

Many-body quantum dynamics simulations with the Ibisco resource

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April 19 2024

Ig-Nobel prize



Ig-Nobel prize



Deepak Chopra

Quantum physics: Why is it hard?

100 SPINS = SIZE 10^{30}

1000 SPINS = SIZE 10^{300}

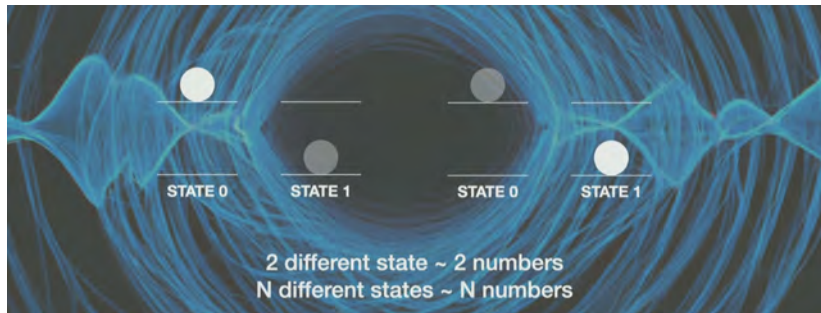
$|\psi\rangle$

2 SPINS = SIZE 4

10 SPINS = SIZE 1024

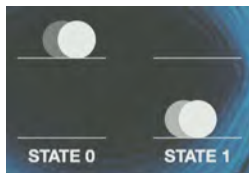
50 SPINS = SIZE 10^{15}

Classical states



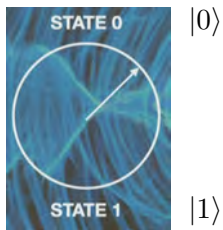
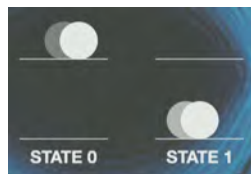
Quantum states

Quantum superposition



Quantum states

Quantum superposition

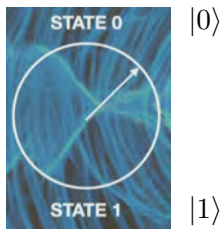


$|0\rangle$

$|1\rangle$

Quantum states

Quantum superposition



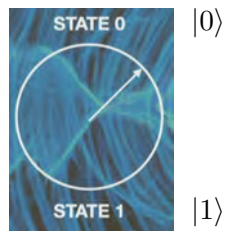
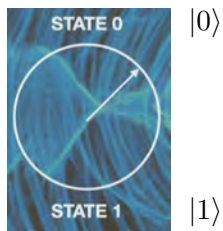
$|0\rangle$

Quantum state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

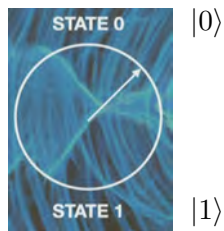
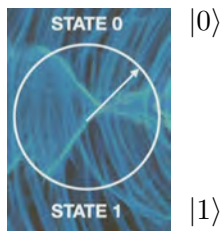
$|1\rangle$

Quantum states



$$|\psi_{\text{combined}}\rangle = A |01\rangle + B |00\rangle + C |10\rangle + D |11\rangle$$

Quantum states



$$|\psi_{\text{combined}}\rangle = A |01\rangle + B |00\rangle + C |10\rangle + D |11\rangle$$

L systems $\implies 2^L$ complex parameters

Quantum physics is hard

$$2^L$$



Quantum physics is hard

$$2^L$$

Schrödinger eq.

$$i \dot{|\Psi\rangle} = \underbrace{\hat{H}}_{2^L \times 2^L!} |\Psi\rangle$$



Quantum physics is hard

$$2^L$$

Schrödinger eq.

$$i |\dot{\Psi}\rangle = \underbrace{\hat{H}}_{2^L \times 2^L!} |\Psi\rangle$$



Expectation

$$A = \langle \Psi | \hat{A} | \Psi \rangle$$

Quantum physics is hard

$$2^L$$

Schrödinger eq.

$$i |\dot{\Psi}\rangle = \underbrace{\hat{H}}_{2^L \times 2^L!} |\Psi\rangle$$



Expectation

$$A = \langle \Psi | \hat{A} | \Psi \rangle$$

Measure

$$|\Psi\rangle \rightarrow |A_n\rangle \quad p_n = |\langle A_n | \Psi \rangle|^2$$

Quantum physics is hard

Schrödinger eq.

$$i|\dot{\Psi}\rangle = \underbrace{\hat{H}}_{2^L \times 2^L!} |\Psi\rangle$$

 2^L 

Expectation

$$A = \langle \Psi | \hat{A} | \Psi \rangle$$

POVM Measure

$$|\Psi\rangle \rightarrow \frac{\hat{K}_n |\Psi\rangle}{\langle \Psi | \hat{K}_n^\dagger \hat{K}_n | \Psi \rangle} \quad p_n = \langle \Psi | \hat{K}_n^\dagger \hat{K}_n | \Psi \rangle$$

How I learned to stop worrying and love the hardness



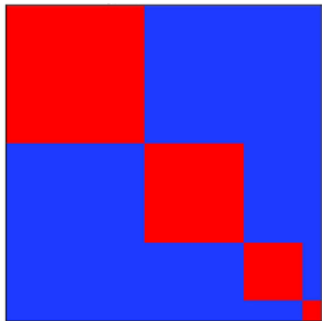
How I learned to stop worrying and love the hardness

Computational resouces



How I learned to stop worrying and love the hardness

Symmetries



How I learned to stop worrying and love the hardness

Algorithms

Uses C_1 , Write ($y_2 := 7 \cdot y_1$, $n := 3$, y_1); $m := m + 1$; Inc(n); Until $\text{abs}(\text{slog}) < \text{eps}$.
 Program $x_1, h, x, y_1, y_2, \text{slog}, \text{eps}, \text{real}$; End
 Inc(n); For $i := 1$ to 168 do write ('-');
 WriteLn; n^2 End; a : array [1..10] of
 Write ('Enter ...'); For $j := 1$ to 3 do
 readLn; $\text{min} := \text{complex}$; y, Eps, k
 $C_3 = B \cdot (-A)$; $\text{min} := \text{mod}$; real ; $n := 0$; $y_2 := 0$;
 WriteLn; For $i := 1$ to 10 do
 i : word; x, Eps ; $\text{Complex} := \text{record}$ n, i, j : integer;
 Type
 Program 2Func; ($A \leftarrow x$) and ($x \leftarrow B$) and (A) WriteLn; Begin
 Repeat $m := 1$; $\text{ReadLn}(a[i], re, a[i], im)$ WriteLn ('| $f_2(x)|N|f_2(x)|N$);
 (name) $y := 1$; $i := 1$; $k := 1$; WriteLn ('Enter the complex number'); For $i := 1$ to 67 do write ('-');
 Begin $S_1 := 1$; $i := 1$; $k := 1$; WriteLn ('The elements are:'); $y_1 := 1/s_1 \cdot (\text{sqr}(1 - \text{sqr}(x)) \cdot \ln(1+x)) / (1-x^2)$
 Uses α ; $S_2 := 1/(1-x) \cdot (S)$; $\text{WriteLn}(\alpha[i], re, \alpha[i], im)$; End; $x_1 := 2 \cdot \text{sqr}(1 - \text{sqr}(x)) \cdot \arctan(x) - y(x)$;
 $C_2 = A \cdot (-B)$; $\text{WriteLn}(\text{Complex} \cdot \text{Mass}$; For $i := 1$ to 200 do write ($x \cdot 5 \cdot 2$);
 $C_4 = (-A) \cdot (-B)$; n, i, j : integer; $[A]_{rc} = 0.0100010101010110$; $\text{eps} := 1e-3$;
 Write $y := y + S_1 \cdot x[k]$; $x_1 := -0.18$;
 End. Var $(1-5-5x-x^2)$ $x_1 := -0.6$; $[A]_{rc} = 1.0100010101010110$; For $i := 1$ to 10 do
 WriteLn ($(2-4-6-x)$); $h := 0.05$; $[A]_{rc} = 1.0101010001000100$; + $\text{sqr}(a[i], im)$;
 $x := x \cdot h$; $x := x_0$; {Form the table for results} If $\text{abs}(x) > 0.001$ then $\text{slog} := 1$;
 $C_2 = A \cdot (-B)$; For $i := 1$ to 67 do write ('-'); $\text{min_module} := 8$; For $j := 1$ to 3 do
 $C_1 = A + B$; End; Repeat $y_2 := y_2 + \text{slog}$; Inc(n); $\text{sqr}(\text{sqr}(a[i], re))$ $y_2 := 0$; $n := 0$; Begin
 $C_4 = (-A) \cdot (-B)$; x^2 Until $\text{abs}(\text{slog}) < \text{eps}$; $\text{min_re} := a[i], re$; $x \cdot \exp((4 \cdot n + 4) \cdot \ln(\text{abs}(x))) /$
 $\text{WriteLn}(\alpha[i], re)$; $\text{min_re} := a[i], re$; $(4 \cdot n - 1) / (4 \cdot h \cdot 3) / (4 \cdot n - 5)$
 WriteLn ('x|f(x)|eps = 0.001|eps = 0.001|eps = 0.000001'); writeLn; For $j := 1$

Publications using Ibisco

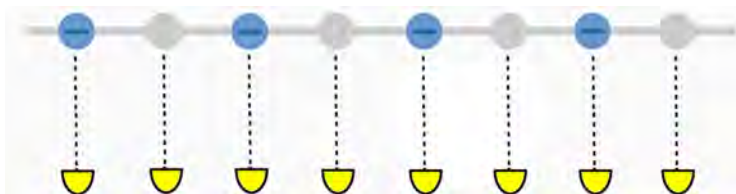
- ▶ Angelo Russomanno, Giulia Piccitto, Davide Rossini,
Entanglement transitions and quantum bifurcations under continuous long-range monitoring,
Physical Review B, **108**, 104313 (2023).
- ▶ Martina Minutillo, Procolo Lucignano, Gabriele Campagnano, and Angelo Russomanno,
Kitaev ring threaded by a magnetic flux: Topological gap, Anderson localization of quasiparticles, and divergence of supercurrent derivative,
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The impact of different unravelings in a monitored system of free fermions,

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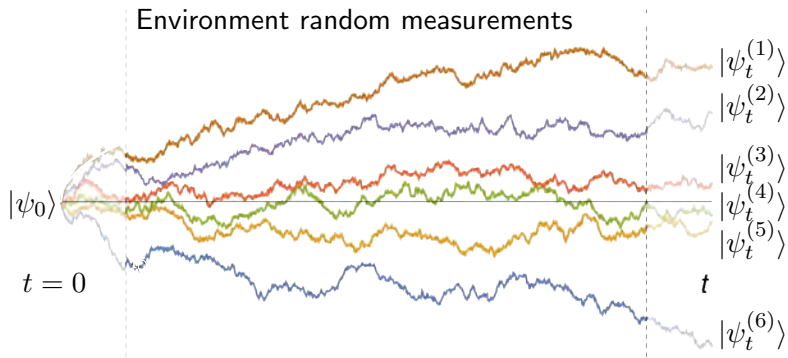
Effect of a classical environment

Environment random measurements



Quantum trajectories

"Brownian motion"



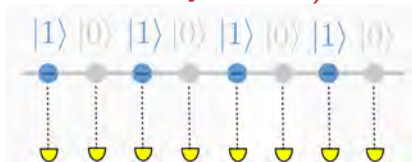
Huge parallelization using MPI

I ❤️
YOU

Coupling to an environment (monitored dynamics)

fermionic Hamiltonian

$$\hat{H} = \frac{1}{2} \sum_{j=1}^L (\hat{c}_j^\dagger \hat{c}_{j+1} + \hat{c}_{j+1}^\dagger \hat{c}_j)$$



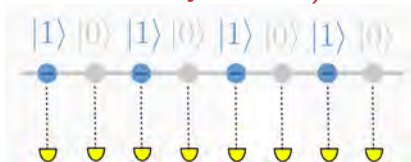
$$\frac{d}{dt} \rho_t = -i[\hat{H}, \rho_t] + \gamma \sum_{j=1}^L \hat{n}_j \rho_t \hat{n}_j - \frac{1}{2} \{ \hat{n}_j, \rho_t \} \quad \text{Lindblad equation}$$

$$\hat{n}_i^2 = \hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$$

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Quantum state diffusion unraveling

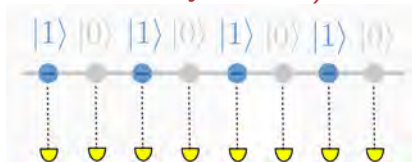
$$\overline{|\psi_t\rangle \langle \psi_t|} = \rho_t$$

$$d|\psi_t\rangle = -i\hat{H}dt|\psi_t\rangle + \underbrace{\sum_{i=1}^L \left(\sqrt{\gamma}[\hat{n}_i - \langle \hat{n}_i \rangle_t] dW_t^i - \frac{\gamma}{2}[\hat{n}_i - \langle \hat{n}_i \rangle_t]^2 dt \right)}_{\text{Non-Hermitian stochastic quadratic Hamiltonian}} |\psi_t\rangle$$

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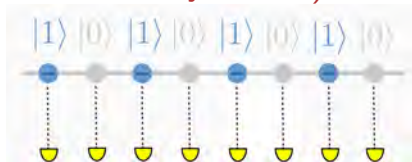
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Gaussian (Wiener process) $\langle dW_t^i dW_{t'}^j \rangle = \delta(t-t') \delta_{ij} dt$

Coupling to an environment (monitored dynamics)

fermionic Hamiltonian

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Gaussian (Wiener process) $\langle dW_t^i dW_{t'}^j \rangle = \delta(t-t') \delta_{ij} dt$

$|\psi_t\rangle$ Slater determinant (discretize time and multiply $L \times L$ matrices)

Coupling to an environment (monitored dynamics)

Trotterize

$$|\psi(t + \delta t)\rangle \propto e^{\sum_j [\delta W_t^j + (2\langle \hat{n}_j \rangle_t - 1)\gamma\delta t]} \hat{n}_j e^{-i\hat{H}\delta t} |\psi(t)\rangle + \mathcal{O}(\gamma^2\delta t^2)$$

- ▶ Slater determinant $|\psi_t\rangle = \prod_k \left(\sum_j U_{jk}(t) \hat{c}_j^\dagger \right) |0\rangle$
 $U_{jk}(t) \ L \times L/2, \ U^\dagger U = \mathbf{1}_{N \times N}.$

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Coupling to an environment (monitored dynamics)

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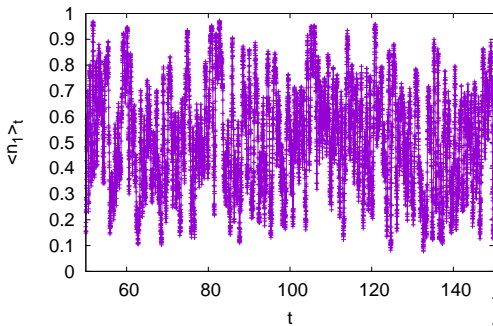
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- ▶ Get $U(t + \delta t)$ with QR decomposition of V'
 (orthonormalization that keeps constant the correlation matrix
 $C = UU^\dagger).$

Coupling to an environment (monitored dynamics)

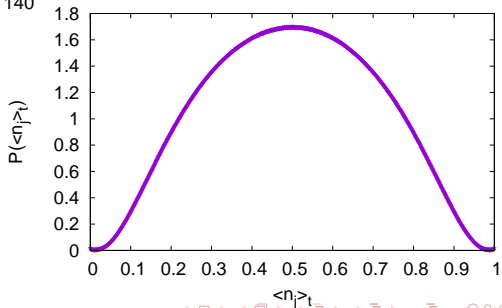
Trotterize

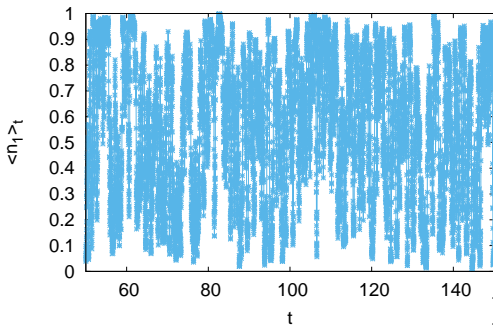
$$|\psi(t + \delta t)\rangle \propto e^{\sum_j [\delta W_t^j + (2\langle \hat{n}_j \rangle_t - 1)\gamma\delta t]} \hat{n}_j e^{-i\hat{H}\delta t} |\psi(t)\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

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 $U_{jk}(t)$ $L \times L/2$, $U^\dagger U = \mathbf{1}_{N \times N}$.
- ▶ Dissipative step $U(t) \rightarrow V = MU(t)$ with $\langle \hat{n}_j \rangle_t$ **Study $\langle \hat{n}_j \rangle_t$**
 $M_{jk} = \delta_{jk} e^{\delta W_t^j + (2\langle \hat{n}_j \rangle_t - 1)\gamma\delta t}$.
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$\gamma = 0.1$ 

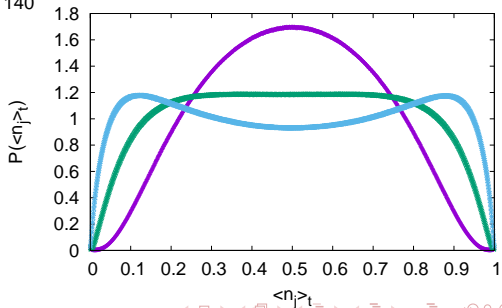
Unimodal distribution

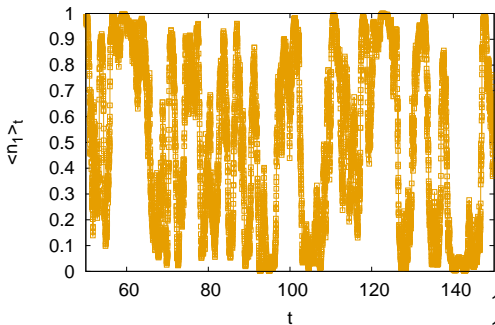
 $L = 128$ 

$\gamma = 0.3$ 

Unimodal distribution

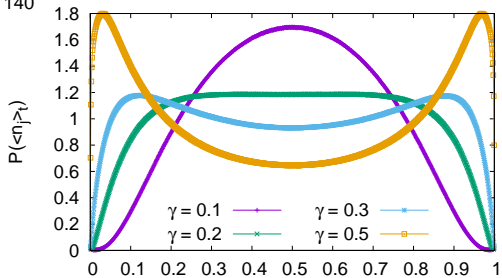
Bimodal distribution

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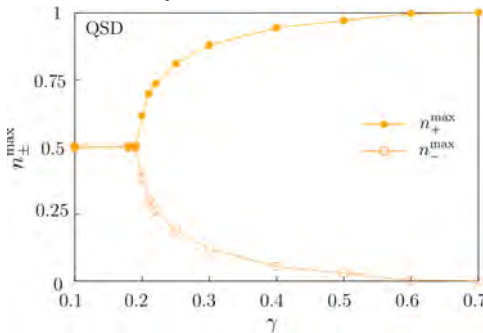
$\gamma = 0.5$ 

Unimodal distribution

Bimodal distribution

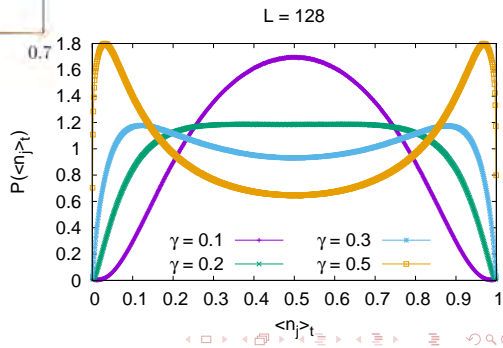
 $L = 128$ 

"Quantum bifurcation"

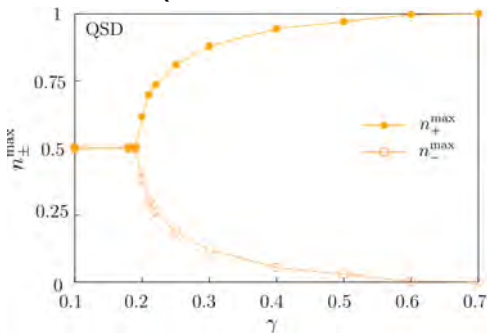


Unimodal distribution

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"Quantum bifurcation"



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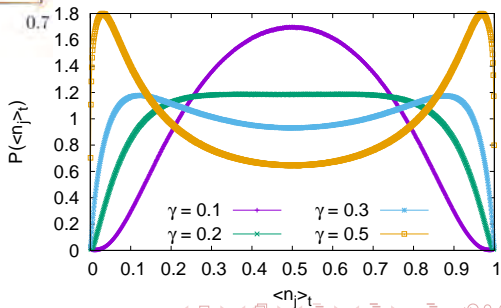
Bimodal distribution

Classical bifurcation
(2nd order transition)Landau free energy $P \sim e^{-\mathcal{F}}$

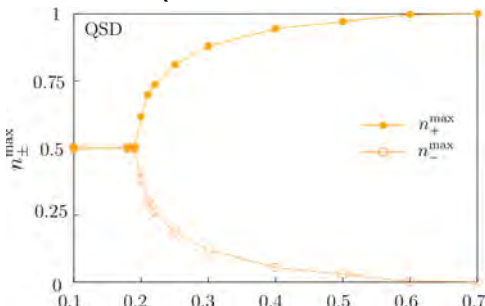
Metropolis algorithm

classical finite-size 2D Ising
magnetization distribution

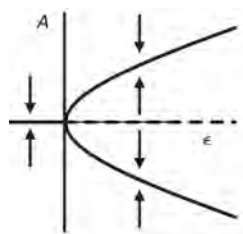
L = 128



“Quantum bifurcation”



$$\tau_0 d_t A = \varepsilon A - gA^3 + \dots$$



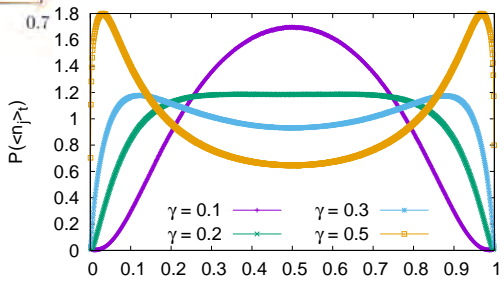
Classical bifurcation in nonlinear dynamics

Classical bifurcation (2nd order transition)

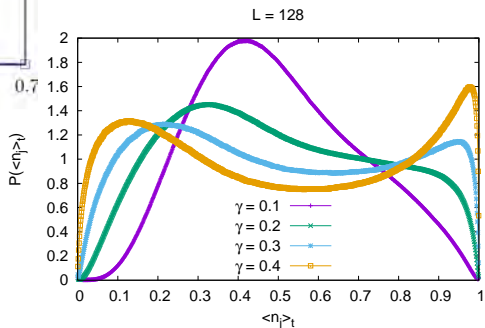
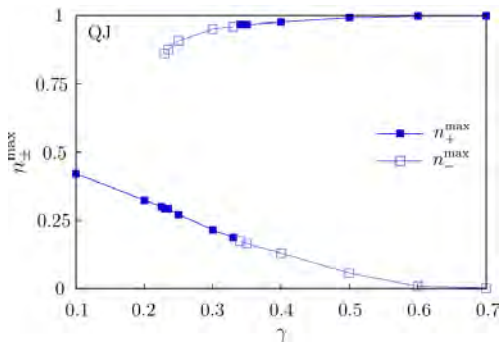
Landau free energy $P \sim e^{-\mathcal{F}}$

Metropolis algorithm
classical finite-size 2D Ising magnetization distribution

L = 128

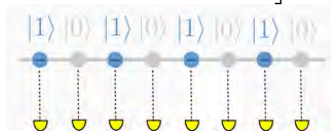


Different (quantum-jump) unraveling

Classical analog
(1st order transition)

Let's get nonintegrable

$$\hat{H} = \sum_{j=1}^L \left[\frac{J}{2} \left(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{H. c.} \right) + \frac{V}{4} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{W}{2} (-1)^j \hat{\sigma}_j^z \right]$$

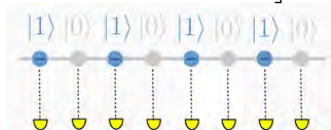


$$|\psi_{t+\delta t}\rangle = (\mathbf{1} - i\hat{H}\delta t) |\psi_t\rangle + \sum_l \left[\delta\xi_l(t) (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \frac{\gamma}{2} \delta t (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2 \right] |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

$$\overline{\delta\xi_j(t)\delta\xi_l(t')} = \delta_{jl}\delta_{tt'}\delta t \quad \text{Gaussian uncorrelated}$$

Let's get nonintegrable

$$\hat{H} = \sum_{j=1}^L \left[\frac{J}{2} \left(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{H. c.} \right) + \frac{V}{4} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{W}{2} (-1)^j \hat{\sigma}_j^z \right]$$



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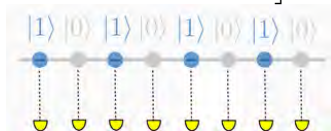
$$\overline{\delta\xi_j(t) \delta\xi_l(t')} = \delta_{jl} \delta_{tt'} \delta t \quad \text{Gaussian uncorrelated}$$

Trotterize

$$|\psi_{t+\delta t}\rangle = \mathcal{N} e^{-i\hat{H}\delta t} e^{\sum_l \left[\delta\xi_l(t) (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \gamma \delta t (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2 \right]} |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

Let's get nonintegrable

$$\hat{H} = \sum_{j=1}^L \left[\frac{J}{2} \left(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{H. c.} \right) + \frac{V}{4} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{W}{2} (-1)^j \hat{\sigma}_j^z \right]$$



$$|\psi_{t+\delta t}\rangle = (\mathbf{1} - i\hat{H}\delta t) |\psi_t\rangle + \sum_l \left[\delta\xi_l(t) (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \frac{\gamma}{2} \delta t (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2 \right] |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

$$\overline{\delta\xi_j(t)\delta\xi_l(t')} = \delta_{jl}\delta_{tt'}\delta t \quad \text{Gaussian uncorrelated}$$

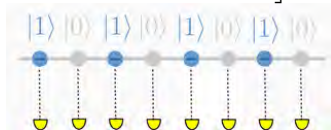
Trotterize

$$|\psi_{t+\delta t}\rangle = \mathcal{N} e^{-i\hat{H}\delta t} e^{\sum_l \left[\delta\xi_l(t) (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \gamma \delta t (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2 \right]} |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

Rotation **symmetry** along $z \implies \hat{S}_z = \frac{1}{2} \sum_l \hat{\sigma}_l^z$ conserved

Let's get nonintegrable

$$\hat{H} = \sum_{j=1}^L \left[\frac{J}{2} \left(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{H. c.} \right) + \frac{V}{4} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{W}{2} (-1)^j \hat{\sigma}_j^z \right]$$



$$|\psi_{t+\delta t}\rangle = (\mathbf{1} - i\hat{H}\delta t) |\psi_t\rangle + \sum_l \left[\delta\xi_l(t) (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \frac{\gamma}{2} \delta t (\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2 \right] |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

$$\overline{\delta\xi_j(t)\delta\xi_l(t')} = \delta_{jl}\delta_{tt'}\delta t \quad \text{Gaussian uncorrelated}$$

Trotterize

$$|\psi_{t+\delta t}\rangle = \mathcal{N} e^{-i\hat{H}\delta t} e^{\sum_l [\delta\xi_l(t)(\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t) - \gamma\delta t(\hat{\sigma}_l^z - \langle \sigma_l^z \rangle_t)^2]} |\psi_t\rangle + \mathcal{O}(\gamma^2 \delta t^2)$$

Rotation **symmetry** along $z \implies \hat{S}_z = \frac{1}{2} \sum_l \hat{\sigma}_l^z$ conservedRestrict to block with $S_z = 0 \implies \dim \mathcal{H}_L = \binom{L}{L/2}$

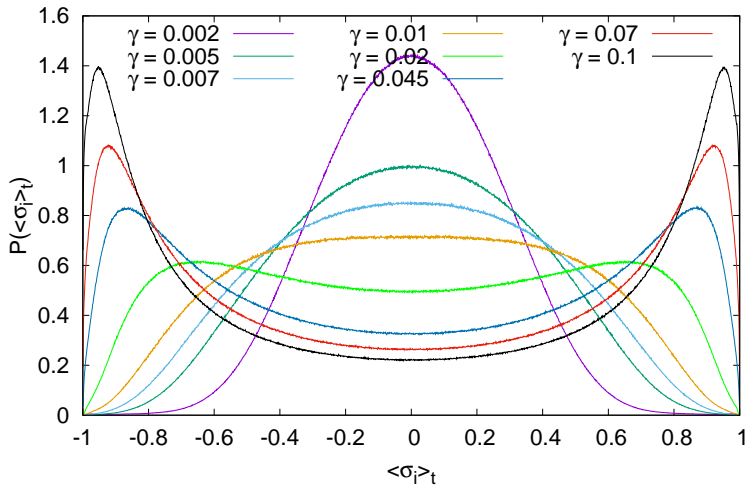
Krylov algorithm

ACM Trans. Math. Softw. 24 (1) (1998)
130–156 (Expokit)

J. Chem. Phys. 85, 5870–5876 (1986)

- ▶ $|\Psi'\rangle = e^{-i\hat{H}\delta t} |\Psi\rangle$.
- ▶ Write the subspace $\text{Span}\{|\Psi\rangle, \hat{H}|\Psi\rangle, \hat{H}^2|\Psi\rangle, \dots, \hat{H}^M|\Psi\rangle\}$ (\hat{H} sparse).
- ▶ Expand the Hamiltonian in this basis (truncate).
- ▶ Converges even for $M \simeq 20$.

Quantum bifurcation



Localization properties

- ▶ Inverse participation ratio $\text{IPR} = \sum_{\{s_j\}} |\langle \{s_j\} | \Psi_t \rangle|^4$.
- ▶ $|\{s_j\}\rangle$ classical spin configurations $\hat{\sigma}_l^z |\{s_j\}\rangle = s_l |\{s_j\}\rangle$.
- ▶ $\text{IPR} \sim 1$ localized.
- ▶ $\text{IPR} \sim 1/\dim \mathcal{H}_L$ fully delocalized.

Localization properties

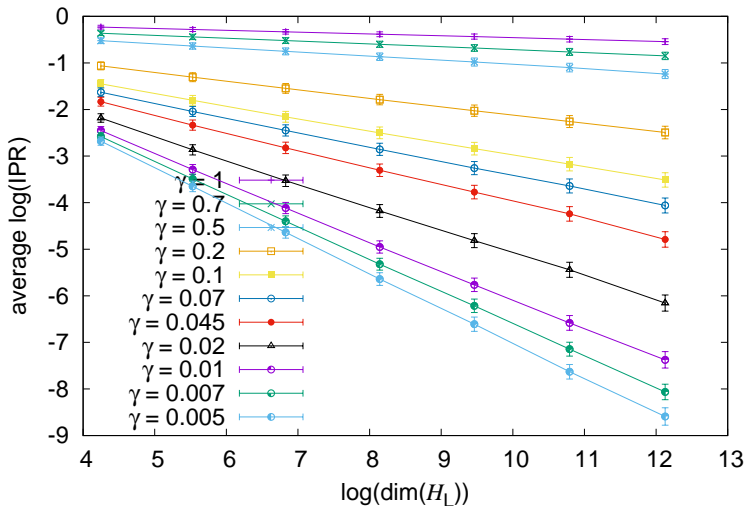
- ▶ Inverse participation ratio

$$\overline{\log(\text{IPR})} = \overline{\log \left(\sum_{\{s_j\}} |\langle \{s_j\} | \Psi_t \rangle|^4 \right)} \quad (\text{average over time and realizations}).$$

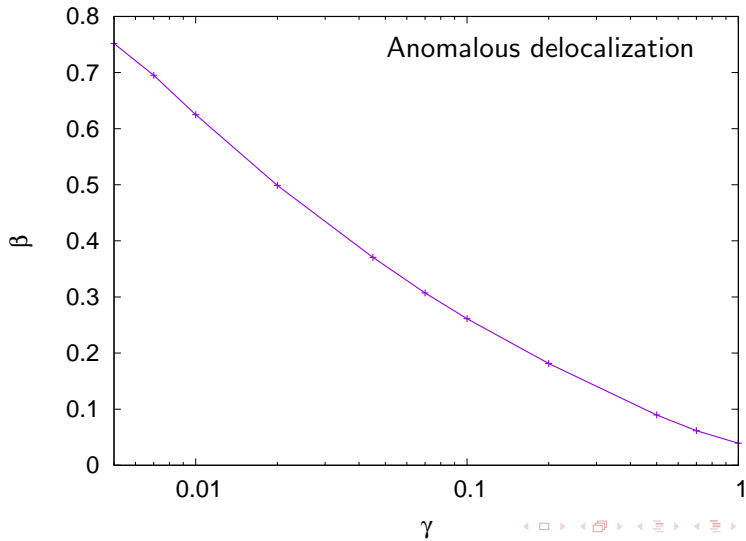
- ▶ $|\{s_j\}\rangle$ classical spin configurations $\hat{\sigma}_l^z |\{s_j\}\rangle = s_l |\{s_j\}\rangle$.
- ▶ $\overline{\log(\text{IPR})} \sim 0$ localized.
- ▶ $\overline{\log(\text{IPR})} \sim -\log \dim \mathcal{H}_L$ fully delocalized.
- ▶ $\overline{\log(\text{IPR})} \sim -\beta \log \dim \mathcal{H}_L$ with $0 < \beta < 1$ anomalously delocalized.

Localization properties

Linear fit $\overline{\log(\text{IPR})} \sim -\beta \log \dim \mathcal{H}_L$



Localization properties



Future challenges

- ▶ Matrix product states and robustness of Many-Body localization to local heating.

- ▶ Neural network states and QAOA dynamics.

Thank you for your attention!

