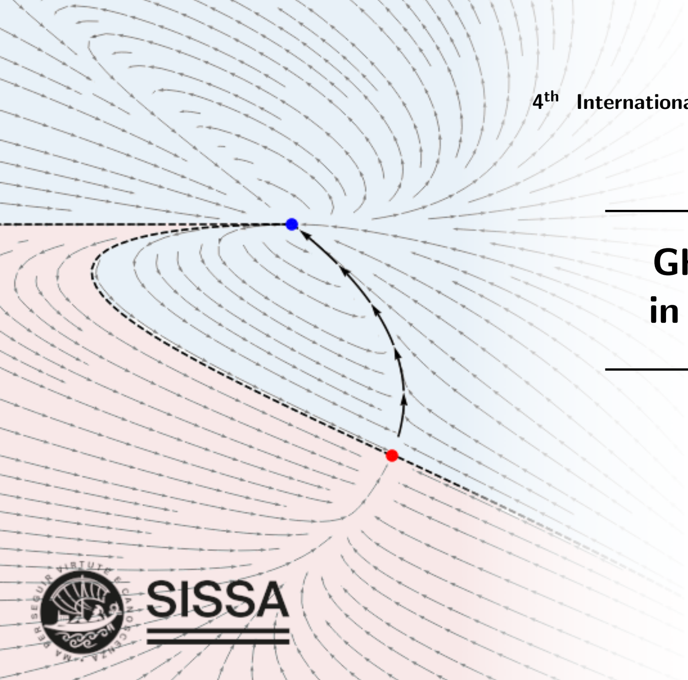

4th International FLAG Workshop: the Quantum and Gravity

9th September 2024

Ghost-driven instabilities in black hole evaporation

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Sezione di Trieste

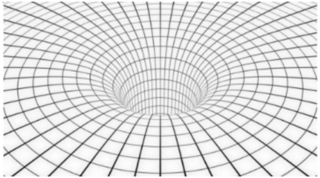
(Provocative) outline:

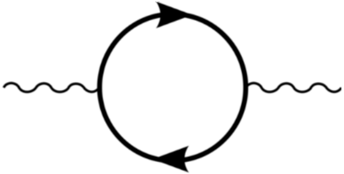
- Choose a theory with quantum and classical ghosts
- Show that ghosts make the Schwarzschild solution unstable
- Argue that black hole evaporation ends with a naked singularity
- Conclusions

Introduction

Black hole evaporation and quadratic gravity

Why black hole evaporation? Semiclassical gravity





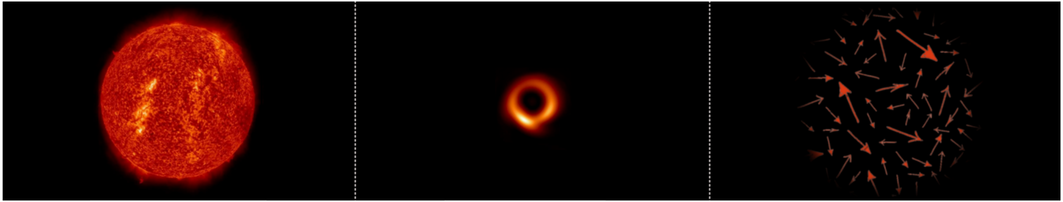
Classical curved spacetime $+$ Quantum Field Theory

$\Downarrow$

Black hole evaporation

Fundamental assumption: $E_{QFT} \ll E_{Quantum\ Gravity}$

Why black hole evaporation? Information paradox

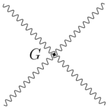


Information is accessible | Information is not accessible | Information is lost

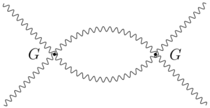
Final stages of evaporation $\Longrightarrow$ $T \to \infty$ $\Longrightarrow$ $E_{QFT} \sim E_{Quantum\ Gravity}$

Solution: quantum correction for gravity?

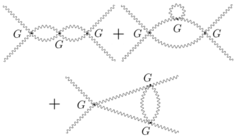
Why quadratic gravity? Perturbative approach



$$\int d^4x\sqrt{-g}\, R$$

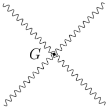


$$\int d^4x\sqrt{-g}\, \left[a\, R^2 + b\, R^{\mu\nu} R_{\mu\nu} + c\, R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}\right]$$



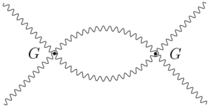
$$\int d^4x\sqrt{-g}\, \left[d\, R^{\mu\nu\rho\sigma} R_{\rho\sigma}{}^{\gamma\delta} R_{\gamma\delta\mu\nu} + e\, R^{\mu\nu} R_{\nu}{}^{\rho} R_{\rho\mu} + \ldots\right]$$

Why quadratic gravity? Perturbative approach



$$\int d^4x\sqrt{-g}\, R$$

} *GR*



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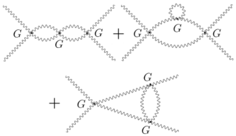
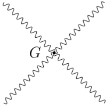
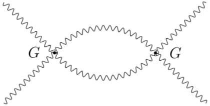


$$\int d^4x\sqrt{-g}\,\left[d\,R^{\mu\nu\rho\sigma}R_{\rho\sigma}{}^{\gamma\delta}R_{\gamma\delta\mu\nu} + e\,R^{\mu\nu}R_{\nu}{}^{\rho}R_{\rho\mu} + \dots\right]$$

Why quadratic gravity? Perturbative approach

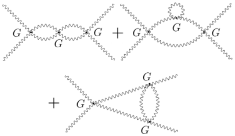


$$\int d^4x\sqrt{-g}\,R$$



$$\int d^4x\sqrt{-g}\,\left[a\,R^2 + b\,R^{\mu\nu}R_{\mu\nu} + c\,R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\right]$$

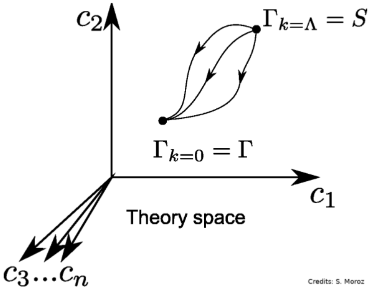
} *Quadratic Gravity*



$$\int d^4x\sqrt{-g}\,\left[d\,R^{\mu\nu\rho\sigma}R_{\rho\sigma}{}^{\gamma\delta}R_{\gamma\delta\mu\nu} + e\,R^{\mu\nu}R_{\nu}{}^{\rho}R_{\rho\mu} + \ldots\right]$$

Why quadratic gravity? Wilsonian approach

Non-renormalizable theory $\implies$ effective theory at low energies



$$\mathcal{I}_{eff} = \int \mathrm{d}^4 x \sqrt{-g} \left[E^4 c_0 + E^2 c_1 R + c_2 R^2 + c_3 R^{\mu\nu} R_{\mu\nu} + c_4 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \frac{c_5}{E^2} R^3 + \ldots \right]$$

Why quadratic gravity? Wilsonian approach

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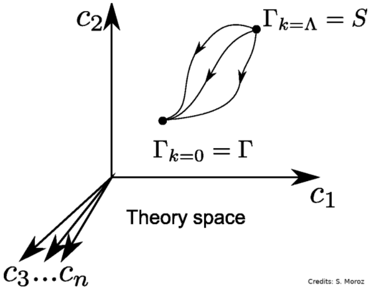

$$\mathcal{I}_{eff} = \int \mathrm{d}^4x\sqrt{-g}\left[\underbrace{E^4c_0 + E^2c_1R}_{GR} + c_2R^2 + c_3R^{\mu\nu}R_{\mu\nu} + c_4R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + \frac{c_5}{E^2}R^3 + \ldots\right]$$

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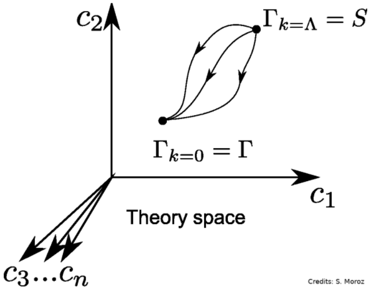

$$\mathcal{I}_{eff} = \int \mathrm{d}^4x\sqrt{-g}\left[E^4 c_0 + \underbrace{E^2 c_1 R + c_2 R^2 + c_3 R^{\mu\nu} R_{\mu\nu} + c_4 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}}_{\textit{Quadratic gravity}} + \frac{c_5}{E^2} R^3 + \dots\right]$$

Quadratic gravity: a classical model for quantum corrections

$$\mathcal{I}_{QG} = \int \mathrm{d}^4x\sqrt{-g}\left[\gamma R - \alpha C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \beta R^2 + \chi\mathcal{G}\right] \begin{cases} S = 2, & m = 0 \\ S = 0, & m_0^2 = \gamma/6\beta \\ S = 2, & m_2^2 = \gamma/2\alpha \end{cases}$$

PRO: general, IR limit of fundamental theories, renormalizable

CON: negative energy states $\implies$ non-unitary theory

First assumption:

Classical solutions as first quantum corrections

Quadratic gravity: a classical model for quantum corrections

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Classical solutions as *first* quantum corrections

Black holes in quadratic gravity

Old and new solutions

Symmetries and weak field limit

Staticity, spherical symmetry:

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2$$

Asymptotic flatness (isolated objects):

$$h(r) \sim 1 - \frac{2\,M}{r} + 2\,S_2^- \frac{\mathrm{e}^{-m_2\,r}}{r}$$
$$f(r) \sim 1 - \frac{2\,M}{r} + S_2^- \frac{\mathrm{e}^{-m_2\,r}}{r}(1 + m_2\,r)$$

Total (ADM) mass: M Yukawa charge S_2^-

Event horizon: internal boundary

Series expansion around horizon radius r_H:

$$h(r) = h_1 (r - r_H) + \sum_{n=2}^{\infty} h_n (r - r_H)^n$$

$$f(r) = f_1 (r - r_H) + \sum_{n=2}^{\infty} f_n (r - r_H)^n$$

Hawking: $T_{BH} = \frac{1}{4\pi}\sqrt{h_1 f_1}$

Wald: $S_{BH} = 16\pi^2\gamma \left(r_H^2 + \frac{2}{m_2^2}(1 - f_1 r_H) \right)$

Nature of singularity: behavior close to the origin

Series expansion around the origin:

$$h(r) = r^t \sum_{n=0}^{\infty} h_{t+n} r^n \qquad\qquad t = \lim_{r\to 0} \frac{\mathrm{d}\log(h(r))}{\mathrm{d}\log(r)}$$

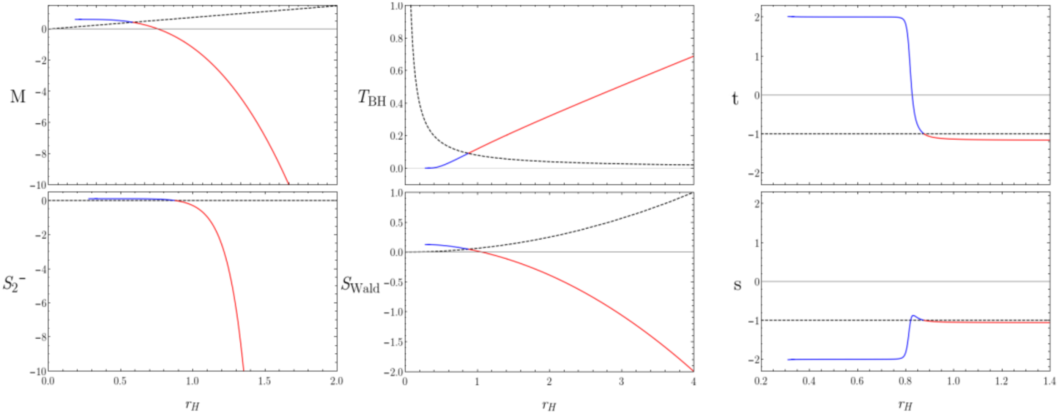
$$\Longrightarrow$$

$$f(r) = r^s \sum_{n=0}^{\infty} f_{s+n} r^n \qquad\qquad s = \lim_{r\to 0} \frac{\mathrm{d}\log(f(r))}{\mathrm{d}\log(r)}$$

Divergent metric: $t = -1,\ s = -1$

Vanishing metric: $t = 2,\ s = -2$

Properties of black holes in quadratic gravity



Linear perturbations and stability

Metric perturbation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(r) + \delta g_{\mu\nu}(r,t) = \bar{g}_{\mu\nu} + \begin{cases} h_{\mu\nu}, & \text{massless tensor} \\ \phi, & \text{massive scalar} \\ \psi_{\mu\nu}, & \text{massive tensor} \end{cases}$$

Reducing the degrees of freedom: $\psi_{\mu\nu} \to \psi_{\mu\nu}(\varphi)$

Regge-Wheeler-Zerilli-like equation:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} + V(r^*)\right)\varphi = 0$$

Linear perturbations and stability

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$$g_{\mu\nu} = \bar{g}_{\mu\nu}(r) + \delta g_{\mu\nu}(r,t) = \bar{g}_{\mu\nu} + \begin{cases} h_{\mu\nu}, & \text{massless tensor} \\ \phi, & \text{massive scalar} \\ \psi_{\mu\nu}, & \text{massive tensor} \quad \Longrightarrow \quad \text{ghosts!} \end{cases}$$

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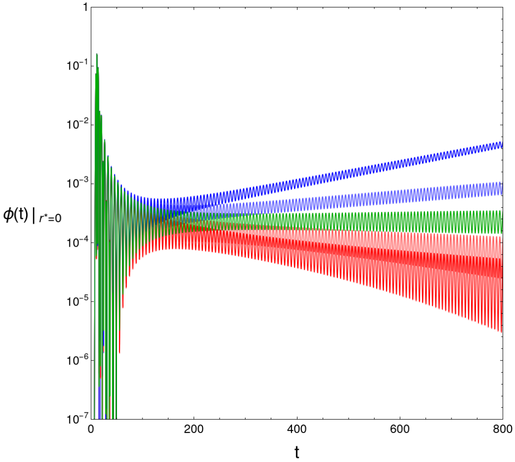
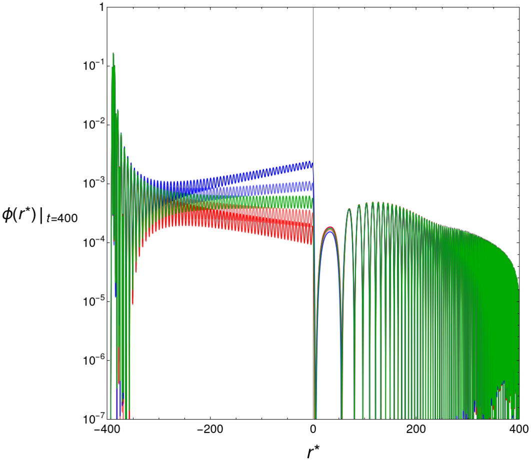
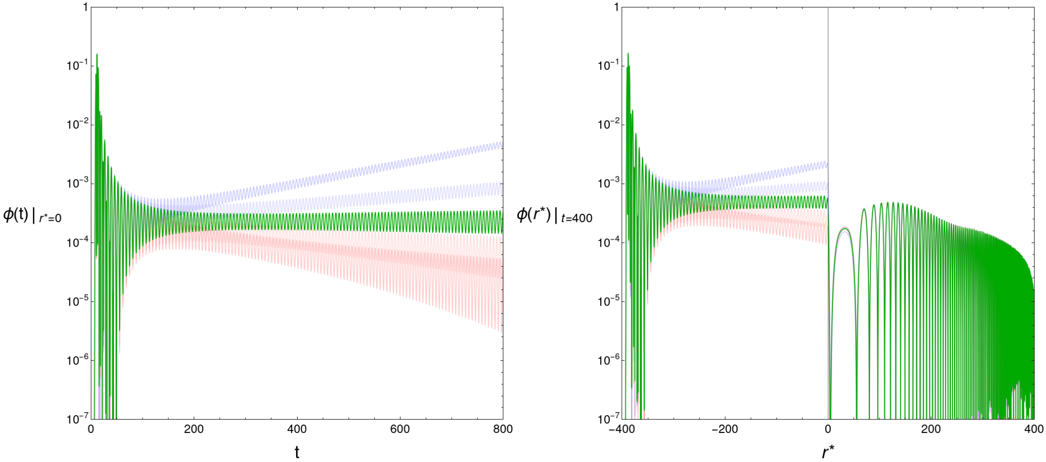
Black hole crossing point: onset of instabilities





Black hole crossing point: a phase transition?



Black hole crossing point: a phase transition?

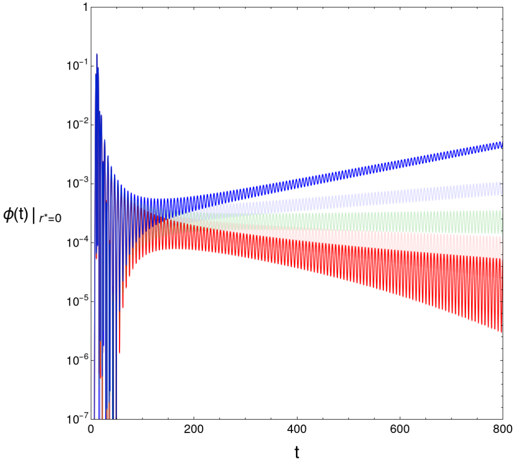
$$\psi_{\mu\nu}(r,t) \sim \psi_{\mu\nu,bo}(r,t)\,\mathrm{e}^{-\frac{t}{\tau}}\mathrm{e}^{-\frac{r}{\xi}}$$

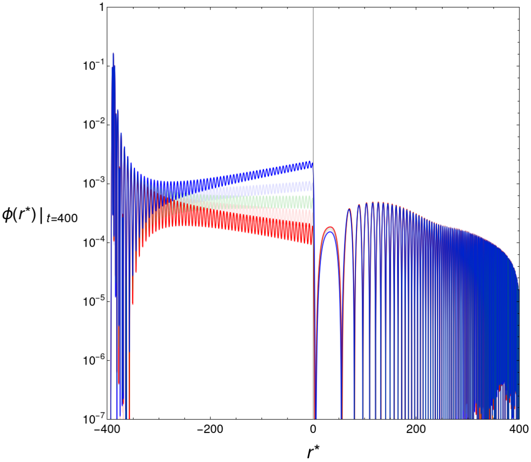
Critical point: $\lambda \to \lambda_c \implies \tau \to \infty,\ \xi \to \infty$

Order parameter: $\psi_{\mu\nu} = \delta R_{\mu\nu} \implies \begin{cases} \lambda < \lambda_c, & R_{\mu\nu} = 0 \\ \lambda > \lambda_c, & R_{\mu\nu} \neq 0 \end{cases}$

Symmetry breaking: $\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = -\frac{1}{3}\theta - R_{\mu\nu}x^{\mu}x^{\nu}$

Black hole phase transition and instabilities





Black hole phase transition and instabilities

$$S_2^- = 0 \to S_2^- < 0 \implies \text{equilibrium phase transition, } \lambda = T_{BH} \implies \text{reversible process}$$

$$S_2^- = 0 \to S_2^- > 0 \implies \text{dynamical phase transition, } \lambda = t \implies \text{irreversible process}$$

Second assumption:

The system will always end up in the unstable phase

Black holes evolution

Unstable dynamics and its endpoint

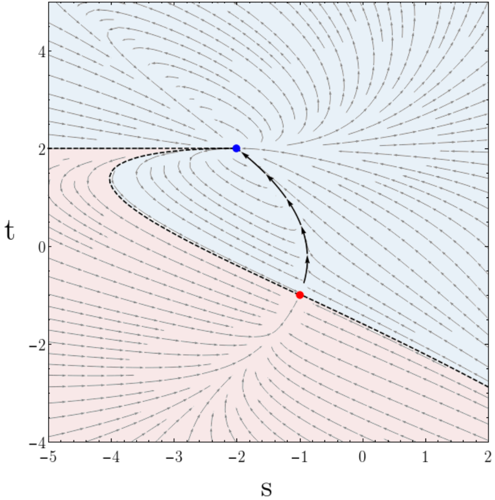
Unstable evolution: nature of the singularity

Equations for t and s in $x = -\log(r)$

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{2}\left(4 + 2t + 4s + t^2 + ts\right)$$

$$\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{1}{2\,(t-2)}\left(8 + 8s - s^2 + 3t^2 + st^2 + 2s^2 t - t^3\right)$$

N.B. valid for time-dependent metric!



Unstable evaporation: an equation for the evolution

No adiabatic expansion $\Longrightarrow$ full time-dependent evolution?

Asymptotic flatness (still isolated objects):

$$f(r,t) \sim 1 - \frac{2\,M(t)}{r} + \frac{1}{r}\int \mathrm{d}r\, r^2 \int \mathrm{d}r'\mathrm{d}t'\, G_{\left(\Box - m_2^2\right)}\left(r,t,r',t'\right)\mathcal{C}\left(T_{\mu\nu}\right)$$

$M(t)$ is the time-dependent ADM (and Misner-Sharp, and Hawking-Hayward) mass

$$\left(\partial_t^2 + m_2^2\right)\partial_t M(t) = \frac{1}{8\alpha}\lim_{r\to\infty} r^2\, T_{tr}(r,t)$$

Educated guess for T_{tr}:

- ghost instability

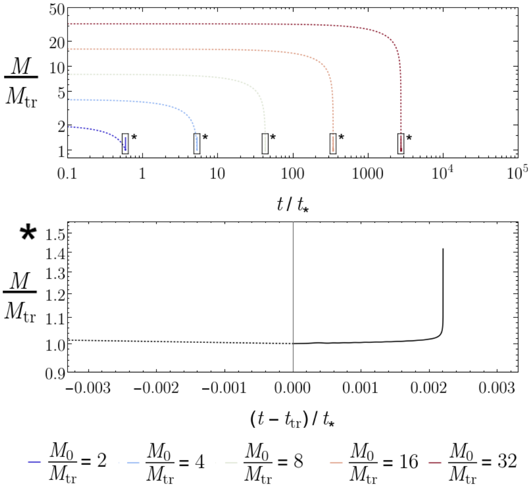
$$\implies \quad T_{tr} \propto e^{\nu t}, \;\; \nu > 0$$

- ghost dominance

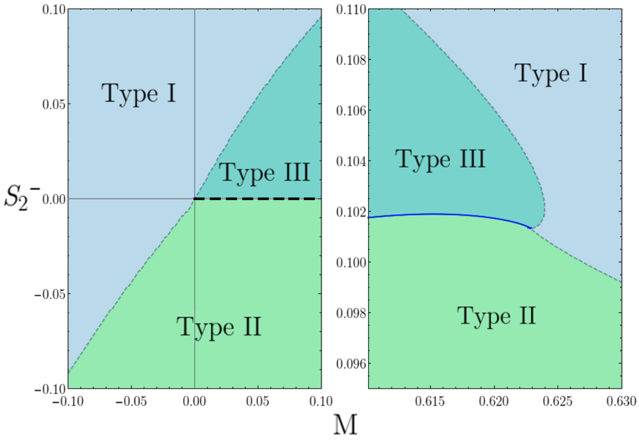
$$\implies \quad T_{tr} > 0$$

Third assumption:

Stop evolution when $r_H \to 0$



Endpoint of evaporation: a naked singularity



"Triple points" $r_H \to 0$:

- $M \to 0$

  $\implies$ Minkowski

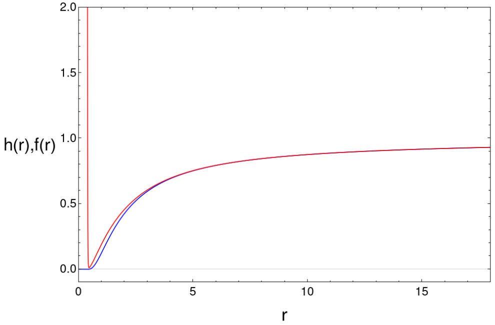
- $M \to M_e$

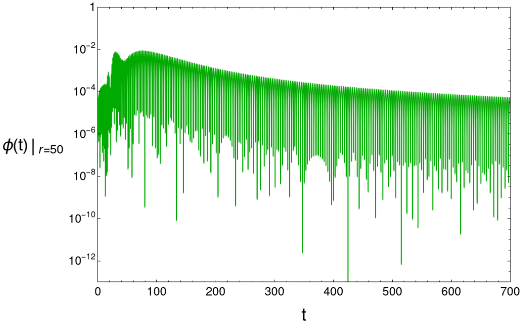
  $\implies$ Singularity

Dynamical system:

$(t, s) = (2, -2)$?

Endpoint of evaporation: a safe naked singularity?





$$z_\infty = \frac{1}{\sqrt{h(r)}} - 1 \to \infty$$

Stable?

Conclusions

Conservative approach:

Information paradox: semiclassical graivty breaks down at high energies

$\Longrightarrow$ inclusion of first order quantum corrections to gravity (even if they have ghosts)

Change of point of view:

Stationary (astrophysical) solution $\Longrightarrow$ instabilities have to be avoided

Dynamical evolution $\Longrightarrow$ instabilities have to be followed

Accpetance of limitations:

A naked singularity cannot be accepted from a theoretical point of view

If allowed by observations it can be a plausible direction


Information paradox
Singularity problem
Naked singularity

What's next? Hope you will tell me!

A no-(scalar) hair theorem and ghosts

$C_{\mu\nu\rho\sigma}$ is traceless $\Longrightarrow$ trace of vacuum e.o.m. is $\left(\Box - m_0^2\right) R = 0$

$$\left\{\begin{array}{l}\text{staticity}\\ \text{asymptotic flatness}\\ \text{presence of event horizon}\end{array}\right. \Longrightarrow \quad R = 0 \text{ in all spacetime}$$

R^2 term is irrelevant $\Longrightarrow$ $C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$ term is crucial (ghosts!)

Numerical methods: shooting method