UNIVERSITY OF TRENTO

INFN
Istituto Nazionale di Fisica Nucleare

TIFPA
Trento Institute for
Fundamental Physics
and Applications

Workshop The Quantum
and Gravity
9th–11th September 2024,

TESTING SCALE-INVARIANT INFLATION AGAINST COSMOLOGICAL DATA

Based on JCAP 07 (2024) 058, C. Cecchini, M. De Angelis, W. Giarè, M. Rinaldi, & S. Vagnozzi

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OUTLINE

1. FUNDAMENTAL SCALE INVARIANCE: MAIN MOTIVATIONS

2. SCALE-INVARIANT GRAVITY

3. INFLATIONARY PREDICTIONS: RESULTS

UNIFYING PRINCIPLE FOR 3 MAIN ISSUES

INFLATION

How to realise naturally flat inflationary potentials without fine-tuning?

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NATURALNESS PROBLEM

How to make divergent quantum correction naturally small?

UNIFYING PRINCIPLE FOR 3 MAIN ISSUES

INFLATION

How to realise naturally flat inflationary potentials without fine-tuning?

NATURALNESS PROBLEM

How to make divergent quantum correction naturally small?

RENORMALIZABILITY

Can we find a new, highly predictive, criterion beyond renormalizability?

FUNDAMENTAL SCALE INVARIANCE

C. Wetterich *Nucl. Phys. B* **964** (2021)
A. Strumia & A. Salvio *J. High Energ. Phys.*, **6** (2017)

Basic idea: a fundamental QFT does not involve any intrinsic parameter with dimension of mass or length

Following Wetterich, we can introduce an explicit mass scale k

$$\underset{\text{Dimension of a mass}}{\textbf{Canonical field}} \longleftarrow \phi = k\,\tilde{\phi} \longrightarrow \underset{\text{Dimensionless}}{\textbf{Scale-invariant field}}$$

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Canonical field
Dimension of a mass

$$\xleftarrow{} \phi = k\,\tilde{\phi} \xrightarrow{}$$

Scale-invariant field
Dimensionless

The corresponding effective actions obey

$$\boxed{k\partial_k \Gamma_k[\phi] = \zeta_k[\phi]}$$

General solution

$$\boxed{k\partial_k \Gamma_k[\tilde{\phi}] = 0}$$

Particular, scaling solution holding when the canonical fields are expressed in terms of the scale-invariant ones

FUNDAMENTAL SCALE INVARIANCE

NATURALLY FLAT POTENTIALS FOR INFLATION

Scale-invariant theory non-minimally coupled to gravity

$$\mathscr{L}_J = \sqrt{-g}\left[\xi\,\phi^2 R - \lambda\phi^4 - \frac{1}{2}(\partial\phi)^2\right]$$

Weyl rescaling from the Jordan to the Einstein frame

$$\mathscr{L}_E = \sqrt{-\tilde{g}}\left[\frac{M_{pl}^2}{2}\tilde{R} - M_{pl}^4\frac{\lambda}{\xi^2} - \frac{1}{2}(\partial\tilde{\phi})^2\right]$$

The potential is flat at tree-level: no fine-tuning

Scale symmetry breaking can occur from quantum corrections

FUNDAMENTAL SCALE INVARIANCE

SOLUTION TO THE NATURALNESS PROBLEM

Coefficients of super-renormalizable terms ⟶ Power-law divergences

Mass dimension < 4
in 3+1 dimensions

Fundamental scale invariance implies only mass dimension 4 Lagrangian terms

- With a dimensional regularization scheme, all the counterterms vanish
- Indication that Nature may prefer dimension-4 operators

FUNDAMENTAL SCALE INVARIANCE

A CRITERION BEYOND RENORMALIZABILITY

For general renormalizable theories the effective action remains well defined in the continuum limit if one employs renormalized fields

$$\text{Renormalized fields} \longleftarrow \phi_{R,i}(x) = k^{d_i} f_i(k)\, \tilde{\phi}_i(x) \longrightarrow \text{Scale-invariant field}$$

FUNDAMENTAL SCALE INVARIANCE

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For general renormalizable theories the effective action remains well defined in the continuum limit if one employs renormalized fields

Renormalized fields ← $\phi_{R,i}(x) = k^{d_i} f_i(k)\, \tilde{\phi}_i(x)$ → **Scale-invariant field**

Theories with fundamental scale invariance:

- Renormalizable
- For some choice of the fields $\tilde{\phi}$ the effective action becomes k-independent
- Exact scaling solutions: no free parameters. High predictive power

SCALE-INVARIANT QUADRATIC GRAVITY

THE MODEL

M. Rinaldi & L. Vanzo *PR D* **94** (2016)

- $\mathscr{L}_{EH} \longrightarrow f(R,\phi)$: scalar-tensor theory of modified gravity

$$\mathscr{L}_J = \sqrt{-g}\left[\underbrace{\frac{\alpha}{36}R^2}_{\text{Higher order term in R}} + \frac{\xi}{6}\phi^2 R - \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4}_{\text{Scalar field}}\right], \qquad \alpha, \lambda, \xi > 0$$

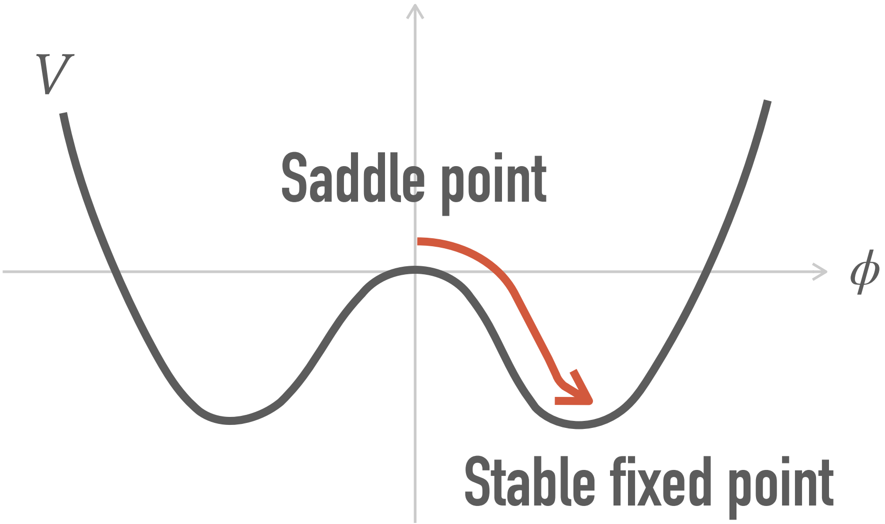
FUTURE INVESTIGATION

- Weyl curvature term C^2
- Running of the paramters
- Coupling to gauge fields

SCALE-INVARIANT QUADRATIC GRAVITY

JORDAN FRAME

The field ϕ is subjected to an effective potential $V_{eff}(\phi) = -\frac{\xi}{6}\phi^2 R + \frac{\lambda}{4}\phi^4$



Saddle point

Stable fixed point

Classical scale-symmetry breaking

The scalar field takes a non-zero VEV at the minimum

$$\langle \phi_0^2 \rangle = \frac{\xi R}{3\lambda}$$

Dynamical generation of a mass scale

Natural identification with the Planck mass

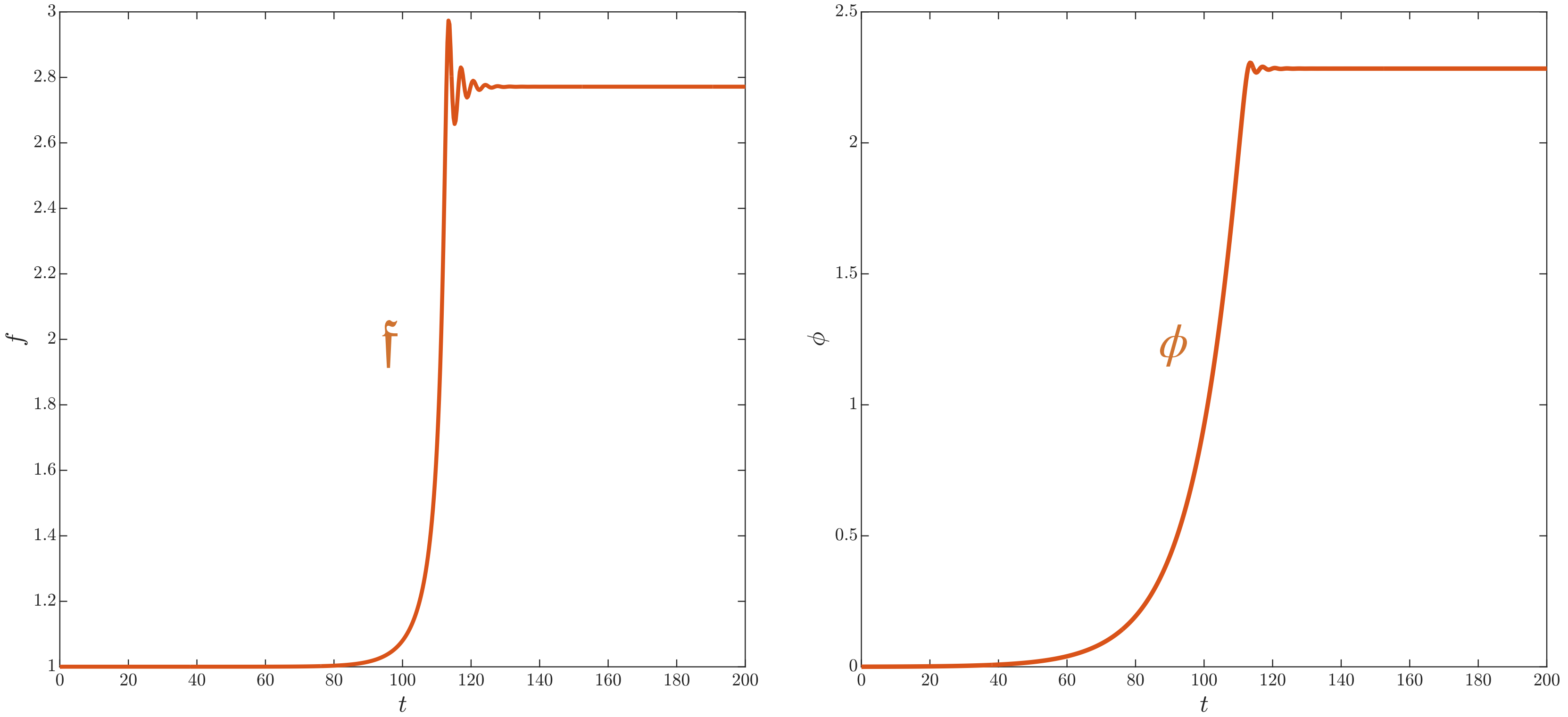
$$\frac{\xi}{6}\phi_0^2 R \equiv \frac{1}{2}M_{pl}^2 R$$

SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME $g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}$

Two dynamical degrees of freedom: are we in multi-field inflation?

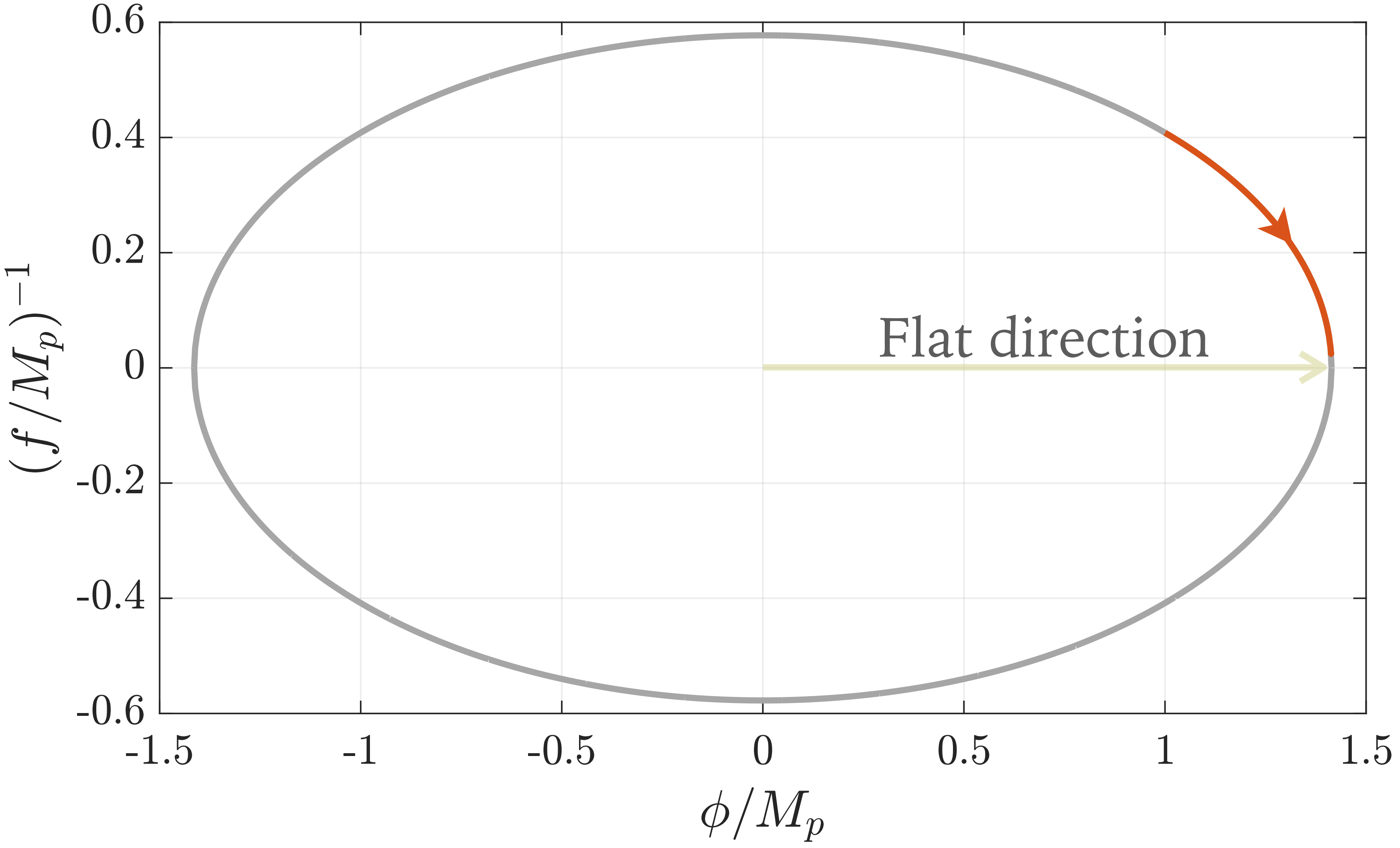
$$\mathscr{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$



SCALE-INVARIANT QUADRATIC GRAVITY

EINSTEIN FRAME: NOETHER'S CURRENT

Noether's current conservation: constraint on the two-fields dynamics



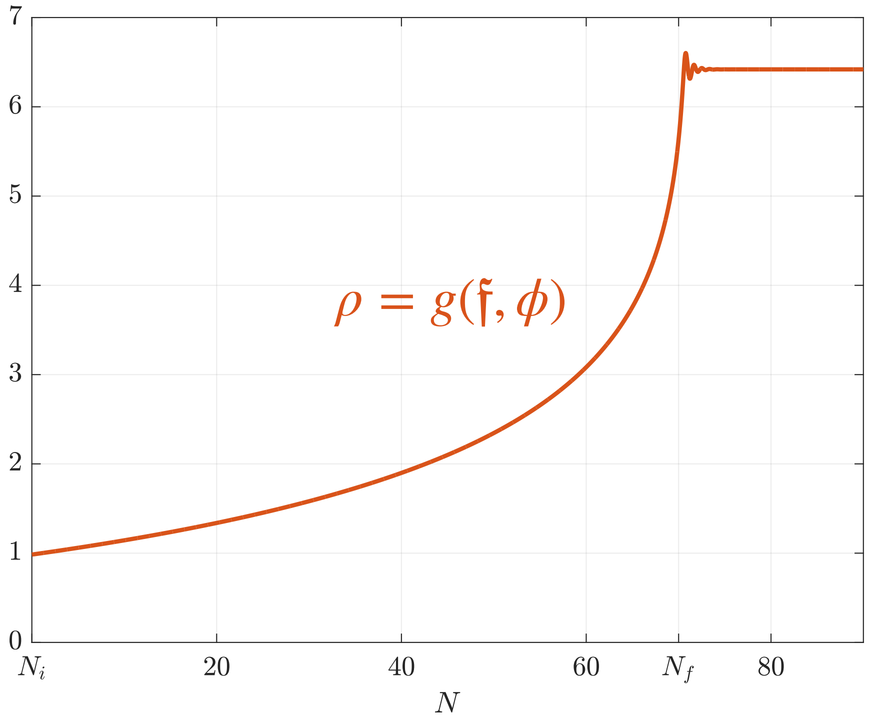
SCALE-INVARIANT QUADRATIC GRAVITY

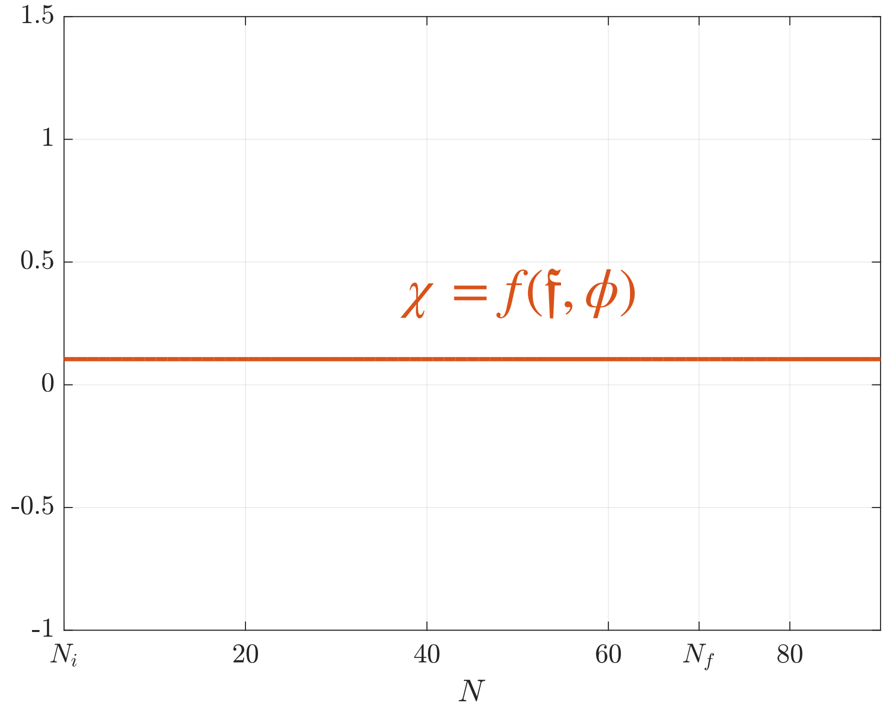
NOETHER'S CURRENT CONSERVATION

J. Garcia-Bellido et al. *PR D* **84** (2011)
G. Tambalo & M. Rinaldi *Gen. Relativ. Gravit.* **49** (2017)

Noether's current conservation can be employed to shift all the dynamics on one field

$$\mathscr{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - 3\mathrm{Cosh}\left[\frac{\rho}{\sqrt{6}M}\right]^2\partial_\mu\chi\partial^\mu\chi - V(\rho)\right)$$





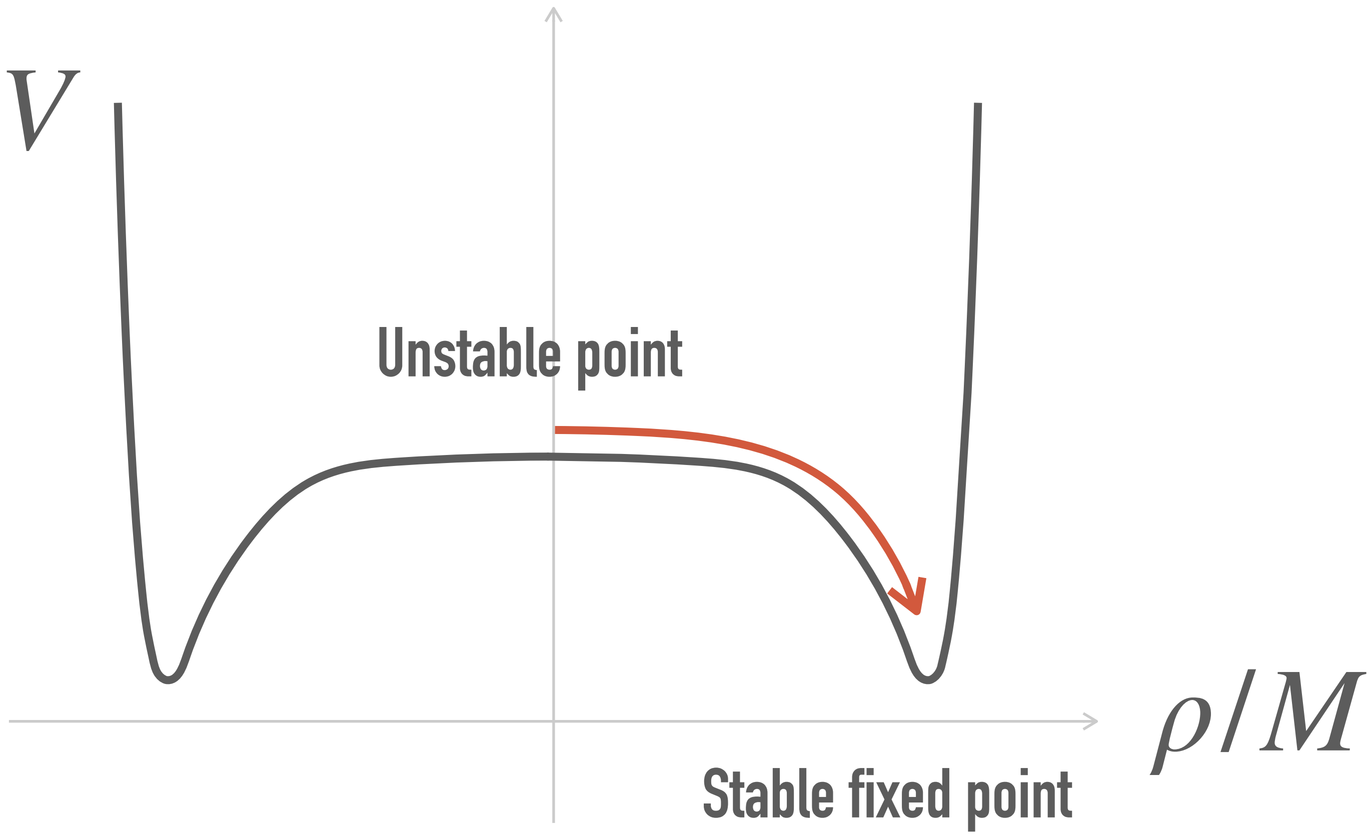
SCALE-INVARIANT QUADRATIC GRAVITY

NOETHER'S CURRENT CONSERVATION: SINGLE-FIELD INFLATION

- Naturally flat plateau: no fine-tuning
- Non-vanishing at the minima

χ **GOLDSTONE BOSON**

ρ **INFLATON**



SCALE-INVARIANT QUADRATIC GRAVITY

NOETHER'S CURRENT CONSERVATION: ENTROPY PERTURBATIONS

Employing Noether's current conservation we show that

$$\delta s = 0$$

- Scale invariance protects from any form of geometrical destabilization

GEOMETRICAL DESTABILIZATION OF INFLATION

S. Renaux-Petel & K. Turzyński *Phys. Rev. Lett.* ***117*** (2016)

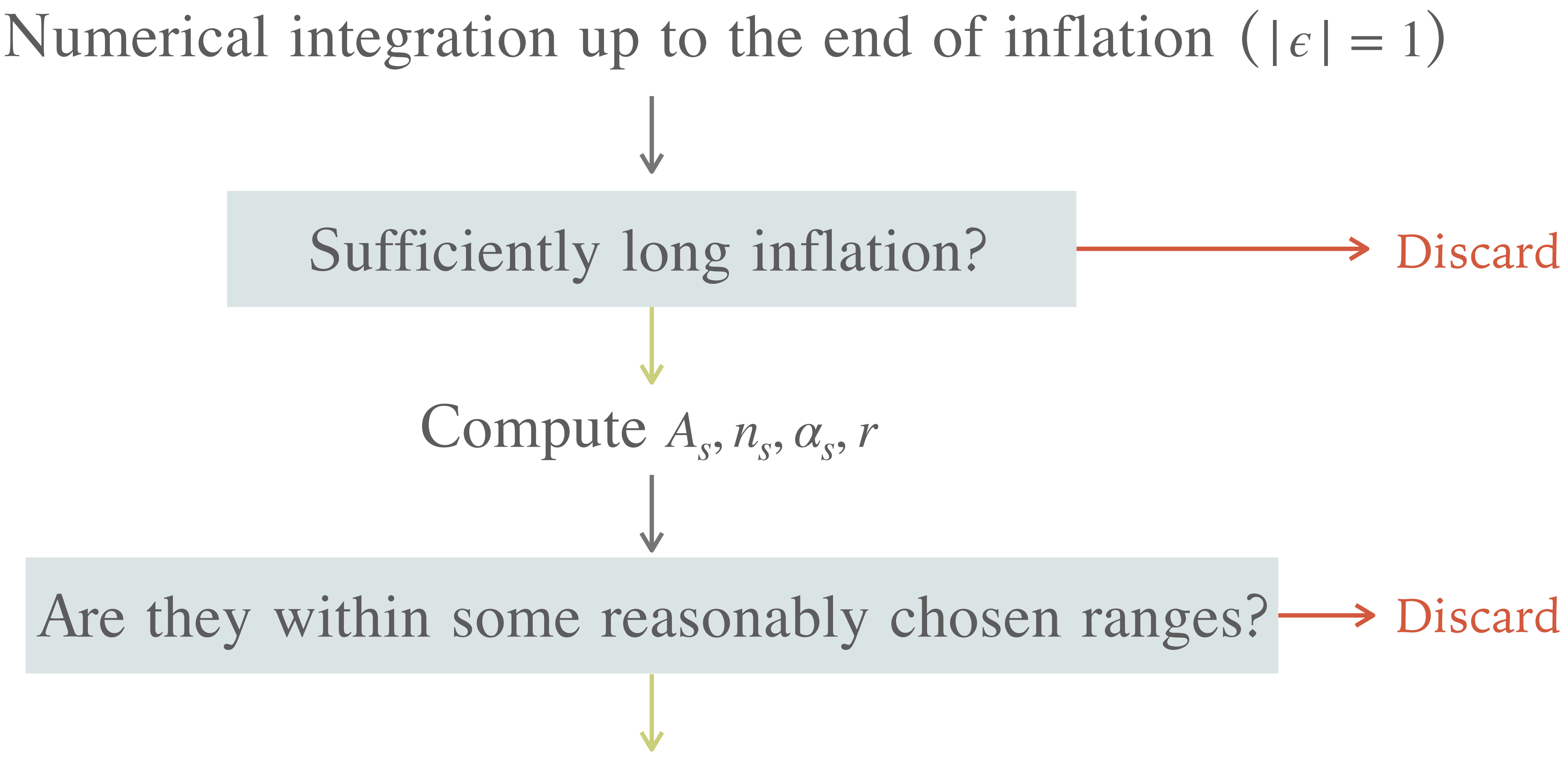
- Multi-field inflation
- Hyperbolic fields' space geometry

→ $m^2_{s(eff)} < 0$: tachyonic instability prematurely ending inflation

INFLATIONARY PREDICTIONS

NUMERICAL ANALYSIS

W. Giarè, M. De Angelis, C. van de Bruck, & E. Di Valentino
JCAP **12** (2023)



Numerical integration up to the end of inflation $\left(|\epsilon| = 1\right)$

Sufficiently long inflation? → Discard

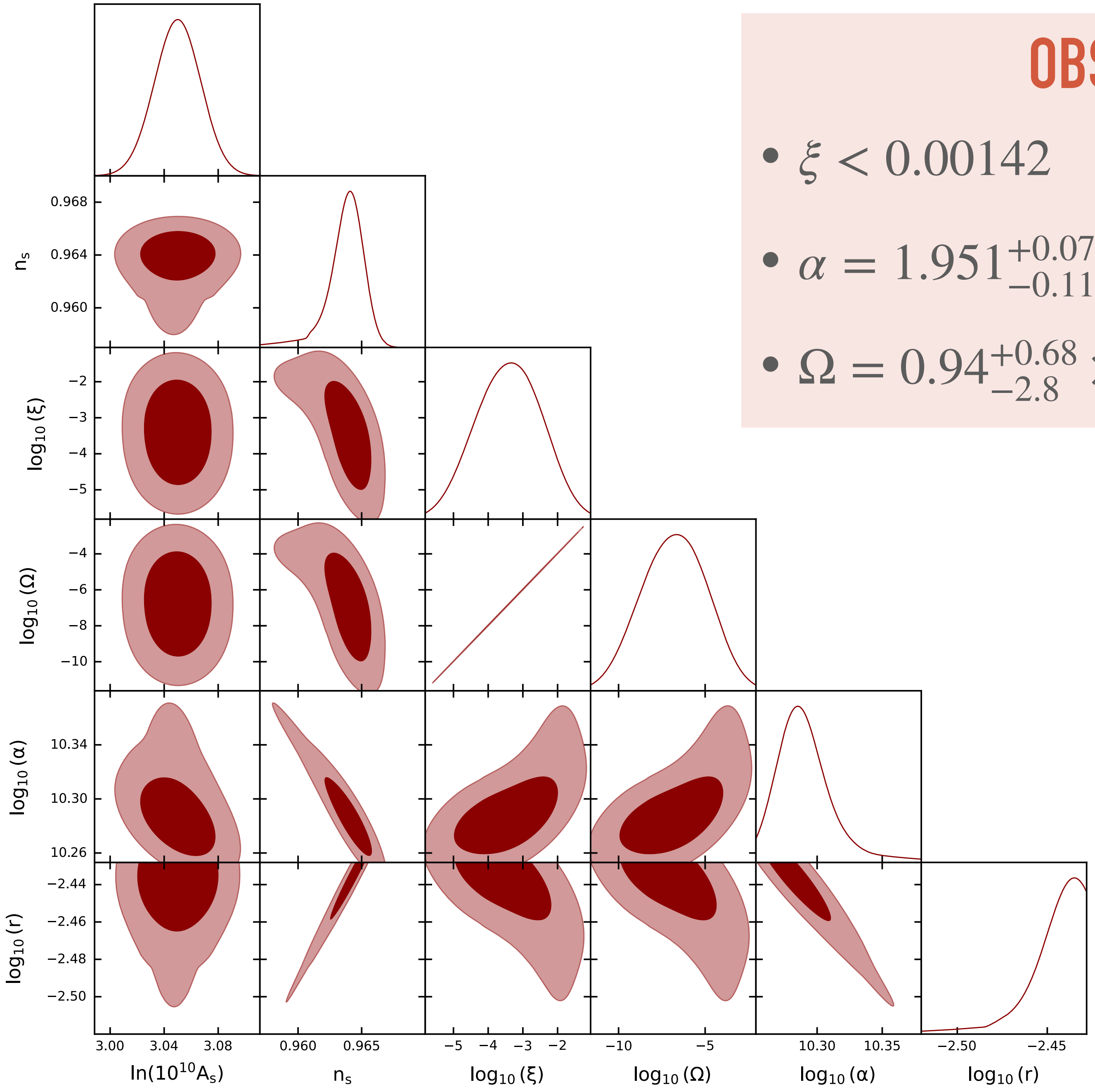
Compute A_s, n_s, α_s, r

Are they within some reasonably chosen ranges? → Discard

Implement CAMB and assign a likelihood based on how well the model agrees with CMB data

INFLATIONARY PREDICTIONS

OBSERVATIONAL CONSTRAINTS



OBSERVATIONAL CONSTRAINTS

- $\xi < 0.00142$
- $\alpha = 1.951^{+0.076}_{-0.11} \times 10^{10}$
- $\Omega = 0.94^{+0.68}_{-2.8} \times 10^{-5}$
- $A_S = (2.112 \pm 0.033) \times 10^{-9}$
- $n_S = 0.9638^{+0.0015}_{-0.0010}$
- $r > 0.00332$

➤ Lower bound on the tensor-to-scalar ratio r, that will be testable from next generation CMB experiments

$$\Omega \equiv \alpha\lambda + \xi^2$$

INFLATIONARY PREDICTIONS

OBSERVATIONAL CONSTRAINTS

- Overall insensitivity to initial conditions
- $\xi < 0.00142$ (95% C.L.): conformal invariance is ruled out
- Strong correlation between Ω and ξ to avoid eternal inflation

INFLATIONARY PREDICTIONS

SCALE INVARIANCE VS STAROBINSKY

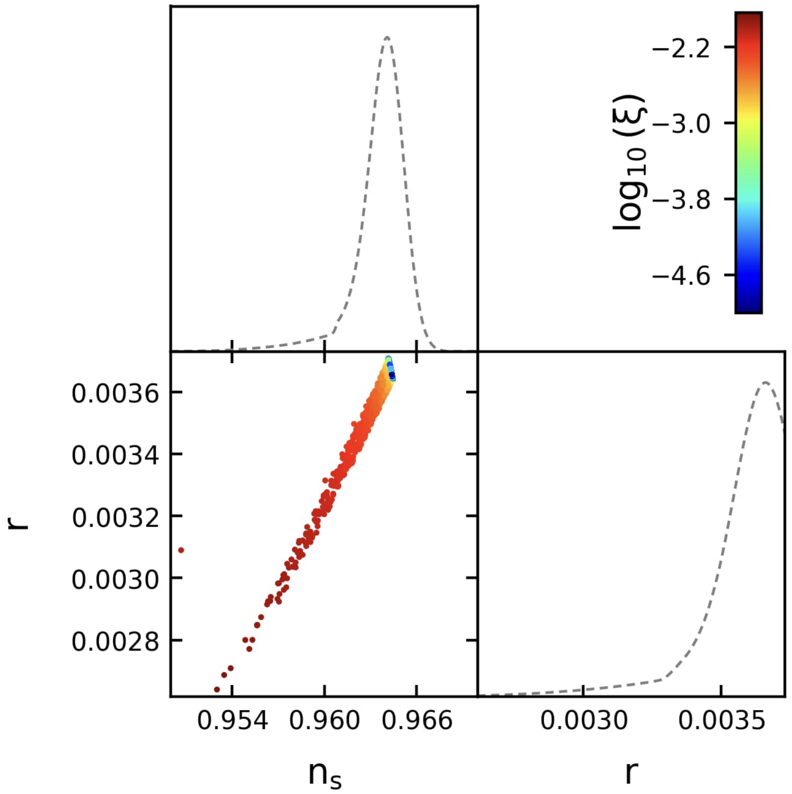
$$\mathscr{L} = \sqrt{-g}\,\frac{M_{pl}^2}{2}\left[R + \underbrace{\frac{R^2}{6M^2}}_{\text{Scale-invariant}}\right]$$

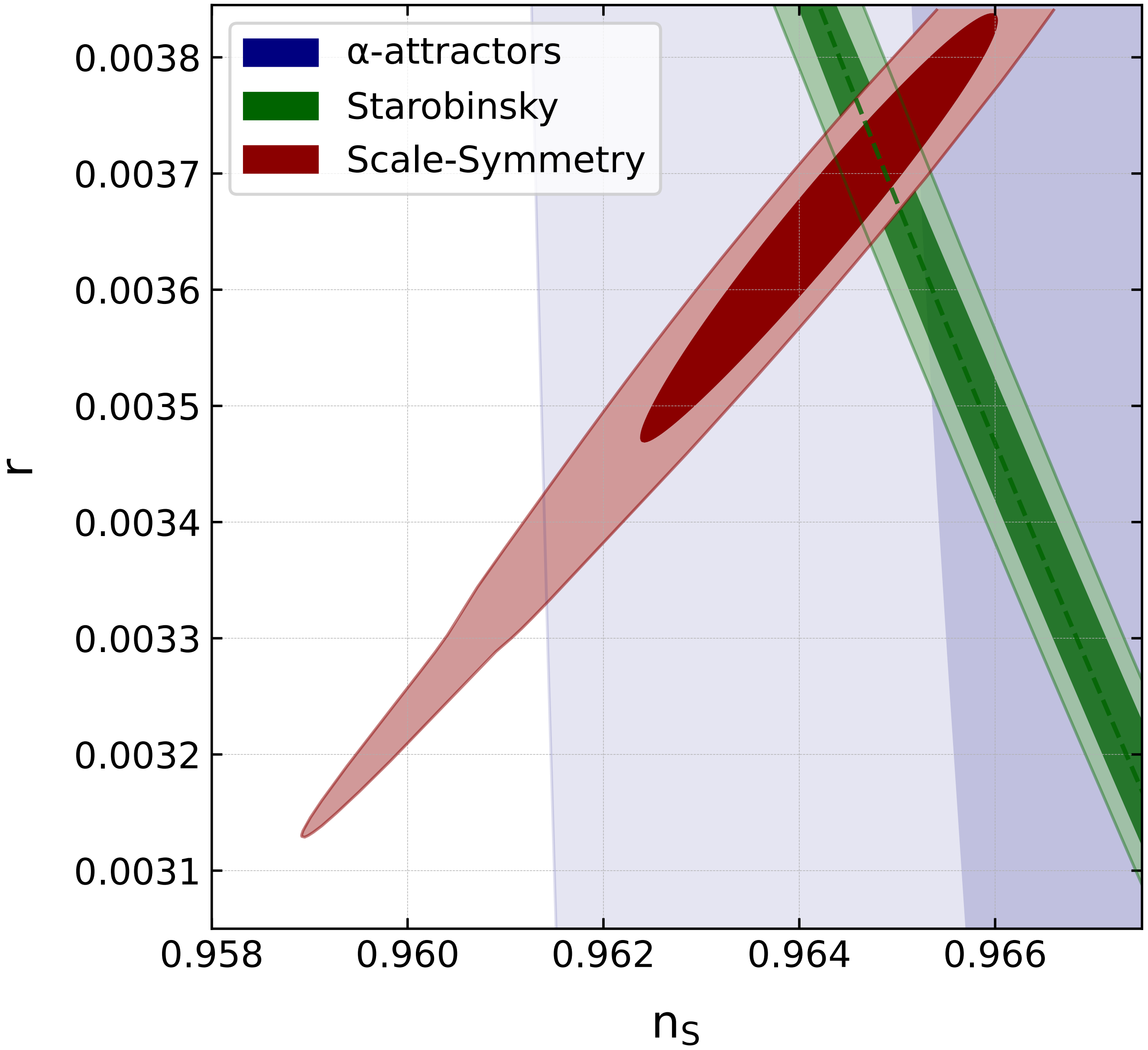
- Starobinsky's model is scale-invariant when the R^2 term dominates!
- Can we discriminate between the two models?

INFLATIONARY PREDICTIONS

SCALE INVARIANCE VS STAROBINSKY

n_s and r are anti-correlated like in Starobinsky's model only at fixed ξ. Overall, they are correlated: it is potentially possible to discriminate between the two models!





SUMMARY

- Fundamental scale invariance as a new theoretical principle beyond renormalizability
- Solution to the naturalness problem and flat potentials for inflation
- Scale-invariant quadratic gravity: Noether's current conservation for single-field dynamics and vanishing entropy perturbations
- Promising numerical result: the model is competitive with Starobinsky

BACKUP SLIDES

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- Scale invariance can be checked explicitely

SCALE TRANSFORMATION

- $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$
- $\bar{\phi}(x) = \ell\phi(\ell x)$

$$\longrightarrow \qquad \bar{\mathscr{L}} = \mathscr{L}$$

SCALE-INVARIANT QUADRATIC GRAVITY

WEYL CORRECTION

Squared Weyl curvature term: conformally-invariant, second order term.
Why don't we add it to the action?

$$\boxed{C^2 = 2R_{\mu\nu}R^{\mu\nu} - \frac{2}{3}R^2 + \mathcal{G}}$$

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Background:

The Weyl curvature term vanishes in a conformally flat spacetime
→ no contribution to the equations of motion

Perturbations:

A. De Felice et al. *PR D* **108**, *123524* (2023)

Weyl-Starobinsky inflation is plagued by ghosts and classical instabilities→ possible drawback also here (ongoing project)

INFLATIONARY PREDICTIONS

LIKELIHOOD

W. Giarè, M. De Angelis, C. van de Bruck, & E. Di Valentino *JCAP* **12** (2023)

Covariance matrix $\boldsymbol{\Sigma}$ and mean value of the parameters μ

↓

Analytical likelihood

DATA

- Planck 2018 temperature and polarisation (TT TE EE) likelihood
- B-modes power spectrum likelihood cleaned for foreground contamination (Bicep/Keck Array Collaboration)

ANALYTICAL LIKELIHOOD

$$\mathscr{L} \propto \exp\left(-\frac{1}{2}\left(\mathbf{x}-\mu\right)^{T}\boldsymbol{\Sigma}^{-1}\left(\mathbf{x}-\mu\right)\right), \quad \mathbf{x} \equiv \left(A_s, n_s, \alpha_s, r\right)$$

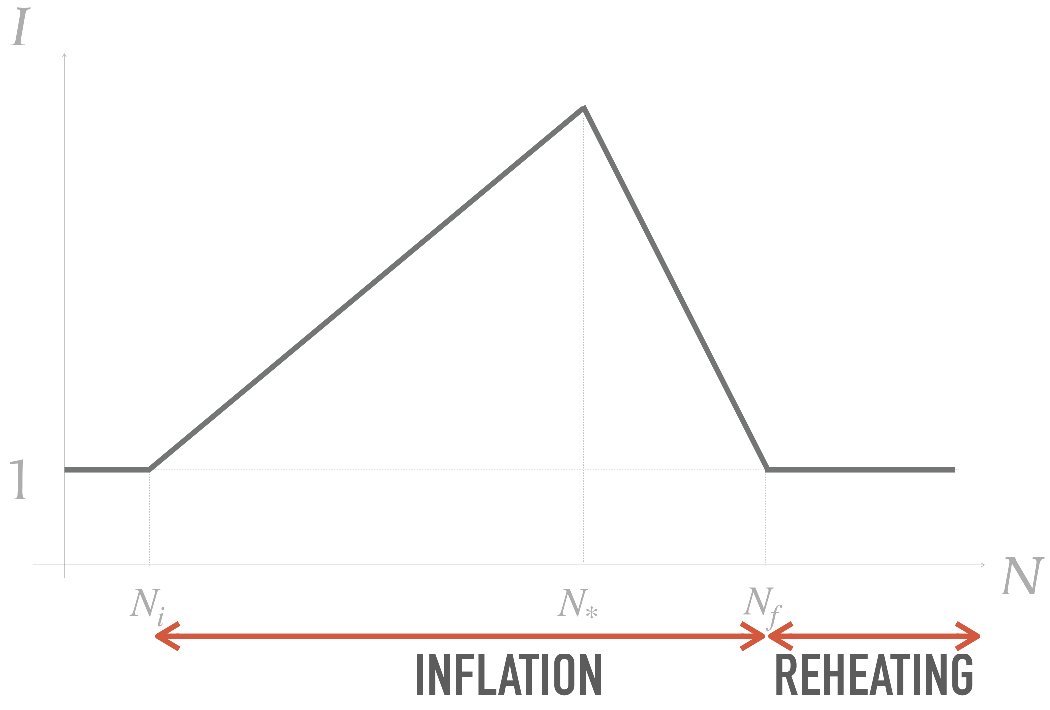
INFLATIONARY PREDICTIONS

MAGNETOGENESIS

C. Cecchini & M. Rinaldi *Phys Dar Univ* **40** (2023)

Modify the Maxwell's action and add helicity to generate primordial magnetic fields through a sawtooth coupling to the inflaton: EM conformal invariance is broken only during inflation → amplification of vector perturbations

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\, I^2[\zeta(t)]\left[F_{\mu\nu}F^{\mu\nu} - \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_E$$



$$I = \begin{cases} \mathscr{C}\left(\frac{a}{a_*}\right)^{\nu_1} & a_i > a > a_* \\ \mathscr{C}\left(\frac{a}{a_*}\right)^{-\nu_2} & a_* > a > a_f \end{cases}$$

INFLATIONARY PREDICTIONS

MAGNETOGENESIS

C. Cecchini & M. Rinaldi *Phys Dar Univ* **40** (2023)

Present-day magnetic field's amplitude and coherence length compatible with bounds on the IGM fields