4th International FLAG Workshop:
The Quantum and Gravity
Catania, September 9-11, 2024

How modified dust collapse can produce a regular black hole

(the cases of Asymptotic Safety and Non Linear Electrodynamics)

Daniele Malafarina
Nazarbayev University, Astana, KZ

NAZARBAYEV UNIVERSITY

4th International FLAG Workshop:
The Quantum and Gravity
Catania, September 9-11, 2024

Based on
DM and B. Toshmatov, Phys. Rev. D 105, L121502 (2022)
A. Bonanno, DM and A. Panassiti, Phys. Rev. Lett. 132, 031401 (2024)

The work was supported by
Nazarbayev University Faculty Development Competitive Research Grant No. 11022021FD2926

Daniele Malafarina
Nazarbayev University, Astana, KZ

NAZARBAYEV UNIVERSITY

4th International FLAG Workshop:
The Quantum and Gravity
Catania, September 9-11, 2024

Plan of the talk

1. Black holes
2. Black hole singularities to probe the limits of GR
3. Dust collapse and (non rotating) black holes
4. Non singular collapse
5. Regular black holes in GR coupled to NLED
6. Regular black holes in Asymptotic Safe gravity

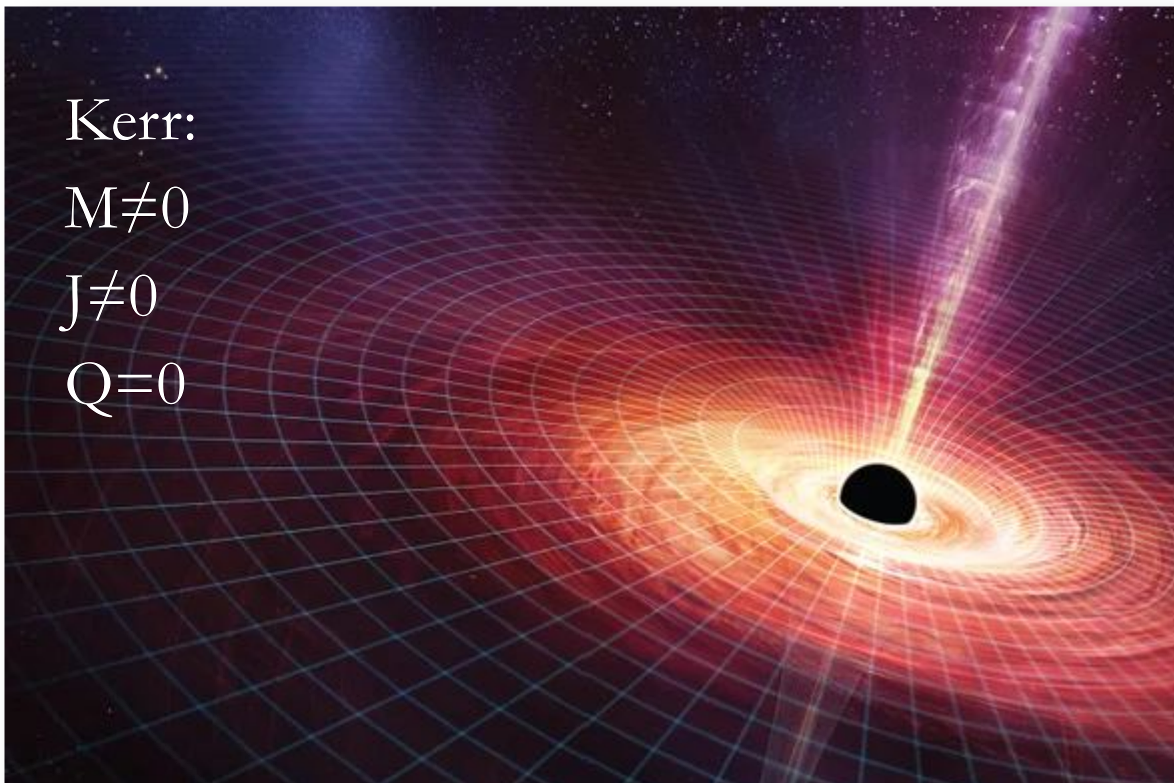
Daniele Malafarina
Nazarbayev University, Astana, KZ

NAZARBAYEV UNIVERSITY

1. Black holes

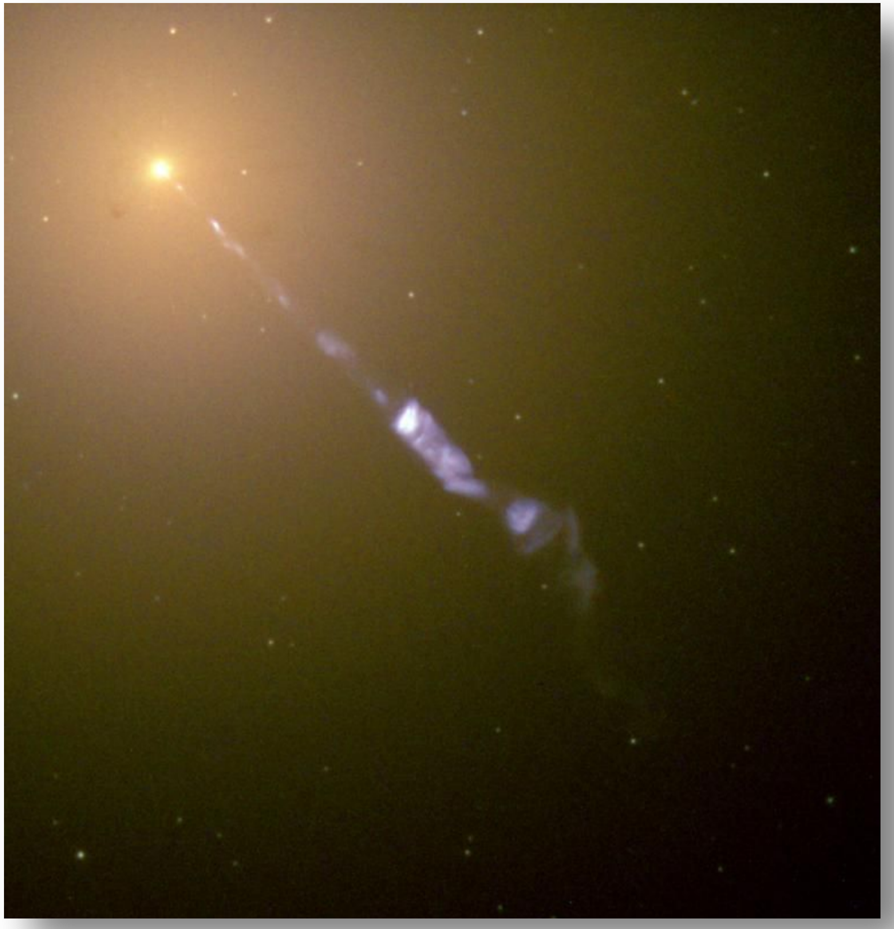
Mathematical black holes

Defined by the presence of a space-time **singularity** covered by an **event horizon**. Black holes in GR are characterized by only 3 parameters: Mass (M), Angular momentum (J) and Charge (Q).

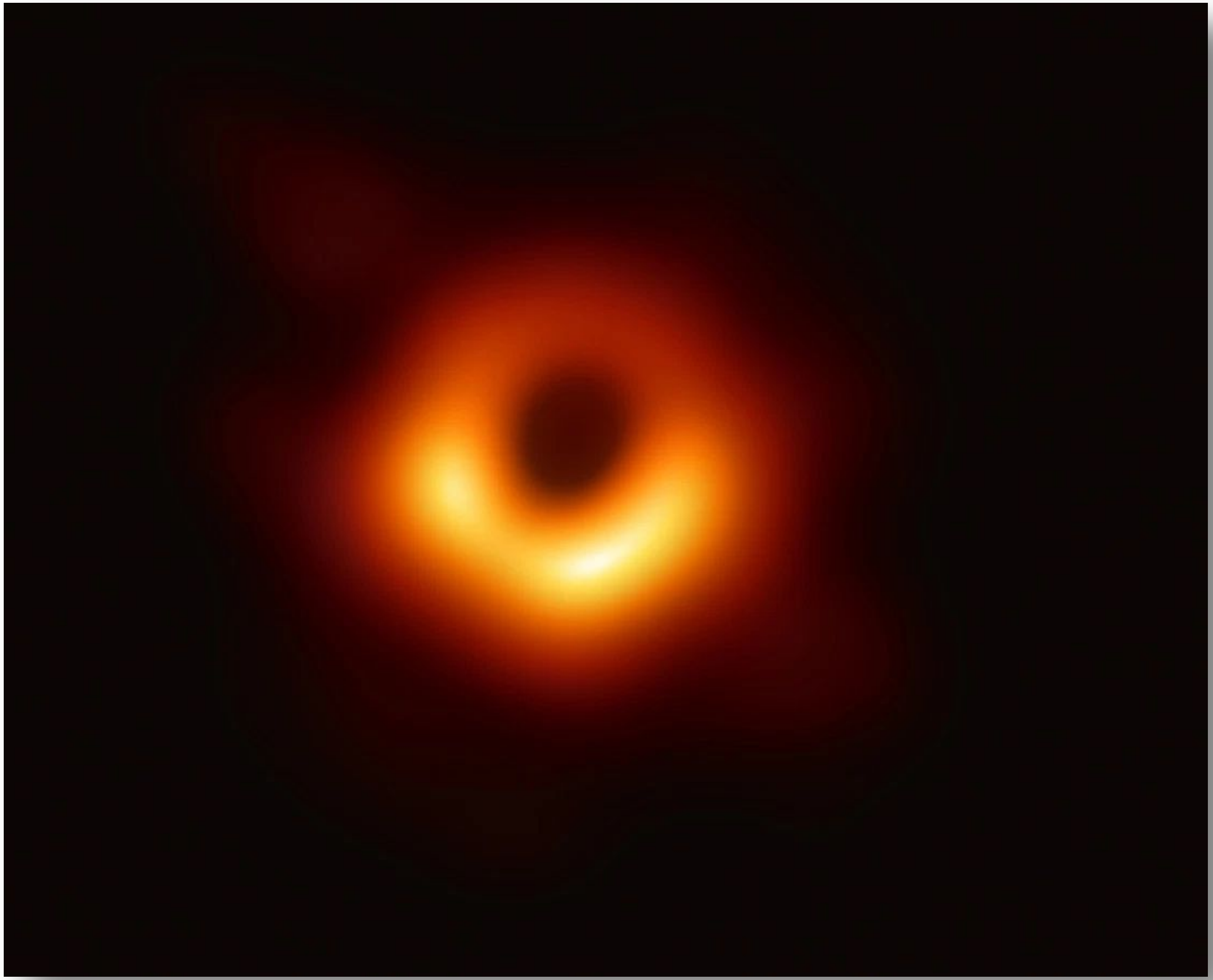


Astrophysical black holes

Evidence from stellar mass X-ray binaries, AGNs, Quasars, Sgr-A*, M87* and gravitational waves. Are they exactly Kerr black holes?



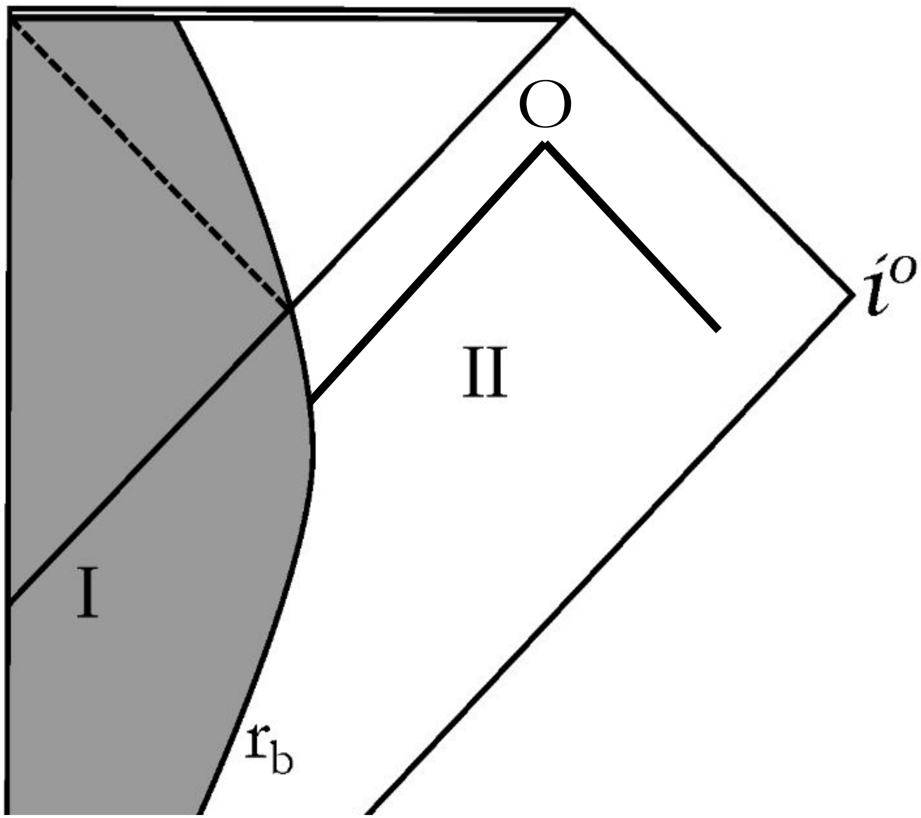
Is this a Kerr black hole?



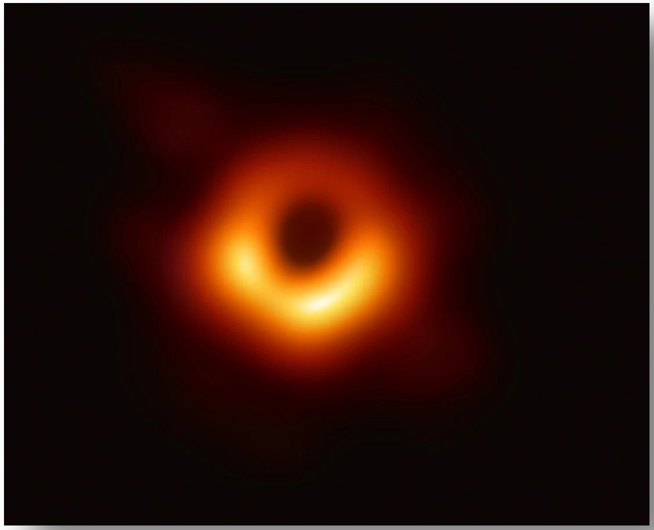
[Event Horizon Telescope Collaboration, Astrophys. J. 875, L1 (2019)]

We can't really say

Any observer (O) outside the black hole (region II) will only ever receive photons from the matter that collapses into it. And very few photons from the vicinity of the horizon.



Is there a horizon?

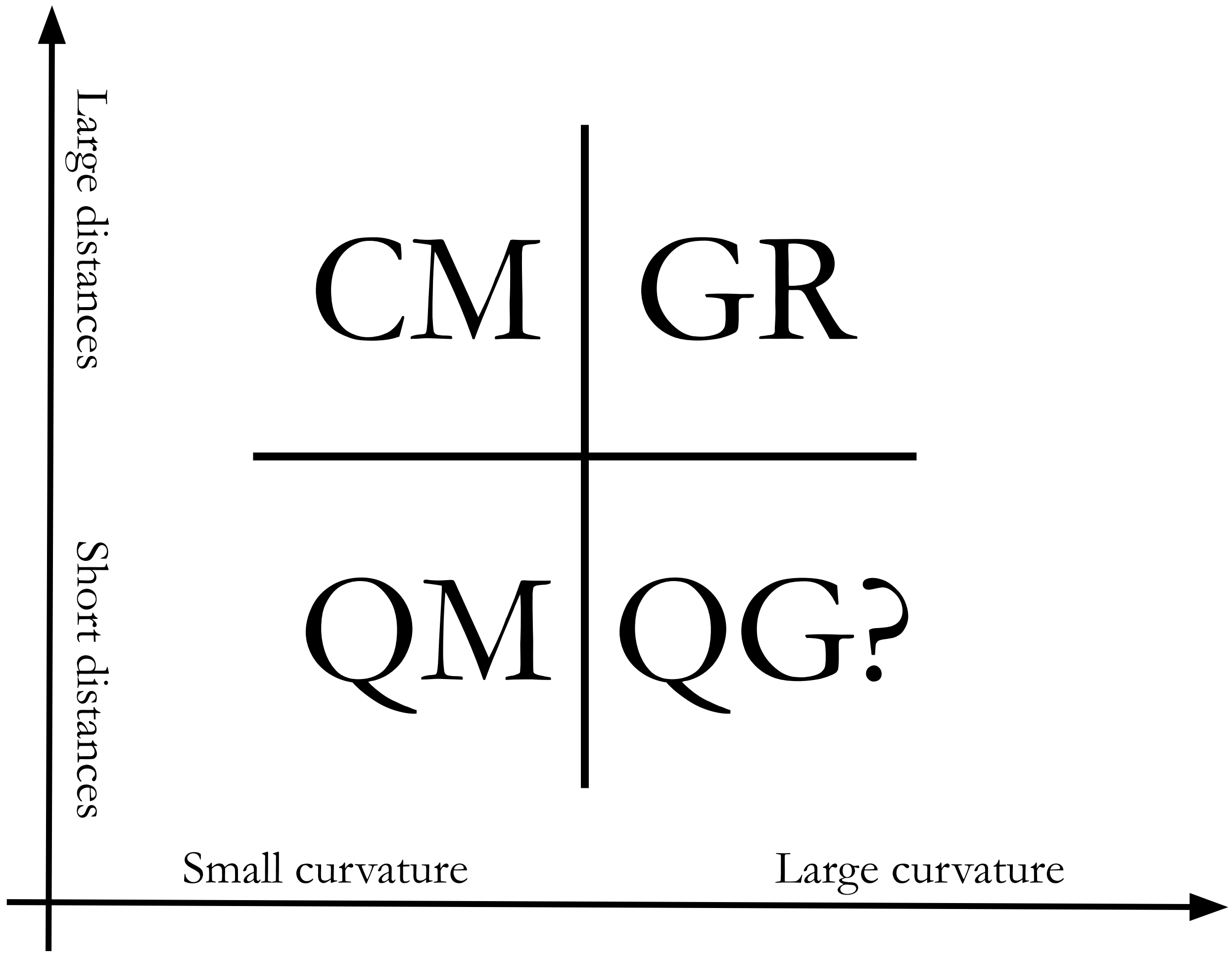


(1) Yes: Then is it a classical GR horizon? What happens near the singularity? Does it last forever?

(2) No: Then why does it look so similar to a black hole? What kind of object is it? Can we tell the difference?

2. Black hole singularities to probe the limits of GR

The limits of GR



Why the Planck scale?

The idea that we must look for departures from GR at Planck scales is based only on dimensional analysis using G, c and ħ.

1. *Is Λ a fundamental constant too?*
   Λ may be regarded as an universal constant in its own right.

   [N. Dadhich, arXiv:1609.02138]

2. *Other scales?*
   Consider for example $m = \left(\frac{\hbar^2\sqrt{\Lambda}}{G}\right)^{1/3} \simeq 0.1 \cdot 10^{-24}\mathrm{gr}$

   [P. Zenczykowski, Found Sci 24, 287 (2019)]

3. *Length? Mass? Density?*
   The effects of GR become important for $2GM/(Rc^2) \simeq 1$. For large M and large R this occurs when the gravitational field is weak.

Can we test our models?

Can signs of a departure from GR appear at classical scales?

An example from QM:

At large scales and for large numbers of particles we use a classical description of mechanics instead of QM. However we can build situations in which QM effects are non negligible at scales that normally would be classical (for example, entanglement in gases with large number of particles or entanglement over large distances).

An example from GR:

Tiny relativistic effects can be measured in the weak gravitational field, for example when they accumulate over time (think about Mercury's perihelion).

Can a similar situation arise for a new theory beyond GR?
If it does it probably has to be near the horizon.

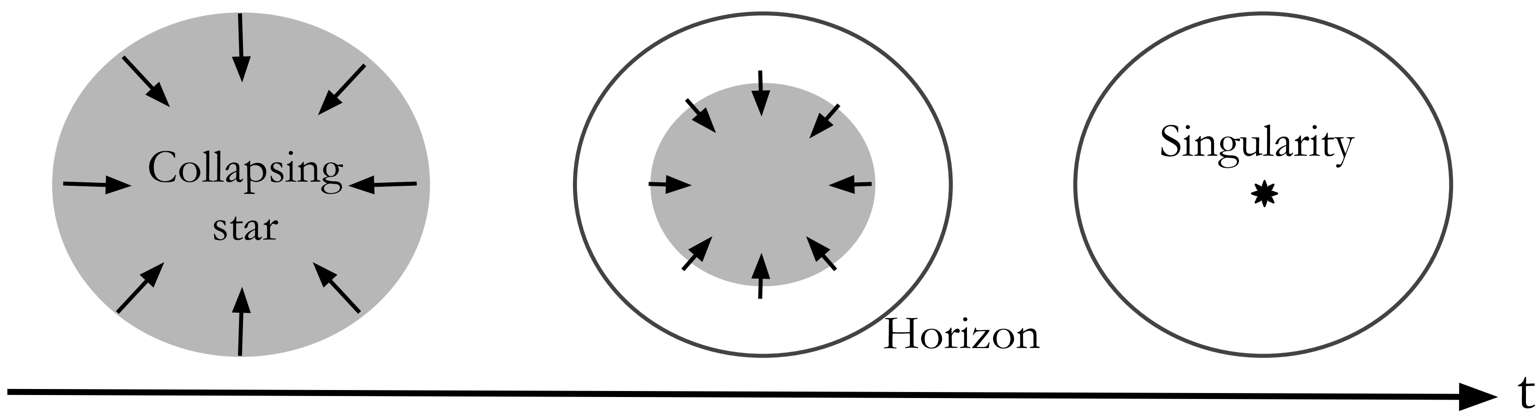
Singularity theorems

In classical GR, ordinary matter compressed in small volumes must form a singularity.

- GR holds
- Matter satisfying energy conditions (EC).
- Globally hyperbolic space-time.
- Trapped surfaces form at some point.

} spacetime singularity

Matter behaves like ordinary matter at all densities until the singularity. GR holds at every energy scale until the formation of the singularity.



Singularities yes or no?

1. The singularity theorems hold and singularities do occur!

   Let's hope they are all hidden.

or

✔2. GR and/or EC break down at some scale and singularities are an artifact of the incompleteness of our description of large curvatures.

Avoiding singularities

To avoid singularities we may consider:

1. *Violation of EC* $\Longrightarrow$ *Corrections to* $< T_{\mu\nu} >$
   Matter at extreme densities may behave in a way that violates the classical energy conditions.

And/Or

✔ 2. *Modifications to GR* $\Longrightarrow$ *Corrections to* $< G_{\mu\nu} >$
   UV modifications to GR in the 'strong field' become non negligible.

What phenomena?

Cosmology

There is only one universe. Only one initial singularity.

In principle there exist geodesics from the initial singularity leading to observers here and now. In practice is not so easy.

✔ *Black hole candidates*

There seem to be plenty of black holes. Many singularities.

If black hole singularities are hidden behind the horizon then there are no geodesics from the singularity leading to observers here and now.

Can we still look for such effects in astrophysical black holes?

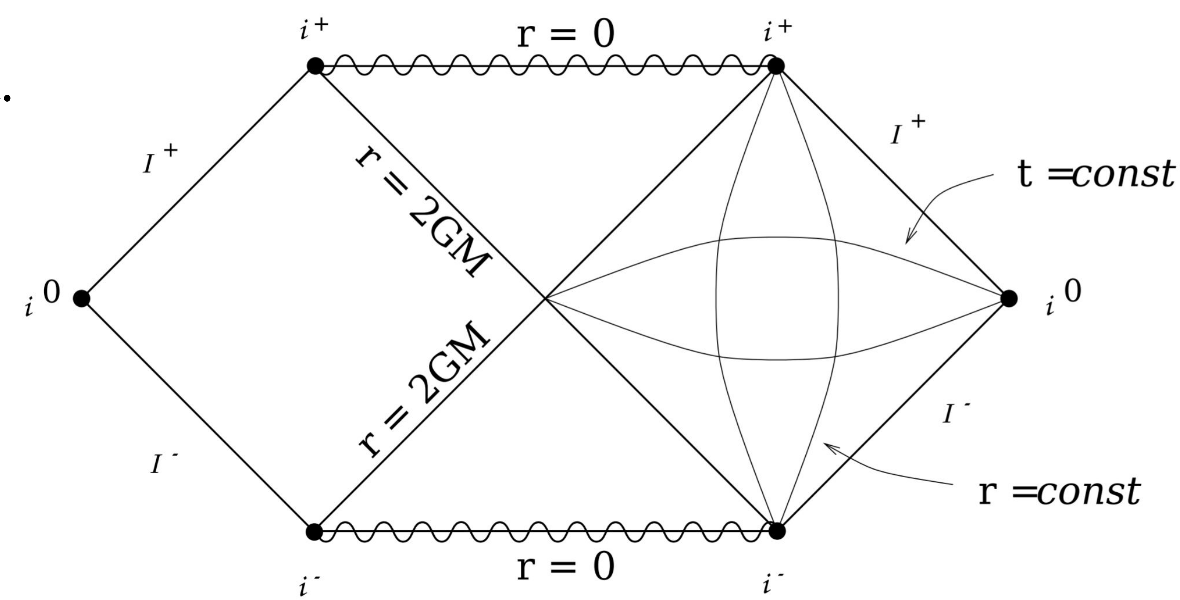
3. Dust collapse and (non rotating) black holes

Schwarzschild

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2$$

$$f(R) = 1 - \frac{2\mathcal{M}(R)}{R}$$

- $\mathcal{M}$(R)=M=const.
- Event horizon at R=2M.
- Singularity at R=0.



Lemaitre coordinates

Describe radially infalling particles in Schwarzschild as well as collapsing dust particles in OSD.

$$\begin{cases} d\tau = \quad dT + \dfrac{g(R)}{f(R)} dR \\ d\rho = dT + \dfrac{1}{g(R) f(R)} dR \end{cases}$$

with

$$g = \sqrt{1 - f}$$

Radial infall

The line element becomes

$$ds^2 = -d\tau^2 + \frac{2\mathcal{M}(R)}{R} d\rho^2 + R(\rho, \tau)^2 d\Omega^2$$

A particle located at $\varrho = \varrho_0 =$const falls radially along $B_0(\tau) = R(\varrho_0, \tau)$ according to

$$\frac{dB_0}{d\tau} = -\sqrt{\frac{2\mathcal{M}(B_0)}{B_0}} = -\sqrt{\frac{2M}{B_0}}$$

Gravitational collapse

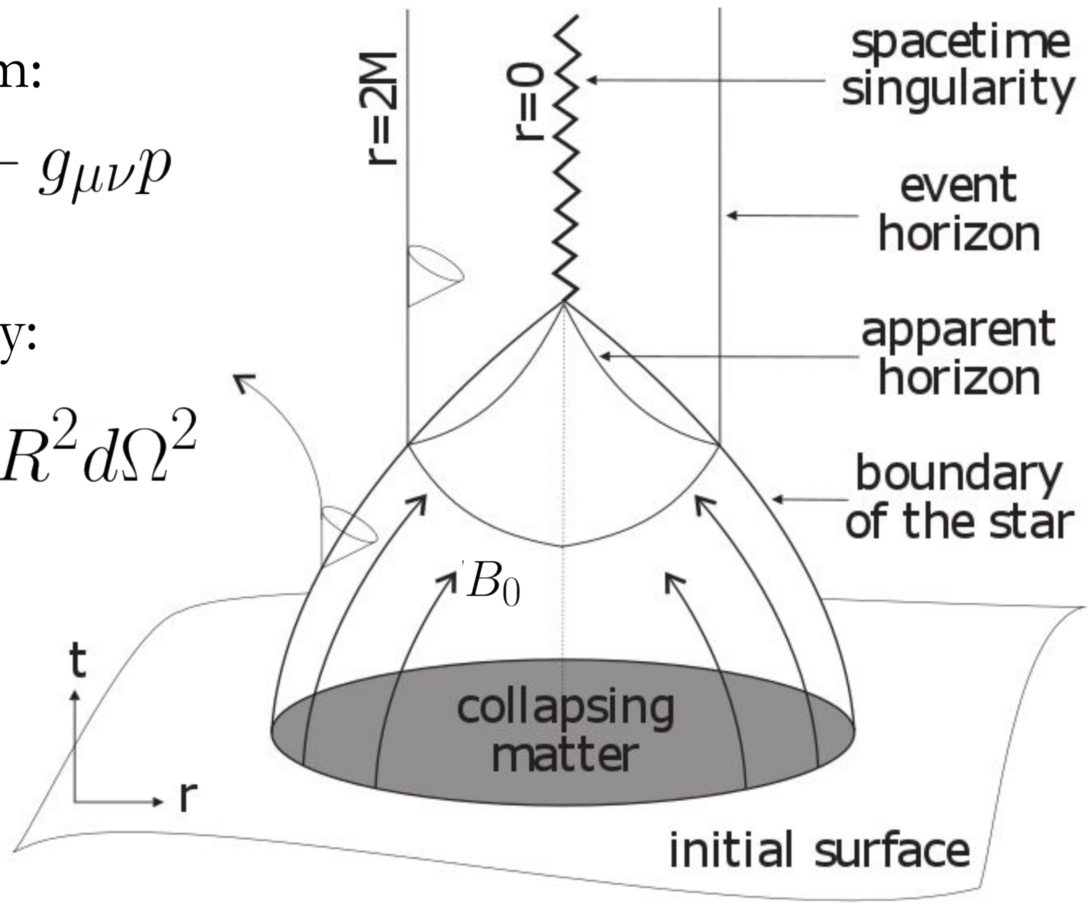
The collapsing interior is made of a perfect fluid with energy density ϵ and pressure p.

Energy-momentum:

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + g_{\mu\nu}p$$

Spherical symmetry:

$$ds^2 = g_{ab}dx^a dx^b + R^2 d\Omega^2$$



Interior field equations

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}dr^2 + R^2 d\Omega^2$$

Einstein's equations

$$\epsilon = \frac{F'}{R^2 R'}$$

$$p = -\frac{\dot{F}}{R^2 \dot{R}}$$

$$\dot{R}' = R'\dot{\psi} + \dot{R}\nu'$$

Misner-Sharp mass F

$$F = R(1 - e^{-2\psi}R'^2 + e^{-2\nu}\dot{R}^2)$$

Bianchi identity

$$\nu' = -\frac{p'}{\epsilon + p}$$

Scaling

$$\begin{cases} R = ra \\ e^{2\psi} = \frac{R'^2}{1-kr^2} \\ F = r^3 m \end{cases}$$

Homogeneous dust

$$ds^2 = -dt^2 + \frac{a^2}{1-kr^2}dr^2 + (ra)^2 d\Omega^2$$

Einstein's equations

$$\epsilon = \frac{F'}{R^2 R'} = \frac{3m}{a^3}$$

$$p = -\frac{\dot{F}}{R^2 \dot{R}} = 0 \Rightarrow F(r)$$

$$\dot{R}' = R'\dot{\psi} + \dot{R}\nu' = \dot{a}$$

Misner-Sharp mass F

$$F = ra(kr^2 + r^2\dot{a}^2)$$

Bianchi identity

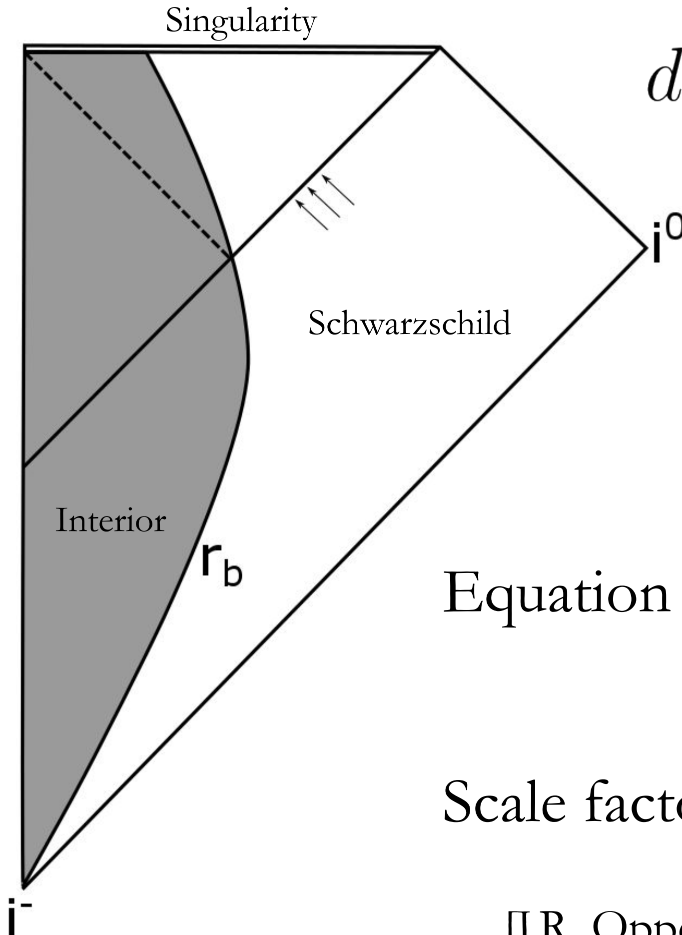
$$\nu' = 0$$

Scaling

$$\begin{cases} R = ra \\ e^{2\psi} = \frac{a^2}{1-kr^2} \\ F = r^3 m_0 \end{cases}$$

Oppenheimer - Snyder - Datt

Homogeneous (ϵ(t)), marginally bound (k=0) dust (p=0)



$$ds^2 = -dt^2 + a^2dr^2 + (ra)^2d\Omega^2$$

Density: $\epsilon(t) = \frac{3m_0}{a^3}$

Pressure: $p(t) = 0$

Equation of motion: $\dot{a} = -\sqrt{\frac{m_0}{a}}$

Scale factor: $a(t) = \left(1 - \frac{3}{2}\sqrt{m_0}t\right)^{2/3}$

[J.R. Oppenheimer and H. Snyder, Phys. Rev. 56, 455 (1939);
S. Datt, Z. Phys. 108, 314 (1938)]

Trapped surfaces

The condition for the formation of trapped surfaces is

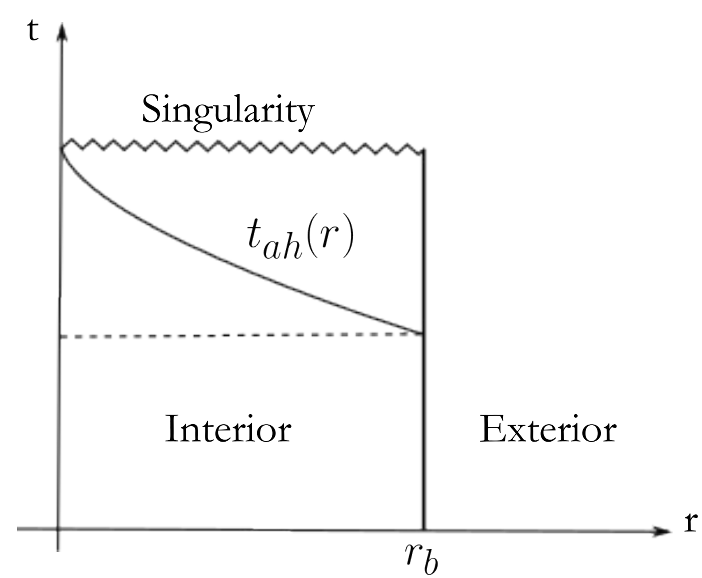
$$1 - \frac{F}{R} = 0$$

For the exterior Schwarzschild metric this corresponds to R=2M. For the interior OSD the apparent horizon curve is given by

$$r_{ah}(t) = \sqrt{\frac{a(t)}{m_0}} = \frac{1}{\dot{a}(t)}$$

or inversely

$$t_{ah}(r) = \frac{2}{3\sqrt{m_0}} - \frac{2}{3} m_0 r^3$$



The horizon

Inside the collapsing matter ($r<r_b$) the apparent horizon is

$$r_{\rm ah}(t) \simeq 1/\dot{a}$$

OSD (dotted line):

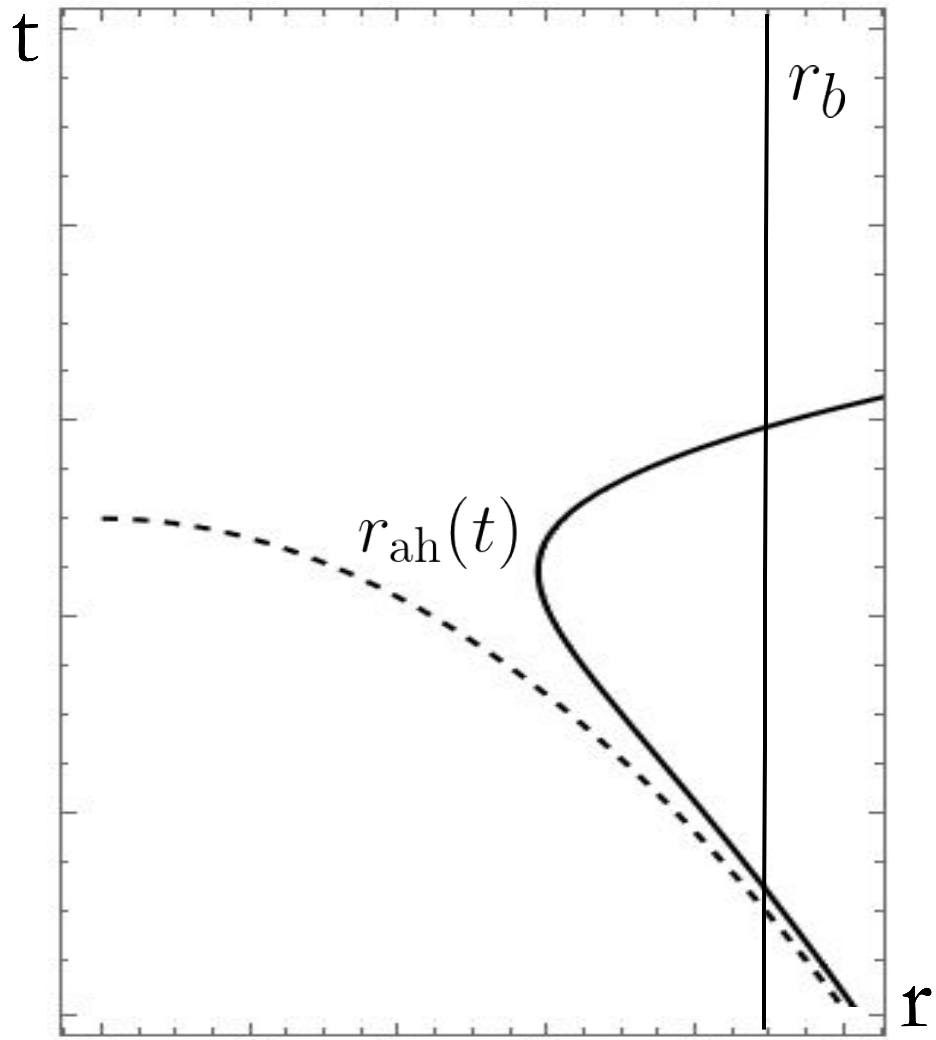
$\dot{a}\rightarrow\infty \Rightarrow r_{ah}\rightarrow 0$ in a finite time.

r_{ah} crosses the boundary r_b once.

Modified OSD (thick solid line):

$\dot{a}\rightarrow 0 \Rightarrow r_{ah}\rightarrow\infty$.

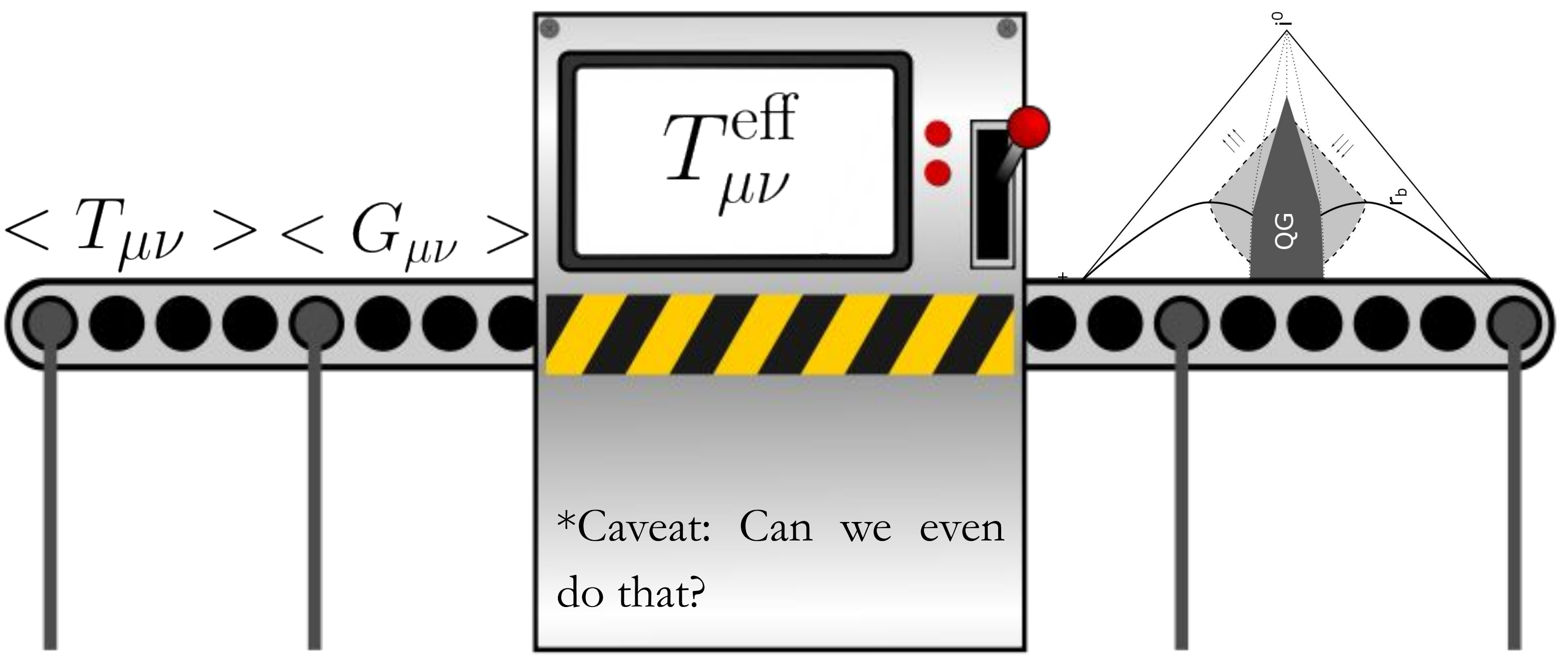
r_{ah} must cross the boundary r_b (vertical solid line) twice.



4. Non singular collapse

A semi-classical cookbook*

- Choose an approach to modify GR.
- Determine the effective energy-momentum.
- Plug it in a model of classical collapse.
- Study the behaviour of the scale factor and horizon.
- Compare with the classical case.
- Is there some observable effect?



Semi-classical limits

How to proceed in the absence of a valid theory beyond GR?

Consider the modifications to Einstein's equations coming from some approach

$$G_{\mu\nu} + < G_{\mu\nu} > = 8\pi\kappa (T_{\mu\nu} + < T_{\mu\nu} >)$$

Define an effective energy-momentum tensor containing the modifications to geometry

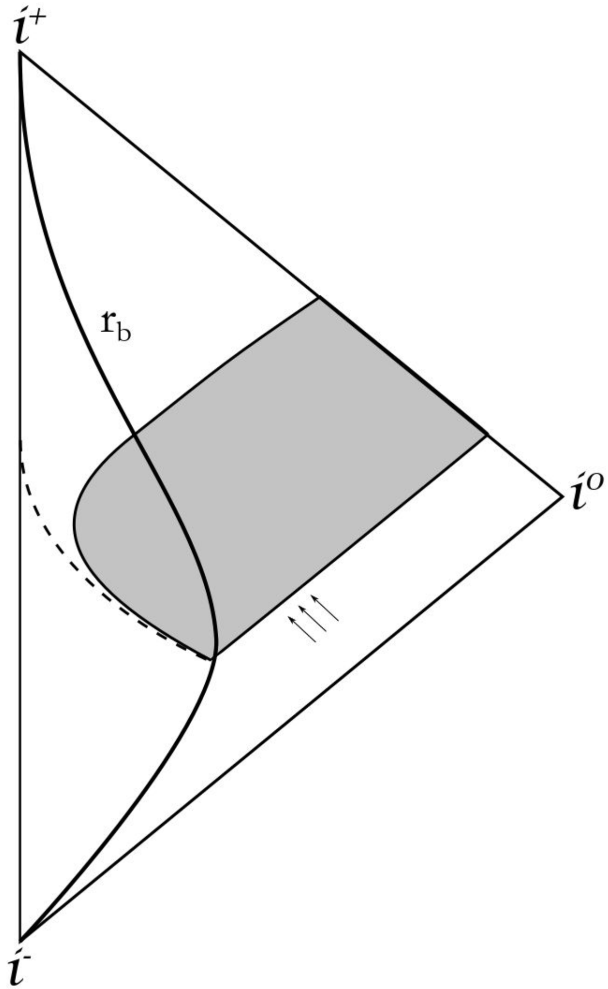
$$8\pi\kappa T^{\text{eff}}_{\mu\nu} = 8\pi\kappa (T_{\mu\nu} + < T_{\mu\nu} >) - < G_{\mu\nu} >$$

Solve Einstein's equations for the non physical effective matter source

$$G_{\mu\nu} = 8\pi\kappa T^{\text{eff}}_{\mu\nu}$$

How good is this framework to describe small scale effects beyond GR?

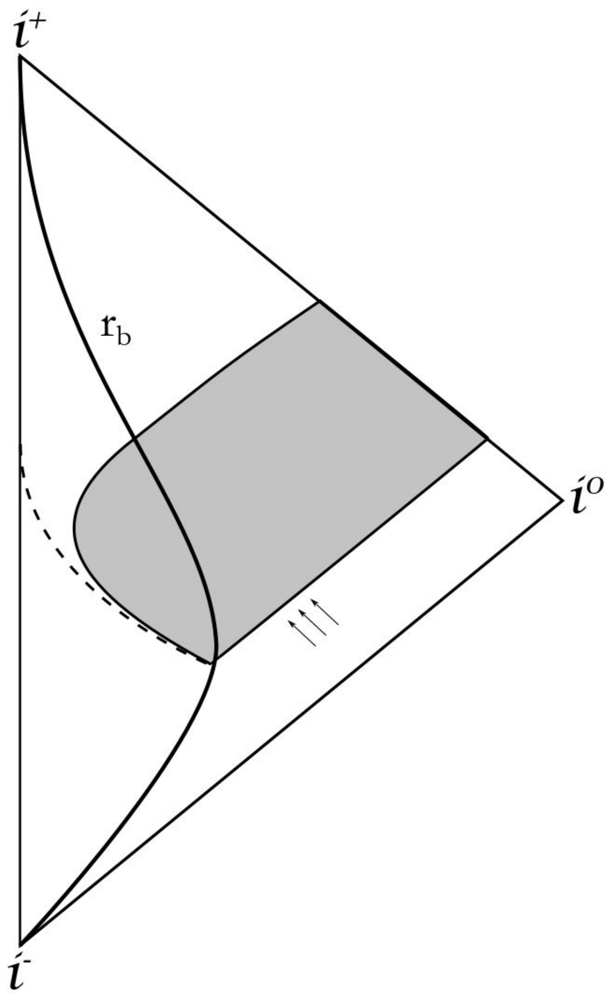
The aim…



Find a solution for dust collapse that generalizes OSD and has the following features:

- Not singular.
- Obtained from a viable action.
- Produces a regular black hole.
- The energy-momentum is 'well behaved'.

…and how to achieve it



- GR fails during collapse at a certain scale (density, size, energy…)

- At that scale the system must be described by a new theory such as AS or GR+NLED.

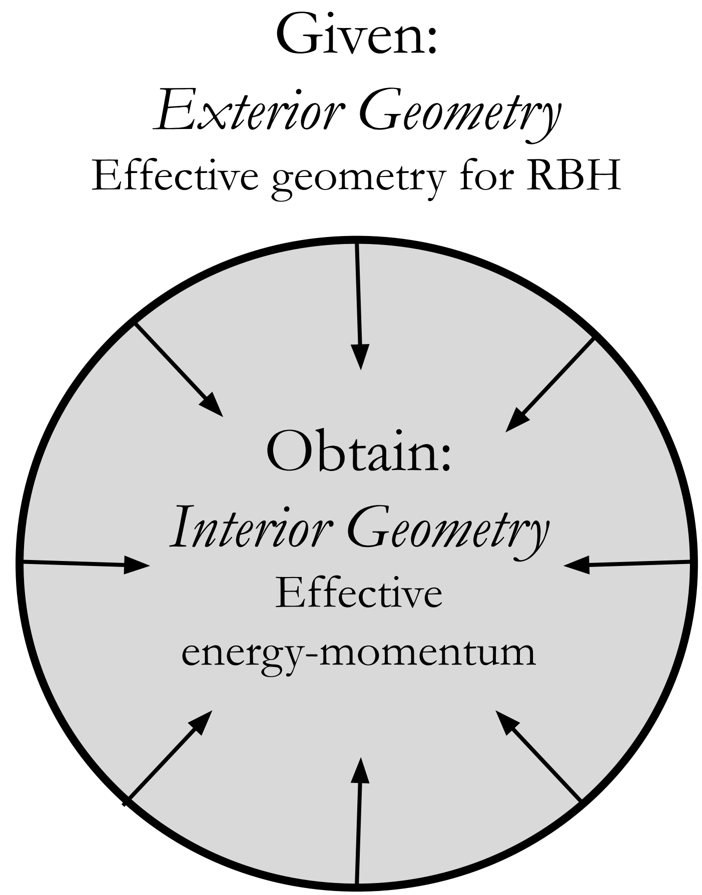
- Repulsive effects are modeled via an effective energy-momentum.

- The effects are present in the matter and possibly in the exterior too.

- The horizon forms but collapse does not produce a singularity.

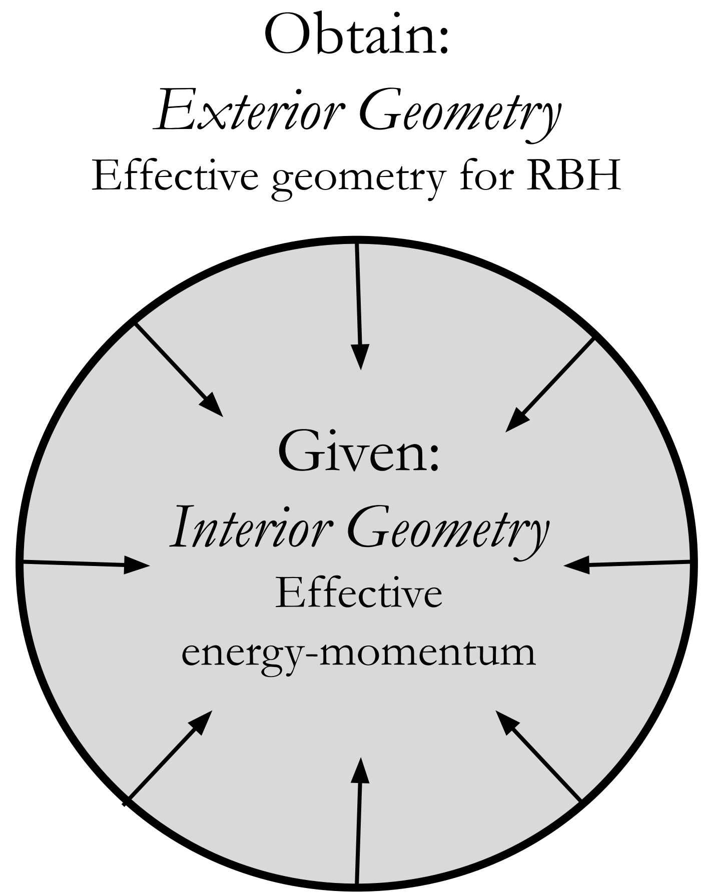
From the exterior (NLED)...

- What metric describes the space-time in the exterior? We choose a regular black hole (RBH) in NLED.
- Is there a semi-classical dust collapse model that produces the RBH as final state?
- What are the properties of its effective energy-momentum?



…or from the interior (AS)

- What metric describes the space-time in the interior? We choose an Asymptotically Safe modification of GR.
- We obtain a semi-classical dust collapse model for which the singularity is avoided.
- What is the corresponding exterior metric? Is it a RBH?



Semi-classical dust collapse

The action for GR with semi-classical corrections is

$$\mathcal{A} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}\,(\mathrm{R} - \mathcal{L}_{\mathrm{Dust}} - \mathcal{L}_{\mathrm{corr}})$$

And we obtain the energy momentum tensor as

$$\mathrm{T}^{\mathrm{eff}}_{\kappa\lambda} = \mathrm{T}^{\mathrm{Dust}}_{\kappa\lambda} + \mathrm{T}^{\mathrm{corr}}_{\kappa\lambda}$$

Where the correction in general will also have non vanishing effective pressures.

Effective energy-momentum

In general the effective density can be written as

$$\epsilon_{\rm eff} = \epsilon + \alpha_1 \epsilon^2 + \alpha_2 \epsilon^3 + \ldots$$

With the higher order contributions negligible at lower densities.

All coefficients α_i must depend on one parameter (such as the UV cutoff of GR). Then we may take the effective density as

$$\epsilon_{\rm eff} = \epsilon \left[1 \pm \left(\frac{\epsilon}{\epsilon_{\rm cr}} \right)^{\beta} \right]^{\gamma}$$

Where the critical density depends on the model and is not necessarily of the order of the Planck density.

Equation of motion

The equation of motion for the scale factor then takes the form

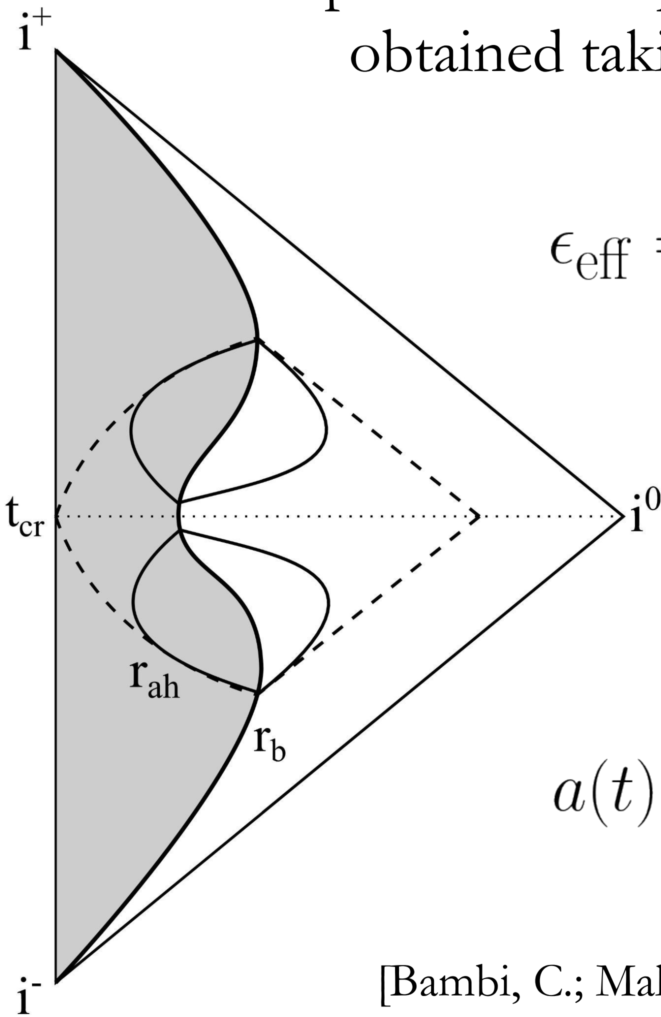
$$\dot{a} = -\sqrt{\frac{m_0}{a}\left[1 \pm \left(\frac{a_{\rm cr}}{a}\right)^{3\beta}\right]^{\gamma}}$$

With the critical scale given by $\dfrac{3m_0}{a_{\rm cr}^3} = \epsilon_{\rm cr}$

We must take $\beta=1$ in order to have $\alpha_1 \neq 0$.

The LQG bounce…

A simple model inspired by Loop Quantum Gravity is obtained taking the minus sign and γ=1.



$$\epsilon_{\rm eff} = \frac{3m_0}{a^3}\left(1 - \frac{\epsilon}{\epsilon_{\rm cr}}\right) \qquad p_{\rm eff} = -\frac{\epsilon^2}{\epsilon_{\rm cr}}$$

$$\dot{a} = -\sqrt{\frac{m_0}{a}\left(1 - \frac{a_{\rm cr}^3}{a^3}\right)}$$

$$a(t) = \left[a_{\rm cr}^3 + \left(\sqrt{1 - a_{\rm cr}^3} - \frac{3}{2}\sqrt{m_0}t\right)^2\right]^{\frac{1}{3}}$$

[Bambi, C.; Malafarina, D.; Modesto, L. Phys. Rev. D 2013, 88, 044009]

…is not a regular BH

Collapse halts and bounces at a finite size.
The effective density vanishes at the time of the bounce.

The exterior black hole spacetime corresponding to the LQG collapse is not regular

$$\mathcal{K} = \frac{48M^2}{R^6}\left(39\frac{q_*^6}{R^6} - 10\frac{q_*^3}{R^3} + 1\right)$$

The Kretschmann scalar $\mathcal{K}$ diverges for R going to zero.
However, the equation for radial infall of shows that particles bounce at $R=q_*$ without reaching $R=0$.

5. Regular black holes in GR coupled to NLED

GR coupled to NLED

The action for GR coupled to NLED is

$$\mathcal{A} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}\,(\mathrm{R} - \mathcal{L}_{\mathrm{NLED}}(\mathrm{F}))$$

with

$$\mathcal{L}_{\mathrm{NLED}} = \frac{4\mu}{\alpha}\frac{(\alpha\mathrm{F})^{(\nu+3)/4}}{[1-(\alpha\mathrm{F})^{\nu/4}]^{1+\mu/\nu}}$$

$\mathrm{F}^{\kappa\lambda}$ is the Faraday tensor and $\mathrm{F} = \mathrm{F}_{\kappa\lambda}\mathrm{F}^{\kappa\lambda}$

[Z.-Y. Fan and X. Wang, Phys. Rev. D 94, 124027 (2016)]

Regular black holes in NLED

The line element for a black hole in GR coupled to NLED has

$$f(R) = 1 - \frac{2\mathcal{M}(R)}{R} \qquad \mathcal{M}(R) = \frac{M\,R^{\mu}}{(R^{\nu} + q_{*}^{\nu})^{\mu/\nu}}$$

- Where q_* is the NLED charge.
- The black hole is regular (i.e. no singularity at R=0) for $\mu \geq 3$.
- Maxwell's electrodynamics is obtained for μ=-1 and ν=1 and it gives the Reissner-Nordstrom black hole.
- Other notable examples are:
  - Hayward black hole for $\mu = \nu = 3$,
  - Bardeen black hole for $\mu = 3$, $\nu = 2$.

Hayward black hole

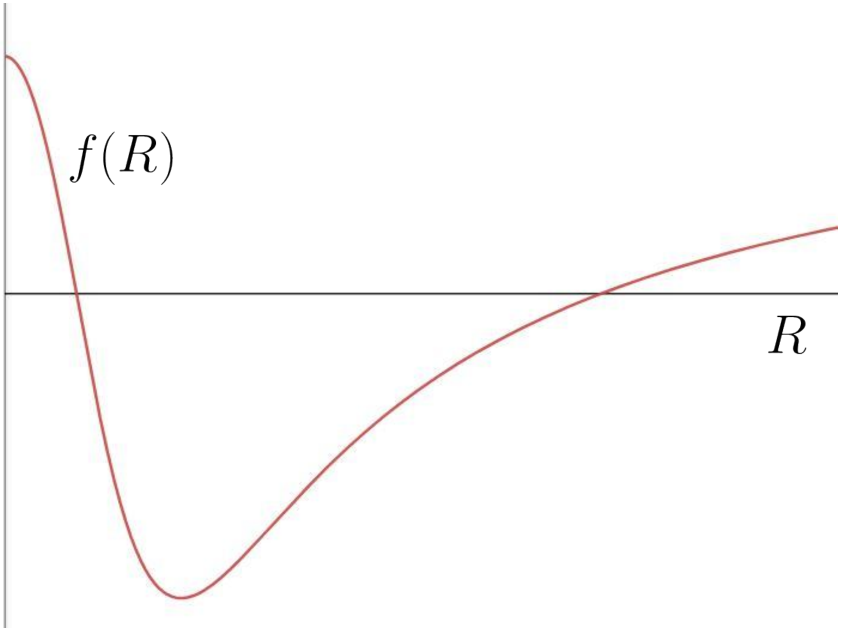
The Hayward black hole is given by $\mu=\nu=3$

$$f(R) = 1 - \frac{2M}{R}\left(\frac{1}{1+q_*^3/R^3}\right)$$

- It has 2 horizons if

$$M^3 > \frac{27}{32}q_*^3$$

- The black hole is regular, the Kretschmann scalar is finite at R=0.



[S. A. Hayward, Phys. Rev. Lett. 96, 031103 (2006)]

Lemaitre coordinates

Apply the transformation to Lemaitre coordinates.
A particle at $\varrho=\varrho_0$=const falls radially along $B_0(\tau)=R(\varrho_0,\tau)$

$$\frac{dB_0}{d\tau} = -\sqrt{\frac{2M}{B_0}\left(1+\frac{q_*^\nu}{B_0^\nu}\right)^{-\mu/\nu}}$$

This looks similar to a marginally bound dust collapse with semi-classical corrections.

Equation of motion

The equation of motion for the scale factor then takes the form

$$\dot{a} = -\sqrt{\frac{m_0}{a}\left[1 \pm \left(\frac{a_{\rm cr}}{a}\right)^{3\beta}\right]^{\gamma}}$$

With the critical scale given by $\dfrac{3m_0}{a_{\rm cr}^3} = \epsilon_{\rm cr}$

We must take $\beta=1$ in order to have $\alpha_1 \neq 0$.

Equation of motion

The equation of motion for the scale factor of semi-classical collapse was

$$\dot{a} = -\sqrt{\frac{m_0}{a}\left[1 \pm \left(\frac{a_{\rm cr}}{a}\right)^{3\beta}\right]^{\gamma}}$$

To recover the NLED black hole we need to consider the plus sign and take:

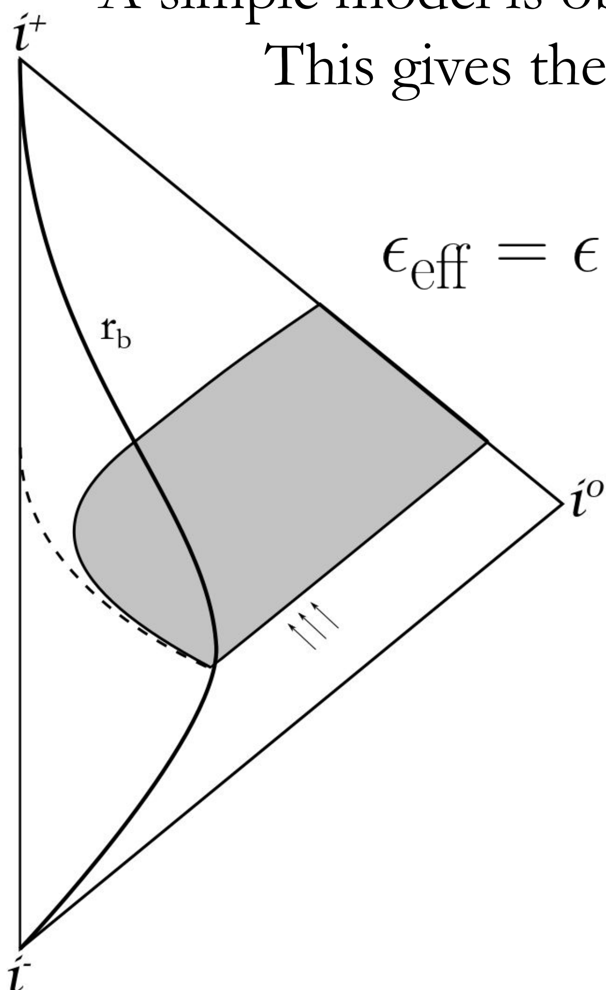
(1) $\gamma=-\mu/\nu$,

(2) $\nu=3\beta$ (then $\beta=1$ implies $\nu=3$).

The Hayward black hole is then given by $\gamma=-1$.

The Hayward RBH

A simple model is obtained taking the plus sign and γ=-1.
This gives the Hayward RBH in the exterior.



$$\epsilon_{\text{eff}} = \epsilon \cdot \sum_{n=0}^{\infty} \left(-\frac{\epsilon}{\epsilon_{\text{cr}}} \right)^n = \epsilon \left(1 - \frac{\epsilon}{\epsilon + \epsilon_{\text{cr}}} \right)$$

$$p_{\text{eff}} = -\epsilon^2 \frac{\epsilon_{\text{cr}}}{(\epsilon_{\text{cr}} + \epsilon)^2}$$

$$\dot{a} = -\sqrt{m_0 \frac{a^2}{a^3 + a_{\text{cr}}^3}}$$

[Malafarina, D.; Toshmatov B., Phys. Rev. D 2022, 105, L121502]

Settles to a regular BH

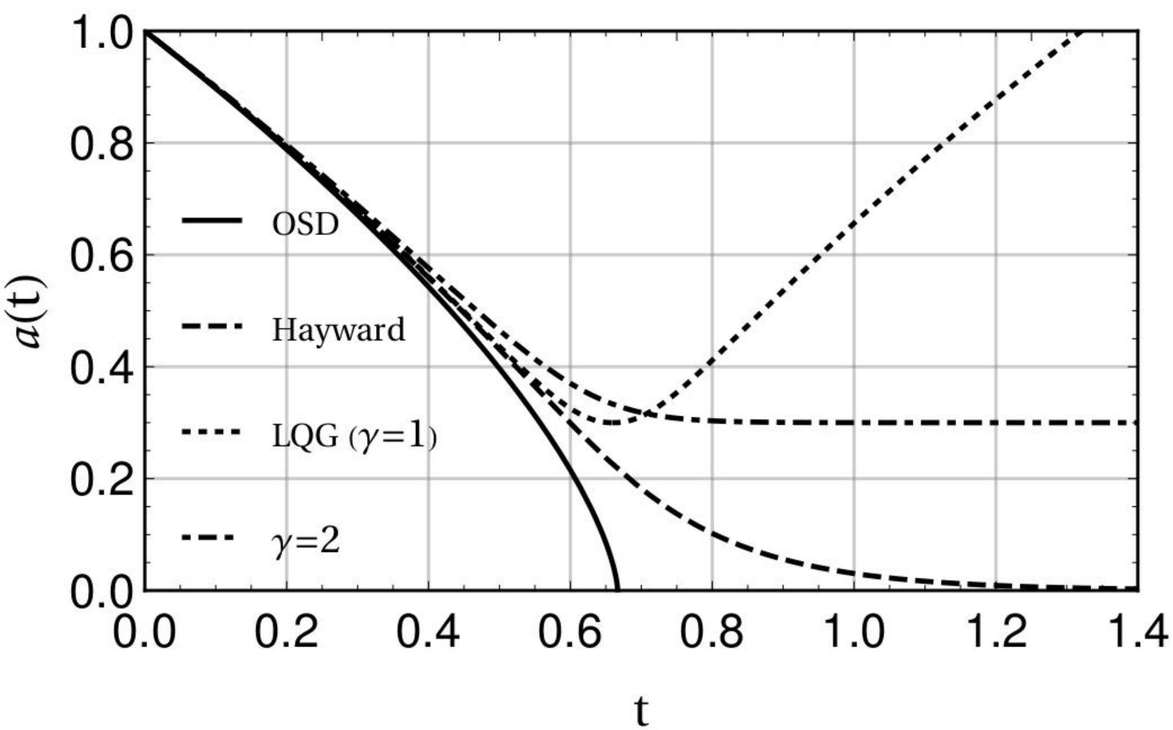
The exterior space-time is described by the Hayward RBH solution.

The interior settles asymptotically to a non singular configuration of zero volume.

Effects of the modified theory are negligible near the event horizon.

The apparent horizon has a minimum and crosses the boundary twice.

The final configuration has two horizons (inner and outer).



6. Regular black holes in Asymptotic Safety

The action

We look for a theory in which the gravitational coupling $G(\epsilon)$ is not constant and depends on the energy-density

$$\mathcal{A} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\, [R + 2\chi(\epsilon)\mathcal{L}_{\mathrm{Dust}}]$$

With $\chi(\epsilon)$ a multiplicative gravity-matter coupling for which

$$\chi(0) = 8\pi G_N$$

where G_N is Newton's constant.

(In what follows we adopt units for which $\chi(0)=1$)

[M. A. Markov and V. F. Mukhanov, Nuovo Cim. B 86, 97 (1985)]

Variable G and variable Λ

The field equations take the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\partial(\chi\epsilon)}{\partial\epsilon}T_{\mu\nu} + \frac{\partial\chi}{\partial\epsilon}\epsilon^2 g_{\mu\nu}$$

Which allow to interpret $\chi(\epsilon)$ as related to a variable gravitational coupling $G(\epsilon)$ and a variable cosmological constant $\Lambda(\epsilon)$ via

$$8\pi G(\epsilon) = \frac{\partial(\chi\epsilon)}{\partial\epsilon}, \quad \Lambda(\epsilon) = -\frac{\partial\chi}{\partial\epsilon}\epsilon^2$$

Then fixing $G(\epsilon)$ we obtain $\chi(\epsilon)$ and $\Lambda(\epsilon)$.

Asymptotic Safety

We choose $G(\epsilon)$ from Asymptotic Safety as

$$G(\epsilon) = \frac{G_N}{1 + \xi\epsilon}$$

with the dimensionful parameter ξ obtained from the AS ultraviolet cutoff.

[A. Bonanno and M. Reuter, Phys. Rev. D 62, 043008 (2000)]

Then for $\chi(\epsilon)$ and $\Lambda(\epsilon)$ we get

$$\chi(\epsilon) = \frac{\log(1+\xi\epsilon)}{\xi\epsilon}$$

$$\Lambda(\epsilon) = \frac{\log(1+\xi\epsilon)}{\xi} - \frac{\epsilon}{1+\xi\epsilon}$$

Equation of motion

The equation of motion for the scale factor then takes the form

$$\dot{a} = -\sqrt{\frac{\log(1 + 3m_0\xi/a^3)}{3\xi}a^2}$$

At large times we have that

$$a(t) \sim e^{-t^2/4\xi}$$

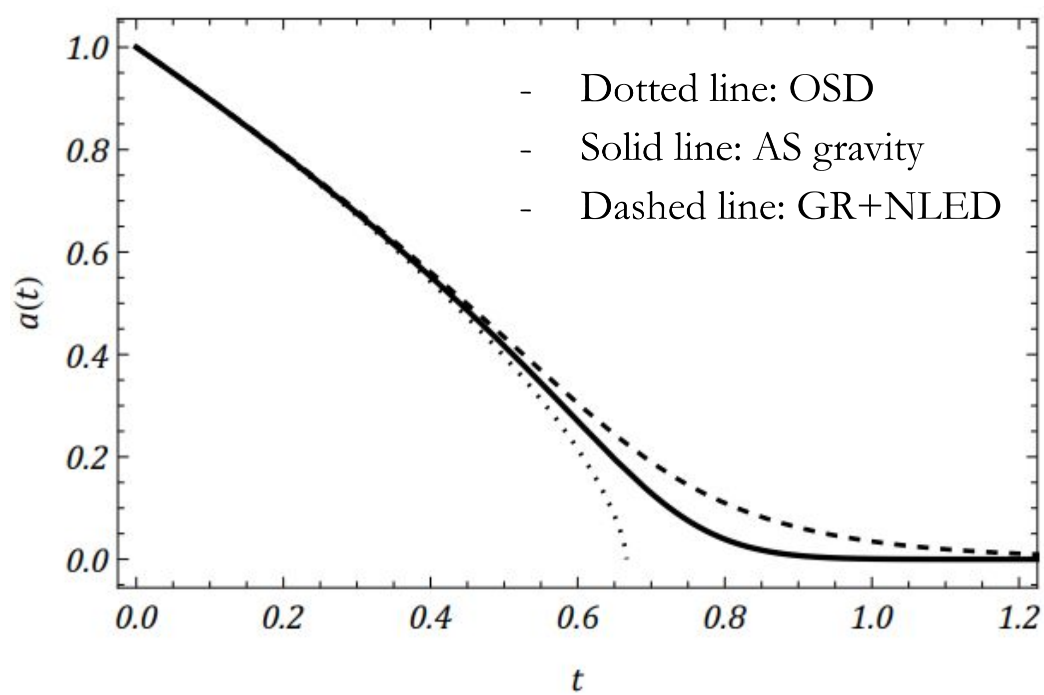
Which shows that the scale factor goes to zero asymptotically and thus the singularity is never achieved.

[A. Bonanno, DM and A. Panassiti, Phys. Rev. Lett. 132, 031401 (2024)]

A new regular black hole

By matching with the exterior we obtain the line element for the new regular black hole from AS as

$$f(R) = 1 - \frac{R^2}{3\xi}\log\left(1 + \frac{6M\xi}{R^3}\right)$$



4th International FLAG Workshop:
The Quantum and Gravity
Catania, September 9-11, 2024

Thank you

Daniele Malafarina
Nazarbayev University, Astana, KZ

NAZARBAYEV UNIVERSITY