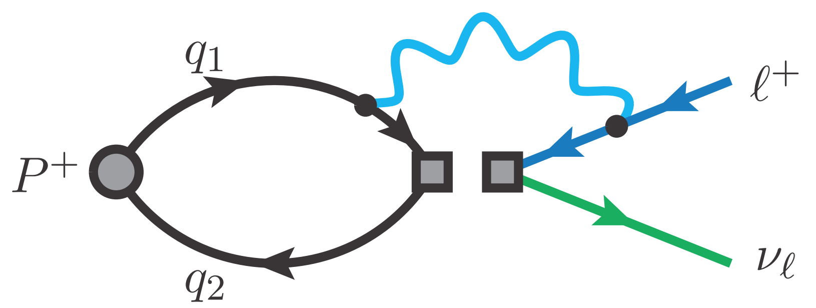
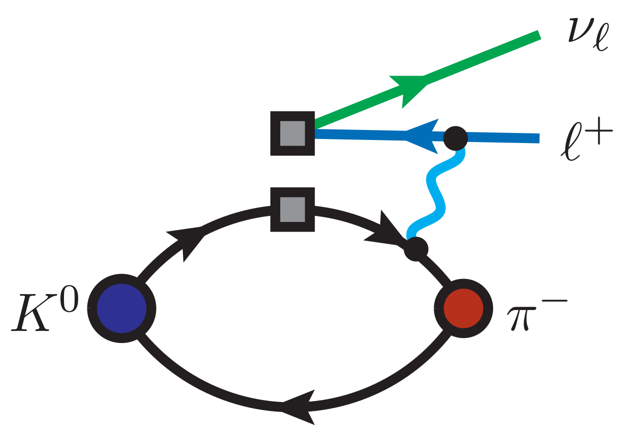
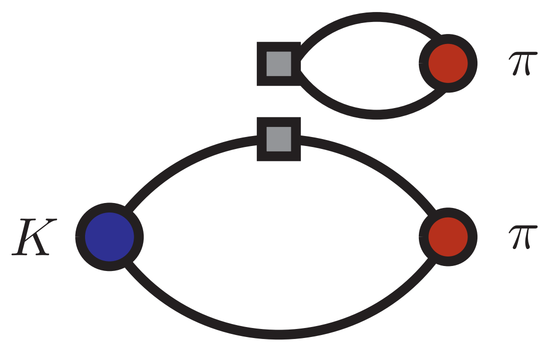
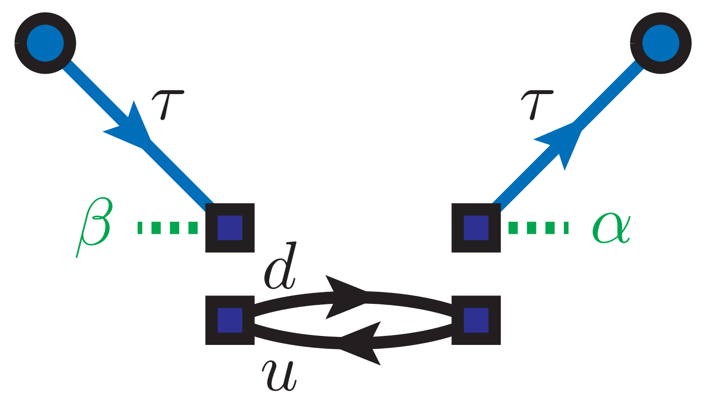
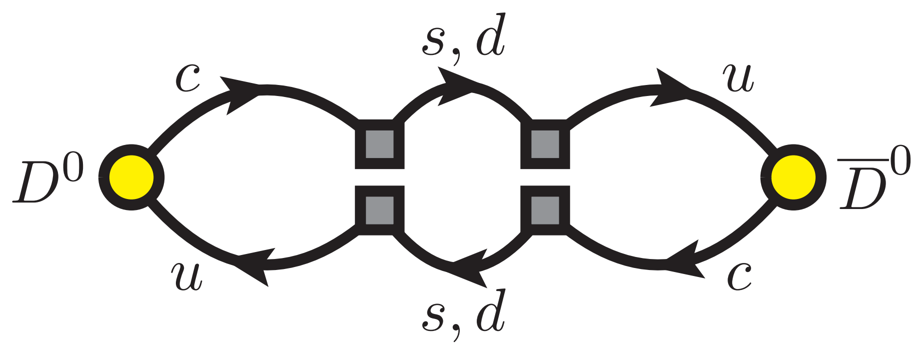



# Lattice QCD and hadronic uncertainties: status and prospects

Matteo Di Carlo

12th November 2024

CERN

Funded by the European Union

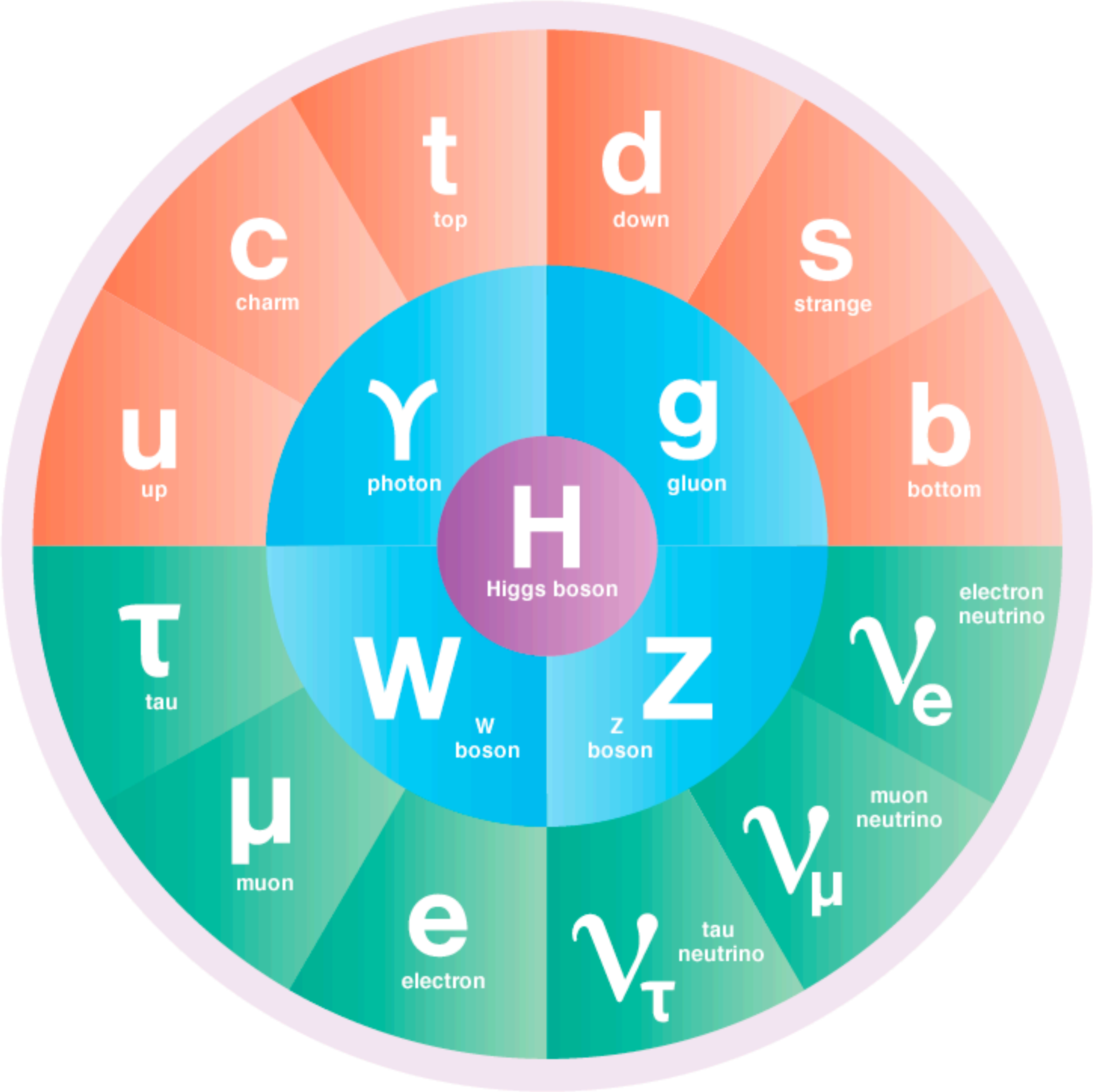
Workshop Italiano sulla Fisica ad Alta Intensità (WIFAI 2024)
Bologna, 12-15 November 2024

- The **Standard Model** (SM) works very well, but it leaves many questions still unanswered

  why three generations?
  why this quark mass hierarchy?
  why this pattern in quark mixing?
  why such matter-antimatter asymmetry?
  …

- We want to look for **new physics effects**: deviations from SM expectations could be a signal of new physics!

- Need for precise and controlled **experimental measurements** and **theoretical predictions**




[https://www.symmetrymagazine.org/standard-model/]

# Flavour physics

**Flavour physics** offers opportunities to test the Standard Model and probe new physics effects

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

in the Standard Model:

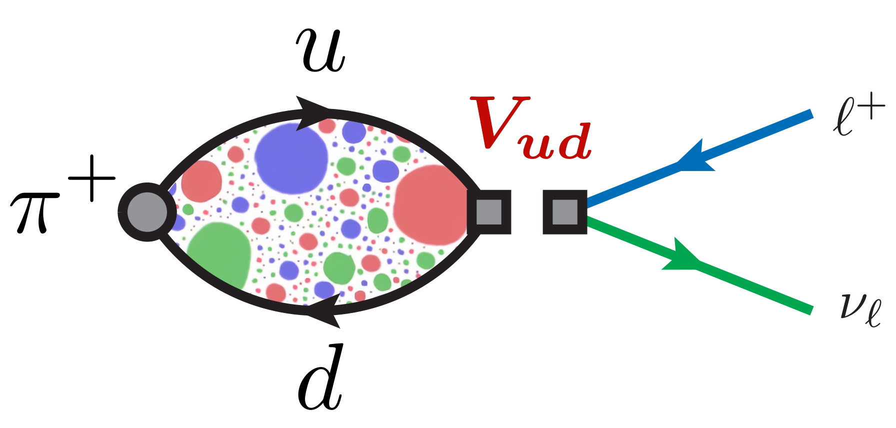
3 mixing angles + 1 CPV phase

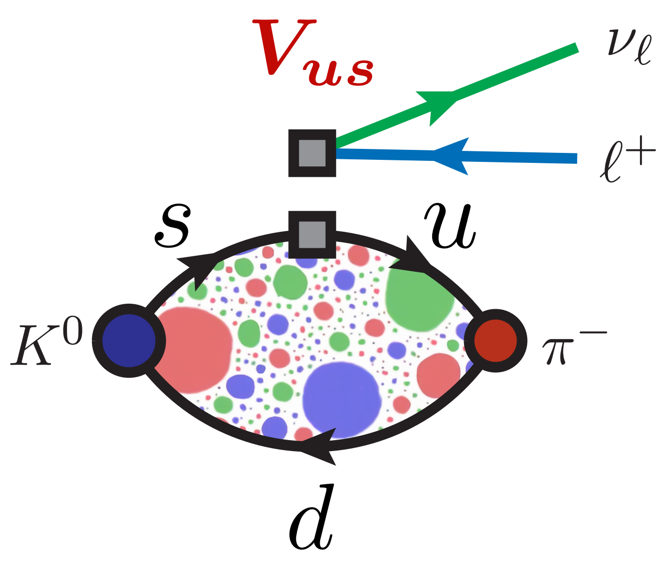
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Matrix elements can be extracted e.g. from leptonic and semileptonic decays of hadrons

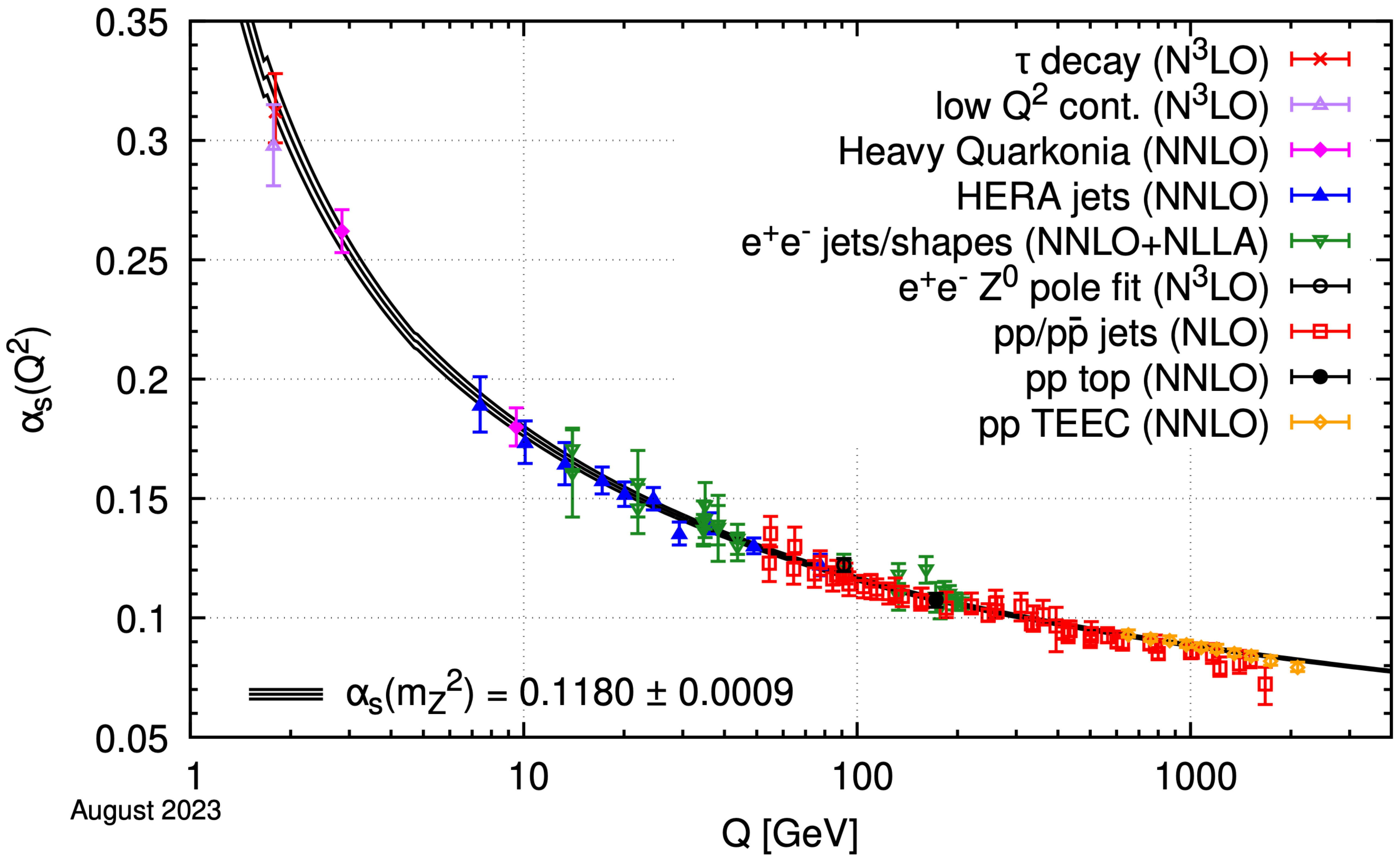
$$\underbrace{\frac{\Gamma\left[K \to \ell\nu_\ell(\gamma)\right]}{\Gamma\left[\pi \to \ell\nu_\ell(\gamma)\right]}}_{\text{experiments}} \propto \boxed{\left|\frac{V_{us}}{V_{ud}}\right|^2} \underbrace{\left(\frac{f_K}{f_\pi}\right)^2}_{\text{QCD}}$$

$$\underbrace{\Gamma\left[K \to \pi\ell\nu_\ell(\gamma)\right]}_{\text{experiments}} \propto \boxed{|V_{us}|^2} \underbrace{|f_+^{K\pi}(0)|^2}_{\text{QCD}}$$



# The strong coupling constant




PDG, PRD 110 (2024) 3

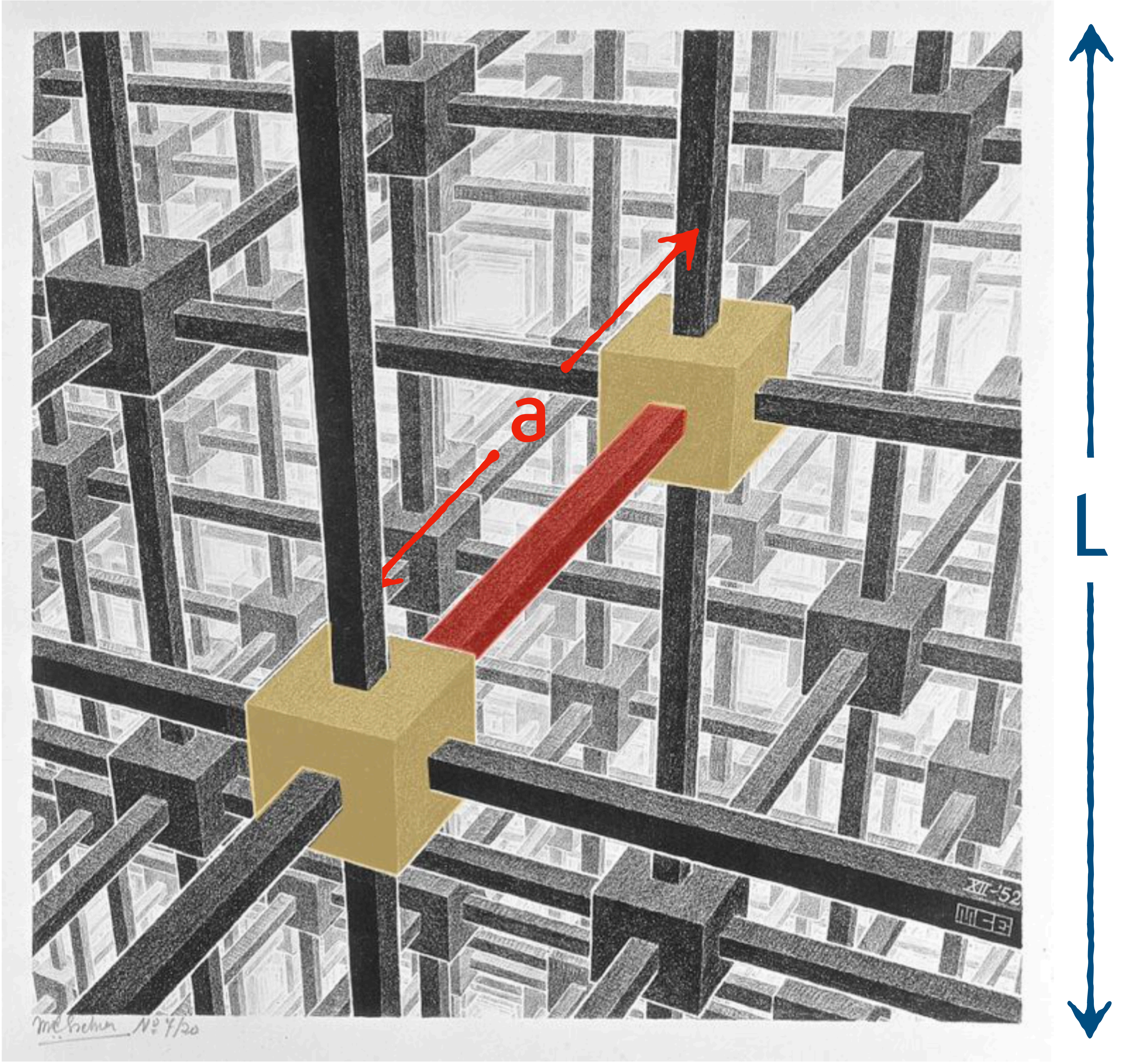
- The strong coupling constant $\alpha_s(Q^2)$ runs with the energy $Q$
- At high energies $Q \sim m_Z$ the coupling is small:
  - perturbative expansion
  - quarks are **asymptotically free**
- At small energies $Q \sim \Lambda_{\text{QCD}}$ the coupling is strong:
  - non-perturbative
  - quarks are **confined**

# Lattice QCD in a [small] nutshell

- **QCD** on a **discrete** and **finite Euclidean** space-time
- Based on **Feynman path integrals**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]\, \mathcal{O}[\psi,\bar{\psi},U]\, \mathrm{e}^{-S[\psi,\bar{\psi},U]}$$

- Path integral solved using **Monte Carlo methods**
- Physical **QCD** results obtained, after **renormalization**, by taking the **continuum** & **infinite-volume limit**

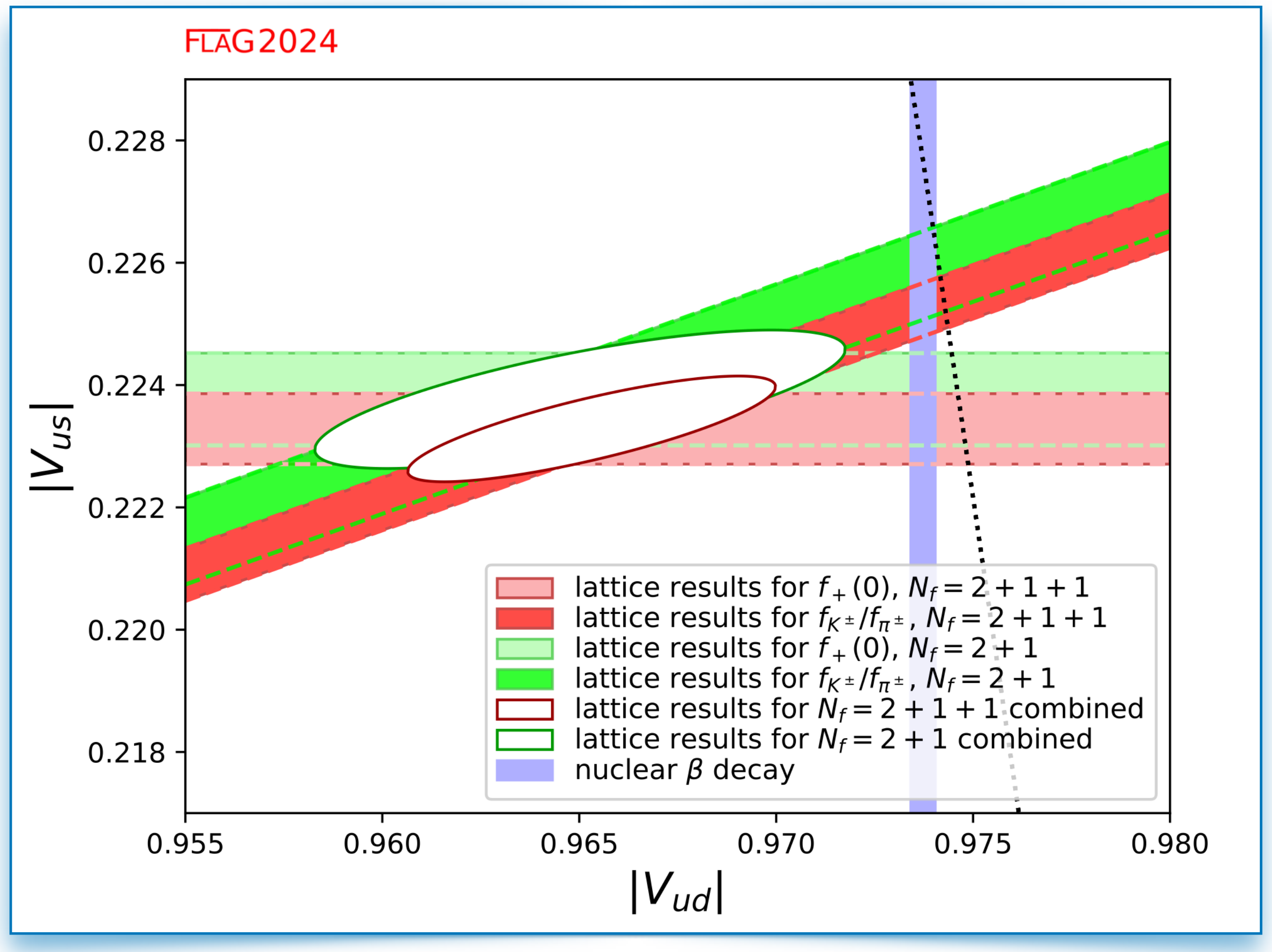



M.C. Escher, "Cubic space division" (1953)

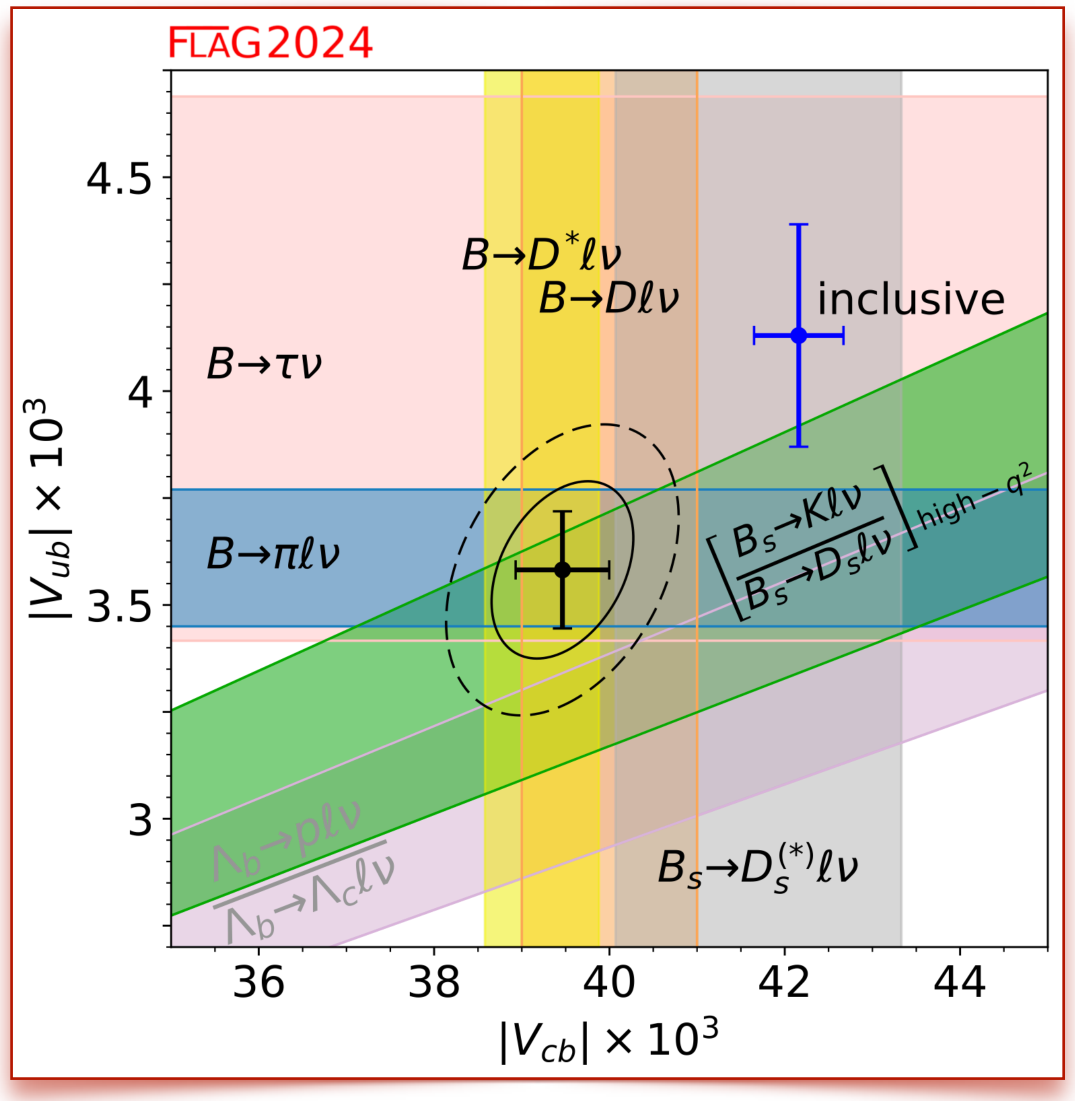
# Plan of the talk

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

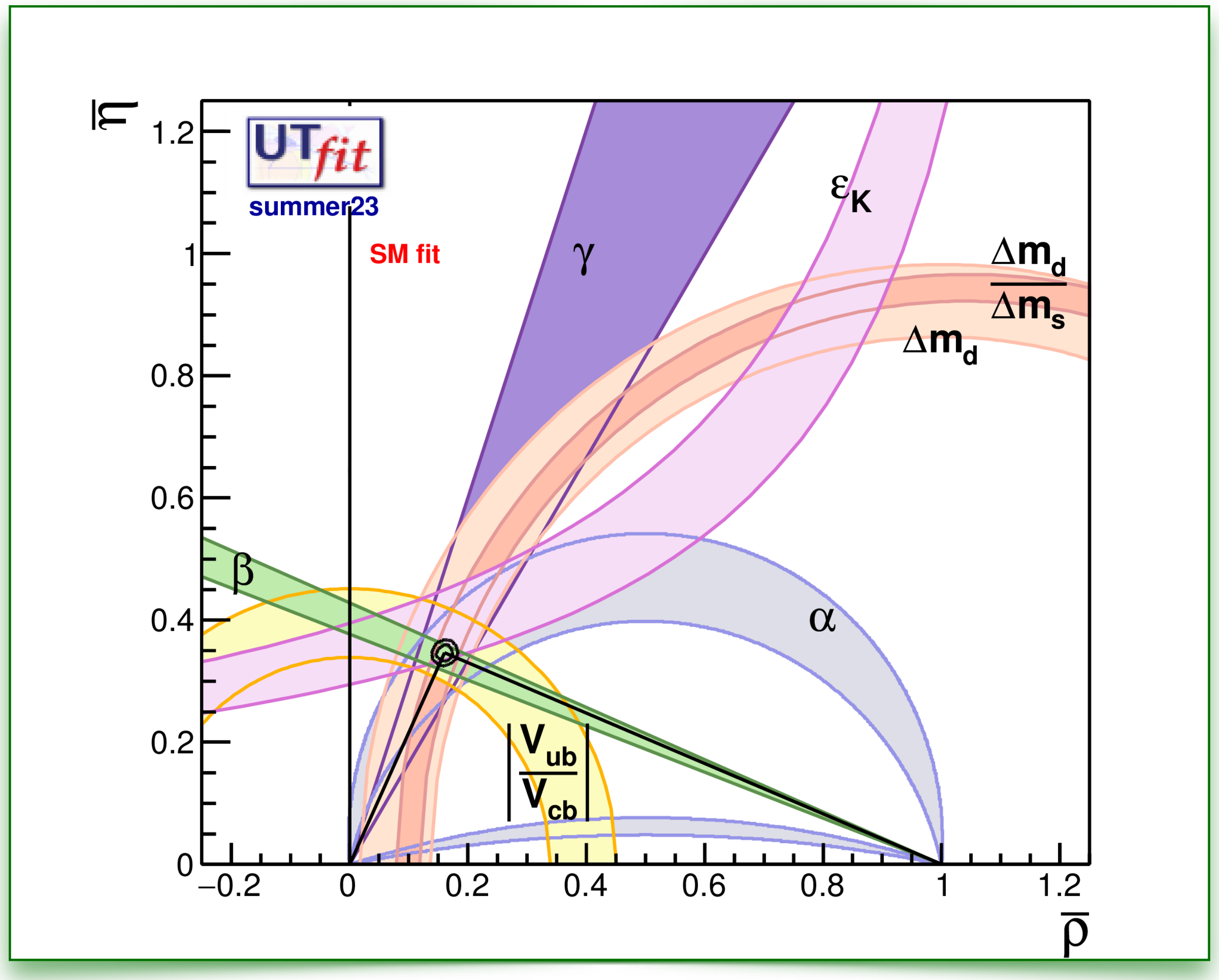
## Cabibbo anomaly




## $|V_{cb}|$ puzzle

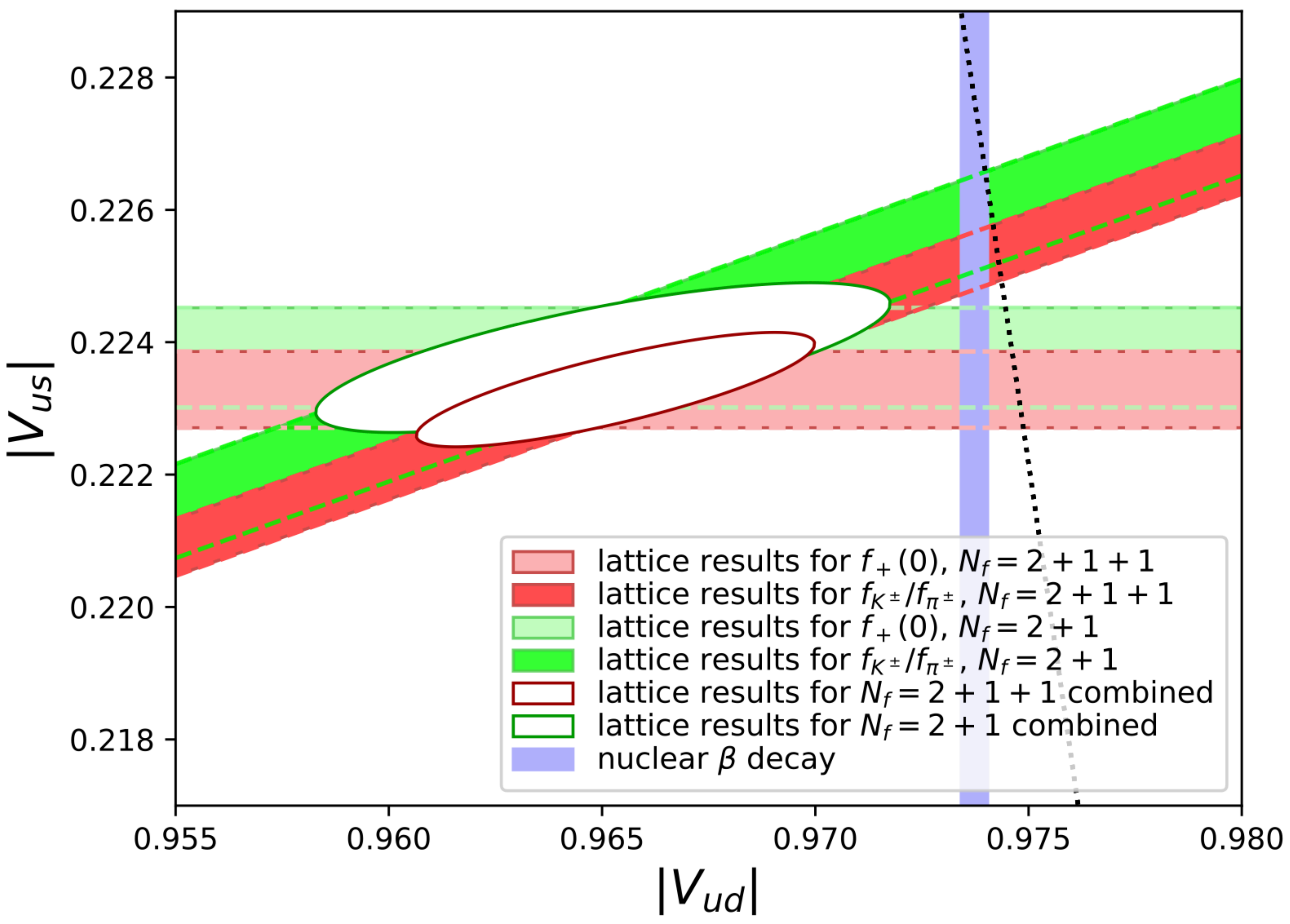



## CPV in neutral mesons



# The Cabibbo anomaly

FLAG2024 FLAG Review 2024, [arXiv:2411.04268]




$$\frac{|\boldsymbol{V_{us}}|}{|\boldsymbol{V_{ud}}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(41)$$

$$|\boldsymbol{V_{us}}|\,|f_+^{K^0\pi^-}(0)| = 0.21654(41)$$

M.Moulson, PoS CKM2016 (2017)
PDG, PTET **2022** (2022)

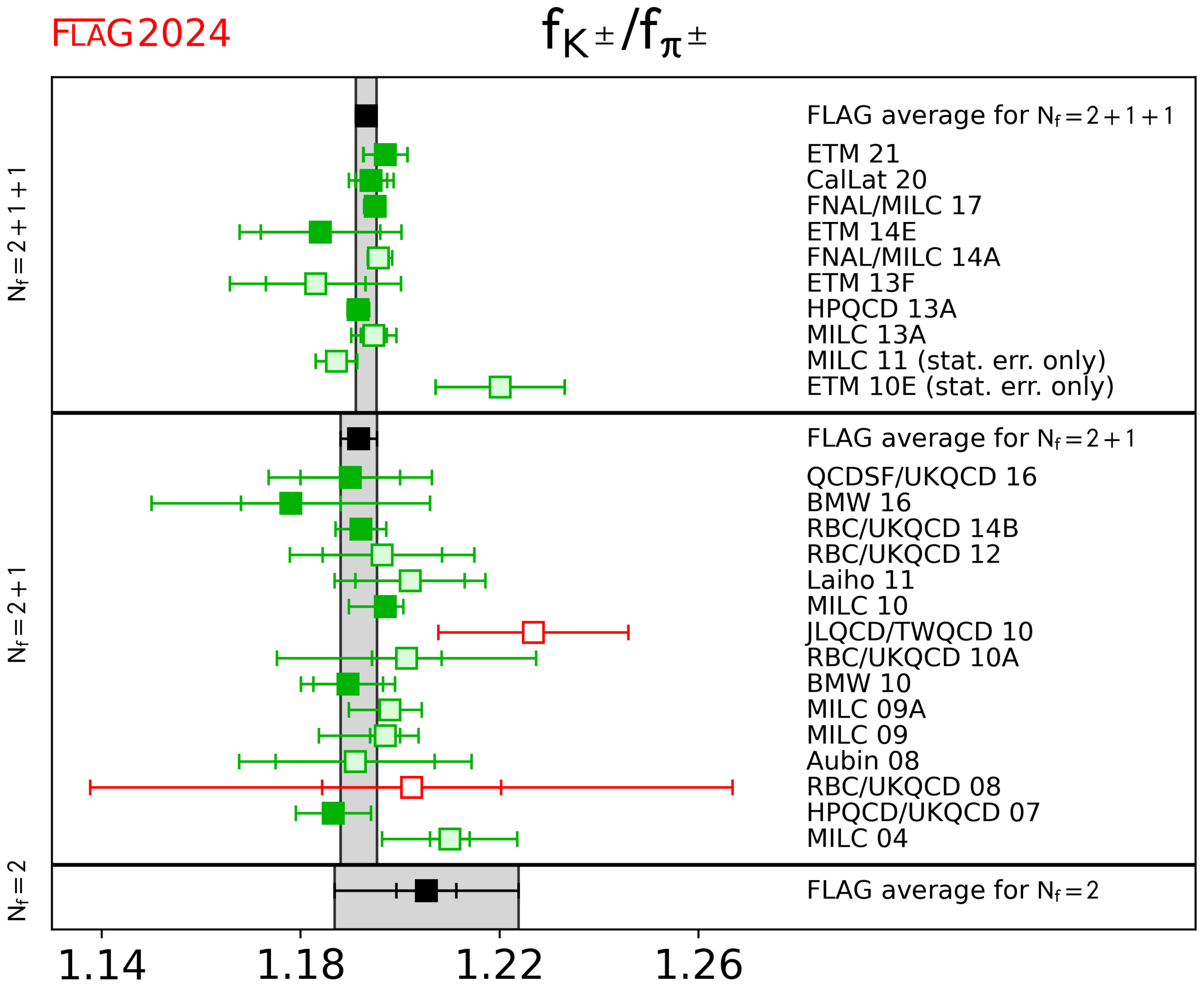
Different **tensions** in the $V_{us}$-$V_{ud}$ plane:

$$|V_u|^2 - 1 = 2.8\sigma$$

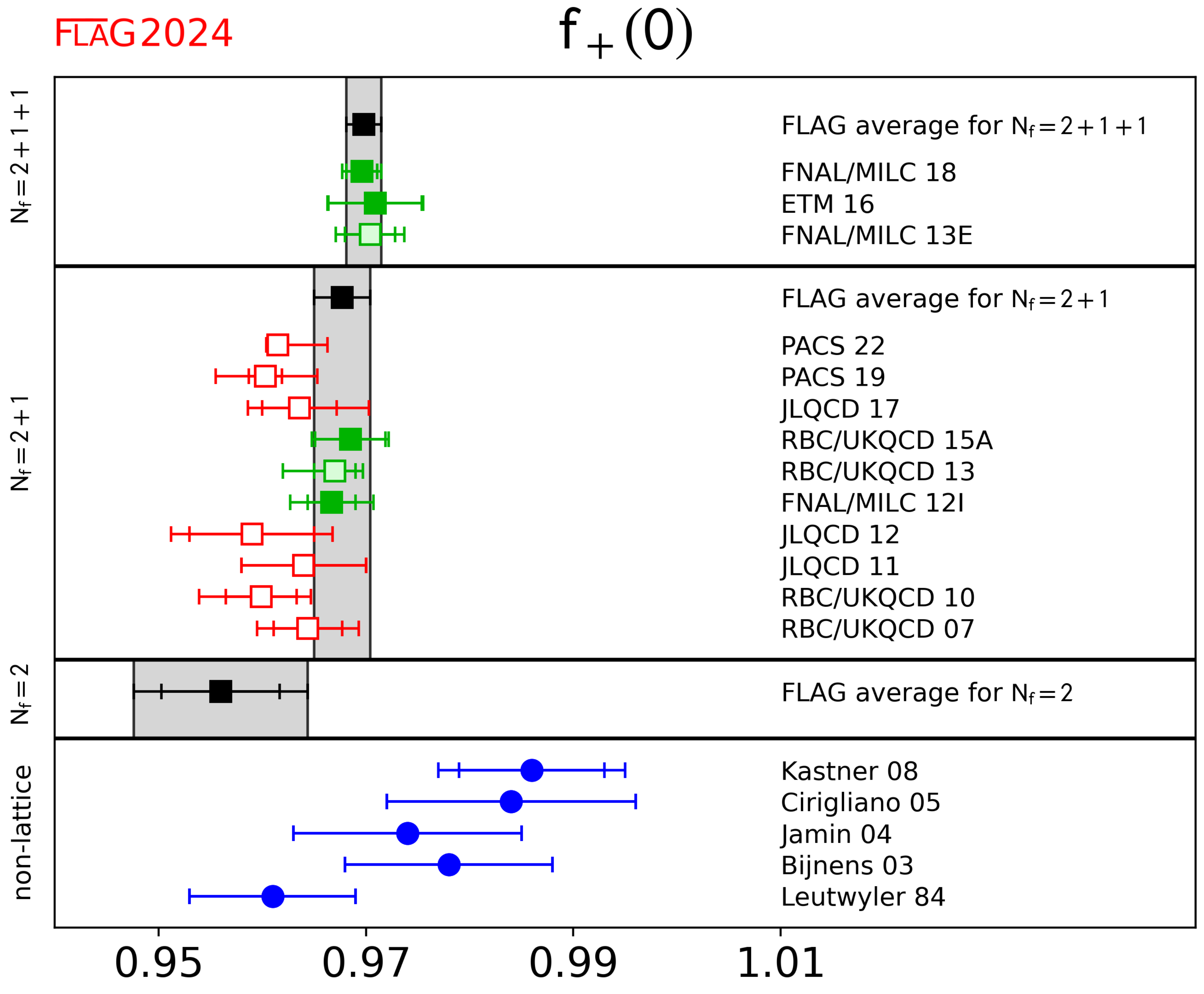
$$|V_u|^2 - 1 = 3.1\sigma \qquad |V_u|^2 - 1 = 1.7\sigma$$

**Experimental** and **theoretical** control of these quantities is of crucial importance to solve the issue

# Lattice QCD inputs




$$f_{K^\pm}/f_{\pi^\pm} = 1.1934\,(19)$$

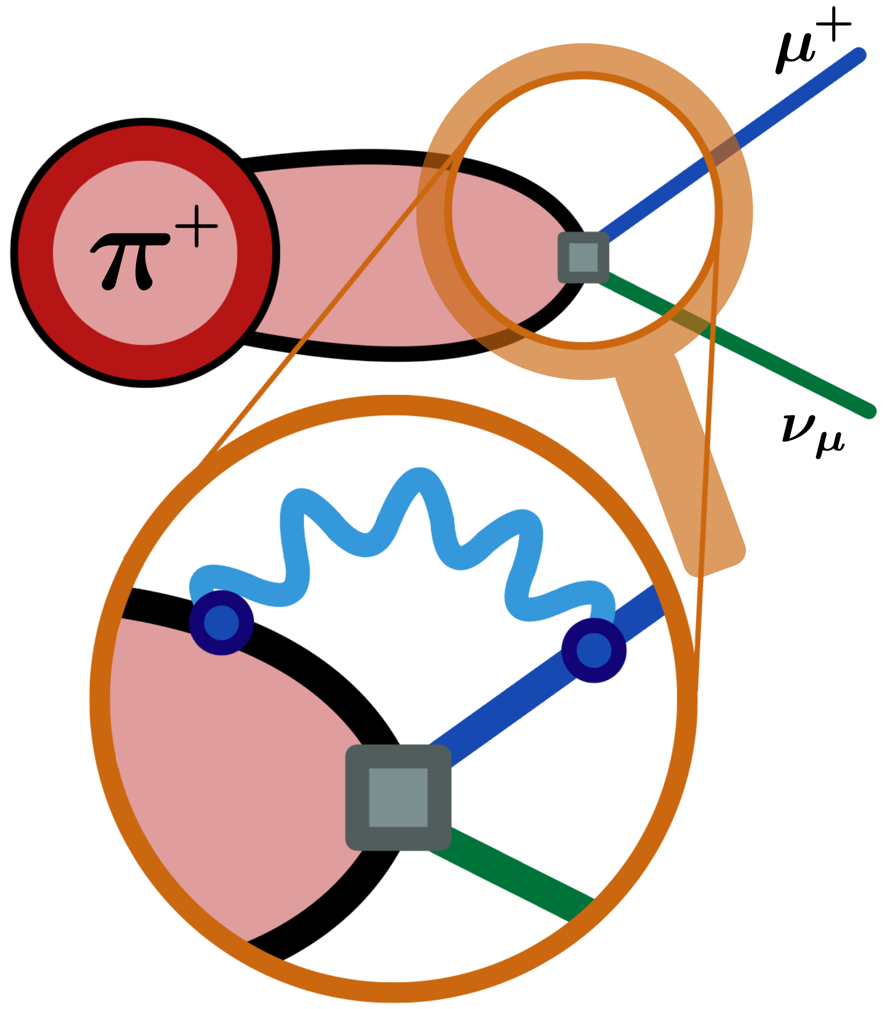



$$f_+^{K\pi}(0) = 0.9698\,(17)$$

FLAG 2024 Flavour Lattice Averaging Group

$f_K/f_\pi$ and $f_+^{K\pi}(0)$ determined from lattice QCD with sub percent precision!

# QED and isospin-breaking effects




Current level of precision requires the inclusion of isospin-breaking corrections due to

- strong effects $[m_u - m_d]_{\rm QCD} \neq 0$
- electromagnetic effects $\alpha \neq 0$

$\sim \mathcal{O}(1\%)$

$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta R_{K\pi}) \qquad \Gamma(K \to \pi\ell\nu_\ell) \propto |V_{us}|^2 \, |f_+^{K\pi}(0)|^2 \, \mathcal{I}^\ell_{K\pi} \, (1 + \delta R^\ell_{K\pi})$$

- results currently quoted in the PDG come from $\chi$PT — V.Cirigliano & H.Neufeld, PLB **700** (2011)
- fully **non-perturbative** (structure dependent) quantities
- first-principle **lattice calculations** are possible!

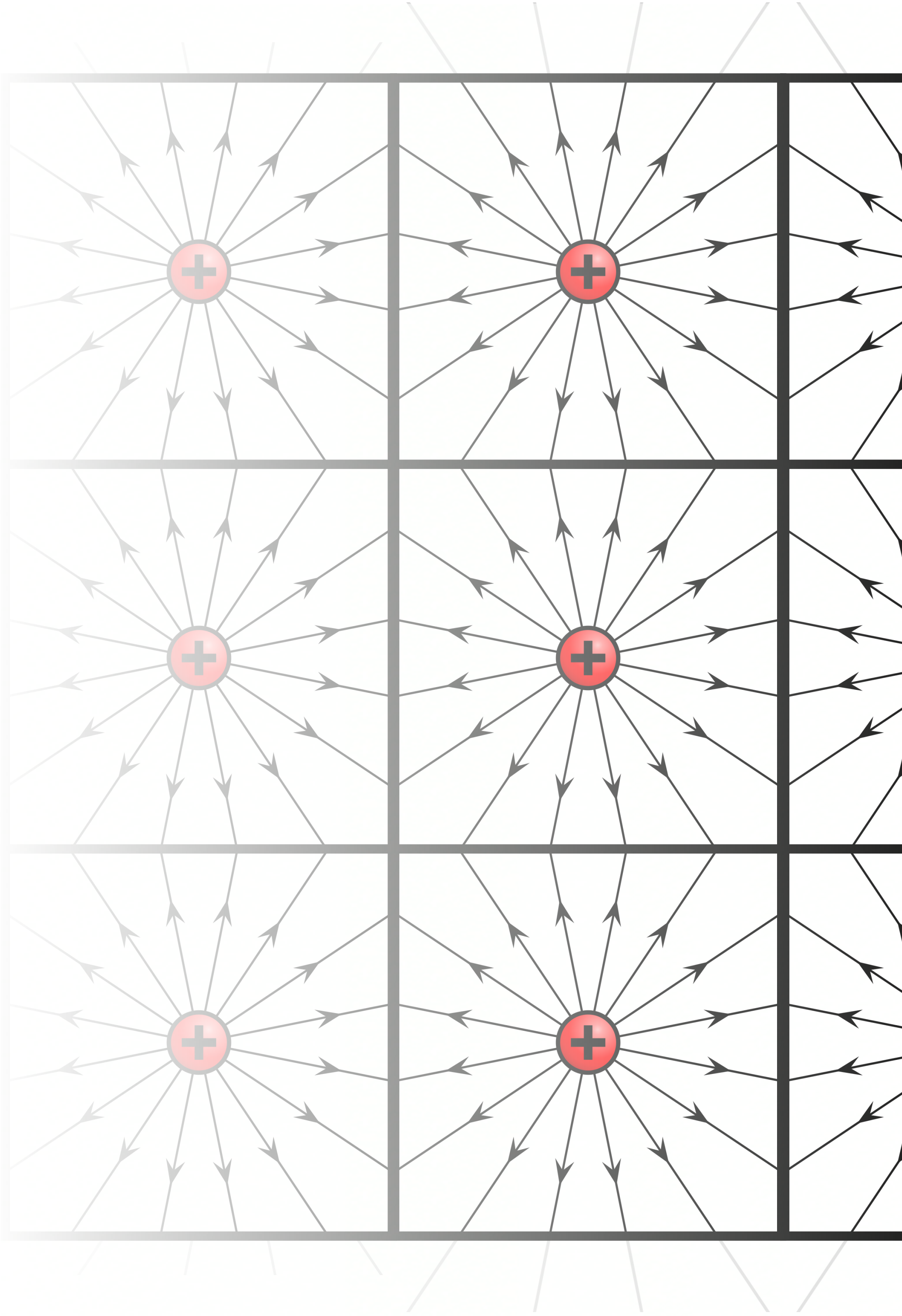
# Lattice QCD + QED



A conceptual challenge: how to define QED in a finite periodic box?

- need to circumvent **Gauss' law:** no charged states in a periodic box
- **finite-volume effects** can be sizeable and power-like
- logarithmic **infrared divergences** arise when studying decays

Problems well studied. Different lattice QED formulations proposed and used.



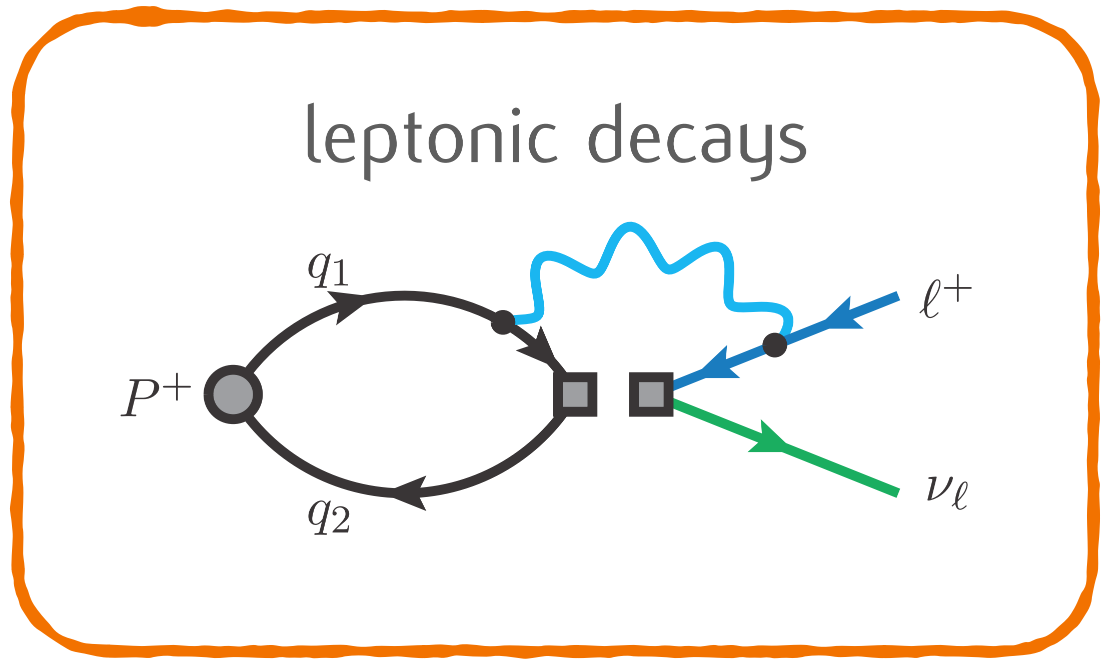
## RM123 approach:

G.M.de Divitiis et al. [RM123], PRD 87 (2013)

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S_{\text{iso}} - \Delta S} = \langle \mathcal{O} \rangle_{\text{iso}} + \langle \Delta S \, \mathcal{O} \rangle_{\text{iso}} + \ldots$$

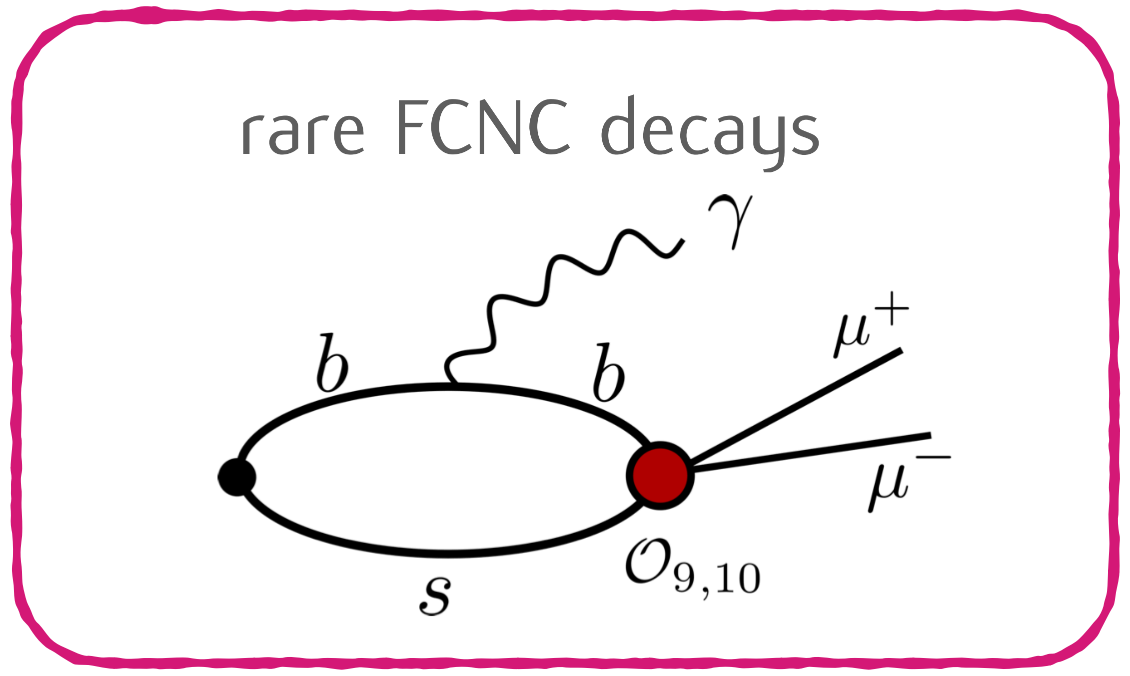
$$\text{“iso”} = \begin{cases} m_{\text{u}} = m_{\text{d}} \\ \alpha_{\text{em}} = 0 \end{cases}$$

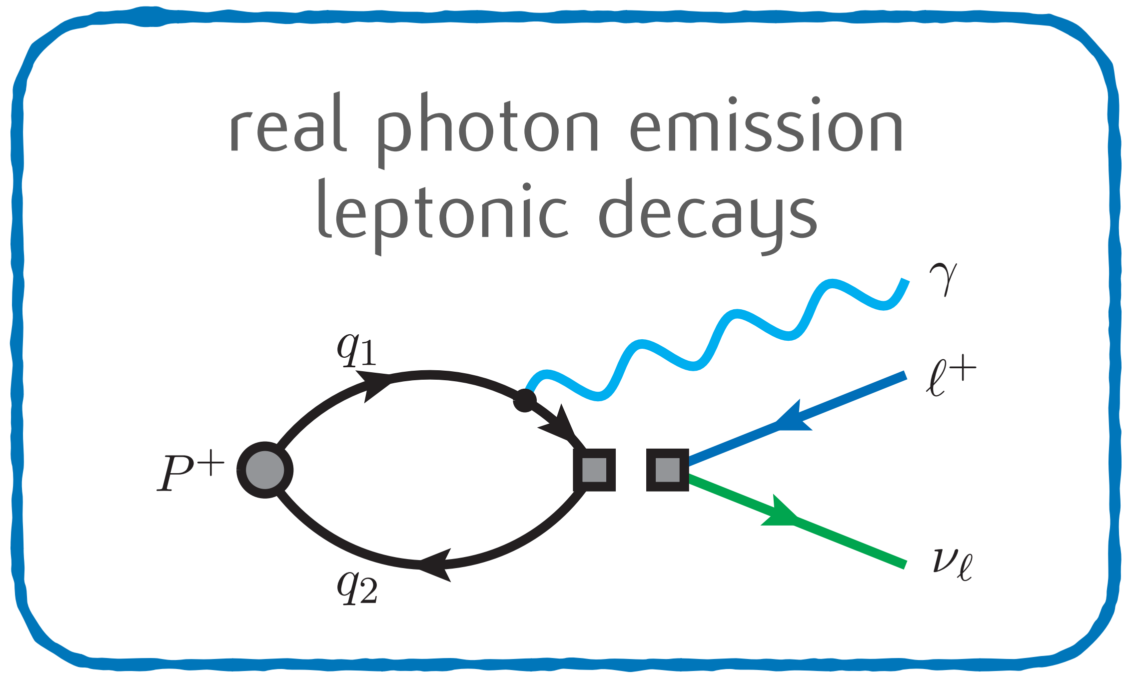
# Weak decays — some recent works




N.Carrasco et al., PRD 91 (2015)
V.Lubicz et al., PRD 95 (2017)
N.Tantalo et al., [1612.00199v2]
D.Giusti et al., PRL 120 (2018)
MDC et al., PRD 100 (2019)
MDC et al., PRD 105 (2022)
P.Boyle, MDC et al., JHEP 02 (2023)
N.Christ et al., [2304.08026]

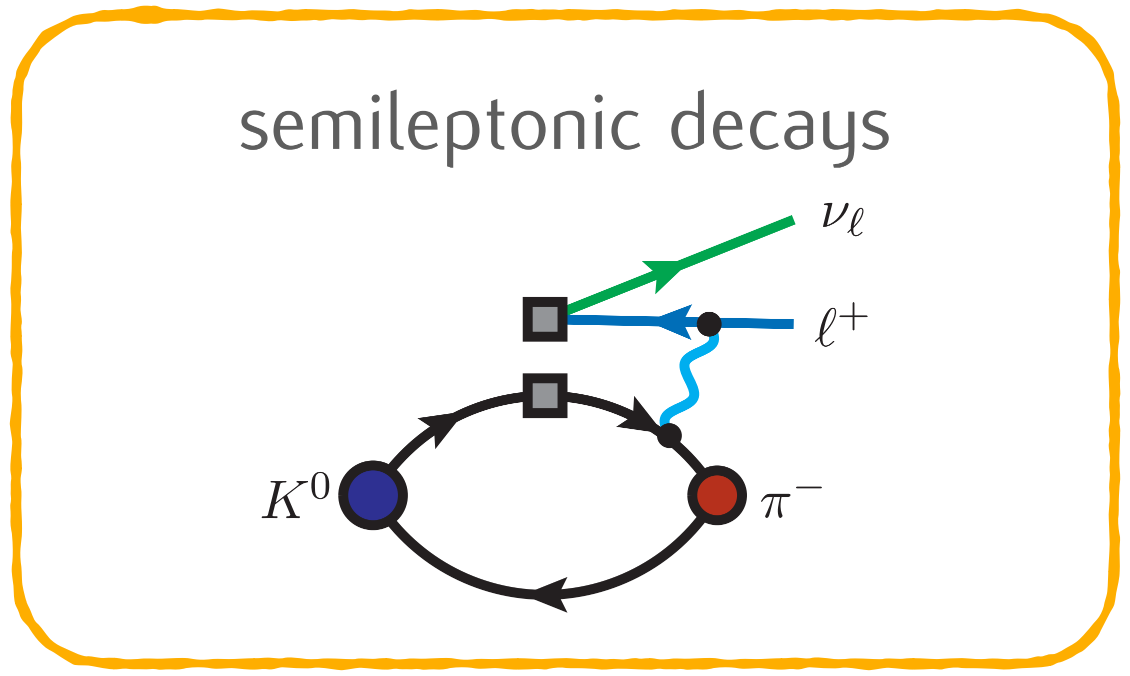
R.Frezzotti et al., [2402.03262]

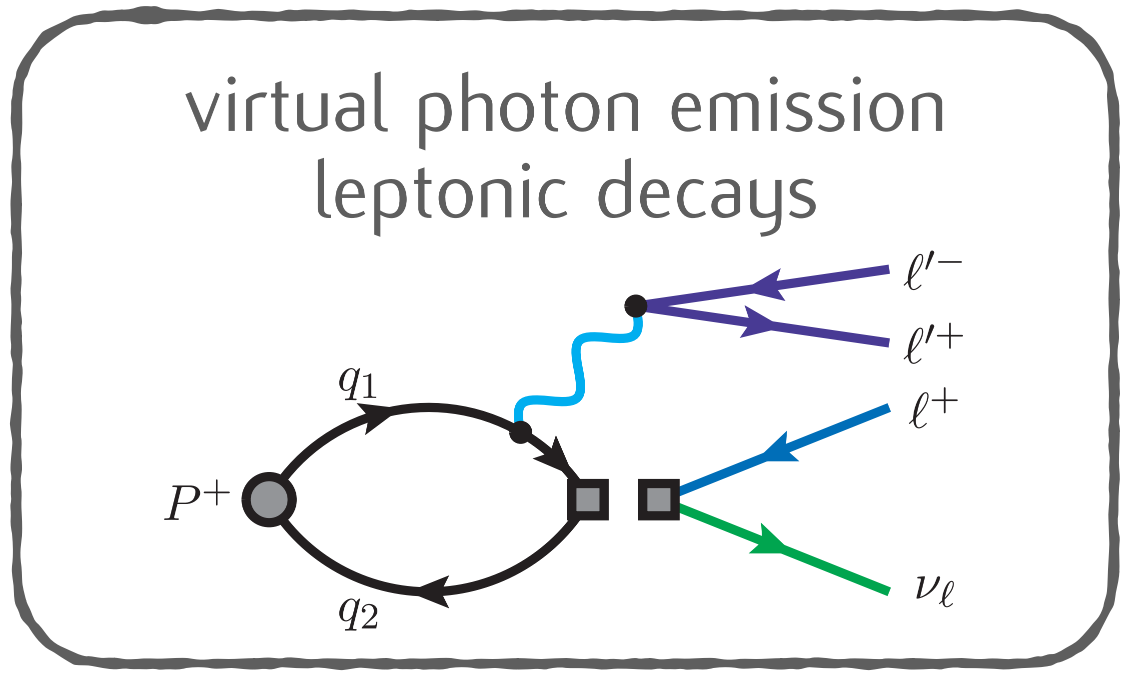



G.M.de Divitiis et al., [1908.10160]
C.Kane et al., [1907.00279 & 2110.13196]
R.Frezzotti et al., PRD 103 (2021)
A.Desiderio et al., PRD 102 (2021)
D.Giusti et al., [2302.01298]
R.Frezzotti et al., [2306.05904]

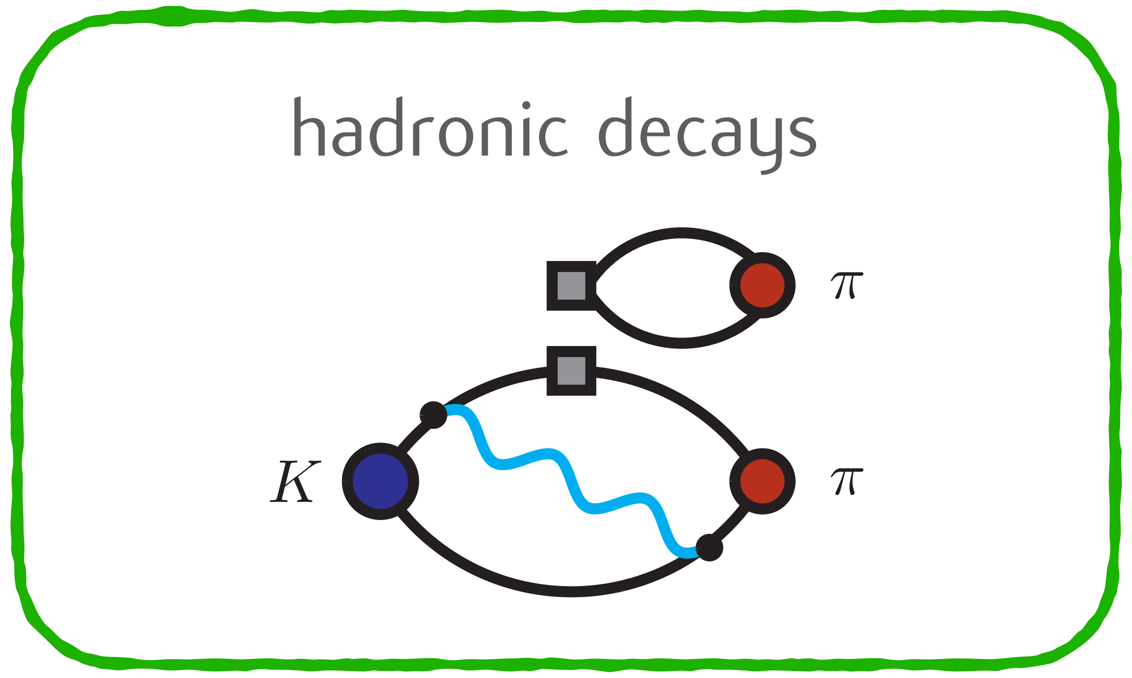
C.Sachrajda et al., [1910.07342]
N.Christ et al., PRD 108 (2023)
N.Christ et al., [2402.08915]




G.Gagliardi et al., Phys. Rev. D 105 (2022)
R.Frezzotti et al., [2306.07228]

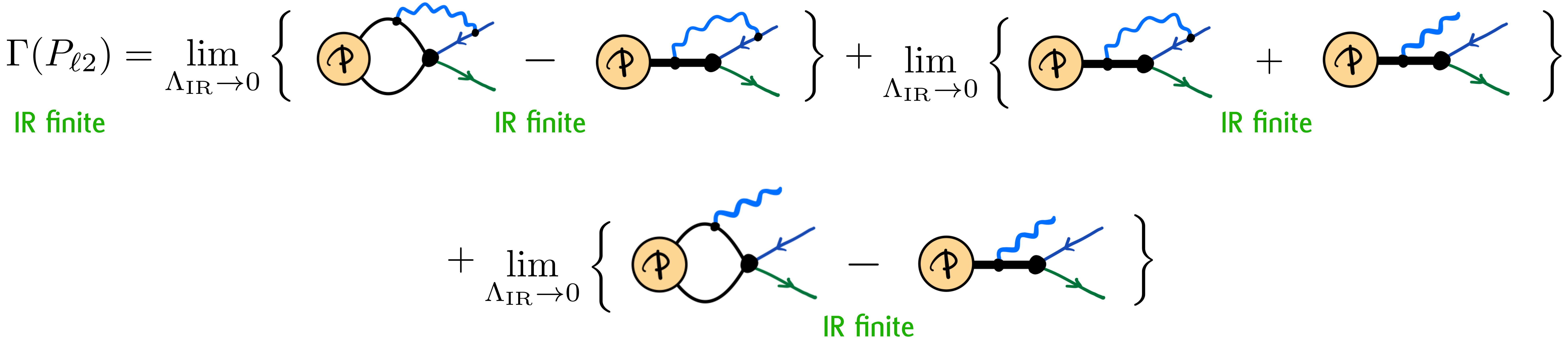
R.Abbott et al., PRD 102 (2020)
Z.Bai et al., PRL 115 (2015)
N.Christ et al., PRD 106 (2022)
N.Christ & X.Feng, EPJ Web Conf. 175 (2018)
Y.Cai & Z.Davoudi, [1812.11015]



# Leptonic decay rate at $\mathcal{O}(\alpha)$

F.Bloch & A.Nordsieck, PR **52** (1937)
N.Carrasco et al., PRD **91** (2015)
V.Lubicz et al., PRD **95** (2017)
N.Tantalo et al. [1612.00199v2]
MDC et al., PRD **105** (2022)

## The RM123+Soton recipe (2015)



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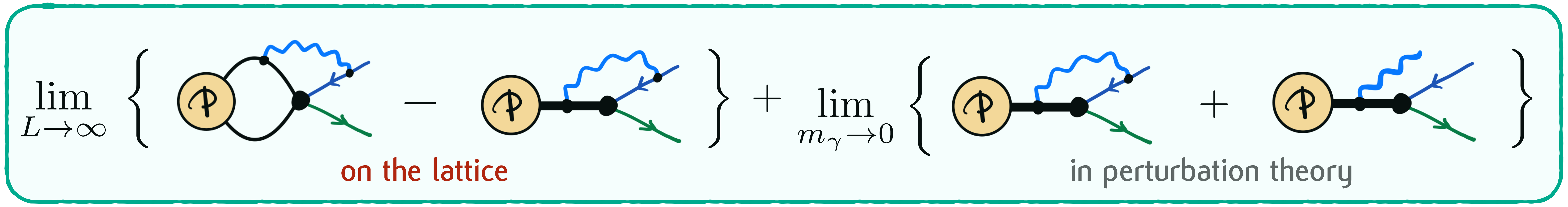
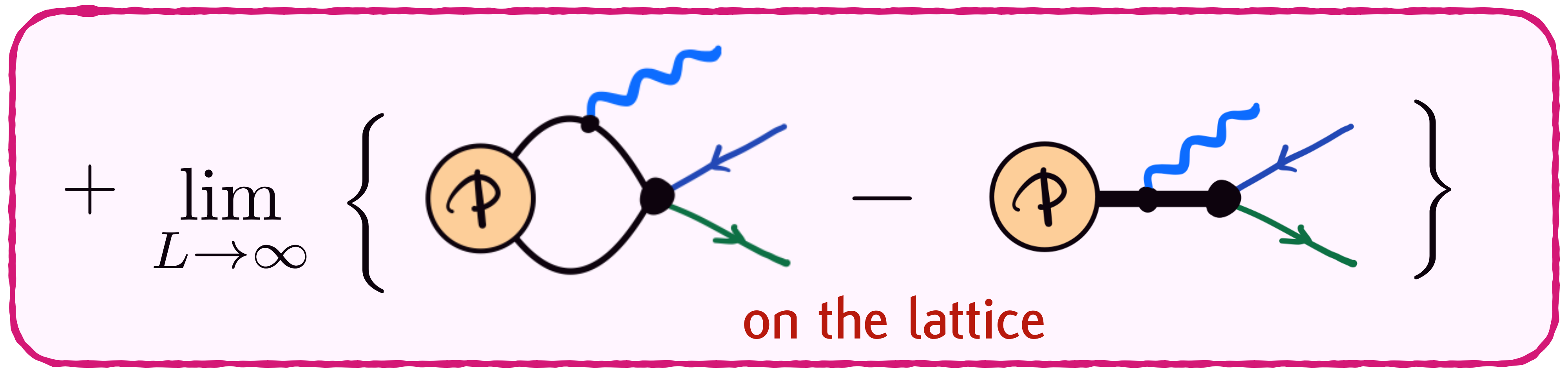
N.Tantalo et al. [1612.00199v2]
MDC et al., PRD **105** (2022)

## The RM123+Soton recipe (2015)

$\Gamma(P_{\ell 2}) =$




enough for $K_{\mu 2}$ and $\pi_{\mu 2}$

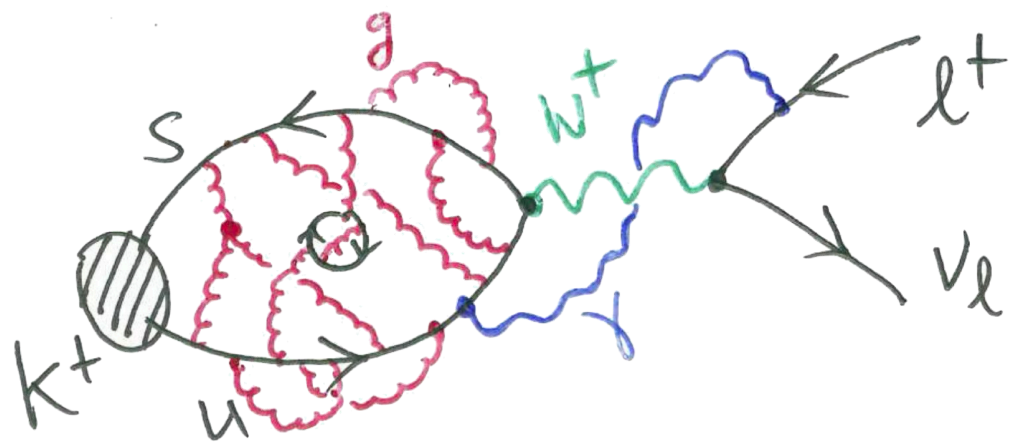
D. Giusti et al., PRL 120 (2018)
MDC et al., PRD 100 (2019)
P.Boyle, MDC et al., JHEP 02 (2023)

relevant for $K_{e2}$ and $\pi_{e2}$
& decays of heavier mesons

G.M.de Divitiis et al., [1908.10160]
R.Frezzotti et al., PRD **103** (2021)
A.Desiderio et al., PRD **102** (2021)

C.Kane et al., [1907.00279 & 2110.13196]
D.Giusti et al., [2302.01298]
R.Frezzotti et al., [2306.05904]

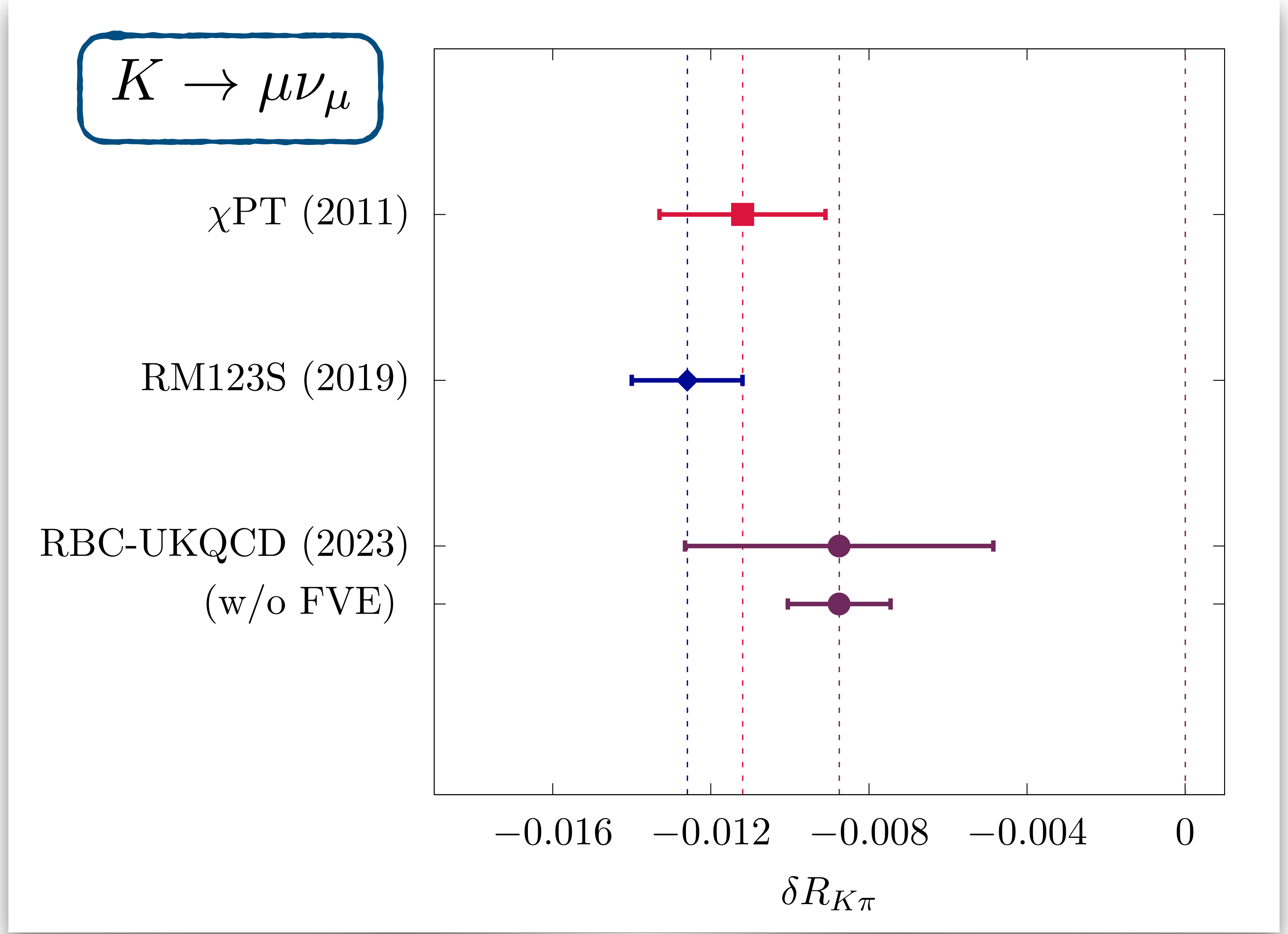
# Results for $\delta R_{K\pi}$




V.Cirigliano et al., PLB **700** (2011)
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)

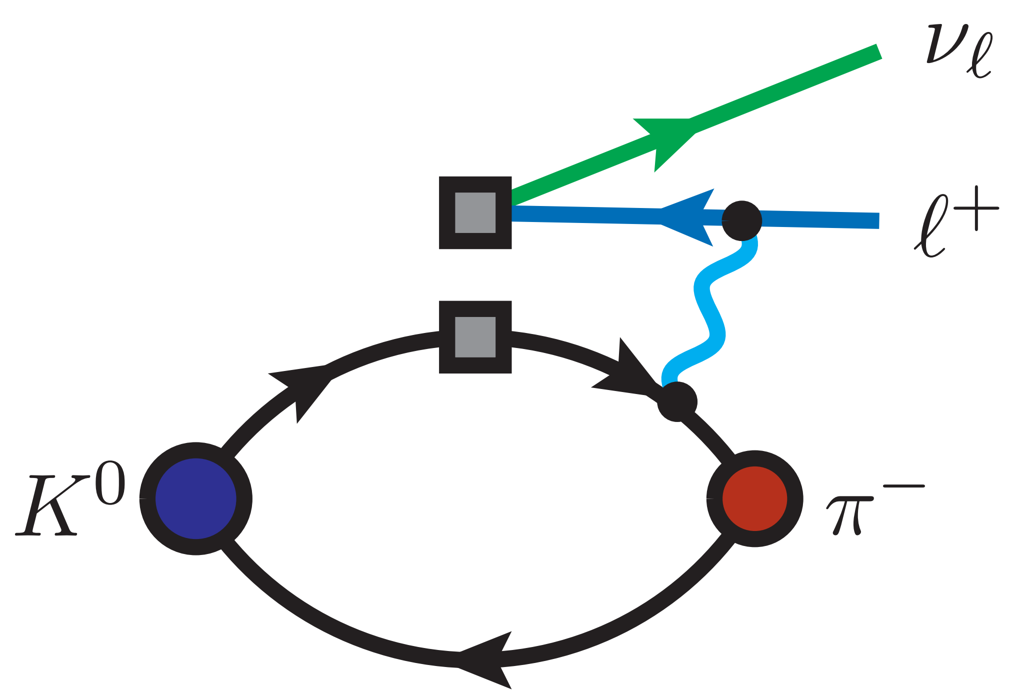
- ■ $\delta R_{K\pi} = -0.0112\,(21)$
- ◆ $\delta R_{K\pi} = -0.0126\,(14)$
- ● $\delta R_{K\pi} = -0.0086\,(13)(39)_{\text{vol.}}$

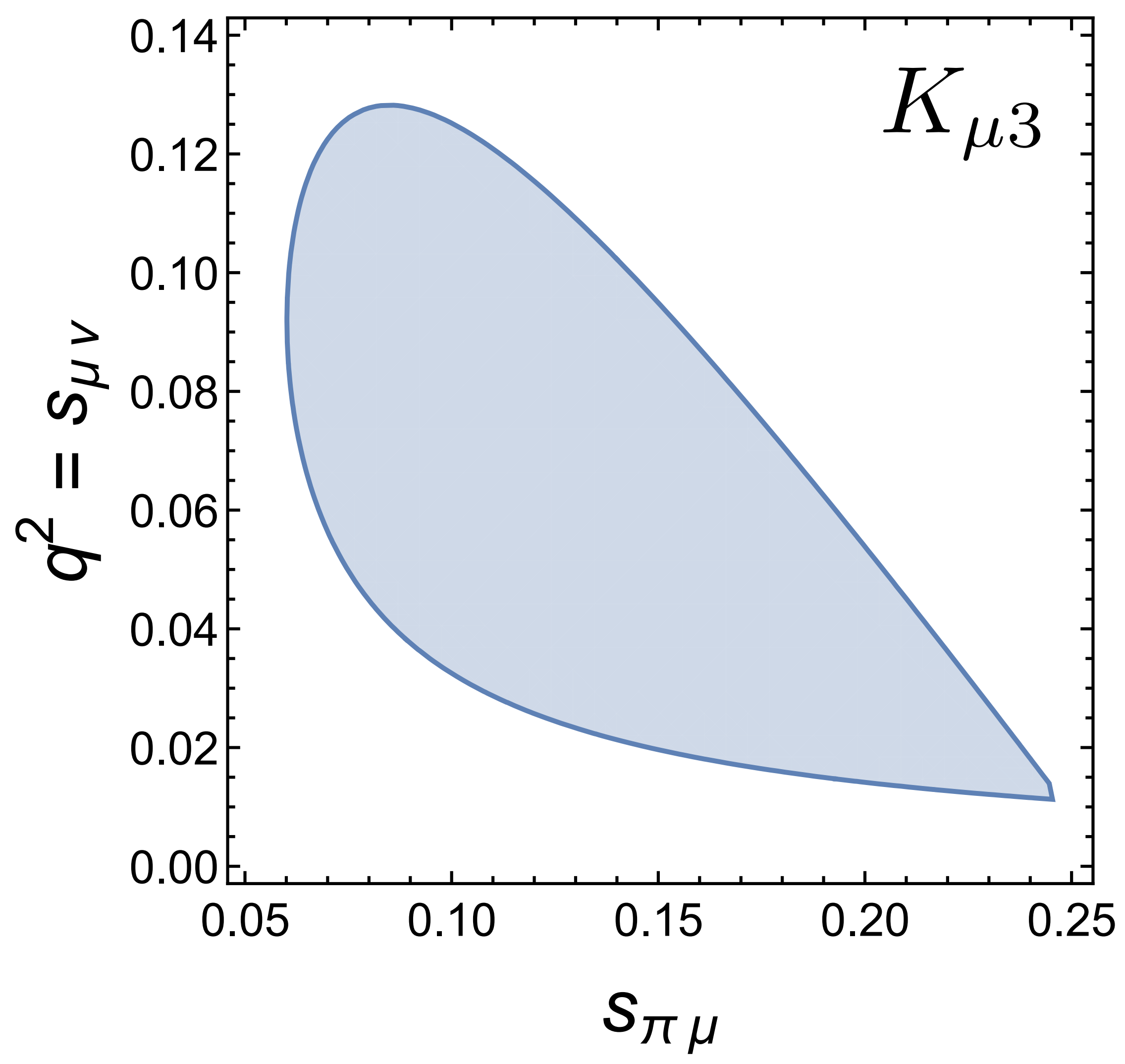
$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta R_{K\pi})$$




- Good evidence that $\delta R_{K\pi}$ **can be computed from first principles non-perturbatively on the lattice!**
- **RBC-UKQCD error** dominated by a large systematic uncertainty related to **finite-volume effects** (!)
  **Work in progress** to improve the result.
- **Errors on** $|V_{us}|/|V_{ud}|$ from **theoretical** inputs could become comparable with those from **experiments**

# Semileptonic kaon decays




To go beyond current precision, we need to include **isospin-breaking effects** computed from first principles lattice QCD+QED.

Significant additional **difficulties** compared to leptonic decays:

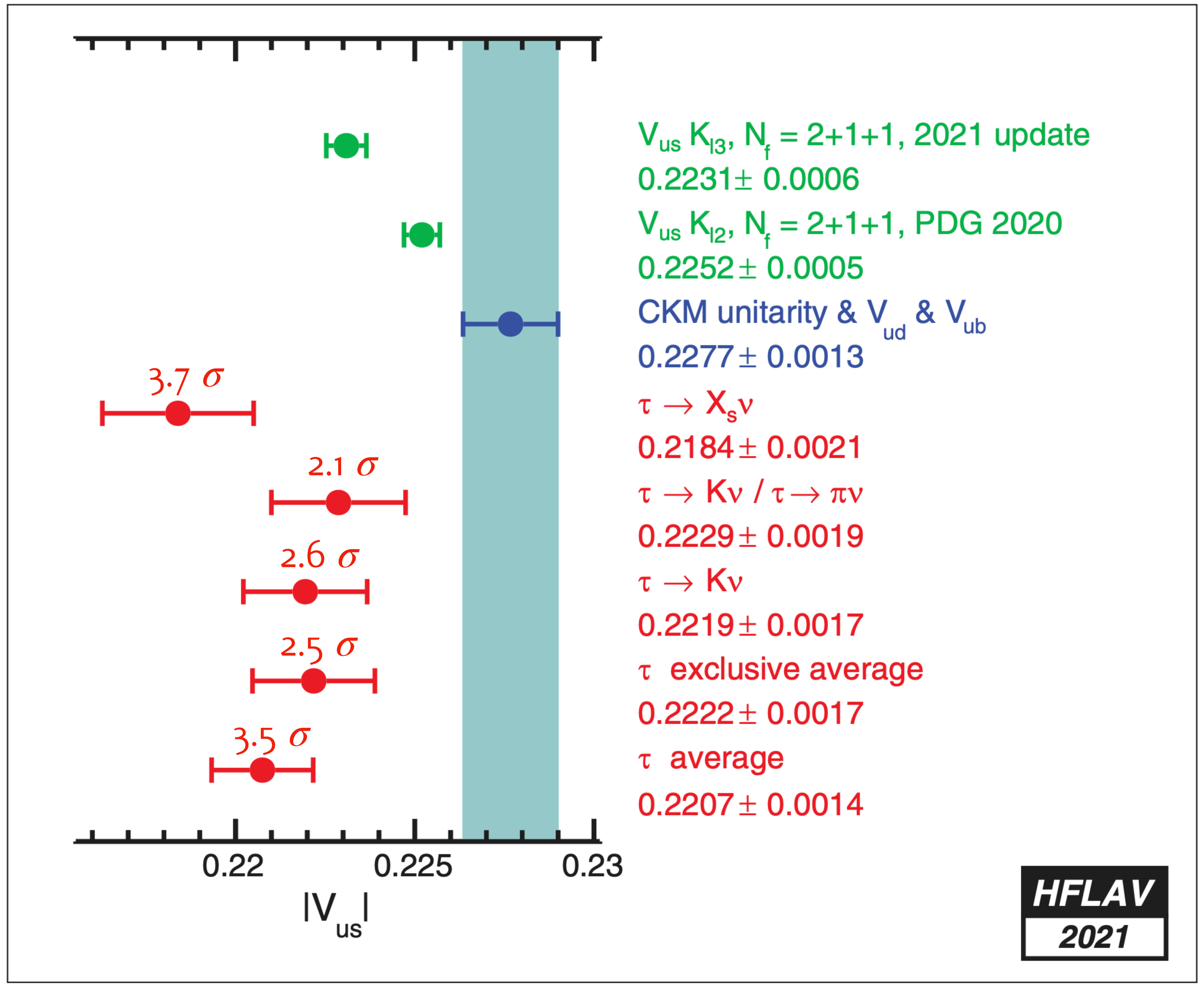
- integration over **three-body phase-space**
- problems of **analytical continuation** from Euclidean to Minkowski when intermediate on shell $\pi$-$\ell$ states are lighter than external ones

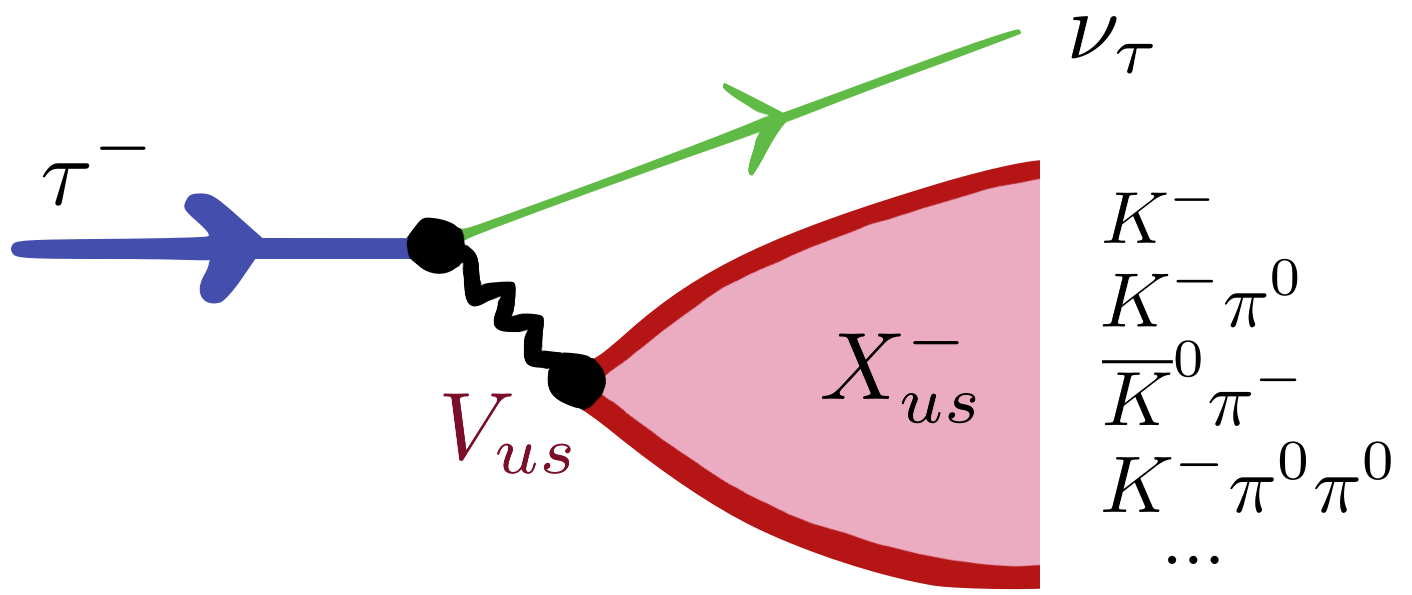
Proper **finite-volume QED formalism is still missing**, but solutions are under study by different groups.

Recent **QED$_\infty$ proposal**: N.Christ et al., PRD 108 (2023) & PoS LATTICE2023 (2024) 266

# Inclusive hadronic $\tau$ decays

Alternative determinations of $|V_{us}|$ can be obtained from inclusive **hadronic $\tau$ decays**




- And yet another **puzzle**: lower value of $|V_{us}|_{\tau-\text{incl.}}$
- Inclusive $\tau \to X_{us}\nu_\tau$ result in HFLAV plot obtained using truncated operator product expansion (OPE)
- Exclusive channels give results larger than $|V_{us}|_{\tau-\text{incl.}}$ but smaller than that obtained imposing CKM unitarity

# Inclusive hadronic $\tau$ decays

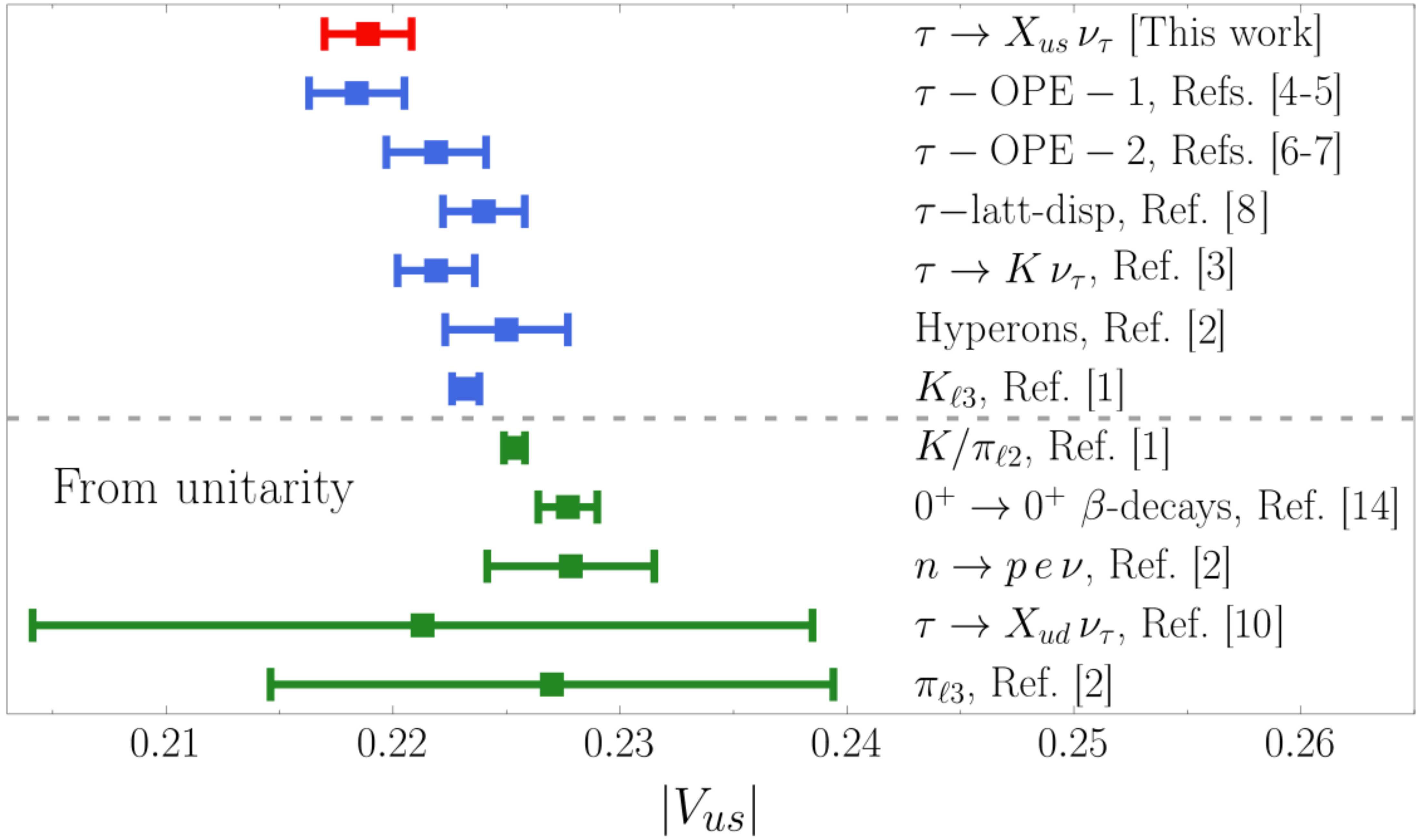
A.Evangelista et al. (ETMC), PRD **108** (2023)
C.Alexandrou et al. (ETMC), PRL **132** (2024)

Recent calculation obtains **inclusive decay rate** using **smeared spectral densities** reconstructed from finite-volume **Euclidean lattice correlators** M.Hansen, A.Lupo & N.Tantalo, PRD **99** (2019)

$$\rho(\omega) = \langle \tau^- | H_{\rm w}^{us}\,(2\pi)\delta(\mathbb{H}-\omega)\,H_{\rm w}^{us}|\tau^-\rangle$$

$$\hat\rho_L(E,\epsilon) = \int_0^\infty \frac{d\omega}{2\pi}\,\Delta_\epsilon(E,\omega)\,\rho_L(\omega) = \sum_{t=0}^{T} g_t(E,\epsilon)\,C_L(t)$$

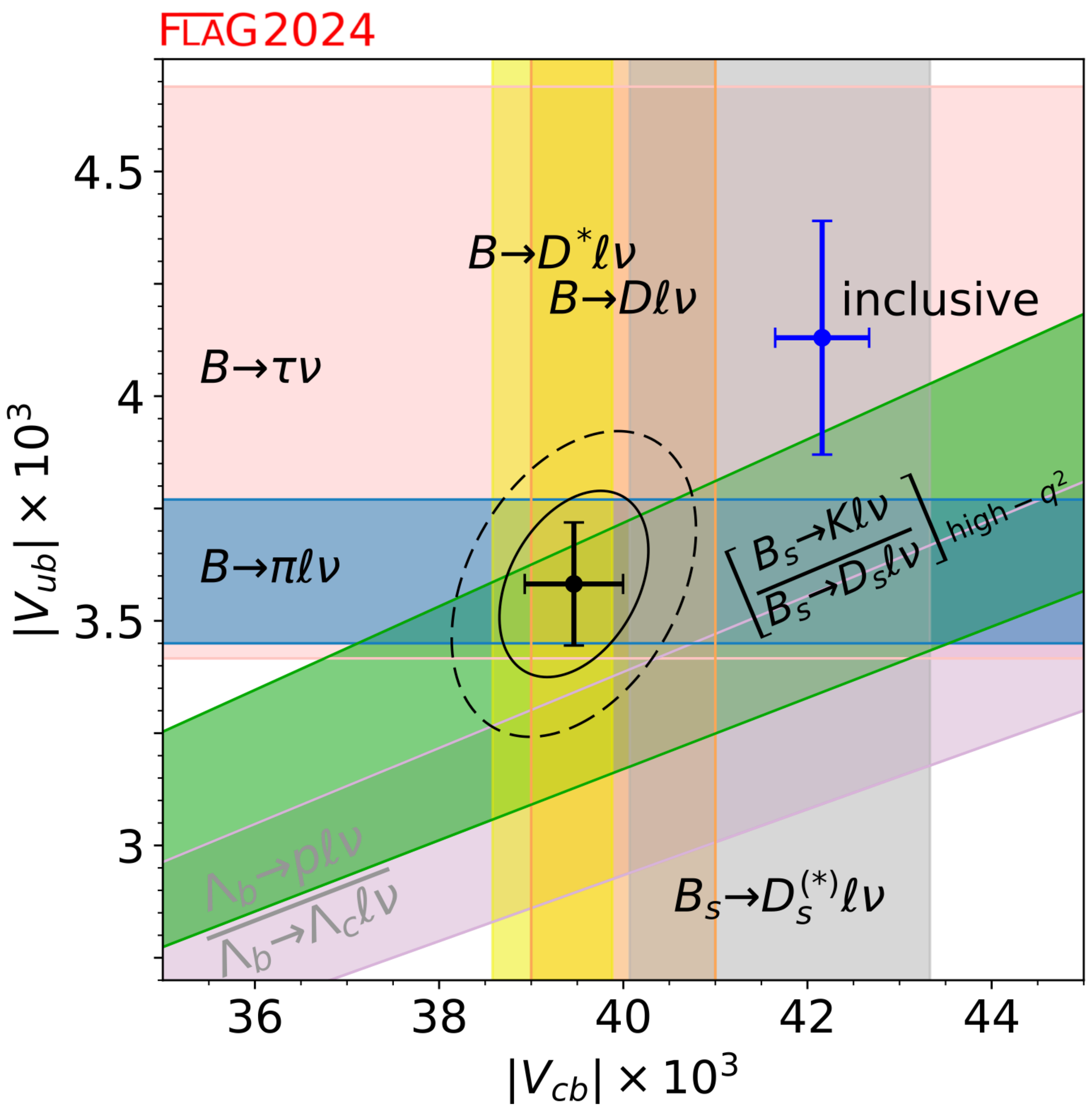
$$\Rightarrow\ \Gamma(\tau\to X_{us}\nu_\tau) = \lim_{\epsilon\to 0}\lim_{L\to\infty}\frac{\hat\rho_L(m_\tau,\epsilon)}{2m_\tau}$$




› **Next steps:** inclusion of QED and strong isospin-breaking effects

Intense activity on **inverse problems in lattice QCD** recently discussed at workshop Lattice@CERN 2024

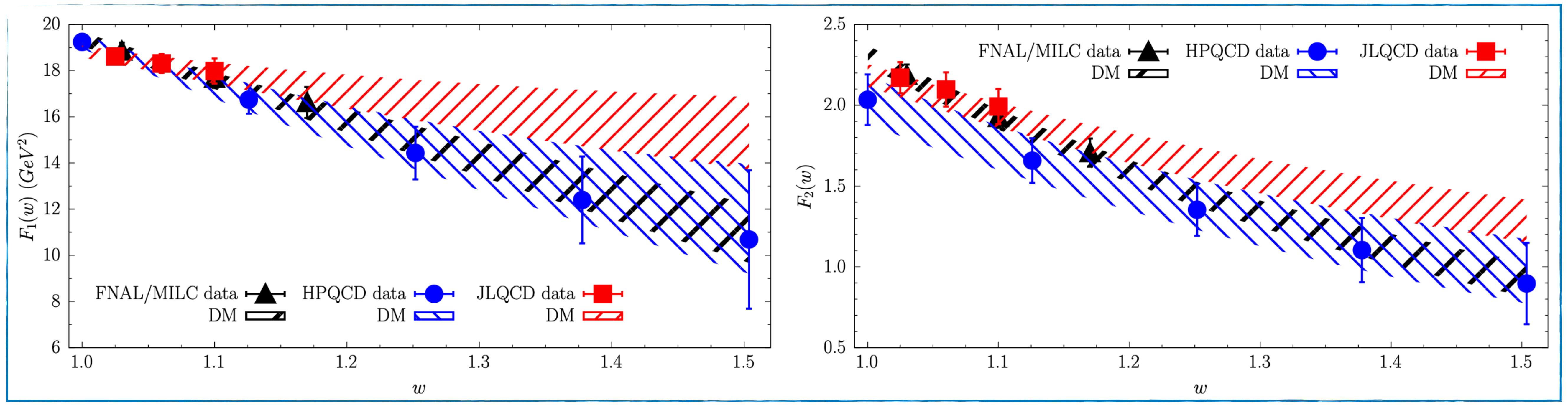
# The $|V_{cb}|$ puzzle



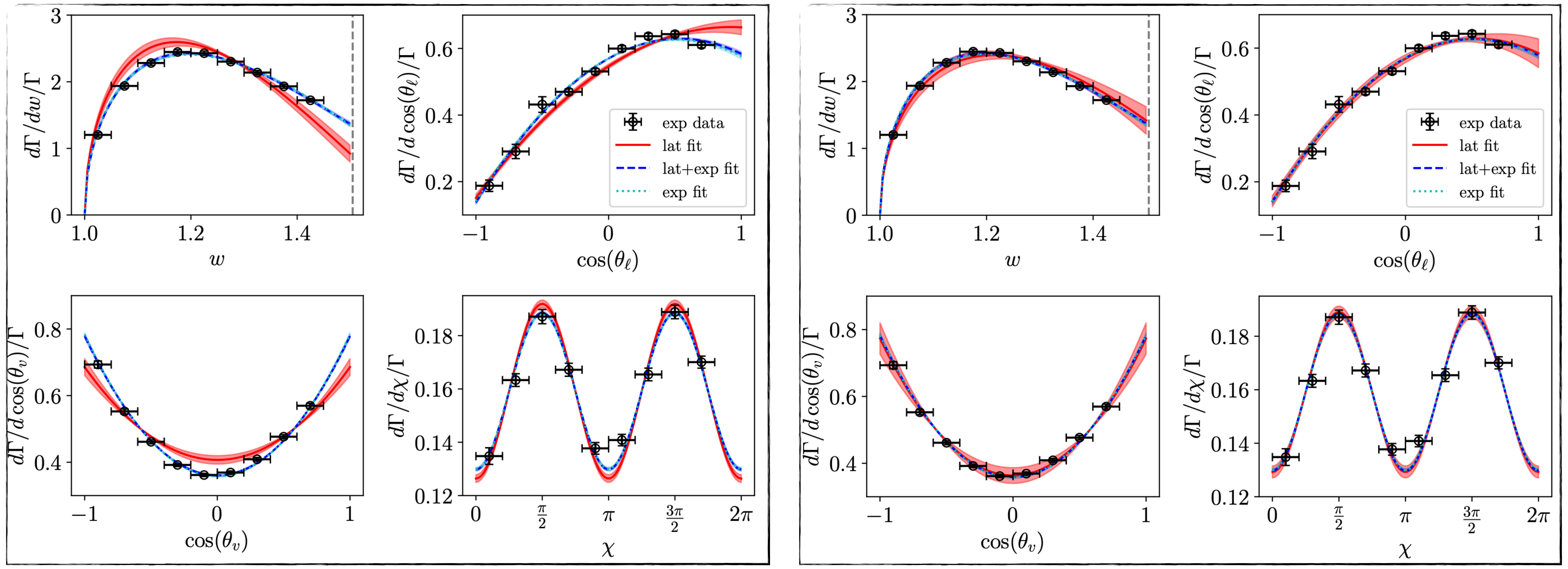

- Long-standing tension between **inclusive** and **exclusive** determinations of $|V_{cb}|$
- Most precise **experimental results** come from $B \to D^* \ell \nu$ decay (Belle and Belle II)
- **Form factors** are computed in lattice QCD by 3 collaborations: **FNAL/MILC**, **HPQCD** and **JLQCD**
- There are some **tensions in the form-factor shapes**, which are currently under investigation

# Comparison of lattice results

G.Martinelli et al., EPJC **84** (2024) 4

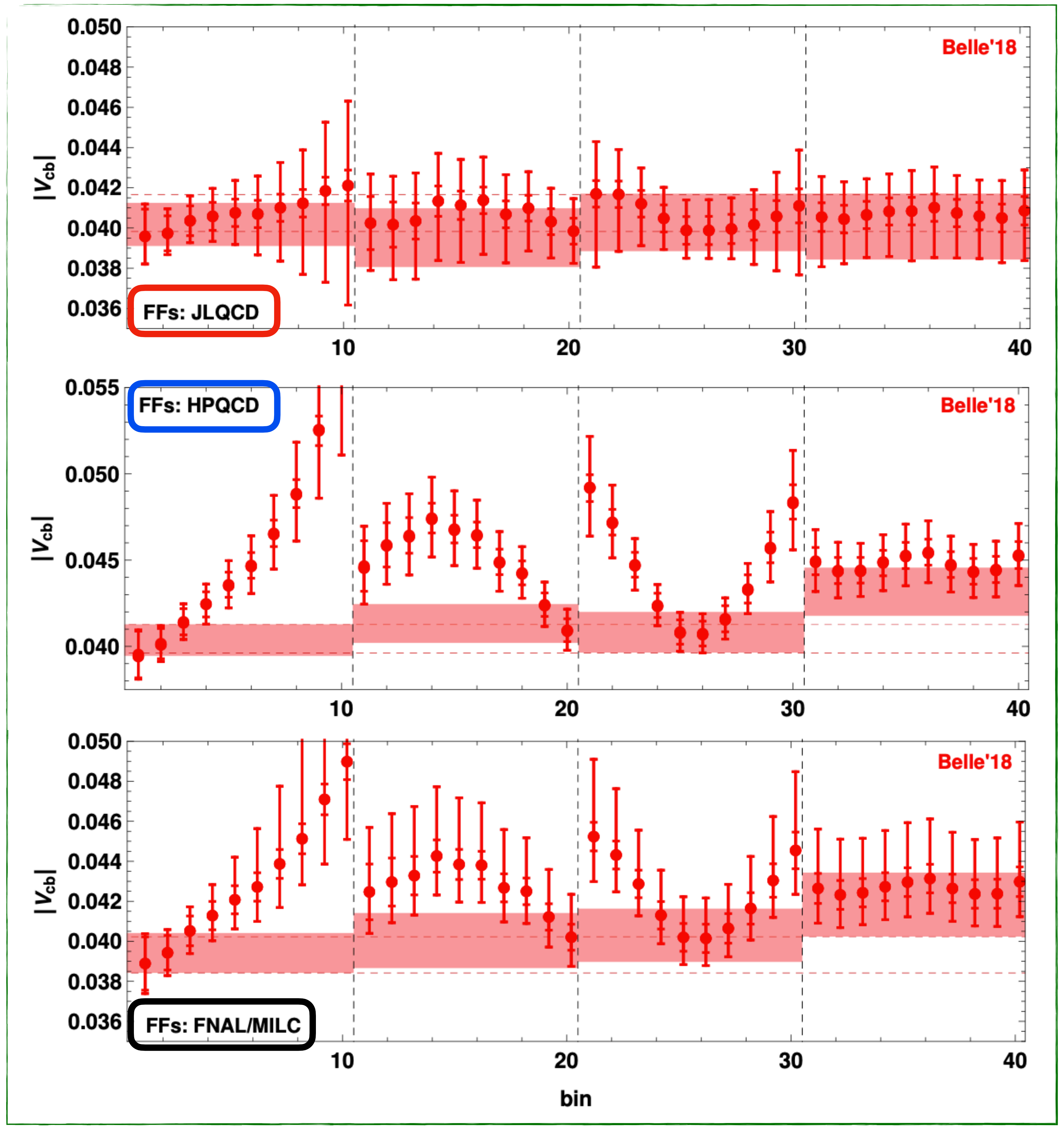



M.Bordone & A.Jüttner, arXiv:2406.10074




FNAL/MILC & HPQCD

JLQCD




M.Jung @ Siegen 2024

# Inclusive decays on the lattice

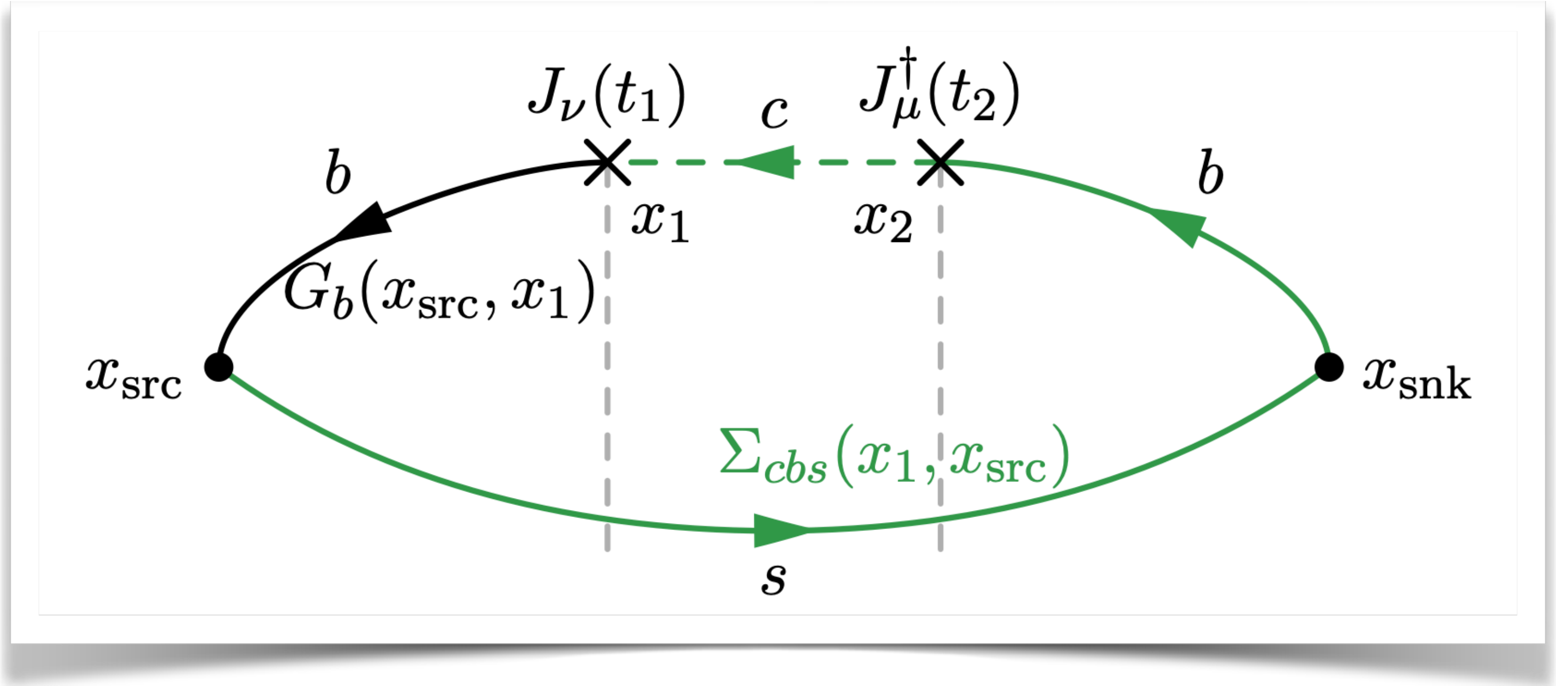
- Interesting proposal by P.Gambino & S.Hashimoto PRL 125 (2020) for lattice calculations of **inclusive semileptonic decays:** alternative way to get independent determination of $V_{cb}$.

- **Use spectral reconstruction techniques** to evaluate

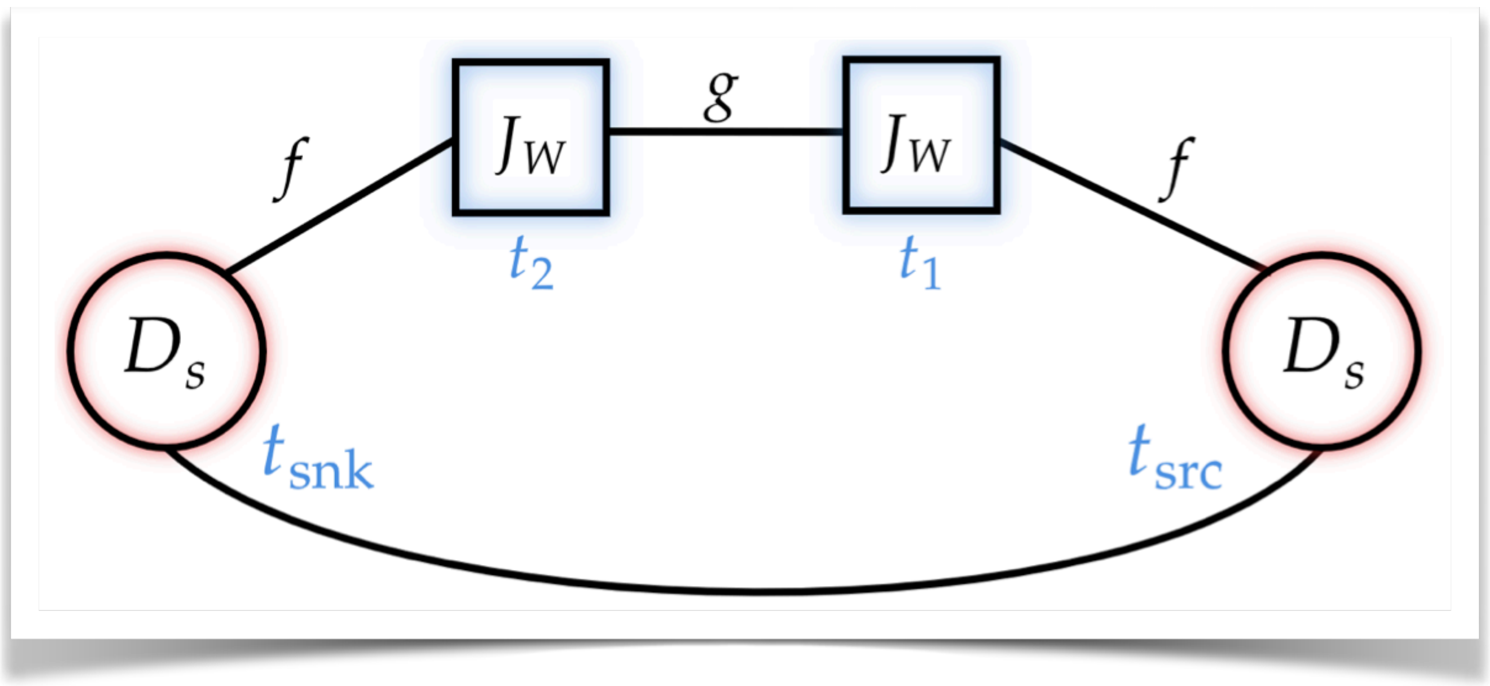
$$\frac{d\Gamma}{dq^2 dq_0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu} \longrightarrow W^{\mu\nu} \sim \sum_X \langle B|J^\mu|X\rangle\langle X|J^{\nu\dagger}|B\rangle$$

- Ongoing work using different approaches on B and D meson decays

A.Barone et al., JHEP **07** (2023) 145




A.De Santis & C.Groß (ETMC) @ Lattice 2024



# CP violation in neutral kaons

flavour eigenstates

$$i\frac{\partial}{\partial t}\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}$$

weak eigenstates

$$|K_{L,S}\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(\bar{\epsilon}\,|K_\pm\rangle + |K_\mp\rangle\right)$$

CP eigenstates

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\overline{K}^0\rangle\right)$$

**Neutral mesons** can mix because the **flavour eigenstates** are different from the **weak eigenstates**

- "Indirect" **CP violation** in the **mixing**

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$$

PDG, PTET **2022** (2022)

- "Direct" **CP violation** in the **decay**

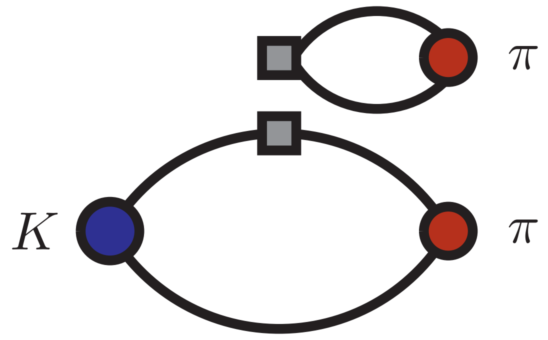
$$\eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H}_{\rm w} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_{\rm w} | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_{\rm w} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_{\rm w} | K_S \rangle} = \epsilon - 2\epsilon'$$

$$\mathrm{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}$$

PDG, PTET **2022** (2022)

# Direct CP violation in $K \to \pi\pi$




If **isospin-symmetry** is conserved, then the CP violation parameters can be expressed as

$$\frac{\epsilon'}{\epsilon} = \frac{i e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon} \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \left[ \frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right]$$

$$A_I = \langle (\pi\pi)_I | H_W^{\Delta S=1} | K \rangle$$

$\delta_I =$ $\pi\pi$ scattering phase shifts

( $I =$ isospin)

1. **RBC-UKQCD** performed **first calculation** of $\epsilon'$ in 2015 — Z.Bai et al., PRL **115** (2015)
2. **Improved result** in 2020: 3.5x more statistics + improved systematics — R.Abbott et al., PRD **102** (2020)

lattice: $\mathrm{Re}(\epsilon'/\epsilon) = 21.7\,(2.6)_{\mathrm{stat.}}(8.0)_{\mathrm{sys.}} \times 10^{-4}$

experiments: $\mathrm{Re}(\epsilon'/\epsilon) = 16.6\,(2.3) \times 10^{-4}$

# Direct CP violation in $K \to \pi\pi$

## Systematic error budget

(from C.Kelly @Lattice2023)

- (~12%) Perturbation theory in Wilson coeffs to match 3f – 4f weak EFT at $m_c$
  - Improve with 4f calculation (active charm) : computationally infeasible?
  - Non-perturbative calculation of matching matrix : investigation underway

    [M.Tomii, *PoS* LATTICE2018 (2019) 216]
- (~23%) Lack of EM+isospin-breaking contributions in lattice calculation
  - Lattice measurement of these effects extremely challenging but approach is being formulated. [*Phys.Rev.D* 106 (2022) 1, 014508] [Christ, *PoS* LATTICE2021 (2022) 312]

    V.Cirigliano et al., JHEP **02** (2020)
- (~12%) Use of single lattice spacing to compute I=0 amplitude
  - Repeat calculation with multiple, finer lattice spacings: **my current focus**

- Intense work by RBC-UKQCD to reduce ~12% error due single lattice spacing (C.Kelly @Lattice2024) + parallel ongoing project using different computational approach (M.Tomii @Lattice2024)

- IB correction will soon become relevant! (but very tricky to compute on the lattice) Usually O(1%), but the "$\Delta I = 1/2$ rule" can give a ~20x enhancement in $\epsilon'/\epsilon$.

# Indirect CP violation in kaon mixing

$$\epsilon = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right)$$

$$\tan(\phi_\epsilon) = \frac{\Delta M_K}{\Delta\Gamma_K/2}$$

$$\Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$$

$$\Delta M_K = M_{K_L} - M_{K_S}$$

The quantity $M_{\bar{0}0}$ splits into a short and long distance parts

$$M_{\bar{0}0} = \langle \overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle = \langle \overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{SD} + \langle \overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{LD}$$

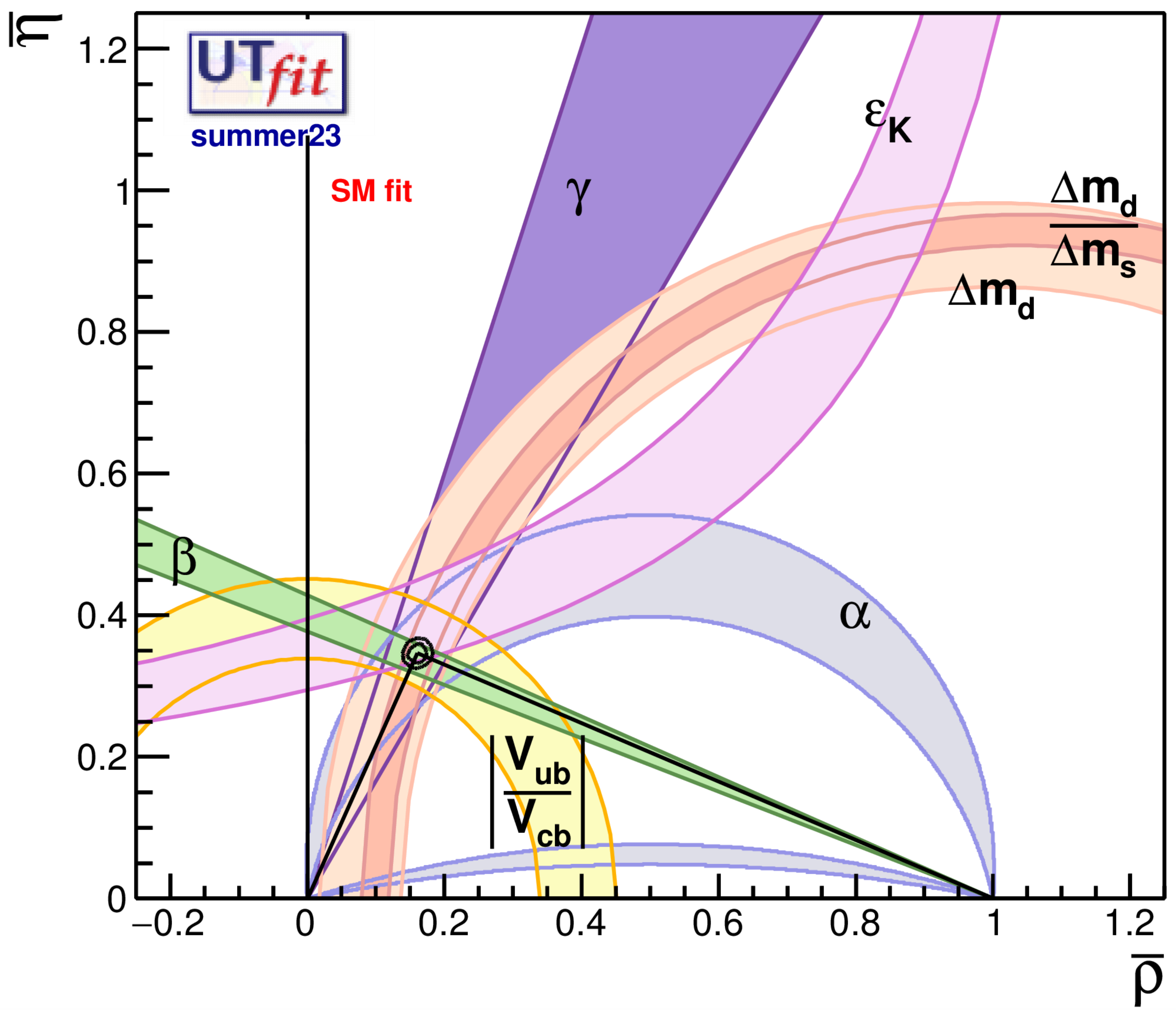
$$= \langle \overline{K}^0|\mathcal{H}_\mathrm{w}^{\Delta S=2}|K^0\rangle + \mathcal{P}\sum_n \frac{\langle \overline{K}^0|\mathcal{H}_\mathrm{w}^{\Delta S=1}|n\rangle\langle n|\mathcal{H}_\mathrm{w}^{\Delta S=1}|K^0\rangle}{M_K - E_n}$$

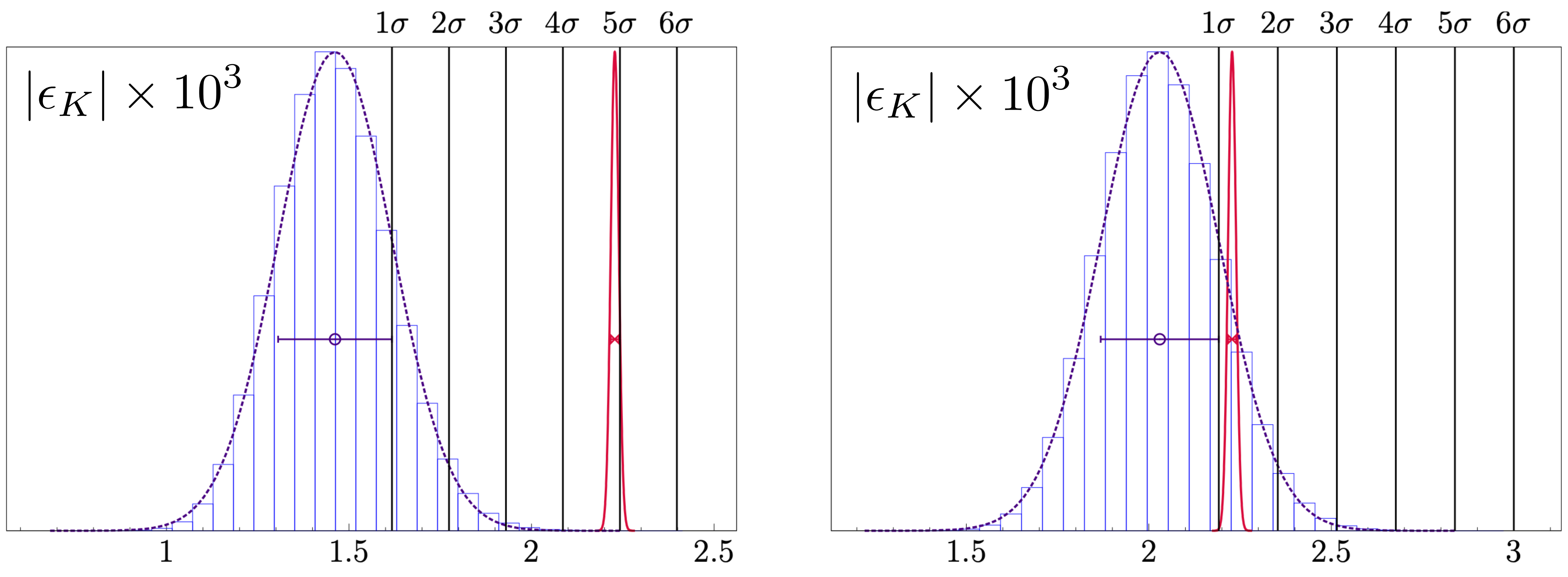
On the **lattice**, we can compute both:

- $\langle \overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{SD}$ e.g. P.A.Boyle et al. PRD 110 (2024) ←— dominating contribution
- $\langle \overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{LD}$ Z.Bai et al. PRD 109 (2024)

# CP violation in neutral kaons

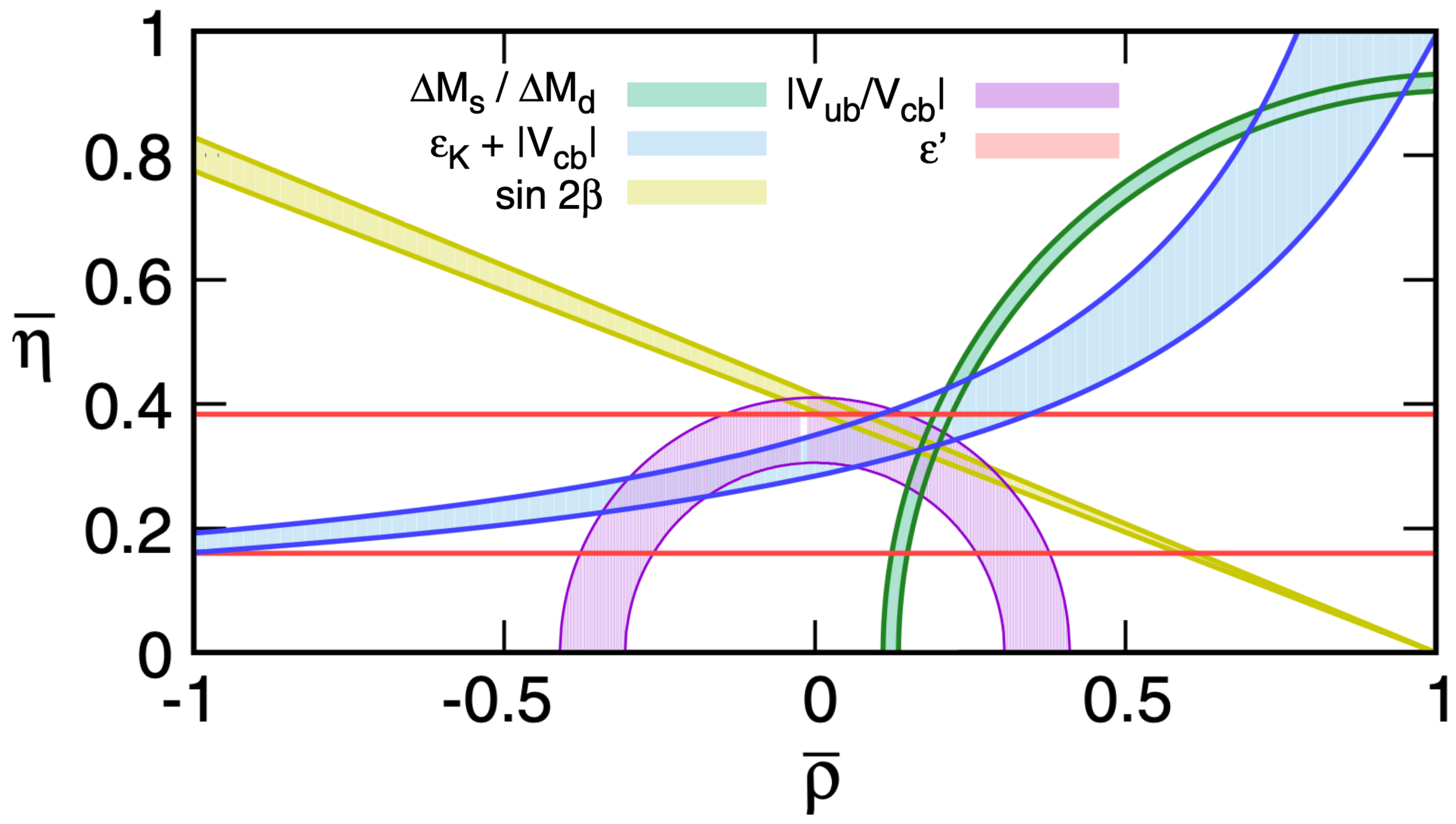
- $\mathrm{Re}(\epsilon'/\epsilon)$ currently at **40% precision.**
  Significant improvements expected in next couple of years
- $(\epsilon_K)_{\mathrm{LD}}$ also at **40% precision.**
  Errors of ~10% can be achieved on the long term
- Lattice inputs to $(\epsilon_K)_{\mathrm{SD}}$ can be computed with high precision, but **overall uncertainty is dominated by $|V_{cb}|$**
- **$|V_{cb}|$-puzzle** affects the SM prediction for $|\epsilon_K|$




**(a)** Exclusive $|V_{cb}|$ (FNAL/MILC-22, BGL) **(b)** Inclusive $|V_{cb}|$ (HFLAV-23, 1S scheme)

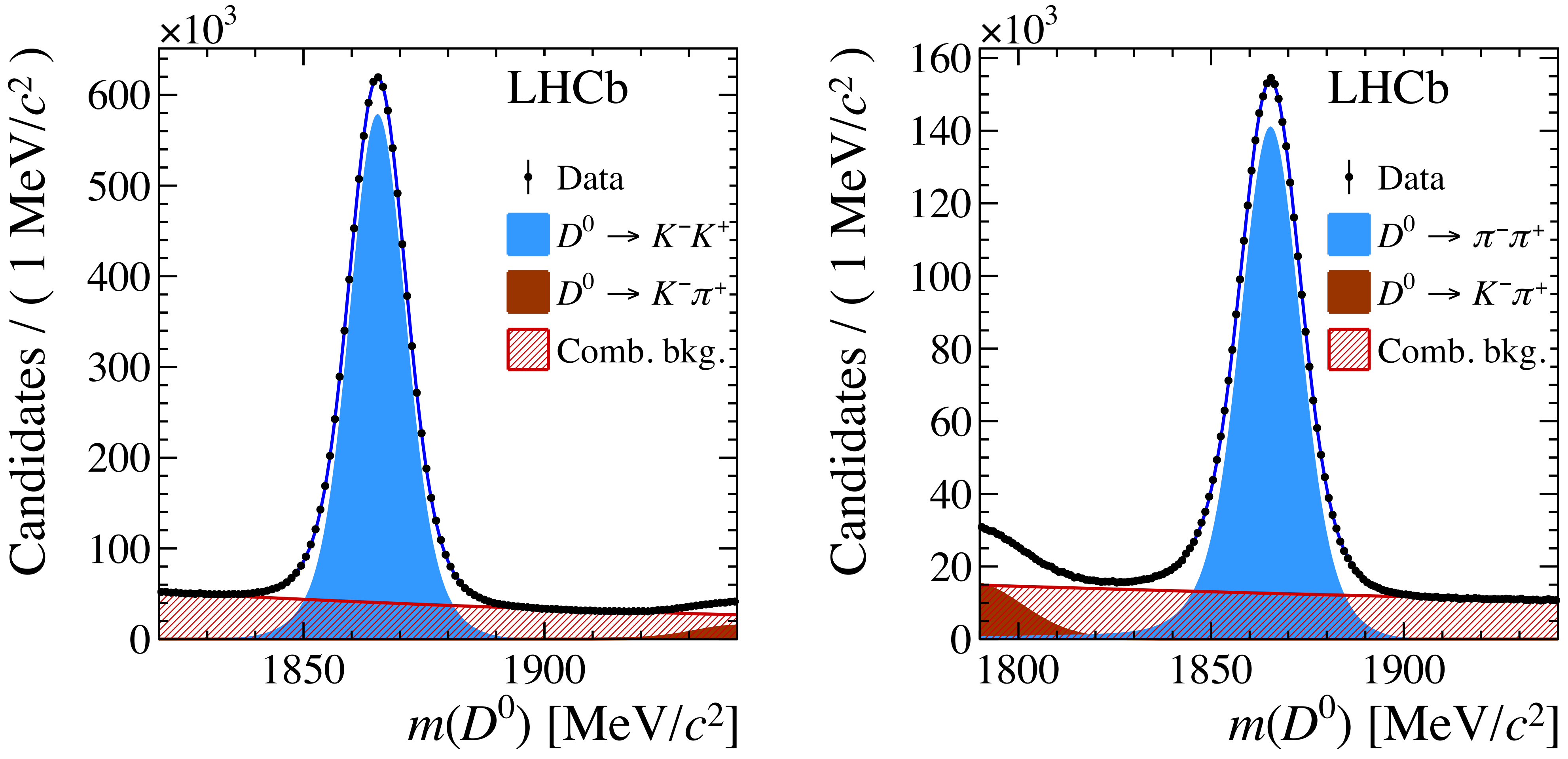
S.Jwa et al., PoS LATTICE2023 (2024) 160




R.Abbott et al., PRD **102** (2020)

# The charm sector

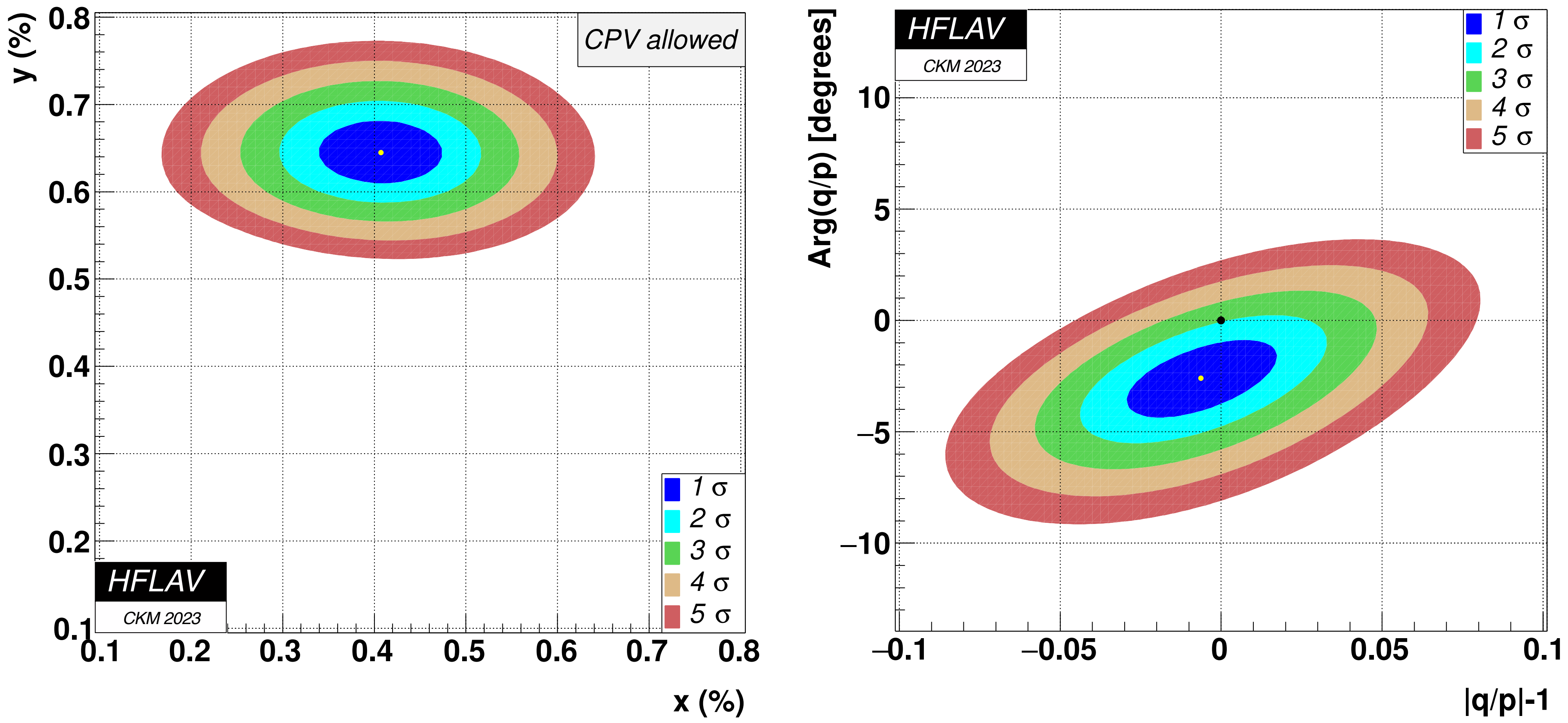
## Hadronic $D$ decays




LHCb, PRL **122** (2019)

- $\Delta A_{\mathrm{CP}} = A_{\mathrm{CP}}(K^-K^+) - A_{\mathrm{CP}}(\pi^-\pi^+)$
  $= (-15.4 \pm 2.9) \times 10^{-4}$

## $D^0$-$\overline{D}^0$ mixing

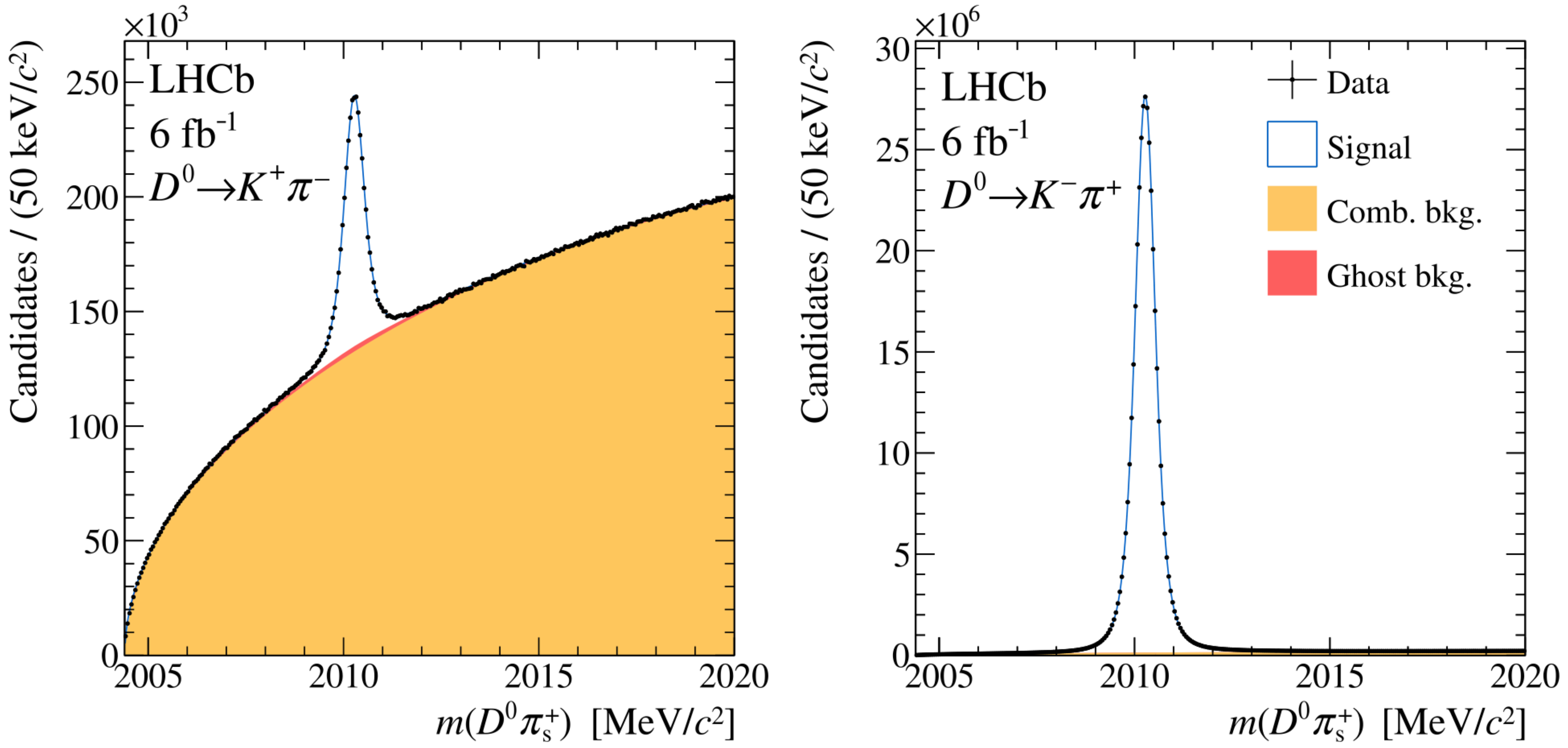



HFLAV, PRD **107** (2023)

- no-mixing point excluded at > 11.5 $\sigma$
- no evidence for indirect CP violation

# The charm sector

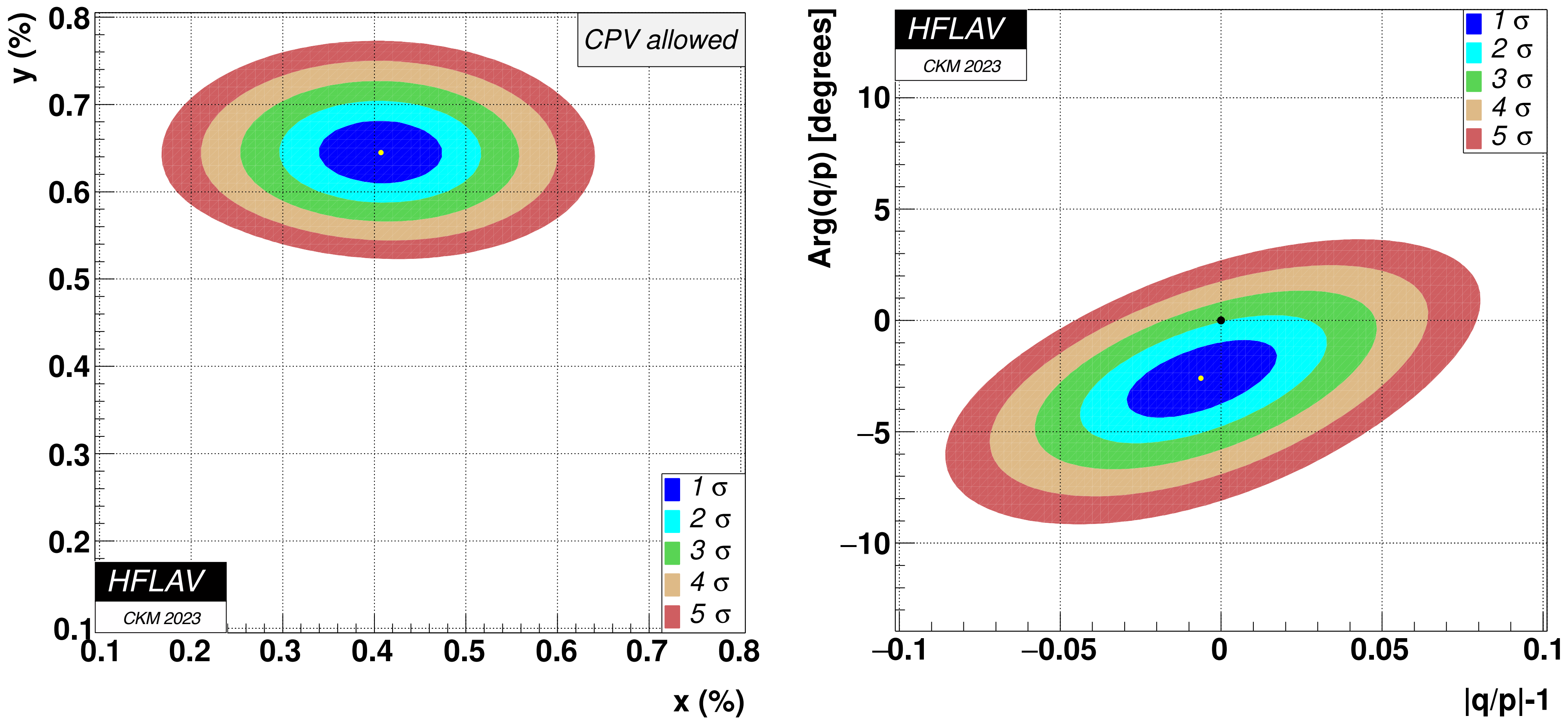
## Hadronic $D$ decays




LHCb, [arXiv:2407.18001]

- Measure of CP violating parameters in $D^0 \to K^\pm\pi^\mp$

## $D^0$-$\overline{D}^0$ mixing




HFLAV, PRD **107** (2023)

- no-mixing point excluded at > 11.5 $\sigma$
- no evidence for indirect CP violation

# Towards hadronic D decays on the lattice

- First exploratory calculation of $D \to K\pi$ at $SU(3)_F$ symmetric point F.Joswig et al., PoS LATTICE2022 (2023)
- Difficult calculation with **many challenges**

$$A(D \to h_1 h_2) = \mathcal{C}^{\mathsf{LL}}_{n,L,h_1 h_2} \left[ \lim_{a \to 0} Z^{\overline{\mathrm{MS}}} \langle n, L | \mathcal{H}_W | D, L \rangle \right]$$

- Non-perturbative renormalization of four-quark operators ●●●●○
- Reliable creation of excited multi-hadron final states ●●●○○
- Removal of discretization effects (enhanced by the charm mass) ●●●○○
- Formalism to relate finite-volume matrix elements to the amplitudes ●●●●○
- Extraction of the matrix element from three-point functions ●●○○○

from M.T.Hansen @Lattice2023

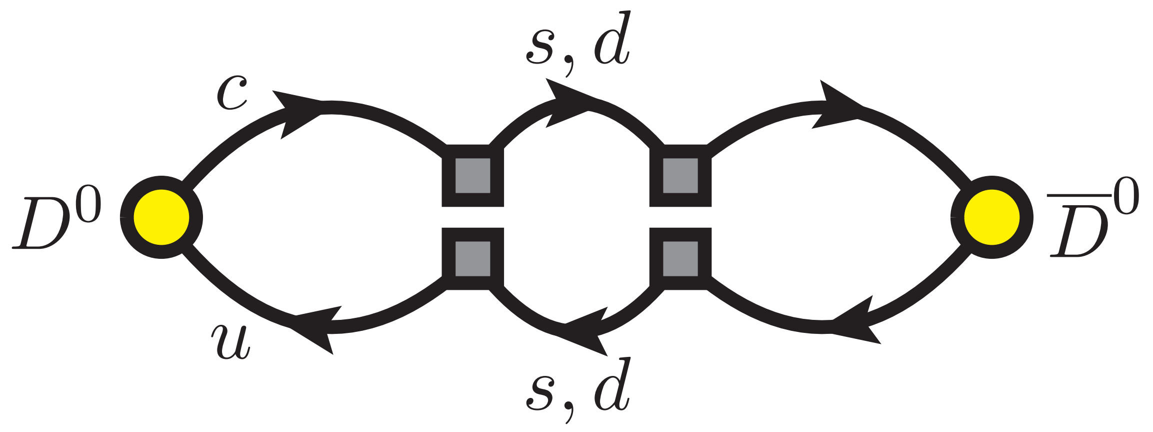
# $D^0$-$\overline{D}^0$ mixing on the lattice?

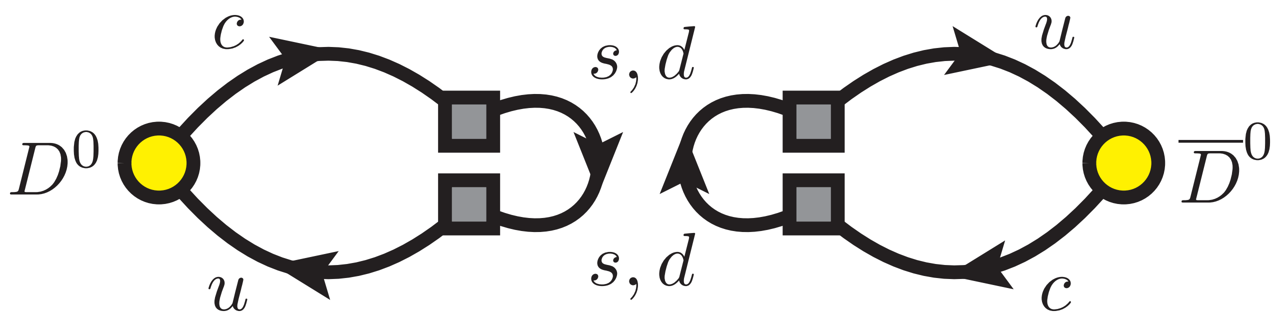
- **long distance** contributions dominate
- huge number of intermediate states makes **direct calculations** impossible
- we can take advantage of recent developments in **spectral density reconstruction techniques**
- use $i\epsilon$-prescription as a "**smearing**" of spectral densities related to the mixing parameters

$$\rho(\omega) = \langle \bar{D}^0 | H_\mathrm{w}\, (2\pi)\delta(\mathbb{H} - \omega)\, H_\mathrm{w} | D^0 \rangle$$

$$\hat{\rho}_L(E, \epsilon) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_L(\omega)}{\omega - E - \mathrm{i}\epsilon}$$
$$= \sum_{t=0}^{T} \left\{ g_t^\mathrm{R}(E, \epsilon) + \mathrm{i}\, g_t^\mathrm{I}(E, \epsilon) \right\} C_L(t)$$

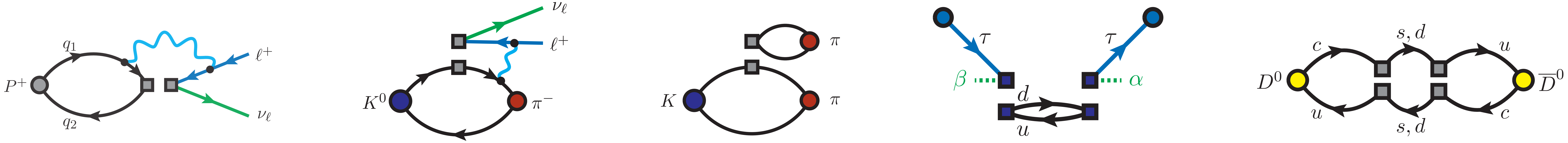
$$M_{12} \propto \lim_{\epsilon \to 0} \lim_{L \to \infty} \mathrm{Re}[\hat{\rho}_L(m_D, \epsilon)]$$
$$\Gamma_{12} \propto \lim_{\epsilon \to 0} \lim_{L \to \infty} \mathrm{Re}[\hat{\rho}_L(m_D, \epsilon)]$$



# Conclusions

- **Flavour physics** offers unique opportunities for **indirect searches** of New Physics
- **Lattice QCD** is at a mature stage on many flavour observables and is now in the **precision era**:
  towards physical pion masses, QED & isospin-breaking effects, physical b quarks, ...
- Progress expected on **Cabibbo anomaly**, $V_{cb}$ **puzzle** and **CPV in kaons** in next years.
  Maybe some of the tensions will be clarified?
- **Developments of new techniques & algorithmic advances** make it possible to study **long-distance observables** considered before unaccessible!



# Thank you

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# Backup slides

# Prospects for $|V_{us}/V_{ud}|$

## An exercise on the error budget

$$\left|\frac{V_{us}}{V_{ud}}\right|^2 = \left[\frac{\Gamma(K_{\ell 2})}{\Gamma(\pi_{\ell 2})}\frac{M^3_{K^+}}{M^3_{\pi^+}}\frac{(M^2_{K^+}-M^2_{\mu^+})^2}{(M^2_{\pi^+}-M^2_{\mu^+})^2}\right]_{\text{exp}} \cdot \left[\frac{f_{K,0}}{f_{\pi,0}}\right]^2 (1+\delta R_{K\pi})$$

- Using our new result $\delta R_{K\pi} = -0.0086\,(13)(39)_{\text{vol.}}$

| $[f_{K,0}/f_{\pi,0}]$ | | $\lvert V_{us}/V_{ud}\rvert$ |
|---|---|---|
| FLAG21 2+1 average | 1.1930 (33) | $0.23154\ (28)_{\text{exp}}\ (15)_{\delta R}\ (45)_{\delta R,\text{vol.}}\ (65)_{f_P}$ |

- Using RM123S result $\delta R_{K\pi} = -0.0126\,(14)$

| $[f_{K,0}/f_{\pi,0}]$ | | $\lvert V_{us}/V_{ud}\rvert$ |
|---|---|---|
| FLAG19 2+1+1 average | 1.1966 (18) | $0.23131\ (28)_{\text{exp}}\ (17)_{\delta R}\ (35)_{f_P}$ |