Functional renormalisation of UV-safe gauge theories coupled to matter

Daniele Rizzo

In collaboration with

Daniel Litim

During my visit at

Sussex University
Brighton, UK

Paper to appear on the ArXiv (hopefully) soon!

4th International FLAG Workshop
The Quantum and Gravity
Catania, Italy.

NARODOWE
CENTRUM
NAUKI

NATIONAL
CENTRE
FOR NUCLEAR
RESEARCH
ŚWIERK

daniele.rizzo@ncbj.gov.pl **10/09/2024**

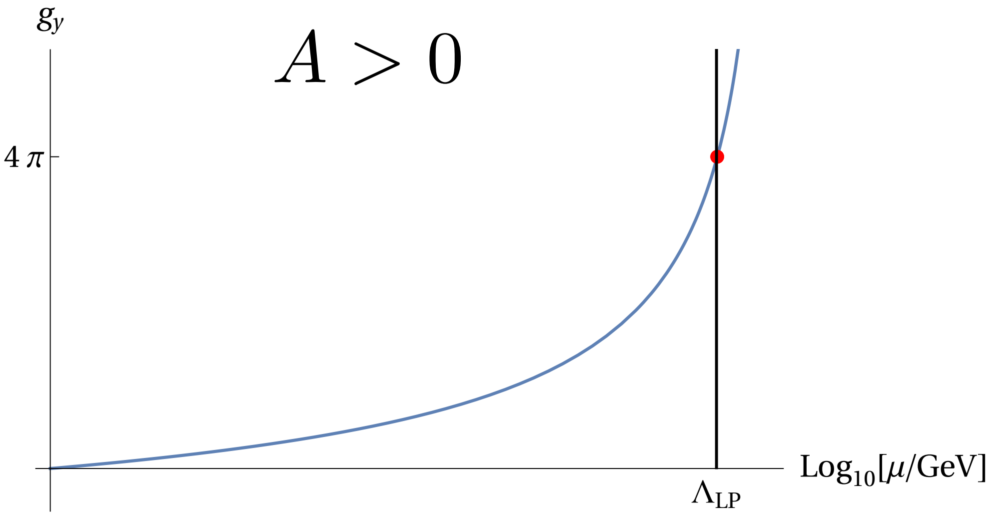
Asymptotic Behaviors

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\, g^2$$

daniele.rizzo@ncbj.gov.pl

Asymptotic Behaviors

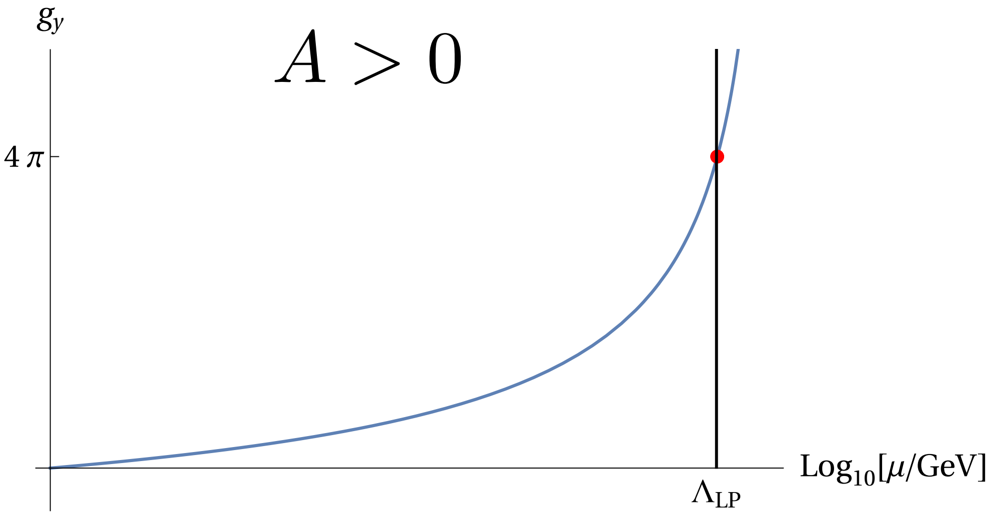
$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2$$

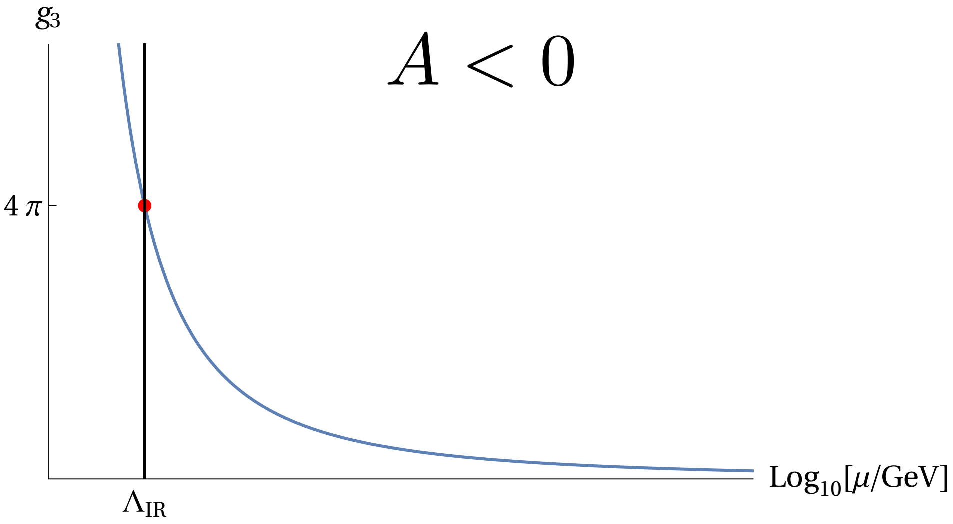


Landau pole

Asymptotic Behaviors

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\, g^2$$





Landau pole

Asymptotic freedom

daniele.rizzo@ncbj.gov.pl

Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

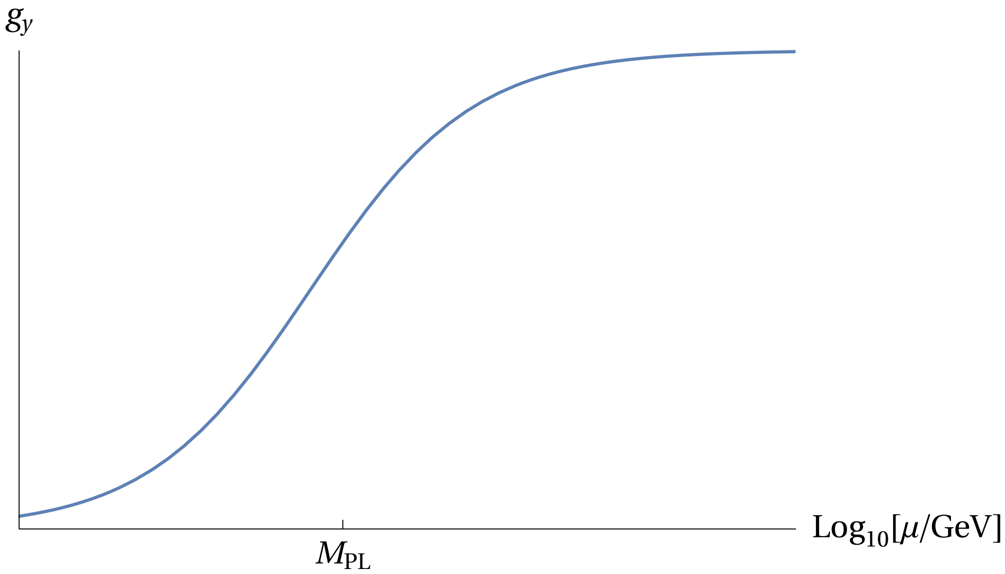
There is a specific value $g^* = -B/A$

$$\downarrow$$

$$\beta(g^*) = 0$$

$$\downarrow$$

Fixed Point!

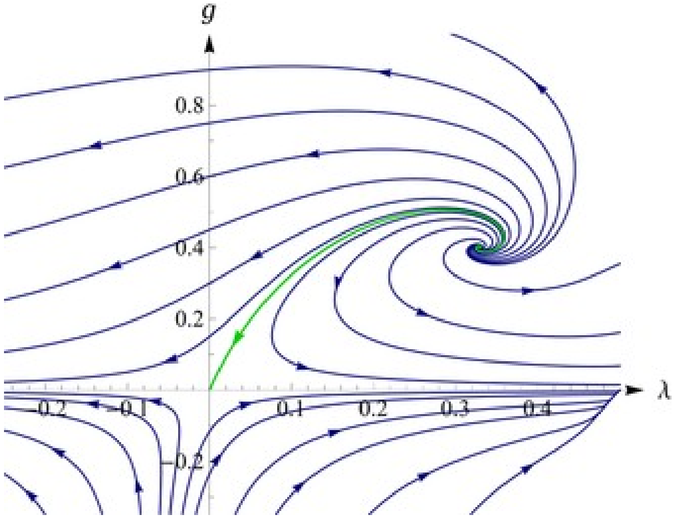


Asymptotic safety

Asymptotic Safety in Quantum Gravity

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g)+2\Lambda)$$



Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

Large uncertainties when computed analytically.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, …]

Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

Renormalization Group Equations in the Trans-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}} - \boxed{g\,f_g}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}} - \boxed{y\,f_y}$$

In our project are determined by matching the low-energy data.

A list of paper exploring such possibility for different BSM scenarios:

1803.04027
1810.08461
1911.00012
2003.08401
2005.03661
2007.03567
2012.15200
2112.08972
2204.00866
2206.02686
2204.09008
2209.07971
2209.14268

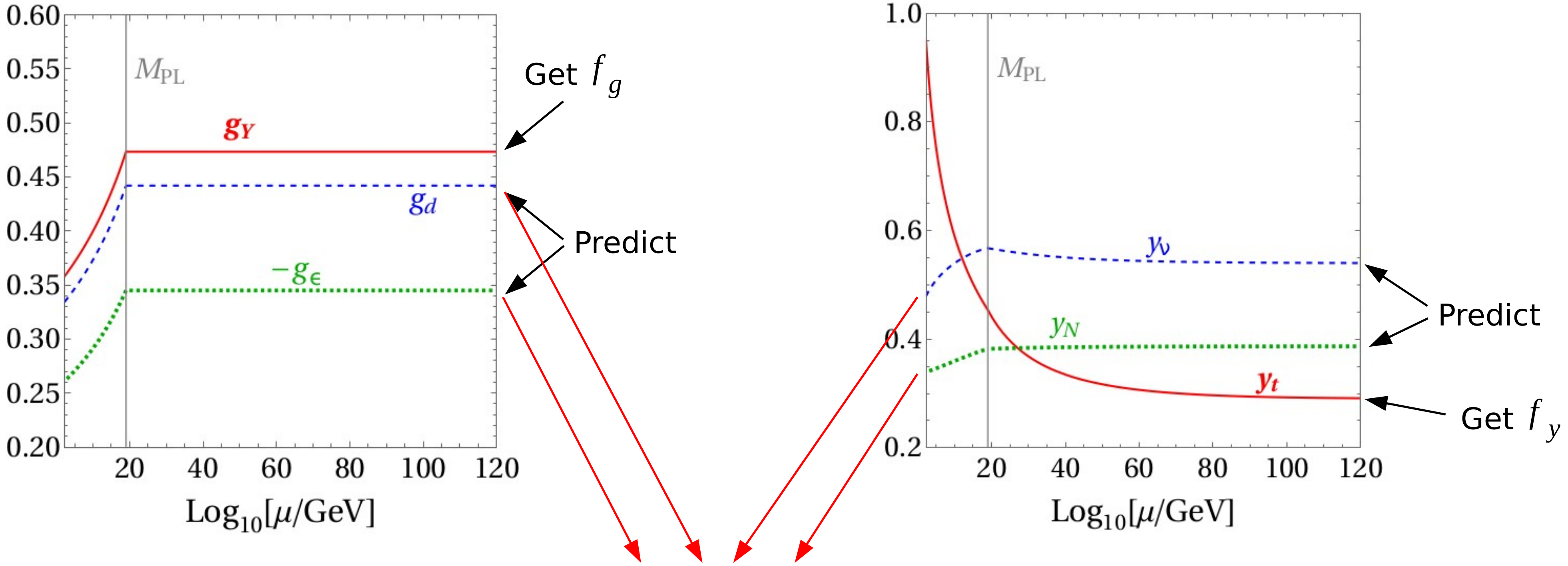
Asymptotic Safety in Quantum Gravity

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_{\nu}N\left(\tilde{\epsilon}H^{*}\right)^{\dagger}L - \frac{1}{2}Y_{N}SNN + \text{H.c.}$$

Gauge couplings

Yukawa couplings



Get f_g

Predict

Predict

Get f_y

Phenomenology!

cf. *e.g.* Chikkaballi, Kotlarski, Kowalska, **DR**, Sessolo JHEP (2023).

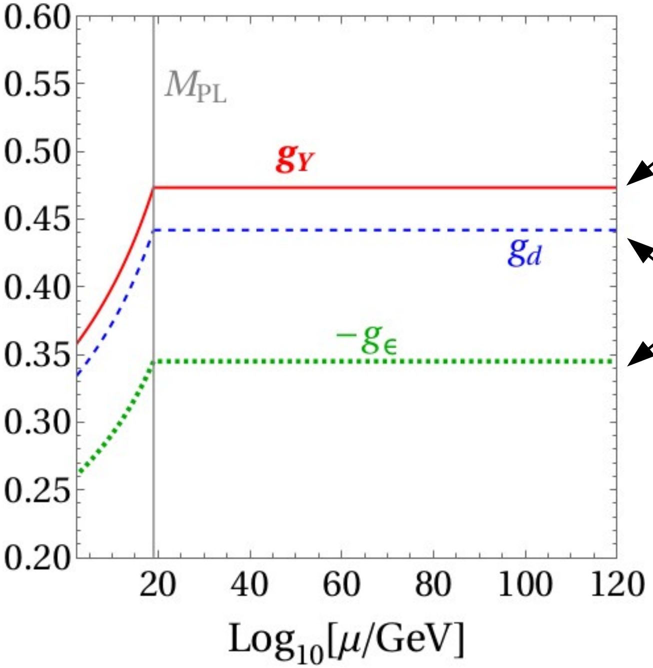
EPJC: 10.1140

How robust are particle physics predictions in asymptotic safety?

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

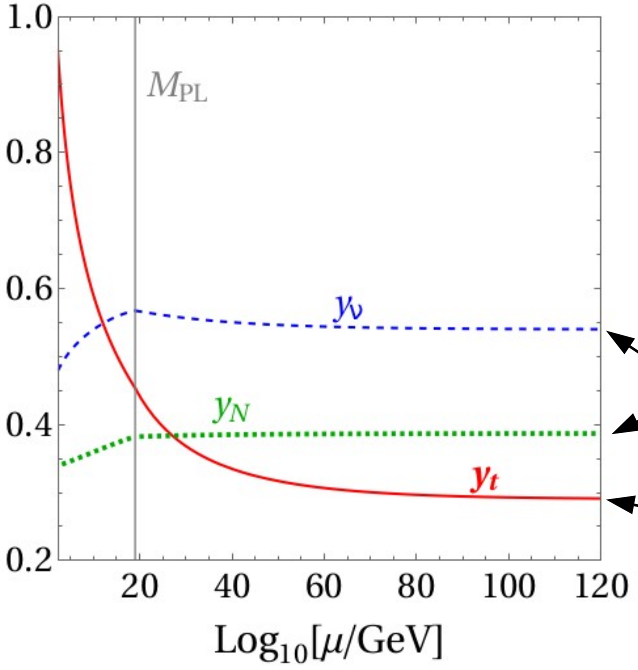
Gauge couplings



Get f_g

Predict

Yukawa couplings



Predict

Get f_y

Sources of uncertainties →

1 - Computations of the beta functions are performed at 1-loop level.

2 - Planck scale is set arbitrarily at 10^{19} GeV.

3 - Gravity decouples instantaneously at the Planck scale.

Gauge-Yukawa Theory

Litim, Sannino (2014)

daniele.rizzo@ncbj.gov.pl

Gauge-Yukawa Theory

Gauge $F^a_{\mu\nu}\,(a = 1, ..., N_C^2 - 1)$

Fermions $Q_i\,(i = 1, ..., N_F)$

Scalars $H \in N_F \times N_F$

$$\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\, i D\, Q\right)$$

$$-y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right) - u\,\mathrm{Tr}\left(H^\dagger H\right)^2 - v\left(\mathrm{Tr}\, H^\dagger H\right)^2$$

Litim, Sannino (2014)

Conformal Window

Under perturbative expansion, the theory has an Ultra Violet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Bond et al. 1710.07615
Litim et al. 2307.08747

Conformal Window

Under perturbative expansion, the theory has an Ultra Violet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Let us be a little bit more quantitative and ask the questions:

- For what values of the Veneziano parameter do we actually have a fixed point?
- What can cause a fixed point to disappear?

The values of the Veneziano parameter for which the fixed point exist is called

CONFORMAL WINDOW

Conformal Window & Vacuum Stability

The conformal window can close because of:

- Vacuum Stability
- A Fixed Point merger

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Conformal Window & Vacuum Stability

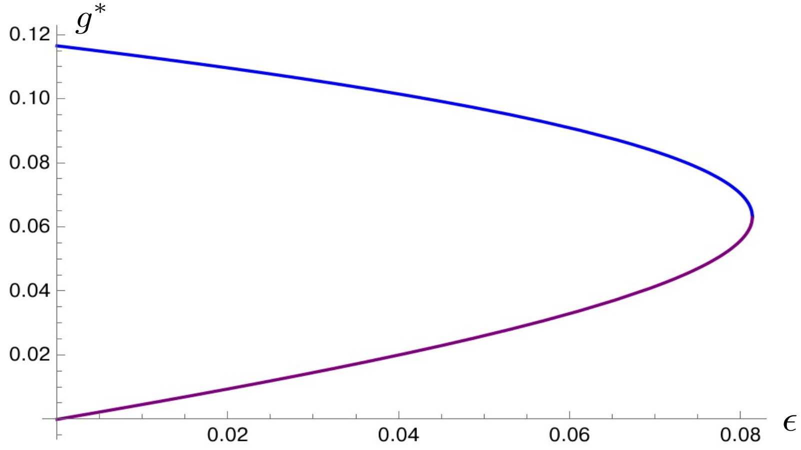
The conformal window can close because of:

- Vacuum Stability
- A Fixed Point merger

Disclaimer: Fixed Point merger

$$A + B\,g + g^2 = 0$$

$$g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$

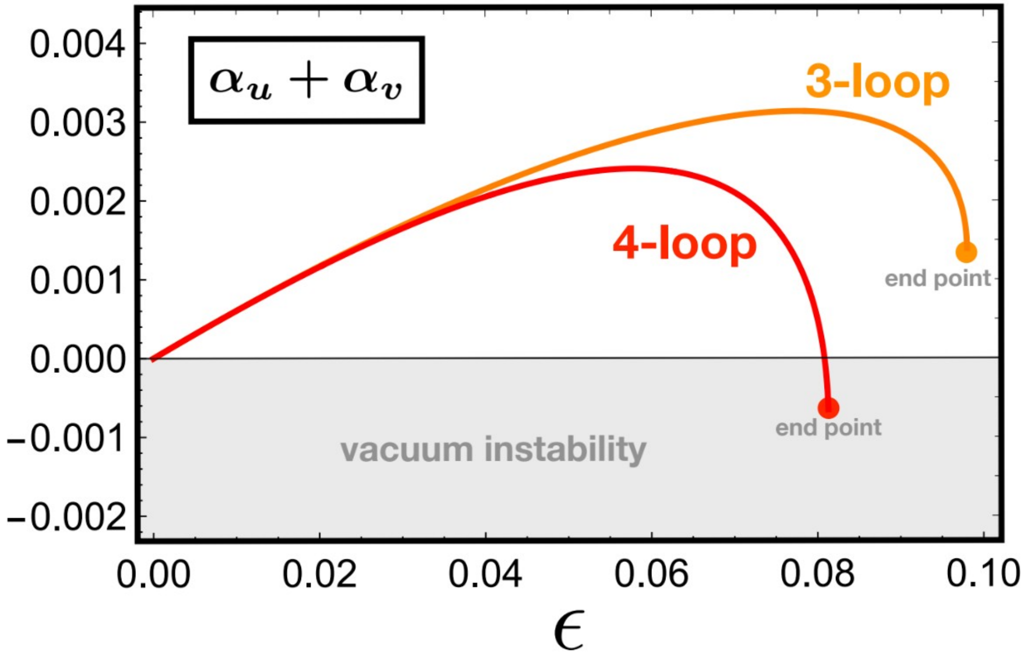


daniele.rizzo@ncbj.gov.pl

Conformal Window & Vacuum Stability

The conformal window can close because of:

- Vacuum Stability
- A Fixed Point merger

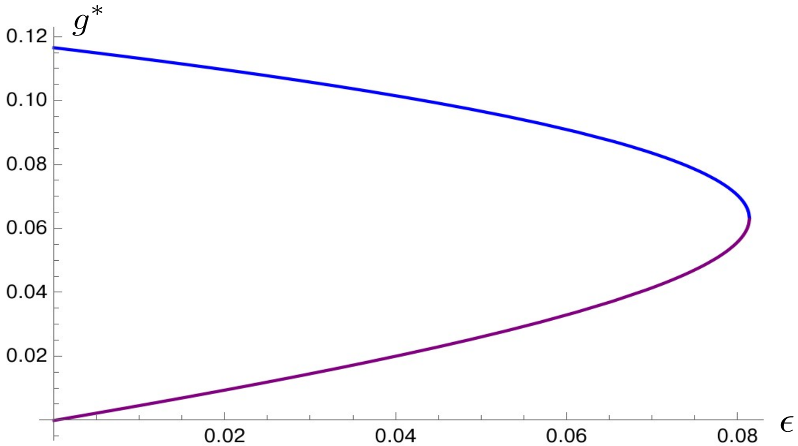


I thank Zaan for the plot.

Disclaimer: Fixed Point merger

$$A + B\,g + g^2 = 0$$

$$g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$



daniele.rizzo@ncbj.gov.pl

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ)$$

daniele.rizzo@ncbj.gov.pl

Beyond marginal operators

$$v\,\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\,\left(H^\dagger H\right)^2$$

$$y\,\mathrm{Tr}(\bar{Q} H Q)$$

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \sum_m \alpha_m (\mathrm{Tr} H^\dagger H)^{m-2}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q} H Q)$$

daniele.rizzo@ncbj.gov.pl

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n(\mathrm{Tr}H^\dagger H)^{n-2}(\mathrm{Tr}H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \sum_m \alpha_m(\mathrm{Tr}H^\dagger H)^{m-2}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ) \longrightarrow \sum_l Y_l\,(\mathrm{Tr}H^\dagger H)^l\;\mathrm{Tr}(\bar{Q}HQ)$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \boxed{\sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2}$$

$$U(\mathrm{Tr} H^\dagger H)$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \sum_m \alpha_m (\mathrm{Tr} H^\dagger H)^{m-2}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q} H Q) \longrightarrow \sum_l Y_l\,(\mathrm{Tr} H^\dagger H)^l\,\mathrm{Tr}(\bar{Q} H Q)$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^{\dagger}H\right)^{2} \longrightarrow \underset{U(\mathrm{Tr}H^{\dagger}H)}{\boxed{\sum_{n}\gamma_{n}(\mathrm{Tr}H^{\dagger}H)^{n-2}(\mathrm{Tr}H^{\dagger}H)^{2}}}$$

$$u\,\mathrm{Tr}\left(H^{\dagger}H\right)^{2} \longrightarrow \underset{C(\mathrm{Tr}H^{\dagger}H)}{\boxed{\sum_{m}\alpha_{m}(\mathrm{Tr}H^{\dagger}H)^{m-2}}}\,\mathrm{Tr}(H^{\dagger}H)^{2}$$

$$y\,\mathrm{Tr}(\bar{Q}HQ) \longrightarrow \sum_{l}Y_{l}\,(\mathrm{Tr}H^{\dagger}H)^{l}\;\mathrm{Tr}(\bar{Q}HQ)$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \underbrace{\boxed{\sum_n \gamma_n(\mathrm{Tr}H^\dagger H)^{n-2}(\mathrm{Tr}H^\dagger H)^2}}_{U(\mathrm{Tr}H^\dagger H)}$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \underbrace{\boxed{\sum_m \alpha_m(\mathrm{Tr}H^\dagger H)^{m-2}}}_{C(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ) \longrightarrow \underbrace{\boxed{\sum_l Y_l\,(\mathrm{Tr}H^\dagger H)^l}}_{Y(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(\bar{Q}HQ)$$

Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$ $\partial_t C(\mathrm{Tr} H^\dagger H)$ $\partial_t Y(\mathrm{Tr} H^\dagger H)$

daniele.rizzo@ncbj.gov.pl

Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$ $\partial_t C(\mathrm{Tr} H^\dagger H)$ $\partial_t Y(\mathrm{Tr} H^\dagger H)$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$ $\partial_t C(\mathrm{Tr} H^\dagger H)$ $\partial_t Y(\mathrm{Tr} H^\dagger H)$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Regulator

$$R_k = Z_k\,(k^2 - q^2)\,\Theta(k^2 - q^2)$$

daniele.rizzo@ncbj.gov.pl

Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

daniele.rizzo@ncbj.gov.pl

Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

And compute the flow:

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

Flow

We define dimension-less couplings:

$$U = k^4 u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

And compute the flow:

$$\partial_t u = \underset{\uparrow}{-4u} + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

Canonical dimension

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

And compute the flow:

$$\partial_t u = -4u + (2 + \eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1 + u' + 4\rho\, c} + \frac{1}{1 + u'}\right) - \frac{2N_C}{N_F}\frac{1}{1 + \rho\, y^2}$$

Canonical dimension

Anomalous dimension

Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

And compute the flow:

$$\partial_t u = -4u + (2 + \eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1 + u' + 4\rho\, c} + \frac{1}{1 + u'}\right) - \frac{2N_C}{N_F}\frac{1}{1 + \rho\, y^2}$$

Canonical dimension

Anomalous dimension

Quantum corrections from the scalar potential

Flow

We define dimension-less couplings:

$$U = k^4\, u \qquad \mathrm{Tr}\, H^\dagger H = \rho\, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau\, k^4$$

And compute the flow:

$$\partial_t u = -4u + (2 + \eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1 + u' + 4\rho\, c} + \frac{1}{1 + u'}\right) - \frac{2N_C}{N_F}\frac{1}{1 + \rho\, y^2}$$

Canonical dimension

Anomalous dimension

Quantum corrections from the scalar potential

Quantum corrections from the yukawa

daniele.rizzo@ncbj.gov.pl

Flow

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\,\frac{1}{1+\rho\, y^2}$$

Tuğba Büyükbeşe, PhD Thesis

$$\begin{aligned}\partial_t c = {} & 2\eta_H c + (2+\eta_H)\rho c' - \frac{2N_C}{N_F}\,\frac{y^4}{\left(1+\rho y^2\right)^3} \\ & + \frac{1}{2}\left(-\frac{128\rho^3c^5}{\left(1+u'\right)^3\left(1+4\,\rho\, c+u'\right)^3} + \frac{64\rho^2c^3\left(c-\rho c'\right)}{\left(1+u'\right)^2\left(1+4\,\rho\, c+u'\right)^3} - \frac{8\rho c c'}{\left(1+4\,\rho\, c+u'\right)^3}\right. \\ & \left. - \frac{48\rho^2c^2c'}{\left(1+u'\right)\left(1+4\,\rho\, c+u'\right)^3} + \frac{16c^2}{\left(1+4\,\rho\, c+u'\right)^3} - \frac{2c'}{\left(1+4\,\rho\, c+u'\right)^2}\right)\end{aligned}$$

Tuğba Büyükbeşe, PhD Thesis

$$\begin{aligned}\partial_t y = {} & -3\alpha_g y(0) + \frac{1}{2}(2\eta_\psi + \eta_H)y + (2+\eta_\phi)\rho y' - \frac{1}{2}\left(\frac{y'}{\left(1+4\,\rho\, c+u'\right)^2} + \frac{y'}{\left(1+u'\right)^2}\right) \\ & + \frac{y^3}{2\left(1+\rho y^2\right)\left(1+4\rho\, c+u'\right)}\left(\frac{1}{1+4\rho\, c+u'} + \frac{1}{1+\rho y^2}\right) - \frac{y^3}{2\left(1+u'\right)\left(1+\rho y^2\right)}\left(\frac{1}{1+\rho y^2} + \frac{1}{1+u'}\right)\end{aligned}$$

Fixed Point

$\partial_t u = 0$ $\partial_t c = 0$ $\partial_t y = 0$

Fixed Point

$$\partial_t u = 0$$

$$\Big\downarrow$$

$$u(\rho) = \sum_{n=0}^{N\to\infty} \alpha_n \rho^{n+1}$$

$$\partial_t c = 0$$

$$\Big\downarrow$$

$$c(\rho) = \sum_{n=1}^{N\to\infty} \gamma_n \rho^{n-1}$$

$$\partial_t y = 0$$

$$\Big\downarrow$$

$$y(\rho) = \sum_{n=0}^{N\to\infty} y_n \rho^{n}$$

Fixed Point

$$\begin{tikzcd}
\partial_t u = 0 \arrow[d] & \partial_t c = 0 \arrow[d] & \partial_t y = 0 \arrow[d] \\
u(\rho) = \sum_{n=0}^{N\to\infty} \alpha_n \rho^{n+1} \arrow[d] & c(\rho) = \sum_{n=1}^{N\to\infty} \gamma_n \rho^{n-1} \arrow[d] & y(\rho) = \sum_{n=0}^{N\to\infty} y_n \rho^{n} \arrow[d] \\
\partial_t \alpha_n = 0 & \partial_t \gamma_n = 0 & \partial_t y_n = 0
\end{tikzcd}$$

Fixed Point & Power Counting in ε

Dim 4:

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Fixed Point & Power Counting in ε

Dim 4: $\alpha_h^* \equiv \gamma_1^*$ $\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$ $\alpha_y \equiv y_0^{2,*}$ $\approx \epsilon$

Fixed Point & Power Counting in ε

Dim 4: $\alpha_h^* \equiv \gamma_1^*$ $\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$ $\alpha_y \equiv y_0^{2,*}$ $\approx \epsilon$

Dim 6:

Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
Dim 6:	γ_2^*	α_2^*	y_1^*	**?**

Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
Dim 6:	γ_2^*	α_2^*	y_1^*	**?**

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
Dim 6:	γ_2^*	α_2^*	y_1^*	**?**

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
Dim 6:	γ_2^*	α_2^*	y_1^*	$\approx \epsilon^3$

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2\left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2}y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
Dim 6:	γ_2^*	α_2^*	y_1^*	$\approx \epsilon^3$

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

Dim 2n:

daniele.rizzo@ncbj.gov.pl

Fixed Point & Power Counting in ε

Dim 4: $\alpha_h^* \equiv \gamma_1^*$ $\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$ $\alpha_y \equiv y_0^{2,*}$ $\approx \epsilon$

Dim 6: γ_2^* α_2^* y_1^* $\approx \epsilon^3$

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

Dim 2n: $\partial_t \lambda_{2n} = (2n-4)\lambda_{2n} + \lambda_{2n}\left(A\,\epsilon + \mathcal{O}(\epsilon^2)\right) + B\epsilon^n$

Fixed Point & Power Counting in ε

Dim 4: $\alpha_h^* \equiv \gamma_1^*$ $\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$ $\alpha_y \equiv y_0^{2,*}$ $\approx \epsilon$

Dim 6: γ_2^* α_2^* y_1^* $\approx \epsilon^3$

$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

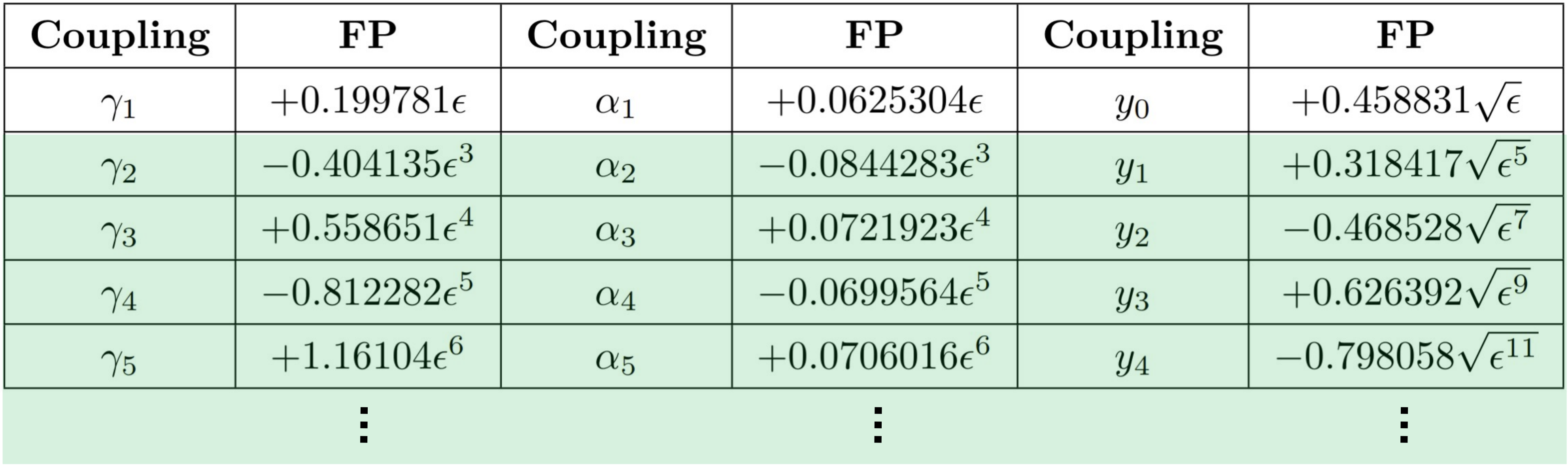
Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

Dim 2n: $\partial_t \lambda_{2n} = (2n-4)\lambda_{2n} + \lambda_{2n}\left(A\,\epsilon + \mathcal{O}(\epsilon^2)\right) + B\epsilon^n$ $\approx \epsilon^n$

Fixed Point & Power Counting in ε



Coupling	FP	Coupling	FP	Coupling	FP
γ_1	$+0.199781\epsilon$	α_1	$+0.0625304\epsilon$	y_0	$+0.458831\sqrt{\epsilon}$
γ_2	$-0.404135\epsilon^3$	α_2	$-0.0844283\epsilon^3$	y_1	$+0.318417\sqrt{\epsilon^5}$
γ_3	$+0.558651\epsilon^4$	α_3	$+0.0721923\epsilon^4$	y_2	$-0.468528\sqrt{\epsilon^7}$
γ_4	$-0.812282\epsilon^5$	α_4	$-0.0699564\epsilon^5$	y_3	$+0.626392\sqrt{\epsilon^9}$
γ_5	$+1.16104\epsilon^6$	α_5	$+0.0706016\epsilon^6$	y_4	$-0.798058\sqrt{\epsilon^{11}}$
	$\vdots$		$\vdots$		$\vdots$

Vacuum stability at the UV fixed point

Vacuum stability at the UV fixed point

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_c}{N_F} D^2\,\rho^2 \log(1 + D\,\rho)$$

$A \equiv 2\alpha_1^*$

$B \equiv 2\alpha_1^* + 4\gamma_1^*$

$D \equiv \alpha_y^*$

daniele.rizzo@ncbj.gov.pl

Vacuum stability at the UV fixed point

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_c}{N_F} D^2\,\rho^2 \log(1 + D\,\rho)$$

$$c^*(\rho) = \gamma_1^* + \frac{C^2}{4}\log(1 + B\rho) + \dots$$

$A \equiv 2\alpha_1^*$ $\quad B \equiv 2\alpha_1^* + 4\gamma_1^*$ $\quad C \equiv B - A$ $\quad D \equiv \alpha_y^*$

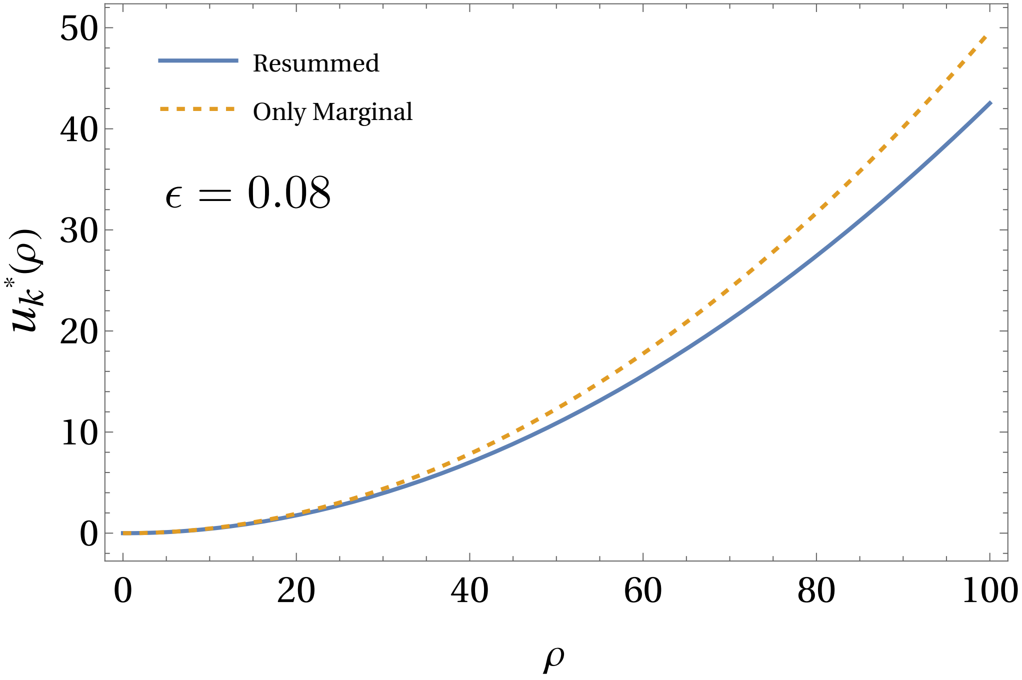
Vacuum stability at the UV fixed point

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^*\rho^2 + \frac{A^2\,\rho^2}{4}\log(1+A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1+B\,\rho) - \frac{N_c}{N_F}D^2\,\rho^2\log(1+D\,\rho)$$

$$c^*(\rho) = \gamma_1^* + \frac{C^2}{4}\log(1+B\rho) + \dots$$

$$A \equiv 2\alpha_1^* \qquad B \equiv 2\alpha_1^* + 4\gamma_1^*$$



daniele.rizzo@ncbj.gov.pl

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

θ_{γ_1}	4.03859ϵ	θ_{α_1}	2.94059ϵ	θ_{y_0}	2.73684ϵ
θ_{γ_2}	$2+5.50889\epsilon$	θ_{α_2}	$2+4.41089\epsilon$	θ_{y_1}	$2+2.83872\epsilon$
θ_{γ_3}	$4+6.97919\epsilon$	θ_{α_3}	$4+5.88119\epsilon$	θ_{y_2}	$4+4.30901\epsilon$
θ_{γ_4}	$6+8.44949\epsilon$	θ_{α_4}	$6+7.35148\epsilon$	θ_{y_3}	$6+5.77931\epsilon$
θ_{γ_5}	$8+9.91978\epsilon$	θ_{α_5}	$8+8.82178\epsilon$	θ_{y_4}	$8+7.24961\epsilon$
	$\vdots$		$\vdots$		$\vdots$

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

$$\alpha_{n-1}\,(\mathrm{Tr} H^\dagger H)^n$$

$$\gamma_{n-1}\,(\mathrm{Tr} H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2$$

$$y_n\,(\mathrm{Tr} H^\dagger H)^n\,\mathrm{Tr}(\bar{Q} H Q)$$

daniele.rizzo@ncbj.gov.pl

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

Critical Exponents

$$\alpha_{n-1}\,(\mathrm{Tr}H^\dagger H)^n$$

$$\theta_{\alpha_{n-1}} = (2n-4) + n\gamma_M$$

$$\gamma_{n-1}\,(\mathrm{Tr}H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2$$

→

$$y_n\,(\mathrm{Tr}H^\dagger H)^n\,\mathrm{Tr}(\bar{Q}HQ)$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

Critical Exponents

$$\alpha_{n-1}\,(\mathrm{Tr}H^\dagger H)^n \qquad \theta_{\alpha_{n-1}} = (2n-4)\,+\,n\gamma_M$$

$$\gamma_{n-1}\,(\mathrm{Tr}H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2 \Rightarrow \theta_{\gamma_{n-1}} = (2n-4)\,+\,(n-2)\gamma_M + \gamma_m$$

$$y_n\,(\mathrm{Tr}H^\dagger H)^n\,\mathrm{Tr}(\bar{Q}HQ)$$

daniele.rizzo@ncbj.gov.pl

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

Critical Exponents

$$\alpha_{n-1}\,(\mathrm{Tr} H^\dagger H)^n \qquad \theta_{\alpha_{n-1}} = (2n-4) + n\gamma_M$$

$$\gamma_{n-1}\,(\mathrm{Tr} H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2 \Rightarrow \theta_{\gamma_{n-1}} = (2n-4) + (n-2)\gamma_M + \gamma_m$$

$$y_n\,(\mathrm{Tr} H^\dagger H)^n\,\mathrm{Tr}(\bar{Q} H Q) \qquad \theta_{y_n} = 2n + n\gamma_M + \left(\frac{\eta_H}{2} + \eta_Q\right)$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Conclusion

- It is not completely understood whether the conformal window of gauge-Yukawa theories closes because of vacuum instability or because of two FPs merging.

- The inclusion of beyond marginal operators can spoil the stability of the scalar potential.

- We have computed the FP of infinitely many higher dimensional operators at leading order in ϵ and found a power counting argument.

- Because of the power counting, it was possible to perform a re-summation of the scalar potential and study the stability for high values of the field: the potential remains stable!

daniele.rizzo@ncbj.gov.pl