

The peculiar velocity bispectrum

Funded by the European Union NextGenerationEU

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redshift independently **and the set of the set**

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• The power spectrum of peculiar velocities has been proposed as a powerful tool to

- redshift independently
- complement traditional galaxy clustering

[Davis & Scrimgeour 2014; Watkins & Feldman 2015]

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Maximum redshift, z_{max}

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improvement on constraints on the growth of structure relative to galaxy clustering

[Carrick et al. 2015; Qin et al. 2019; Adams & Blake 2020; Said et al. 2020; Lai et al. 2022]

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Redshift, z_{max}

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- redshift independently
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- What about the bispectrum, then?

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Polyspectra formalism

peculiar velocities, …):

• Given an observable cosmological perturbation field X (e.g. galaxy number counts, λ *X* (e.g. galaxy number counts,

density contrast and the velocity divergence ✓ := r *· v*. The density and velocity can then

be expanded about the linear solutions, which correspond to time dependent scalings of the initial density field. Hence, it has been shown that in Fourier space we can write

Polyspectra formalism

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	-
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• Fourier-space summary statistics for *N*-point correlation functions [polyspectra] where I is the Dirac-delta distribution. By 'polyspectrum' I shall refer to a generic *n*-point \mathbb{R}^n

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$\mathcal{A} = (\mathcal{A} - \mathcal{A})^3 \mathcal{A}$ (*b*) $D_-(\mathcal{A})$ $\langle X(\boldsymbol{k}_1) X(\boldsymbol{k}_2) \rangle = (2\pi)^3 \delta_{\rm D}(\boldsymbol{k}_{12}) P_X(\boldsymbol{k}_1)$ $^{3}\delta_{\rm D}(\bm{k}_{12})\,P_{X}(\bm{k}_{1})$

- peculiar velocities, …): For a generic observable *X*, be it fluctuations in galaxy number counts (GC, as per terms of the second of the international counts (GC, as per terms of the international counts of the international counts (GC, as per ter
	-
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		-

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Polyspectra formalism *q* **12** Z d3*q*

• Given an observable cosmological perturbation field X (e.g. galaxy number counts, λ *X* (e.g. galaxy number counts, $2^{\frac{2}{\alpha}}$

relation between the wave-vectors makes the dependence upon one of them redundant, much in the same way

• Fourier-space summary statistics for *N*-point correlation functions [polyspectra] where I is the Dirac-delta distribution. By 'polyspectrum' I shall refer to a generic *n*-point \mathbb{R}^n $\begin{bmatrix} 0 & 0 \end{bmatrix}$ or galaxy statistics for report correlation functions $\begin{bmatrix} 1 & 0 \end{bmatrix}$

$\mathcal{A} = (\mathcal{A} - \mathcal{A})^3 \mathcal{A}$ (*b*) $D_-(\mathcal{A})$ $\mathcal{L} \mathcal{L} \mathcal{$ $^{3}\delta_{\rm D}(\bm{k}_{12})\,P_{X}(\bm{k}_{1})$

 $d = (\Omega - \lambda)^3 \lambda (\mathbf{I}_2 - \lambda \mathbf{D} (\mathbf{I}_2 - \mathbf{I}_3))$ $\begin{array}{lll} \hbox{b)} & \partial \text{D}(\bm{\kappa}_{123}) \, D_X(\bm{\kappa}_1,\bm{\kappa}_2) \[2mm] \hbox{b)} \end{array}$ $\langle X(\mathbf{k}_1) X(\mathbf{k}_2) X(\mathbf{k}_3) \rangle = (2\pi)^3 \delta_{\rm D}(\mathbf{k}_1, \mathbf{k}_2)$ $\frac{3}{5}\delta_{\rm D}(\bm{k}_{123})\,B_{X}(\bm{k}_{1},\bm{k}_{2})$

- peculiar velocities, …): For a generic observable *X*, be it fluctuations in galaxy number counts (GC, as per terms of the second of the international counts (GC, as per terms of the international counts of the international counts (GC, as per ter
	-
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initial density field. Hence, it has been shown that in Fourier space we can write

Polyspectra formalism *q* **12** Z d3*q*

as the power spectrum is *PX*(*k*1) and not *PX*(*k*1*, k*2). $\langle X(\boldsymbol{k}_1) X(\boldsymbol{k}_2) X(\boldsymbol{k}_3) \rangle = (2\pi)$

• Bispectrum ¹I opt for the use of *BX*(*k*1*, k*2), instead of the ubiquitous *BX*(*k*1*, k*2*, k*3), to emphasise that the closure

• Given an observable cosmological perturbation field X (e.g. galaxy number counts, λ *X* (e.g. galaxy number counts, $2^{\frac{2}{\alpha}}$

$$
\langle X(\mathbf{k}_1) X(\mathbf{k}_2) \rangle = (2\pi)
$$

Standard perturbation theory primer

• Small perturbations can be treated perturbatively

- Small perturbations can be treated perturbatively
- As long as the matter distribution can be described as an irrotational fluid (i.e. no shell crossing), gravitational instability can be fully described by the density contrast *δ* and the velocity divergence *θ*

Standard perturbation theory primer

- Small perturbations can be treated perturbatively
- As long as the matter distribution can be described as an irrotational fluid (i.e. no shell crossing), gravitational instability can be fully described by the density contrast δ and the velocity divergence θ
- In the spherical collapse model (sensible for small perturbations), solutions at any order remain separable in time and scale

Standard perturbation theory primer

(*k*) *,* (2.5)

$$
\delta(\boldsymbol{k},z)=\sum_{m=1}^{\infty}D^{m}(z)\,\delta^{(m)}(\boldsymbol{k})\\ \theta(\boldsymbol{k},z)=-\mathcal{H}(z)\,f(z)\,\sum_{m=1}^{\infty}D^{m}(z)\,\theta^{(m)}(\boldsymbol{k})\\
$$

 $\theta(\boldsymbol{k},z)$

where D is tandard perturbation theory primer to a general \mathcal{L}_{max} **We Standard perturbation theory primer** $\frac{1}{2}$ correlator in Fourier space.
Correlator in Fourier space. **Standard perturbation theory primer**

- Small perturbations can be treated perturbatively \bullet Small nerturbations can be treated perturbatively appears on $\frac{1}{2}$ is non-linear level. As a function $\frac{1}{2}$ is useful (and powerful), it is useful
- As long as the matter distribution can be described as an irrotational fluid (i.e. no shell crossing), gravitational instability can be fully described by the density $\frac{\partial}{\partial t}$ contrast δ and the velocity divergence θ to approach the non-linear regime perturbatively. By approximating the matter distribution as iong as the matter distribution can be described as an inotational nully (i.e. no t_{total} perturbations can be treated perturbatively. as long as the matter distribution can be described as an irrotational fiuld (i.e. no \blacksquare density contrast δ and the velocity divergence θ : **and the velocity divergence** θ $\frac{1}{\sqrt{2}}$ correspondent about the verborty which corresponds to time dependent scaling of the solutions of the solutions of the solution o
- In the spherical collapse model (sensible for small perturbations), solutions at any order remain separable in time and scale $\begin{bmatrix} 1 & 1 & 1 \ 0 & 1 & 1 \end{bmatrix}$ in the opnotion congpount our stronger of shian perturbations, solutions at any in the spherical collapse model (sensible for small perturbations), solutions at any line space we can write \mathbf{r}_i

Standard perturbation theory primer **where** $\frac{\text{UNIVERSITA}}{\text{DI TORINO}}$

• From the continuity and Euler's equations in Fourier space W_{∞} • From the continuity and Euler's equations in Fourier space

$$
\begin{bmatrix}\n\delta^{(m)}(\mathbf{k}) \\
\theta^{(m)}(\mathbf{k})\n\end{bmatrix} = \int_{\mathbf{q}_1} \cdots \int_{\mathbf{q}_m} \delta^{(1)}(\mathbf{q}_1) \ldots \delta^{(1)}(\mathbf{q}_m) \begin{bmatrix}\nF_m(\mathbf{q}_1, \ldots, \mathbf{q}_m) \\
G_m(\mathbf{q}_1, \ldots, \mathbf{q}_m)\n\end{bmatrix} (2\pi)^3 \delta_D(\mathbf{q}_{1\ldots m} - \mathbf{k})
$$

(*k*)

L

(*k*)

• Hence the tree-level power spectrum and bispectrum $\mathbf{r} = \mathbf{r} \cdot \mathbf{r} + \mathbf$ $\left(\frac{\lambda}{k} \left(\frac{\kappa_1}{2}\right) \right) \left(\frac{\kappa_2}{2}\right) = \left(\frac{\lambda}{k} \right) \left(\frac{\kappa_1}{2}\right) \left(\frac{\kappa_2}{2}\right)$ **F**

■ ● Hence the tree-level nower spectrum and bispectrum $\sqrt{\bf{v}}(L)$ $\bf{v}(L)$ $\bf{v}(L)$ $\sqrt{\bf{v}(1)}$ $\bf{v}(L)$ $\sqrt{\bf{v}(1)}$ $\sqrt{\bf{v}(1)}$ $\sqrt{\bf{v}(1)}$ $\langle X(\boldsymbol{k}_1)\,X(\boldsymbol{k}_2)\rangle = \langle X^{(1)}(\boldsymbol{k}_1)\,X^{(1)}(\boldsymbol{k}_2)\rangle$ **Contribution to the tree-level power spectrum and bispectrum**

$$
\langle X(\mathbf{k}_1) X(\mathbf{k}_2) X(\mathbf{k}_3) \rangle = \langle X(\mathbf{k}_1) X(\mathbf{k}_2) \rangle
$$

\mathbf{r} *F*_{*M*} *acce* **NO** *...* \mathbf{r} pace **...** α

Standard perturbation theory primer, and linear groups of $\frac{1}{2}$ $\frac{\text{UNIVERSITA}}{\text{DI TORINO}}$ where superscript '(*m*)' denotes the *m*th perturbative order, *D* is the linear growth factor, $\frac{1}{2}$ **Here are conformal Hubble factor, and** *f* **and** *f* \overline{a} *de**de**de**de**de**de**de**also***,** *with a***nd** *universi***tà** \mathbf{F} abuse of notation, I have symbol for a function $\mathbb{E}\left\{\mathbb{E}\left[\mathbb{I}\right]\right\}$ With the use of recursions, it is the use of the possible to the possible to express both density contrast to e
The contrast both density contrast both density contrast of the contrast of the contrast of the contrast of th \mathbf{F} above of a function, \mathbf{F} for a function and its Fourier \mathbf{F} With the use of recursion relations, it is then possible to express both density contrast a bushes of notation, I have so the same symbol for a function $\frac{2}{3}\left(\frac{2\pi}{3}\right)^{\frac{3}{2}}$ WITH THE USE OF RECOULS TO REFLECT RECOURSE THE USE OF RECOURSE AND THE USE OF RECOULS AND THE USE OF RECOULS
And the use of the use of the use of the contrast of the use of the contrast of the contrast of the contrast of **Standard perturbation theory primer**

• From the continuity and Euler's equations in Fourier space W_{∞} **and velocity of the continuity and Euler's equations in Fourier space and** \mathbf{r} $\int \delta^{(m)}(\bm{k})$ $\theta^{(m)}(\boldsymbol{k})$ $\overline{}$ = z
Zanada
Zanada *q*1 *...* ^Z *qm* $\delta^{(1)}(\boldsymbol{q}_1) \; \ldots \delta^{(1)}(\boldsymbol{q}_m)$ $\boxed{\theta}$ $\binom{m}{\bm{k}}\binom{\bm{k}}{\bm{k}} = \int_{\bm{q}_1} \cdots \int_{\bm{q}_m} \delta^{(1)}(\bm{q}_1) \, \ldots \delta^{(1)}(\bm{q}_m)$ *q*1 *qm* om the contracts ✓(*m*) (*k*) $\frac{1}{2}$ $J\Gamma$ and Euler's equations in Fourier space *q*1 *qm* (*m*) (*k*) Γ \cdot (*m*) H Ξ α Departmentic in Fouri
 α *q*¹ Id Euler's ec $\frac{1}{2}$ <u>l</u>

$$
\left[\begin{array}{c} \left[\begin{matrix} \delta^{(m)}(\boldsymbol{k}) \\ \theta^{(m)}(\boldsymbol{k}) \end{matrix}\right] = \int_{\boldsymbol{q}_1} \ldots \int_{\boldsymbol{q}_m} \delta^{(1)}(\boldsymbol{q}_1) \ldots \delta^{(1)}(\boldsymbol{q}_m) \begin{bmatrix} F_m(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_m) \\ G_m(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_m) \end{bmatrix} (2\,\pi)^3 \,\delta_\mathrm{D}(\boldsymbol{q}_{1\ldots m} - \boldsymbol{k}) \end{array}\right]
$$

$$
=\langle X^{(1)}(\boldsymbol{k}_1) \, X^{(1)}(\boldsymbol{k}_2) \rangle
$$

 $\braket{X(\bm{k}_1) \, X(\bm{k}_2) \, X(\bm{k}_3)} = \braket{X^{(1)}(\bm{k}_1) \, X^{(1)}(\bm{k}_2) \, X^{(2)}(\bm{k}_3)} + 2 \circlearrowleft_{\bm{k}_i}$ $\langle X({\bm k}_1)\, X({\bm k}_2)\, X({\bm k}_3) \rangle = \langle X^{(1)}({\bm k}_1)\, X^{(1)}({\bm k}_2)\, X^{(2)}({\bm k}_3) \rangle + 2\,\circlearrowleft_{{\bm k}_i}$

1, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$

 $\delta_{\rm g}(\boldsymbol{x}) \coloneqq$

ields • Galaxy clustering is the summary statistics of fluctuations in galaxy number counts

$$
= \frac{n_{\rm g}(\bm{x}) - \bar{n}_{\rm g}}{\bar{n}_{\rm g}}
$$

- - $\delta_{\rm g}(\boldsymbol{x}) \coloneqq$
-

1, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$

, (2.8) overdensity field to the underlying matter density field via a bias expansion of the form [see Stefano Camera The peculiar velocity bispectrum 16 · IX · 2024

ie Galaxy clustering is the summary statistics of fluctuations in galaxy number counts **in**

$$
= \frac{n_{\rm g}(\bm{x}) - \bar{n}_{\rm g}}{\bar{n}_{\rm g}}
$$

• Redshift-space distortions (RSD) arise as we don't know galaxies' distances, but infer $\frac{1}{8}$ **them from their observed redshifts, which include radial peculiar velocities**

bO(*z*) *O*(*x, z*) *,* (2.9)

- - $\delta_{\rm g}(\boldsymbol{x}) \coloneqq$
-

1, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$, for an exhaustive review $\mathcal{L}^{\mathcal{A}}$

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$$
= \frac{n_{\rm g}(\bm{x}) - \bar{n}_{\rm g}}{\bar{n}_{\rm g}}
$$

Galaxy clustering

• However, number density conservation dictates that metry conservation dictates that α isity conservation **unctates that** α $d^3s [1 + \Delta(s)]$

^B(*k*1*, ^k*2)=2 *^Z*(1)(*k*1) *^Z*(1)(*k*2) *^Z*(2)(*k*1*, ^k*2) *^P*(*k*1) *^P*(*k*2)+2 *^kⁱ ,* (2.12)

with *P*(*ki*) the linear matter power spectrum, implicitly defined via

Galaxy clustering

• However, number density conservation dictates that metry conservation dictates that α isity conservation **unctates that** α

$d^3s [1 + \Delta(s)] = d^3x [1 + \delta_g(x)]$

^B(*k*1*, ^k*2)=2 *^Z*(1)(*k*1) *^Z*(1)(*k*2) *^Z*(2)(*k*1*, ^k*2) *^P*(*k*1) *^P*(*k*2)+2 *^kⁱ ,* (2.12)

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Galaxy clustering level, neglecting subdominant light-cone projection e \bullet

- However, number density conservation dictates that metry conservation dictates that α isity conservation **unctates that** α $d^3s [1 + \Delta(s)] = d^3x [1 + \delta_g(\bm{x})]$ **• However, number density conservation dictates that** $\left[1 + \Delta(s)\right] = \alpha^s x \ln$
- Knowing the real-to-redshift space mapping, we construct redshift-space kernels… • Knowing the real-to-redshift space mapping, we constru $\varDelta^{(m)}(\bm{k}) = 1$ *q*1 *...* ^Z *qm* $\delta^{(1)}(\boldsymbol{q}_1)\ldots\delta^{(1)}(\boldsymbol{q}_m) \, \boldsymbol{\mathcal{Z}}^{(m)}$

with *P*(*ki*) the linear matter power spectrum, implicitly defined via

With this all in mind, the galaxy clustering power spectrum and bispectrum at tree level, neglecting subdominant light-cone projection e↵ects, can be expressed as (*q*1*,..., qm*) (2 ⇡) ³ D(*q*1*...m ^k*) *.* (2.14)

^B(*k*1*, ^k*2)=2 *^Z*(1)(*k*1) *^Z*(1)(*k*2) *^Z*(2)(*k*1*, ^k*2) *^P*(*k*1) *^P*(*k*2)+2 *^kⁱ ,* (2.12) ¹ *,* (2.15)

$$
= d^3x [1 + \delta_g(\boldsymbol{x})]
$$

redshift space mapping, we construct redshift-space kernels...

Galaxy clustering level, neglecting subdominant light-cone projection e \bullet \mathbf{a} w cluster ind $UNIVERSITÀ$ $\frac{a_1}{a_1} \left(\frac{a_2}{a_1} \right)^2$ \bullet diaxy ciustering \bullet $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$

- However, number density conservation dictates that metry conservation dictates that α isity conservation **unctates that** α $d^3s [1 + \Delta(s)] = d^3x [1 + \delta_g(\bm{x})]$ **• However, number density conservation dictates that** $\left[1 + \Delta(s)\right] = \alpha^s x \ln$ *s*^k = *x*^k + with subscript the component of a vector along the line-of-sight direction. In the line-of-si $d^3s [1 + \Delta(s)] = d^3s$ $\overline{\mathcal{X}}$ $x^2 + 2x + 2$ $\frac{1}{2}$ **es that** $d^3a[1+A(a)] = d^3a[1+\lambda(a)]$ $\alpha^{s}(1 + \Delta)$
- Knowing the real-to-redshift space mapping, we construct redshift-space kernels… • Knowing the real-to-redshift space mapping, we constru $\varDelta^{(m)}(\bm{k}) = 1$ *q*1 *...* ^Z *qm* $\delta^{(1)}(\boldsymbol{q}_1)\ldots\delta^{(1)}(\boldsymbol{q}_m) \, \boldsymbol{\mathcal{Z}}^{(m)}$ $\frac{d}{d}$ $\frac{d}{d}$ and the determinal parameter of the coordinates from redshift space the redshift space $\frac{d}{d}$ $\mathbf{k}) = \left. \begin{array}{cc} \hbox{ } \hbox{ } \ldots \end{array} \right| \quad \delta^{(1)}(\bm{q}_1) \ldots \delta^{(1)}(\bm{q}_m) \, \mathcal{Z}^{(m)}(\bm{q}_1, \ldots, \bm{q}_m) \, (2 \, \pi)^3 \, \delta_{\Gamma} \right|$ $\int_{\mathbf{q}_1}$ $\int_{\mathbf{q}_m}$ and $\int_{\frac{5}{9}}$ **d b** \bullet Knowing the real-to-redshift space mapping, we construct redshift-space kernels... \mathcal{C} the determinant of the change of coordinates from \mathcal{C} $\Delta^{(m)}(\boldsymbol{k})=\left.\rule{0pt}{12pt}\right|\left.\rule{0pt}{12pt} \ldots\right.\left.\rule{0pt}{12pt}\right.\delta^{(1)}(\boldsymbol{q}_1)\ldots\delta^{(1)}(\boldsymbol{q}_m)\,\mathcal{Z}^{(m)}(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_m)$ $J{\boldsymbol q}_1$ in $J{\boldsymbol q}_m$
- …from which Specifical Library Constitutions of the specific second services of the specific services of the specific services of the specific services of the specific service service services of the specific service service service s $\n *which*\n$ level, neglection substitution which

with *P*(*ki*) the linear matter power spectrum, implicitly defined via

With this all in mind, the galaxy clustering power spectrum and bispectrum at tree level, neglecting subdominant light-cone projection e↵ects, can be expressed as (*q*1*,..., qm*) (2 ⇡) ³ D(*q*1*...m ^k*) *.* (2.14) ˆ *·* r. ˆ *·* r.

$$
P_{\Delta}(\mathbf{k}_1) = \mathcal{Z}^{(1)}(\mathbf{k}_1) \, \mathcal{Z}^{(1)}(-\mathbf{k}_1) \, P(k_1)
$$

$$
d^3s [1 + \Delta(s)] = d^3x [1 + \delta_g(\boldsymbol{x})]
$$

redshift space mapping, we construct redshift-space kernels... ing the real-to-redshift space mapping, we construct redshift-space kernels...

that

-
- $\mathbf{k}_1 \ \mathbf{k}_2 = 2 \ \mathcal{Z}^{(1)}(\mathbf{k}_1) \ \mathcal{Z}^{(1)}(\mathbf{k}_2) \ \mathcal{Z}^{(2)}(\mathbf{k}_1 \ \mathbf{k}_2) \ P(k_1) \ P(k_2) + 2 \gamma_{\mathbf{k}_2}$ $D_{\Delta}(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2) = \angle z$ ² ⁺ *^b*¹ *^F*2(*k*1*, ^k*2) + *^bG*² *^S*2(*k*1*, ^k*2) + *f µ*² $B = 2 \mathcal{Z}^{(1)}(\mathbf{k}_1) \mathcal{Z}^{(1)}(\mathbf{k}_2) \mathcal{Z}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}$ $B_{\Delta}(\boldsymbol{k}_1, \boldsymbol{k}_2) = 2 \, \mathcal{Z}^{(1)}(\boldsymbol{k}_1) \, \mathcal{Z}^{(1)}(\boldsymbol{k}_2) \, \mathcal{Z}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \, P(k_1) \, P(k_2) + 2 \, \circlearrowleft_{\boldsymbol{k}_i} \, \Big\{ \begin{bmatrix} 0 \ 0 \ 0 \ 0 \end{bmatrix} \Big\}$

Galaxy clustering \mathbf{a} w cluster ind $UNIVERSITÀ$ $\frac{a_1}{a_1} \left(\frac{a_2}{a_1} \right)^2$ \bullet diaxy ciustering \bullet $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$

• Galaxy clustering kernels in redshift space with subscript 'k' denoting the component of a vector along the component of a vector along the line-of-sight
The component of a vector along the line-of-sight direction. In the line-of-sight direction. In the line-of-si f_{r} f_{r} f_{r}

With this all in mind, the galaxy clustering power spectrum and bispectrum at tree

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$$
\mathcal{Z}^{(1)}(\boldsymbol{k}_1) = b_1 + f \mu_1^2
$$

$$
\mathcal{Z}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{b_2}{2} + b_1 F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + b_{\mathcal{G}_2} S_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + f \mu_{12}^2 G_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + \frac{f}{2} \mu_{12} k_{12} \left[\frac{\mu_1}{k_1} \mathcal{Z}^{(1)}(\boldsymbol{k}_2) + \frac{\mu_2}{k_2} \mathcal{Z}^{(1)}(\boldsymbol{k}_1) \right]_{\frac{5}{8}}^{\frac{5}{8}}
$$

 \bullet ... from which $\n *which*\n$

that

$$
P_{\Delta}(\mathbf{k}_1) = \mathcal{Z}^{(1)}
$$

^H , (2.10)

with subscript 'k' denoting the component of a vector along the component of a vector along the line-of-sight d
The component of a vector along the line-of-sight direction. In the line-of-sight direction. In the component

 $\frac{\text{p}}{\text{p}}$

 $P_{\Delta}(\mathbf{k}_1) = \mathcal{Z}^{(1)}(\mathbf{k}_1) \, \mathcal{Z}^{(1)}(-\mathbf{k}_1) \, P(k_1)$ $B = 2 \mathcal{Z}^{(1)}(\mathbf{k}_1) \mathcal{Z}^{(1)}(\mathbf{k}_2) \mathcal{Z}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}$ $P_{\mathcal{A}}(\mathbf{k}_1) = \mathcal{Z}^{(1)}(\mathbf{k}_1) \mathcal{Z}^{(1)}(-\mathbf{k}_1) P(k_1)$ $B_{\Delta}(\boldsymbol{k}_1, \boldsymbol{k}_2) = 2 \, \mathcal{Z}^{(1)}(\boldsymbol{k}_1) \, \mathcal{Z}^{(1)}(\boldsymbol{k}_2) \, \mathcal{Z}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \, P(k_1) \, P(k_2) + 2 \, \circlearrowleft_{\boldsymbol{k}_i} \, \Big\{ \begin{bmatrix} 0 \ 0 \ 0 \ 0 \end{bmatrix} \Big\}$

space. From Eq. (2.10), we have *J* = *|*1 + @k*u/H|*, with @^k = *r*

$$
B_{\Delta}(\boldsymbol{k}_1,\boldsymbol{k}_2)=2\,\mathcal{Z}^{(1)}(\boldsymbol{k}_1)\,\mathcal{Z}^{(1)}(\boldsymbol{k}_2)
$$

Galaxy clustering \mathbf{a} w cluster ind $UNIVERSITÀ$ $\frac{a_1}{a_1} \left(\frac{a_2}{a_1} \right)^2$ \bullet diaxy ciustering \bullet $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$

• Galaxy clustering kernels in redshift space with subscript 'k' denoting the component of a vector along the component of a vector along the line-of-sight
The component of a vector along the line-of-sight direction. In the line-of-sight direction. In the line-of-si f_{r} f_{r} f_{r}

With this all in mind, the galaxy clustering power spectrum and bispectrum at tree

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$$
\mathcal{Z}^{(1)}(\mathbf{k}_1) = b_1 + f \mu_1^2
$$

$$
\mathcal{Z}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) = \frac{b_2}{2} + b_1 \underline{F_2(\mathbf{k}_1, \mathbf{k}_2)} + b_{\mathcal{G}_2} \underline{S_2(\mathbf{k}_1, \mathbf{k}_2)} + f \mu_{12}^2 G_2(\mathbf{k}_1, \mathbf{k}_2) + \frac{f}{2} \mu_{12} k_{12} \left[\frac{\mu_1}{k_1} \mathcal{Z}^{(1)}(\mathbf{k}_2) + \frac{\mu_2}{k_2} \mathcal{Z}^{(1)}(\mathbf{k}_1) \right]_{\frac{5}{8}}^{\frac{5}{8}}
$$

 \bullet ... from which $\n *which*\n$

that

$$
P_{\Delta}(\mathbf{k}_1) = \mathcal{Z}^{(1)}
$$

^H , (2.10)

with subscript 'k' denoting the component of a vector along the component of a vector along the line-of-sight d
The component of a vector along the line-of-sight direction. In the line-of-sight direction. In the component

space. From Eq. (2.10), we have *J* = *|*1 + @k*u/H|*, with @^k = *r*

 $P_{\Delta}(\mathbf{k}_1) = \mathcal{Z}^{(1)}(\mathbf{k}_1) \, \mathcal{Z}^{(1)}(-\mathbf{k}_1) \, P(k_1)$ $B = 2 \mathcal{Z}^{(1)}(\mathbf{k}_1) \mathcal{Z}^{(1)}(\mathbf{k}_2) \mathcal{Z}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}$ $P_{\mathcal{A}}(\mathbf{k}_1) = \mathcal{Z}^{(1)}(\mathbf{k}_1) \mathcal{Z}^{(1)}(-\mathbf{k}_1) P(k_1)$ $B_{\Delta}(\boldsymbol{k}_1, \boldsymbol{k}_2) = 2 \, \mathcal{Z}^{(1)}(\boldsymbol{k}_1) \, \mathcal{Z}^{(1)}(\boldsymbol{k}_2) \, \mathcal{Z}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) \, P(k_1) \, P(k_2) + 2 \, \circlearrowleft_{\boldsymbol{k}_i} \, \Big\{ \begin{bmatrix} 0 \ 0 \ 0 \ 0 \end{bmatrix} \Big\}$

 $\frac{\text{p}}{\text{p}}$

Galaxy clustering \mathbf{a} w cluster ind $UNIVERSITÀ$ $\frac{a_1}{a_1} \left(\frac{a_2}{a_1} \right)^2$ \bullet diaxy ciustering \bullet $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$ $\overline{\bullet}$

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The component of a vector along the line-of-sight direction. In the line-of-sight direction. In the line-of-si f_{r} f_{r} f_{r}

 $\Phi^{(1)}({\bm k}_1,{\bm k}_2) = \left(\begin{array}{c|c} b_2 & b_1 \end{array}\right) F_2({\bm k}_1)$ $\mathcal{Z}^{(1)}$ $\mathcal{Z}^{(2)}(\bm{k}_1,\bm{k}_2) = \frac{\bm{b}_2}{2}$ $\frac{2}{2}$ + b_1 $\frac{1}{2}$ (k_1 , k_2) + b_3 ₂ S_2 (k_1 , k_2) + f μ_1^2

 \bullet ... from which $\n *which*\n$

With this all in mind, the galaxy clustering power spectrum and bispectrum at tree

16 · IX · 2024

$$
\mathcal{Z}^{(1)}(\mathbf{k}_1, \mathbf{k}_2) = \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_2\right\}}_{2} \underbrace{\left\{b_1\right\}}_{2} + \underbrace{\left\{b_1\right\}}_{2} \underbrace{\left\{b_2\right\}}_{2} + \underbrace{\left\{
$$

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 $\left(1\right)$

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 $\frac{\text{p}}{\text{p}}$

• Galaxies' (radial) peculiar velocities are inferred from various types of observations (SNeIa luminosity-distance fluctuations, Tully-Fisher or Faber-Jackson relations, …)

Peculiar velocities

- Galaxies' (radial) peculiar velocities are inferred from various types of observations (SNeIa luminosity-distance fluctuations, Tully-Fisher or Faber-Jackson relations, …)
- RSD still arise, because in most cases the underlying peculiar velocity field of matter (as traced by galaxies) is reconstructed by positioning galaxies in a 3D comoving grid constructed from their redshift-space position—same as in galaxy clustering

- Galaxies' (radial) peculiar velocities are inferred from various types of observations (SNeIa luminosity-distance fluctuations, Tully-Fisher or Faber-Jackson relations, …)
- RSD still arise, because in most cases the underlying peculiar velocity field of matter (as traced by galaxies) is reconstructed by positioning galaxies in a 3D comoving grid constructed from their redshift-space position—same as in galaxy clustering
- Analogously to what done before, I computed peculiar-velocity kernels... At this point, using Eq. (2.6), we can introduce peculiar velocity kernels *^U*(*m*) such that

$$
u^{(m)}(\bm{k}) = \int_{\bm{q}_1} \ldots \int_{\bm{q}_m} \delta^{(1)}(\bm{q}_1) \ldots \delta^{(1)}(\bm{q}_m) \, \mathcal{U}^{(m)}(\bm{q}_1, \ldots, \bm{q}_m) \, (2 \, \pi)^3 \, \delta_{\rm D}(\bm{q}_{1...m} - \bm{k})
$$

. (2.26)

$P_u(\mathbf{k}_1) = \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(-\mathbf{k}_1) P(k_1)$ $B = 2 \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(\mathbf{k}_2) \mathcal{U}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}$ $P_u(\mathbf{k}_1) = \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(-\mathbf{k}_1) P(k_1)$ $B_u(\mathbf{k}_1, \mathbf{k}_2) = 2 \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(\mathbf{k}_2) \mathcal{U}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}$

uar veloch (1)(*q*1)*...* (1)(*qm*) *^U*(*m*) (*k*) = ^Z *...* ^Z **Peculiar velocities**

• …from which and, thus, the peculiar velocity power spectrum and bispectrum and bispectrum and bispectrum and bispectrum and bispectrum are \mathcal{A} f_{tom} thus, thus, the peculiar velocity power spectrum and big f_{tom}

 $\begin{array}{c} \epsilon_{\mathrm{CD}} \ \pm \epsilon_{\mathrm{CD}} \end{array}$ $\frac{1}{\sqrt{2}}$

scribed before and take the form

. (2.26)

. (2.26)

uar veloch (1)(*q*1)*...* (1)(*qm*) *^U*(*m*) (*k*) = ^Z *...* ^Z \mathbf{r} *q*1 *...* ^Z **Peculiar velocities** (1)(*q*1)*...* (1)(*qm*) *^U*(*m*)

• …from which and, thus, the peculiar velocity power spectrum and bispectrum and bispectrum and bispectrum and bispectrum and bispectrum are \mathcal{A} f_{tom} thus, thus, the peculiar velocity power spectrum and big f_{tom}

 $\begin{array}{c} \epsilon_{\mathrm{CD}} \ \pm \epsilon_{\mathrm{CD}} \end{array}$ $\frac{1}{\sqrt{2}}$ Equation (2.25) leads to the usual form of the peculiar velocity power spectrum, in full agreement with the literature, whereas Eq. (2.26) is the first original result of this work.

which
\n
$$
P_u(\mathbf{k}_1) = \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(-\mathbf{k}_1) P(k_1)
$$
\n
$$
B_u(\mathbf{k}_1, \mathbf{k}_2) = 2 \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(\mathbf{k}_2) \mathcal{U}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) P(k_1) P(k_2) + 2 \mathcal{O}_{\mathbf{k}_i}
$$

scribed before and take the form

• The kernel for the power spectrum coincides with the literature The semeror the power spectrum coincides with the interature $\frac{18}{2}$ nel for the power spectrum coincid

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$$
\mathcal{U}^{(1)}(\boldsymbol{k}_1) = -\mathrm{i} \mathcal{H} f D \frac{\mu_1}{k_1}
$$

. (2.26)

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- \sim (1) \sim
- The kernel for the bispectrum is a novel result the bispectrum is a nove *k*1 the bispectrum is a nove *k*1 U
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which
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P_u(\mathbf{k}_1) = \mathcal{U}^{(1)}(\mathbf{k}_1) \mathcal{U}^{(1)}(-\mathbf{k}_1) P(k_1)
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$$
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 $\begin{array}{c} \epsilon_{\mathrm{CD}} \ \pm \epsilon_{\mathrm{CD}} \end{array}$ $\frac{1}{\sqrt{2}}$ $\sum_{i=1}^{n}$ $\mathcal{L}_{\mathcal{D}}$ agreement with the literature, whereas Eq. (2.26) is the first original result of this work.

$$
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$$

trum is a novel result

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nel for the bispectrum is a novel result
\n
$$
\mathcal{U}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) = \mathcal{U}^{(1)}(\mathbf{k}_{12}) D \left[G_2(\mathbf{k}_1, \mathbf{k}_2) - \frac{3}{2} f \mu_1 \mu_2 \frac{(k_{12})^2}{k_1 k_2} \right]
$$

scribed before and take the form

• …from which and, thus, the peculiar velocity power spectrum and bispectrum and bispectrum and bispectrum and bispectrum and bispectrum are \mathcal{A} f_{tom} thus, thus, the peculiar velocity power spectrum and big f_{tom} \bullet from which

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Peculiar velocities *uar* veloch (1)(*q*1)*...* (1)(*qm*) *^U*(*m*) (*k*) = ^Z *...* ^Z (*k*) = ^Z *...* ^Z At this point, using Eq. (2.6), we can introduce peculiar velocity kernels *^U*(*m*) such that \mathbf{r} *q*1 *...* ^Z *qm* (1)(*q*1)*...* (1)(*qm*) *^U*(*m*)

which
\n
$$
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$$
\n
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$$

scribed before and take the form

• …from which

g(*q*2) + *k µ ^µ*² This means that, if *p* =: *p*(1) + *p*(2), we can introduce kernels that ultimately read + *...* (3.1) Stefano Camera The peculiar velocity bispectrum 16 · IX · 2024

$p := (1 + \Delta) u$ Z
Z
Z $p := (1$ *µ*1 $\Delta) \, u$

Momentum density nen 1 Z Im density (*ⁿ* 1)! *^u*(*x*) [1 + g(*x*)]

• Momentum density is the density-weighted peculiar velocity field *q*1 ³ D(*q*¹ *^k*) $\mathsf{W}\mathsf{f}$

$$
\mathcal{P}^{(1)}(\bm{k}_1) = \mathcal{U}^{(1)}(\bm{k}_1) \\ \mathcal{P}^{(2)}(\bm{k}_1,\bm{k}_2) = \mathcal{U}^{(2)}(\bm{k}_1,\bm{k}_2) + \mathcal{U}^{(1)}(\bm{k}_{12}) \, D \, \left[\frac{f}{2} \, \mu_1 \, \mu_2 \, \frac{(k_{12})^2}{k_1 \, k_2} + \frac{b_1}{2} \left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2}\right) \, \frac{k_{12}}{\mu_{12}}\right] \, \left[\frac{f}{2} \, \mu_1 \, \mu_2 \, \frac{(k_{12})^2}{k_1 \, k_2} + \frac{b_1}{2} \left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2}\right) \, \frac{k_{12}}{\mu_{12}}\right]
$$

This means that, if *p* =: *p*(1) + *p*(2), we can introduce kernels that ultimately read

, (3.3)

 \hat{k}_1 ⊥

 $\hat{k}_|$

 $\begin{picture}(20,20) \put(0,0){\vector(1,0){15}} \put(15,0){\vector(1,0){15}} \put(15,0){\vector($

-
- # of dof:
• $33Dk_i = 9$

- - *k i*
- -
- ˆ*k*1⊥ ˆ*k*∥ • # of dof:

• 3 3D $k_i = 9$

• But cosmological principle:

• -3 rotations (isotropy)

• -3 translations (homogeneity)

- - *k i*
- -
- ˆ*k*1⊥ ˆ*k*∥ • # of dof:

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- -
- -
- # of dof:

 $3 \, 3D \, k_i = 9$

 But cosmological principle:

 -3 rotations (isotropy)

 -3 translations (homogeneity)

 $+2$ angles w.r.t. line of sight

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Comparing bispectra

[SC 2024 (in prep.)]

Comparing bispectra comparing bispectra

Comparing bispectra

Detectability

$\tilde{P}_X = P_X +$ $\sqrt{ }$ $1/\bar{n}_{\text{g}}(\bar{z}_i)$ if $X = \Delta$ $\sigma_{\rm v}^2$ $\frac{2}{v}$ / $\bar{n}_v(\bar{z}_i)$ if $X = \{u, p\}$, $\frac{2}{v}$

- ⁴ ⇡*/*3 [*r*3(¯*zⁱ z/*2) *^r*3(¯*zⁱ* ⁺ *z/*2)] the comoving volume available in a bin of width *^z* centred on ¯*zⁱ* (assuming full sky coverage for simplicity). Finally, *s^B* is a coecient respec-
- tively equal to 6*,* 2*,* 1 for equilateral, isosceles, and scalene triangles, which accounts for their multiplicity. Let me also emphasise that most of the quantities entering the variance are
-
-
-
-
-

Detectability

 $\tilde{P}_X = (P_X) +$ $\sqrt{ }$ $1/\bar{n}_{\text{g}}(\bar{z}_i)$ if $X = \Delta$ $\sigma_{\rm v}^2$ $\frac{2}{v}$ / $\bar{n}_v(\bar{z}_i)$ if $X = \{u, p\}$, $\frac{2}{v}$

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Detectability Example 2014 [SC 2024 (in prep.)]

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

Central redshift of the bin, \bar{z}_i $\frac{1}{2}$ f

 $-$ 1 u , $=$ Λ p ⁴ ⇡*/*3 [*r*3(¯*zⁱ z/*2) *^r*3(¯*zⁱ* ⁺ *z/*2)] the comoving volume available in a bin of width *^z* centred on ¯*zⁱ* (assuming full sky coverage for simplicity). Finally, *s^B* is a coecient respectively equal to 6*,* 2*,* 1 for equilateral, isosceles, and scalene triangles, which accounts for their multiplicity. Let me also emphasise that most of the quantities entering the variance are redshift-bin dependent, like *k*f(¯*zi*). In Eqs. (4.2) and (4.3), $\tilde{P}_X = P_X +$ $\sqrt{ }$ $1/\bar{n}_{\text{g}}(\bar{z}_i)$ if $X = \Delta$ $\sigma_{\rm v}^2$ $\frac{2}{v}$ / $\bar{n}_v(\bar{z}_i)$ if $X = \{u, p\}$, $\frac{2}{v}$

Detectability Example 2024 (in prep.)]

 0.4 0.6 0.8

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

0*.*2 0*.*4 0*.*6 0*.*8 Central redshift of the bin, \bar{z}_i $\frac{1}{2}$ f

 $-$ 1 u , $=$ Λ p ⁴ ⇡*/*3 [*r*3(¯*zⁱ z/*2) *^r*3(¯*zⁱ* ⁺ *z/*2)] the comoving volume available in a bin of width *^z* centred on ¯*zⁱ* (assuming full sky coverage for simplicity). Finally, *s^B* is a coecient respectively equal to 6*,* 2*,* 1 for equilateral, isosceles, and scalene triangles, which accounts for their multiplicity. Let me also emphasise that most of the quantities entering the variance are the variance are the
The variance are the vari redshift-bin dependent, like *k*f(¯*zi*). In Eqs. (4.2) and (4.3), $\tilde{P}_X = P_X +$ $\sqrt{ }$ $1/\bar{n}_{\text{g}}(\bar{z}_i)$ if $X = \Delta$ $\sigma_{\rm v}^2$ $\frac{2}{v}$ / $\bar{n}_v(\bar{z}_i)$ if $X = \{u, p\}$, $\frac{2}{v}$

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

Detectability Section Contracts

Central redshift of the bin, \bar{z}_i $\frac{1}{2}$ f

 $-$ 1 u , $=$ Λ p centred on ¯*zⁱ* (assuming full sky coverage for simplicity). Finally, *s^B* is a coecient respectively equal to 6*,* 2*,* 1 for equilateral, isosceles, and scalene triangles, which accounts for their multiplicity. Let \mathcal{L} and the quantities entering that most of the variance are t redshift-bin dependent, like *k*f(¯*zi*). In Eqs. (4.2) and (4.3), $\tilde{P}_X = P_X +$ $\sqrt{ }$ $1/\bar{n}_{\text{g}}(\bar{z}_i)$ if $X = \Delta$ $\sigma_{\rm v}^2$ $\frac{2}{v}$ / $\bar{n}_v(\bar{z}_i)$ if $X = \{u, p\}$, $\frac{2}{v}$

Detectability Section Contracts

Central redshift of the bin, \bar{z}_i $\frac{1}{2}$ f

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

Information content \sum_{1404} $\begin{array}{ccc} \text{[SC 2024 (in prep.)]} \ \text{[SC 2024 (in prep.)]} \end{array}$ Upon sy is not going the peculiar velocity of t bis per la contrarily to contrarily to case of the galaxy case of the galaxy clustering bispectrum. The galaxy clustering bispectrum. The galaxy clustering bispectrum. The galaxy clustering bispectrum. The galaxy clusterin

[SC 2024 (in prep.)]

 $Power$ spectrum SNR, $SNR_P(\bar{z}_i)$ Power spectrum SNR, $SNR_{P}(\bar{z}_i)$ $\overline{\mathcal{A}}$ Γ t ower speech all state, bialo $P(\varkappa_l)$

Central redshift of the bin, \bar{z}_i n_A Critr as the interesting of the new ρ of the newly ρ of ρ or ρ

 $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $\frac{11}{\sqrt{2}}$ $\overline{z}_i)$ trum, I performation matrix and information matrix and the growth rate, \mathbf{r} *f*. In general, for a given set of parameters of interest, # = *{*#↵*}*, the per-bin information *I* (\varPi,X) $\frac{1}{\alpha\beta}(I,X)\begin{pmatrix} \bar{z}_i \end{pmatrix} = \frac{\partial\bm{\varPi}_X^{\sf H}(\bar{z}_i)}{\partial\vartheta^{\sf H}}$ $\partial \vartheta_{\alpha}$ $C^{-1}(\bar{z}_i)$ $\partial \boldsymbol{\varPi}_{X}(\bar{z}_i)$ $\partial \vartheta_\beta$ $\mathcal{V} = \{f, b_1, b_2, b_{\mathcal{G}_2}, P_{\text{shot}}, B_{\text{shot}}\}$ clude nuisance parameters for the galaxy biases, and others to account for any residual shot noise in the power spectrum and bispectrum; in other words, # = *{f, b*1*, b*2*, bG*² *, P*shot*, B*shot*}*. $I_{\alpha\beta}^{(\Pi,X)}(z_i) = \frac{\partial \Pi_X(z_i)}{\partial z_i} C^{-1}(z_i) \frac{\partial \Pi_X(z_i)}{\partial z_i}$ $\frac{\partial Q}{\partial \sigma}$ biases, and others to account for any residual show $\frac{\partial v}{\partial \sigma}$ and $\frac{\partial v}{\partial \sigma}$ $N_{\rm{obs}}$ all parameters are allowed to vary freely in each redshift bin. Once the information $N_{\rm{obs}}$ $\frac{m}{2}$ is known, constraints on measurements of the set can be readily set can be rea

spurious information [4]. For this reason, I decided to take a conservative approach and do

not include them at all in the analysis. From what discussed about the signal dependence

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

Information content

[SC 2024 (in prep.)]

Power spectrum SNR, $SNR_{P}(\bar{z}_i)$

Information content

