

STRING THEORY AND LOOP QUANTUM GRAVITY: TRUE THEORY VS “AD HOC” HYPOTHESIS?

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It all starts with Einstein's eq. of General Relativity (GR):

$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{Geometry (GR, gen. covariance)}} = \underbrace{\kappa T_{\mu\nu}(g)}_{\text{Matter (Stand.model, QT)}}$$

maybe the most inconsistent one of the modern physics!
Purely classical on the left, essentially quantum mechanical on the right!

To resolve the inconsistency extend quantum theory to spacetime geometry, that is to gravity (thing we attempt to do since more than a hundred years!).

Almost all the attempts have led to **incurable divergences** at various orders of perturbative development.

String Theory (ST) for the first time led to regular quantization showing a massless level of spin=2 in the foreseen mass spectrum of the strings, but in a spacetime with at least 10+1 dimensions!

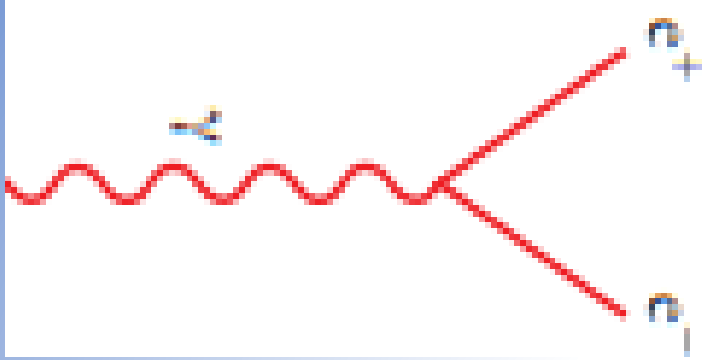
The second successful attempt was suggested by Lee Smolin and Carlo Rovelli, giving rise to **Loop Quantum Gravity** (LQG), which remains bound to the old well known 4-dimensional spacetime.

These just correspond to two of the three roads suggested by Lee Smolin in his book “Three Roads to Quantum Gravity” [1]: ST is the attempt to extend quantum field theory to gravity, LQG on the contrary starts from classical GR and tries to quantize it (so it is called also Quantum General Relativity).

String Theory started in 1968 with an intuition of Gabriele Veneziano, then young researcher at CERN, Geneve, who introduced an Euler Beta function [2] for the transition amplitude in particle collisions. Leonard Susskind found and shew that Veneziano's formula was just describing the movement of a collections of quantum oscillators and proposed the idea that elementary particles should be considered the external aspect of a vibrating microscopic string, open or closed.

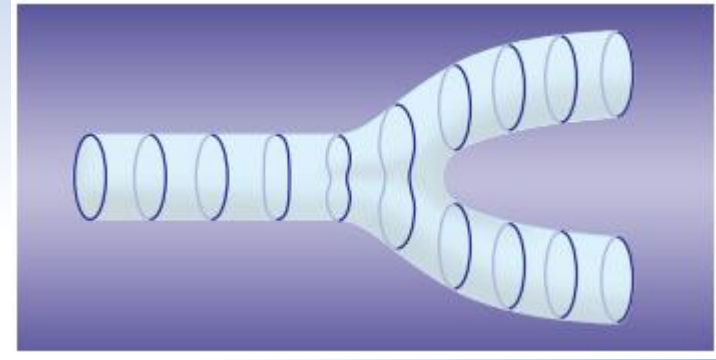
The first results of the replacing point-like particle fields with one-dimensional string fields was the disappearance of the big bang singularity. All matter in the universe could contract no longer at a singular point, but at a minimum along an extended line, so maintaining a finite extension at finite temperature.

It soon turned out that the zero-mass level in closed strings was associated with a tensor field representing a massless particle of spin = 2 which was identified as the graviton. The first step towards quantum gravity had been made starting from a quantum theory.



$$\int f d\tau$$

Fig. 1

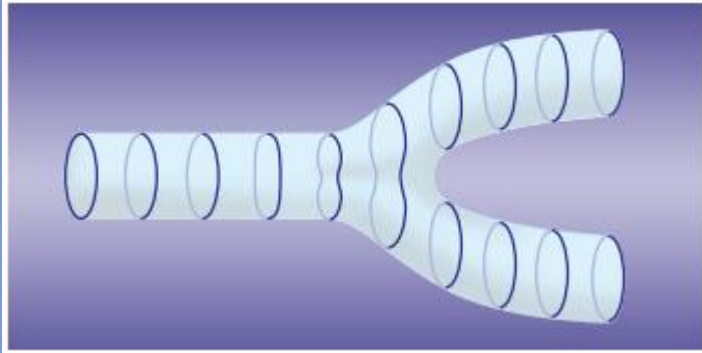


$$\iint g d\tau d\sigma$$

Origin of mathematical difficulties in string theory:

on the left the Feynman diagram relating to the doubling of a gamma ray in an electron-positron pair (in the language of field theory, that is for point particles);

on the right the same diagram, but in the language of closed strings: all world lines are replaced by world sheets; this means changing old line integrals to surface integrals with the consequent much greater mathematical difficulty.



Remember the appearance of the world sheet of the closed string, one of the states of which represents the graviton: we will find a similar appearance in the world sheets of loop quantum gravity

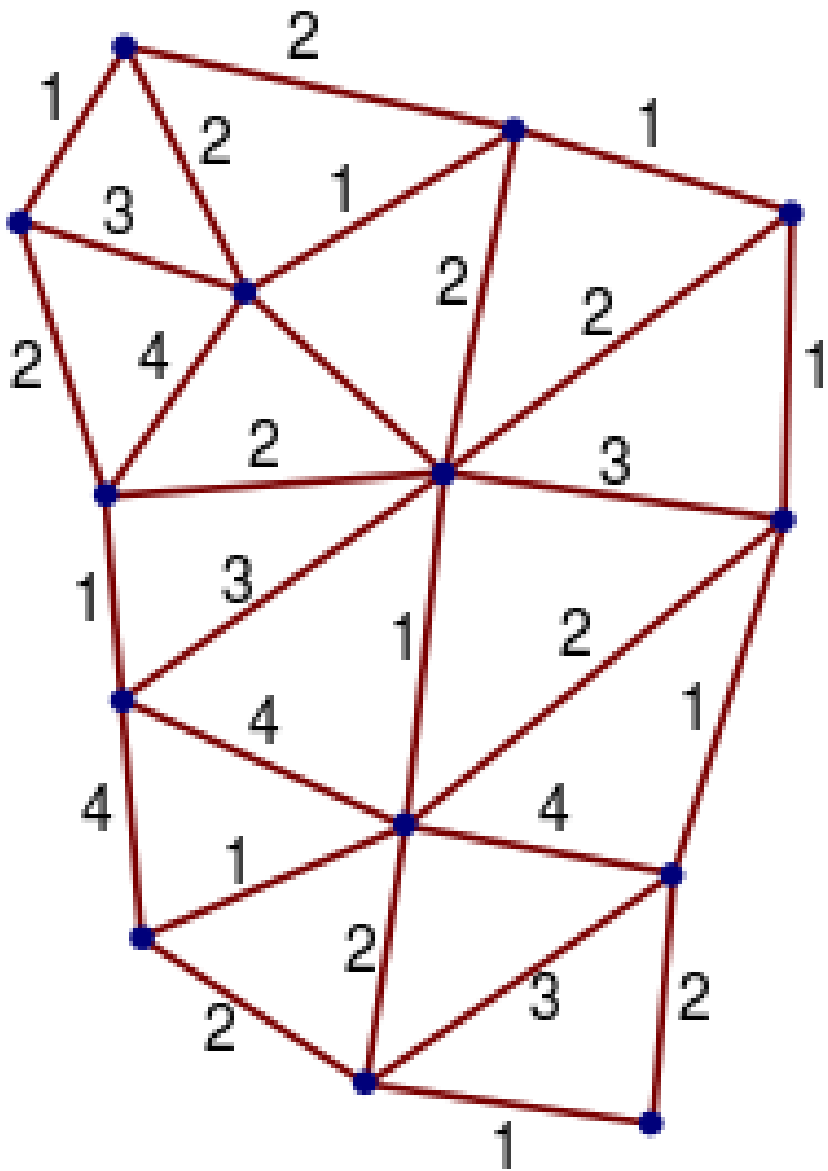
String theory has become, over years, a major field of research. After two revolutions, it took the form of the M theory, the theory of superstrings with 10 spatial dimensions (7 of which are hidden) in addition to time.

It seems to give satisfactory solutions to:

- i. the big bang explosion with inflation scenario (via a big crunch from near zero to maximum spatial curvature and subsequent expansion back to near zero curvature, see figure; the maximum curvature is $2\pi/\ell_{st}$, very big, but finite); $\ell_{st} \sim \ell_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} = 1,6 \cdot 10^{-33}$ cm is the length of a string;
- ii. the black hole information paradox.

Direct competitor of string theory is the one that attempts gravity quantization starting from the classical formulation given by general relativity. This attempt is not simple. While in the case of electromagnetic and nuclear fields the space in which the interactions take place is given and fixed (the background), in the case of gravitational field it is itself the space that in which the interactions take place. This causes problems with quantization, which is actually the quantization of space-time itself with no background.

The first important step was made in 1986 by A. Ashtekar, who introduced the conjugate variables that allowed to rewrite the conservation laws and Einstein's equation in a simple and linearizable form. **Position coordinates** are essentially **angles** while **conjugate momenta** are essentially **spinors**, so that **the basic states** of quantum gravity are **spinors** which, intersecting in various ways, form **spin networks** (introduced by R. Penrose in 1995).



Simple spin network of the type used in loop quantum gravity

Basic spinors tend to form closed curves, that is loops. So the name given to the proposed theory: **Loop Quantum Gravity (LQG)**. Graphs like the one in the figure can be deformed at will, replacing straight segments with curves or rolling the figure on itself in three dimensional space, as long as the intersection nodes remain identical (with two, three or more spinors intersecting there): this is precisely the **diffeomorphism invariance**.

To move to spacetime we need to see the evolution of the network over time. This is represented in the next figure: the tube produced over time by the spin network is now an element of the spin foam; this foam is the true representation of quantum gravity in spacetime. In the foam can be represented dynamic transitions between states of quantum gravity.

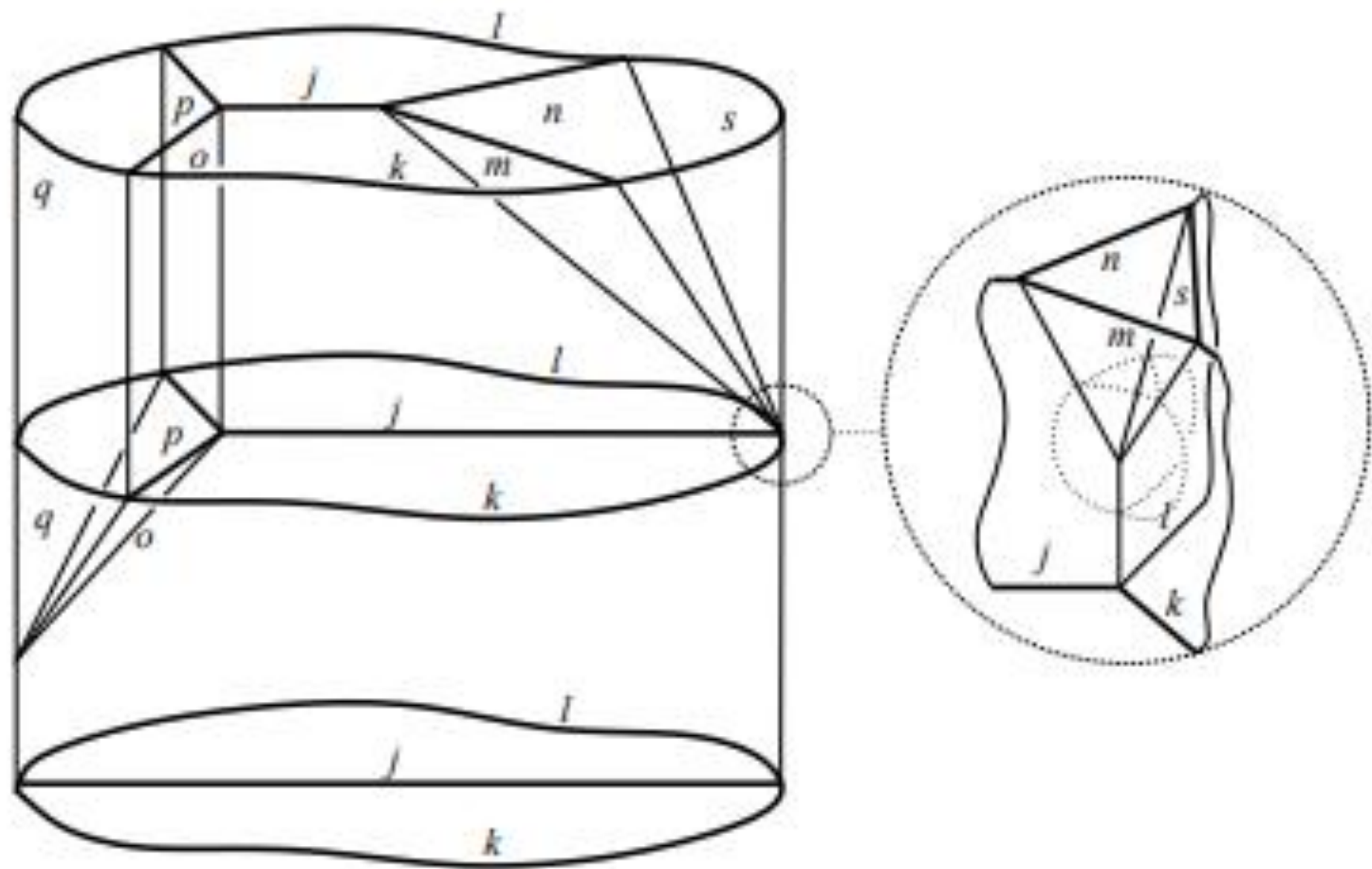
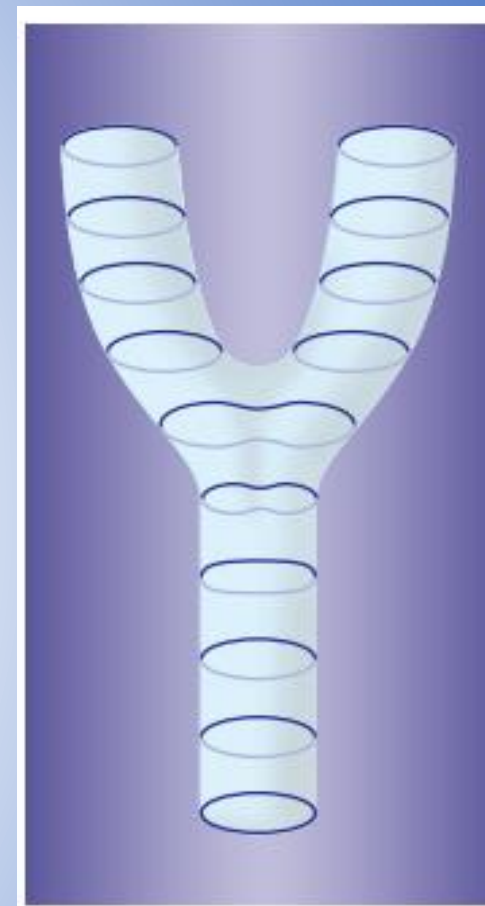
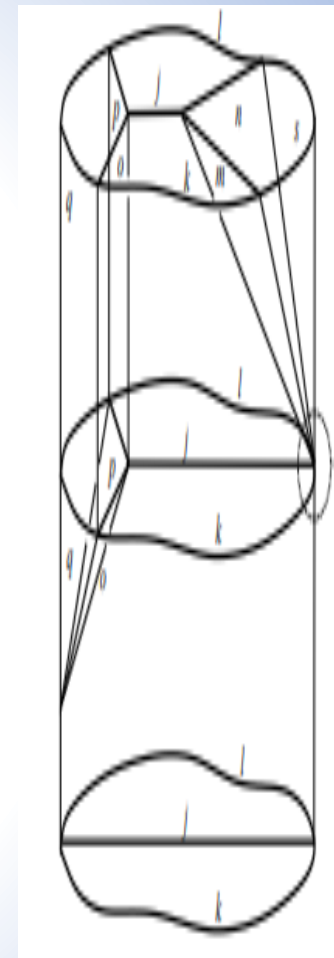


Fig. 11. A *spin foam* as the 'colored' 2-complex representing the transition between three different *spin network* states. A transition vertex is magnified on the right.

Note please the similarity between an element of spin foam and the world sheet of a string. It seems that both theories lead to the same or at least similar results.

This is true for the graviton in strings and quantum gravity, for the information paradox in black holes and probably also for the development of the universe since before the big bang, especially since the maximum curvature achievable in quantum gravity is just the same as that in string theory ($2\pi/l_{\text{Planck}} = 3.9 \times 10^{33} \text{ cm}^{-1}$, big, but finite).



The fact that both theories give similar results reassures us of their ability to produce the quantization we hoped to achieve. But it makes harder for us to discriminate them through experiments.

In fact there is **only one big difference between the two paths to quantum gravity: the number of dimensions of the space in which they operate**, at least 10 in the case of (super)strings, only 3 in the loop quantum gravity. Only when we have devised an experiment capable of giving us the true number of spatial dimensions (rolled up or not) we will be able to prove the truth of one of the two competing theories. As an experimental physicist I suggest investigating in this direction.

And in conclusion I say that the two theories have equal dignity for me (obviously with the addition of the whole standard model to the loop quantum gravity and that I prefer the simpler one, that with 3+1 dimensions of spacetime even if only the experimental results have the right to confirm my inclinations.

Thank you for
your patience