Entanglement in high-energy collisions

Trieste
22 September 2024

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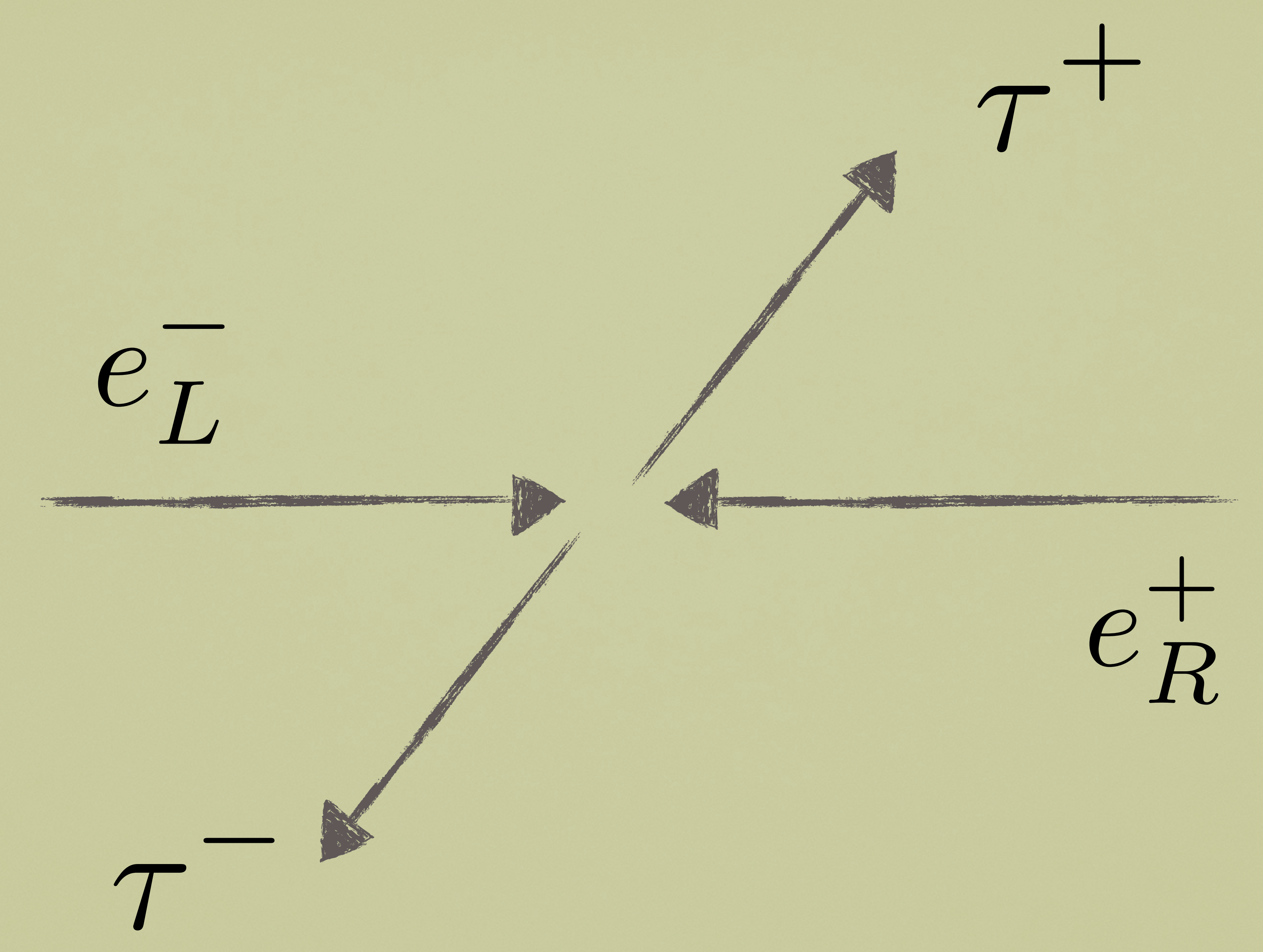
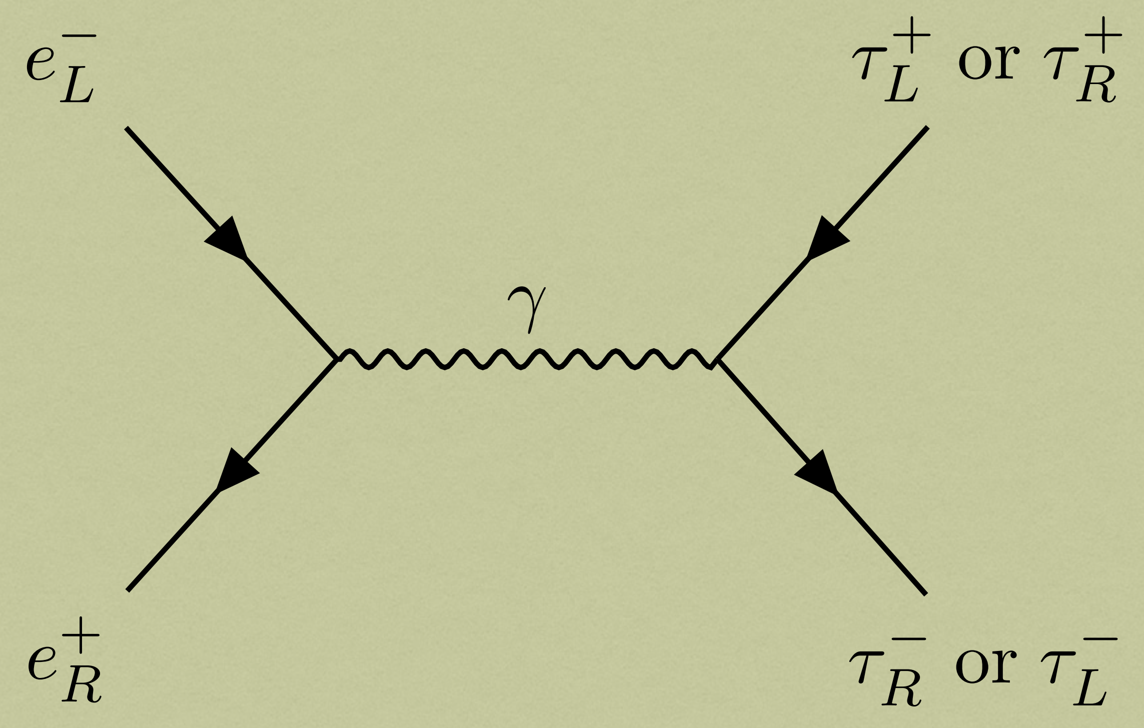


$$\zeta_1 |\tau_L^-\rangle |\tau_L^+\rangle + \zeta_2 |\tau_R^-\rangle |\tau_L^+\rangle + \zeta_3 |\tau_L^-\rangle |\tau_R^+\rangle + \zeta_4 |\tau_R^-\rangle |\tau_R^+\rangle \quad \left(\sum_i |\zeta_i|^2 = 1 \right)$$



$$\underbrace{\Big(1+\cos\Theta\Big)}_{\zeta_2 = D^{(1)}_{1,1}(\Theta)}|\tau_R^-\rangle\,|\tau_L^+\rangle + \underbrace{\Big(1-\cos\Theta\Big)}_{\zeta_3 = D^{(1)}_{1,-1}(\Theta)}|\tau_L^-\rangle\,|\tau_R^+\rangle$$

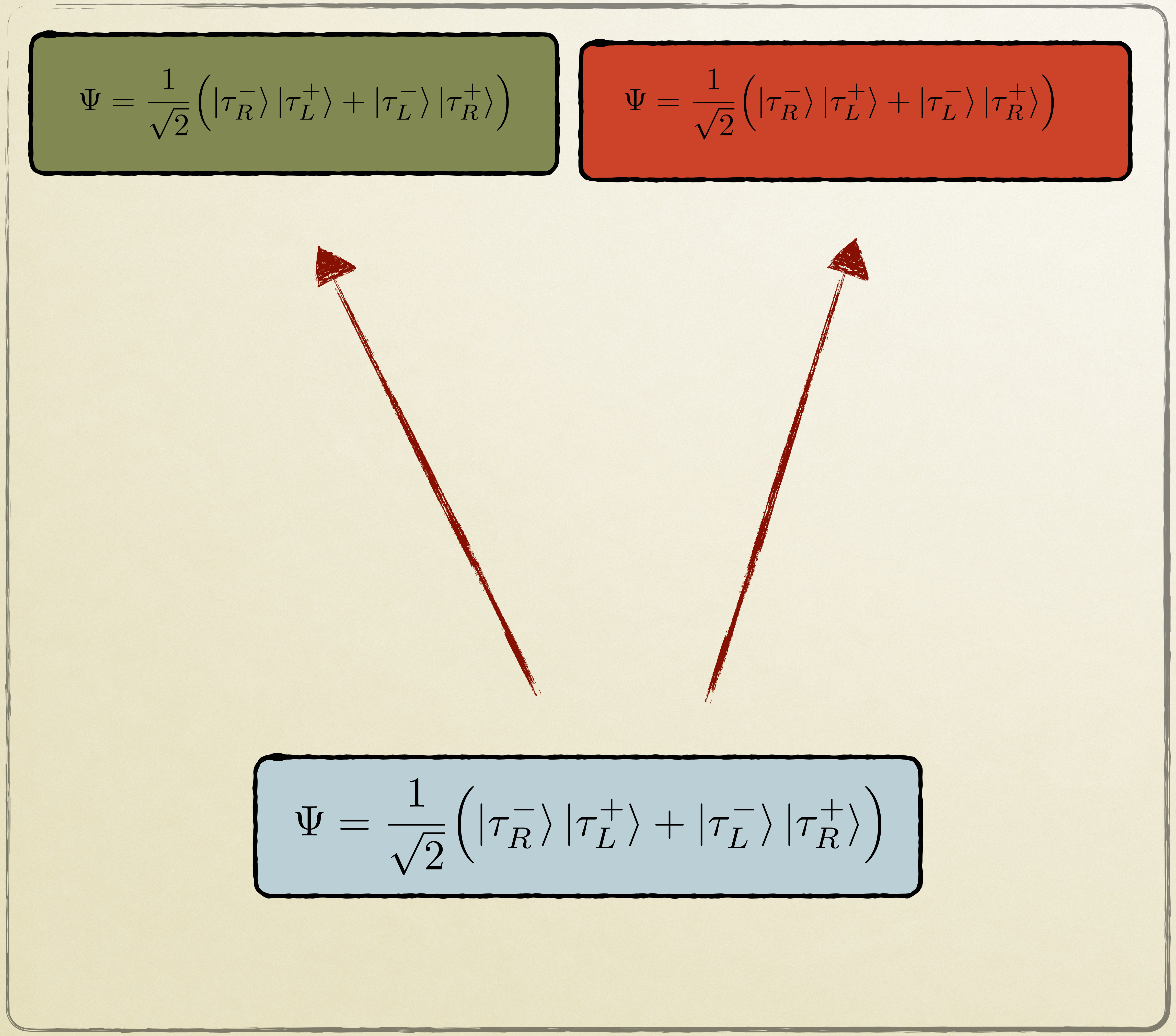
$J=\pm1 \quad J_z=\pm1 \quad (\Theta=0)$	$\lvert\tau_R^-\rangle\,\lvert\tau_L^+\rangle$	separable
$J=\pm1 \quad J_z=0 \quad (\Theta=\pi/2)$	$\frac{1}{\sqrt{2}}\Big(\lvert\tau_R^-\rangle\,\lvert\tau_L^+\rangle + \lvert\tau_L^-\rangle\,\lvert\tau_R^+\rangle\Big)$	entangled (Bell state)

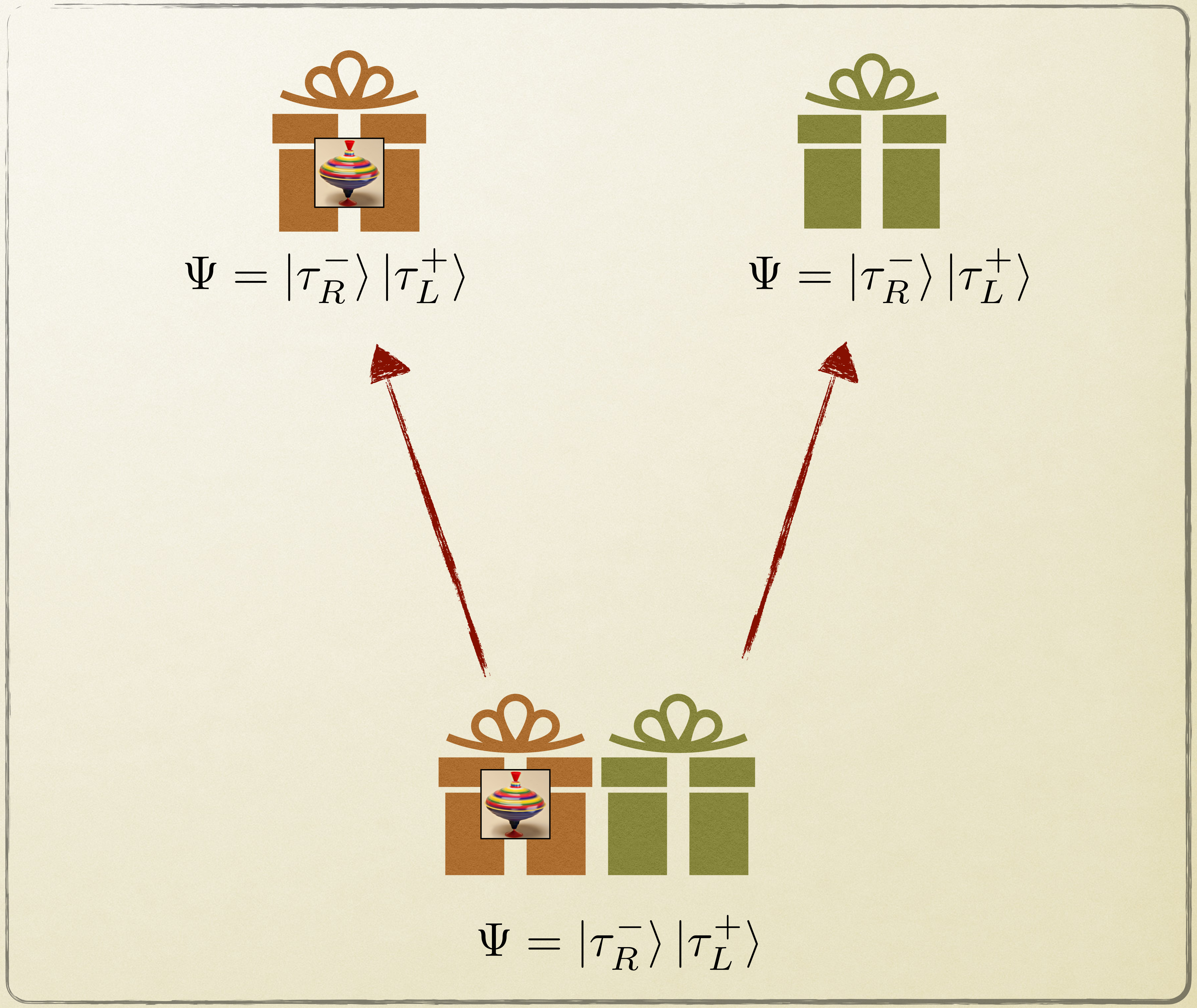
The quantum in quantum field theory

Entanglement

Bell inequality violation

Entanglement





Bell inequality violation

probabilities

$$\mathcal{P}(\uparrow_{\hat{n}_i};-)$$

spin of one tau-lepton up
in the direction n_j

$$\mathcal{P}(\uparrow_{\hat{n}_i};\downarrow_{\hat{n}_j})$$

spin of one tau-lepton up in the direction n_i
other tau-lepton spin down in the direction n_j

Bell inequality violation

stocastic variables

$$\mathcal{P}(\uparrow_{\hat{n}_1}; -) = \int d\lambda\, \eta(\lambda)\, p_\lambda(\uparrow_{\hat{n}_1}; -)$$

$$\int d\lambda\, \eta(\lambda) = 1$$

$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) = \int d\lambda\, \eta(\lambda)\, p_\lambda(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}})$$

$$p_\lambda(\uparrow_{\hat{n}}; \downarrow_{\hat{m}}) = p_\lambda(\uparrow_{\hat{n}}; -)\, p_\lambda(-; \downarrow_{\hat{m}})$$

Bell locality assumption

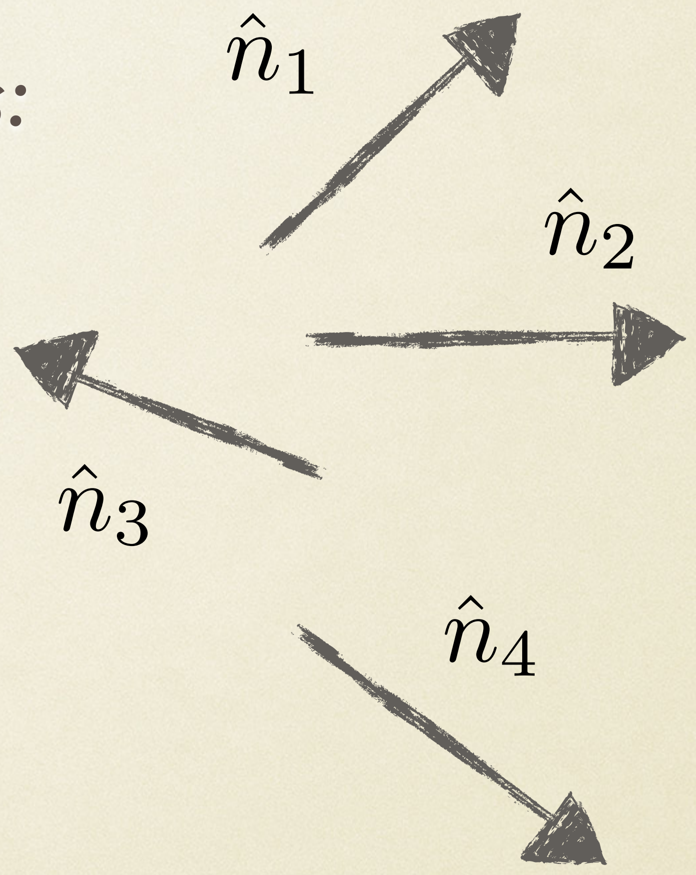
probability independence

Bell inequality violation

any four non-negative numbers

$$x_1 x_2 - x_1 x_4 + x_3 x_2 + x_3 x_4 \leq x_3 + x_2$$

four directions:



$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) \leq \mathcal{P}(\uparrow_{\hat{n}_3};-) + \mathcal{P}(-;\uparrow_{\hat{n}_2})$$

$$\Psi = \ |\tau_R^-\rangle\,|\tau_L^+\rangle \longrightarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i};\uparrow_{\hat{n}_j}) = \frac{1}{4}\,\langle\Psi|(1_{2\times2}+\hat{n}_i\cdot\vec{\sigma})\otimes(1_{2\times2}+\hat{n}_j\cdot\vec{\sigma})|\Psi\rangle = \frac{1}{4}\,(1-\hat{n}_i^z+\hat{n}_j^z-\hat{n}_i^z\hat{n}_j^z)$$

$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{-1}{\sqrt{2}}(\hat{z}+\hat{x}), \quad \hat{n}_3 = -\hat{x}, \quad \hat{n}_4 = \frac{1}{\sqrt{2}}(\hat{z}-\hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) = \frac{1}{2}$$

$$\leq \ \mathcal{P}(\uparrow_{\hat{n}_3};-) + \mathcal{P}(-;\uparrow_{\hat{n}_2}) = 1 - \frac{\sqrt{2}}{4}$$

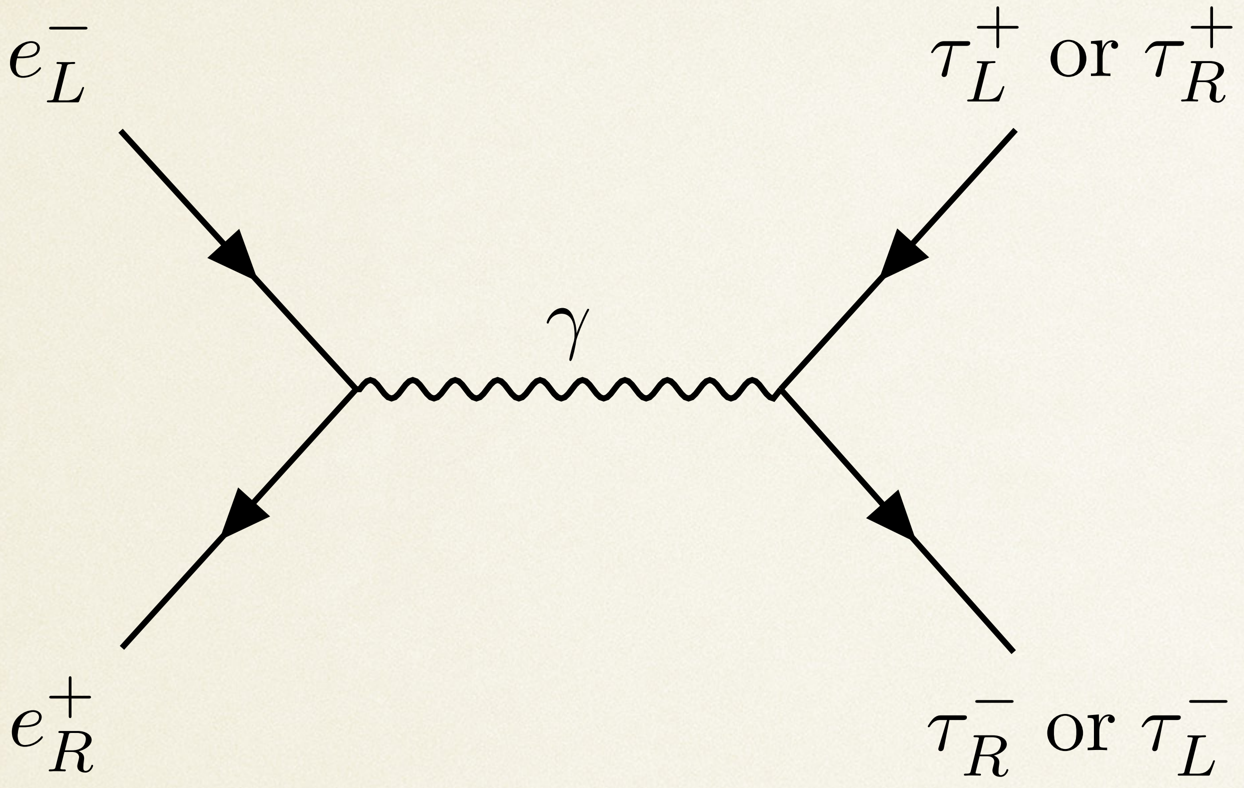
$$\Psi = \frac{1}{\sqrt{2}}\Big(|\tau_R^-\rangle\,|\tau_L^+\rangle + |\tau_L^-\rangle\,|\tau_R^+\rangle\Big) \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i};\uparrow_{\hat{n}_j}) = \frac{1}{4}\,\langle\Psi|(1_{2\times2} + \hat{n}_i\cdot\vec{\sigma})\otimes(1_{2\times2} + \hat{n}_j\cdot\vec{\sigma})|\Psi\rangle = \frac{1}{4}\,(1 + \hat{n}_i^x\hat{n}_j^x + \hat{n}_i^y\hat{n}_j^y - \hat{n}_i^z\hat{n}_j^z)$$

$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{-1}{\sqrt{2}}(\hat{z}+\hat{x}), \quad \hat{n}_3 = -\hat{x}, \quad \hat{n}_4 = \frac{1}{\sqrt{2}}(\hat{z}-\hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) = \frac{1}{2} + \frac{\sqrt{2}}{2}$$

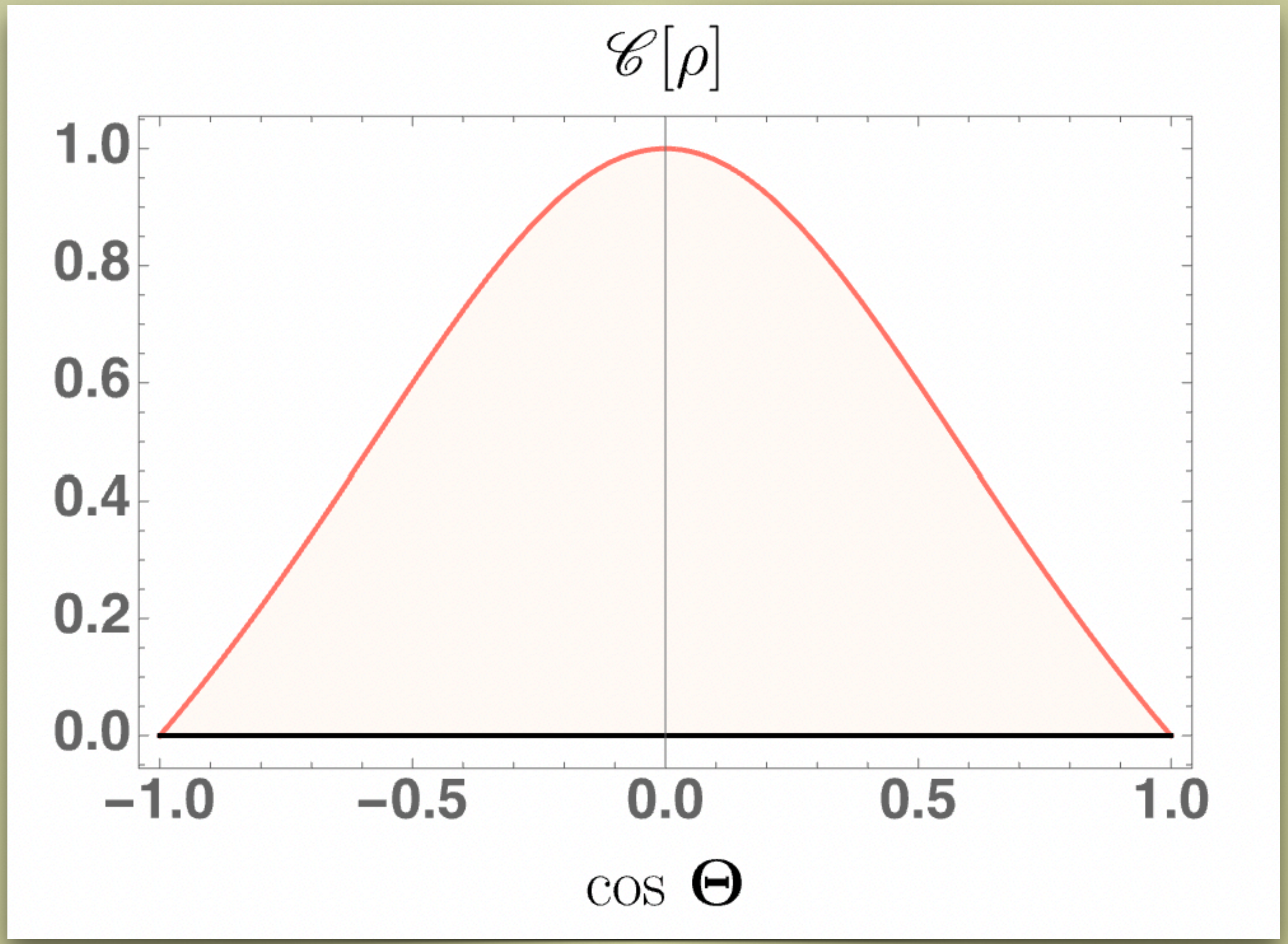
$$\nleq \quad \mathcal{P}(\uparrow_{\hat{n}_3};-) + \mathcal{P}(-;\uparrow_{\hat{n}_2}) = 1$$

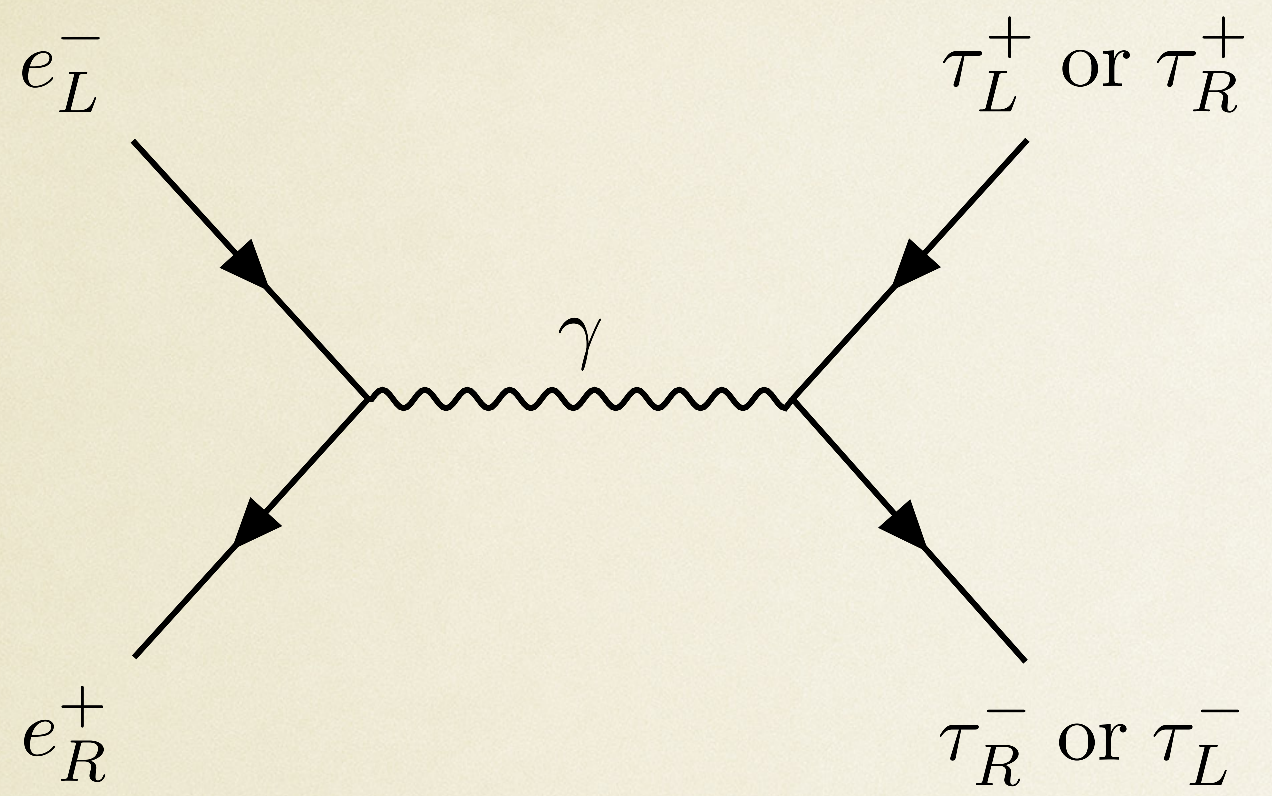


$$\Big(1+\cos\Theta\Big)|\tau_R^-\rangle\,|\tau_L^+\rangle + \Big(1-\cos\Theta\Big)|\tau_L^-\rangle\,|\tau_R^+\rangle$$

Concurrence

$$\mathcal{C}[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2\Theta}{1+\cos^2\Theta}$$





$$\rho = \frac{1}{4}\left[1_2 \otimes 1_2 + \sum_{i=1}^{3} B_i^+ (\sigma_i \otimes 1_2) + \sum_{i=1}^{3} B_j^- (1_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} C_{ij} (\sigma_i \otimes \sigma_j)\right]$$

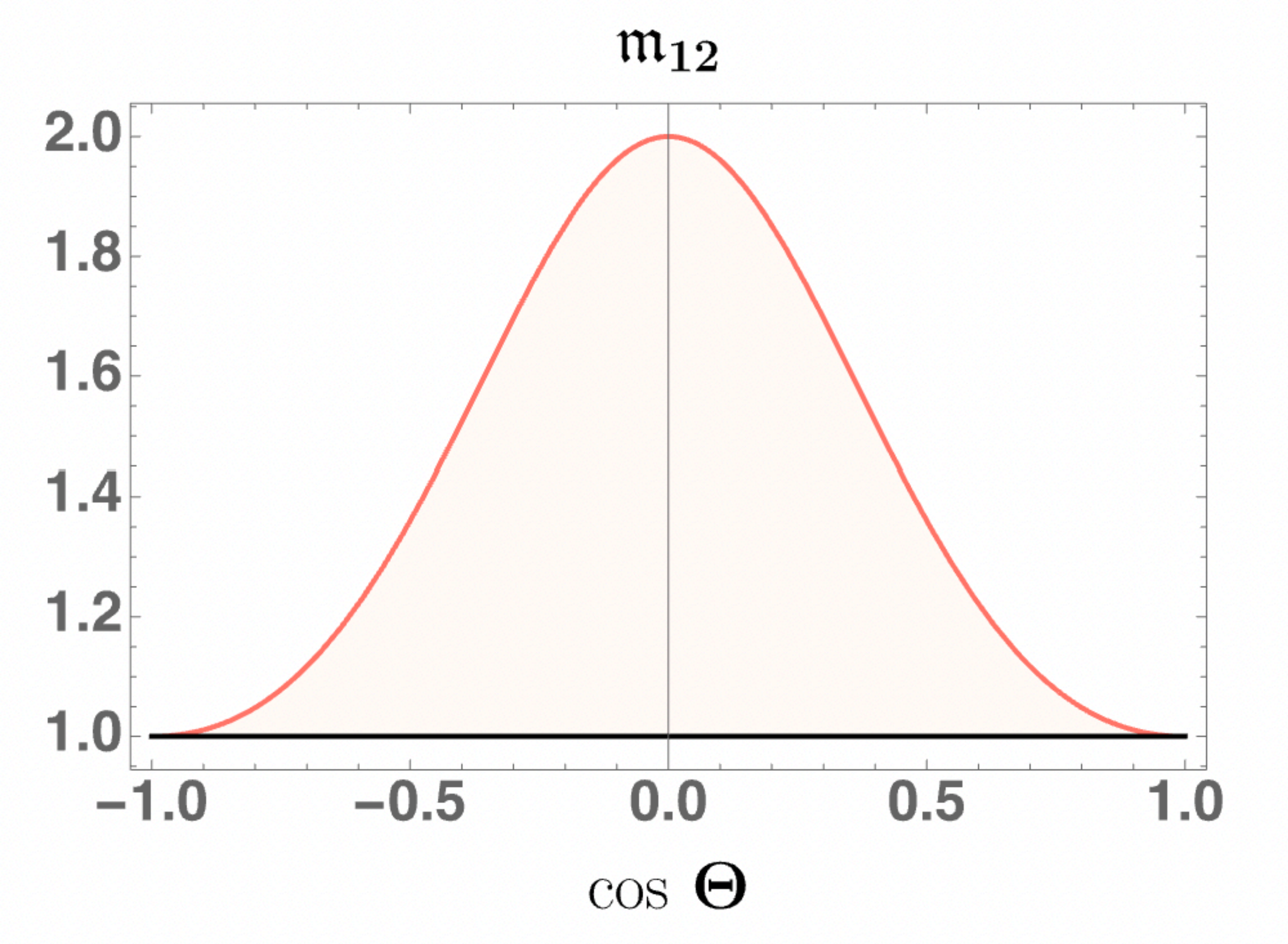
$$C_{ij} = \mathrm{Tr}\left[\sigma\left(\sigma_i \otimes \sigma_j\right)\right]$$

$$CC^T \qquad [m_1,\, m_2,\, m_3]$$

$$\left(1 + \cos\Theta\right) |\tau_R^-\rangle\, |\tau_L^+\rangle + \left(1 - \cos\Theta\right) |\tau_L^-\rangle\, |\tau_R^+\rangle$$

Horodecki condition $\quad \mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4\Theta}{(1 + \cos^2\Theta)^2}$$



Low-energy tests with photons and solid-state devices

A. Aspect, J. Dalibard and G. Rogers, Phys. Rev. Lett. 49 (1982) 5039

J.F. Clauser, M.A. Horne, Phys. Rev. D 10 (1974) 526

J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. 23 (1969) 880

G. Weihs, T. Jennewein, C. Simon, H. Weinfurther and A. Zeilinger, Phys. Rev. Lett. 81 (1998) 5039

W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Phys. Rev. Lett. 81 (1998) 3563

M. Ansmann et al, Nature 461 (2009) 504

PHYSICAL REVIEW LETTERS
Volume 81 | 7 December 1998 | Number 23

Violation of Bell's Inequality under Strict Einstein Locality Conditions

Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
(Received 6 August 1998)

We observe strong violation of Bell's inequality in an Einstein-Podolsky-Rosen-type experiment with independent observers. Our experiment definitely implements the ideas behind the well-known work by Aspect *et al*. We for the first time full... the condition of locality, a central assumption in the derivation of Bell... like separation of the observations is achieved ...ations, by ultrafast and random setting of the ...
[S0031-9007(98)07901-0]

Volume 81, Number 17 | PHYSICAL REVIEW LETTERS | 26 October 1998

Violation of Bell Inequalities by Photons More Than 10 km Apart

W. Tittel,* J. Brendel, H. Zbinden, and N. Gisin
Group of Applied Physics, University of Geneva, 20, Rue de l'Ecole de Médecine, CH-1211 Geneva 4, Switzerland
(Received 10 June 1998)

A Franson-type test of Bell inequalities by photons 10.9 km apart is presented. Energy-time entangled photon pairs are measured using two-channel analyzers, leading to a violation of the inequalities by 16 standard deviations without subtracting accidental coincidences. Subtracting them, a two-photon interference visibility of 95.5% is observed, demonstrating that distances up to 10 km have no significant effect on entanglement. This sets quantum cryptography with photon pairs as a practical competitor to the schemes based on weak pulses. [S0031-9007(98)07478-X]

...e 47, Number 7 | PHYSICAL REVIEW LETTERS | 17 August 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

...ear polarization correlation of the photons emi... ...m. A high-efficiency source provided an i... ...orm new tests. Our results, in excellent ... ...tions, strongly violate the generalized Be... ...realistic local theories. No significant c... ...rizer separations of up to 6.5 m.

Article | Open access | Published: 10 May 2023

Loophole-free Bell inequality violation with superconducting circuits

Simon Storz, Josua Schär, Anatoly Kulikov, Paul Magnard, Philipp Kurpiers, Janis Lütolf, Theo Walter, Adrian Copetudo, Kevin Reuer, Abdulkadir Akin, Jean-Claude Besse, Mihai Gabureac, Graham J. Norris, Andrés Rosario, Ferran Martin, José Martinez, Waldimar Amaya, Morgan W. Mitchell, Carlos Abellan, Jean-Daniel Bancal, Nicolas Sangouard, Baptiste Royer, Alexandre Blais & Andreas Wallraff

Nature **617**, 265–270 (2023) | Cite this article

Selected for a Viewpoint in *Physics*
PRL **115**, 250402 (2015) | PHYSICAL REVIEW LETTERS | week ending 18 DECEMBER 2015

Strong Loophole-Free Test of Local Realism*

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[6]*Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA*
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[8]*ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*
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(Received 10 November 2015; published 16 December 2015)

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7}. We therefore reject the hypothesis that local realism governs our experiment.

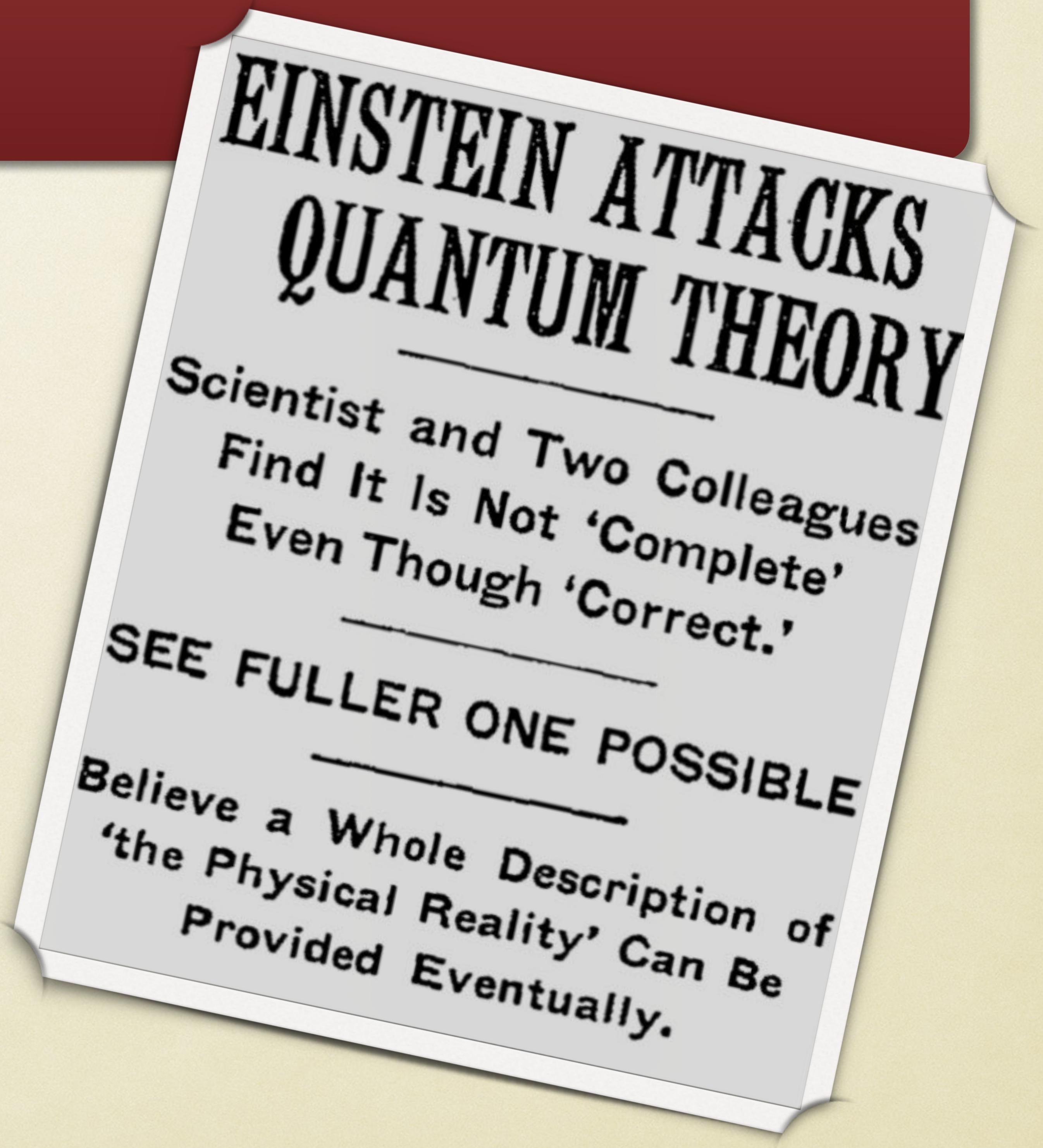
Local, deterministic models satisfy Bell inequality
quantum mechanics does not

Entanglement is just a measurement,
Bell inequality violation is a true discovery

both can be studied at colliders

- high-energy regime
- in the presence of weak and strong interactions
- qubits and qutrits

Where have we already seen entanglement or Bell inequality violation at high energies?



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

New York Times Headline
May 4th, 1935

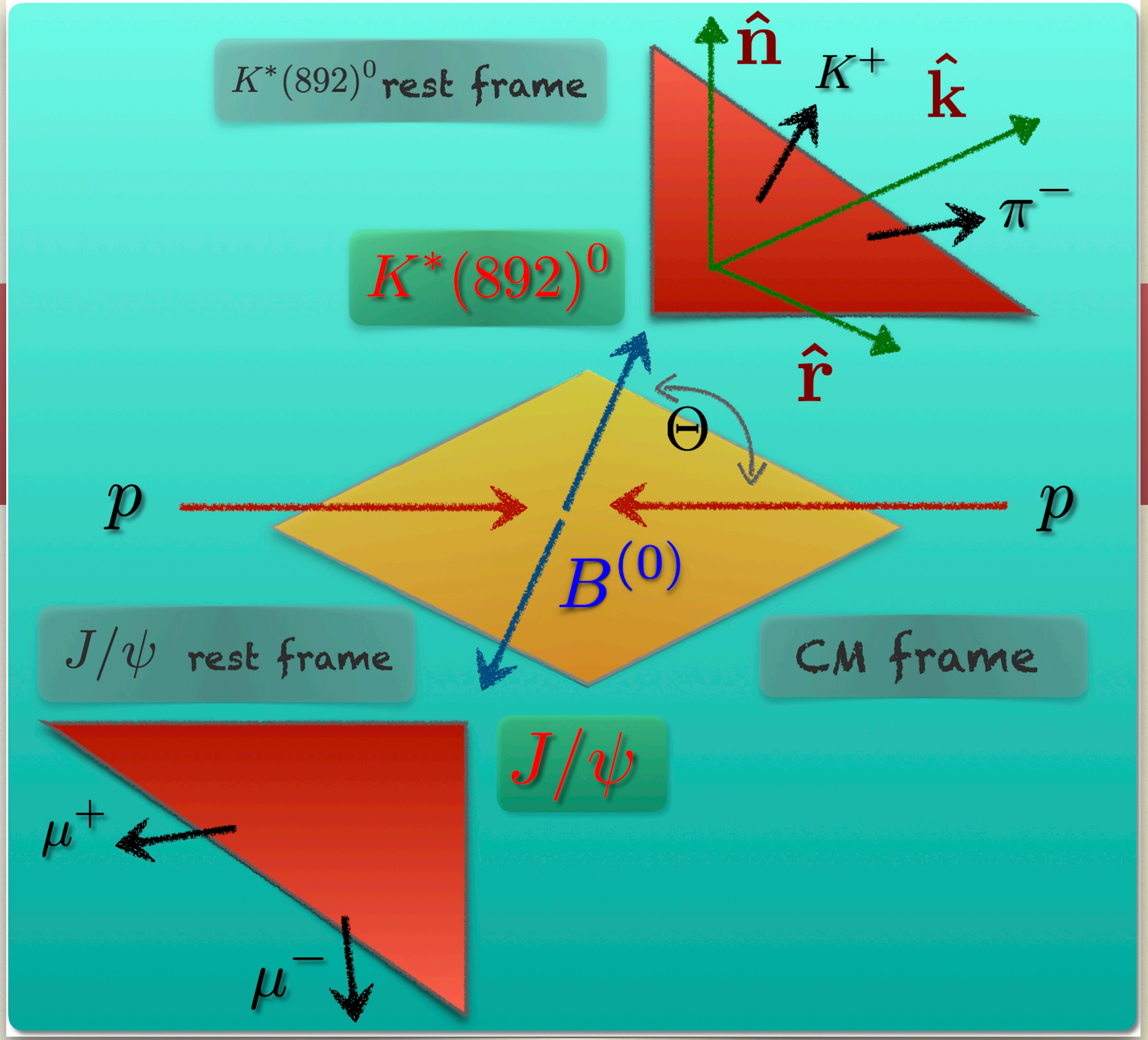
1

B-meson decays

	$\mathscr{E}$	$\mathcal{I}_3$
• $B^0 \to J/\psi\, K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015
• $B^0 \to \phi\, K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$
• $B^0 \to \rho\, K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$
• $B_s \to \phi\,\phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$
• $B_s \to J/\psi\, \phi$ [23]	0.731 ± 0.032	2.462 ± 0.080

entanglement

Bell inequality



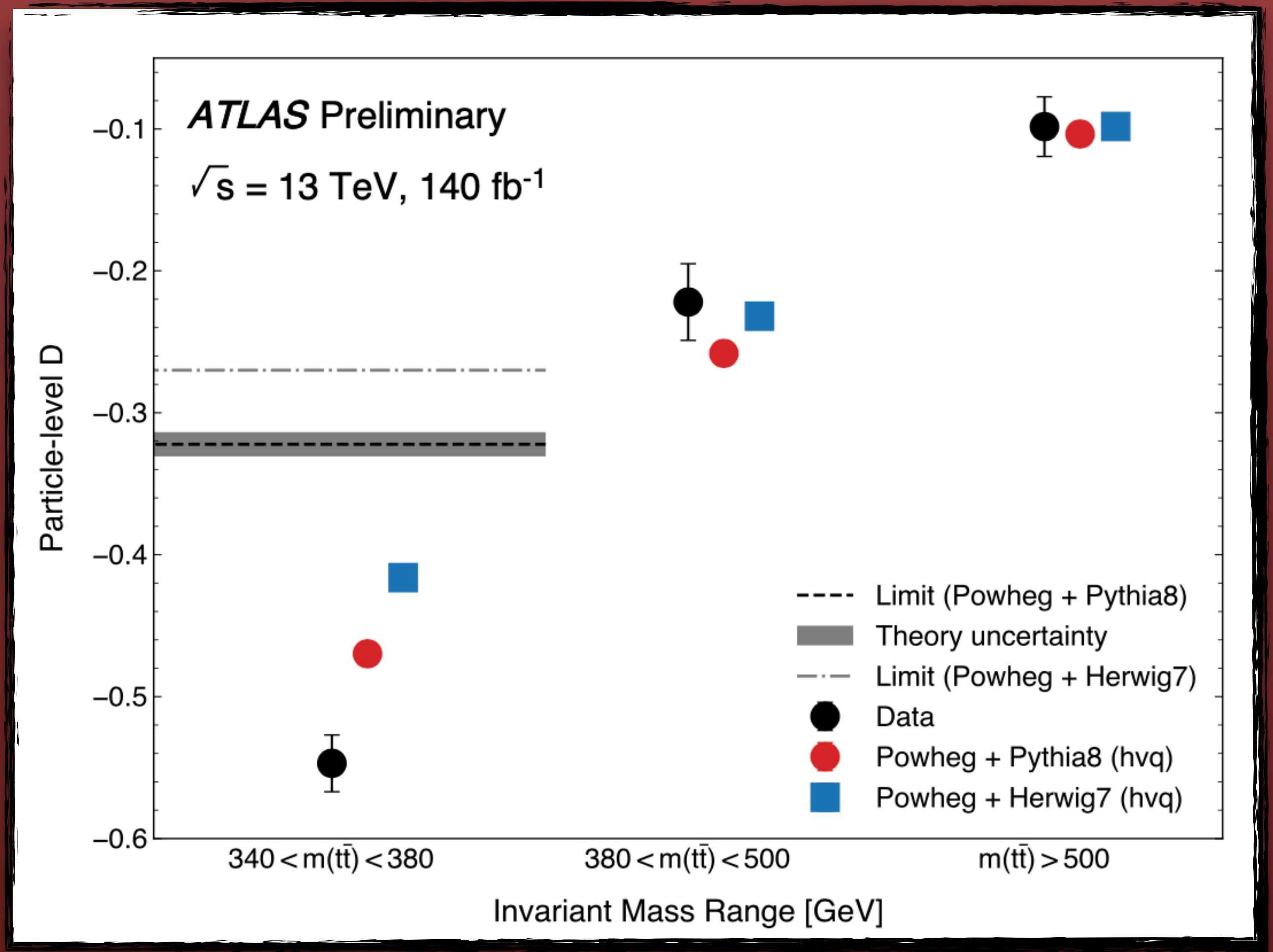
MF, R. Floreanini, E. Gabrielli and L. Marzola, Phys. Rev D 109 (2024) 3, L031104

Pairs of top quarks

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907

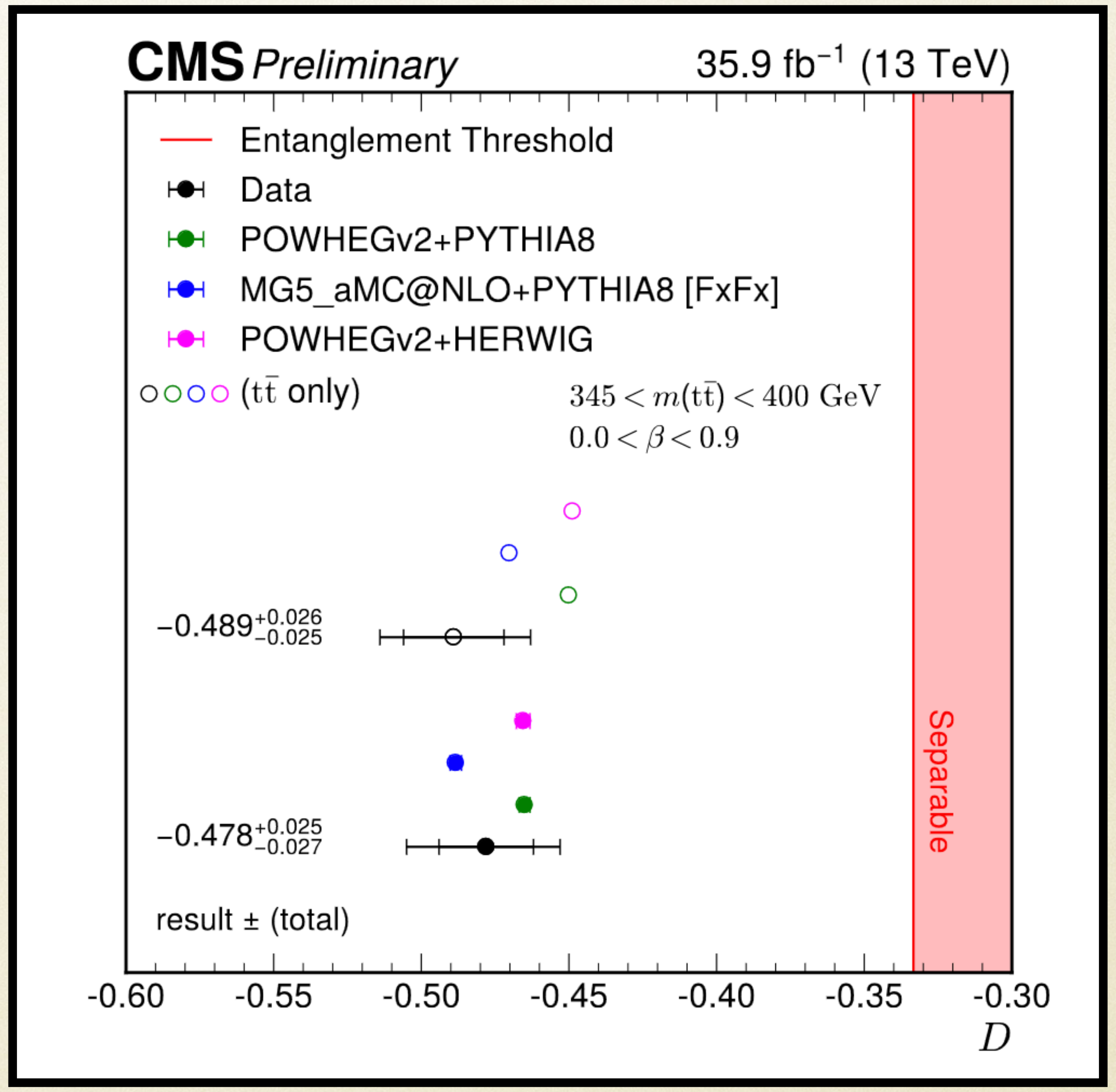
$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\Big(1 - D\cos\phi\Big)$$



$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, arXiv:2311.07288



$$D = -0.478^{+0.025}_{-0.027}$$

CMS Collaboration, CMS-TOP-23-001

3

Charmonium spin-0 states

$$\boxed{\eta_c \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \chi_c^0 \to \Lambda + \bar{\Lambda}}$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},\tfrac{1}{2}\rangle$$

Concurrence $\mathscr{C} = 1$

Horodecki condition $\mathfrak{m}_{12} = 2$

N. A. Tornqvist, *Suggestion for Einstein-podolsky-rosen Experiments Using Reactions Like $e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$*, Found. Phys. **11** (1981) 171–177.

N. A. Tornqvist, *The Decay $J/\psi \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ as an Einstein-Podolsky-Rosen Experiment*, Phys. Lett. A **117** (1986) 1–4.

S. P. Baranov, *Bell's inequali decays $\eta_c \to \Lambda\bar{\Lambda}$, $\chi_c \to \Lambda\bar{\Lambda}$ an* Phys. G **35** (2008) 075002.

$$\boxed{\chi_c^0 \to \phi + \phi}$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

Entropy $\mathscr{E}[\rho] = 0.531 \pm 0.0021$ (255σ)

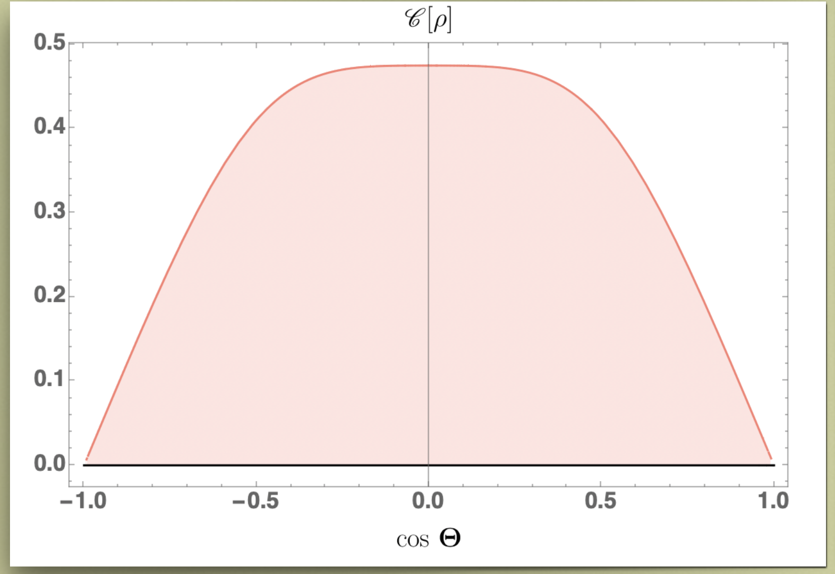
Bell operator $\mathrm{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.2961 \pm 0.0165$ (18σ)

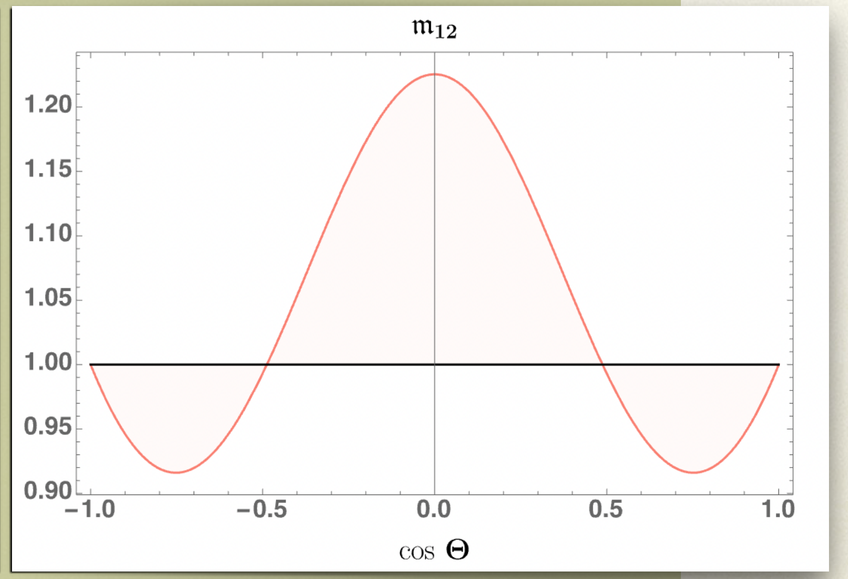
BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$*, *JHEP* **05** (2023) 069, [arXiv:2301.12922].

Charmonium spin-1 states

$$J/\psi \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}$$

$$\begin{aligned}
|\psi_\uparrow\rangle &\propto w_{\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \\
|\psi_\downarrow\rangle &\propto w_{-\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \\
|\psi_0\rangle &\propto w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,,
\end{aligned}$$





Concurrence $\quad \mathscr{C} = 0.475 \pm 0.0039 \quad (122\sigma)$

Horodecki condition $\quad \mathfrak{m}_{12} = 1.225 \pm 0.004 \quad (56\sigma)$

BESIII Collaboration, M. Ablikim et al., *Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ-$\bar{\Lambda}$ Pairs*, *Phys. Rev. Lett.* **129** (2022), no. 13 131801, [arXiv:2204.11058].

Bell inequality violation

decay	$\mathfrak{m}_{12}$	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

quantum tomography at future colliders

recent studies

MF, R. Floreanini, E. Gabrielli and L. Marzola, Eur. Phys.J. C 83 (2023) 823 $W^+W^-,\ ZZ$

M.M. Altakach, P. Lamba, F. Maltoni, K. Mawatari, K. Sakurai, Phys. Rev. D 107 (2023) 093002 $H \to \tau^+\tau^-$

R. Aoude, E. Madge, F. Maltoni, and L. Mantani, JHEP 12 (2023) 017 $e^+e^- \to W^+W^-,\ ZZ$

Y. Wu, R. Ding, S. Qian, AM Levin, A. Ruzi and Q. Li, arXiv:2408.05429 $H \to ZZ$ (muon collider)

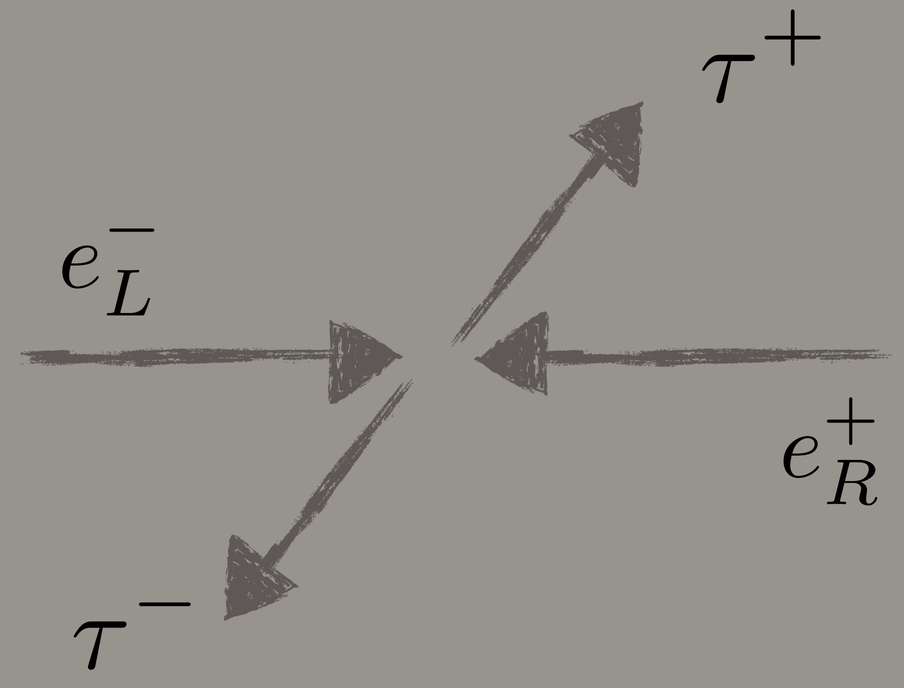
F. Maltoni, C. Severi, S. Tentori and E. Vryonidou, arXiv:2404.08049 $t\bar{t}$

and a review article

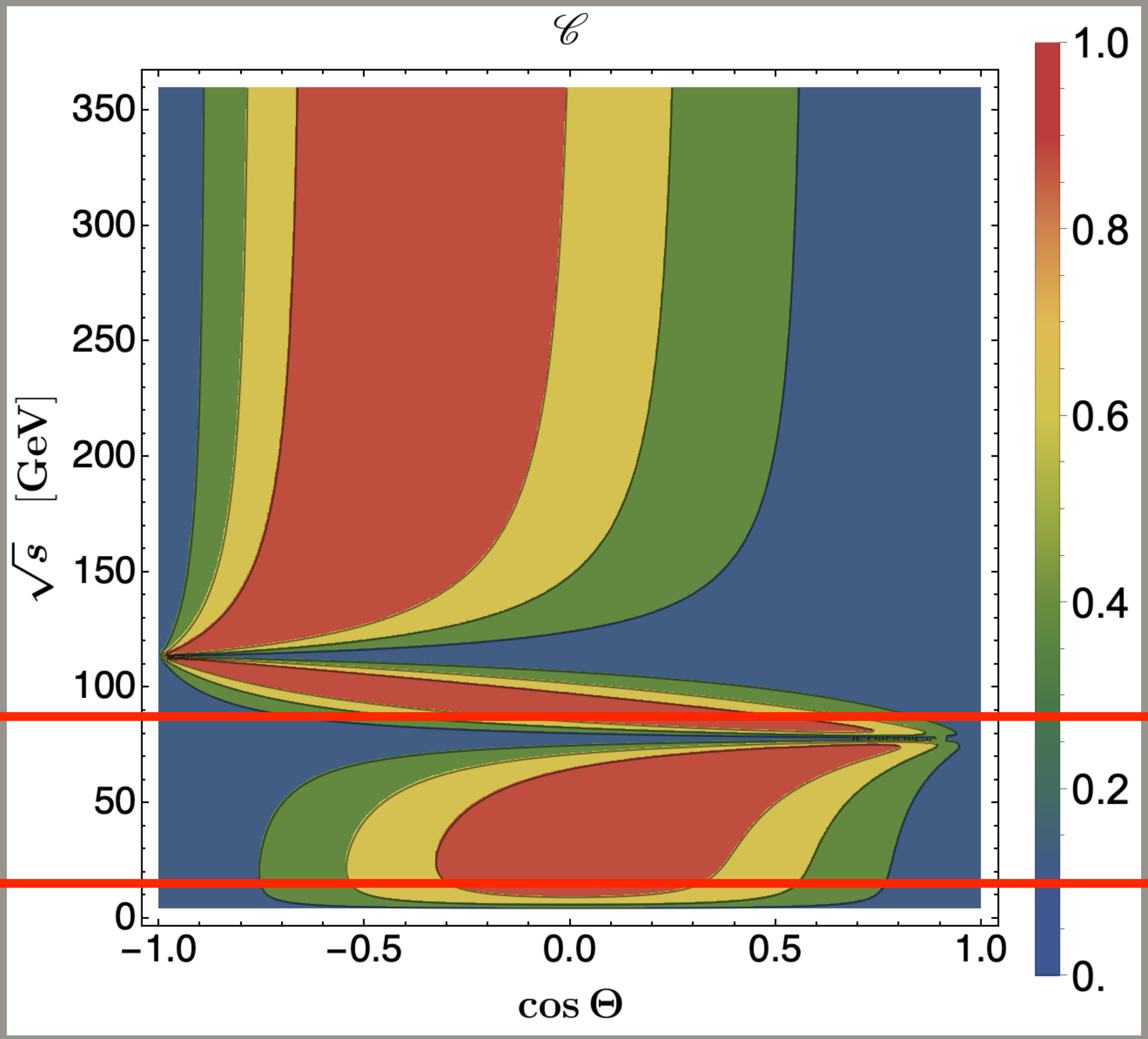
A. Barr, MF, R. Floreanini, E. Gabrielli and L. Marzola, Progress in Particle and Nuclear Physics 139 (2024) 104134

Tau-lepton

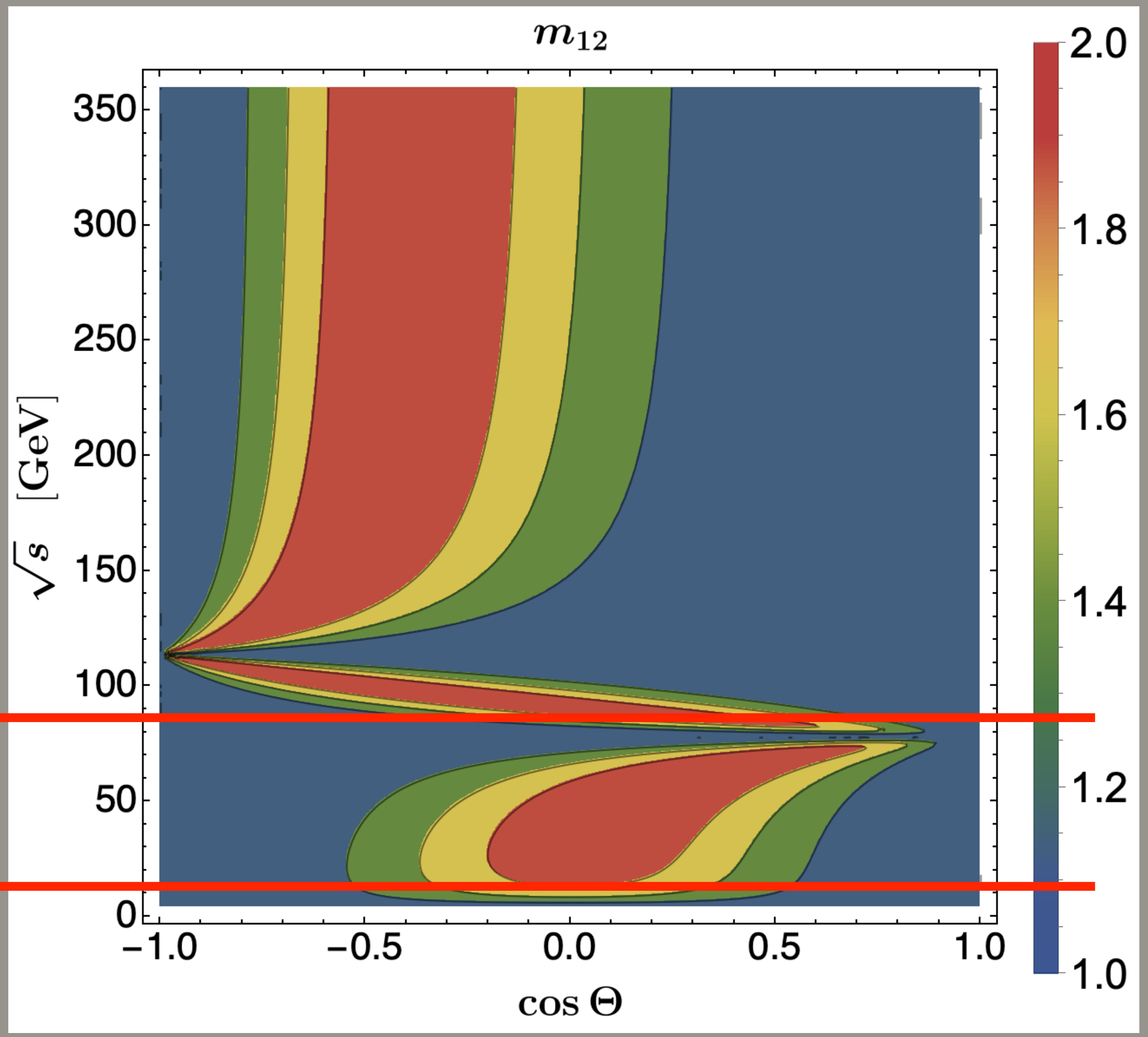
- heaviest lepton
- non-vanishing impact parameter
- simple hadronic decays



Concurrence



Horodecki condition



FCC-ee

Belle II

$$e^+e^- \to Z,\gamma \to \tau^+\tau^- \to \pi^+\pi^-\nu_\tau\,\bar{\nu}_\tau$$

$$p^\mu_{\tau^+} + p^\mu_{\tau^-} = p^\mu_{e^+e^-}$$

$$\begin{array}{ccc} (p_{\tau^+} - p_{\pi^+})^2 = m_\nu^2 = 0 & \text{and} & (p_{\tau^-} - p_{\pi^-})^2 = m_\nu^2 = 0 \\ p^2_{\tau^+} = m^2_\tau & \text{and} & p^2_{\tau^-} = m^2_\tau\,. \end{array}$$

$$\mathbf{d}_{min} = \mathbf{d} + \frac{[(\mathbf{d}\cdot\mathbf{n}_+)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_-]\,\mathbf{n}_- + [(\mathbf{d}\cdot\mathbf{n}_-)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_+]\,\mathbf{n}_+}{1-(\mathbf{n}_-\cdot\mathbf{n}_+)^2}\,.$$

statistical errors

FCC Collaboration, A. Abada et al., *FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2*, *Eur. Phys. J. ST* **228** (2019), no. 2 261–623.

We model the detector resolution with the following uncertainties:

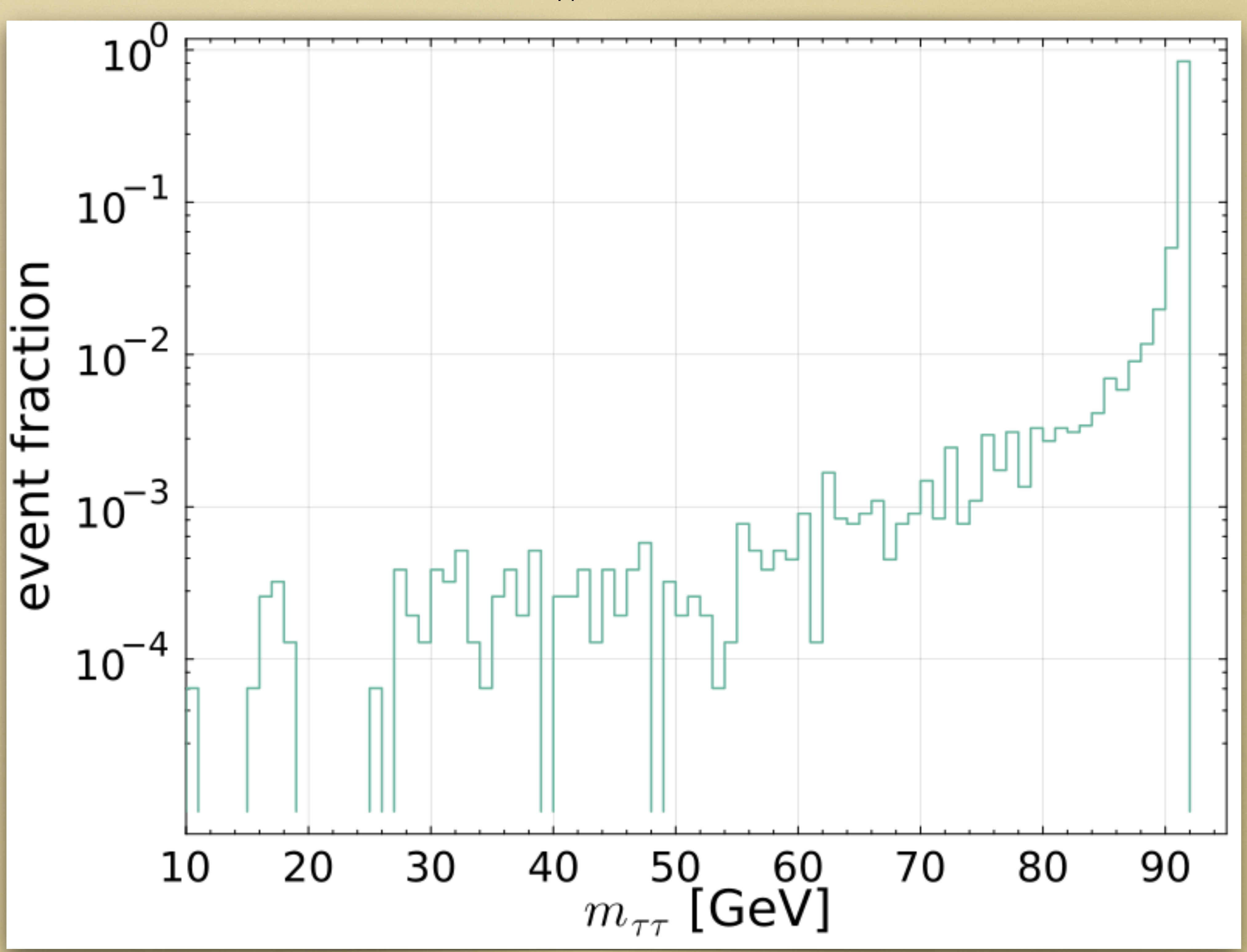
$$\frac{\sigma_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{\text{GeV}} \quad \text{and} \quad \sigma_{\theta,\phi} = 0.1 \times 10^{-3}\,\text{rad}$$

for the tracks proper and

$$\sigma_b = 3\,\mu\text{m} \oplus \frac{15\,\mu\text{m}}{\sin^{2/3}\Theta}\frac{\text{GeV}}{p_T}$$

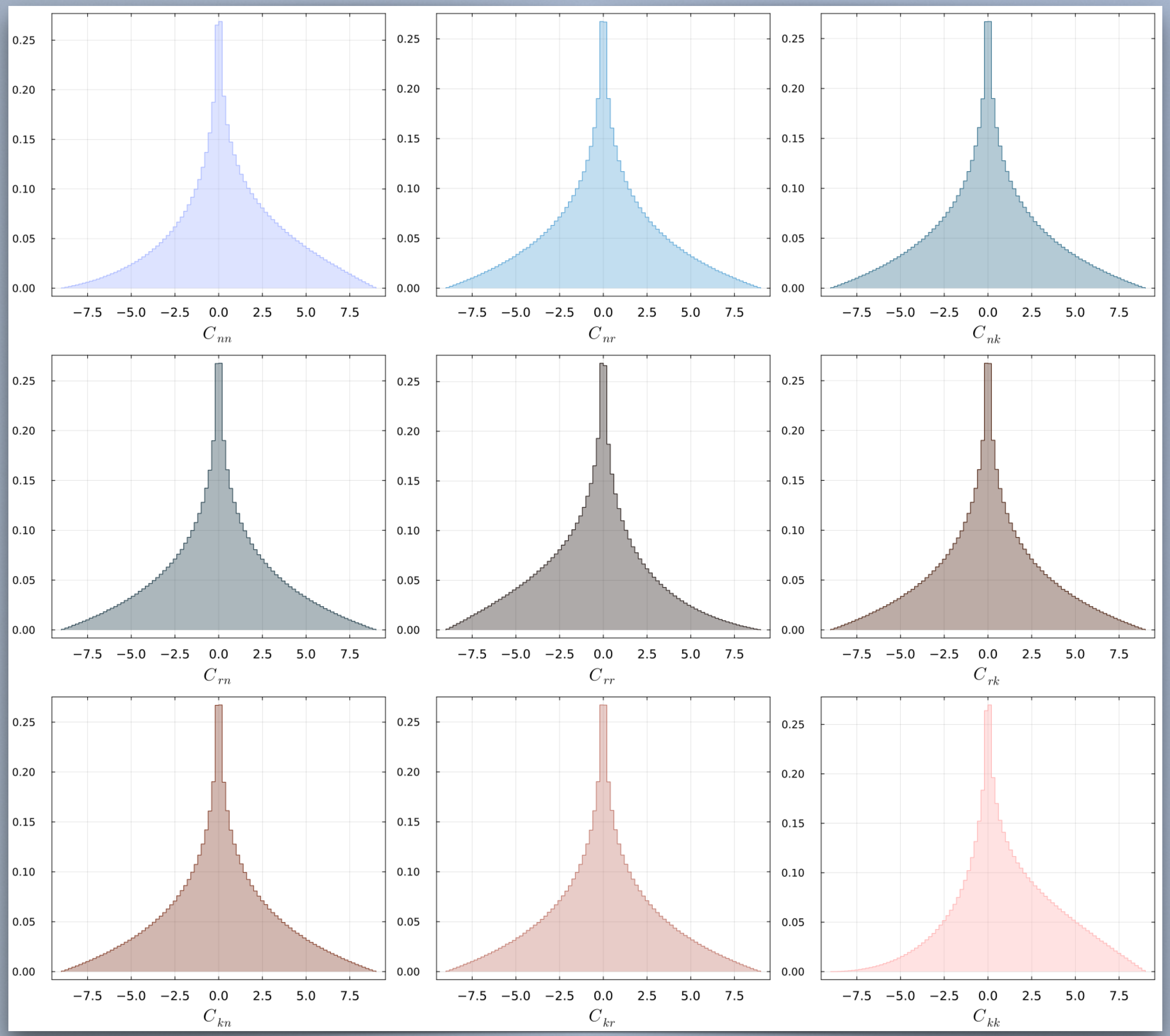
more statistical errors

C. Bierlich et al., *A comprehensive guide to the physics and usage of PYTHIA 8.3*, *SciPost Phys. Codeb.* **2022** (2022) 8, [arXiv:2203.11601].

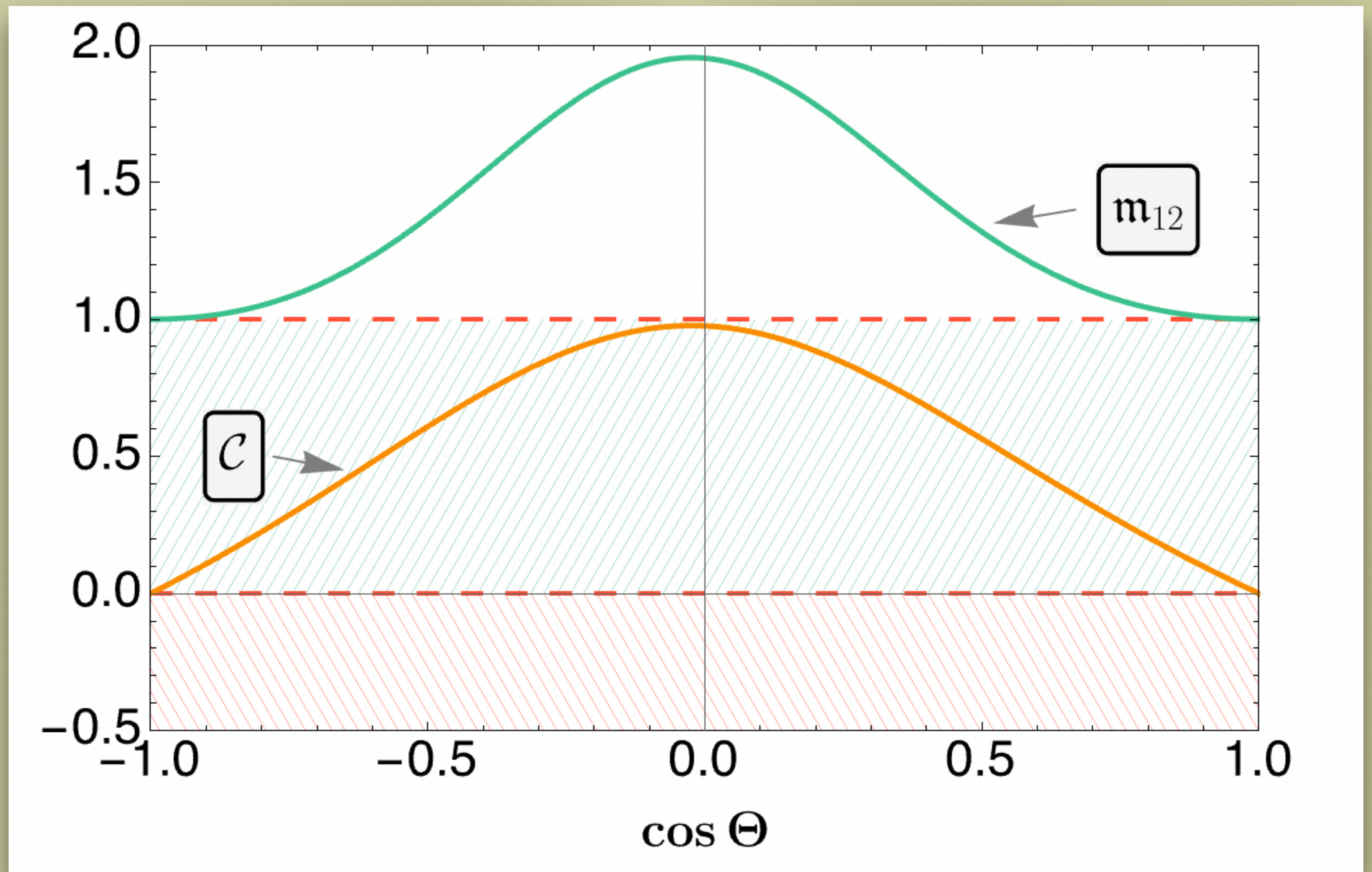


plus systematic errors

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_i^+ \, d\cos\theta_j^-} = \frac{1}{4}\left(1 + \mathrm{C}_{ij}\,\cos\theta_i^+\,\cos\theta_j^-\right)$$



$$\rho = \frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1} + \sum_i \mathrm{B}_i^+\,(\sigma_i\otimes\mathbb{1}) + \sum_j \mathrm{B}_j^-\,(\mathbb{1}\otimes\sigma_j) + \sum_{i,j}\mathrm{C}_{ij}\,(\sigma_i\otimes\sigma_j)\right]$$



$$\mathscr{C} = 0.4805 \pm 0.0063|_{\text{stat}} \pm 0.0012|_{\text{syst}}\,,$$

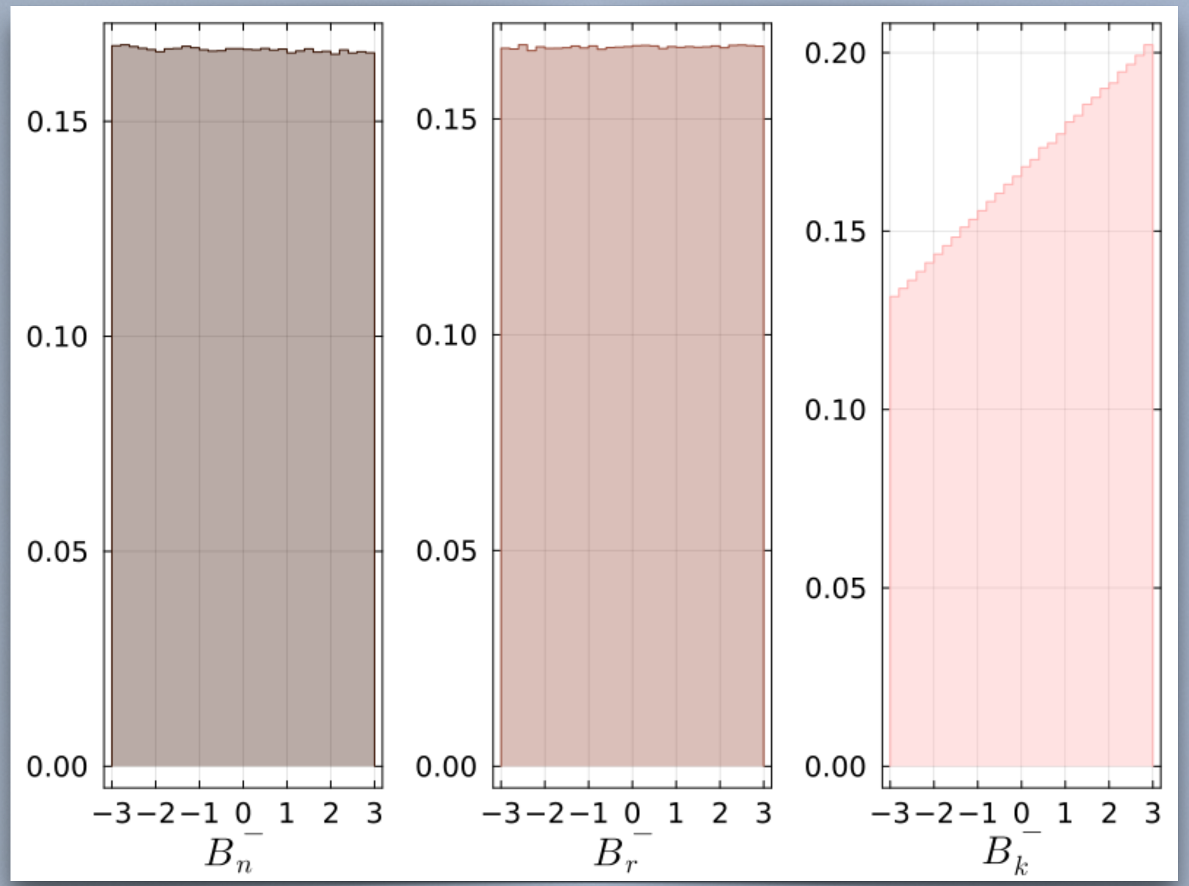
$$\mathfrak{m}_{12} = 1.239 \pm 0.017|_{\text{stat}} \pm 0.008|_{\text{syst}}\,,$$

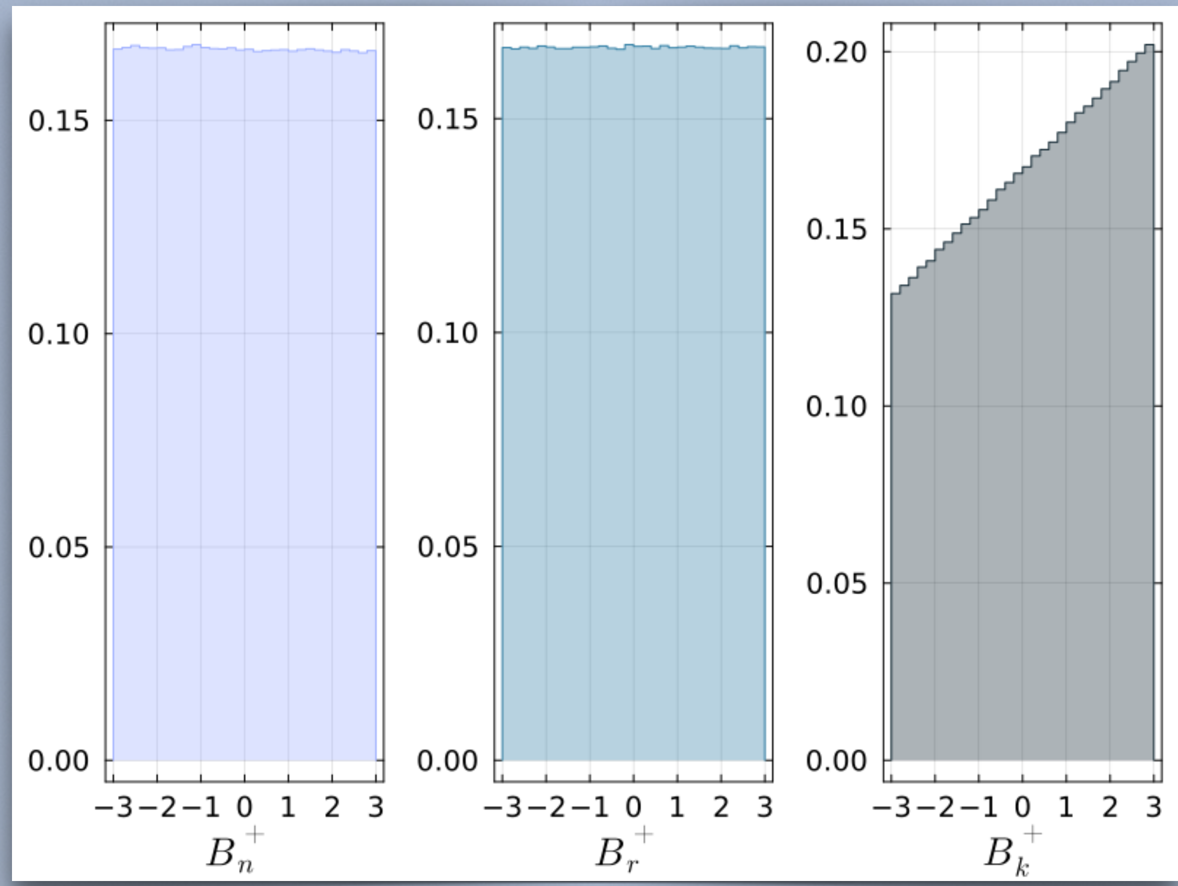
MF and L. Marzola, arXiv:2405.09201

J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079, [arXiv:1405.0301].

K. Hagiwara, T. Li, K. Mawatari, and J. Nakamura, *TauDecay: a library to simulate polarized tau decays via FeynRules and MadGraph5*, *Eur. Phys. J. C* **73** (2013) 2489, [arXiv:1212.6247].

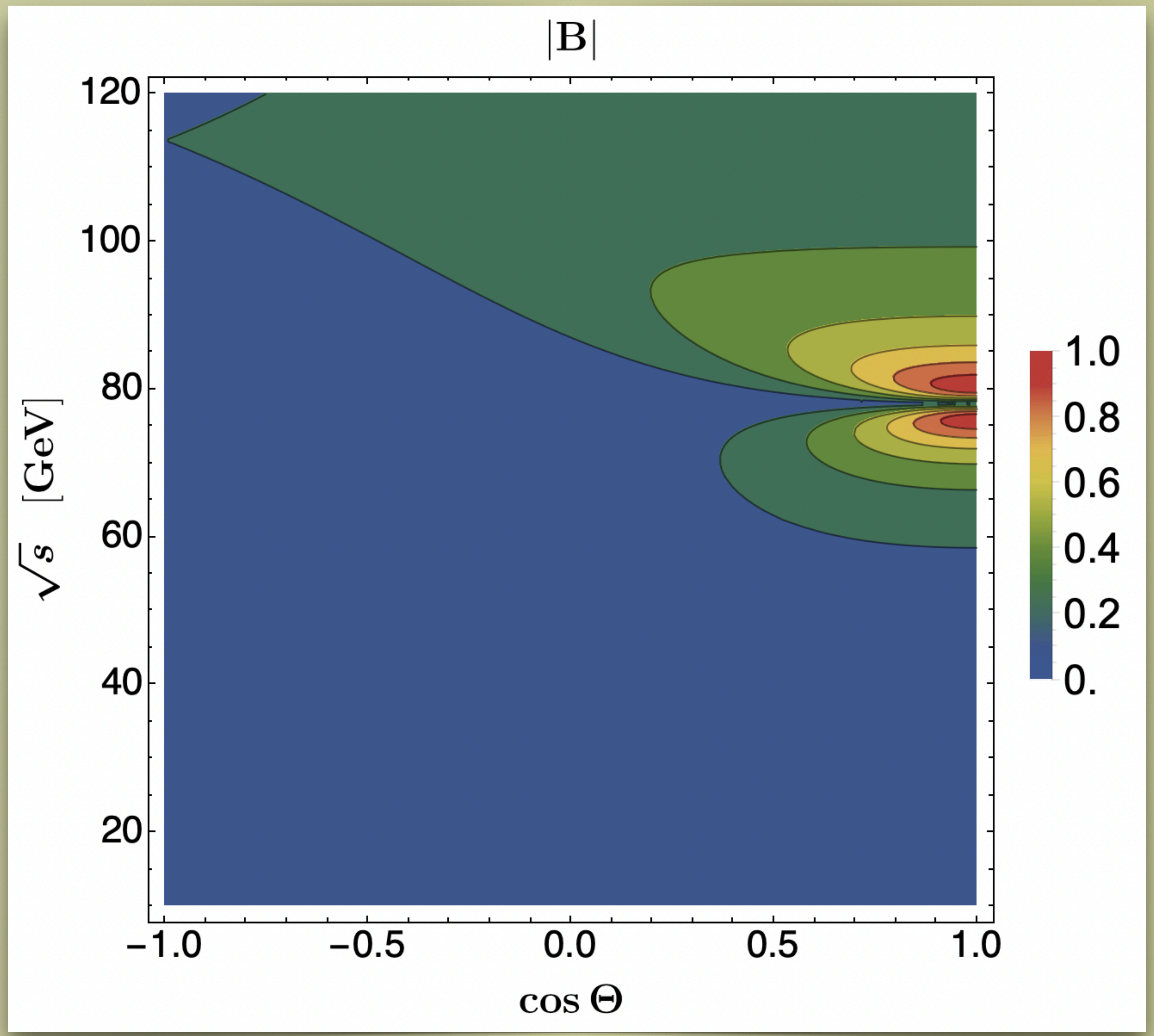
$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^{\pm}} = \frac{1}{2}\left(1 \mp \mathrm{B}_i^{\pm}\cos\theta_i^{\pm}\right)$$





$$P_\tau(\cos\Theta) = \frac{\mathcal{A}_\tau\left(1+\cos^2\Theta\right) + 2\cos\Theta\,\mathcal{A}_e}{1+\cos^2\Theta + 2\cos\Theta\,\mathcal{A}_\tau\mathcal{A}_e},$$

$$\mathcal{A} = \mathcal{A}_e = \mathcal{A}_\tau = \frac{2\left(1-4\sin^2\theta_W\right)}{1+\left(1-4\sin^2\theta_W\right)^2},$$



$$\langle P\rangle_\tau = \frac{1}{2}\left(\mathrm{B}_k^+ + \mathrm{B}_k^-\right) = 0.2203 \pm 0.0044|_{\mathrm{stat}} \pm 0.0008|_{\mathrm{syst}},$$

$$\sin^2\theta_W = 0.2223 \pm 0.0006|_{\mathrm{stat}} \pm 0.0001|_{\mathrm{syst}},$$

MF and L. Marzola, arXiv:2405.09201

putting quantum observables to work

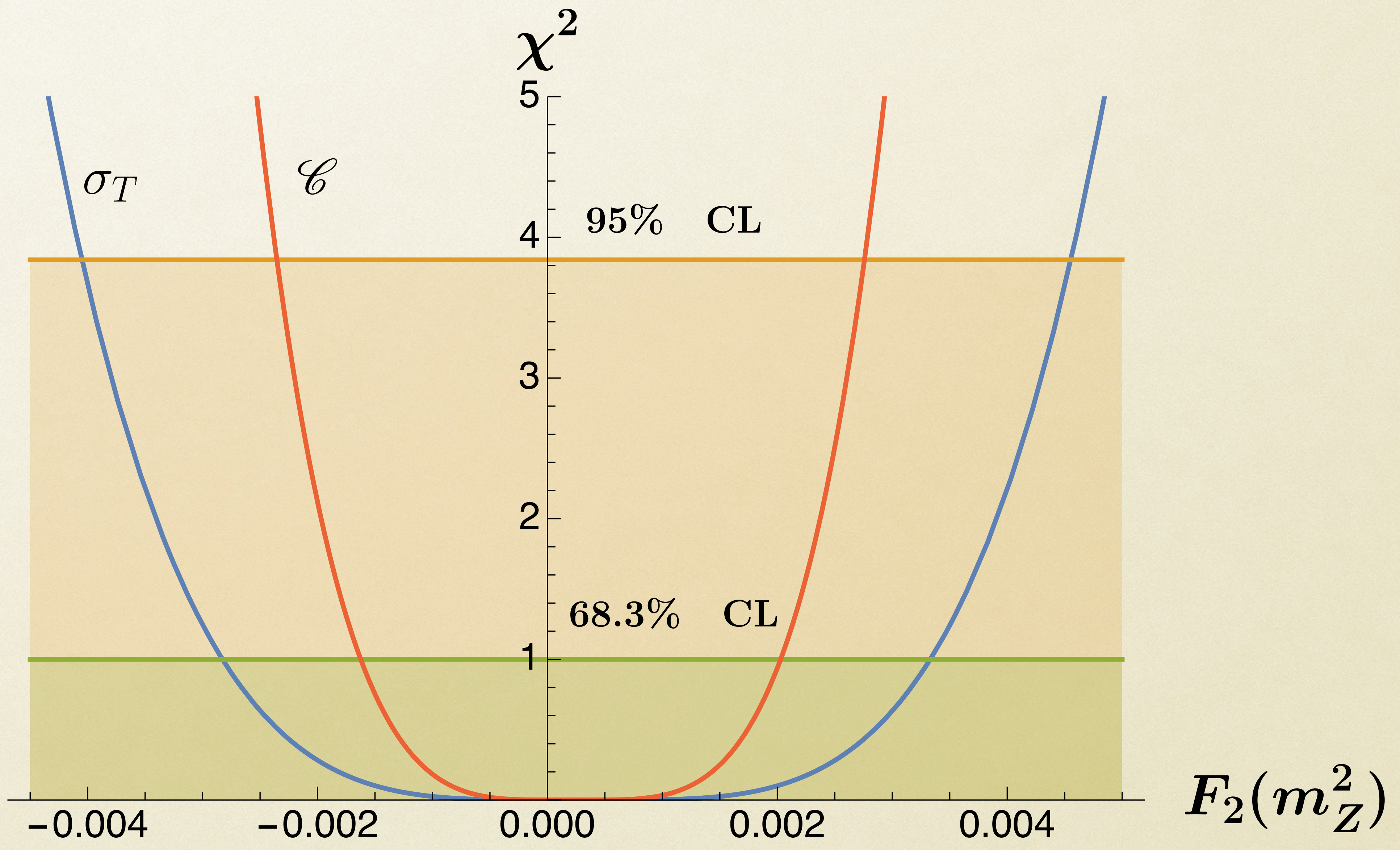
$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\,\Gamma^{\mu}(q^2)\,\tau\,Z_\mu(q) =$$

$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\left[\gamma^\mu F_1^V(q^2) + \gamma^\mu\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau}F_3(q^2)\right]\tau\,Z_\mu(q)$$

$$\mathscr{C}_{odd} = \sum_{i<j}\left|\,\mathrm{C}_{ij} - \mathrm{C}_{ji}\,\right|,$$

$$\sigma_T = \frac{1}{64\pi^2\,s}\int \mathrm{d}\Omega\frac{|\mathcal{M}|^2}{4}\sqrt{1-\frac{4m_\tau^2}{s}}\,,$$

$$\mathscr{C} = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$$



$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\,\Gamma^{\mu}(q^2)\,\tau\,Z_{\mu}(q) =$$

$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\left[\gamma^{\mu}F_1^V(q^2)+\gamma^{\mu}\gamma_5F_1^A(q^2)+\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_{\tau}}F_2(q^2)+\frac{\sigma^{\mu\nu}\gamma_5q_{\nu}}{2m_{\tau}}F_3(q^2)\right]\tau\,Z_{\mu}(q)$$

$\mathscr{O}_a$	σ_a^I	limits I (L = 17.6 fb^{-1})	σ_a^{II}	limits II (L = 150 ab^{-1})
$\mathscr{C}$	0.006	$-0.002 \leq F_2(m_Z^2) \leq 0.003$	0.001	$-0.001 \leq F_2(m_Z^2) \leq 0.001$
$\mathscr{C}_{odd}$	0.009	$-0.001 \leq F_3(m_Z^2) \leq 0.001$	0.006	$-0.0004 \leq F_3(m_Z^2) \leq 0.0005$
σ_T	0.05 pb	$-0.009 \leq C_1^V \leq 0.010$	0.02 pb	$-0.004 \leq C_1^V \leq 0.004$
σ_T	0.05 pb	$-0.001 \leq C_1^A \leq 0.001$	0.02 pb	$-0.0004 \leq C_1^A \leq 0.0004$

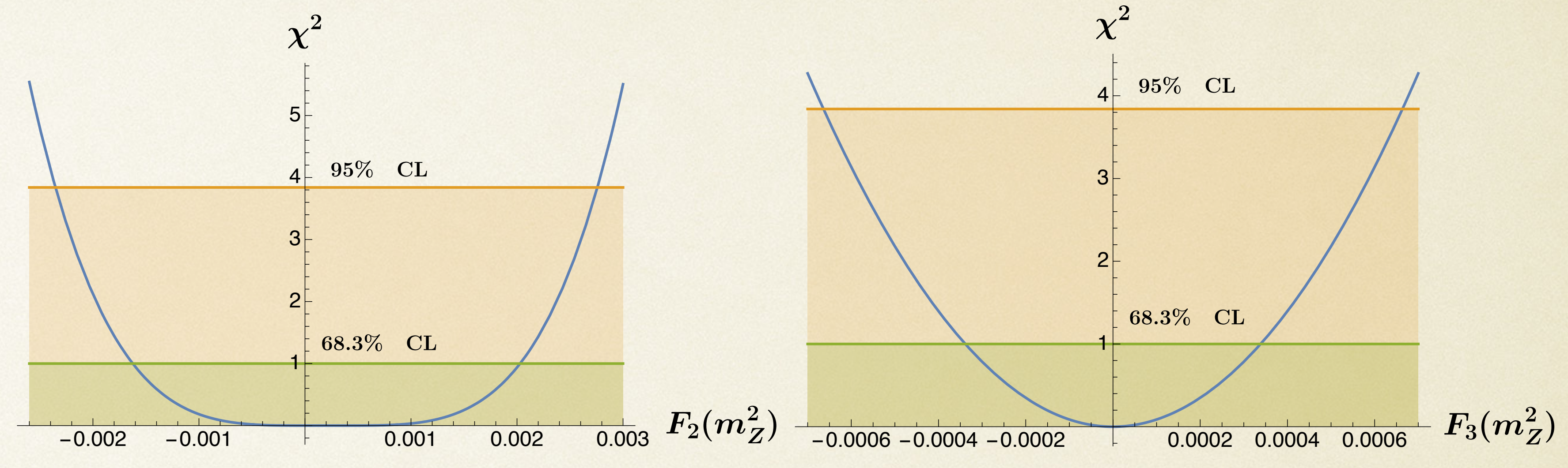


Figure 3.4: χ^2 test for the form factor $F_2(m_Z^2)$ and $F_3^V(m_Z^2)$. The limits for $F_2(m_Z^2)$ are obtained by means of the concurrence, those for the form factor $F_3^V(m_Z^2)$ by means of the operator $\mathscr{C}_{odd}$.

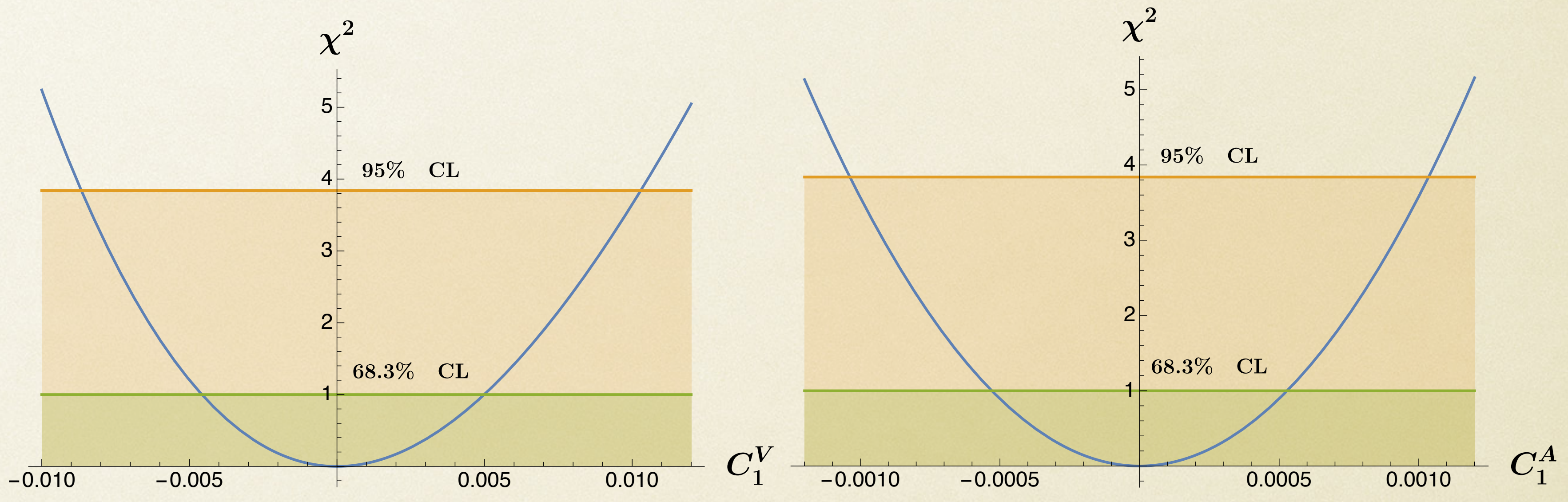


Figure 3.5: χ^2 test for the form factor $F_1^V(m_Z^2)$ (left) and $F_1^A(m_Z^2)$ (right). Both limits are obtained by means of the cross section.

MF and L. Marzola, arXiv:2405.09201