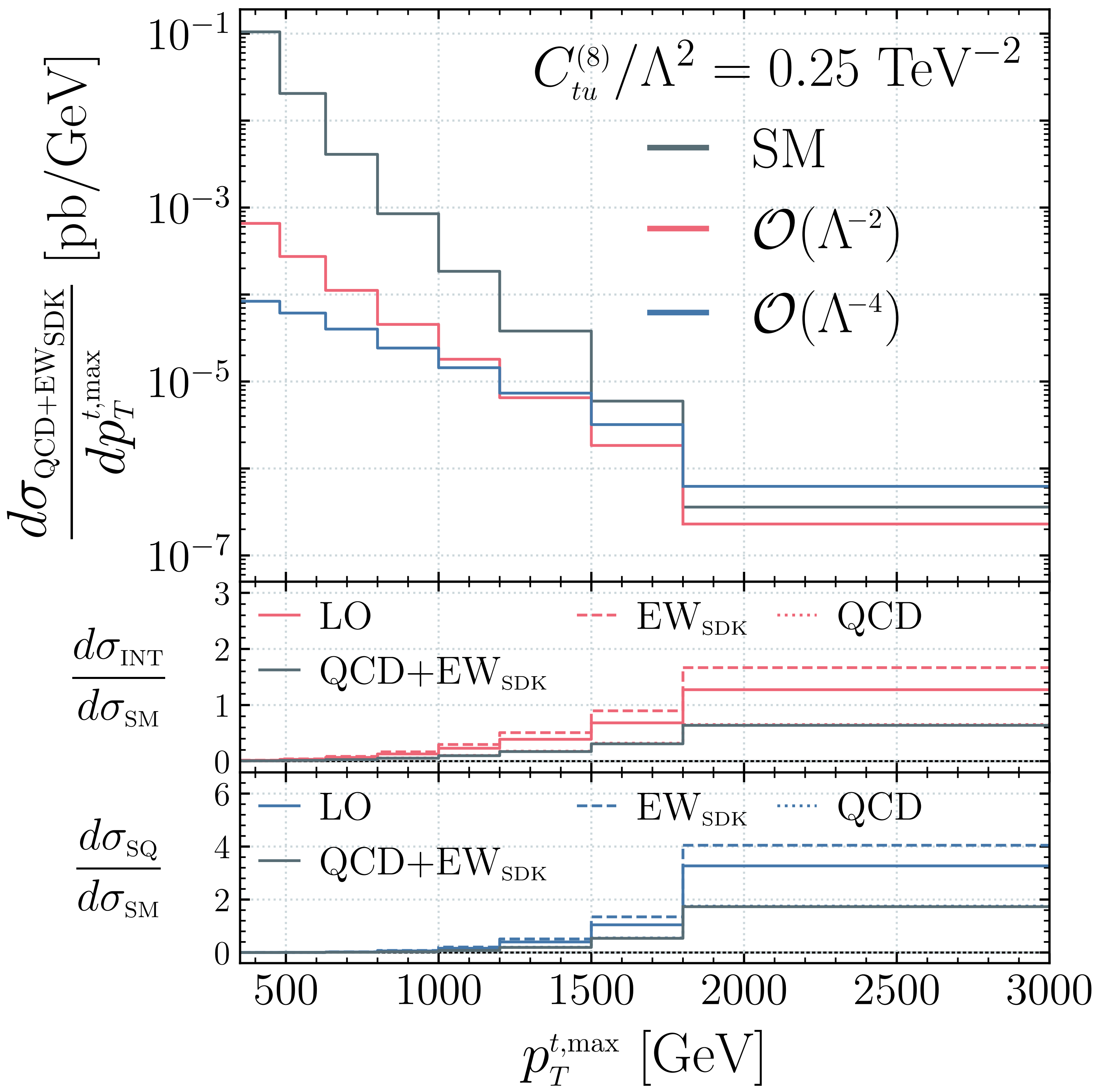
It is a general pattern: radiation of heavy bosons is much less important than loops!

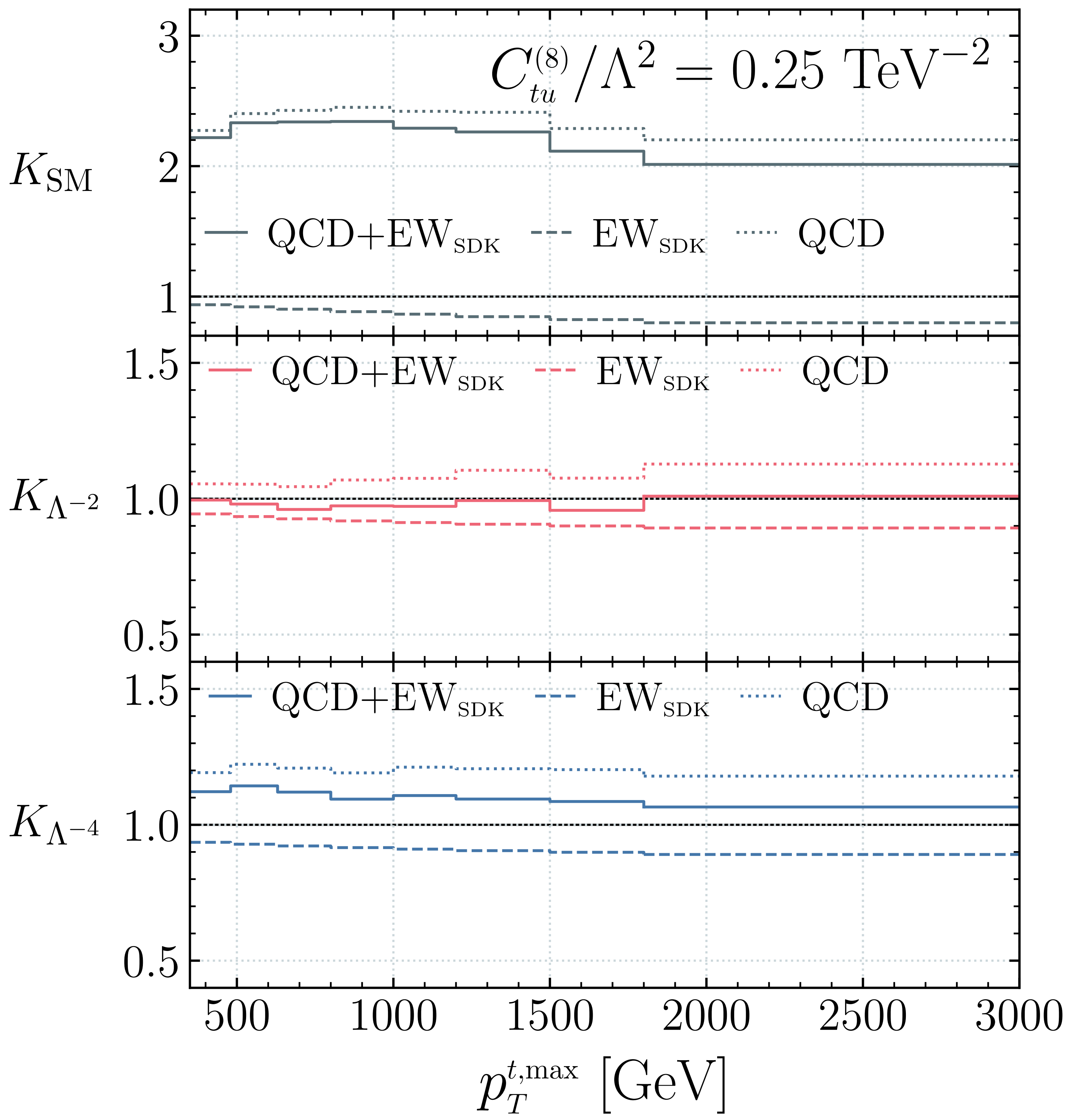
Ma, DP, Zaro **TODAY**

EW Sudakov and SMEFT: again $t\bar{t}$

Only Four-Fermion operators are considered in the study.

$$\mathcal{O}^8_{tu} = (\bar{t}\gamma^\mu T^A t)(\bar{u}_i \gamma_\mu T^A u_i)$$





LHC

Both QCD and EW corrections are different for SM, SM-SMEFT interference, and SMEFT^2 contributions of dim-6.

QCD and EW cancel each other: both are important.

K-factors can be different in SM and BSM!

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro: in preparation

EW Sudakov and SMEFT: again $t\bar{t}$

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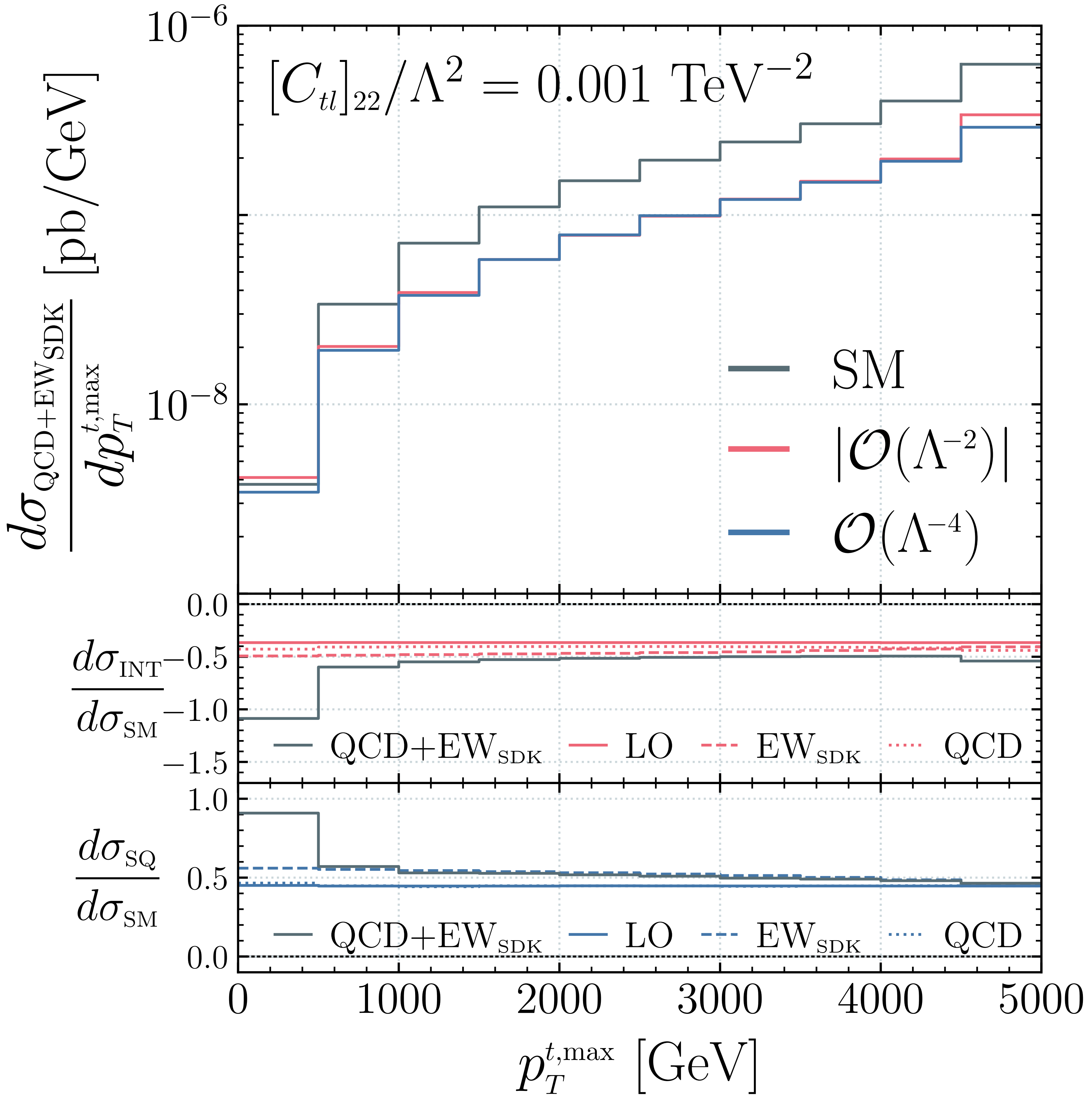
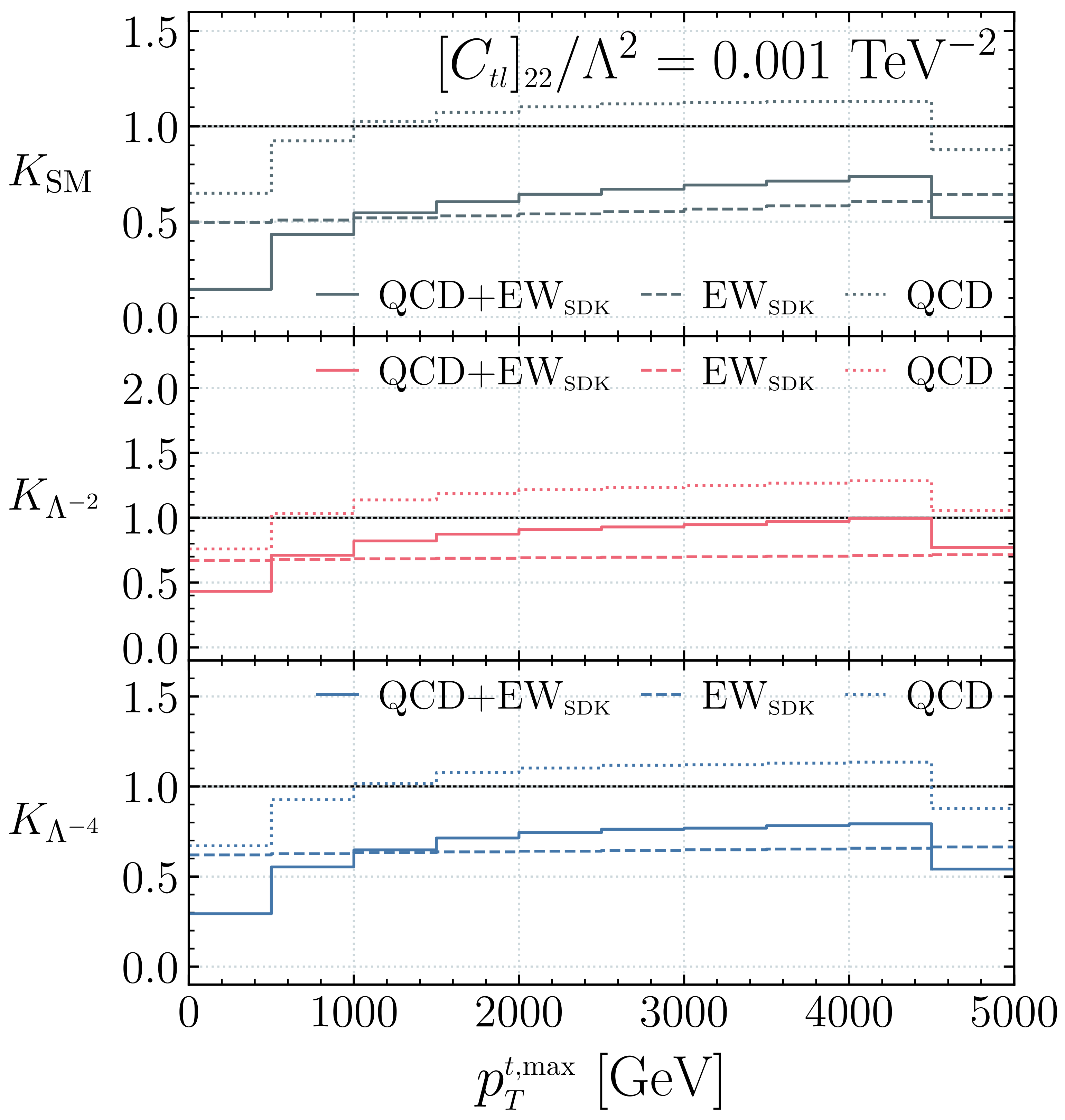
$$\mathcal{O}_{tl} = (\bar{t}\gamma^{\mu}t)(\bar{l}_i\gamma_{\mu}l_i)$$





10 Tev μ-coll

Both QCD and EW corrections are different for SM, SM-SMEFT interference, and SMEFT^2 contributions of dim-6.

QCD and EW cancel each other: both are important.

K-factors can be different in SM and BSM!

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro: in preparation

CONCLUSION

- EW corrections are mandatory for phenomenology at future colliders, especially for high energies. **Not only for the SM also for BSM!**
- There are many interesting open questions/issues on this subject.
- In the talk we have **focussed on the comparison between Sudakov approximation and the exact NLO EW for direct production at high energy**, both available only for the SM.
- **Sudakov logs** are the dominant contribution of EW corrections at high energy and they are a **good approximation** of them, but only **IF**: single logs present, logs among invariants present, correct scheme $\mathrm{SDK}_{\mathrm{weak}}$ adopted, mass-suppressed contributions negligible etc.
- **Heavy-Boson Radiation** has an impact, but not always so large and typically **smaller than** the **virtual** contributions.
- **Resummation may be mandatory for sensible results** in many configurations and in general for precision.
- Effects in the SM may be altered by BSM (see SMEFT example). Still EW is important.

EXTRA SLIDES

Our revisitation and automation: Amplitude level

We have **revisited** and **automated** in aMG5 the **Denner&Pozzorini algorithm** for the evaluation of one-loop EW Sudakov corrections to amplitudes (*Denner, Pozzorini '01*). In particular we have introduced the **following novelties**.

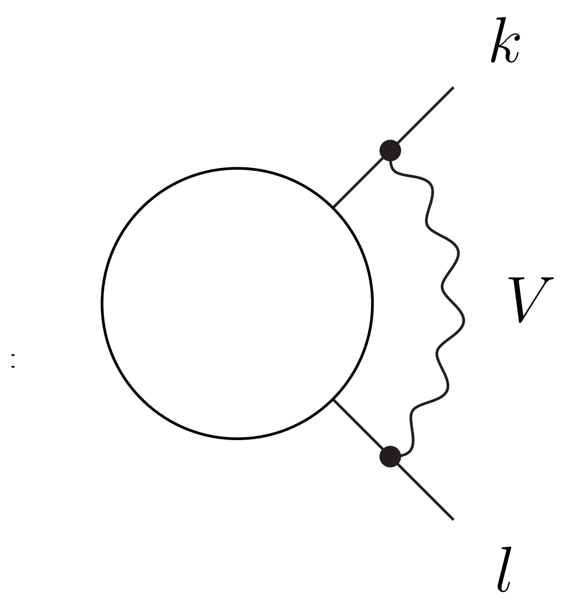
- **IR QED** divergencies are dealt with **via D**imensional **R**egularisation, with strictly massless photons and light fermions.

- **Additional logarithms** that involve ratios between invariants, and therefore **angular** dependences, are taken into account.

- We correctly take into account an **imaginary term** that was **previously omitted** in the literature. Relevant for $2 \to n$ processes with $n > 2$

- Moving to the level of interferences of tree and one-loop amplitudes, we take into account NLO EW contributions originating from **QCD loops on top of subleading LO terms**.

Derivation of LSC and SSC



Denner&Pozzorini

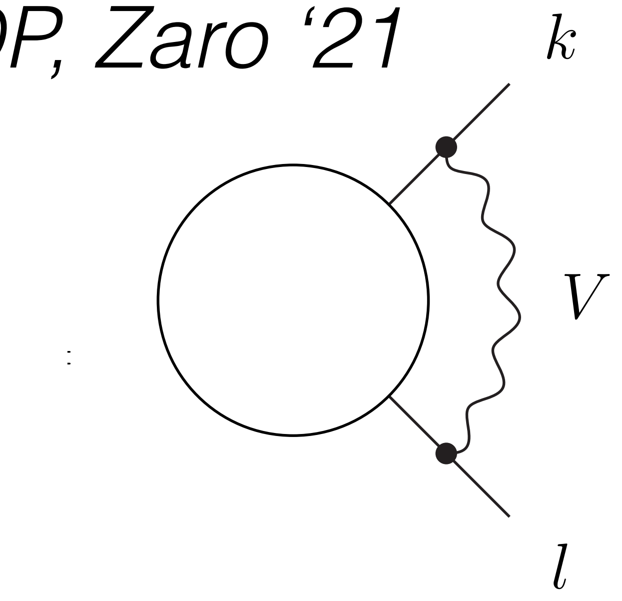
$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$\boxed{=}\ \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\textbf{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\textbf{SSC}}\ \boxed{+\cdots}$$

$$L(s) \equiv L(s, M_W^2) \qquad \text{and} \qquad l(s) \equiv l(s, M_W^2)$$

The relation $\boxed{r_{kl} = r_{k'l'} = s}$ is used in all logs, unless they multiply $l(s)$.

DP, Zaro '21



Our approach: $\cancel{r_{kl} = r_{k'l'} = s}$ **in the expressions**

$$C_0(p_k, p_l, M, M_k, M_l) \quad \Rightarrow \quad L(|r_{kl}|, M^2)\boxed{- 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)}$$

Previously omitted imaginary term

$$L(|r_{kl}|, M^2) - \boxed{2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)} =$$

$$= L(s, M^2) + 2l(s, M^2)\left(\log\frac{|r_{kl}|}{s}\boxed{- i\pi\Theta(r_{kl})}\right) + L(|r_{kl}|, s) - \boxed{2i\pi\Theta(r_{kl})l(|r_{kl}|, s)} =$$

$$\boxed{=}\ \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\textbf{LSC}} + \underbrace{2l(s)\left(\log\frac{|r_{kl}|}{s} - \boxed{i\pi\Theta(r_{kl})}\right)}_{\textbf{SSC}} +$$

$$\underbrace{2l(M_W^2, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s) - \boxed{2i\pi\Theta(r_{kl})l(|r_{kl}|, s)}}_{\mathrm{SSC}^{s\to r_{kl}}} + \cdots$$

The conceptual derivation relies on the assumption $s = r_{kl}$, but is not actually used in the expressions. Therefore, further angular dependencies are taken into account.

New angular dependences via ratios among invariants

Calculation set up for showcasing some results

$\mu^+\mu^- \longrightarrow X$, where X is a generic final state involving W, Z, t, H, ℓ. Thus direct production, no VBF considered.

ISR Treatment: we use the LL PDF for the muon only

$$\Gamma_{\rm LO}(z) = \frac{\exp\left(3\beta_S/4 - \gamma_E\beta_E\right)}{\Gamma\left(1+\beta_E\right)}\,\beta_E(1-z)^{\beta_E-1} - \frac{1}{2}\beta_H(1+z) + \mathcal{O}(\alpha^2)$$

Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22

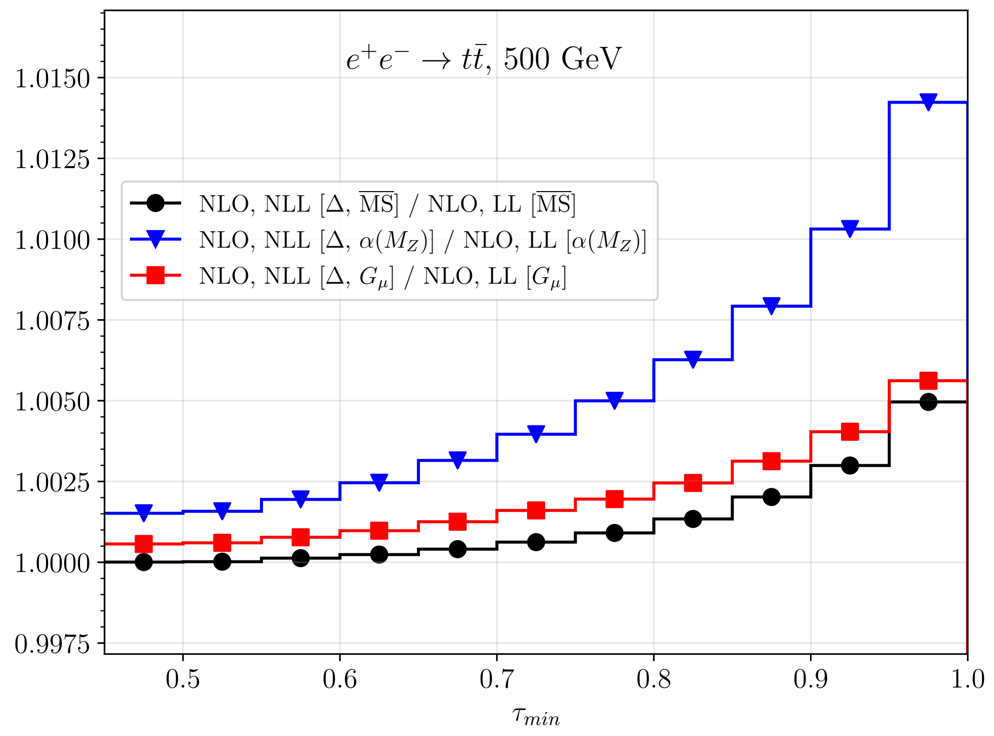
- Beta scheme:

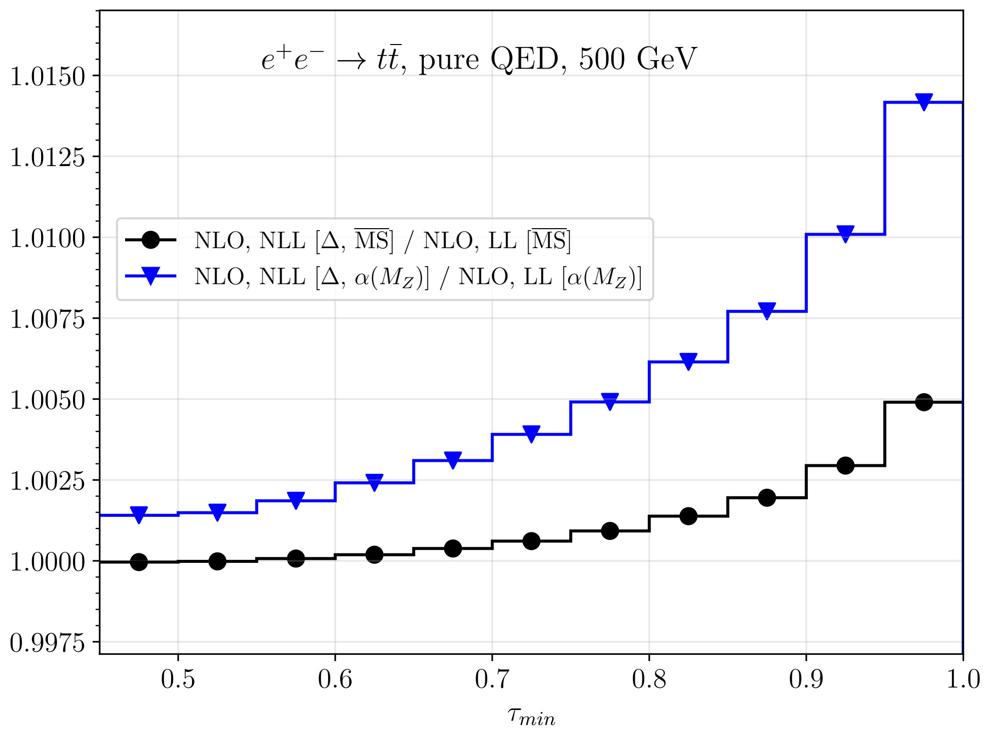
$$\beta_E = \beta_S = \beta_H = e_e^2\beta\,.$$

- Eta scheme:

$$\beta_E = \beta_S = e_e^2\beta\,, \qquad \beta_H = e_e^2\eta\,.$$

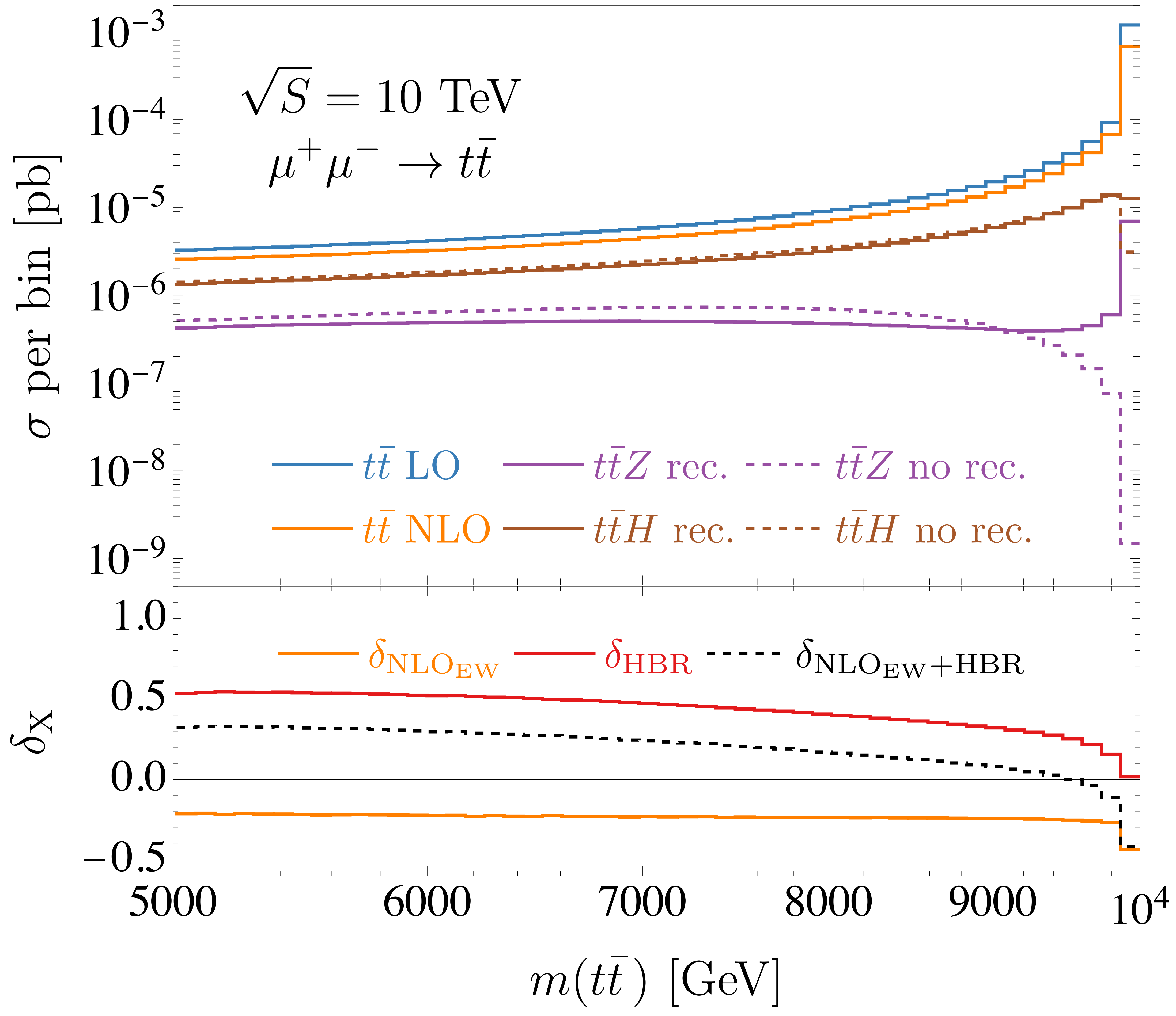
$$\eta = \frac{\alpha}{\pi}\log\frac{\mu^2}{m^2}\,, \qquad \beta = \frac{\alpha}{\pi}\left(\log\frac{\mu^2}{m^2} - 1\right)$$

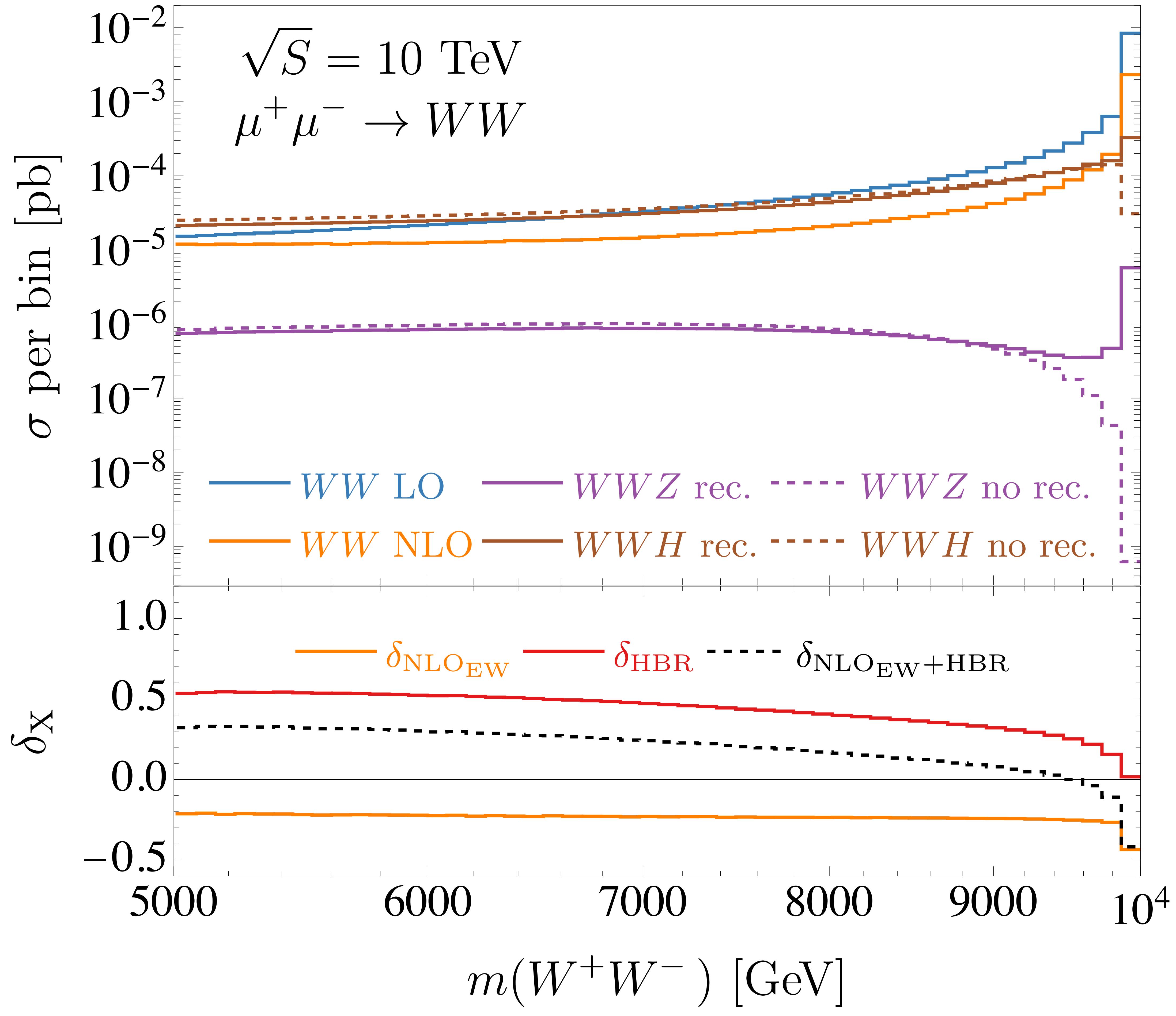




For precision physics the scheme adopted and the NLL accuracy (*Frixione, Stagnitto '23*) are mandatory.

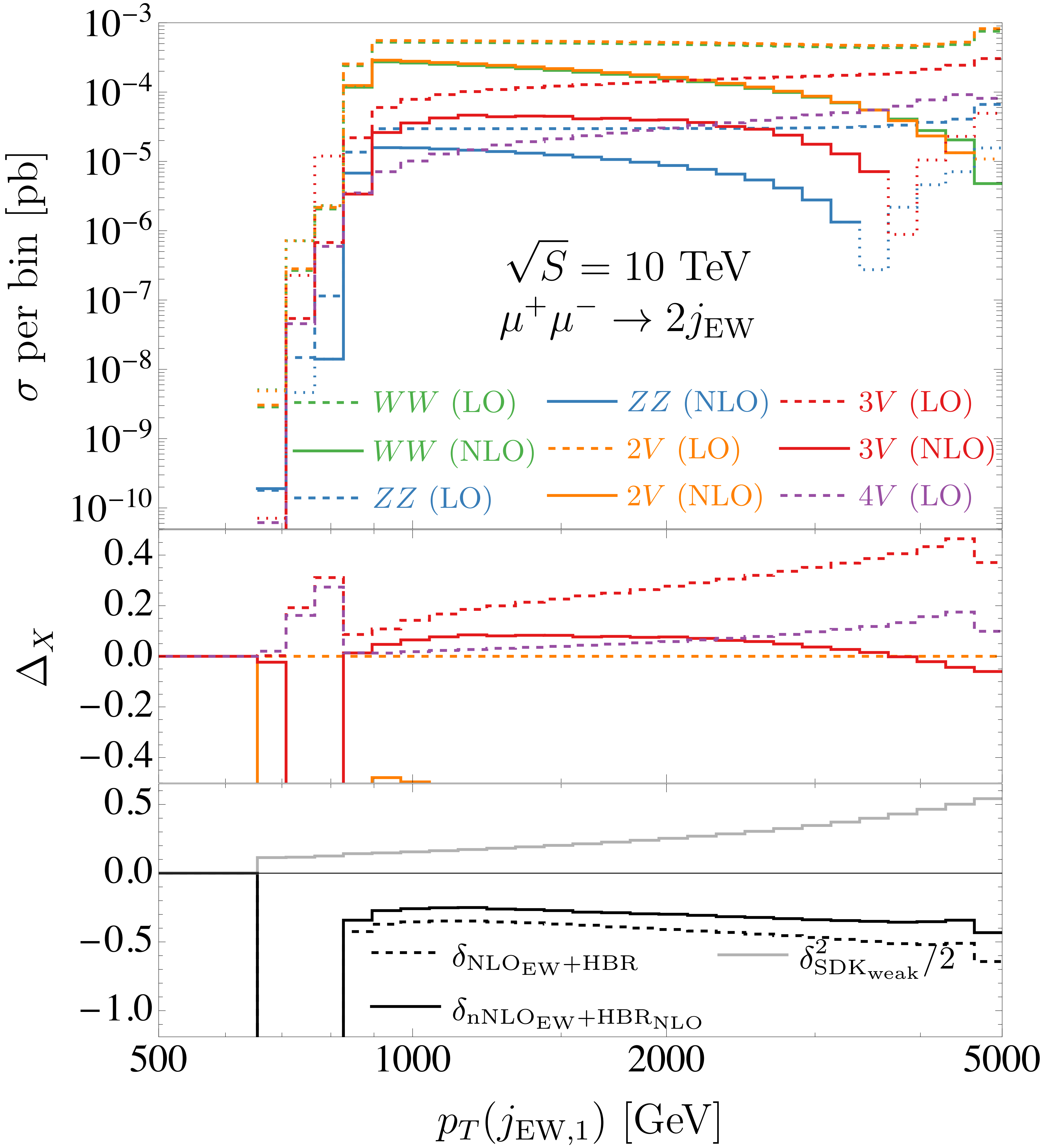
more on the very small effects from Z and H radiation





Ma, DP, Zaro **TODAY**

EW jets



$$\sigma_X(2j_{\rm EW}) \equiv \sigma_X(2V) \qquad \text{for } X = \text{LO, NLO EW, SDK}_{\rm weak}$$

$$\sigma_{\rm HBR}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(3V)\,,$$
$$\sigma_{\rm NLO_{EW}+HBR}(2j_{\rm EW}) \equiv \sigma_{\rm NLO_{EW}}(2V) + \sigma_{\rm LO}(3V)$$

$$\sigma_{\rm nNLO_{EW}+HBR_{NLO}}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(2V)\left(1+\delta_{\rm NLO_{EW}}+\frac{\delta^2_{\rm SDK_{weak}}}{2}\right) + \sigma_{\rm NLO_{EW}}(3V) + \sigma_{\rm LO}(4V)\,.$$

$$\Delta_X(2V) \equiv \frac{\sigma_X(2V)-\sigma_{\rm LO}(2V)}{\sigma_{\rm LO}(2V)}$$
$$\Delta_X(3V) \equiv \frac{\sigma_X(3V)}{\sigma_{\rm LO}(2V)}$$
$$\Delta_X(4V) \equiv \frac{\sigma_X(4V)}{\sigma_{\rm LO}(2V)}$$

It is a general pattern: radiation of heavy bosons is much less important than loops!

Ma, DP, Zaro **TODAY**

Cross-sections: our approach.

FOR WHAT EW SUDAKOV ARE USEFUL?
For providing a very **good approximation of NLO EW** in the **high-energy** limit.

HOW SHOULD ONE PERFORM THE CALCULATION IN THE HIGH-ENERGY LIMIT?
Photons have to be **always clustered with massless charged particle for IR**-safety reasons. But from an experimental point of view, **at high energy also clustering tops and W bosons with photons** is very reasonable, either if you imagine to tag heavy object directly or via their massless decay products.

The **QED Logs,** involving s and λ^2 (or Q^2), **cancel against their real-emission counterparts and PDF counterterms**. The only one surviving are those from tops in vacuum polarisation for external (not tagged) photons, both in the initial and final state:

$$\mathrm{SDK}_{\mathrm{weak}}$$

Almost all the contributions of QED are removed
(e.g. $C_{\mathrm{EW}}(k) \rightarrow C_{\mathrm{EW}}(k) - Q_k^2$, $Q_k^2 = 0$),
but NOT in the parameter renormalisation δ^{PR}.

DP, Zaro '21

Implementation

Born amplitude: $\mathcal{M}_0^{i_1\ldots i_n}(p_1,\ldots,p_n)$

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\ldots i_n}(p_1,\ldots,p_n) = \underbrace{\mathcal{M}_0^{i'_1\ldots i'_n}(p_1,\ldots,p_n)}_{\text{other tree-level amplitudes}}\underbrace{\delta_{i'_1 i_1\ldots i'_n i_n}}_{\text{the logs}}$

Born process: $\boxed{\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n)\to 0}$

$$Z \longrightarrow \chi\,,\qquad W^\pm \longrightarrow \phi^\pm\,,$$

GBE theorem for longitudinal W and Z bosons.

$$\varphi_{i_1}(p_1)\ldots\varphi_{i'_k}\ldots\varphi_{i_n}(p_n)\to 0\,,$$

$$Z \longleftrightarrow A\,,\qquad H \longleftrightarrow \chi\,.$$

Relevant for LSC and C contributions.

$$\varphi_{i_1}(p_1)\ldots\varphi_{i'_k}\ldots\varphi_{i'_l}\ldots\varphi_{i_n}(p_n)\to 0$$

$$f_\sigma \longleftrightarrow f_{-\sigma}\,,\quad H \longleftrightarrow \phi^\pm\,,\quad \chi \longleftrightarrow \phi^\pm\,,\quad A \longleftrightarrow W^\pm\,,\quad Z \longleftrightarrow W^\pm\,.$$

Relevant for SSC charged contributions.

Amplitudes with one or 2 different external particles w.r.t. the Born have to be generated.

Organisation of the logs in the algorithm

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[\boxed{C^{\mathrm{ew}}_{i'_k i_k}(k)}\boxed{L(s)} - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\boxed{l(s)} + \delta_{i'_k i_k}Q_k^2\boxed{L^{\mathrm{em}}(s,\lambda^2,m_k^2)}\right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

Charge for $U(1)_{QED}$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_w^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_W^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_W^2}\right)\right]\boxed{l(s)} + Q^2_{f_\sigma}\boxed{l^{\mathrm{em}}(m^2_{f_\sigma})}\right\},$$

$$\boxed{L(s)} \equiv L(s, M_W^2) \quad \text{and} \quad \boxed{l(s)} \equiv l(s, M_W^2)$$

$$\boxed{l^{\mathrm{em}}(m_f^2)} := \frac{1}{2}l(M_W^2, m_f^2) + l(M_W^2, \lambda^2) \qquad \boxed{L^{\mathrm{em}}(s,\lambda^2,m_k^2)} := 2l(s)\log\left(\frac{M_W^2}{\lambda^2}\right) + L(M_W^2,\lambda^2) - L(m_k^2,\lambda^2)$$

The full EW is present between s and M_W^2, while only QED is present between M_W^2 and λ^2.

So the QED contribution is split between the intervals $(s, M_W^2) + (M_W^2, \lambda^2)$. But the division at M_W^2 is simply determined by convenience, in parallel with the weak case. In this case M_W^2 is just a technical parameter and not a physical quantity.

Cross-sections: standard approach in the literature

SDK_0

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[\boxed{C^{\mathrm{ew}}_{i'_k i_k}(k)}\boxed{L(s)} - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\boxed{l(s)} + \cancel{\delta_{i'_k i_k} Q_k^2 L^{\mathrm{em}}(s, \lambda^2, m_k^2)}\right]$$

Casimir for the entire
$SU(2)_L \times U(1)_B$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_W^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_W^2}\right)\right]\boxed{l(s)} + \cancel{Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma})}\right\}$$

$\boxed{L(s)} \equiv L(s, M_W^2)$ and $\boxed{l(s)} \equiv l(s, M_W^2)$

The logarithms between M_W^2 and the infrared scale are simply removed. Equivalently in the case of DR, logarithms involving M_W^2 and the IR regulator Q^2.

Easy, but not very well motivated.

We will denote in the following this approach as SDK_0.

Purely Weak

1. Calculate the δ^{PR} in eq. (2.12) as in the standard SDK approach.

2. For each external particle φ_{i_k} in (2.9), set
$$Q_k = I^A(k) = 0\,. \tag{4.1}$$
This step alone has the effect of eliminating all the terms tagged as "em", with the exception of $\delta Z^{\mathrm{em}}_{AA}$. It also eliminates all the SSC terms and C terms that lead to SL originating from photons, with the exception of those related to transverse W bosons.

3. For each external particle φ_{i_k} in (2.9), perform the replacement
$$C^{\mathrm{ew}}_{i'_k i_k}(k) \longrightarrow C^{\mathrm{ew}}_{i'_k i_k}(k) - Q_k^2\,, \tag{4.2}$$
with the value of Q_k^2 before enforcing eq. (4.1). This, in combination with eq. (4.1), has the effect of eliminating the DL due to photons.

4. Perform the replacement
$$b^{\mathrm{ew}}_W \longrightarrow b^{\mathrm{ew}}_W - 11/3\,. \tag{4.3}$$
This has the effect of eliminating for the transverse W bosons the C terms that lead to SL originating from photons.

5. Set
$$\delta Z^{\mathrm{em}}_{AA} = 0\,, \tag{4.4}$$
and perform the replacement
$$b^{\mathrm{ew}}_{AA} \longrightarrow b^{\mathrm{ew}}_{AA} + \frac{4}{3}\sum_{f,i,\sigma\neq t} N_{\mathrm{C}}^f Q_{f_\sigma}^2 = b^{\mathrm{ew}}_{AA} + 80/9\,. \tag{4.5}$$
This has the effect of eliminating, for the photons, the C terms that lead to SL originating from light fermions.

6. Calculate the remaining terms in eq. (2.12) with the new redefinitions of steps 2–5.