Local invariances in metric-affine theories and the heat kernel for non-minimal second-order operators

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4th International FLAG Workshop: The Quantum and Gravity, 11/09/2024 Catania

Based on ongoing works

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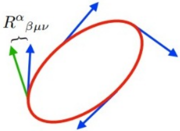
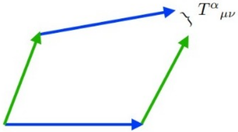
Outline

Introduction

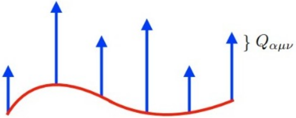
Part 1



The rotation of a vector transported along a closed curve is given by the curvature: General Relativity.



The non-closure of parallelograms formed when two vectors are transported along each other is given by the torsion: Teleparallel Equivalent of General Relativity.



The variation of the length of a vector as it is transported is given by the non-metricity: Symmetric Teleparallel Equivalent of General Relativity.

Image from Universe **5** (2019) 7, 173, J. Beltrán Jiménez, L. Heisenberg, T. Koivisto

Introduction

Part 2

Metric-affine gravity extends the field content of GR retaining its original spirit, and its RG flow is not known

New degrees of freedom may arise in the deep UV ($2^{\pm}$, $1^{\pm}$, $0^{\pm}$), and change the physical properties of the theory

By letting the spin-connection be a dynamical field we obtain non-vanishing torsion

Introduction

Part 3

When the affine-connection is independent of metric, we are in the realm of MAGs. The "curvatures" are

$$
\begin{aligned}
R^{\rho}{}_{\sigma\mu\nu} &= \partial_\mu \Gamma^{\rho}{}_{\sigma\nu} + \Gamma^{\rho}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\sigma\nu} - (\nu \leftrightarrow \mu) \\
T^{\rho}{}_{\nu\mu} &= \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu} \\
Q_{\mu\nu\rho} &= -D_\rho g_{\mu\nu}
\end{aligned}
$$

Splitting: $\Gamma^{\rho}{}_{\nu\mu} = \mathring{\Gamma}^{\rho}{}_{\nu\mu} + \Phi^{\rho}{}_{\nu\mu} = \mathring{\Gamma}^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\nu\mu} + N^{\rho}{}_{\nu\mu}$.

Contortion: $K^{\rho}{}_{\nu\mu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\nu\mu})$;

Distortion: $N^{\rho}{}_{\nu\mu} = \frac{1}{2}(Q_{\mu}{}^{\rho}{}_{\nu} + Q_{\nu}{}^{\rho}{}_{\mu} - Q_{\mu\nu}{}^{\rho})$

Further symmetries in MAGs: why?

The most general torsion theory up to dimension four terms has 94 free parameters

Moreover, the RG flows of such theories is mostly unknown

Thus, we want to study non-linear infinitesimal transformations of the torsion that yield a closed algebra

Two widely studied cases of non-linear symmetries: GR and higher-spins

Further symmetries in MAGs: previous studies

Projective transformations, e.g., $\delta T^\rho{}_{\mu\nu} = \delta^\rho{}_\nu v_\mu - \delta^\rho{}_\mu v_\nu$

Extending Weyl: $\delta g_{\mu\nu} = 2\sigma g_{\mu\nu}$, $\delta T^\rho{}_{\mu\nu} \propto \delta^\rho{}_\nu \partial_\mu \sigma - \delta^\rho{}_\mu \partial_\nu \sigma$ (see Gregorio's talk)

Axial transformation: $\delta T^\rho{}_{\mu\nu} = \varepsilon^{\lambda\rho}{}_{\mu\nu}\partial_\lambda\varphi$ (Shapiro, Phys.Rept. 357 113 (2002))

Last year's proposal:

$$\delta T^\rho{}_{\mu\nu} = \nabla^\rho \nabla_\mu \xi_\nu + \frac{1}{d-1}\delta^\rho{}_\mu(\Box \xi_\nu - R^\lambda{}_\nu \xi_\lambda) - (\nu \leftrightarrow \mu)$$

(See gr-qc/2312.16681)

Affine infinitesimal transformations

We are interested in transformations of the form $\delta T = \nabla\Phi + \mathrm{const}\; T \cdot \Phi$

T counts as one derivative, while Φ as zero

Tensor analysis: $\Phi = \left(A_{\mu\nu},\, \Pi_{\mu\nu},\, S_{\mu\nu},\, \phi\right)$

∇S and $\nabla\phi$ are Hook Antisymmetric (HA), so we impose HA also on $T \cdot S$ and $T \cdot \phi$

$A_{\mu\nu}$ parameterizes the HA transf., Π the Totally Antisymmetric (TA) one

We require closure, and get two non-trivial solutions

One generalizes Weyl's type of transformation

$$\delta_\phi T^\rho{}_{\mu\nu} = \delta^\rho{}_\nu \partial_\mu \phi - \delta^\rho{}_\mu \partial_\nu \phi + z \Big[T^\rho{}_{\mu\nu} + T_\mu{}^\rho{}_\nu - T_\nu{}^\rho{}_\mu - \frac{3}{d-1} \left(\delta^\rho{}_\nu T^\alpha{}_{\mu\alpha} - \delta^\rho{}_\mu T^\alpha{}_{\nu\alpha} \right) \Big] \phi$$

$$[\delta_{\alpha_1}, \delta_{\alpha_2}] = 0$$

Algebras

The non-abelian one

Now we allow transf.'s to be algebraically reducible

The second: mixes HA and TA sectors

$$\delta_A T^\rho{}_{\mu\nu} = 3\nabla^\rho A_{\mu\nu} + \nabla_\mu A^\rho{}_\nu - \nabla_\nu A^\rho{}_\mu + \varsigma \left[2\, T_{[\mu\nu]}{}^\lambda A^\rho{}_\lambda + \frac{2}{3} T_{[\mu}{}^{\rho\lambda} A_{\nu]\lambda} + \frac{2}{3} T^\lambda{}_{\mu\nu} A^\rho{}_\lambda + \frac{4}{3} T^\lambda{}_{[\mu}{}^\rho A_{\nu]\lambda} - \frac{10}{3} T^\rho{}_{[\mu}{}^\lambda A_{\nu]\lambda} \right]$$

$$[\delta_{A_1}, \delta_{A_2}] = \frac{4\,\varsigma}{3} \delta_{[A_1, A_2]} \qquad\qquad [\delta_A, \delta_\xi^{\mathrm{diff}}] = 0$$

Invariant actions

Inv. under δ_ϕ leaves 33 couplings left in the torsion action

Inv. under δ_A yields only 2 ind. couplings form the 92 starting ones. With $\epsilon = \pm 1$

$$\mathcal{L}_T = (\nabla T)^2 + RT^2 + \frac{\epsilon}{\sqrt{\varsigma}\,\xi} T \nabla R + \epsilon \frac{\sqrt{\varsigma}}{\xi} T^2 \nabla T + \frac{\varsigma}{\xi^2} T^4$$

Mass terms are ruled out and coupling to matter fields is forbidden

Tools for working out the RG flow

We focus on the most-general second-order diff. operator

$$\hat{F}(\nabla) = \hat{\Box} - \lambda \hat{N}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu + \hat{v}^\mu \hat{\nabla}_\mu + \hat{E}$$
$$= \hat{F}_v(\nabla) - \lambda \hat{N}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu$$

We only require $\hat{\nabla}_\mu \hat{N}^{\alpha\beta} = 0$

Examples for $\hat{v}^\mu = 0$ in Barvinsky, Vilkovisky, Phys.Rept. 119 (1985) and Grogh et Al., PoS EPS-HEP2011 124 (2011)

Formal developments

See DeWitt, Dynamical theory of groups and fields

We use the heat kernel representation, i.e.,

$$(\hat{F}_v + i\partial_s)\hat{K}(x,x';s) = 0\,, \qquad \hat{K}(x,x';0) = \delta(x,x')$$

$$\hat{K}_v(x,x';s) \approx \frac{i\mathcal{D}^{\frac{1}{2}}(x,x')}{(4\pi i s)^{\frac{d}{2}}} \mathrm{e}^{i\frac{\sigma(x,x')}{2s}} \sum_{n=0}^{\infty} (is)^n \hat{\mathfrak{a}}_n(x,x')\,.$$

$$\sigma = \frac{1}{2}\sigma_\mu\sigma^\mu\,, \qquad \sigma_\mu \equiv \nabla_\mu\sigma\,,$$

$$\mathcal{D}(x,x') = -\det(-\partial_\mu\partial'_\nu\sigma)\,, \qquad \mathcal{D}(x,x') = |g(x)|^{1/2}|g(x')|^{1/2}D(x,x')\,.$$

Recursive relation

$$n\hat{\mathfrak{a}}_n + \sigma^\mu(\nabla_\mu + \frac{1}{2}v_\mu)\hat{\mathfrak{a}}_n = D^{-1/2}\hat{F}_v\left(D^{1/2}\hat{\mathfrak{a}}_{n-1}\right)\;;$$

$$\sigma^\mu(\nabla_\mu + \frac{1}{2}v_\mu)\hat{\mathfrak{a}}_0 = 0\,, \qquad \hat{\mathfrak{a}}_0(x,x) = \hat{\mathbb{1}}\,.$$

Formal developments

First results

The heat kernel for $\hat{F}_v(\nabla)$ is found by the squaring procedure of Obukhov (see Nucl.Phys.B 212 (1983))

Checked this result calculating the coincidence limit of $\hat{\mathfrak{a}}_2$ starting from the modified recurrence relation

For non-minimal operators this trick cannot be applied

There are tensor structures that cannot be generated by a squaring procedure, as it was expected

The final result is very long (I'm still transcribing it from the notebook)

Conclusions and outlooks

1. Local invariances parametrized by rank> 0 tensors highly restrict the parameter space

   Work out the particle content of the theory

   Are these models renormalizabile? What is their RG flow? Are these symmetries anomalous?

2. Still writing down the final formula

   Heat kernel coefficients for higher-derivative gravity coupled to second-order "matter" fields

The end

Thank you for your attention!

Questions are welcome

Invariant actions, part 2

The Lagrangian

The terms that contribute to the flat space particle content are

$$\mathcal{L} \supset (\nabla_\alpha T_{\mu\nu\rho})\nabla^\alpha T^{\mu\nu\rho} - \frac{5}{3}(\nabla_\alpha T_{\mu\nu\rho})\nabla^\alpha T^{\nu\mu\rho} - \frac{2}{3}(\nabla_\lambda T^{\lambda\mu\nu})\nabla_\alpha T^\alpha{}_{\mu\nu}$$
$$+ \frac{1}{3}(\nabla_\alpha T^{\mu\alpha\nu})\nabla_\lambda T_\nu{}^\lambda{}_\mu - \frac{4}{3}(\nabla_\alpha T^{\alpha\mu\nu})\nabla_\alpha T_{\mu\nu}{}^\alpha - \frac{1}{3}(\nabla_\lambda T^{\mu\lambda\nu})\nabla_\alpha T_\mu{}^\alpha{}_\nu$$
$$- \frac{2\epsilon}{\sqrt{\varsigma\,\xi}} T^{\rho\mu\nu}\nabla_\mu R_{\nu\rho} - \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + \frac{2}{\varkappa^2}R$$

$\epsilon = \pm 1$, ξ is the remaining coupling constant and ς is the parameter of the transformation

Torsion mass terms are absent, so the analysis of ghosts presence is more complicated (see Sezgin, van Nieuwenhuizen PRD 21 3269 (1980) and Baldazzi, Percacci, Melichev Annals Phys. 438 168757 (2022))

Formal developments

Perturbation theory

$$\hat{D}(\nabla) \equiv \hat{\Box} - \lambda \hat{N}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu$$

$$\hat{D}(n)\hat{K}(n) = (n^2)^2 \hat{\mathbb{1}} \,, \qquad \hat{D}(\nabla)\hat{K}(\nabla) = \hat{\Box}^2 + \hat{K}_1(\nabla) \,.$$

$$\hat{F}(\nabla)\hat{K}(\nabla) = \hat{\Box}^2 + \hat{M}(\nabla)$$

$$\hat{M}_0 = \hat{\mathbb{1}} \,, \qquad \hat{M}_{p+1} = \hat{M}\hat{M}_p + [\hat{\Box}, \hat{M}_p] \,.$$

Formal developments

Perturbation theory

$$iW(\lambda) = iW(0) - \frac{1}{2}\int_0^\lambda d\lambda' \sum_{p=0}^{4} \mathrm{Tr}\left[(-1)^p \frac{dF(\lambda')}{d\lambda'} K(\lambda') M_p(\lambda') \frac{\hat{\mathbb{1}}}{\Box^{m(p+1)}}\right] + \dots$$

$$\hat{D}(\nabla) = \Box + f(\lambda)\hat{A}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu$$

$$W(\lambda) \supset \int_0^\lambda d\lambda' \int \sqrt{-g}\,\hat{v}^4\hat{A}^4\hat{N}$$