Rotational holographic transport in AdS/CMT

4th FLAG WORKSHOP: The quantum and gravity

Sept 9-11, 2024

Catania

Pedro Meert

AdS/CMT

AdS/CFT correspondence

Duality between gravity theory in D+1 dimensions and CFT (without gravity) in D dimensional space-time.

Couplings are inversely proportional:

strong gravity ↔ weak field theory

weak gravity ↔ strong field theory

Verified within string theory, canonical example

$$AdS_5 \times S^5 \leftrightarrow \mathcal{N} = 4 \text{ SYM}$$

AdS/CMT

More generally referred to as gauge/gravity duality. Essentially an extrapolation of the AdS/CFT correspondence, as we don't know one side of the theory.

(Conjectures that) Weakly coupled gravity at low energy, e.g. General Relativity, in asymptotically AdS background is dual to a strongly coupled approximately* conformal field theory in one lower dimension.

Black hole in the bulk brings the theory at the boundary to a non-zero temperature.
The thermodynamics of the BH and field theory are the same.

*The theory becomes conformally invariant when the bulk is AdS.

Holographic superconductor

Holographic Superconductors

Sean A. Hartnoll[♭], Christopher P. Herzog[♯] and Gary T. Horowitz[♮]

PHYSICAL REVIEW D **78,** 065034 (2008)

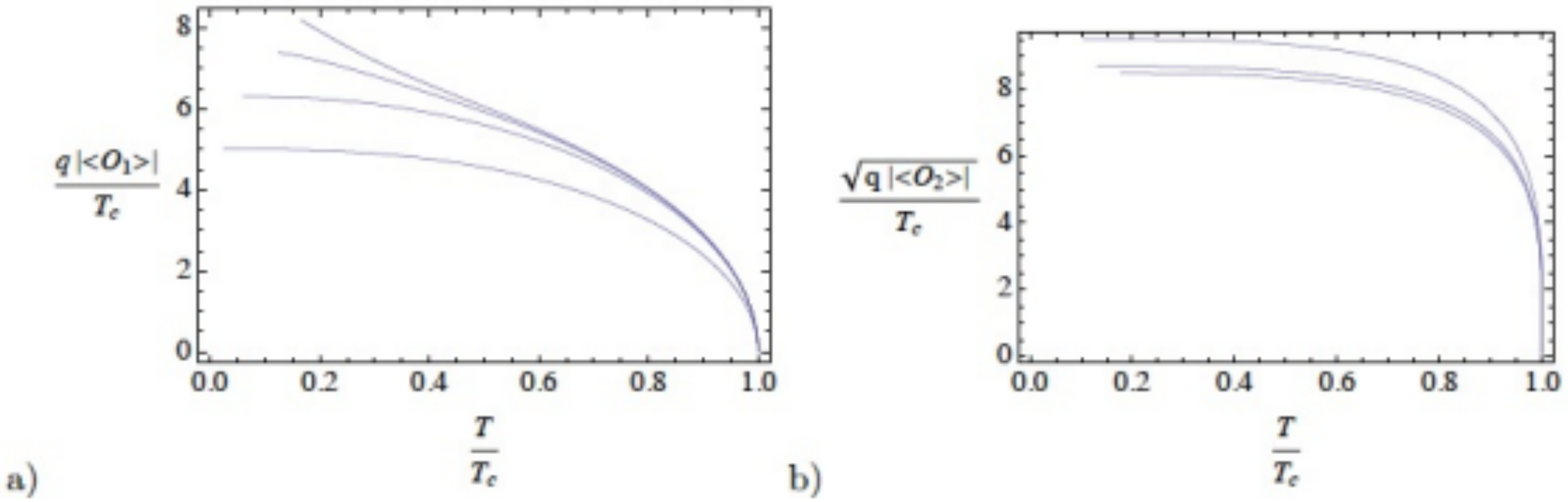
Breaking an Abelian gauge symmetry near a black hole horizon

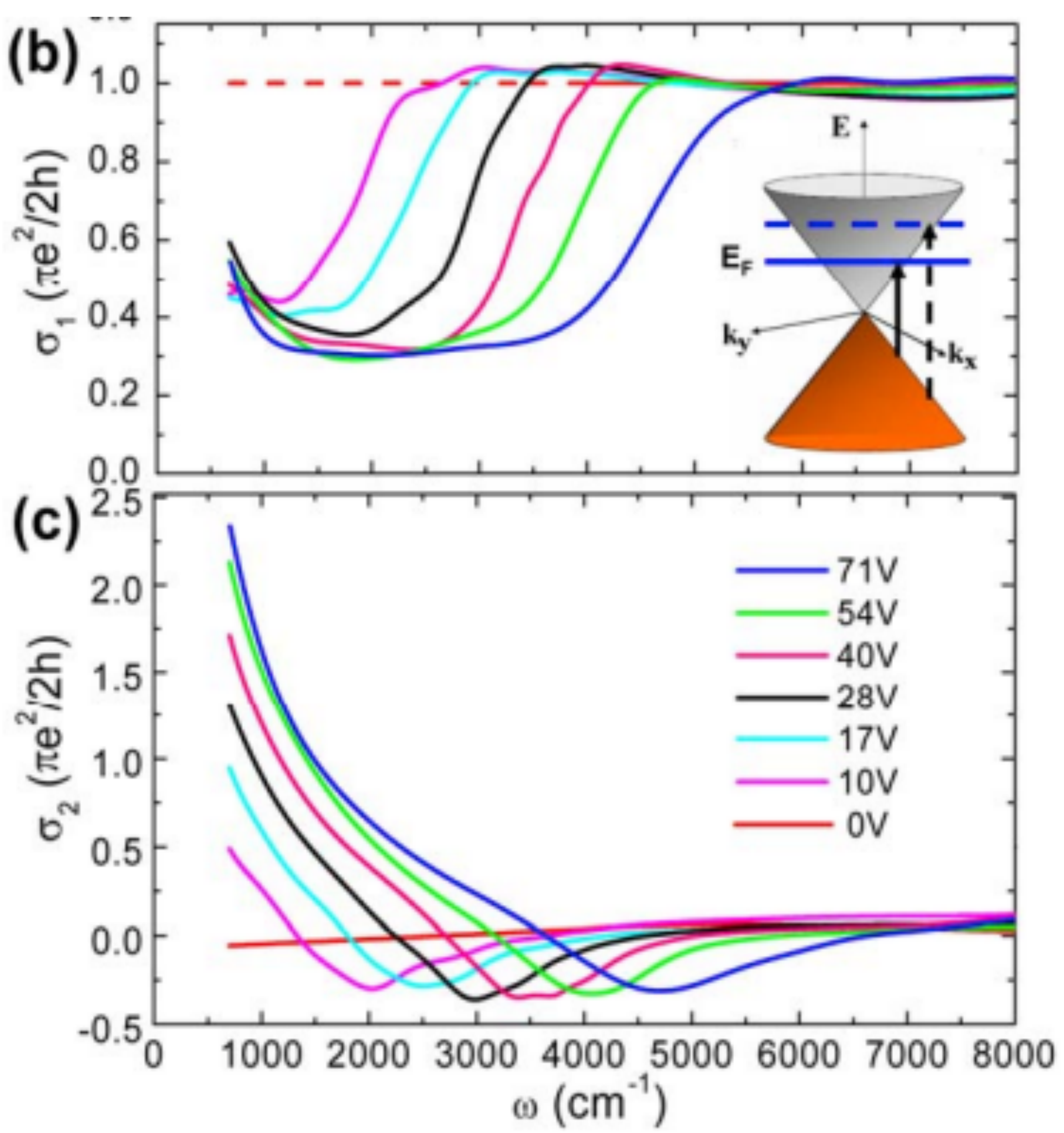
Steven S. Gubser

Lectures on holographic methods for condensed matter physics

Sean A Hartnoll

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2 - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2 - V(|\phi|)$$





Graphene conductivity experiment
Nature Physics 4, 532 (2008)

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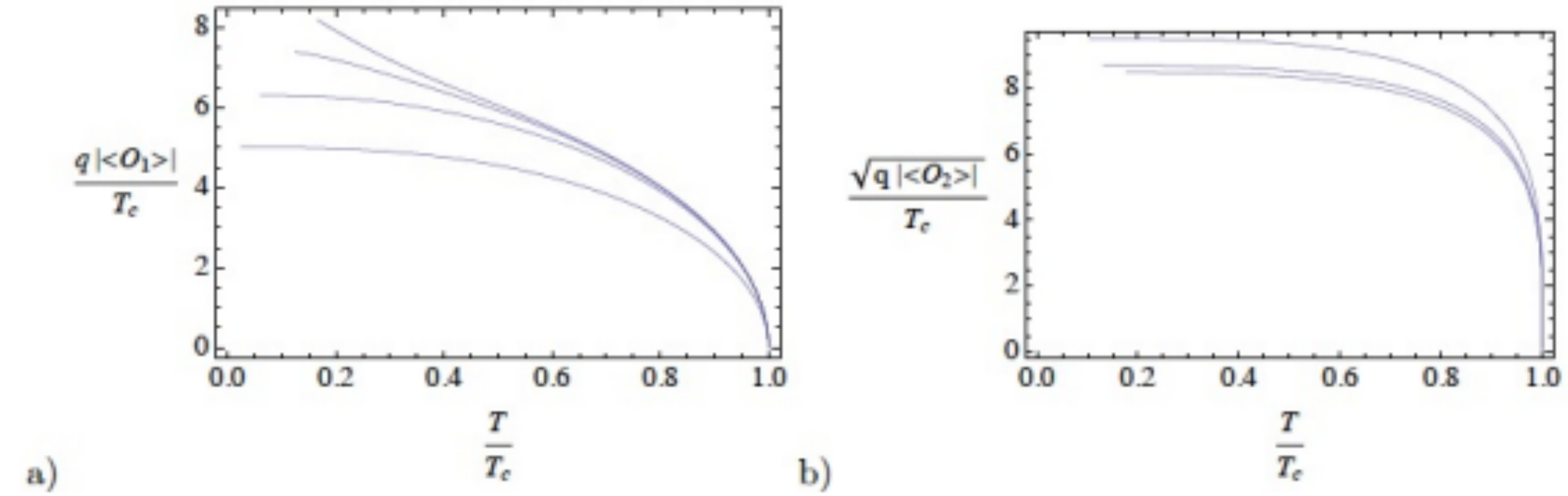

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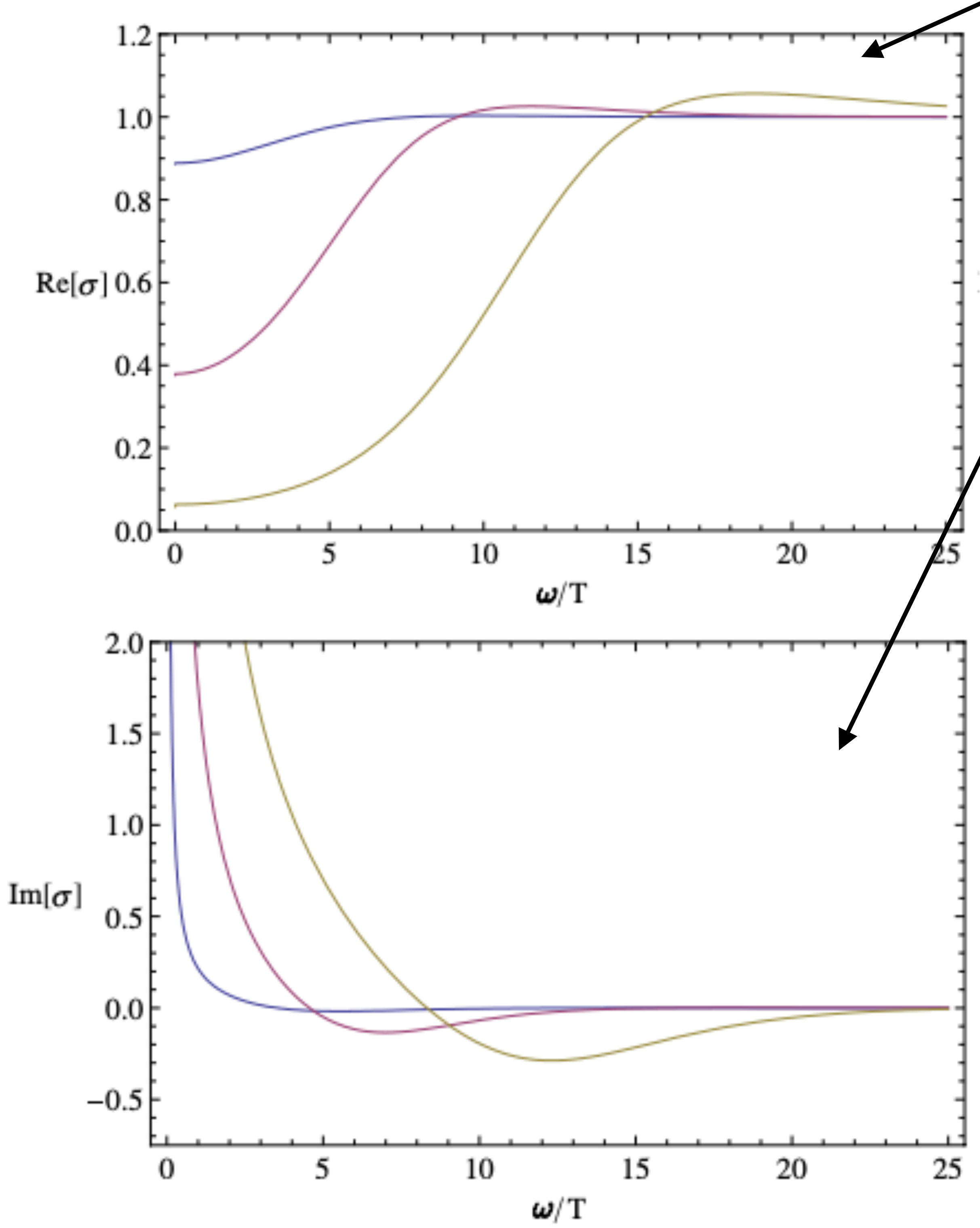

Conductivity calculated from holographic model

Lectures on holographic methods for condensed matter physics

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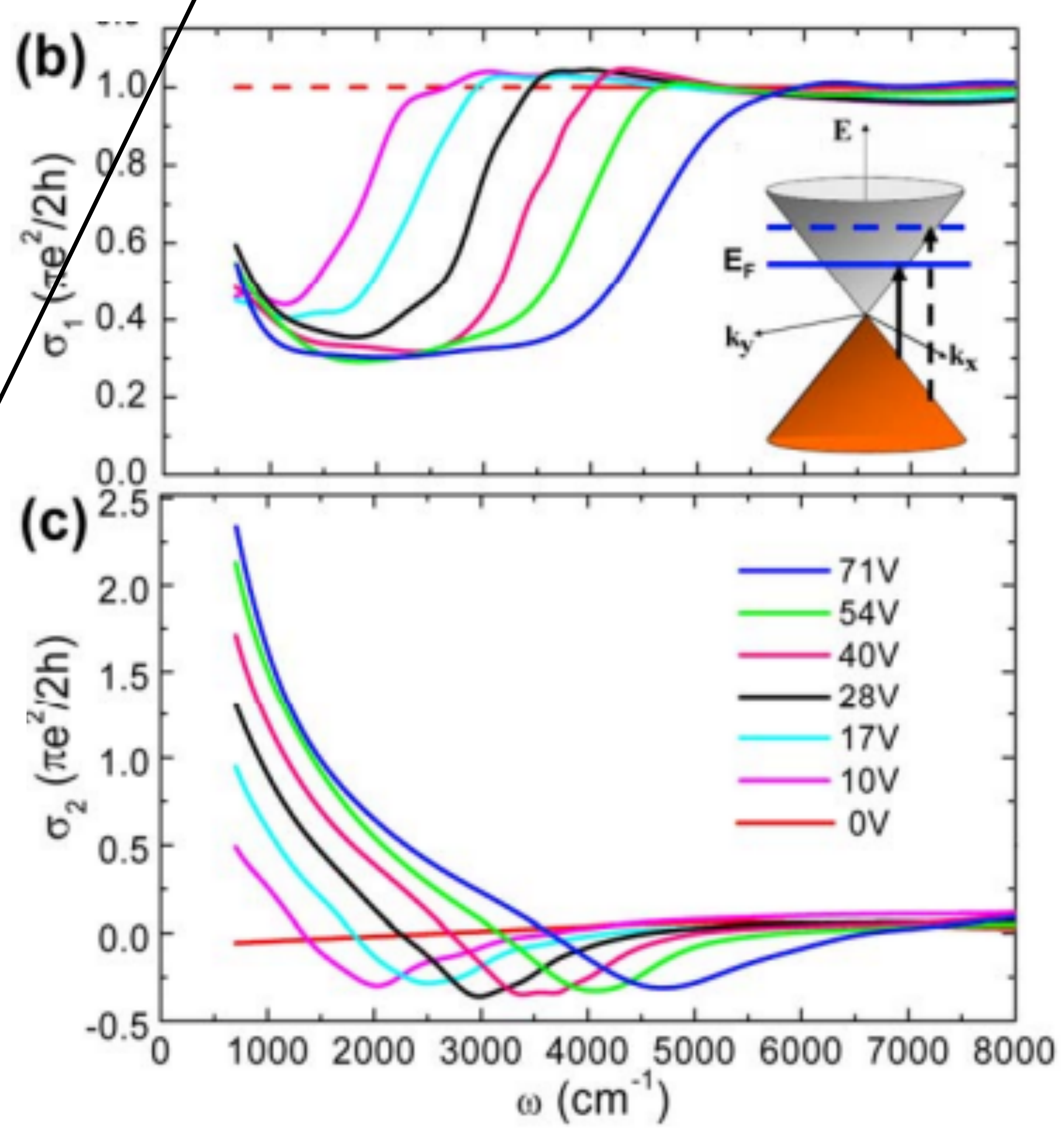

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Holographic superconductor

General model

$$\mathscr{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4g^2}F^2 - \left|\left(\nabla - iqA\right)\phi\right|^2 - m^2\left|\phi\right|^2$$

- Negative cosmological constant, $d = 3$, $V = 0$.
- Scalar field ϕ is associated with the order parameter, distinguishing the normal and superconducting phases.
- The electromagnetic field introduces another scale, setting the critical temperature. It is also used to compute the conductivity as it describes a conserved current.

Holographic superconductor

Probe approximation: use gravity as background.

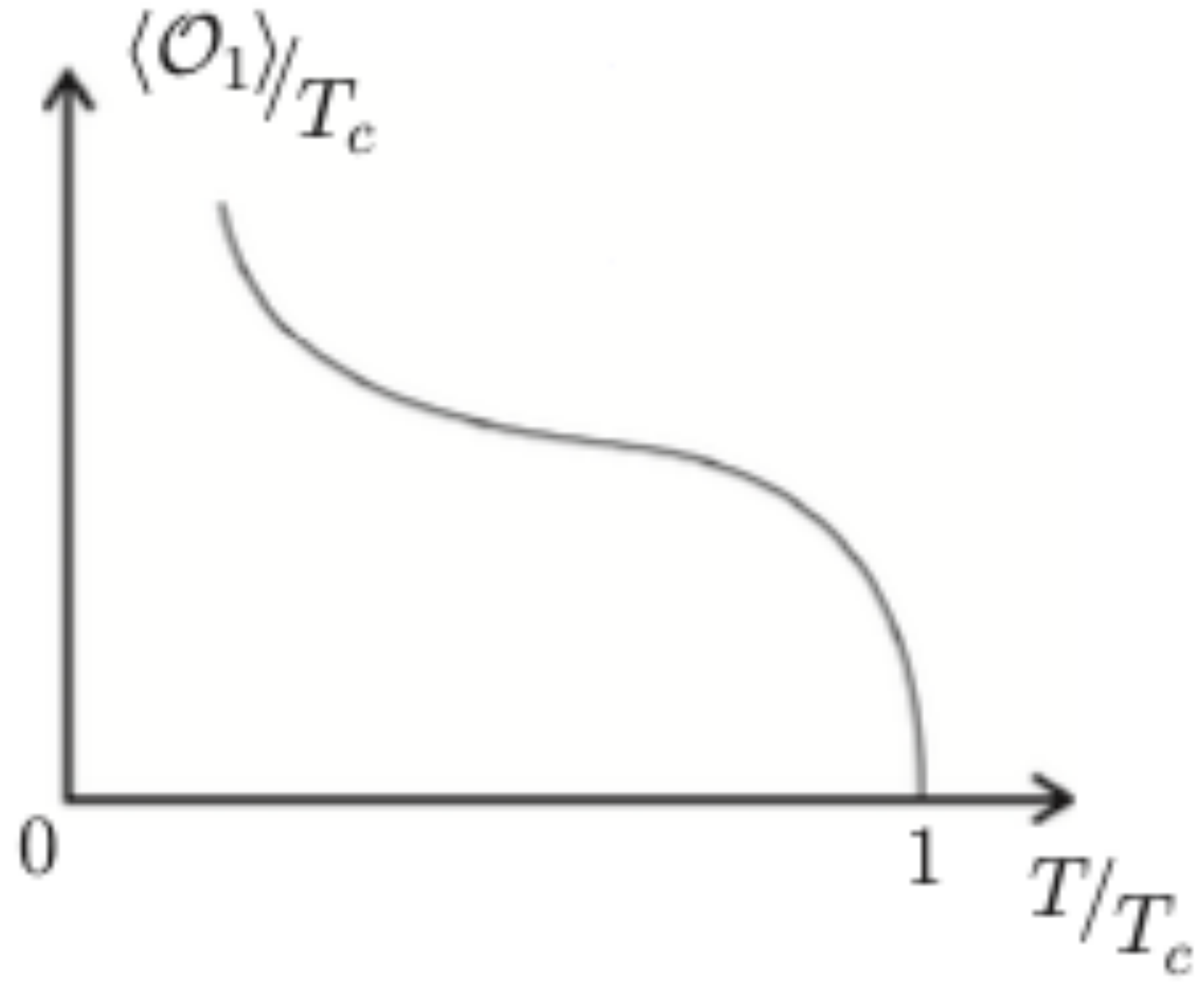
Generically, the asymptotic form of the solutions read:

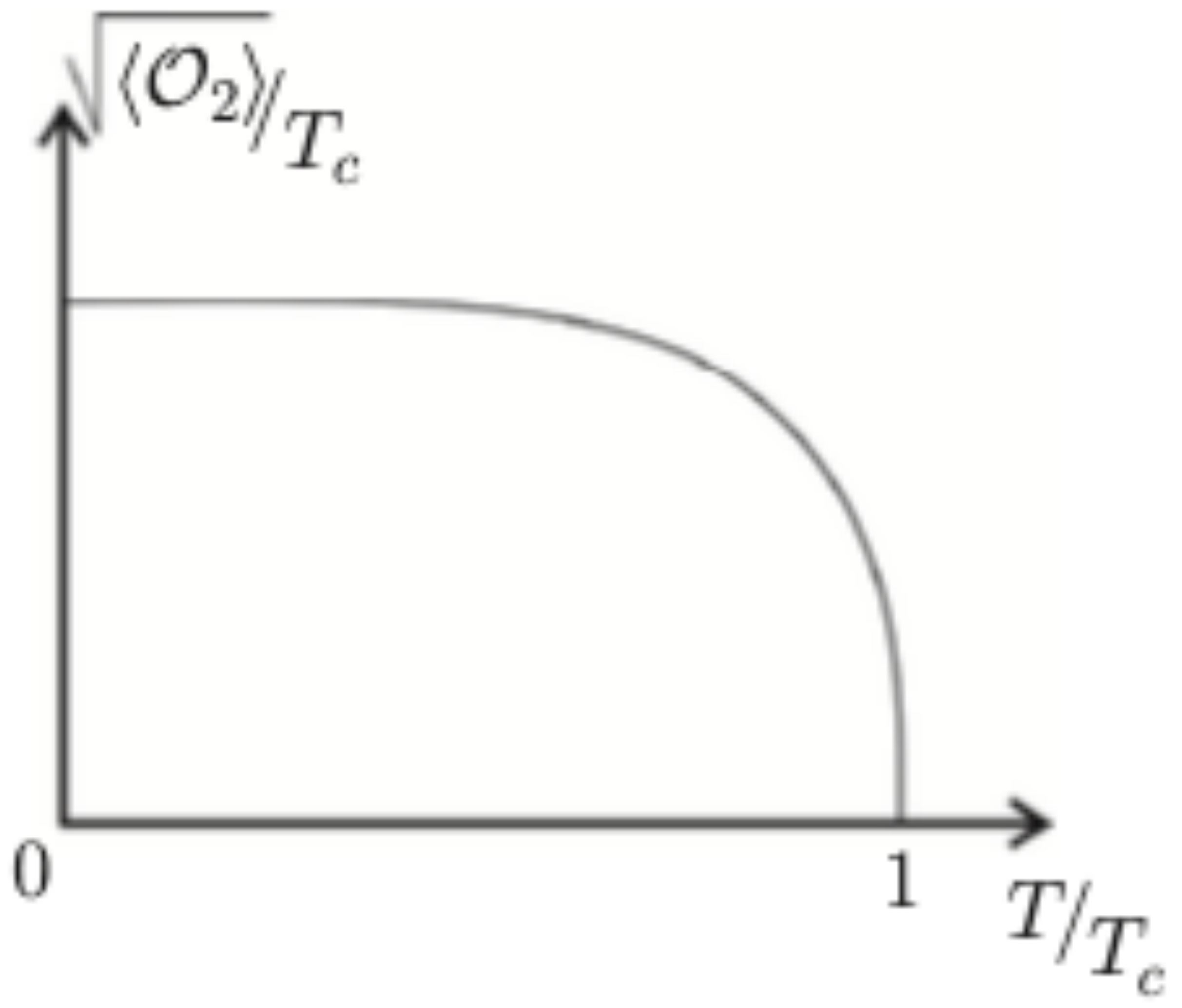
$$\phi(r) \simeq \frac{\phi^{(1)}}{r} + \frac{\phi^{(2)}}{r^2} + \dots$$

VEVs for operators (condensates) $\langle \mathcal{O}_i \rangle \sim \phi^{(i)}$

$$A_0(r) \simeq \mu - \frac{\rho}{r} + \dots$$

Chemical potential and charge density $\mu \sim A_0^{(0)}\,,\ \rho \sim A_0^{(1)}$





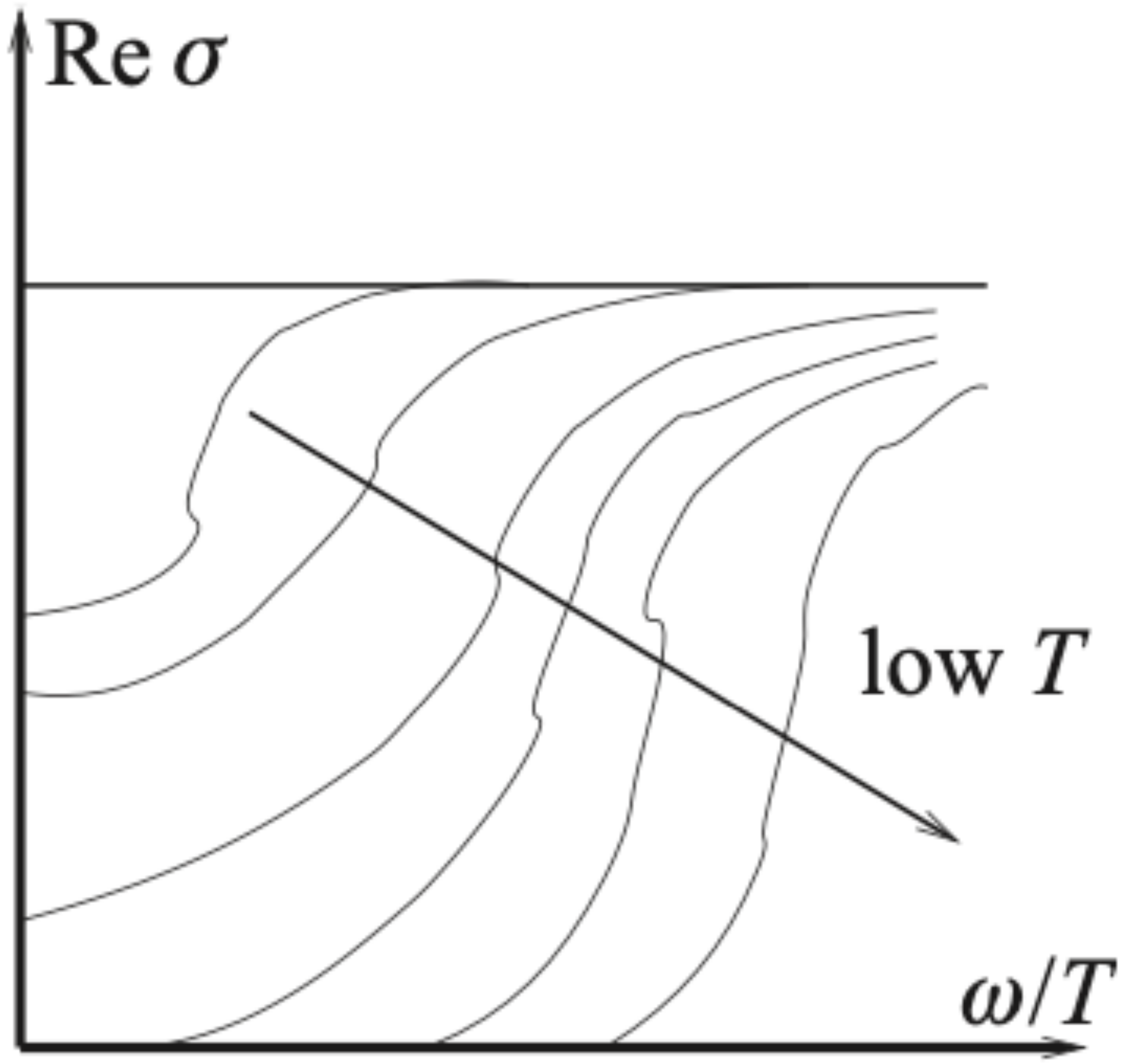
Holographic superconductor

Conductivity:

Introduce the perturbation $\delta A_x = \delta A_x(r)\, e^{i\omega t}$

Asymptotic solution form $A_x = A_x^{(0)} + \frac{\langle J_x \rangle}{r} + \ldots$

Linear response for electric current $\sigma(\omega) = \frac{\langle J_x \rangle}{i\omega A_x^{(0)}}$



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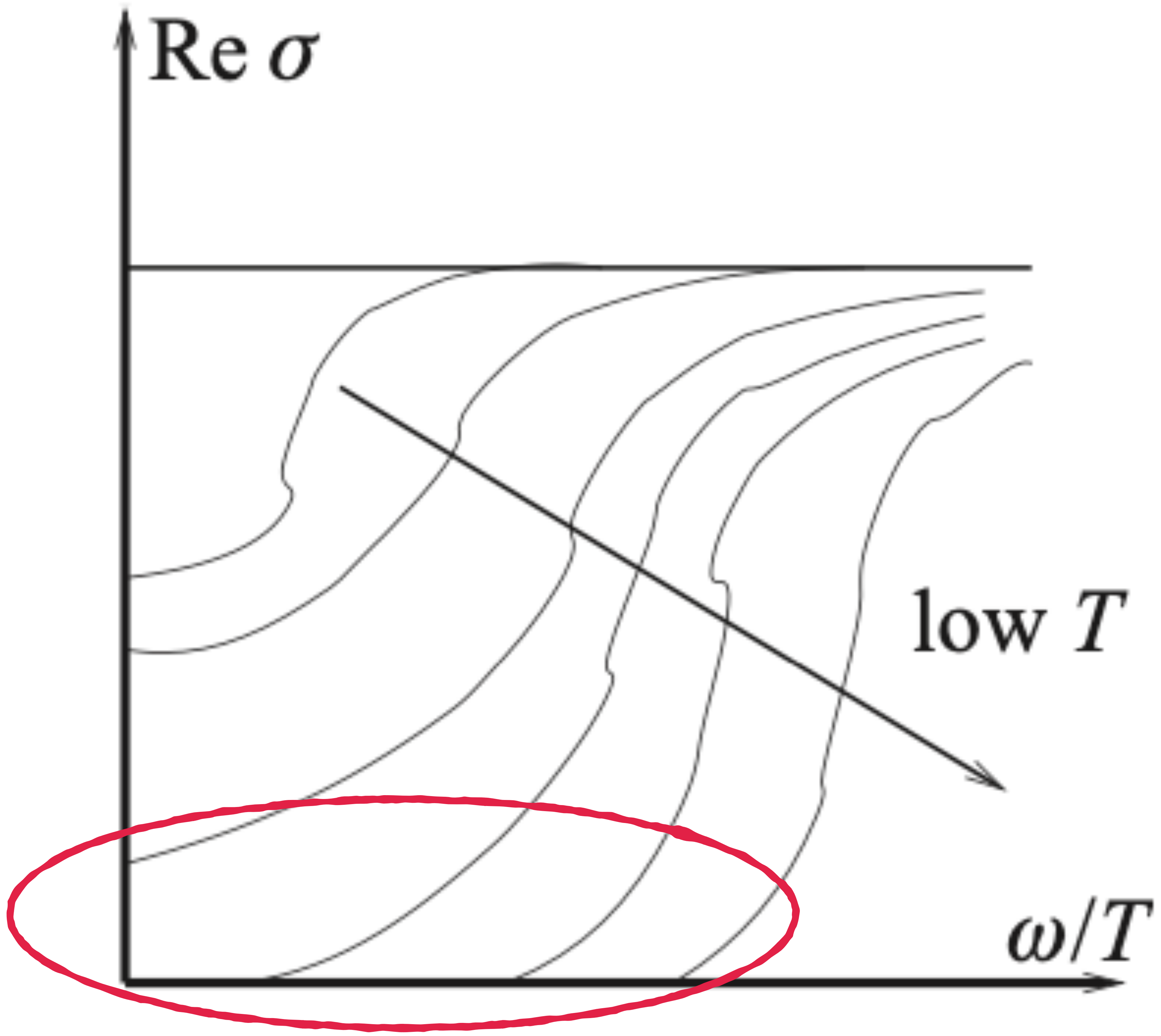
Gap: $\frac{\hbar\omega}{k_B T_C} \simeq 8.4$



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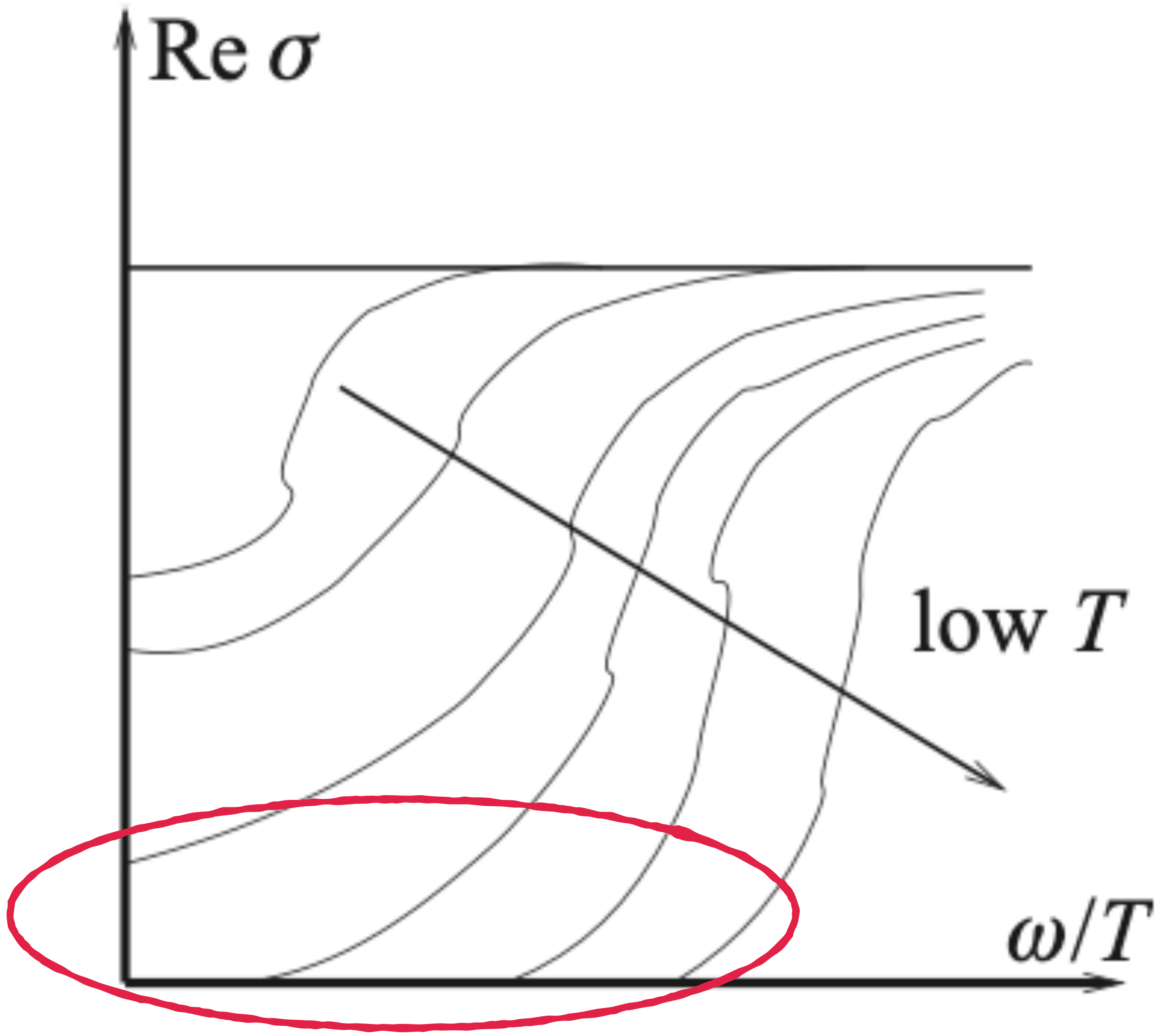
Gap for BCS: $\frac{\hbar\omega}{k_B T_C} \simeq 3.5$



Holographic superconductor

Summary:

- Superconducting instability (scalar field)
- Critical temperature (chemical potential)
- Conductivity (breaking translation invariance of gauge field)
- Energy gap ~ 8

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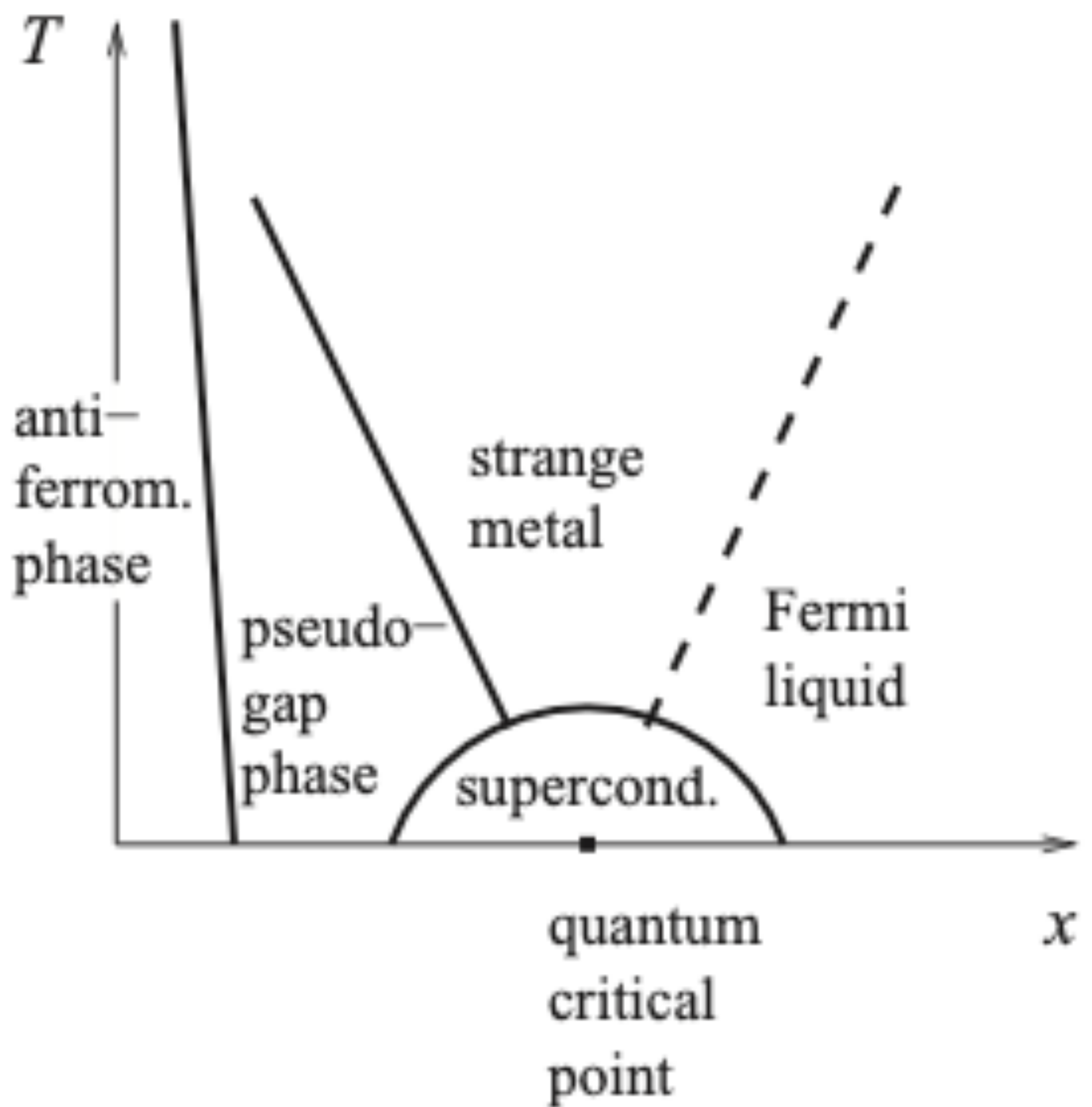
High T_C superconductor?



Rotating holographic SC

AdS-Kerr-Newman black hole: rotating, electrically charged black hole solution including the negative cosmological constant.

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left[dt - \frac{a}{\Xi}\sin^2\theta d\phi\right]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\sin^2\theta\Delta_\theta}{\rho^2}\left[adt - \frac{\left(r^2+a^2\right)}{\Xi}d\phi\right]^2$$

$$A = -\frac{Qr}{\rho^3}\left(dt - \frac{a}{\Xi^2}\sin^2\theta d\phi\right)$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta_r = \left(r^2+a^2\right)\left(1+l^{-2}r^2\right) - 2mr + Q^2$$

$$\Delta_\theta = 1 - l^{-2}a^2\cos^2\theta$$

$$\Xi = 1 - l^{-2}a^2$$

Mass m

Electric charge Q

AdS radius l

Angular momentum density a

Event horizon r_+ such that $\Delta_r\left(r_+\right) = 0$

Rotating holographic SC

AdS-Kerr-Newman thermodynamics:

Temperature

$$\beta \equiv T^{-1} = \frac{4\pi\left(r_+^2 + a^2\right)}{r_+\left(1 + a^2 l^{-2} + 3r_+^2 l^{-2} - \left(a^2 + Q^2\right) r_+^{-2}\right)}$$

Entropy

$$S = \frac{\pi\left(r_+^2 + a^2\right)}{\Xi}$$

Conserved charges

$$J = \frac{am}{\Xi^2} \qquad M = \frac{m}{\Xi^2} \qquad \mathcal{Q} = \frac{Q}{\Xi}$$

"Thermodynamic" angular velocity

$$\Omega = \lim_{r \to r_+} \omega - \lim_{r \to \infty} \omega = \frac{a\left(1 + r_+^2 l^{-2}\right)}{r_+^2 + a^2}$$

Thermodynamic potential

$$I = \frac{\beta}{4l^2\Xi}\left[-r_+^3 + l^2\Xi r_+ + \frac{l^2\left(a^2 + Q^2\right)}{r_+} + \frac{2l^2Q^2r_+}{\left(r_+^2 + a^2\right)}\right] = \beta G\left(T, \Omega, \Phi\right)$$ ←Gibbs free energy

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$\neq 0$ in AdS

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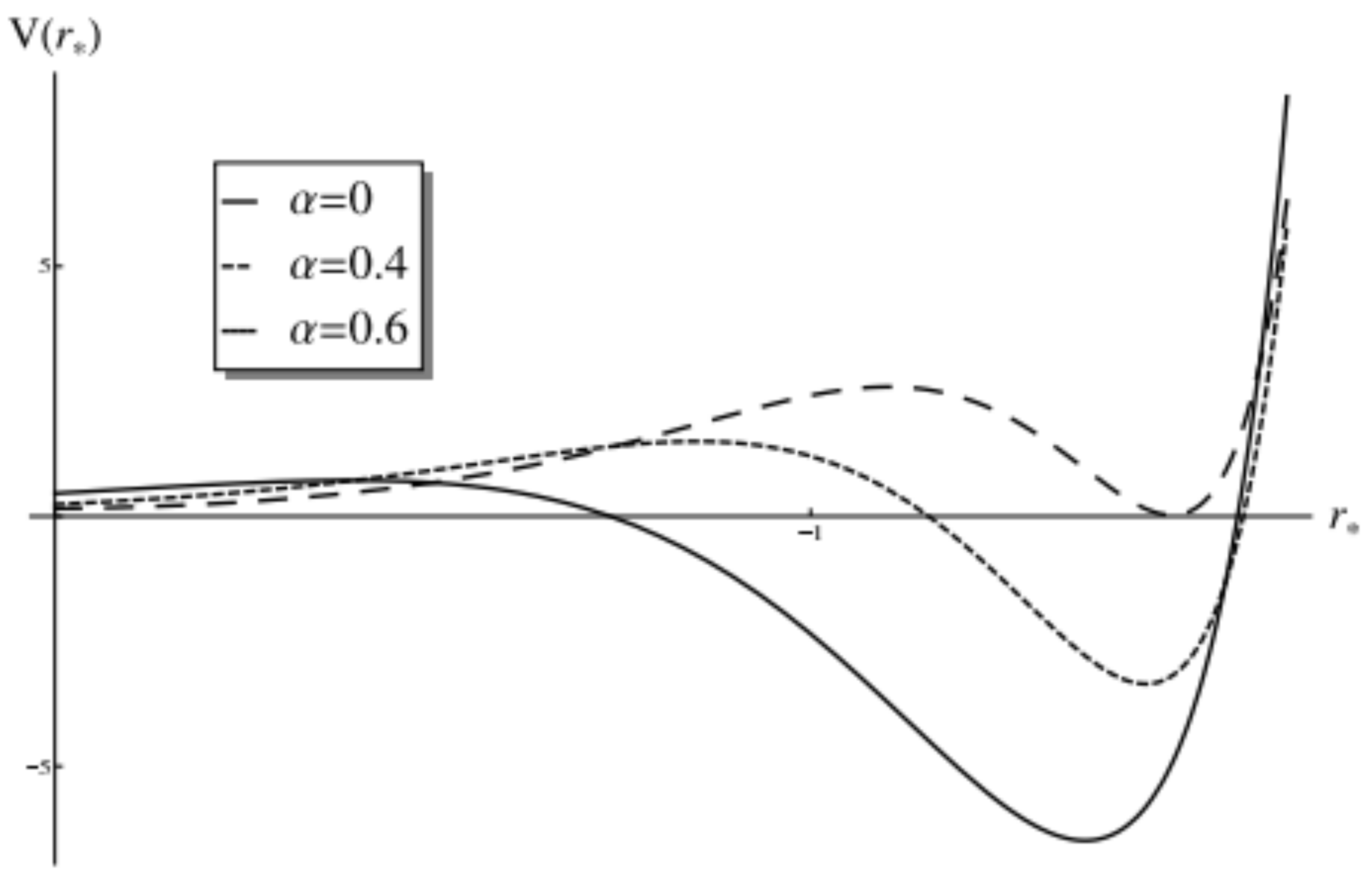
Rotating holographic SC

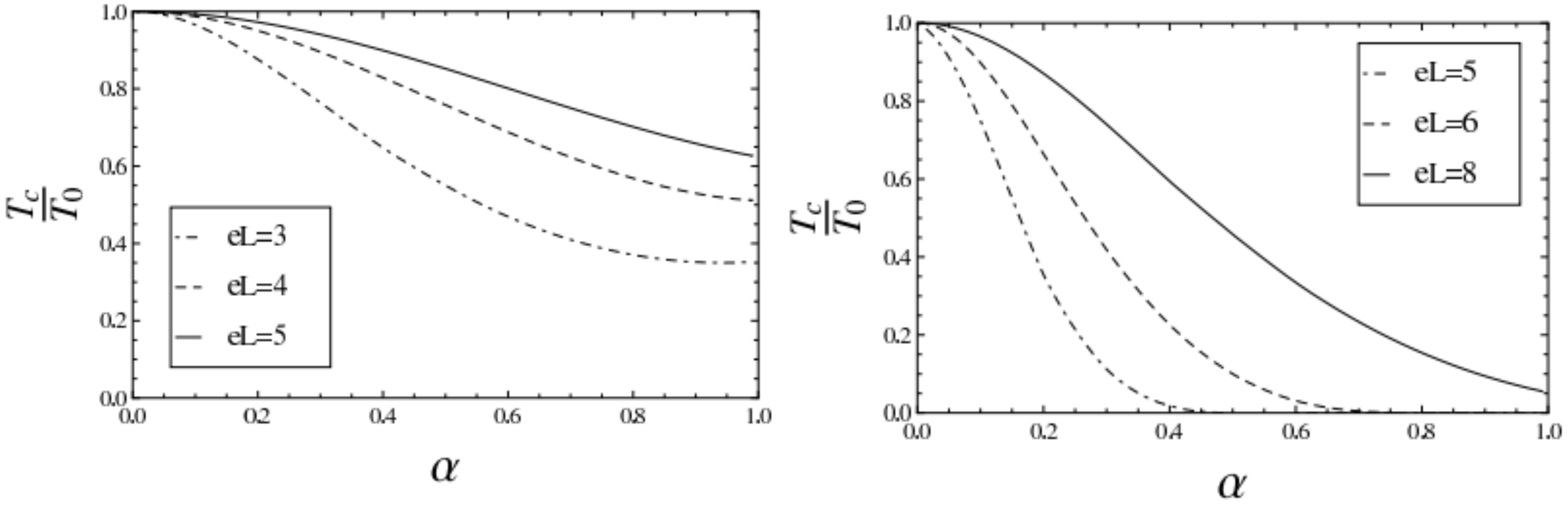
Charged scalar in AdS-KN background.

$$\frac{\mathscr{L}}{\sqrt{-g}} = -\frac{1}{4}F^2 - \frac{1}{2}\left|\left(D - m_\phi\right)\phi\right|^2$$

Can only solve numerically.

Potential changes according to rotation:





$\alpha = al^{-1}$

T_0 critical temperature for $a = 0$

Ref.: Sonner, J, Phys. Rev. D 80, 084031

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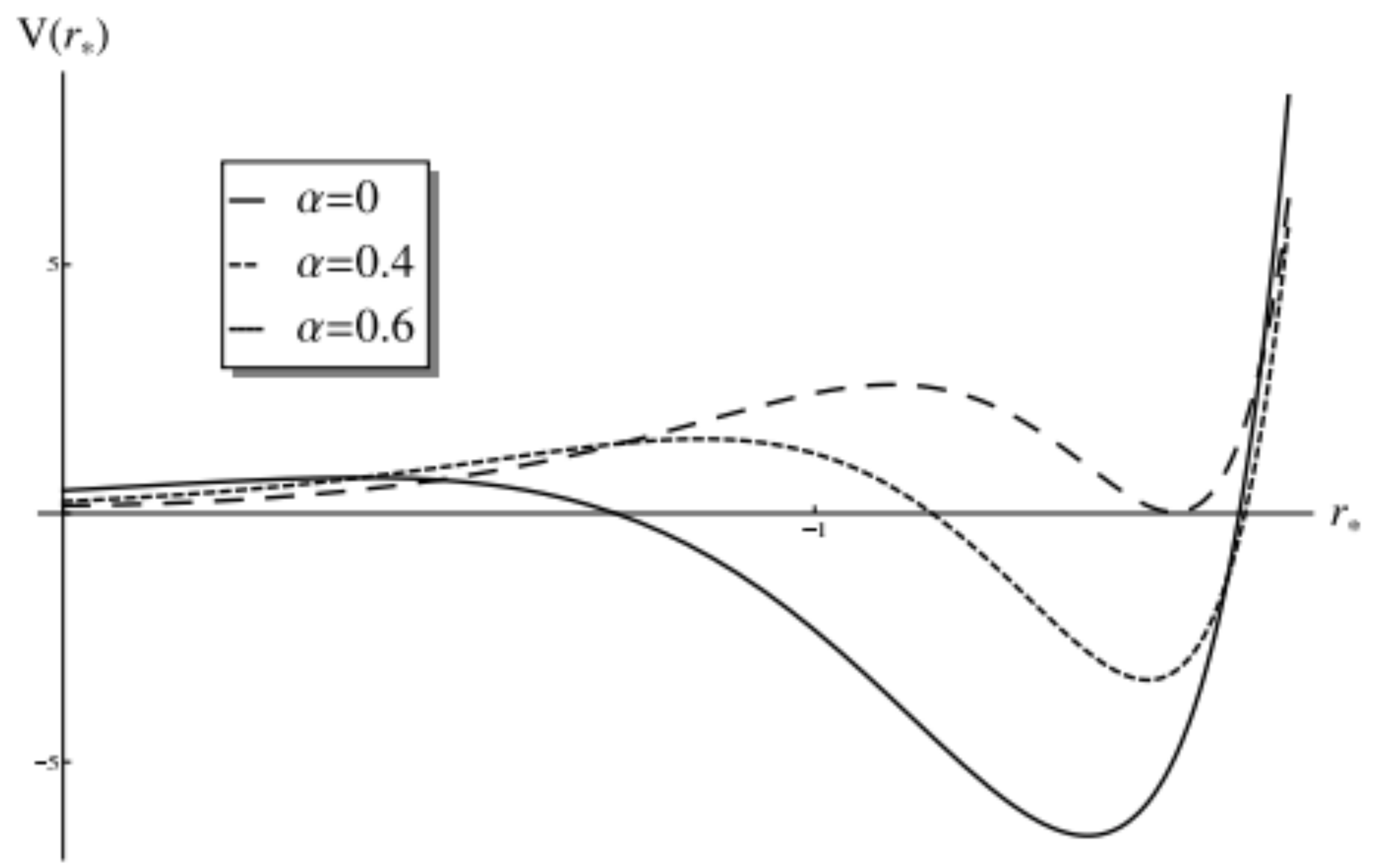

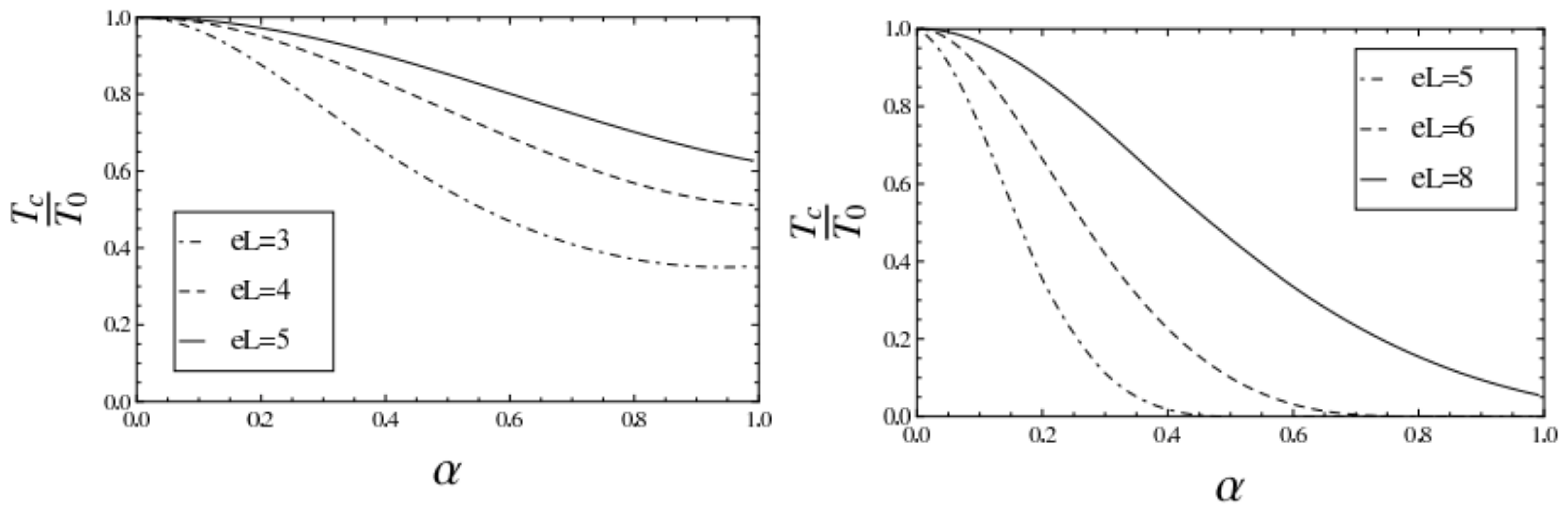

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Rotation suppresses super conducting phase!

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Rotating matter transport

Transport coefficients

Holographic transport coefficients can be computed using linear response theory.

One introduces a perturbation in the bulk that couples to an operator at the boundary theory, we mentioned the electric conductivity

$$\delta \mathcal{J} = \sigma \delta E$$

The moment of inertia is similar

$$\delta J = I \delta \Omega$$

Only depends on the thermodynamics of the black hole, and we know these quantities exactly

Rotating matter transport

However…

Are given in terms of the black hole parameters

$$\Omega = \frac{a\left(1 + r_+^2 l^{-2}\right)}{r_+^2 + a^2}$$

$$J = \frac{am}{\Xi^2}$$

Need to use approximations and numerical methods to obtain thermodynamics

$$G = G\left(T, \Omega, \mathcal{Q}\right) \implies \begin{cases} r_+ = r_+\left(T, \Omega, \mathcal{Q}\right) \\ a = a\left(T, \Omega, \mathcal{Q}\right) \\ Q = Q\left(T, \Omega, \mathcal{Q}\right) \end{cases}$$

Exact solution (probably) impossible

Rotating matter transport

Solve for r_+ approximating for $T \approx 0$, black hole close to extremality

$$T\left(r_0 + \delta r\right) \approx A\left(a, Q\right)\delta r, \qquad r_+ \mapsto r_0 + \delta r$$

$$T(r_0) = 0$$

$$\eta = \sqrt{a^4 + l^4 + 14a^2l^2 + 12Q^2l^2}$$

$$A\left(a, Q\right) = \frac{3\left[\eta^2 - \eta\left(a^2 + l^2\right)\right]}{l^2\pi\left(l^2 - 5a^2 - \eta\right)\left(a^2 + l^2 - \eta\right)}$$

Charges Q and $\mathcal{Q}$ are trivially solved

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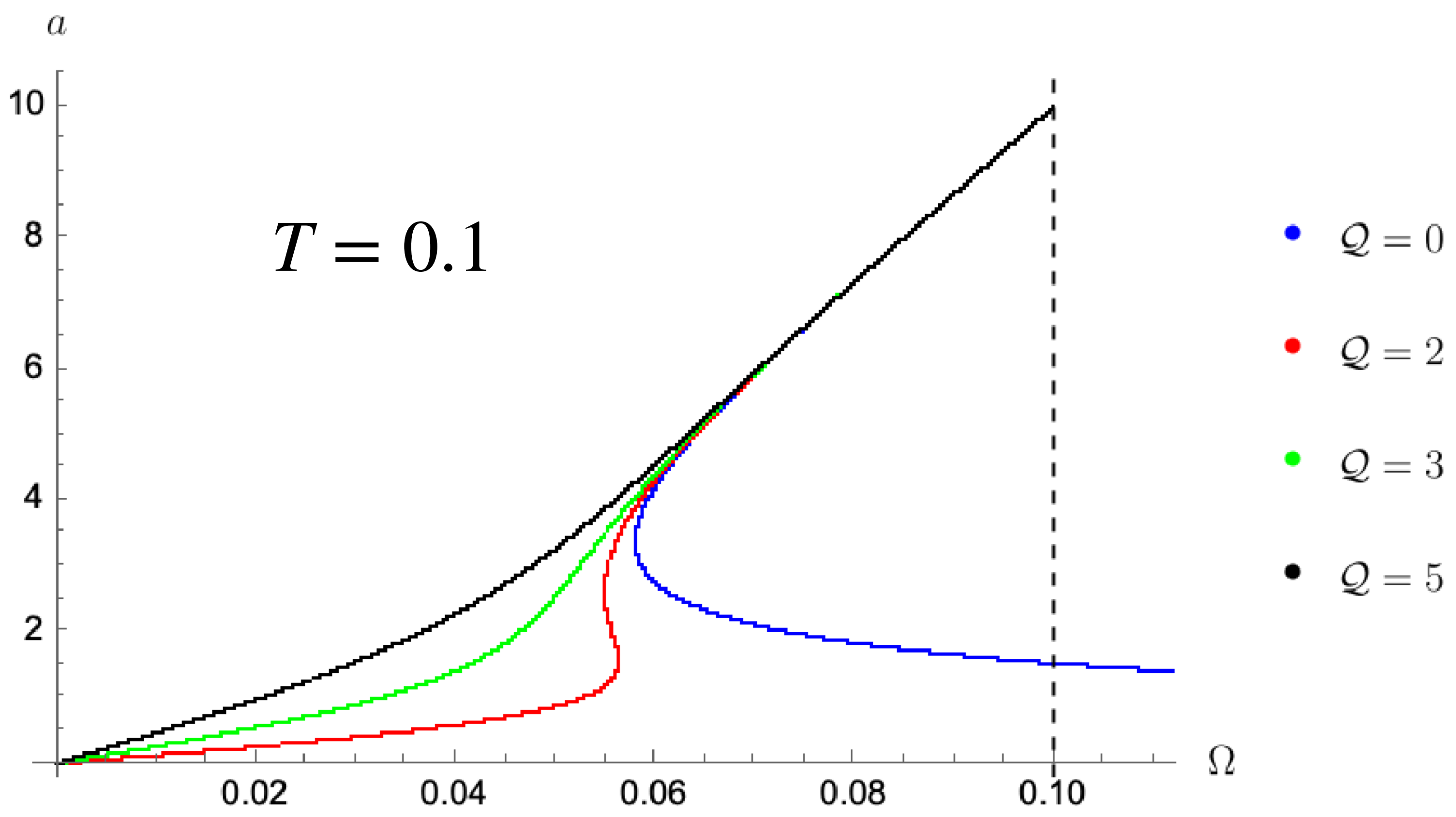
$l = 100$, or $\Lambda \sim 10^{-2}$



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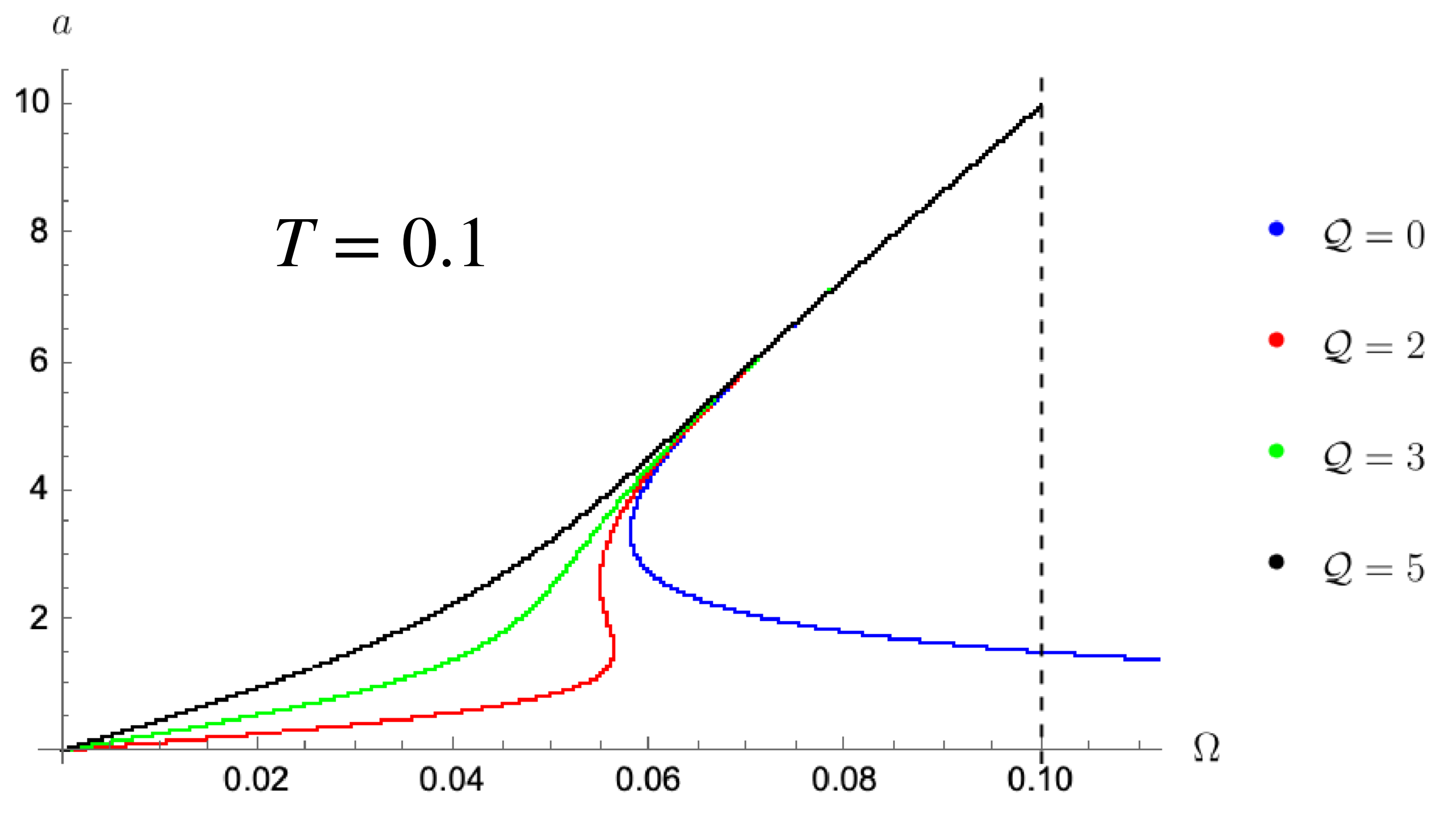
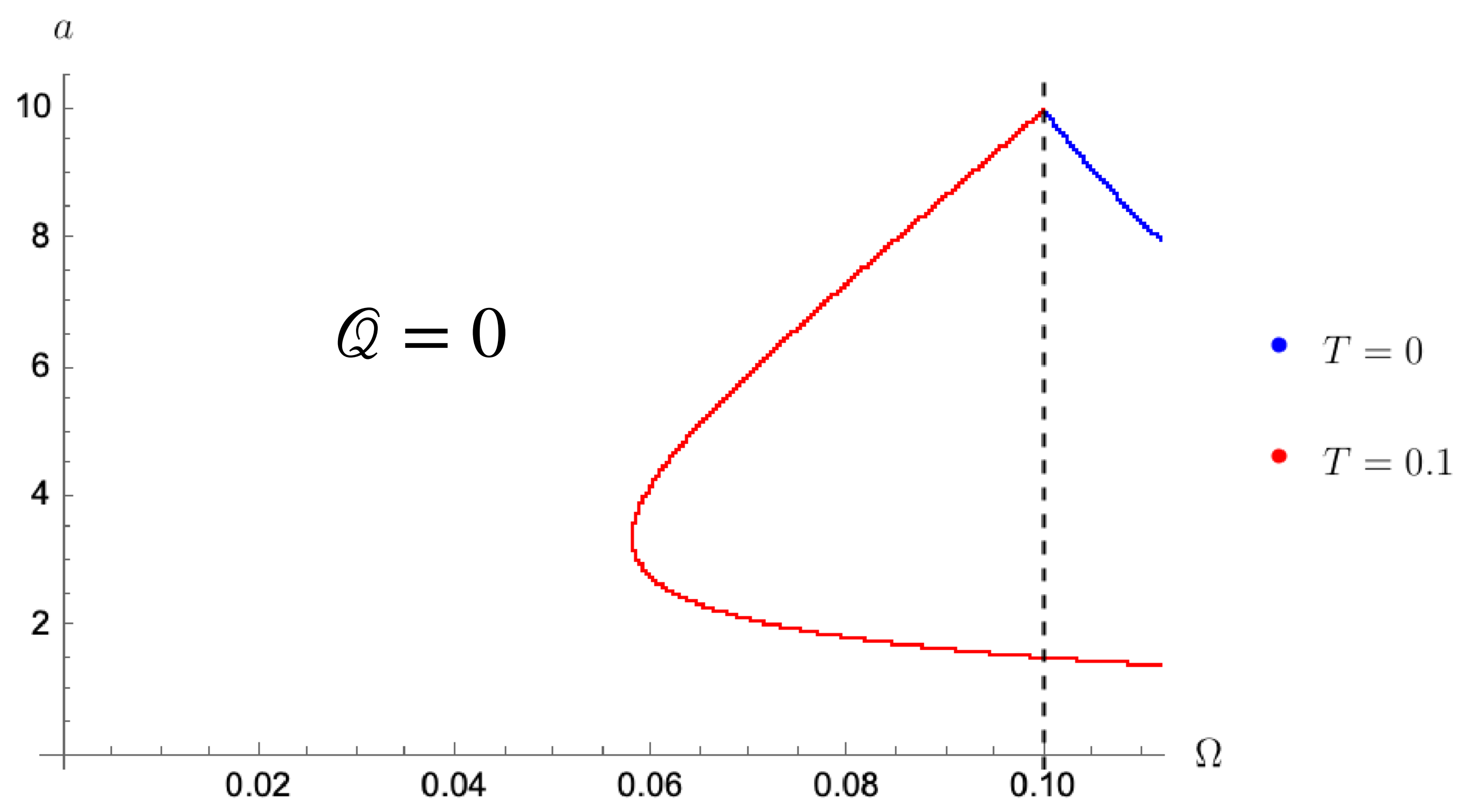
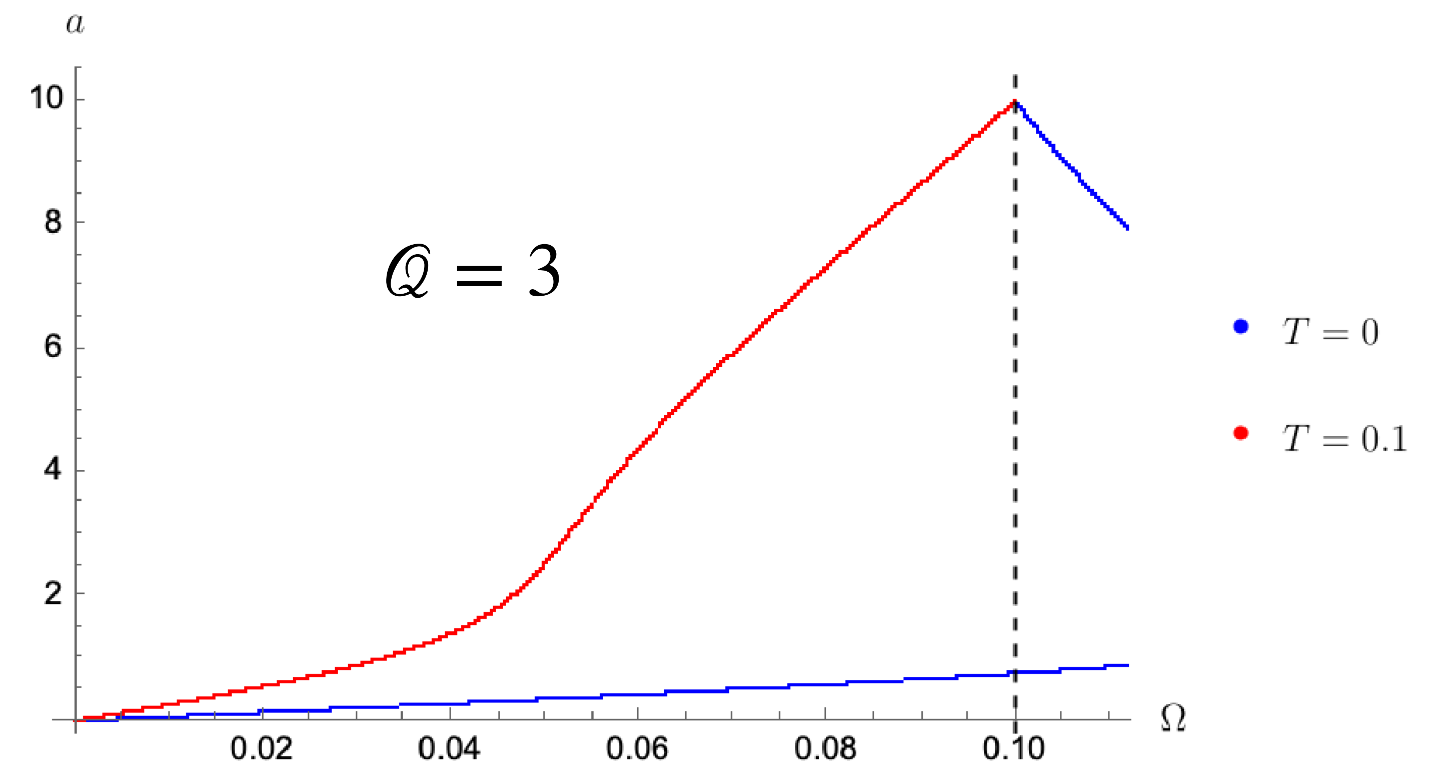
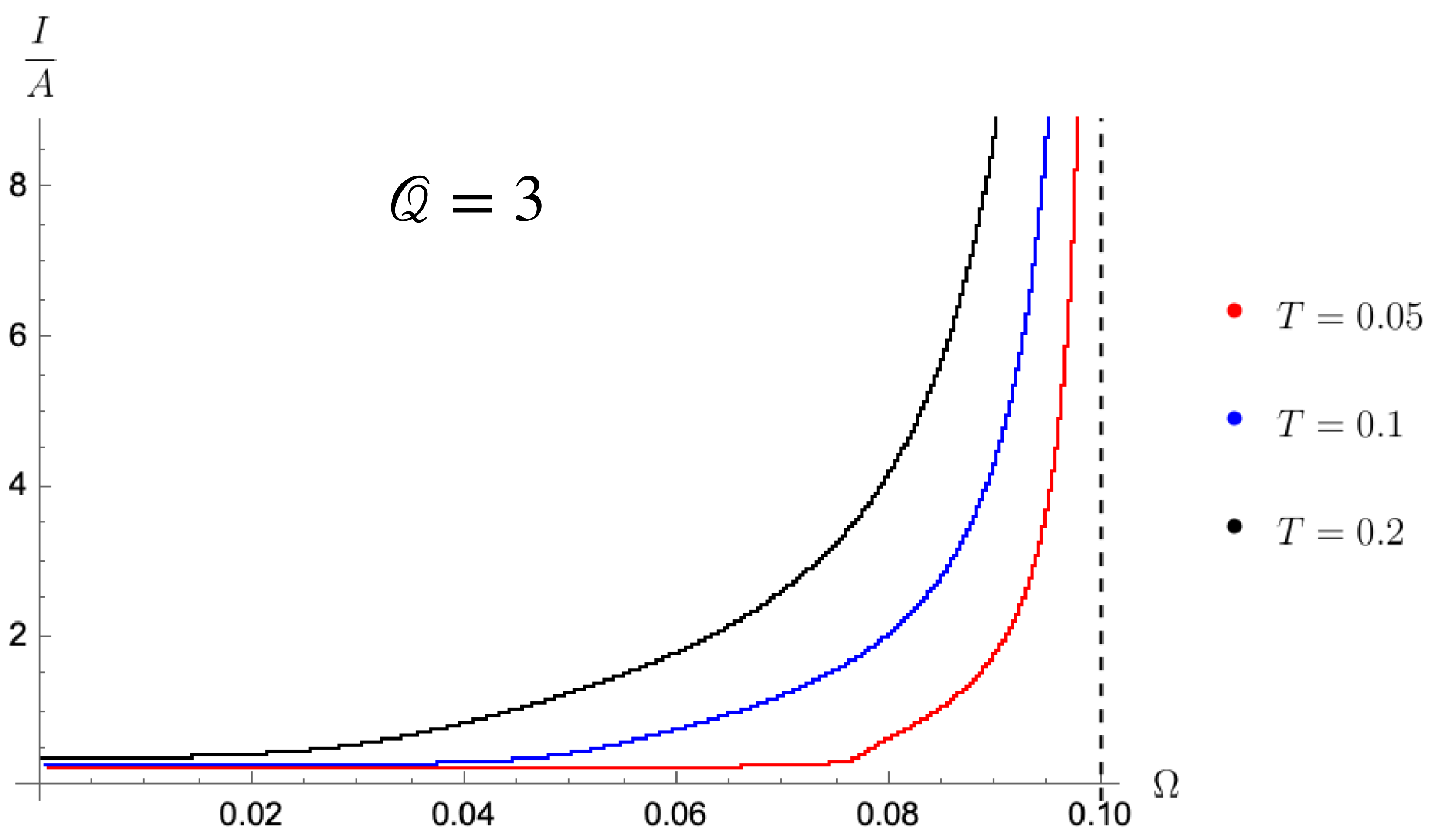
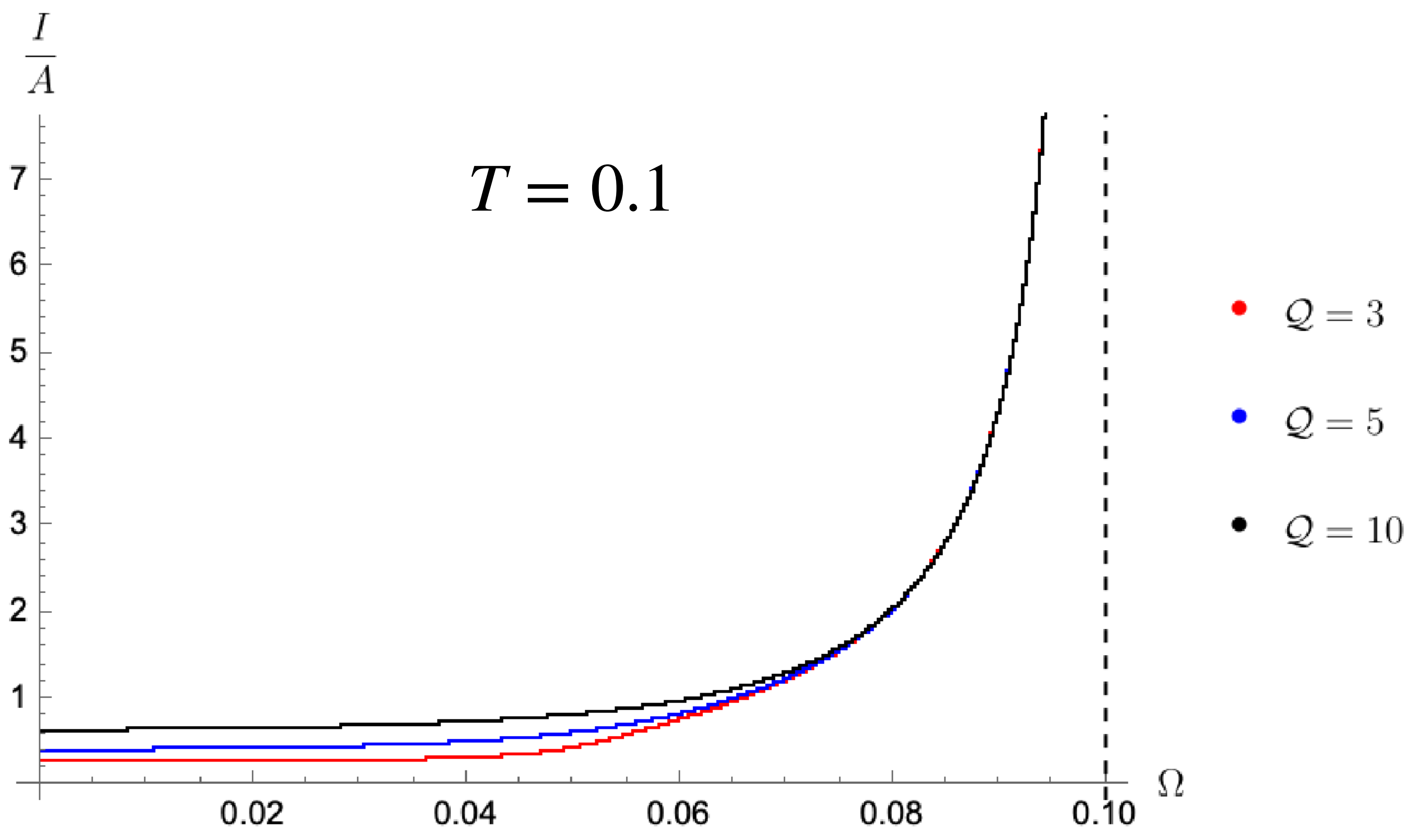






Rotating matter transport





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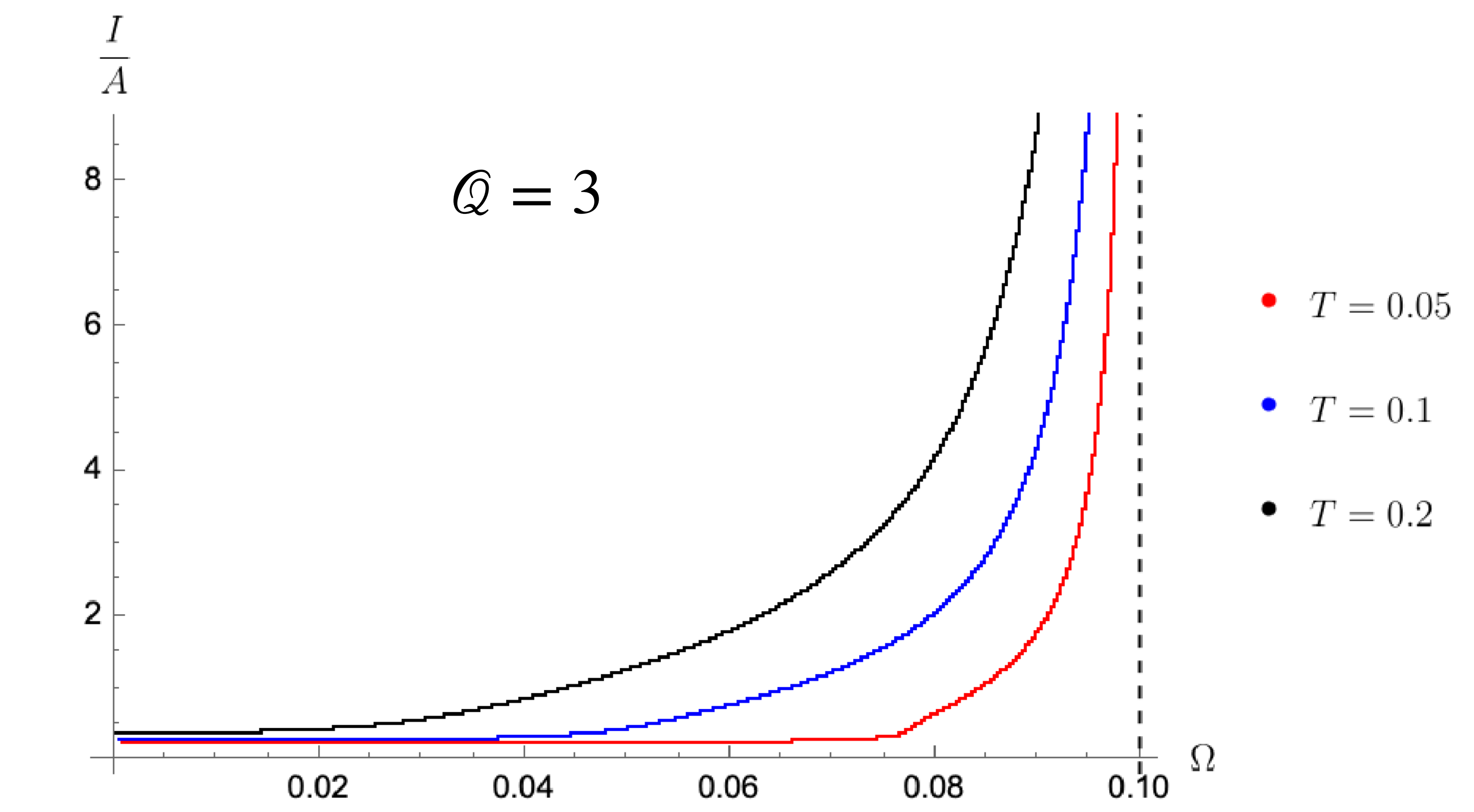

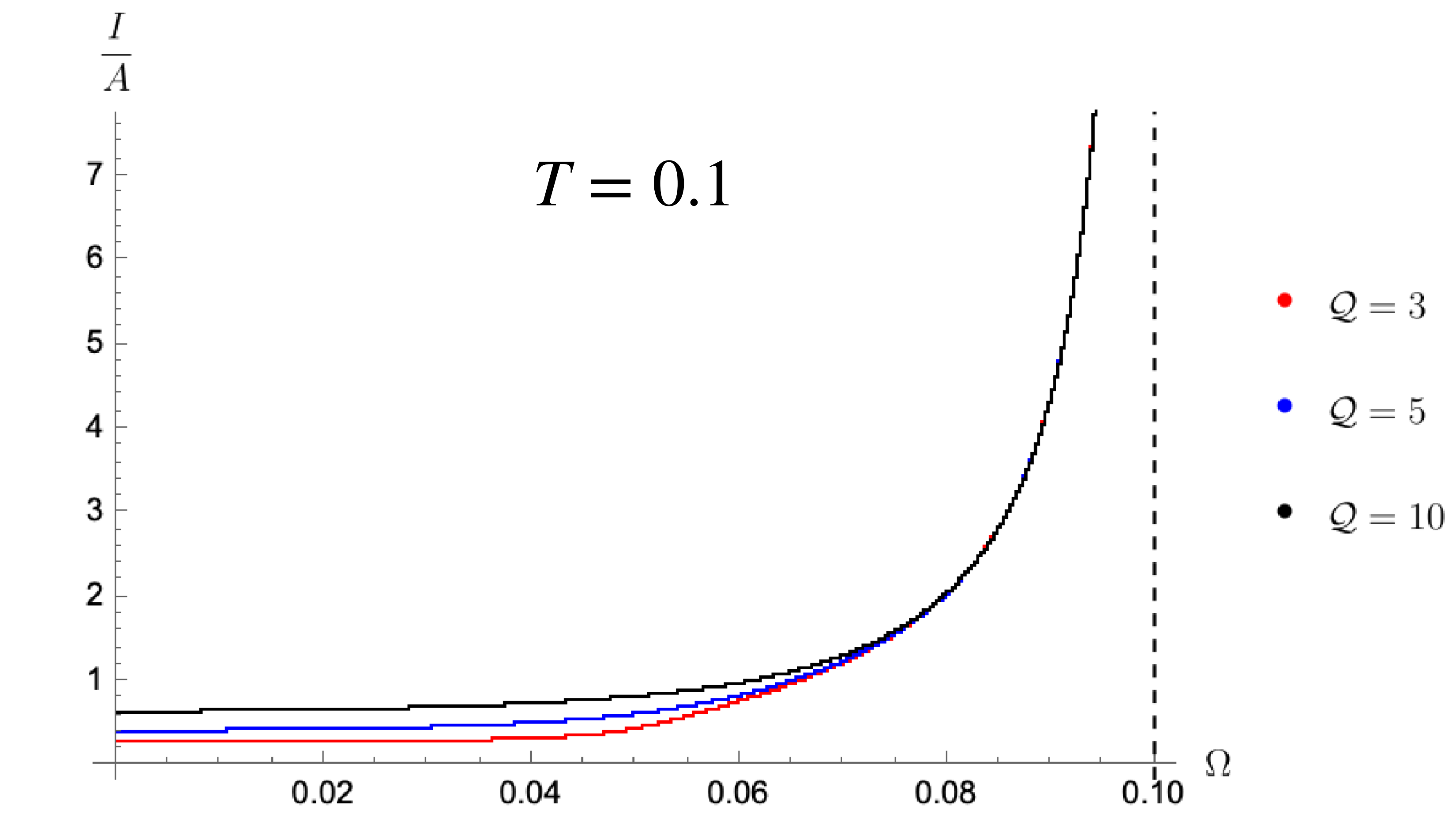

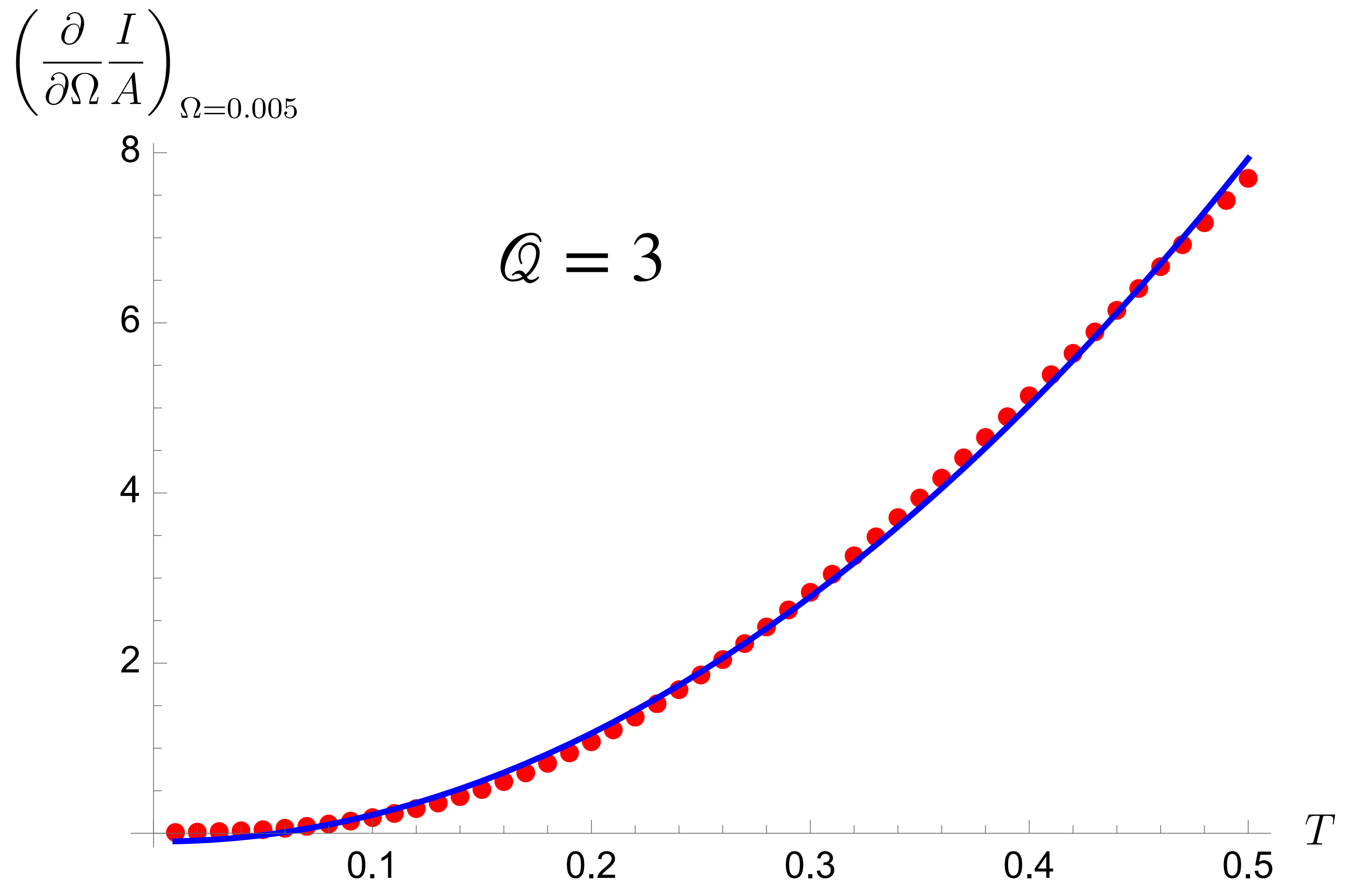

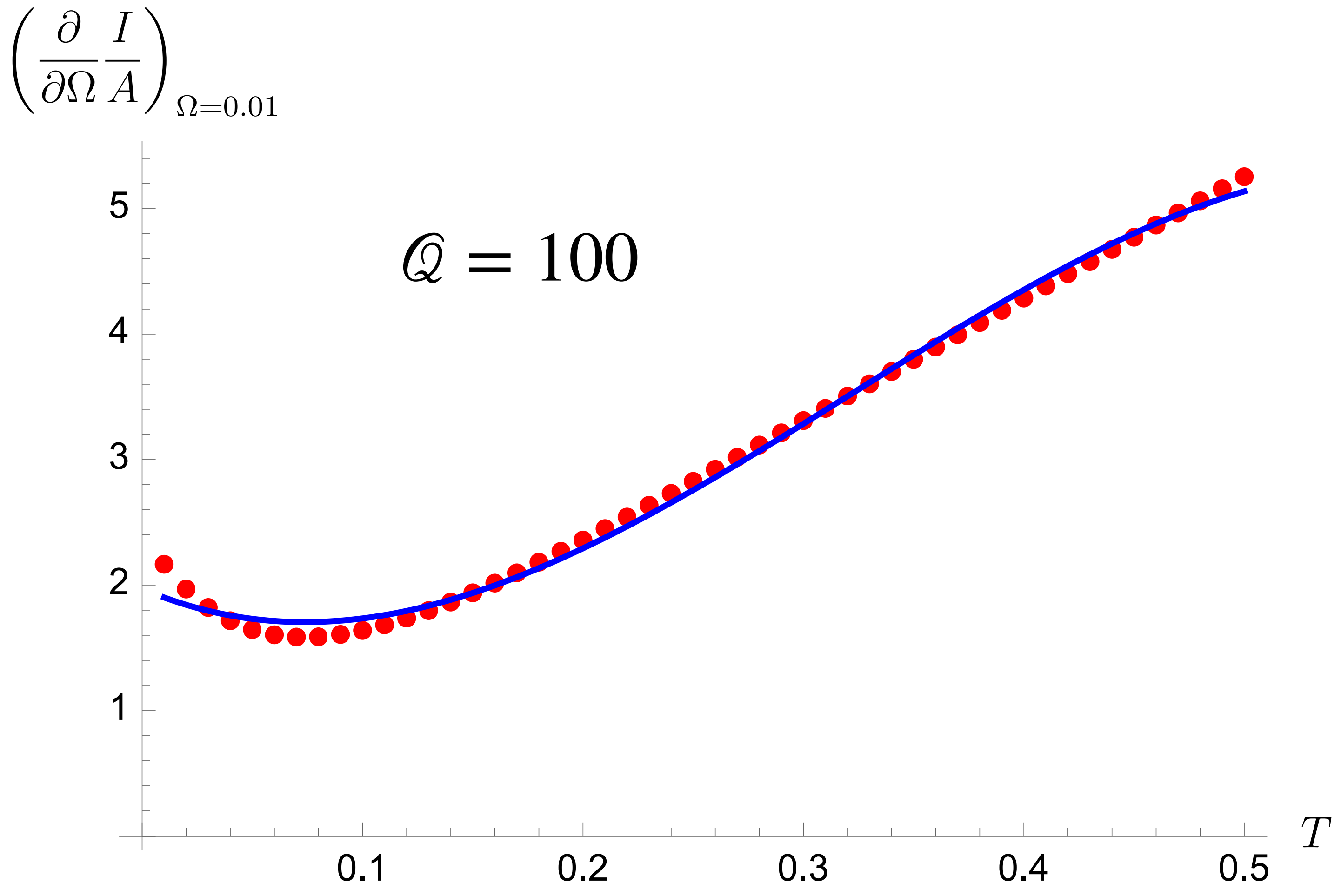

Conclusions

- For an electrically charged system the moment of inertia changes slightly as the temperature is increased from zero.
- More interesting cases appear as charge is higher. This is expected as we are relaxing the rotation parameter, given the low-temperature regime. However this might be unrealistic.
- Work in progress: considering the interaction of the rotating holographic SC with magnetic field.
- It is not clear how to compute the conductivity, as it requires breaking of translation invariance.

Thank you!

arXiv: 2402.04194

A microscopic model..?

From “Holographic superconductors” by Hartnol, Horowitz and Herzog:

Second, **there is a natural way to promote our phenomenological holographic superconductor into a full microscopic description: If we had realised our model as a limit of string theory**, then the potential for ψ would be completely fixed and there would be no free parameters. **We would have a concrete CFT that underwent a superconducting phase transition at a critical temperature specified by the background charge density.** Furthermore, in this theory, the AdS/CFT correspondence allows us to compute all the quantities for this superconductor which would normally follow from a BCS-like treatment: the gap as a function of temperature, the frequency dependent conductivity, the magnetic penetration depth, etc. We have shown how to use AdS/CFT to compute these quantities in this paper and we see that the ‘feel’ of the computation is completely different from weakly coupled BCS-like theories. **Nonetheless, AdS/CFT applied to a model embedded in string theory would be an honest-to-goodness microscopic computation of these quantities in a well-defined theory.**

JHEP12 (2008) 015, pp. 35-36

A little more about the SC instability

In AdS the mass of the scalar field can be negative, this is what triggers the instability of the scalar field.
The near horizon geometry normally is AdS space-time in less dimensions than the boundary, this allows for a range of masses where the scalar field is unstable near the horizon, but is stable at the boundary.

Breitenlohner - Freedman bound

$$m^2 \geq -\frac{9}{4L^2}$$