Impact of perturbative counterterms on black holes

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with
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Based on arXiv:2311.15739 and arXiv:2412.XXXX

Outline

- Motivation
- Corrected spacetime metric
- Thermodynamic properties
- Concluding remarks

Motivation

- Goal: change the equations of motion due to higher-order operators

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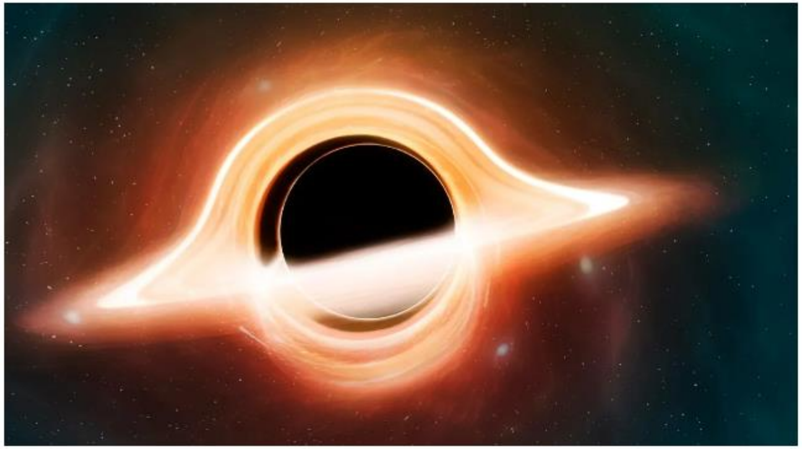
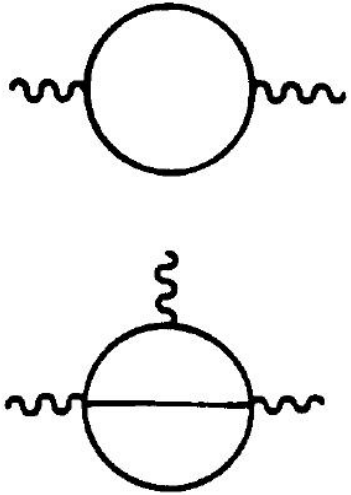
- UV behavior of GR





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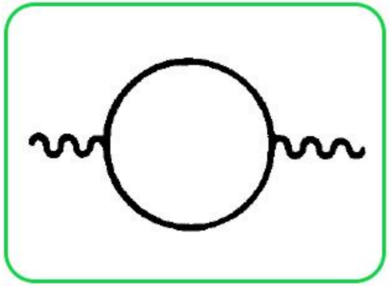

$$\mathcal{L}^{(1)}_{\infty} = \frac{1}{\epsilon}\,\frac{1}{(4\pi)^2}\int d^4x\,\sqrt{|g|}\left(\frac{R^2}{60}+\frac{7}{10}\,R^{\mu\nu}\,R_{\mu\nu}\right)$$

t'Hooft & Veltman '74

For BH in quadratic gravity, see
Lü, Perkins, Pope & Stelle '15
Bonanno & Silveravalle, '22
Held & Zhang '23

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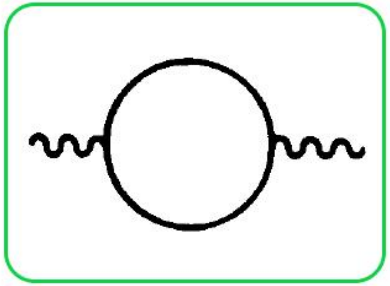

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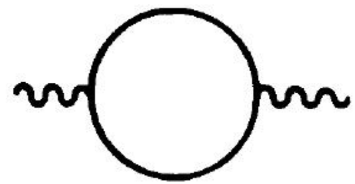

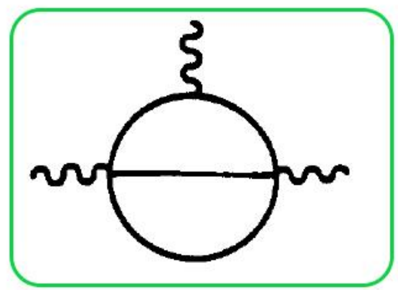

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- UV behavior of GR

$$R\,\Box\,R\,,\; R_{\mu\nu}\,\Box\,R^{\mu\nu}\,,\; R^3$$
$$R\,R_{\mu\nu}\,R^{\mu\nu}\,,\, R_{\mu\nu}\,R^{\nu}_{\alpha}\,R^{\alpha\mu}\,,\; R\,R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta}$$
$$R_{\mu\alpha}\,R_{\nu\beta}\,R^{\mu\nu\alpha\beta}\,,\; R_{\mu\nu}{}^{\alpha\beta}\,R_{\alpha\beta}{}^{\kappa\rho}\,R_{\kappa\rho}{}^{\mu\nu}$$

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$$R_{\mu\alpha}\,R_{\nu\beta}\,R^{\mu\nu\alpha\beta}\;,\boxed{R_{\mu\nu}{}^{\alpha\beta}\,R_{\alpha\beta}{}^{\kappa\rho}\,R_{\kappa\rho}{}^{\mu\nu}}$$

Motivation

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$$\sqrt{-g}\, R_{\mu\nu}{}^{\alpha\beta}\, R_{\alpha\beta}{}^{\kappa\rho}\, R_{\kappa\rho}{}^{\mu\nu}$$

Motivation

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- UV behavior of GR

$$\mathcal{L}_{\infty}^{(2)} = \frac{1}{\epsilon}\frac{209}{2880}\frac{1}{(4\pi)^2}\int d^4x\,\sqrt{|g|}\,C_{\mu\nu}{}^{\rho\sigma}\,C_{\rho\sigma}{}^{\alpha\beta}\,C_{\alpha\beta}{}^{\mu\nu}$$

  - Divergence not of EH form
  - Addition to C^3 operator to bare action
    new free parameter
    GR is perturbatively non-renormalizable

Goroff & Sagnotti '85

Including the two-loop counterterm in the action

Supplemented the Einstein-Hilbert action with the Goroff-Sagnotti term

$$S = \frac{1}{16\pi\, G} \int_{\mathcal{M}} d^4x \, \sqrt{|g|} \, \left[R + G^2 \, \lambda \, C_{\mu\nu}{}^{\kappa\gamma} \, C_{\kappa\gamma}{}^{\rho\sigma} \, C_{\rho\sigma}{}^{\mu\nu} \right]$$

The theory contains:

- 2 gravitational constants $G \,, \lambda$
- Ricci Scalar R
- Weyl tensor $C_{\mu\nu\alpha\beta}$

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The sixth-order field equations following from this action are the following monsters

$$\begin{aligned}
H_{\mu\nu} &= \frac{16\pi}{\sqrt{|g|}\,\tilde{\lambda}} \frac{\delta S}{\delta g^{\mu\nu}} \\
&= \frac{\lambda}{G}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + R_{\mu\nu} C^2 - 2RC\ {}^{\alpha\beta\gamma} C_{\nu\alpha\beta\gamma} + 2 g_{\mu\nu} C^{\alpha\beta\gamma\delta} \Box C_{\alpha\beta\gamma\delta} \\
&\quad - \frac{3}{2} R_{\nu}{}^{\alpha\beta\gamma} C_{\beta\gamma\delta\rho} C_{\mu\alpha}{}^{\delta\rho} - \frac{1}{2} g_{\mu\nu} C_{\alpha\beta}{}^{\rho\lambda} C^{\alpha\beta\gamma\delta} C_{\gamma\delta\rho\lambda} - \frac{3}{2} R_{\mu}{}^{\alpha\beta\gamma} C_{\beta\gamma\delta\rho} C_{\nu\alpha}{}^{\delta\rho} \\
&\quad + 6 C_{\beta\gamma\delta\rho} C_{\mu}{}^{\alpha\beta\gamma} C\ {}^{\delta\rho} + 6 R^{\alpha\beta} C_{\mu\alpha}{}^{\gamma\delta} C_{\nu\beta\gamma\delta} - 2 \nabla_{\mu} C^{\alpha\beta\gamma\delta} \nabla_{\nu} C_{\alpha\beta\gamma\delta} \\
&\quad + \frac{3}{2} C_{\nu}{}^{\alpha\beta\gamma} \nabla_{(\alpha} \nabla_{\delta)} C^{\delta}{}_{\mu\gamma\beta} - \frac{3}{2} C^{\alpha\beta\gamma\delta} \nabla_{\beta} \nabla_{(\mu} C_{\nu)\alpha\gamma\delta} - 6 \nabla_{\gamma} C_{\nu\alpha\beta\gamma} \nabla^{\delta} C_{\mu}{}^{\alpha\beta\gamma} \\
&\quad + \frac{3}{2} C_{\mu}{}^{\alpha\beta\gamma} \nabla_{(\alpha} \nabla_{\delta)} C^{\delta}{}_{\nu\gamma\beta} - \frac{1}{2} C^{\alpha\beta\gamma\delta} \nabla_{(\mu} \nabla_{\nu)} C_{\alpha\beta\gamma\delta} + 3 \nabla_{(\alpha} C_{\mu}{}^{\alpha\beta\gamma} \nabla_{\delta)} C^{\delta}{}_{\nu\gamma\delta} \\
&\quad - 3 C_{\nu}{}^{\alpha\beta\gamma} \nabla_{\delta} \nabla_{\mu} C^{\delta}{}_{\alpha\gamma\beta} - 3 C_{\mu}{}^{\alpha\beta\gamma} \delta_{\gamma} \nabla_{\nu} C^{\delta}{}_{\alpha\gamma\beta} - \frac{3}{2} g_{\mu\nu} C^{\alpha\beta\gamma\delta} \nabla_{(\delta} \nabla_{\rho)} C_{\alpha\beta\gamma}{}^{\rho} \\
&\quad + \frac{3}{2} C^{\beta\gamma\alpha}{}_{(\mu} \Box C_{\nu)\alpha\beta\gamma} - \frac{3}{2} \nabla_{(\delta} C^{\delta}{}_{\alpha\gamma\beta} \nabla_{\nu)} C_{\mu}{}^{\alpha\beta\gamma} - \frac{3}{2} \nabla_{(\delta} C^{\delta}{}_{\alpha\gamma\beta} \nabla_{\mu)} C_{\nu}{}^{\alpha\beta\gamma} \\
&\quad + 3 g_{\mu\nu} \left(\nabla_{\alpha} C^{\alpha\beta\gamma\delta} \nabla_{\rho} C^{\rho}{}_{\beta\delta\gamma} - \nabla_{\delta} C_{\alpha\beta\gamma\rho} \nabla^{\rho} C^{\alpha\beta\gamma\delta} + \frac{2}{3} \nabla_{\rho} C_{\alpha\beta\gamma\delta} \nabla^{\rho} C^{\alpha\beta\gamma\delta} \right)
\end{aligned}$$

Including the two-loop counterterm in the action

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We will explore static and spherically symmetric spacetimes metrics. In Schwarzschild form

$$ds^2 = -h(\bar{r})\,dt^2 + \frac{d\bar{r}^2}{f(\bar{r})} + \bar{r}^2\left(d\theta^2 + \sin^2(\theta)\,d\phi^2\right)$$

The H field equation tensor takes the form

$$\bar{H}_{\mu\nu} = \begin{pmatrix} \bar{H}_{tt}(\bar{r}) & 0 & 0 & 0 \\ 0 & \bar{H}_{rr}(\bar{r}) & 0 & 0 \\ 0 & 0 & \bar{H}_{\theta\theta}(\bar{r}) & 0 \\ 0 & 0 & 0 & \bar{H}_{\theta\theta}(\bar{r})\,\sin^2\bar{\theta} \end{pmatrix}$$

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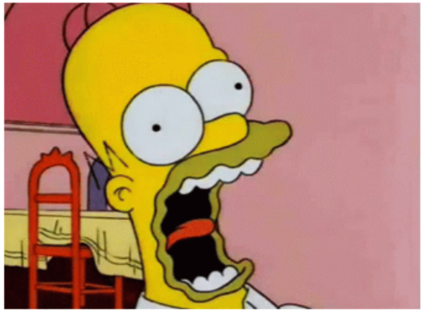

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After some derivative reductions, f''(r̄) becomes,

```
(2 ⋯2⋯ (h[rr] f′[rr] / (f[rr] rr^2) + 2 h[rr] (2-2 f[rr]+rr ⋯1⋯′[rr]) / (f[rr] rr^3) + ⋯129⋯ + ⋯1⋯^2 rr^2 ⋯1⋯^2 ⋯1⋯ (⋯1⋯) / ((2 ⋯1⋯-rr ⋯1⋯) ⋯1⋯ (⋯1⋯)) +
   f[rr]^2 (⋯1⋯) / (12 ⋯1⋯^2 ⋯2⋯ (⋯1⋯+⋯1⋯) (h[rr]^2 (-4+4 f[rr]-2 rr f′[rr])-f[rr] rr^2 ⋯1⋯^2+h[rr] rr (rr f′[rr] h′[rr]-2 f[rr] (h′[rr]-rr ⋯1⋯″[rr])))))) / (3 ⋯6⋯
 (36 (-1)^(2/3) GG^6 δδ^3 f[rr]^10 h[rr]^20 rr^29 f′[rr] + ⋯4⋯ + ⋯1⋯ - i 3^(1/6) (-i + √3) √(GG^6 ⋯4⋯ (h[rr]^2 (-8 rr^2 + 27 GG^2 δδ ⋯1⋯′[rr]^2) + 54 ⋯5⋯ ⋯1⋯′[rr] + 27 GG^2 δδ ⋯1⋯[rr]^2 ⋯1⋯′[rr]^2))
  (-9 GG^4 δδ^2 f[rr]^6 h[rr]^12 rr^17 f′[rr] - 9 ⋯5⋯ ⋯1⋯ + √3 √(GG^6 ⋯4⋯ (h[rr]^2 (-8 rr^2 + 27 ⋯2⋯ ⋯1⋯^2) + ⋯1⋯ + 27 ⋯4⋯)))^(2/3)))
```

Including the two-loop counterterm in the action

Idea: use Kundt geometry. They include *spherically symmetric BH* which can be written as

$$ds^2 = \Omega^2(r)\left[d\theta^2 + \sin^2(\theta)\,d\phi^2 - 2du\,dr + \mathcal{H}(r)\,du^2\right]$$

Svarc, Podolsky, Pravda & Pravdova '18
Podolsky, Svarc, Pravda & Pravdova '20
Pravdova, Pravda & Ortaggio '23

Relation to the Schwarzschild form of the BH metric

$$\bar{r} = \Omega(r) \quad , \quad t = u - \int \frac{dr}{\mathcal{H}(r)}.$$

$\Omega^2(r)$ and $\mathcal{H}(r)$ are related to $f(\bar{r})$ and $h(\bar{r})$ via,

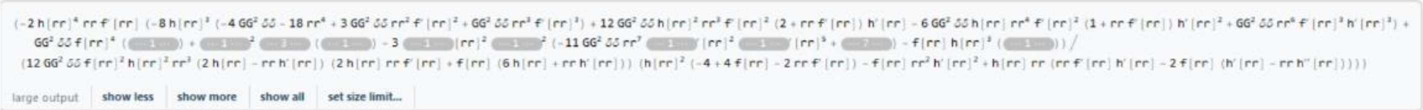
$$h(\bar{r}) = -\Omega^2(r)\,\mathcal{H}(r) \quad , \quad f(\bar{r}) = -\left(\frac{\Omega'(r)}{\Omega(r)}\right)^2 \mathcal{H}(r).$$

Advantage: this new form of the BH metric ***simplifies a lot*** the field equations

Equations of motion in Schwarzschild coordinates

$$f''(\bar{r}) =$$

$$h^{(3)}(\bar{r}) =$$



Equations of motion in Kundt coordinates

We reduce the field equations to a simpler system of *two equations for two metric functions* $\mathcal{H}, \Omega$. Moreover, the *conformal to Kundt* geometry generates an *autonomous system*

$$\mathcal{H}^{(3)} = \frac{G^2\,\lambda\,(3\mathcal{H}'\Omega' + \Omega(\mathcal{H}'' - 1))\,(2 + \mathcal{H}'')^2 - 54\mathcal{H}\Omega^3(\Omega')^2 - 18\mathcal{H}'\Omega'\Omega^4 - 18\Omega^5}{3G^2\,\lambda\,\Omega\,\mathcal{H}'\,(2 + \mathcal{H}'')}$$

$$\Omega'' = \frac{G^2\,\lambda\,(2 + \mathcal{H}'')^3 - 18\Omega^4(2 + \mathcal{H}'') - 108\Omega^3\Omega'\mathcal{H}'}{108\mathcal{H}\Omega^3}\,.$$

Local solutions to the field equations

B. Holdom '06
C. De Rham, J. Francfort & J. Zhang '20
B. Knorr & A. Platania '22

- The asymptotic region

$$f(\bar{r}) = 1 - \frac{2GM}{\bar{r}} + \frac{72M^2\,G^2\,\lambda}{\bar{r}^6} - \frac{128M^3\,G^2\,\lambda}{\bar{r}^7} + \mathcal{O}(\lambda^2)$$
$$h(\bar{r}) = 1 - \frac{2GM}{\bar{r}} + \frac{24M^2\,G^2\,\lambda}{\bar{r}^6} - \frac{32M^3\,G^2\,\lambda}{\bar{r}^7} + \mathcal{O}(\lambda^2)$$

- Asymptotically flat solution
- Corrections start at $\bar{r}^{-6}$
- Power series converges in the exterior

Local analysis at spatial infinity

- The asymptotic region

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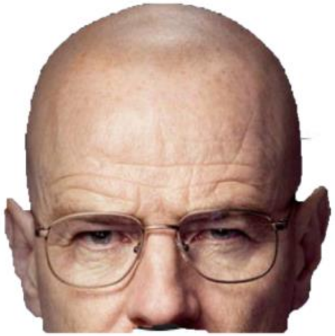
Remarkably, the inclusion of the Goroff-Sagnotti counterterm *does not generate quantum hair -* asymptotically flat, static, spherical solutions are still determined *in terms of a single field parameter*



Local analysis at the horizon

The [n,p] = [0,1] (and [n,p] = [0,2]) family: Near the horizon

From $\mathcal{H}(r) = (r - r_0)^p \, (b_0 + b_1 \, (r - r_0) + \ldots)$ we observe that

the class [n,p] = [0,1] has $\mathcal{H}(r) \sim b_0 \, \Delta + \mathcal{O}(\Delta^2)$ with a single root
the black hole has a horizon at $r_h = r_0$

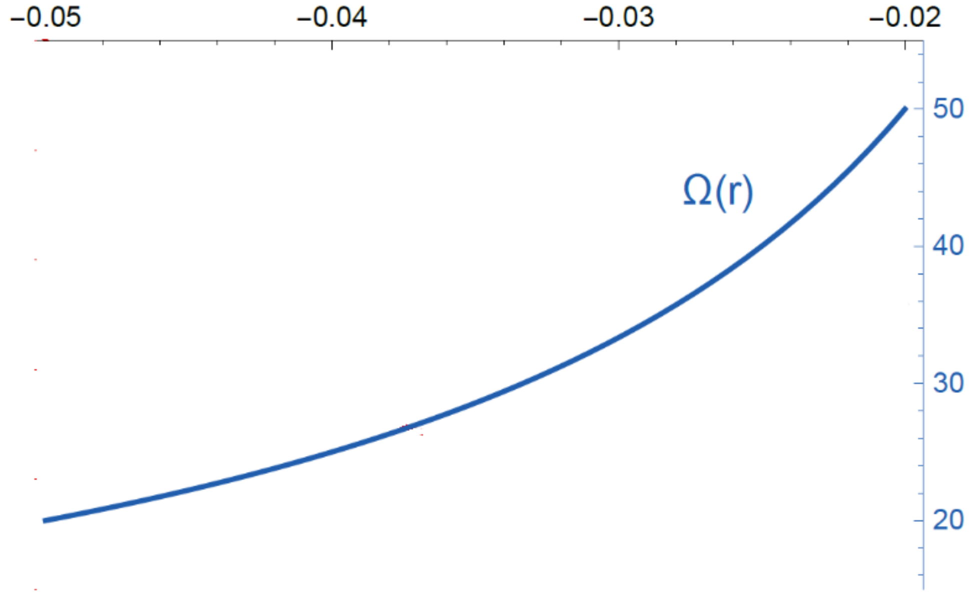
the class [n,p] = [0,2] has $\mathcal{H}(r) \sim b_0 \, \Delta^2 + \mathcal{O}(\Delta^3)$ with a doble root
the black hole has an extreme horizon
but it has 0 free parameters

Numerical integration

Numerical integration

Behaviour of the metric functions

typical functions $\Omega(r)$ and



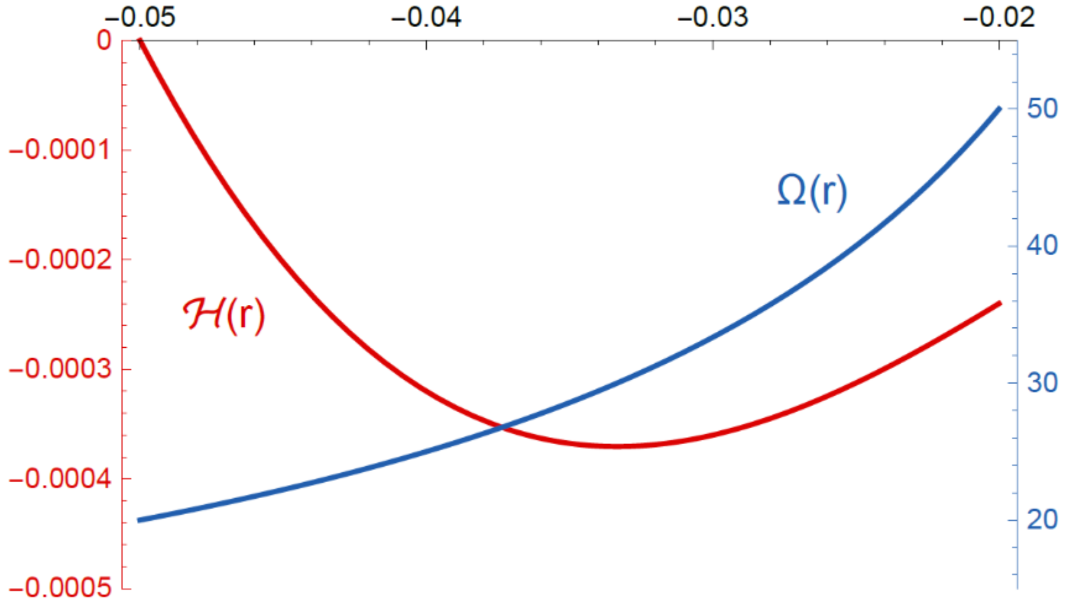
$r_{\text{ini}} = -0.002$
$M = 10$
$\lambda = 0.1$

Recall that $\mathcal{H}(r) = 0$ identifies the *black-hole horizon* at r_h

Numerical integration

Behaviour of the metric functions

typical functions $\Omega(r)$ and $\mathcal{H}(r)$



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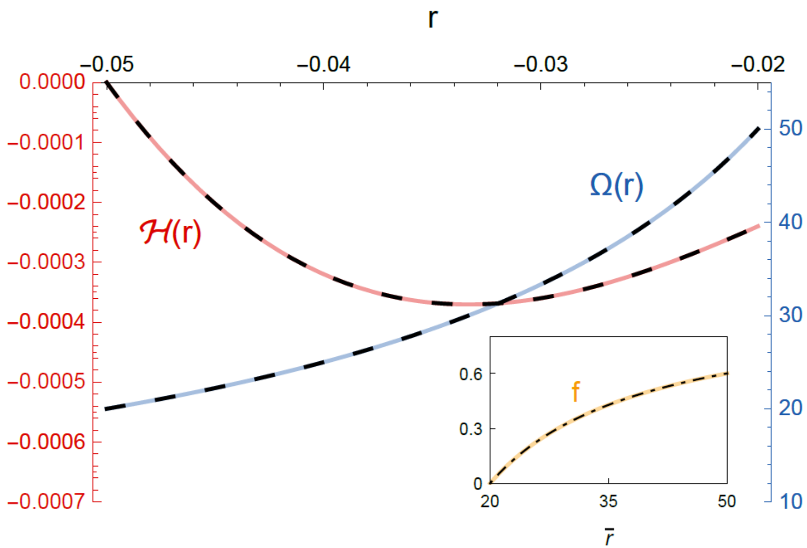
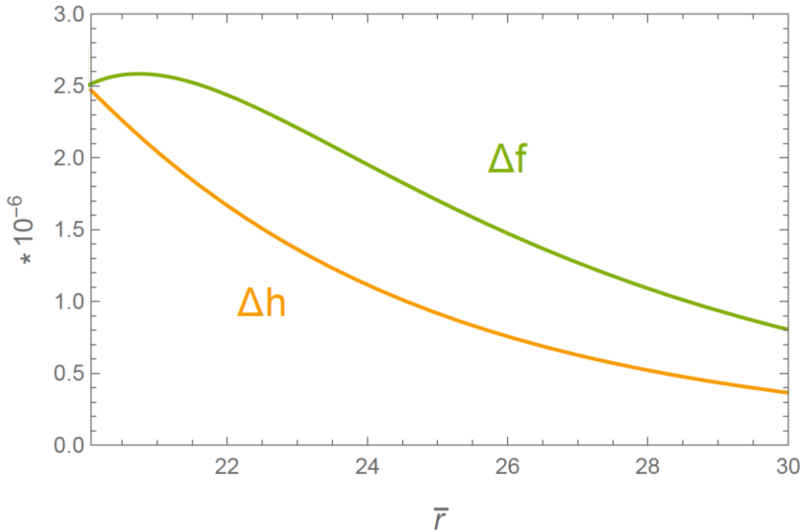
Numerical integration





Our numerical integration follows Schwarzschild solution closely

Local analysis at the core

B. Holdom '02, arXiv:020219

- Local analysis of solutions at $\bar{r} = 0$:
  - Analysis is very complicated!
  - Non-linearity of equations lead to many solutions branches
  - Preliminary result:

All curvature invariants are finite

Observational features

- Thermodynamics

The temperature of the BH is defined as

$$T = \frac{\kappa}{2\pi}$$

Surface gravity in conformal-to-Kundt coordinates

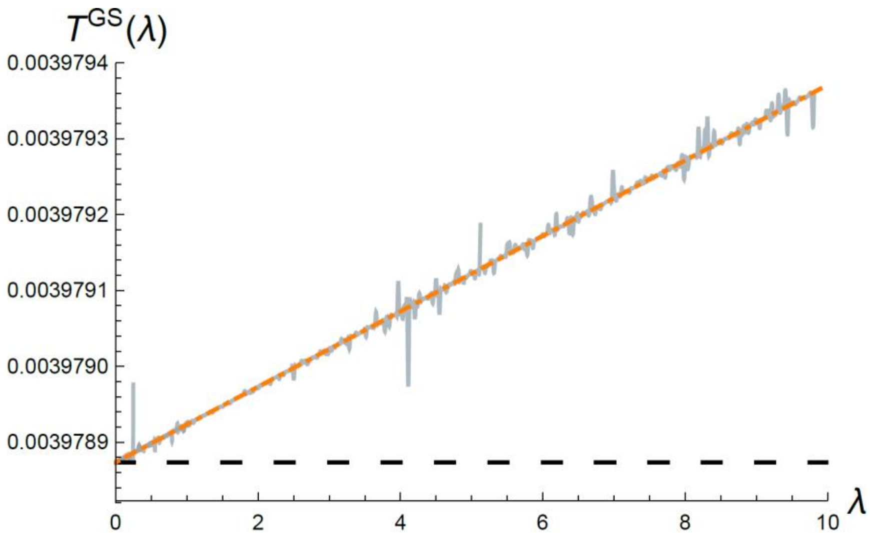
$$\kappa = -\frac{1}{2}\left(\mathcal{H}' + 2\,\mathcal{H}\,\frac{\Omega'}{\Omega}\right)\Big|_{r=r_h}$$

$$\longrightarrow \quad \boxed{T = -\frac{1}{4\pi}\,\mathcal{H}'(r_h)}$$

Observational features

- Thermodynamics

$$T = -\frac{1}{4\pi}\mathcal{H}'(r_h)$$



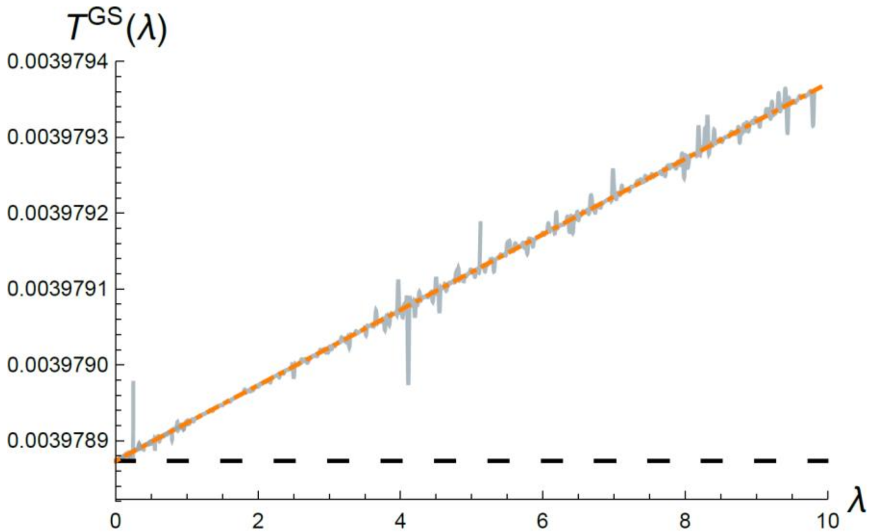
For $\lambda > 0 \ \rightarrow \mathrm{T}(\lambda) > T_{Sch}$

For $\lambda > 0 \ \rightarrow r(\lambda) < r_{Sch}$ Lu & Lyu '20

Observational features

- Thermodynamics

$$\mathrm{T}(\lambda) > T_{Sch}$$



For $\lambda > 0 \ \rightarrow \mathrm{T}(\lambda) > T_{Sch}$

For $\lambda > 0 \ \rightarrow r(\lambda) < r_{Sch}$ Lu & Lyu '20

Observational features

Wald '93
Iyer & Wald '94

- Entropy

Wald entropy formula

$$S_W = -\frac{1}{8}\int_+ d^4x\sqrt{h}\,\frac{\partial L}{\partial R^{abcd}}\epsilon^{ab}\epsilon^{cd}$$

Wald formalism

$$\delta H_+ = T\delta S$$

Observational features

- Entropy

Wald entropy formula

$$S_W = -\frac{1}{8}\int_+ d^4x\sqrt{h}\,\frac{\partial L}{\partial R^{abcd}}\epsilon^{ab}\epsilon^{cd}$$

Applied to our case

$$S_W(\lambda) = \frac{A}{4G}\left[1 + \frac{\lambda G}{6\bar{r}_h^2}\left(2+\mathcal{H}''\right)^2\right]$$

- An explicit expression
- If $\lambda = 0$ one recovers the standard expression for Schwarzschild BHs.
- For smaller black holes the deviation from $S = \frac{A}{4G}$ is larger
- Applications of Wald formalism $\delta H_+ = \delta H_\infty$ to (re)derive the first law of thermodynamics

Fan & Lu '15
Feng, Liu, Lu & Pope '15

Conclusions

- Key results

New class of spherically symmetric BH solutions generated by the inclusion of the Goroff-Sagnotti counterterm that expels Schwarzschild from the solution space

Asymptotically flat, static and spherically symmetric solutions are still determined in terms of the asymptotic mass.

$$T(\lambda) > T_{Sch} \;\&\; r(\lambda) < r_{Sch}$$

- Present and future work

Extension of the numerical integration below the horizon

Further studies of thermodynamic properties of this black hole (entropy, free energy, specific heat)

Including angular momentum?

Thank you for your attention!