Computational fluid dynamics of complex multiphase systems through electrohydrodynamic effect (EHD) for microfluidic applications

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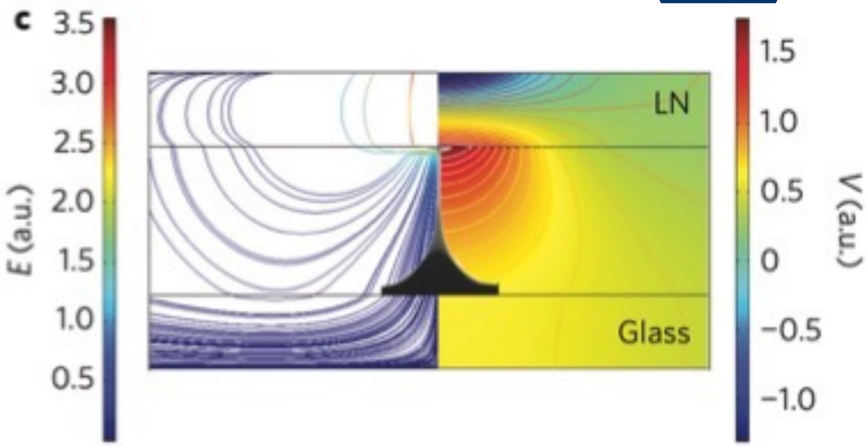
ElectroHydroDynamics (EHD)

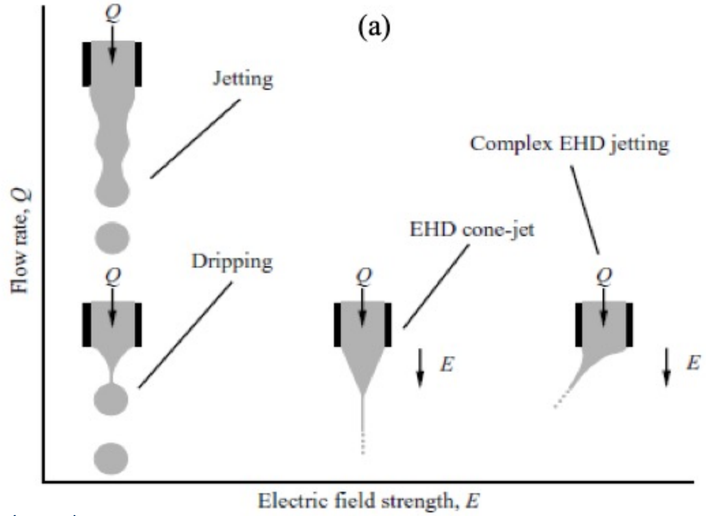
- ❑ Electrohydrodynamic (EHD) describes the motion of liquids subjected to electric fields.

- ❑ Typically, the liquid will be set in motion by electrical stresses, thereby modifying the geometry and charge distribution, which in turn modifies the electric field.

- ❑ The variety of variables affecting EHD often makes difficult to investigate and predict its operation because the difficulty in getting an exact formulation of the dependence by parameters such as:

  - ➢ Applied voltage
  - ➢ Flow rate and/or volume
  - ➢ Electrode configuration
  - ➢ Liquid properties

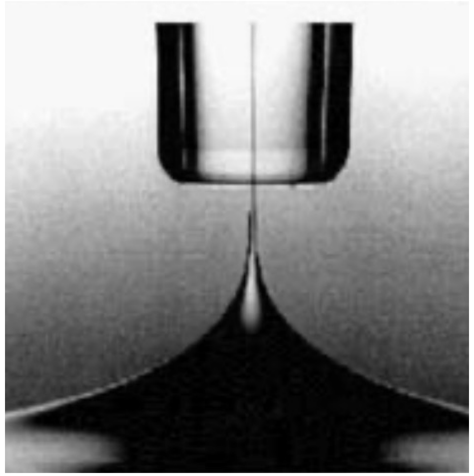




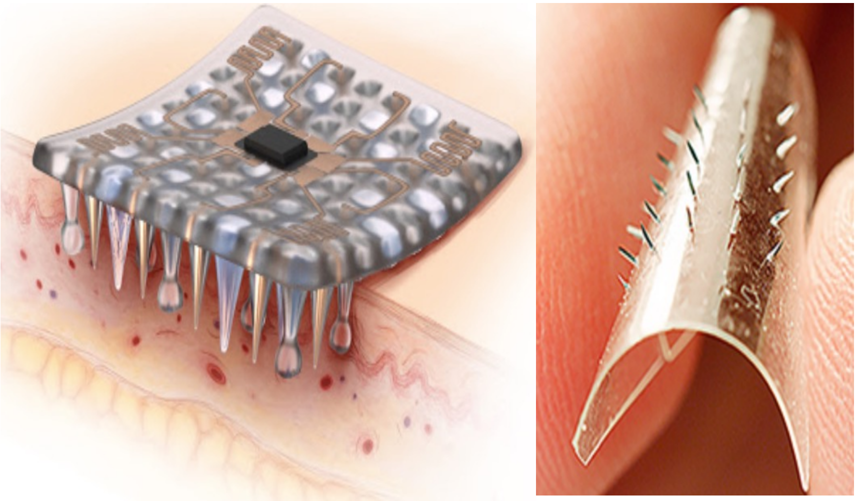
[1] *Electrohydrodynamic (EHD) printing for Advanced Micro/Nano Manufacturing: Current Progresses, Opportunities, and Challenges - Yiwei Han, Jingyan Dong (2018)*

ElectroHydroDynamics (EHD) applications

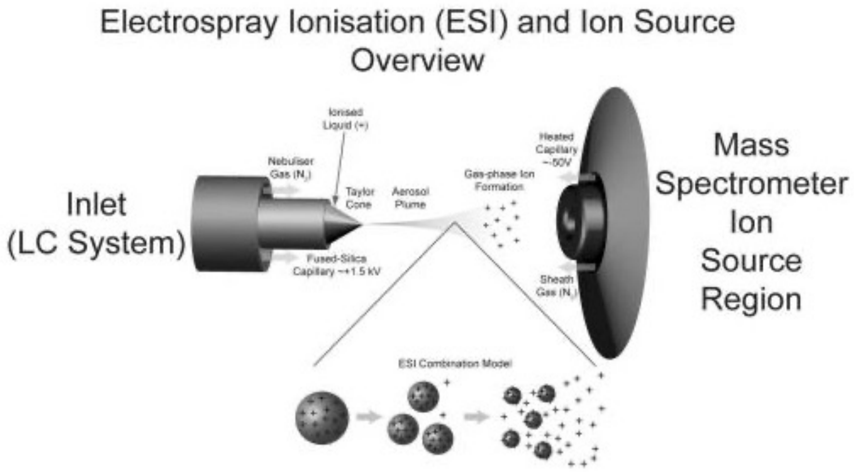
- The EHD effect can be used in numerous applications, especially in microscale systems, such as:



Ink-jet printing



Micro needle



Electrospray for mass spectroscopy

[2] Rahmat, Koca & Yildiza. Additive Manufacturing. (2017)
[3] Higuera, Ibáñez & Hijano, Loscertales. Journal of Aerosol Science. (2013)

Constitutive equations

- A computational method is required to simulate the process, involving the solution of governing equations for fluid flow (specifically, conservation of mass and momentum). Additionally, it necessitates the tracking of the interface between different fluid phases.

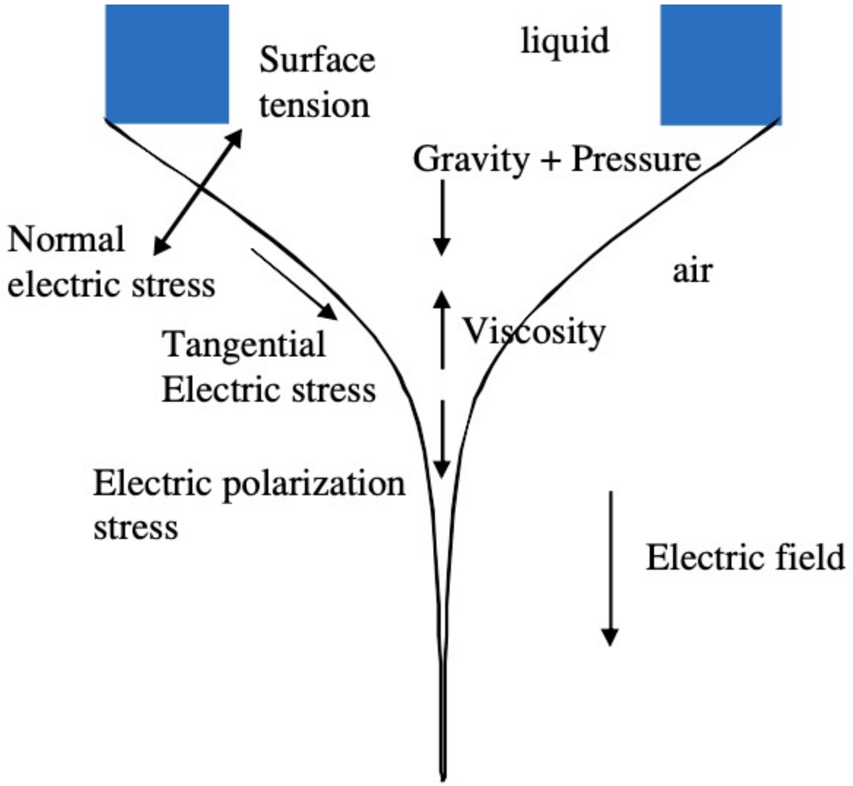
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right] = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho\, \mathbf{g} + \boldsymbol{F_E} + \boldsymbol{F_{ST}}$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \qquad \boldsymbol{J} = K\boldsymbol{E} + \rho_e \boldsymbol{u}$$

$$\nabla \cdot (\varepsilon \boldsymbol{E}) = \rho_e \qquad \boldsymbol{F_e} = \nabla \cdot \mathbb{T}_e = \rho_e \boldsymbol{E} - \frac{1}{2} E^2 \nabla \varepsilon$$

Coulomb Force ($\rho_e \boldsymbol{E}$) — Dielectric Force ($\frac{1}{2} E^2 \nabla \varepsilon$)



Fluid-Fluid interface tracking VOF

- In a two-phase flow, the interface between the phases is moving, the interface can be tracked using different methods among which the VOF (Volume of Fluid)

- This method is based on a volume fraction f where $f = 0$ for the cells filled with fluid 1, $0 < f < 1$ for the cells filled with both fluids and $f = 1$ for the cells filled with fluid 2. The volume fraction f is a scalar function whose transport equation in a standard form is as follows:

$$\frac{\partial f}{\partial t} + \nabla \cdot (\boldsymbol{u} f) = 0$$

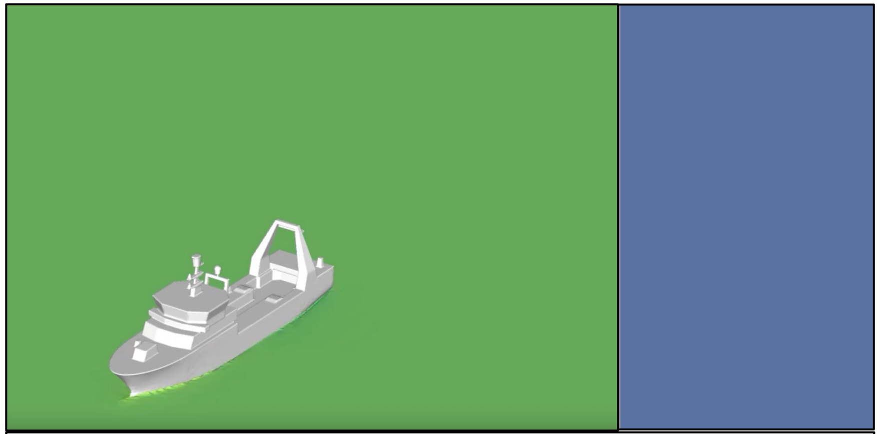
- Two immiscible fluids are considered as a single effective fluid in the whole computational domain:

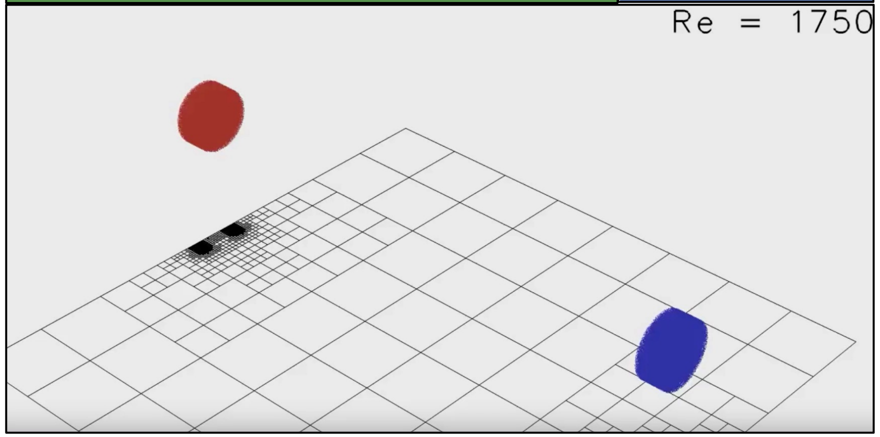
$$\rho = f\rho_o + \rho_i(1 - f) \qquad \mu = f\mu_o + \mu_i(1 - f)$$

$$\frac{1}{\varepsilon} = \frac{f}{\varepsilon_o} + \frac{(1 - f)}{\varepsilon_i} \qquad K = K_i(1 - f)$$

Solver: Basilisk

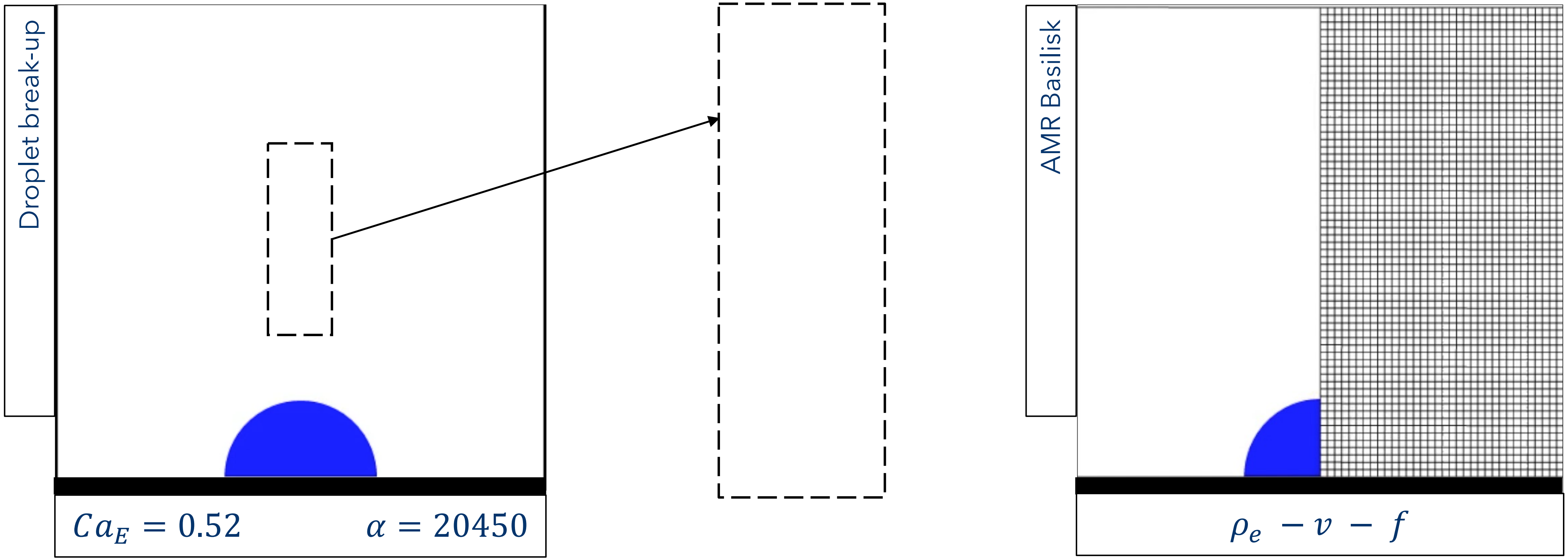
- We are exploring a software (Basilisk, http://basilisk.fr) specialized for multiphase systems
- It is a Free Software program for the solution of the partial differential equations describing fluid flow.
- A brief summary of its main features:
  - Solves the time-dependent incompressible variable-density Euler, Stokes or Navier-Stokes equations
  - Adaptive mesh refinement: the resolution is adapted dynamically to the features of the flow
  - Flexible specification of additional source terms
  - Volume of Fluid advection scheme for interfacial flows
  - Multiphase electrohydrodynamics





Basilisk main feature

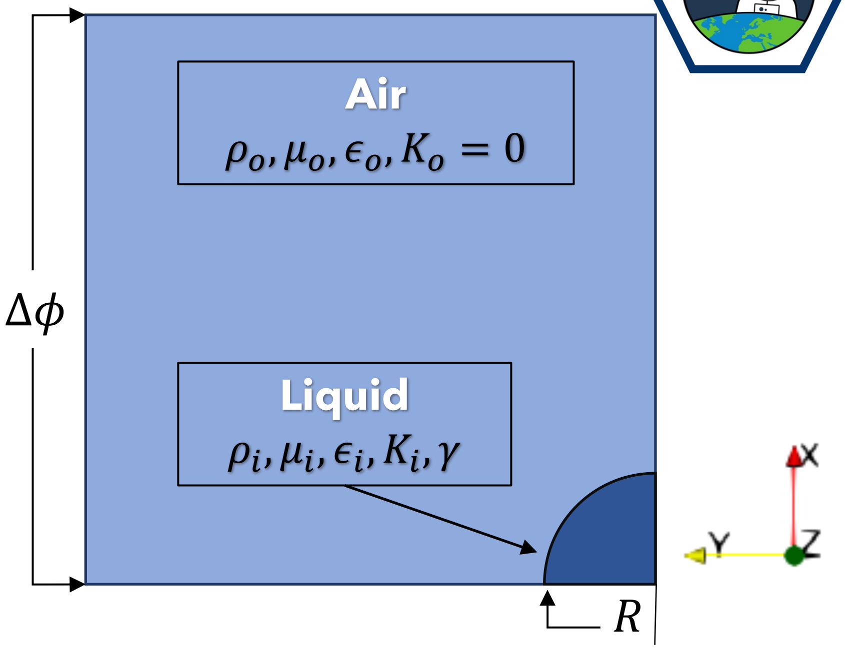
- Short computational time and Adaptive Mesh Refinement (AMR)



Dimensionless parameters

- Fluids property:

	$\mu\ [Pa \cdot s]$	$\rho [Kg/m^3]$	$\varepsilon [C^2/(N \cdot m^2)]$	$K\ [S/m]$	$\gamma\ [N/m]$
Air	$1.80 \cdot 10^{-5}$	1.295	$8.85 \cdot 10^{-12}$	$1.05 \cdot 10^{-14}$	
Water	$1 \cdot 10^{-3}$	997	$7.08 \cdot 10^{-10}$	$5.40 \cdot 10^{-6}$	0.071
PDMS	$2.80 \cdot 10^{-3}$	1110	$2.40 \cdot 10^{-11}$	$3.44 \cdot 10^{-15}$	0.020



$$v = \frac{Vol_{drop}}{R^3} \qquad \epsilon_r = \frac{\varepsilon_1}{\varepsilon_2} \qquad \rho_r = \frac{\rho_2}{\rho_1} \qquad \mu_r = \frac{\mu_1}{\mu_2}$$

$$Ca_E = \sqrt{\frac{E^2 \cdot \varepsilon_1 \cdot R}{\gamma}} = \frac{[electric\ forces]}{[interfacial\ tension\ forces]}$$

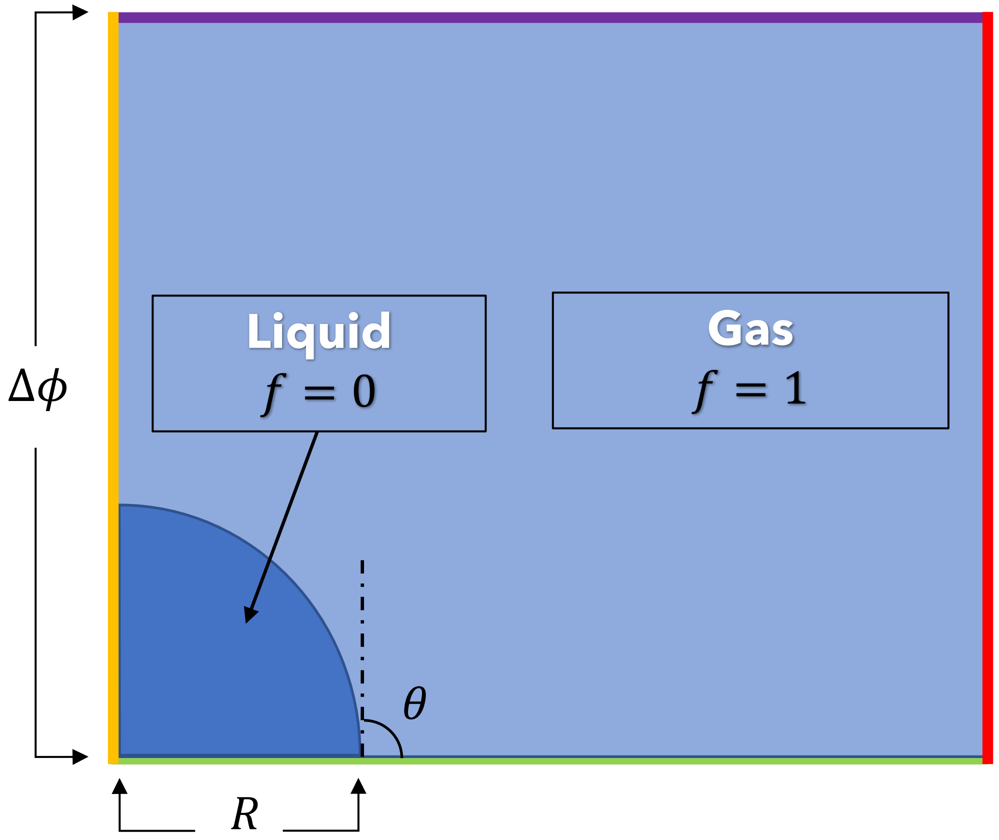
$$Oh = \frac{\mu_1}{\sqrt{\rho \gamma R}} = \frac{[viscous\ forces]}{[inertia \cdot interafacial\ tension\ forces]}$$

$$\alpha = \sqrt{\frac{K_i \cdot \rho_i \cdot R^3}{\epsilon_o^2 \cdot \gamma}} = \frac{[capillary\ time]}{[electrical\ relaxation\ time]}$$

$$G = \frac{g \rho R^2}{\gamma} = \frac{[gravitational\ forces]}{[interafacial\ tension\ forces]}$$

Model problem

- Sessile or Pendant drop influenced by uniform electric field

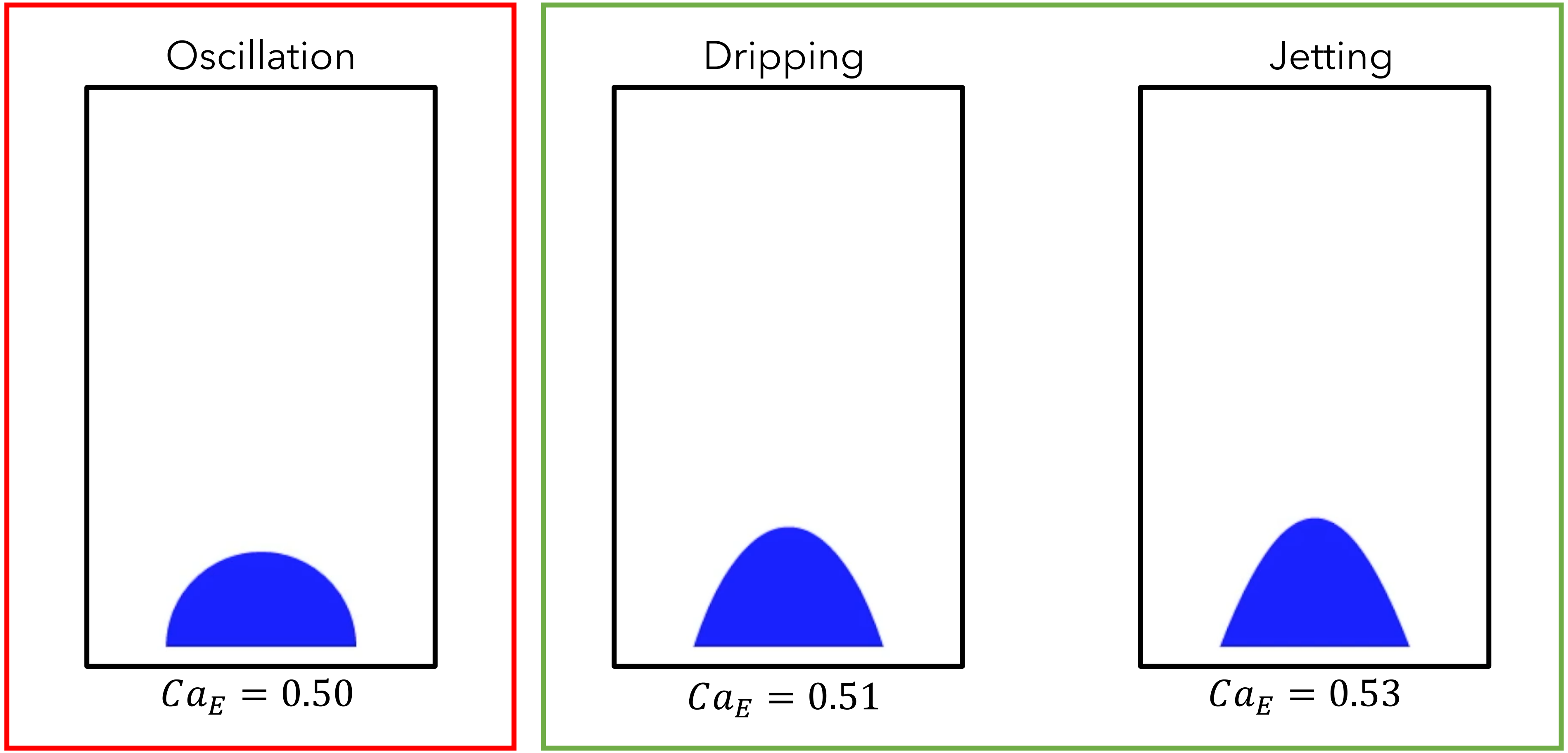


- Boundary conditions for fluid dynamics (f):
  - Outflow on the outlet boundary
  - Outflow on the upper boundary
  - No slip and fixed position of triple point lower wall
  - Axial symmetry on the axis of symmetry

- Boundary conditions for the electric equations (ϕ):
  - Zero Gradient at the outlet boundary
  - Fixed value of "$V=V_0$" at the upper boundary
  - Fixed value of "V=0" at the lower wall
  - Axial symmetry on the axis of symmetry

Dimensionless parameters

- Behaviour of sessile/pendant drop between a uniform electric field.



SubCritical regime

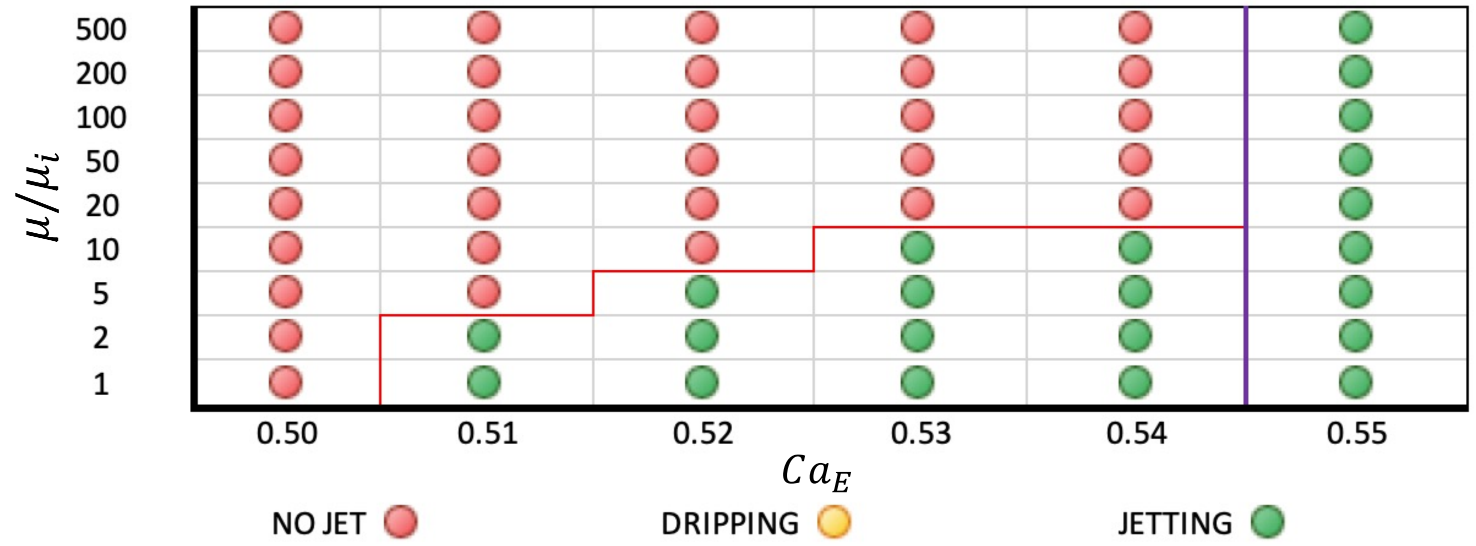
SuperCritical regimes

Droplets dynamic

- Expression to identify the critical electric field [1]:

$$\frac{Electical\ stress}{Surface\ Tension} = \frac{\epsilon_{outer} E_{cri}^2}{\gamma / R} \propto \frac{R^3}{Vol} = Shape\ Parameter$$

Viscosity Effect ?!



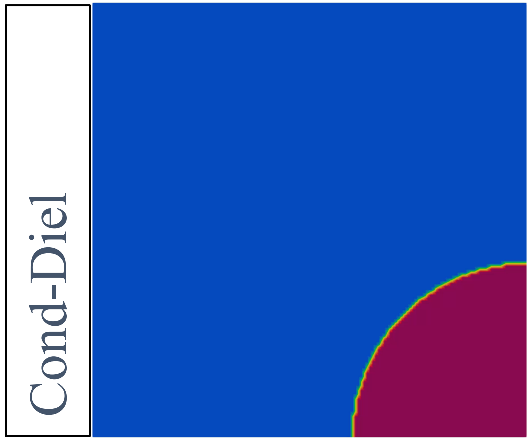
- For starting viscosity $\mu_i = 0.001\ Pa \cdot s$:
  - Law prediction

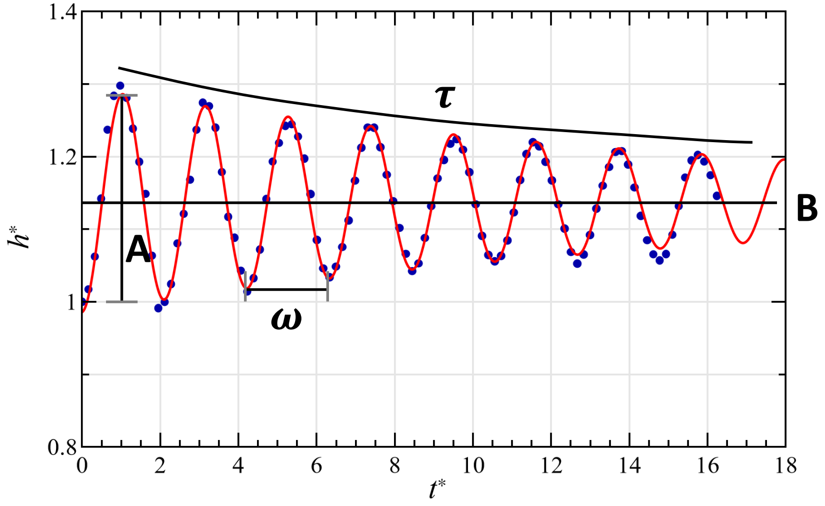
$$E_{cr} = \sqrt{\frac{R^3}{Vol} \cdot \frac{2}{\pi} \cdot \frac{\gamma / R}{\epsilon_{outer}}}$$

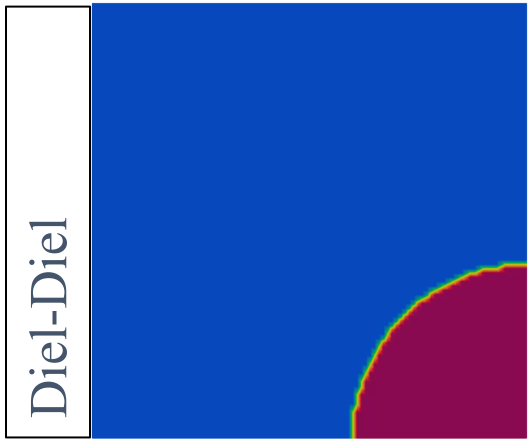
  - Simulation results

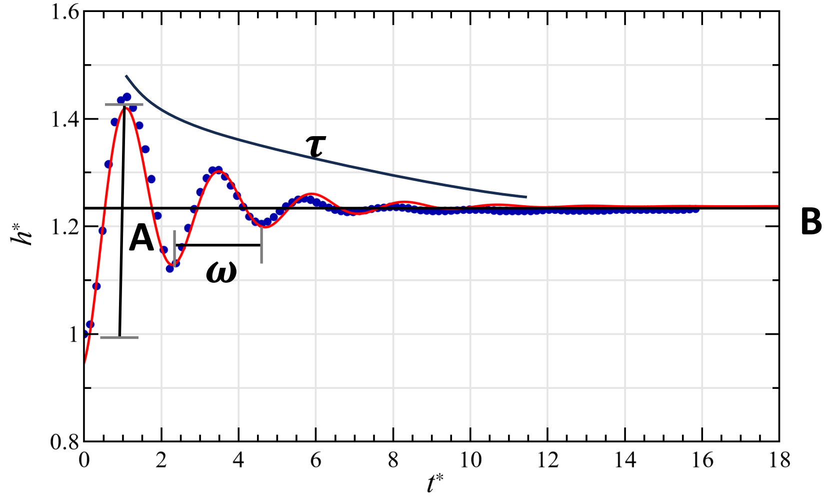
[4] Beroz J, Hart A.J., Bush J.W.M. Stability limit of electrified droplets. Phys Rev Lett. 2019;**122** (24)

SubCritical regime









$$h^* = B + A \cdot e^{\left(-\frac{t^*}{\tau}\right)} \cdot cos(wt^* + \phi)$$

A → Amplitude of the oscillation.

B → Steady value reached by the system.

τ → Rate at which the exponential tends to 0.

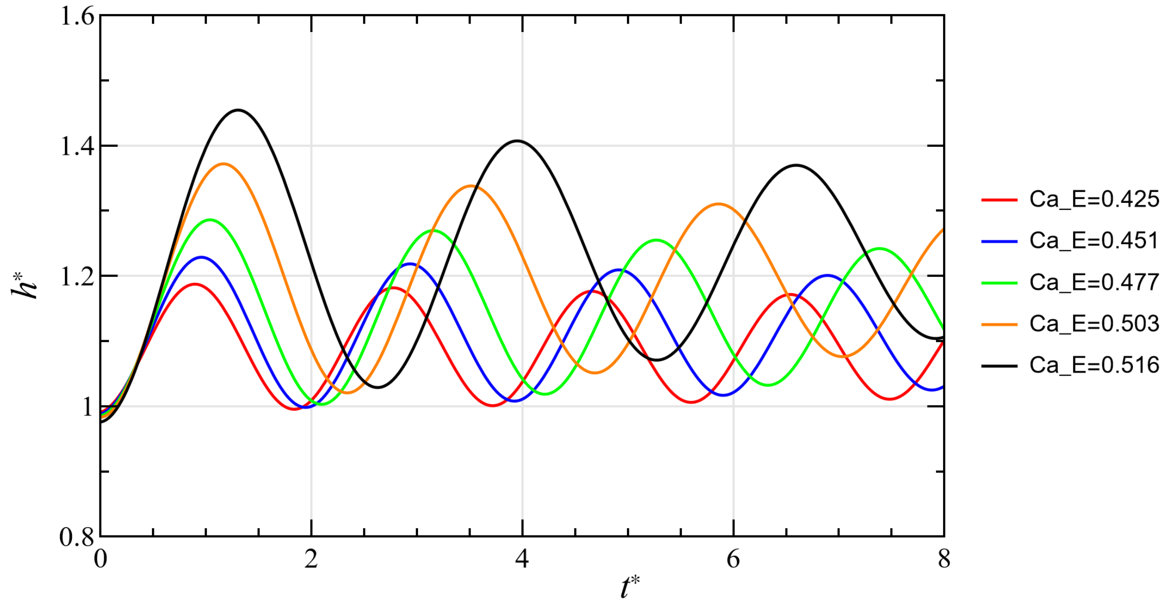
ω → Frequency of the sinusoidal oscillation.

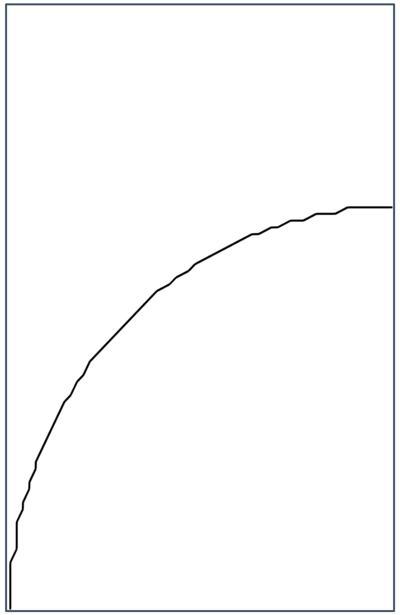
φ → Phase of the sinusoidal oscillation.

$t^* = \sqrt{\frac{t^2\gamma}{\rho R^3}}$ and $h^* = \frac{h}{R}$

[5] Ferrera, C., Lopez-Herrera, J. M., Herrada, M. A., et al. (2013). Dynamical behavior of electrified pendant drops. Physics of Fluids, 25(1).

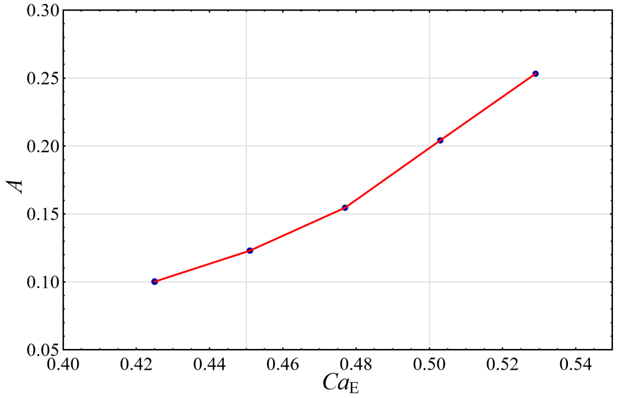
Cond-Diel: Electric Field effect (Ca_E)

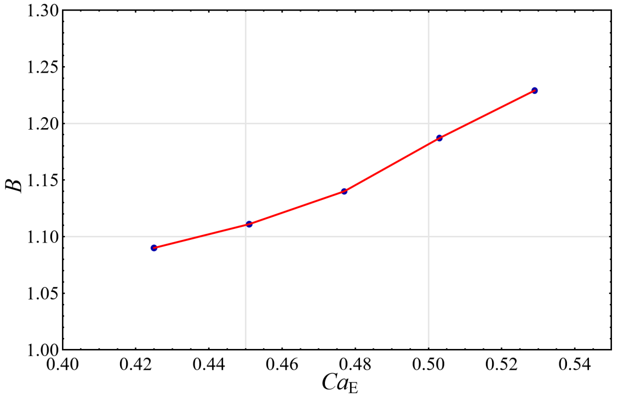


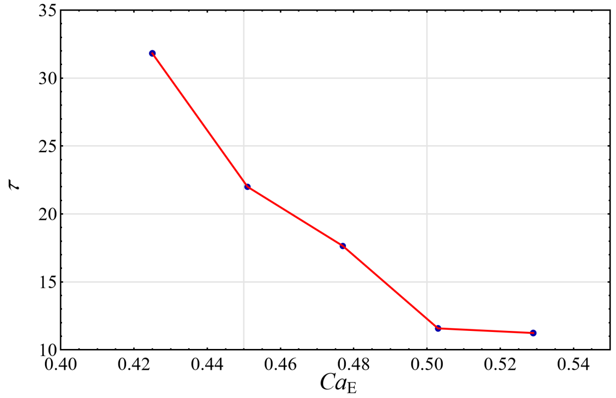


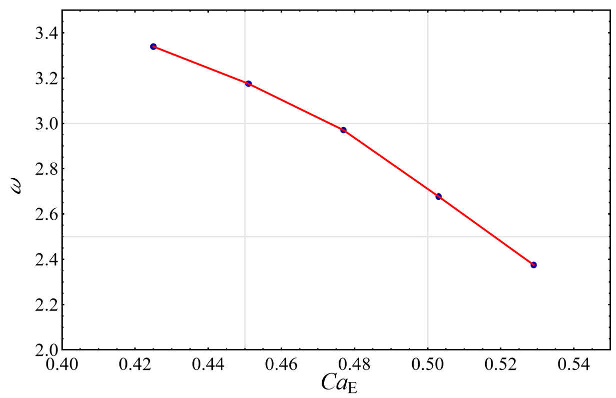
$Oh_i = 0.00686$; $\alpha = 2045$;
$\upsilon = 2.09$; $\epsilon_r = 80$; $\mu_r = 0.018$;

Ca_E	A	B	τ	ω	ϕ
0,425	0,1002	1,090	31,82	3,339	3,286
0,451	0,1230	1,111	22,00	3,175	3,235
0,477	0,1546	1,140	17,65	2,970	3,182
0,503	0,2042	1,187	11,58	2,677	3,130
0,516	0,2532	1,229	11,24	2,375	3,149

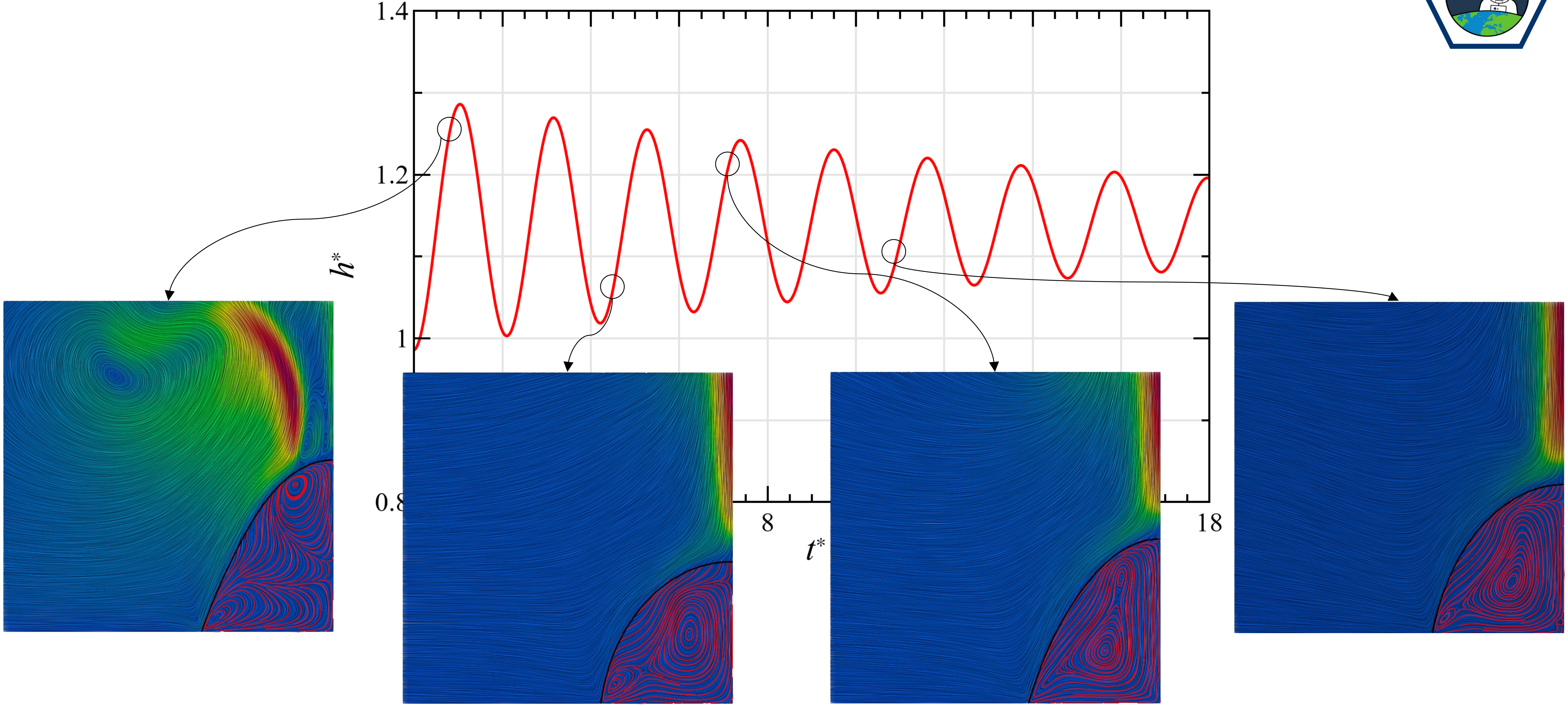




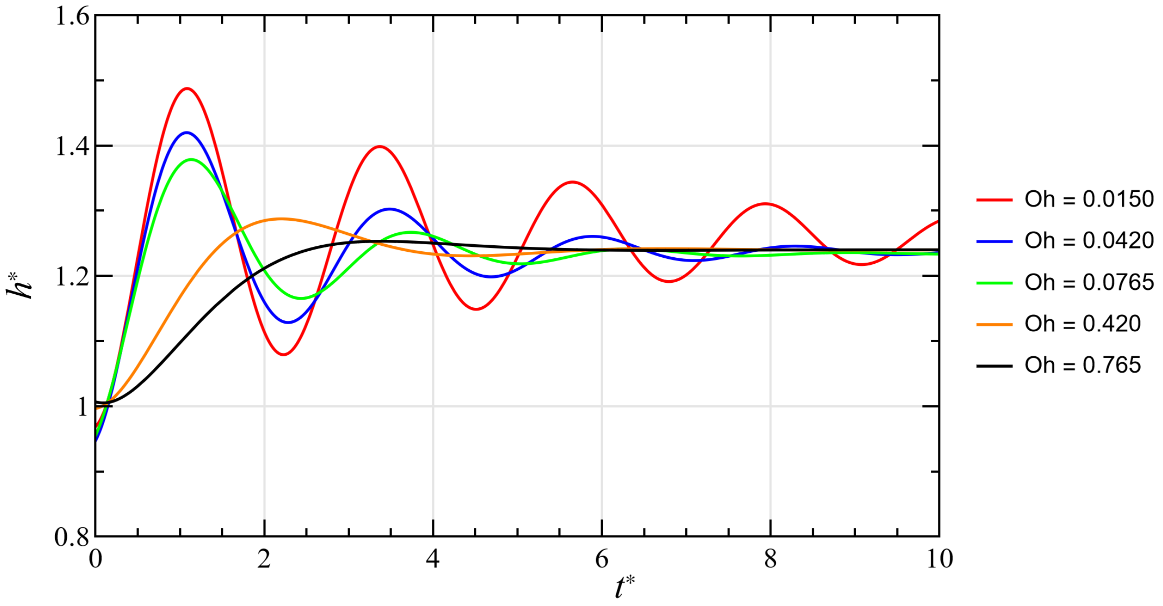


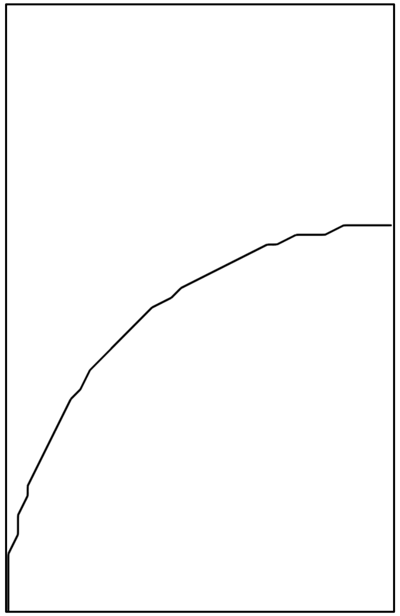


Cond-Diel: Velocity Field ($Ca_E = 0.477$)



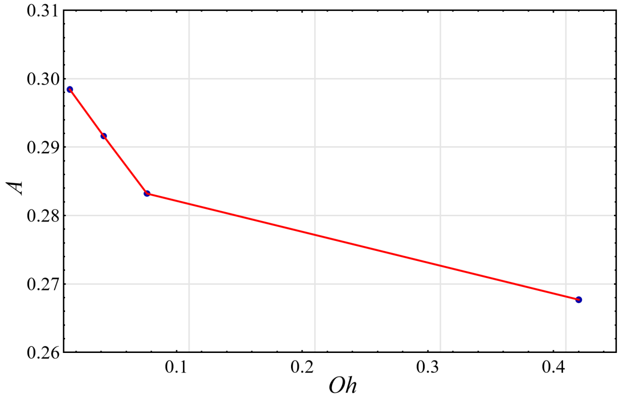
Diel-Diel: Viscosity effect (Oh_i)

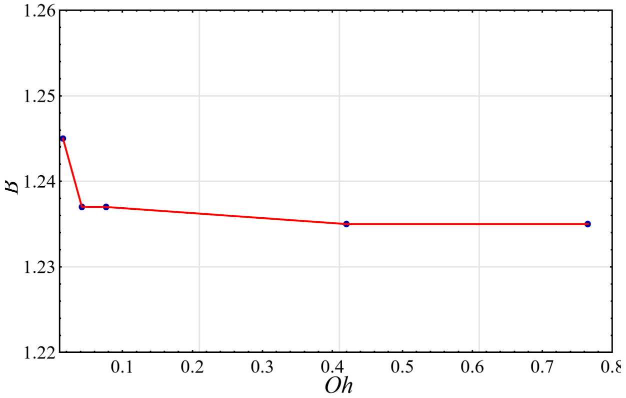


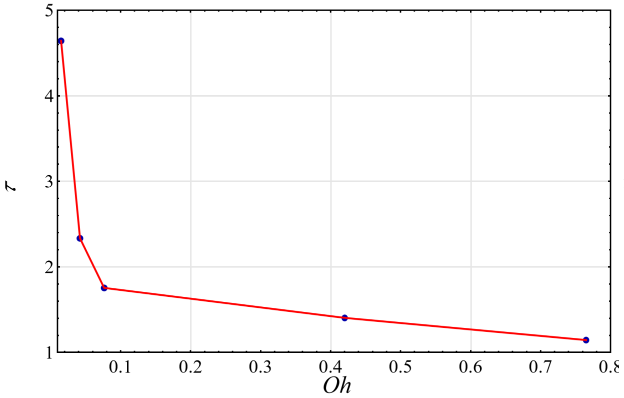


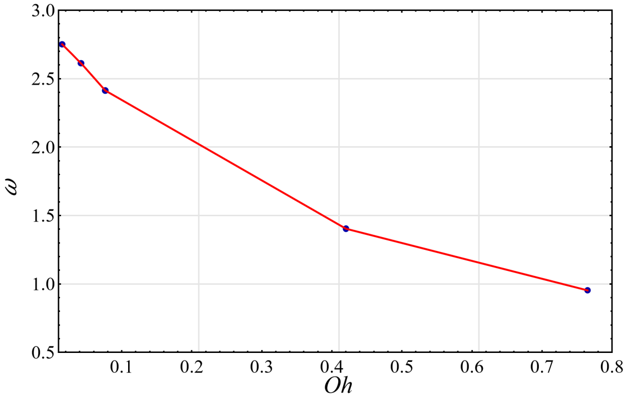
$$Ca_E = 1.74; \quad \alpha = 2045;$$
$$\upsilon = 2.09; \quad \epsilon_r = 80;$$

Oh	A	B	τ	ω	ϕ
0,0150	0,2984	1,246	4,641	2,751	3,221
0,0420	0,2937	1,237	2,333	2,613	3,298
0,0765	0,2832	1,234	1,754	2,413	3,315
0,420	0,2677	1,240	1,368	1,403	3,130
0,765	0,3445	1,240	1,144	0,9531	3,149

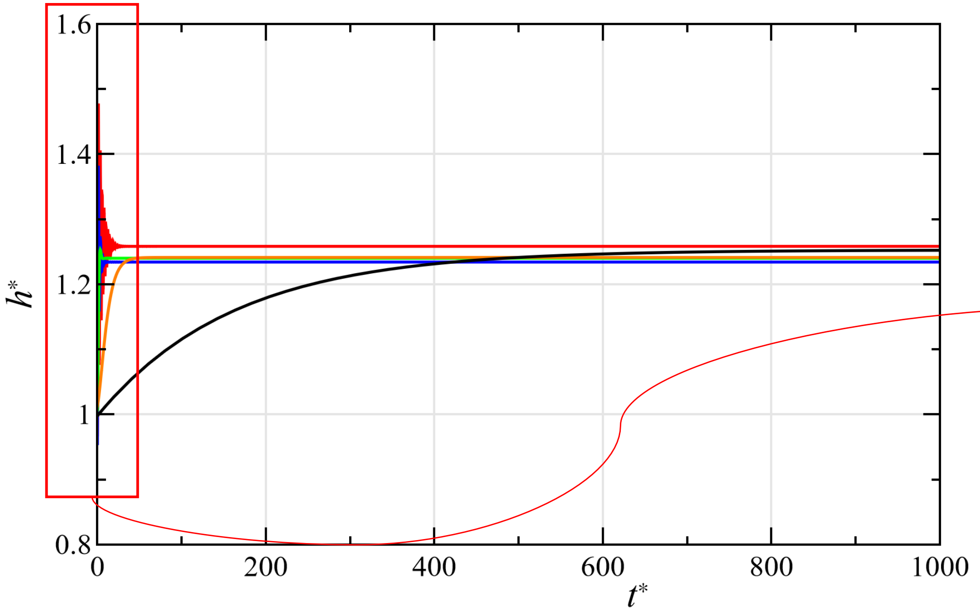


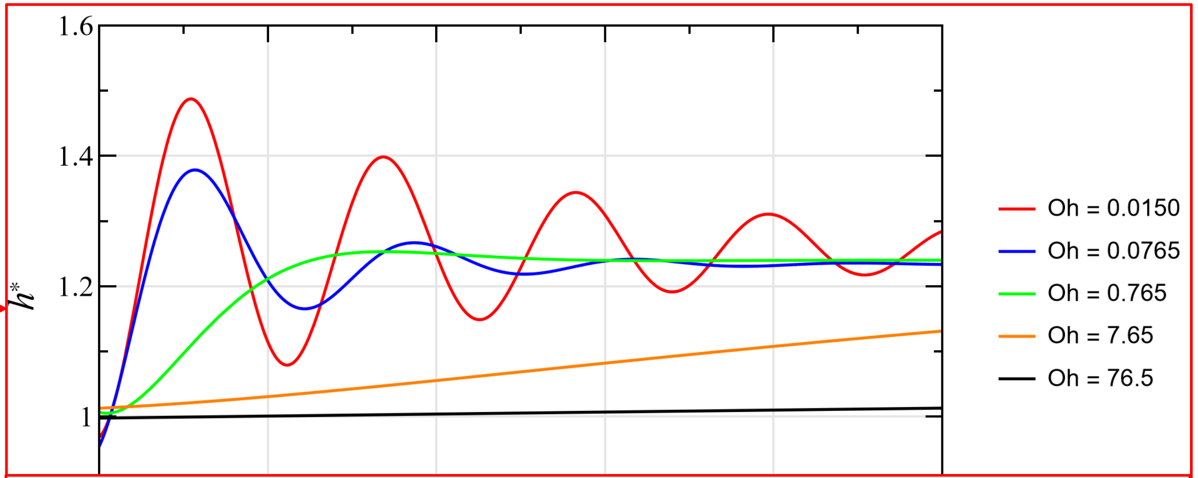




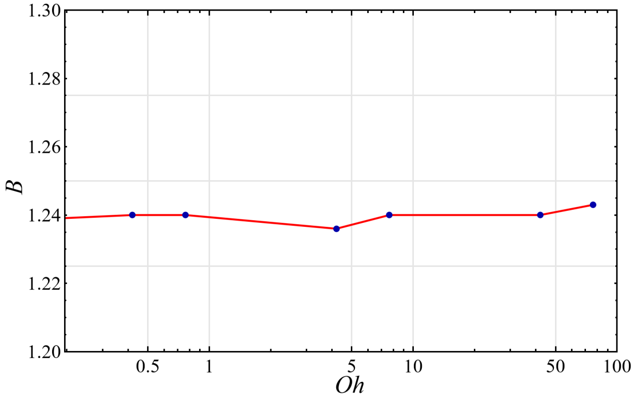


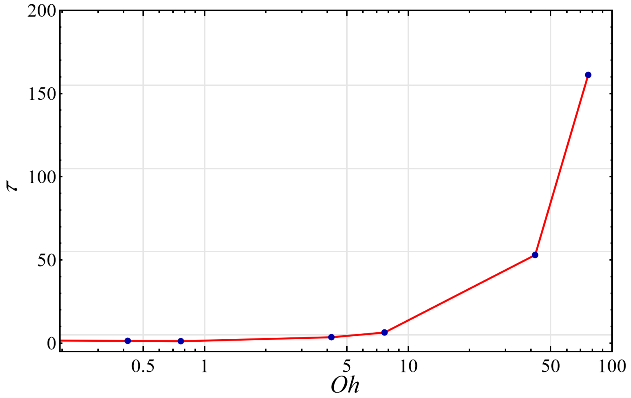
Diel-Diel: Viscosity effect (Higher Oh_i)

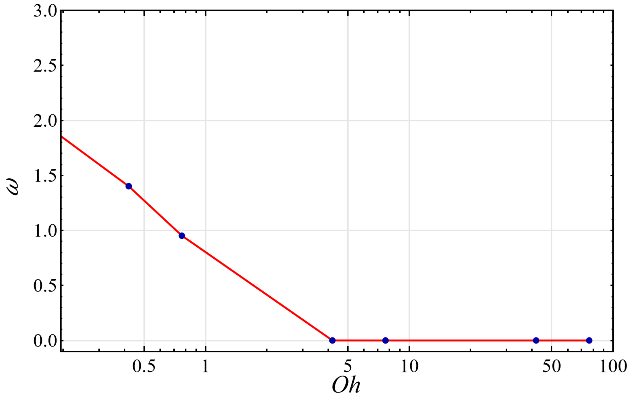




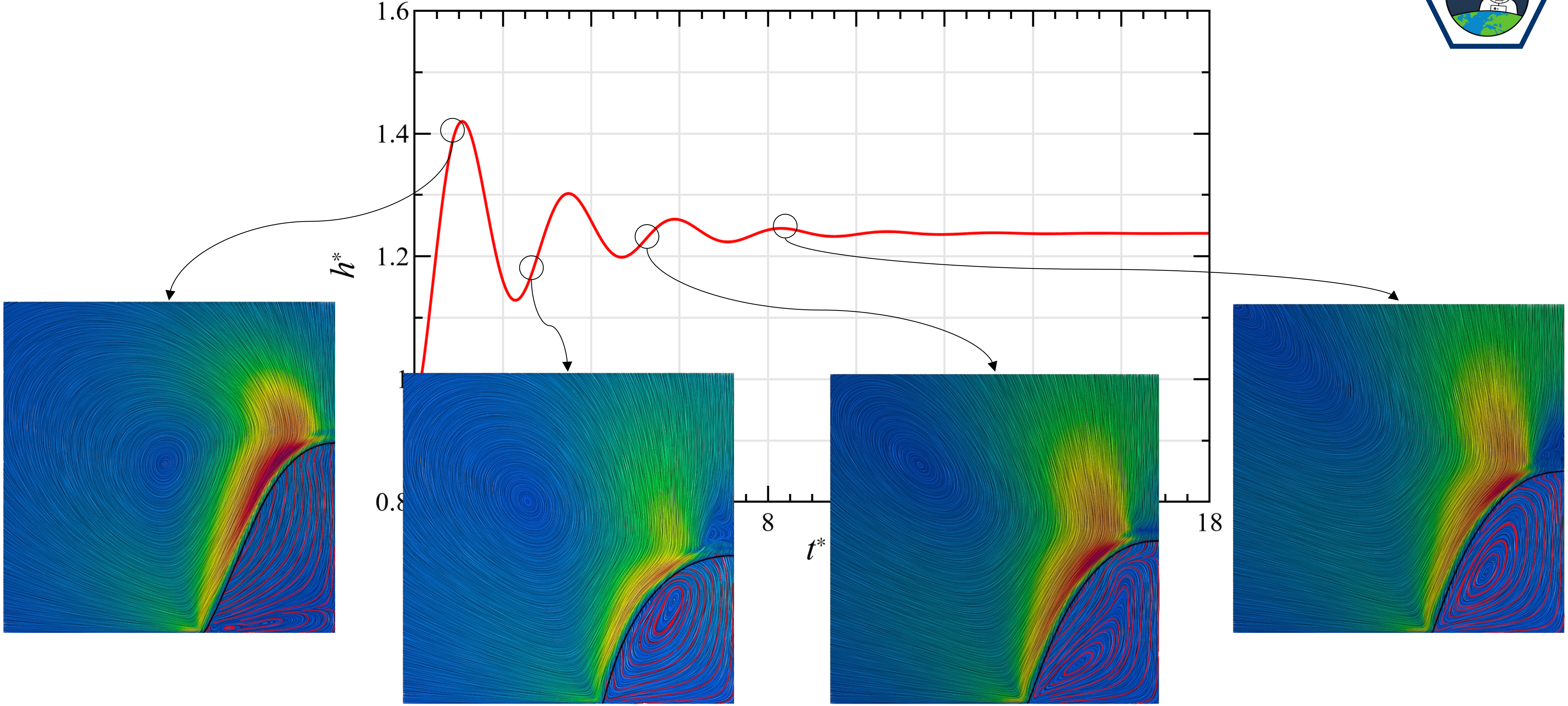
For Higher Onhesorge number, the oscillatory behavior is lost.





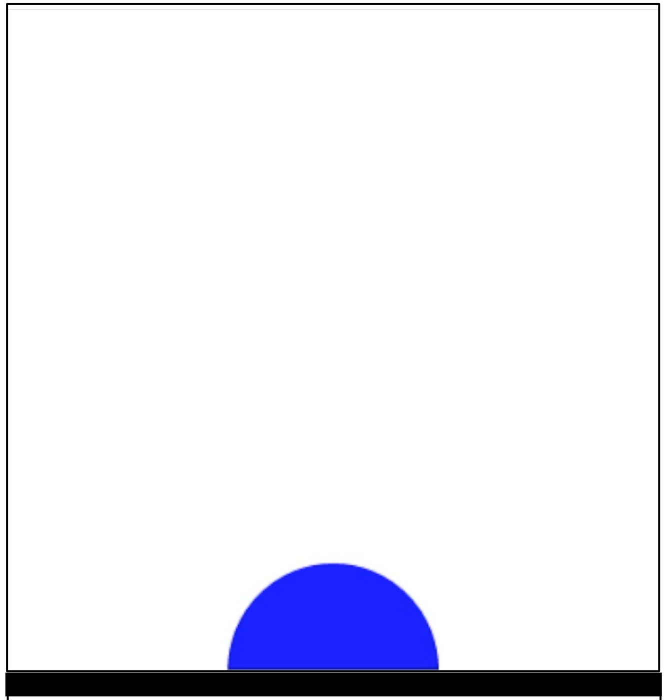


Diel-Diel: Velocity Field ($Oh_i = 0.0765$)

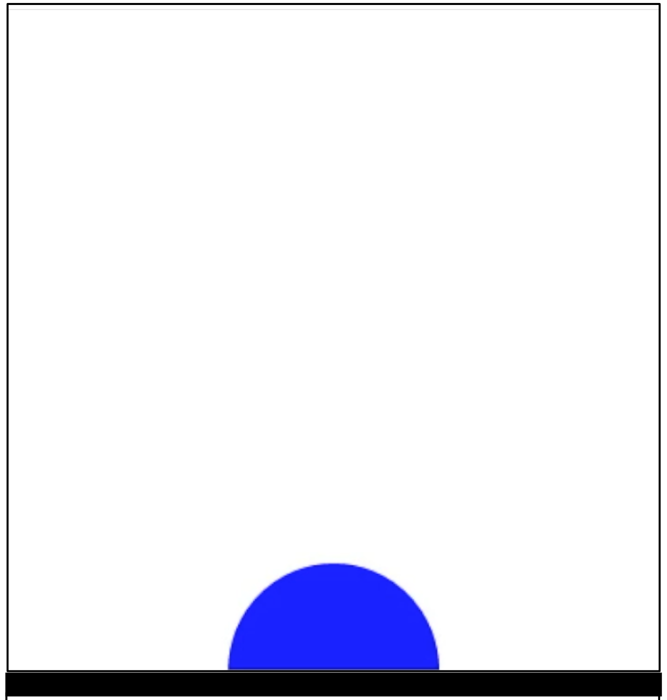


Electrical Conductivity (α)

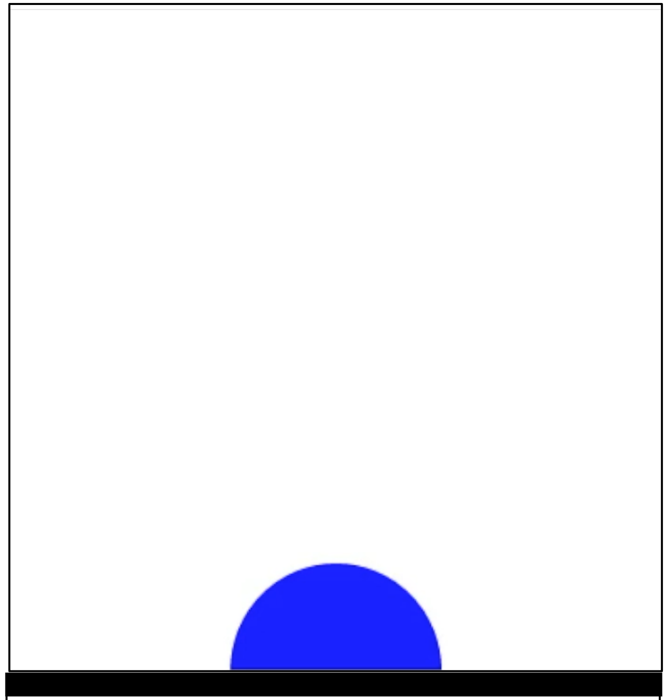
$Ca_E = 0.52;$ $\alpha = 2045;$ $\epsilon_r = 80;$ $\upsilon = 2.09;$ $\mu_r = 0.018;$



α



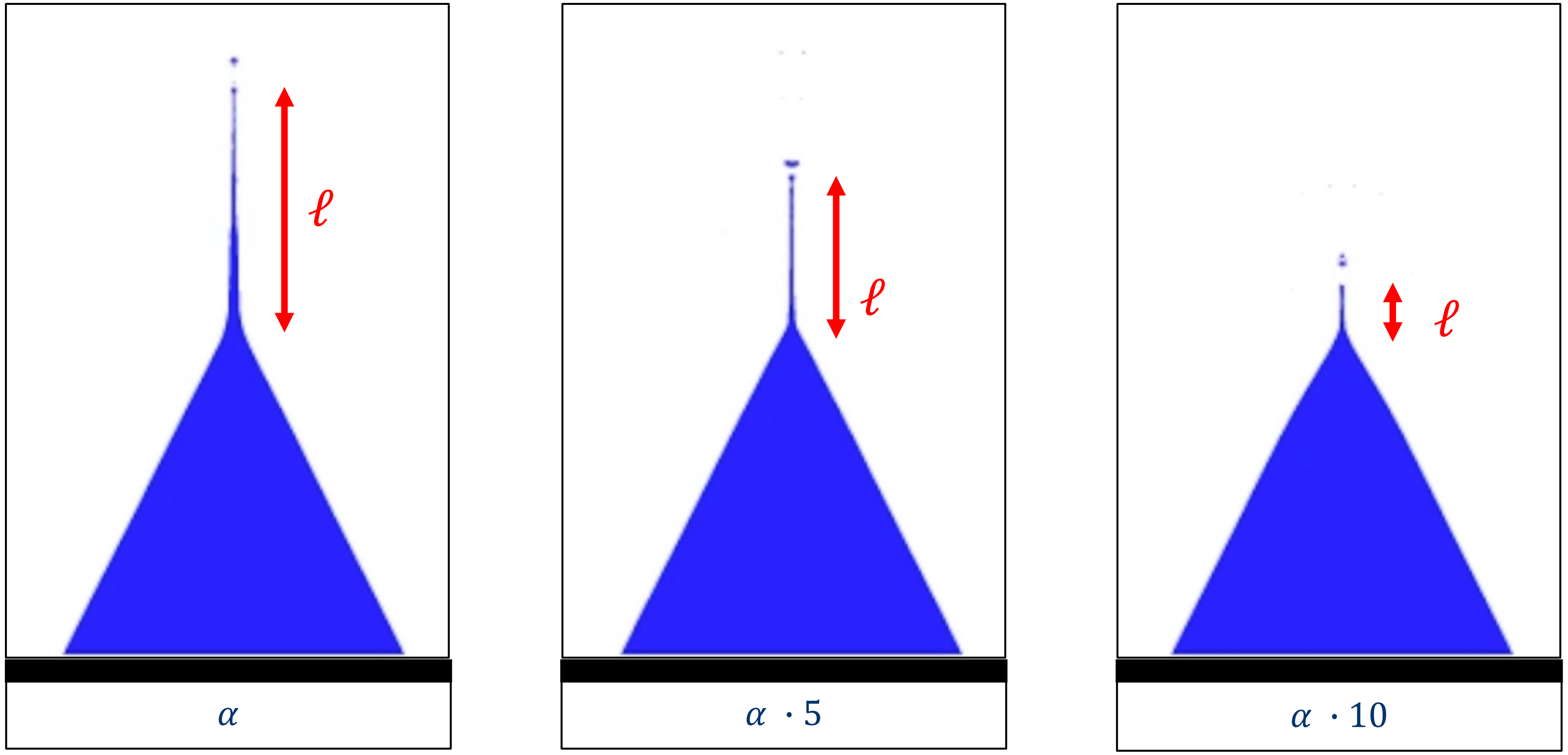
$\alpha \cdot 5$



$\alpha \cdot 10$

Electrical Conductivity (α)

$Ca_E = 0.52$; $\alpha = 2045$; $\epsilon_r = 80$; $\upsilon = 2.09$; $\mu_r = 0.018$;



- Lenght first drop ejection (ℓ) [7]:

$$\ell \propto \frac{\epsilon_{\mathrm{i}}}{K_{\mathrm{i}}}$$

- Emitted droplet's diameter:

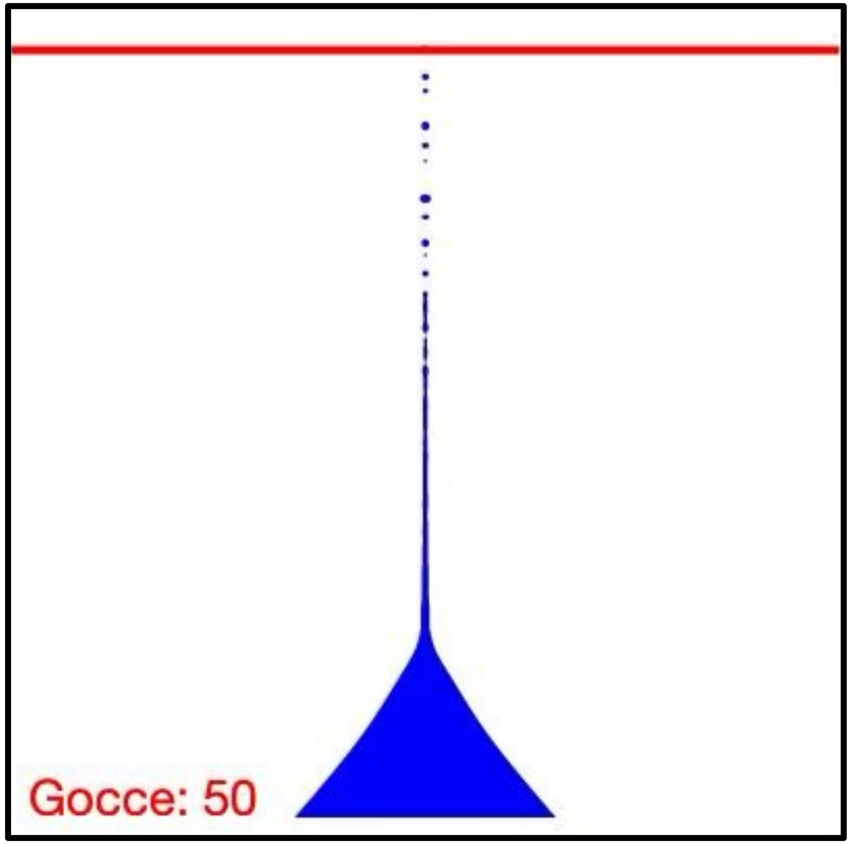
$$d_{drop} \propto \frac{1}{K_{\mathrm{i}}}$$

[7] Ganán-Calvo A. M, López-Herrera J. M. The onset of electrospray: The universal scaling laws of the first ejection. Sci. Rep (6). 2016.

SuperCritical regime: Droplets count

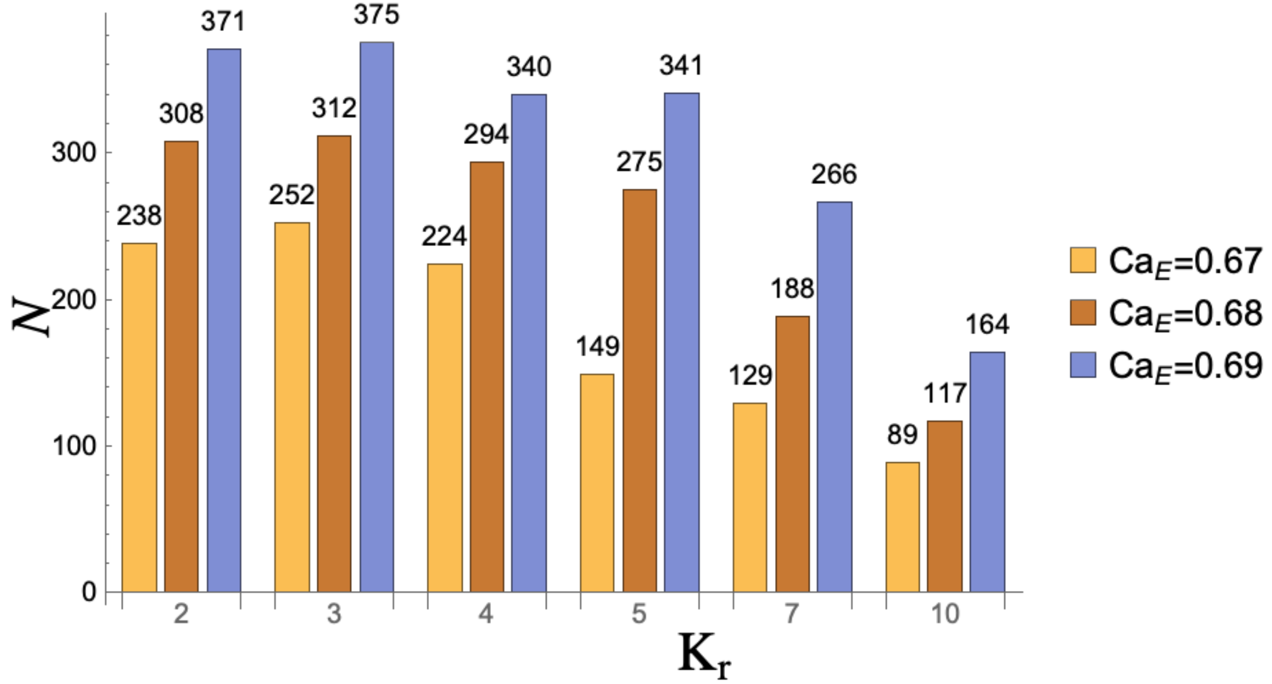
- Matlab code:

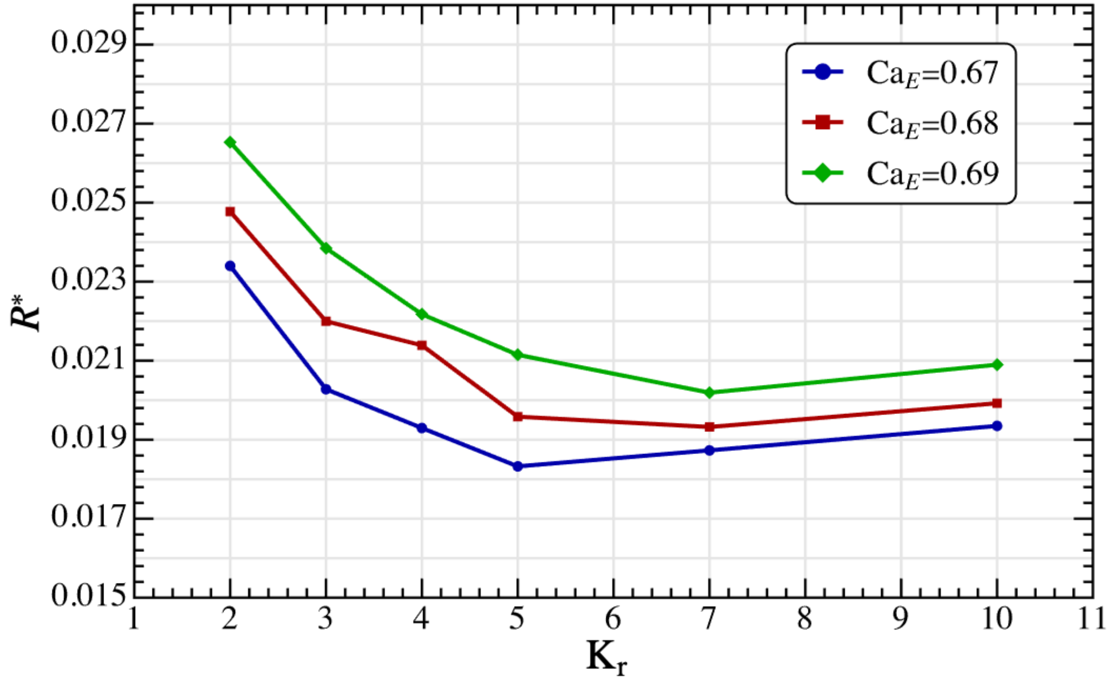
```
% Impostazioni
videoPath = 'shape_k2e67.mp4';
altezza_target = 20; % Altezza desiderata per il controllo del pixel
soglia_colore = 250; % Soglia per considerare un pixel "blu"

…

% Analizza ogni frame del video
while hasFrame(videoObj)
% Leggi il frame corrente
frame = readFrame(videoObj);

% Estrai il valore del pixel nella colonna centrale all'altezza target
pixel_centrale = frame(altezza_target, colonna_centrale, :);

% Verifica la transizione da blu a bianco
if any(pixel_centrale < soglia_colore)
if inGoccia
contatoreGocce = contatoreGocce + 1;
inGoccia = false; % Resetta il flag quando si passa da blu a bianco
end
else
inGoccia = true; % Imposta il flag quando il pixel è blu
End
```



Electrical conductibility (α) for $\upsilon = 1.3$

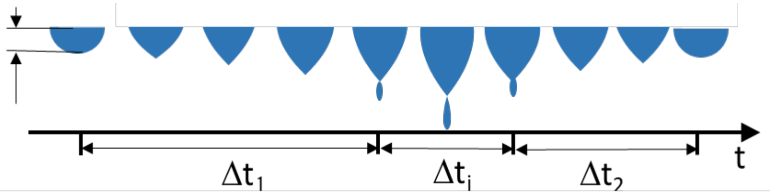
- We analyse the number of drops generated and the average radius:

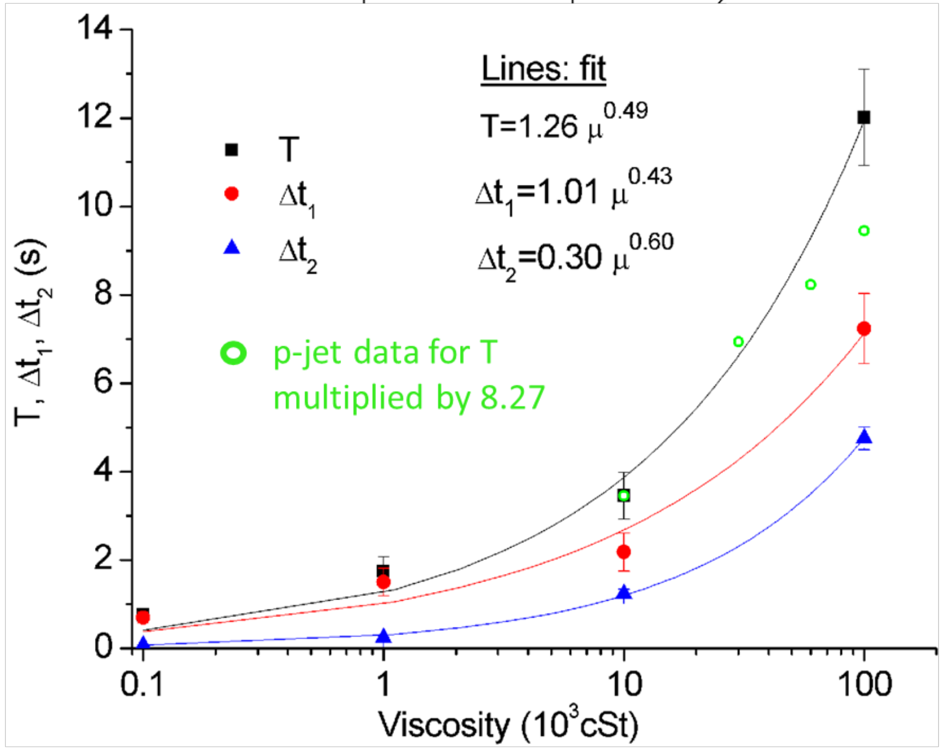




$\upsilon = 1.3$; $\mu_i = 0.018$; $\epsilon_r = 80$; $\alpha = 2045$; Ca_E;

Comparison with experiments: Δt_1

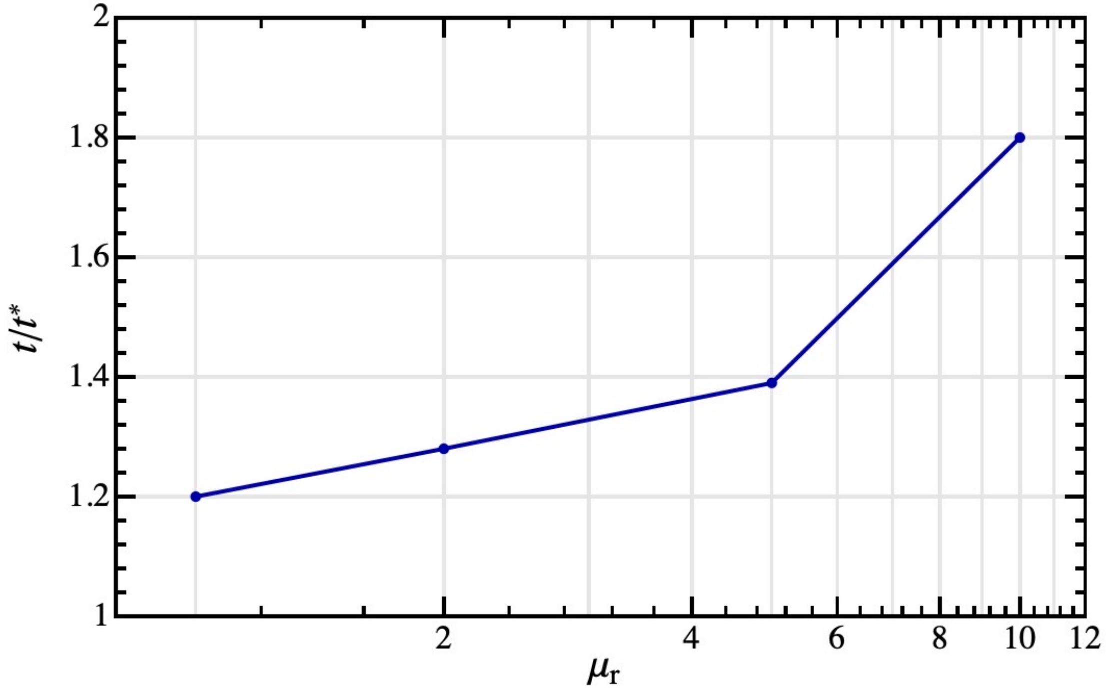




Experiment

<u>Characteristic times</u>

Δt_1 – meniscus elongation and formation of the cone

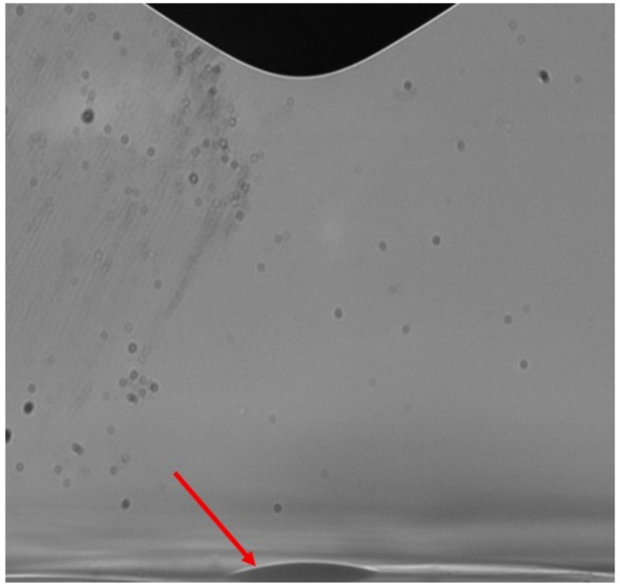


$R_o = 0.2\ mm$; $\theta = 90°$; $\mu_i = 0.001\ Pa \cdot s$;
$K_i = 2.7 \cdot 10^{-4} S/m$; $E = 3.42 \cdot 10^6\ V/m$

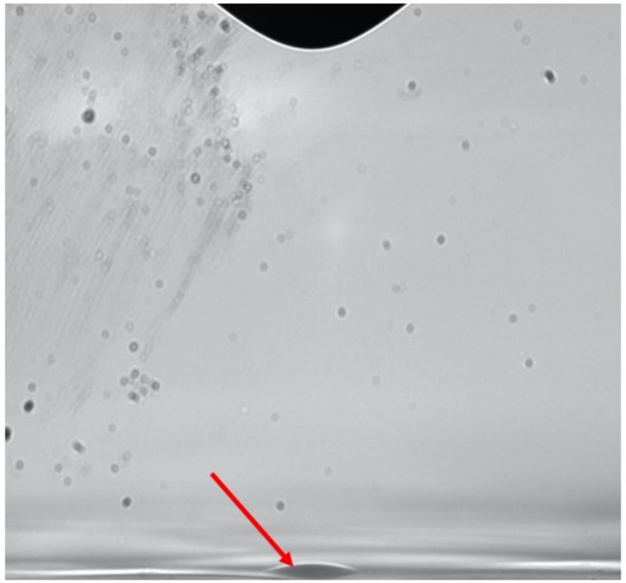
Comparison with experiments: Vol. deposited

- Amount of PDMS deposited after 5 jets:

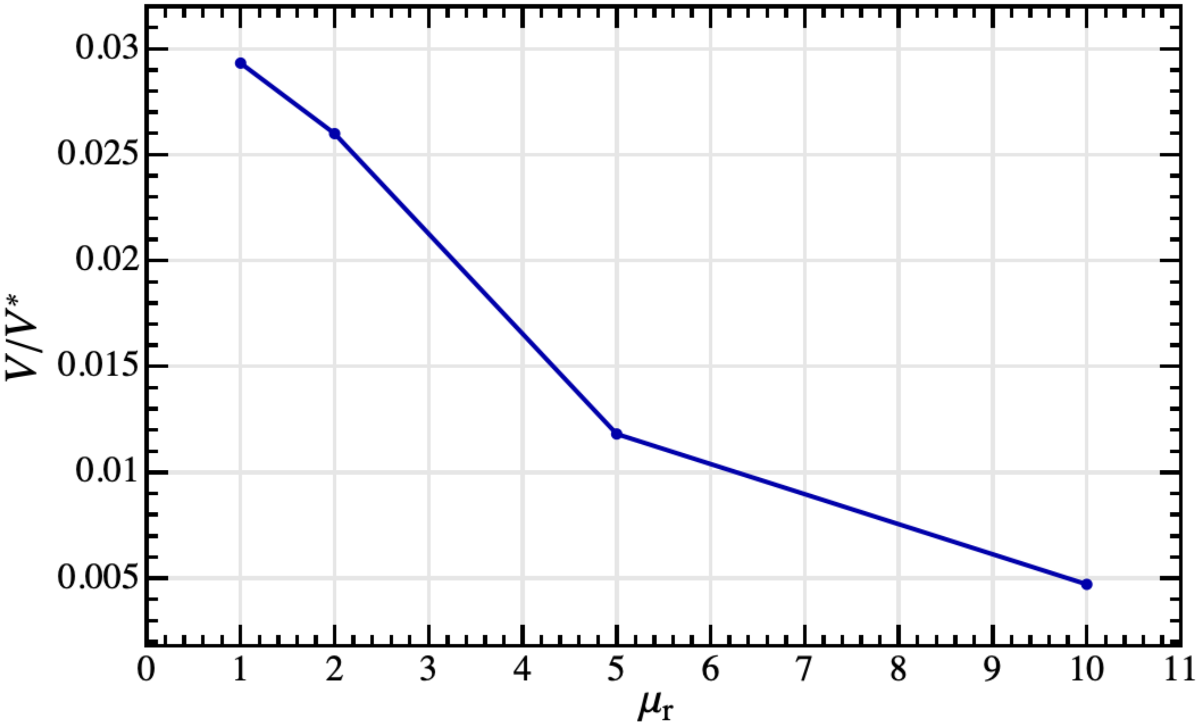
μ = 1000 cSt



μ = 100000 cSt



Experiment



$R_o = 0.2\,mm$; $\theta = 90°$; $\mu_i = 0.001\,Pa \cdot s$;
$K_i = 2.7 \cdot 10^{-4} S/m$; $E = 3.42 \cdot 10^{6}\,V/m$

Summary & Conclusions

- The fluid dynamics of a **sessile drop**let influenced by an **electric field**.

- We used a **dimensionless analysis** to get information about it and have modelled by **Basilisk**.

- The results are **consistent** with the **literature** and add new informations.

- For **SubCritical** regime:

  The effect of **Electrical capillary number** (Ca_E), the **Ohnesorge number** (Oh_i) and **Velocity Field** (u^*) has been investigated.

- For **SuperCritical** regime:

  The effect of **Electrical capillary number** (Ca_E) and **Electrical Conductivity** (α) has been investigated.

- The results are in **agreement** with the **experiments**.

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Thanks for your attention