Nonsingular black holes: effective models and observational signatures

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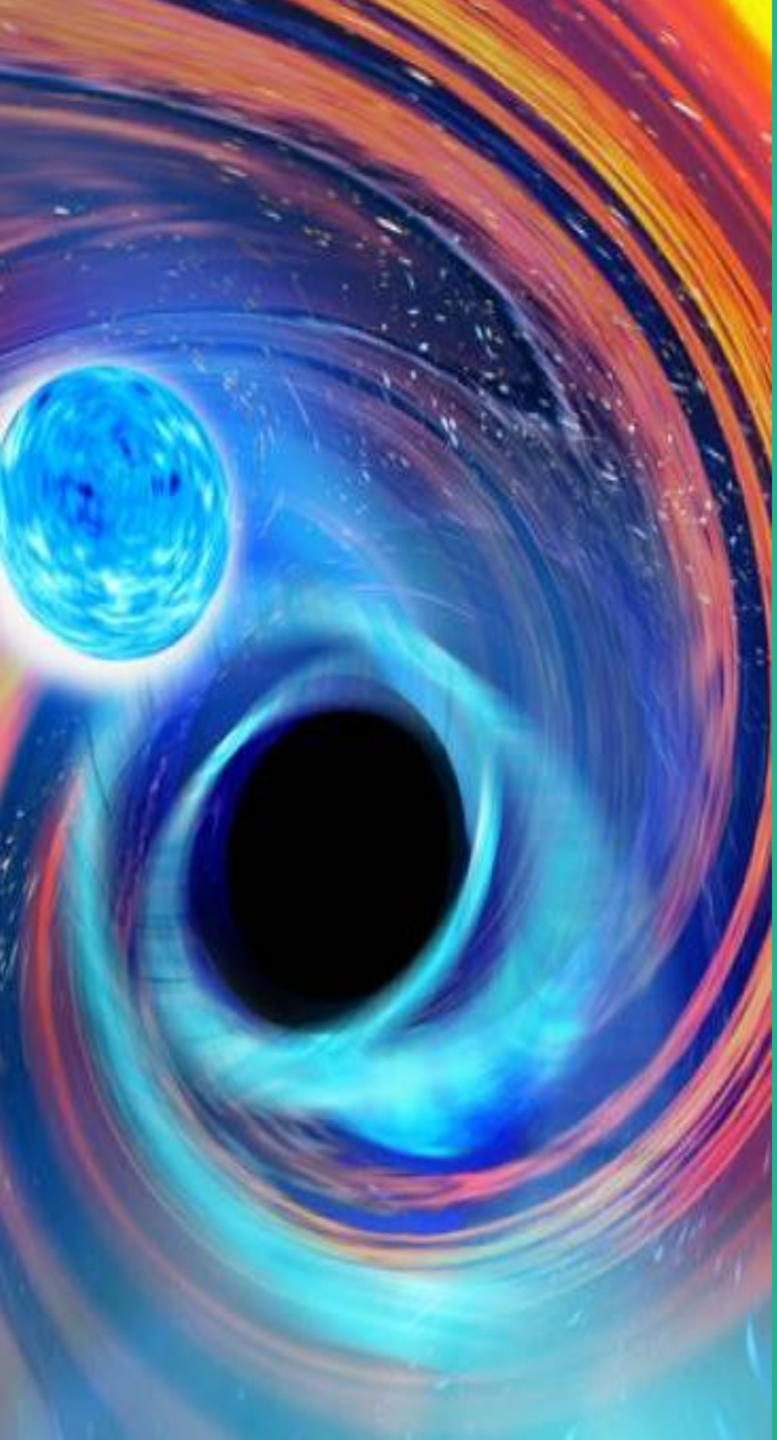
Mainly based on*:*

M.C. M. Oi and A. Sanna, Effective models of Nonsingular quantum black holes , PRD 106 (2022);
M.C. M. F. De Laurentis, I. De Martino, R. Della Monica, M. Oi and A. Sanna, Are Nonsingular black holes with superPlankian hair ruled out by S2 star data? PRD 107 (2023);
M.C. et al, Cosmological coupling of nonsingular black holes, JCAP11 (2023);
M.C. R. Murgia, M. Pitzalis and A. Sanna, Quasi local masses and Cosmological coupling of black holes and mimickers, JCAP03 (2024)



Why Nonsingular black holes (NSBHs)?

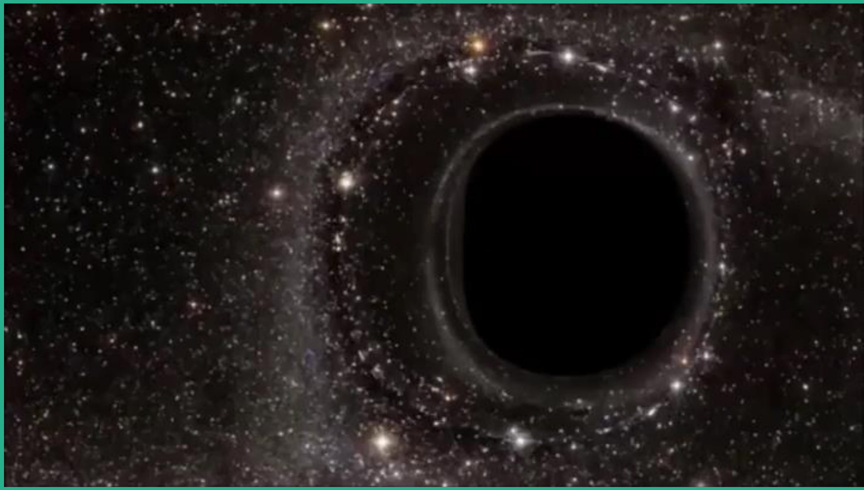
- Singularity problem in GR and Penrose theorem
- BH thermodynamics and Hawking effect also predicts breakdown of classical description: $T \propto \frac{1}{M}$
- Even though not always recognized BH information problem is tightly linked to singularity problem
- Conventional wisdom: Quantum Gravity (QG) effects at Planck scale l_P remove the singularity

$\Downarrow$

FULLY QUANTUM DRESSED BHs SHOULD BE NONSINGULAR OBJECTS

- Present observation (GW, BH imaging, orbits of stars near supermassive BHs) are fully compatible with nonsingular nature of BHs

Nonsingular BH solutions



- Show up in several contexts:
  - As effective models interpolating between a regular geometry near r=0 and a Schwarzschild (SCHW) one at $r=\infty$ (Bardeen 1968, Hayward 2006, GR coupled with nonlinear electrodynamics- Dymnikova 2004…..)
  - As QG, UV-corrected, geometries
    - ✓ String theory (Horowitz 1990)
    - ✓ Loop quantum gravity (Modesto 2004)
    - ✓ Non commutative geometry (Nicolini et al 2006)
    - ✓ In the Functional renormalization group approach (Bonanno-Reuter 2000, see M. Pitzalis talk)
    - ✓ Nonlocal theories of gravity (Modesto 2011)
  - In the emergent gravity and corpuscular gravity scenario (Dvali 2014, Casadio et al 2016, Cadoni 2022)

Effective NSBH models as emergent gravity

- We consider static, spherically symmetric solutions sourced by anisotropic fluid (simplest way to circumvent Penrose theorems)
- Regularity conditions for the metric and the pressures and presence of horizons imply the form for the metric

$$ds^2 = -A(r)dt^2 + A(r)^{-1}dr^2 + d\Omega^2$$

with a de Sitter (dS) form of the UV geometry

$$A(r) = 1 - \frac{r^2}{L^2} + O(r^3)$$

- The spacetime interpolates between a regular dS "Quantum" UV geometry near r=0 and Asymptotic SCHW one endowed with a quantum hair l:

$$A(r) = 1 - \frac{2GM}{r} + \alpha\frac{l^2}{r^2} + \beta\frac{3}{r^3} + O(r^{-4})$$

- The interpolating function A(r) relates the dS length L to the presence of QG gravity effects at horizon scales characteristic by the quantum hair l

- Similarly to Verlinde's emergent galactic dynamics the "mesoscopic" scale l is generated from L and the gravitational radius R_S associated to the mass M by the scaling law

$$l = R_S^{1/3} L^{2/3}$$

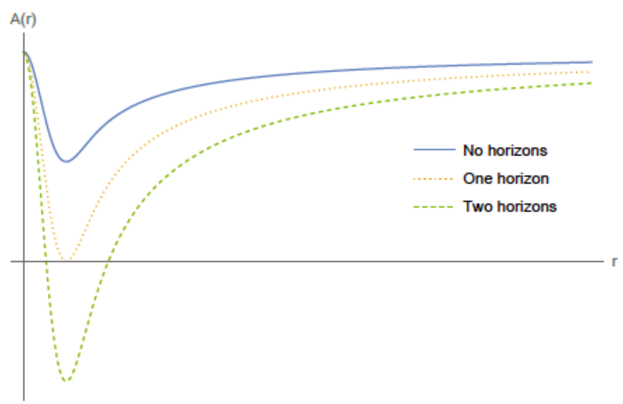
- The form of A(r) is now constrained by reasonable physical requirements (regularity, presence of at most two event horizons…
- Notice: l can be both Planckian or superPlanckian

General class of models

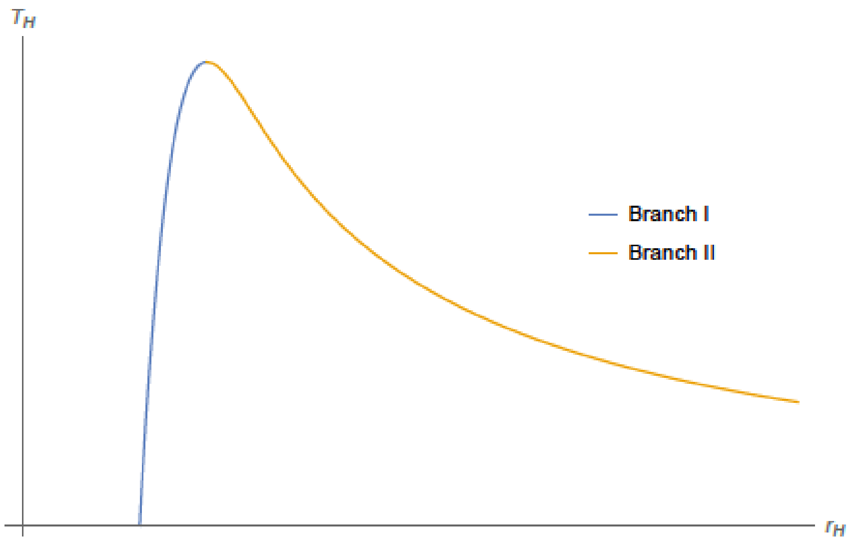
- We have a broad class of models parametrized by the function A(r). It contains as particular cases the NSBH proposed so far
- One can also classify the models in terms of the first non vanishing hair term subleading with respect to the SCHW one (e.g l^2/r^2, l^3/r^3 and so on)

General Features

✓ Presence of two event horizons and causal structure similar to the Reissner-Nordstrom BH. We have a critical value α_c of the parameter $\alpha = R_S/l$ discriminating between NSBH, extremal BH and Quantum horizonless Stars :



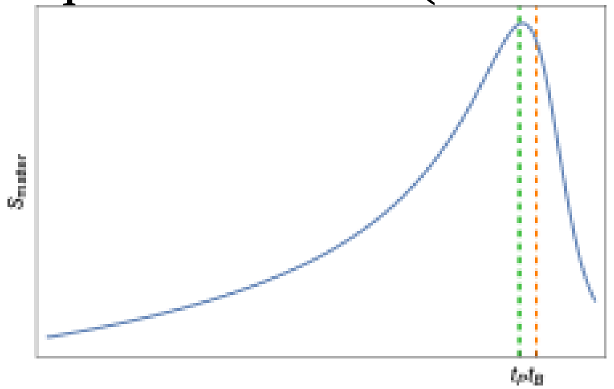
- ✓ Second order thermodynamical phase transition



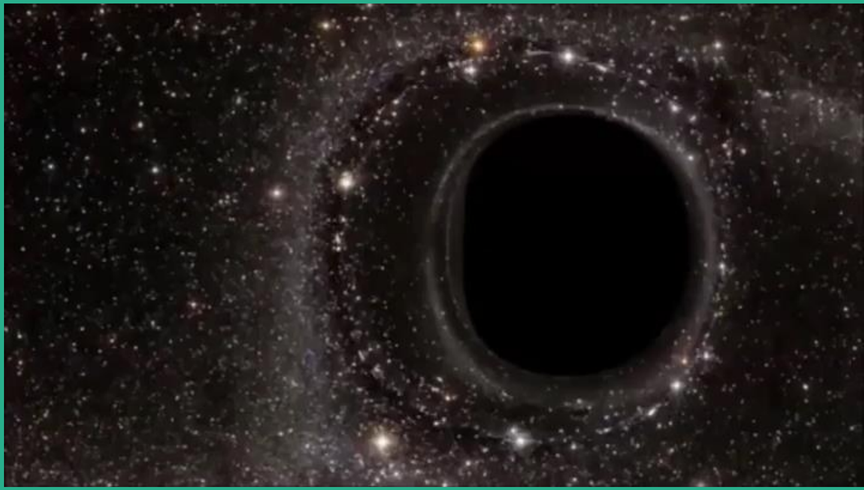
- ✓ In branch I the specific heat is positive (negative in branch II corresponding to the classical SCHW BH)
- ✓ Black holes with $l \approx R_S$ energetically preferred
- ✓ l now can be both Planckian or Superplanckian but latter are thermodynamically preferred
- ✓ Extremal black hole (T=0) is stable
- ✓ Violation of the area law

Black hole information problem

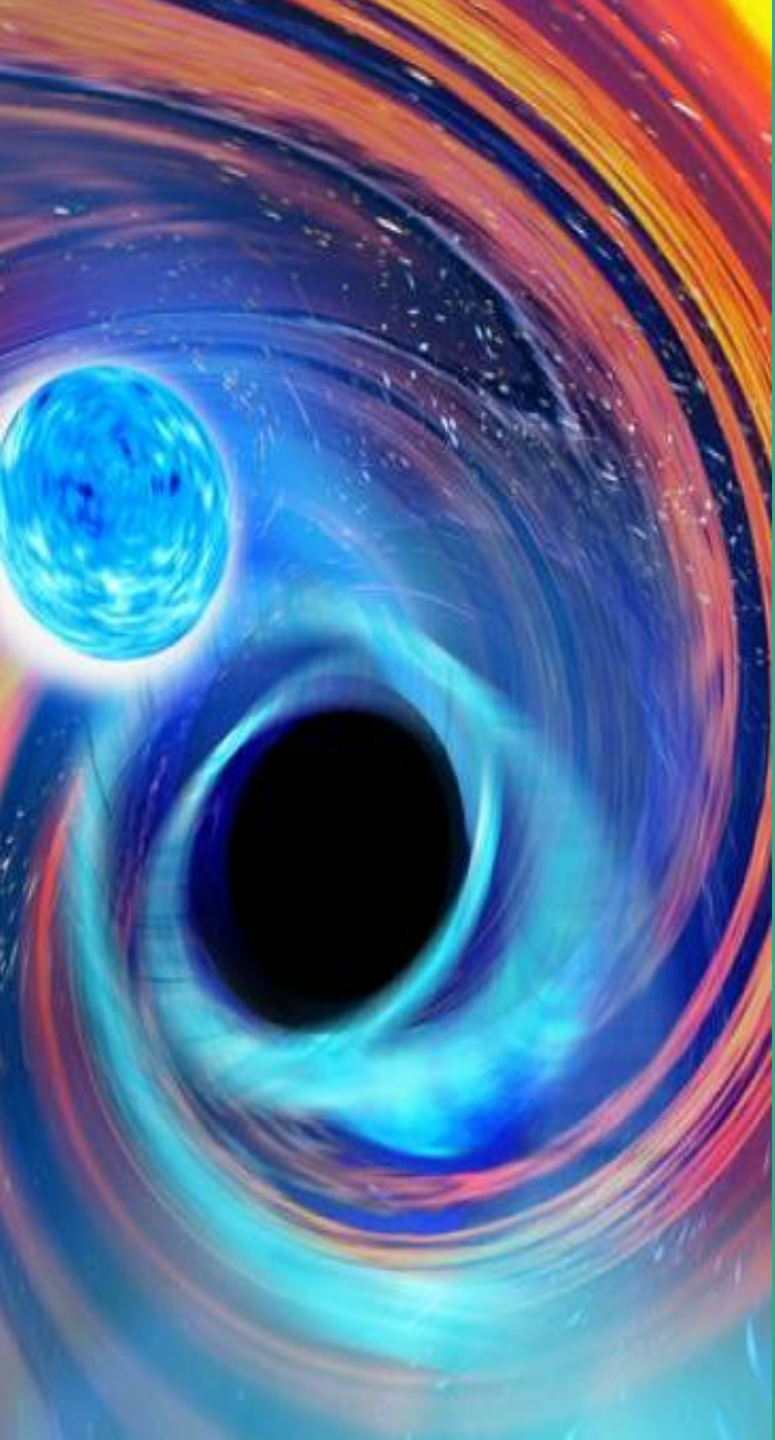
- It has been suggested the breakdown of unitarity in the BH evaporation process to be traced back to the presence of the singularity
- We have tested (JHEP 06 2023) this conjecture in the 2D version of the nonsingular Hayward black hole coupled to conformal matter by solving the exact semiclassical dynamics (including back reaction)
- We have found that the entanglement entropy of the radiation initially grows reaches a maximum and then goes down toward zero (Page curve), leaving behind a pure quantum state (extremal black hole)



OBSERVATIONAL SIGNATURES



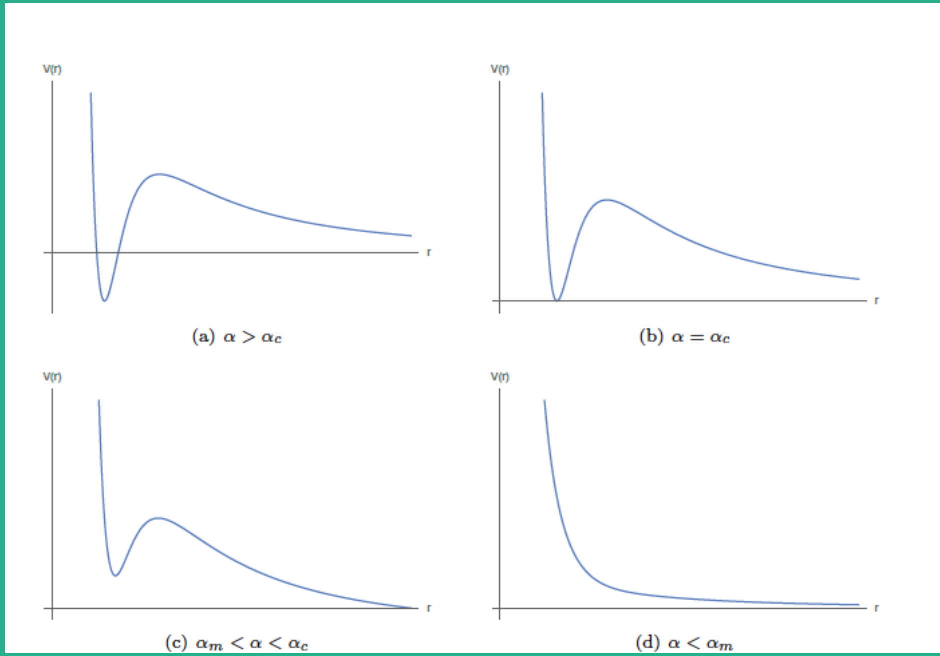
- Local near horizon observations of quantum hair l of NSBH trough :
  - Orbits of light and planets near supermassive galactic black holes
  - Gravitational wave emission in the ringdown phase of binary stellar black hole merging
- Recently discovered global feature (see *M.C. et al,* JCAP11 (2023))
  - Detection of NSBH trough their coupling to large scale cosmological dynamics
- **NOTICE**:
  - Local near horizon observations are expected to detect only superplanckian hair (Planckian hair are suppressed by inverse powers of Planck mass). Owing to its nonlocal character Cosmological coupling is sensitive to both Panckian and superplackian quantum (and even fully classical) deformations



PHOTON ORBITS

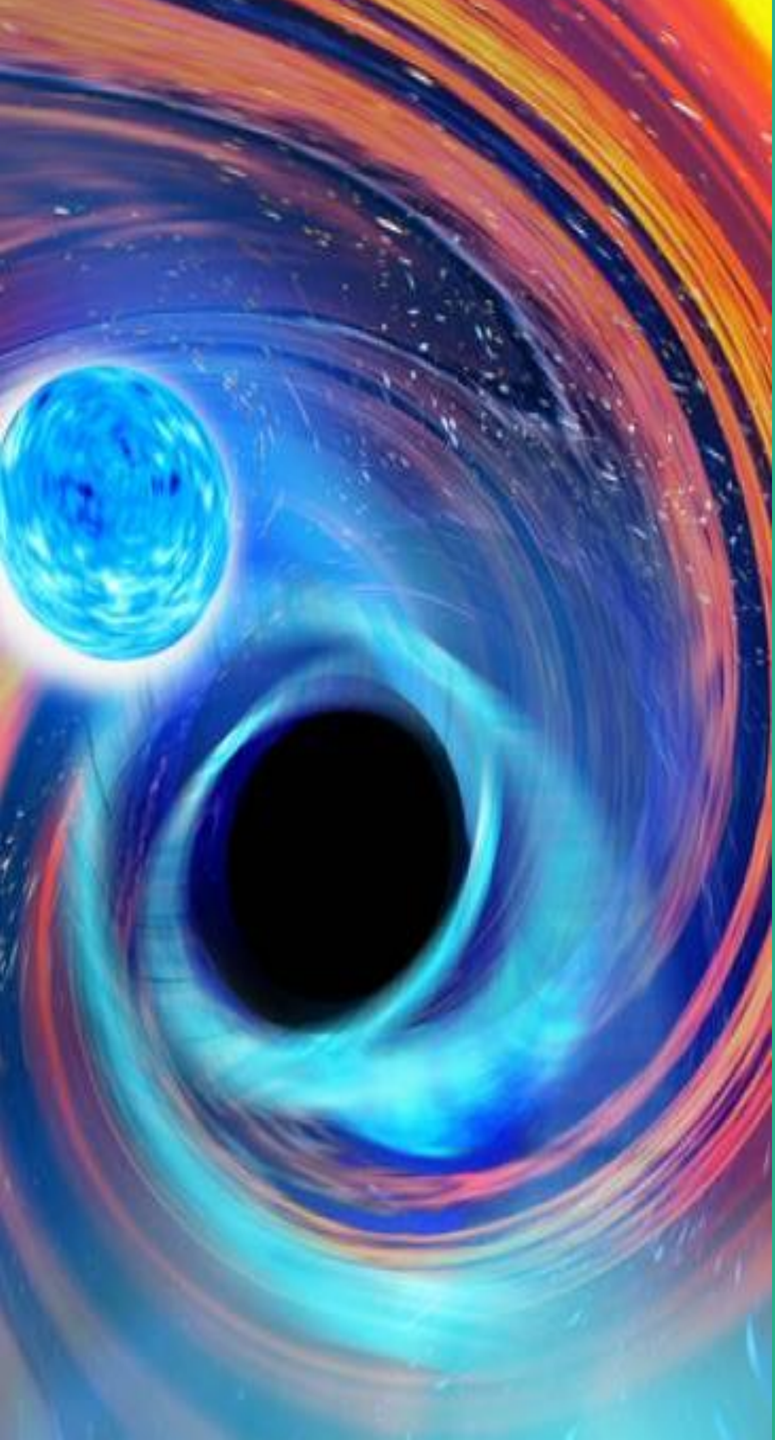
- The form Null geodesics for NSBH depend crucially from the value of the parameter $\alpha = R_S/l$

there is a minimal value of α below which photon rings disappear (quantum star).



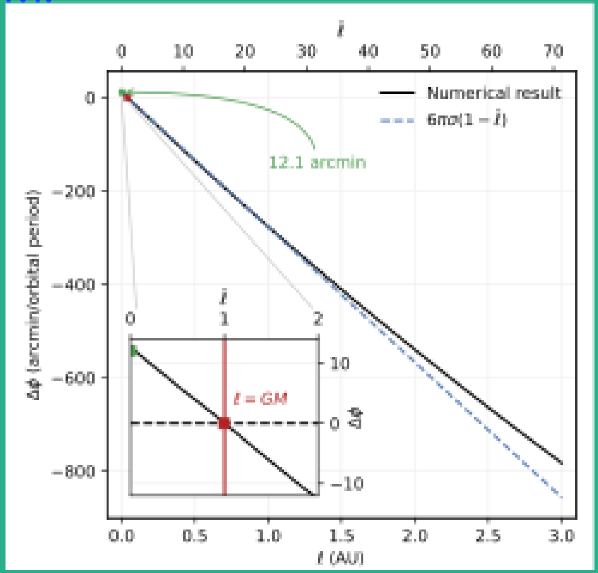
(a) $\alpha > \alpha_c$ (b) $\alpha = \alpha_c$ (c) $\alpha_m < \alpha < \alpha_c$ (d) $\alpha < \alpha_m$

- When there are horizons we have the usual unstable photon ring, which is quantitatively different from the that of the SCHW case
- These features in principle can be tested for supermassive BH using the techniques of the EHT



ORBITS OF STARS

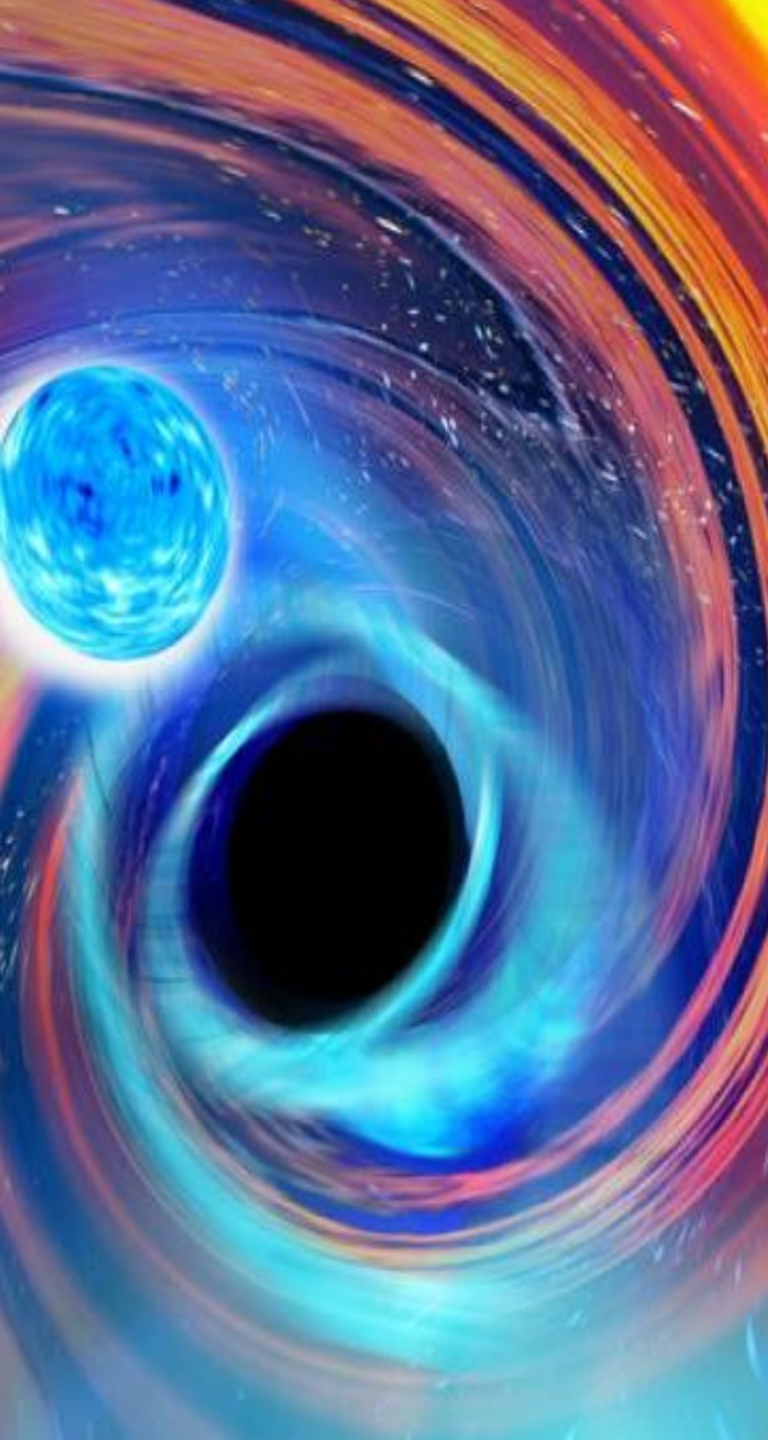
- For timelike geodesics we may have strong deviations from the SCHW case. The precession angle scales linearly with l and becomes retrograde for l>GM.



- We have checked our theoretical prediction using the available data for the orbits of the S2 star around SgRA* for the model

$$f(r) = 1 - \frac{2GMr^2}{(r+\ell)^3},$$

- We found an upper bound for l around 0.47 GM still allowing for superplanckian quantum hair



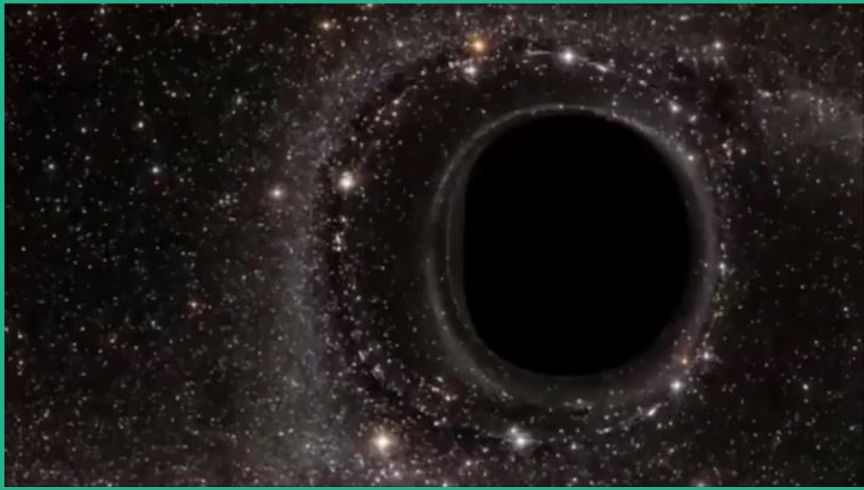
QUASI NORMAL MODES

Quasi Normal modes (QNMs) frequencies depend both on the mass M and on the quantum hair l . In the eikonal approximation :

$$\omega_{\mathrm{R}} = \frac{l}{r_{\mathrm{m}}}\sqrt{A(r_{\mathrm{m}})}$$

$$\omega_{\mathrm{I}} = -\left(n+\frac{1}{2}\right)\frac{1}{\sqrt{2}}\sqrt{A(r_{\mathrm{m}})r_{\mathrm{m}}\left|\left(\frac{A'(r)}{r}\right)'_{\mathrm{m}}\right|}$$

This will have well-defined signature in GW generated in the ringdown phase of the two-BHs merging. Next generation GW detectors like ET will be sensitive enough to detect this effect.



COSMOLOGICAL COUPLING

Do local gravitational systems couple to large scale cosmological dynamics?

- Fully dynamical extremely non-trivial question: description of transition from local inhomogeneities to homogeneity at large-scale cosmological scales, huge separation of scales virial radius – Hubble radius (10^{-3}-10^{10} pc)
- The mass/radius relation $M = R_S/2G$ for BH suggests answer is yes for BHs
- First attempt to address the question:
- McVittie (1933): cosmological FRWL

  embedding of the Schwarzschild solution,
- but entails conceptual and interpretative

  problems (horizons, energy, decoupling..),
Nolan (1999), Faraoni er al. (2012)

McVittie's Cosmological embedding of Schwarzschild (SCHW) solution

- Starts from SCHW solution in isotropic coordinates

$$ds^2 = \left(\frac{1 - m/2r_1}{1 + m/2r_1}\right)^2 dt^2 - \frac{1}{c^2}\left(1 + \frac{m}{2r_1}\right)^4 \{dr_1{}^2 + r_1{}^2(d\theta^2 + \sin^2\theta d\phi^2)\};$$

- Embedding is performed assuming: (1) Spherical symmetry, (2) Absence of fluxes, (3) Sources given by Isotropic fluid, (4) metric reduces to SCHW when expressed in terms of the OBSERVER's (proper) coordinate $r_1 = a\, r$

$$\mathrm{d}s^2 = -\frac{\left(1 - \frac{Gm(t)}{2r}\right)^2}{\left(1 + \frac{Gm(t)}{2r}\right)^2}\mathrm{d}t^2 + a^2\left(1 + \frac{Gm(t)}{2r}\right)^4 (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2)\,,$$

With $m(t) = m_0/a(t),$

- Issue remained almost silent until the theoretical works of Crocker and coll. (2019-20):

  Perturbative approach and averaging procedure gives pressure term in the BH interior which trough Stress energy tensor conservation couples BH mass to scale factor

$$M = M(a_0)\left(\frac{a}{a_0}\right)^k, -3 \leq k \leq 3$$

- ➢ Tested with sample of SMBHs in elliptical galaxies at different Redshits: data showed preference for k=3, (BH as source of Dark energy?)
- ➢ Claim faced significant criticism: formalism flawed from the beginning, state of matter in the BH interior cannot mimic DE, very notion of coupling physically implausible

- Nonetheless scaling behaviour of M with $-3 \leq k \leq 3$ seems quite general and robust result at least at leading order: it is what one expect from esistence of an effective fluid description and causality
- KEY QUESTION IS: **What is the value of k?**
- In order to answer the question in its full generality we need:

➢ work in a quite general solid theoretical GR framework:

COSMOLOGICAL EMBEDDING OF GENERIC COMPACT OBJECTS, USE SOURCES DESCRIBED BY ANISOTROPIC FLUIDS $\longrightarrow$ allows for nonsingular BH mimickers, like nonsingular BH with a de Sitter core

➢ Test the theoretical result with different set of dates and samples, possibly coming from different physical channels

Cosmological Embedding of compact Spherically symmetric objects

- Use the metric parametrization

$$ds^2 = a(\eta)\left[- e^{\alpha(\eta,r)} dt^2 + e^{\beta(\eta,r)} dr^2 + r^2 d\Omega^2\right]$$

- Allows cosmological embedding of small scales inhomogeneities in large scales cosmological dynamics and their coupling at intermediate scales
- Compact object sourced by anisotropic fluid:

  $T_{\mu\nu} = (-\varrho, P_r, P_T, P_T)$

- Allows for nonsingular solution (circumvent Penrose singularity theorems)

- Einstein Equations allow for two classes of solutions: (1) $\dot{\alpha}=0$ (2) $\dot{\beta}=0$

Only (1) describe cosmological embedding of Compact objects

$$e^{-\beta(r,\eta)} = g(r) a^{r\alpha'} \,;$$

$$\frac{\dot{a}^2}{a^2}\left(3 - r\alpha'\right) e^{-\alpha} + \frac{1 - e^{-\beta} + r\beta' e^{-\beta}}{r^2} = 8\pi G a^2 \rho \,;$$

$$\frac{e^{-\beta} + r e^{-\beta}\alpha' - 1}{r^2} + e^{-\alpha}\left(-2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 8\pi G a^2 p_{\parallel}$$

$$\dot{\rho} + \frac{\dot{a}}{a}\left(3\rho + 3p_{\parallel} + r p'_{\parallel}\right) = 0\,,$$

System must be closed imposing form of $\varrho = \varrho(\eta, r)$ and of EoS $P_r = P_r(\varrho)$

We have two extremal regimes:

- $r \to \infty,\ e^{\alpha},\ e^{-\beta} \to 1.$ describes FRWL cosmology

$$3\frac{\dot{a}^2}{a^2} = 8\pi G a^2 \rho_1(\eta)\,;$$
$$\frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} = 8\pi G a^2 p_1(\eta)\,,$$

- (Almost) constant time

$$\frac{1 - e^{-\beta} + r\beta' e^{-\beta}}{r^2} = 8\pi G a_i^2\,\rho\,;$$
$$\frac{e^{-\beta} + r e^{-\beta}\alpha' - 1}{r^2} = 8\pi G a_i^2\, p_{\parallel}\,;$$
$$\dot{\rho} = 0\,,$$

Describes Spherically symmetric compact object at initial time (object becomes cosmologically coupled)

General equations describe the cosmological coupling of the local compact object. Integrating on the proper volume $a^3 L^3$ (L ⟶ size of the object) gives the MISNER-SHARP (MS) mass

$$M(\eta) = 4\pi a^3(\eta) \int_0^L \mathrm{d}r\, r^2\, \rho(r,\eta)$$
$$= \frac{4\pi}{3}\rho_1 a^3 L^3\, e^{-\alpha(L)} + M(a_i)\frac{a}{a_i}\left[1 - e^{-\beta_0(L)} a^{k_L}\right]$$

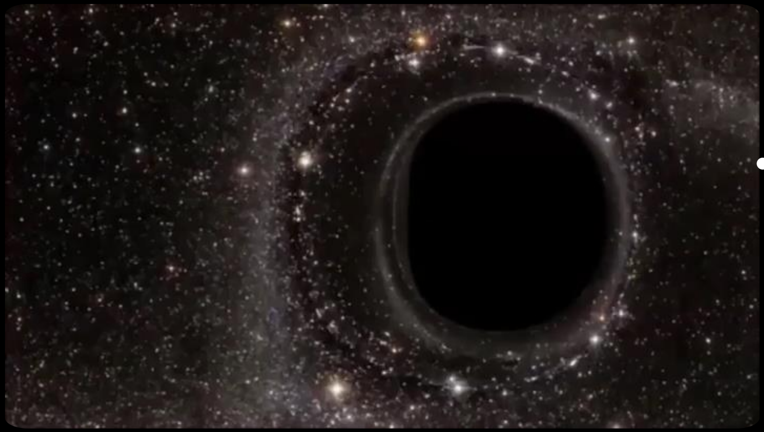
- First term depends on backround cosmological energy density ⟶ can be neglected
- Second term gives **UNIVERSAL LINEAR GROW**

k=1 for Crocker’s scaling law

originated by **the LOCAL CURVATURE** of the object!!

- Linear scaling naively expected from the relation $M = R_S/2G$
- Third term is a nonuniversal subleading term $k(r) \equiv r\alpha'(r)$.

SUMMARY



- For the usual singular BH (Schwarzschild) sum of the second and third term gives ZERO $\longrightarrow$ NO COSMOLOGICAL COUPLING
- For nonsingular BH third term is ZERO $\longrightarrow$ UNIVERSAL LINEAR COSMOLOGICAL SCALING of the mass
- For horizonless BH mimickers (STARS) both terms different from zero UNIVERSAL SCALING CORRECTED BY SUBLEADING TERMS

OBSERVATIONAL EVIDENCE OF COSMOLOGICAL COUPLING OF BH MASSES ⟶ SMOKING GUN OF THEIR NONSINGULAR NATURE

WHAT IS THE OBSERVED BH MASS?

- We would like to understand the previous results in physical terms
- This brings into the play the question about the right definition of physical mass for a cosmologically embedded BH
- For spherically symmetric compact objects we have two different definition of mass
  - Non-Local ADM mass, defined as surface integral (charge) on the asymptotic spacetime boundary
  - Misner-Sharp (MS) mass, gives a covariant quasi-local mass, particular case of quasi-local Hawking –Hayward mass

Writing the ST metric as $ds^2 = -A(T,R)dT^2 + B(T,R)dR^2 + R^2 d\Omega^2$ we have (Faraoni 2015)

$$M_{MS} = \frac{R}{2G}\left(1 - g^{\mu\nu}\nabla_\mu R \nabla_\nu R\right) = \frac{R}{2G}\left(1 - g^{RR}\right) = \frac{R}{2}\left(1 - B^{-1}\right) .$$

- For a generic AF ST ADM and MS mass coincide only at spacial infinity. Outside the compact body only when the stress-energy tensor is zero

Key question: Which is the right definition of mass for a cosmologically embedded compact object?

- Non local ADM mass ⟶ rather problematic in the generic case because of the embedding in FRWL ST (no timelike boundary)
- Usual argument: huge separation of scales implies decoupling of the small-large scale limits ⟶ we can safely treat a cosmological embedded black hole as an eternal one

- Decoupling argument strictly valid only for the SCHWAZSCHILD-DE SITTER ST, for which global static coordinate exist, for the other cases small-large scale limits use different radial coordinates related by time-dependent coordinate transformations

- The MS mall encodes the local properties of the energy of gravitational systems independently from ST asymptotics $\longrightarrow$ physical mass tested by astrophysical observations
- MS mass = ADM mass for SCHW and dS-SCHW
- For NSBH correctly reproduces our CC mass

RECENT RESULTS: further evidence of BH CC

- Recently Faraoni and Rinaldi (arxiv. 2407.14549) have shown that the event horizon of a static black hole cannot be embedded in a time-dependent geometry
- An eternal black hole (singular or non singular) cannot exist in a cosmological environment

⇓

- Cosmological coupling is unavoidable !!

✓ It can come with no mass shift but with formation of a cosmological apparent horizon: k=0, singular black hole)

✓ or with both apparent horizon and mass shift (k=1, nonsingular black hole)

STATUS OF ASTROPHYSICAL OBSERVATIONS

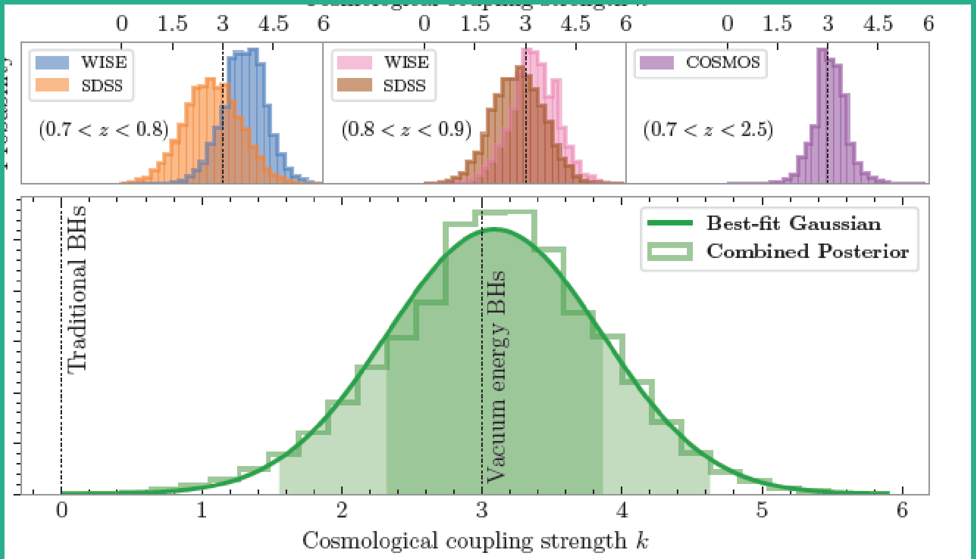
- The observational situation is quite intricate with different analysis pointing to quite different values of the slope parameter k in the allowed range $-3 \leq k \leq 3$

- Tension between expected and observed BH masses at all scale and epochs solved by CC?
- Cosmological scaling of BH masses have been tested using different sets of data and different samples
- Generically, one must compare the mass spectrum of two homogeneous black hole samples at different redshifts in order to detect shift to higher BH masses at low redshift
- Sets of data and samples must be chosen in such way not have other mass growing channels (like e.g accretion) apart from cosmological shift

SMBHs AT CENTER OF ELLIPTICAL GALAXIES

(Farrah et al. (2023))

- They took five high-redshift (HR) samples for SMBH in elliptical galaxies from WISE, SDSS and COSMOS and one low-redshift (LR) sample with $0 < z < 2.5$
- Determine the value of k needed to align HR and LR samples in the M_{BH}-M_* plane and then posterior distribution for k
- Found a preferred value k ~3 at 90% CL, k=0 excluded at 99.98% CL

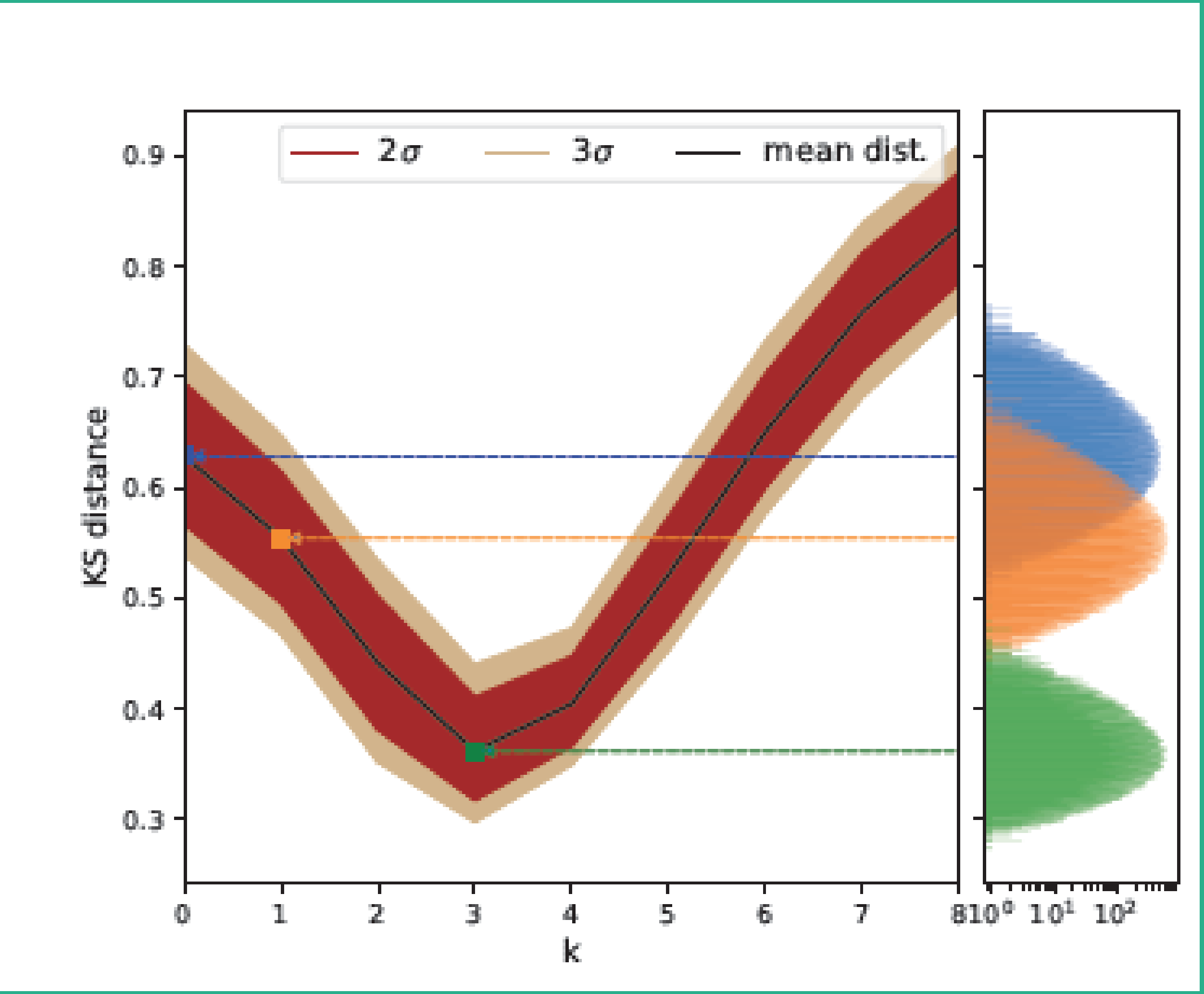


SMBHs AT CENTER OF ELLIPTICAL GALAXIES

(M.C., A. Sanna,…M. Branchesi (2023))

- Improves the analysis of Farrah et al. (uncertainties on mass measurements taken into account). Subset of data with a much smaller HR sample (108 object) obtained with physical cuts that make the HR and LR sample homogeneous
- Statistical analysis based on estimation of minimal Kolmorogov-Smirne (KS) distance between LR and shifted HR sample
- For each value of k in [0,8] simulate 10^5 random realizations and compute the KS distance and 2σ and 3σ confidence limits

- Preferred value by KS test is k=3, smallest KS distance, confirming the Farrah et al result. k=0 and k=1 are consistent within 2σ one with the other, but at tension with best fit !



HIGHER REDSHIFT AGN AND AND QUASAR DATA FROM THE JWSTS

(Lei et al (2023))

- Uses the recently, from the JWST, detected more then 180 AGN and Quasars at z ~ 4.5-7 in early-type host galaxies

- Posterior probability distribution for k centered around $k = -0.94 \pm 1.19$ at 68% CL. k=3 excluded at 3σ CL. k=1 compatible within 2σ

LIGO-VIRGO MASS DISTRIBUTION

(Crocker et al (2021))

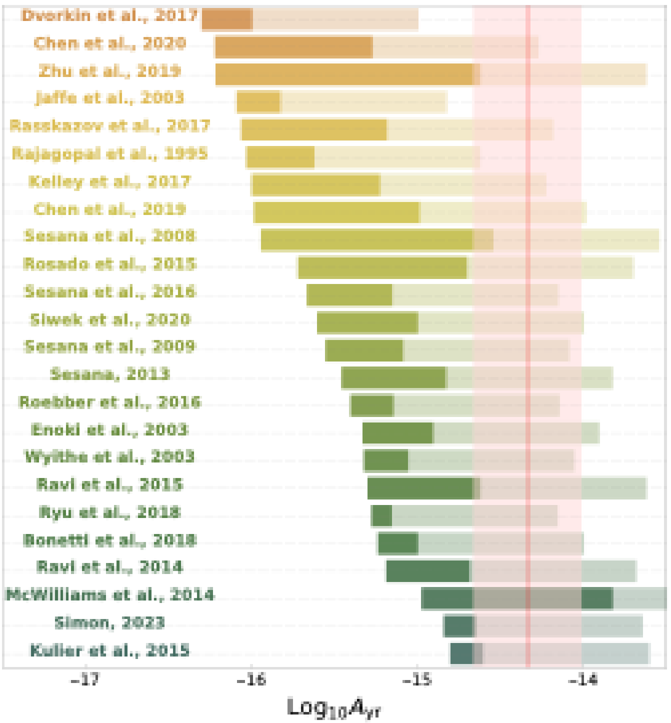
- Uses a single-parameter model for reproducing the mass distribution of over 50 isolated compact binary mergers detected by the LIGO-VIRGO collaboration
- Mass spectrum of these objects is difficult to explain using isolated-binary stellar evolution. Pair-instability mass transfer causes a dearth in the mass spectrum between 50-120 solar masses.
- The detected LIGO-VIRGO mass spectrum have a broad distribution that contaminates the pair-instability mass gap
- The discrepancy could be explained by cosmological mass coupling .The single parameter model indicates a very mild preference for k=0.5

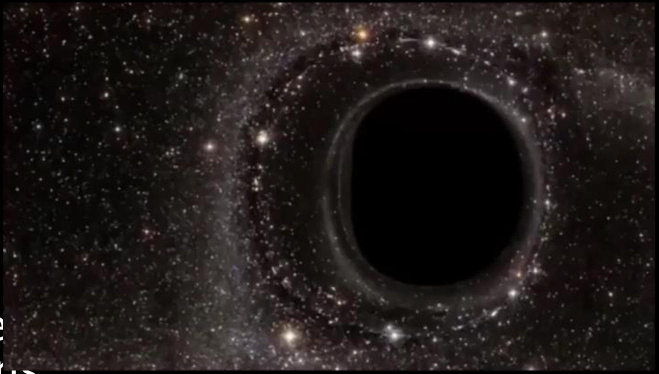
GW OBSERVATIONS AND MINIMAL FORMATION MASS

(L. Amendola et al (2023))

- Uses the dataset of LIGO-VIRGO-KRAGA (72 events) assuming (1) the GW detected come from merging of black holes with stellar progenitors (2) The minimal mass for stellar progenitor is 2 solar masses
- Estimate the probability that at least one of BH among the observed has mass below the threshold (2)
- A tension at level of 2.6 σ - 4σ for the k=3 value is found
- The values k=0.5 and k=1 are essentially not constrained

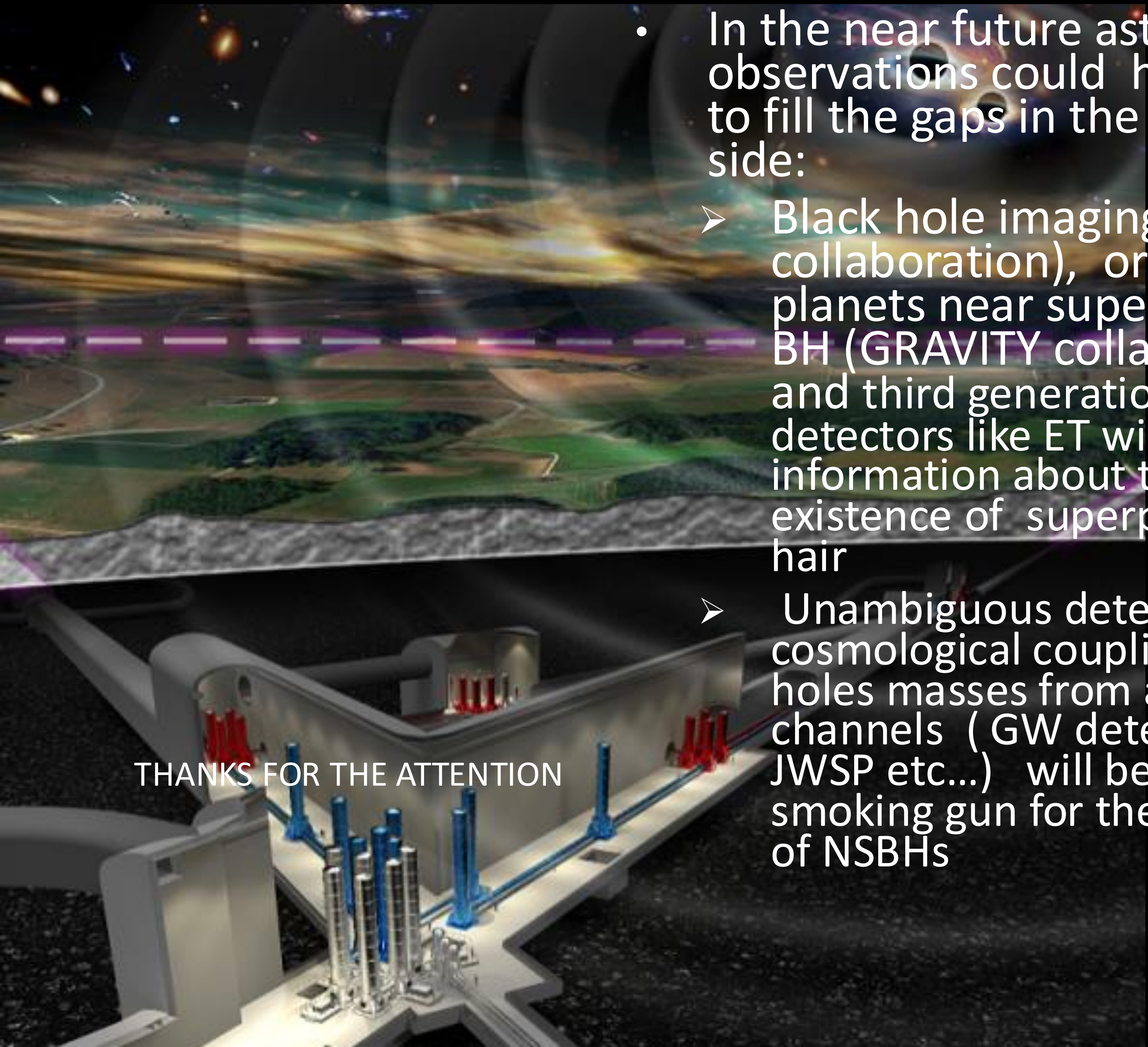
Very recent result: amplitude of Stocastic GW background from merging of supermassive BH by NANOGRAV strongly indicates k>0 (M. Calzà et al. arxiv:2409.01801)





CONCLUSIONS

- NSBH are in agreement with the most basic physical requirement (absence of singularities), have a lot nice features and are not ruled out by present observations
- But: their **theoretical** status is still controversial
  - The anisotropic fluid description can only be an effective long wavelength description of some fundamental microscopic physics
  - It is very difficult to obtain them from local Lagrangians (only known case nonlinear electrodynamics)
  - For this reason there is no universal solution like the SCHW one
  - It is likely that their origin can be traced back to QG effects (at Planckian or superplanckian scales)
  - But: very few examples are known
  - Promising direction: FRG approach (see Mirko's talk)
  - Generalization to rotating solutions (Newmann-Janis)
  - Some other controversial technical issue: Cauchy horizon and mass inflation



- In the near future astrophysical observations could help a lot to fill the gaps in the theory side:
  - Black hole imaging (EHT collaboration), orbits of planets near supermassive BH (GRAVITY collaboration) and third generation of GW detectors like ET will give information about the existence of superplankian BH hair
  - Unambiguous detection of cosmological coupling for black holes masses from a variety of channels (GW detectors, JWSP etc…) will be the smoking gun for the existence of NSBHs

THANKS FOR THE ATTENTION