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Istituto Nazionale di Fisica Nucleare

Gauge invariant quantum backreaction in U (1) axion inflation

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Outline

- Motivation
- U(1)-axion inflation model
- Second order solution in U(1)-axion inflation
- Numerical analysis
- Discussion and Conclusions

Motivation

- ➔ Inflation ⇒ remains the most attractive theory to explain the Cosmic Microwave Background observations.

- ➔ One of the particle physics models which can lead to a quasi-exponential expansion are in the class of axion-like particles.

- ➔ Axion Inflation ⇒ when an axion-like inflation field is coupled to an Abelian gauge field through a Chern-Simons interaction.

  - ◆ In the presence of this coupling, gauge fields are copiously produced by the rolling of inflation; Due to the Schwinger effect, these strong electric fields produce pairs of charged particles and antiparticles and quickly form an ultrarelativistic plasma.

  - ◆ Which in turn, backreacts on the inflaton equation of motion (EOM).
    Such a plasma efficiently screens electric fields and, therefore, strongly influences the generation and evolution of the gauge fields, especially near the end of inflation and during reheating.

U(1)-axion inflation model

Let us consider a pseudo-scalar inflaton coupled to U(1) abelian gauge fields described by the following action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g\phi}{4}F^{\mu\nu}\tilde{F}_{\mu\nu}\right], \quad (1)$$

Axion-like field **Gauge-fields** **Interaction term**

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}, \quad F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha, \quad \text{and} \quad A_\mu$ are the gauge fields.

U(1)-axion inflation model: what is the standard procedure in the literature?

1) Consider a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) background metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 \,, \qquad (2)$$

with a(t) the scale factor, and we will evaluate the impact of the gauge fields production on the background Euler-Lagrange and Einstein equations.

Notation:

$$\mathbf{E} = -\frac{\mathbf{A}'}{a^2} \,, \qquad \mathbf{B} = \frac{\nabla \times \mathbf{A}}{a^2} \,, \qquad (3)$$

one obtains

$$\phi'' + 2aH\phi' - \nabla^2\phi + a^2 V_\phi(\phi) = g a^2 \mathbf{E}\cdot\mathbf{B} \,, \qquad (4)$$

$$\mathbf{E}' + 2aH\mathbf{E} - \nabla \times \mathbf{B} = -g\phi'\mathbf{B} - g\nabla\phi \times \mathbf{E} \,, \qquad (5)$$

$$\nabla \cdot \mathbf{E} = -g(\nabla\phi)\cdot\mathbf{B} \,, \qquad (6)$$

while from the Bianchi identities one obtains

$$\mathbf{B}' + 2aH\mathbf{B} + \nabla \times \mathbf{E} = 0 \,, \qquad (7)$$

$$\nabla \cdot \mathbf{B} = 0 \,, \qquad (8)$$

U(1)-axion inflation model: what is the standard procedure in the literature?

To understand the impact of the gauge fields on the Einstein equations one has to evaluate their energy-momentum tensor

$$T_{00}^{(F)} = \frac{\mathbf{E}^2 + \mathbf{B}^2}{2}, \qquad (14)$$

$$T_{ij}^{(F)} = -E_i E_j - B_i B_j + \delta_{ij} \frac{\mathbf{E}^2 + \mathbf{B}^2}{2}. \qquad (15)$$

Taking into account only the backreaction of the gauge fields on the background, we can then rewrite the Friedmann equations in the Hartree approximation as

$$H^2 = \frac{1}{3m_{Pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle}{2}\right], \qquad (17)$$

$$\dot{H} = -\frac{1}{2m_{Pl}^2}\left[\dot{\phi}^2 + \frac{2}{3}\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle\right], \qquad (18)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = g\langle \mathbf{E} \cdot \mathbf{B} \rangle. \qquad (19)$$

Inflation field homogeneous $\nabla\phi = 0$

U(1)-axion inflation model: what is the standard procedure in the literature?

The impact of the gauge fields on the inflationary background is calculated in the Coulomb gauge $(A_0 = 0 \text{ and } \nabla \cdot \vec{A} = 0)$

$$\frac{\partial^2 \mathbf{A}}{\partial \tau^2} - \nabla^2 \mathbf{A} - g\phi' \boldsymbol{\nabla} \times \mathbf{A} = 0 \,. \qquad (9)$$

Then one promotes A(τ, x) to a quantum operator

$$\hat{\mathbf{A}}(\tau, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} \left[\boldsymbol{\epsilon}_\lambda(\mathbf{k}) A_\lambda(\tau, \mathbf{k}) \hat{a}_\lambda(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} + \text{h.c.} \right] , \qquad (10)$$

Finally, the equation of motion Fourier modes A± reads

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} A_\pm(\tau, k) + \left(k^2 \mp k g \phi'\right) A_\pm(\tau, k) = 0 \,. \qquad (12)$$

U(1)-axion inflation model: homogeneous case

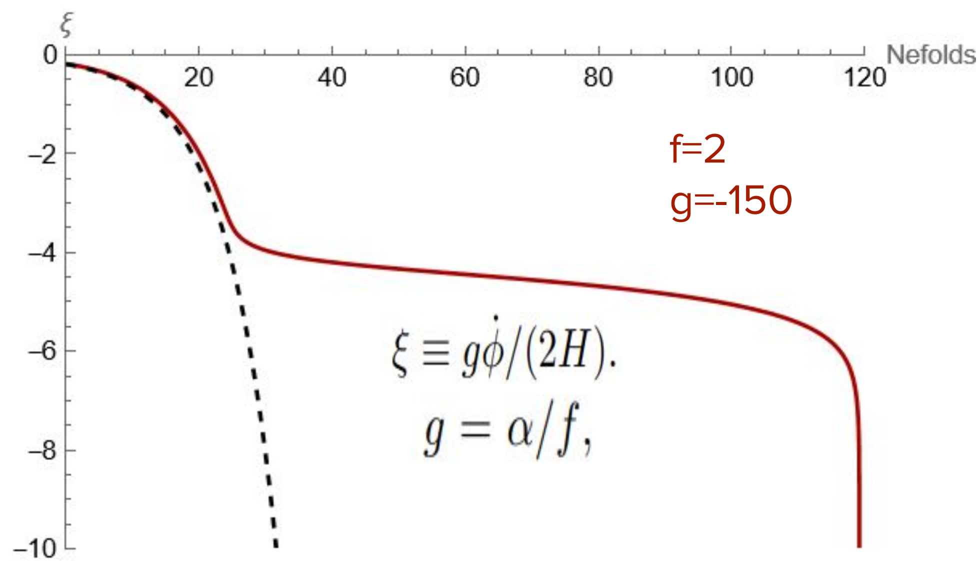
$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] \,,$$

Natural inflation

How to obtain the energy density and helicity contributions?

1) de Sitter background
2) the analytical solution of Eq.(12) is given in terms of Whittaker W − functions

$$A_{\pm}(\tau, k) = \frac{1}{\sqrt{2k}} e^{\pm\pi\xi/2} W_{\pm i\xi, \frac{1}{2}}(-2ik\tau) \,.$$



Without backreaction

3) Approximation:

$$\langle \vec{E} \cdot \vec{B} \rangle \simeq -2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi} \quad , \quad \langle \frac{\vec{E}^2 + \vec{B}^2}{2} \rangle \simeq 1.4 \cdot 10^{-4} \frac{H^4}{\xi^3} e^{2\pi\xi}$$

$$\phi' + \frac{V_\phi}{V} = 0$$

→

$$3H^2 = V(\phi)$$

Barnaby et al JCAP04 (2011) 009

Second order solution in U(1)-axion inflation

Gauge invariant quantum backreaction

One can construct a general non-local observable by taking quantum averages of a scalar field S(x) over a space-time hypersurface ΣA0 where A(x), another time-like scalar field, assumes a constant value A0.

The above **physical gauge invariant observable** is then defined by

$$\langle S \rangle_{A_0} = \frac{\langle \sqrt{|\overline{\gamma}(t_0, \mathbf{x})|}\ \overline{S}(t_0, \mathbf{x}) \rangle}{\langle \sqrt{|\overline{\gamma}(t_0, \mathbf{x})|} \rangle},$$

the determinant of the induced 3-dimensional metric

It is possible to show that the effective scale factor satisfies the following gauge independent effective equation

the expansion scalar of the time-like congruence

$$\left(\frac{1}{a_{eff}} \frac{\partial a_{eff}}{\partial A_0} \right)^2 = \frac{1}{9} \left\langle \frac{\Theta}{\sqrt{-\partial^\mu A \partial_\mu A}} \right\rangle^2_{A_0},$$

A is homogeneous

with $\Theta = \nabla_\mu n^\mu$ $\quad n^\mu = -\frac{\partial^\mu A}{\sqrt{-\partial^\nu A \partial_\nu A}},$

settles then the class of observers

The space-time dynamics can be studied in any gauge, simply ***by solving the matter and Einstein equations*** *in the chosen gauge.*

Second order solution in U(1)-axion inflation

Considering cosmological perturbation theory up to the second order, so we write the general perturbed metric around a FLRW space-time as

$$g_{00} = -1 - 2\alpha - 2\alpha^{(2)} ,$$
$$g_{i0} = -\frac{a}{2}(\beta_{,i} + B_i) - \frac{a}{2}\left(\beta^{(2)}_{,i} + B^{(2)}_i\right),$$
$$g_{ij} = a^2\left[\delta_{ij}\left(1 - 2\psi - 2\psi^{(2)}\right) + D_{ij}\left(E + E^{(2)}\right) + \frac{1}{2}(\chi_{i,j} + \chi_{j,i} + h_{ij}) + \frac{1}{2}\left(\chi^{(2)}_{i,j} + \chi^{(2)}_{j,i} + h^{(2)}_{ij}\right)\right],$$

At the same time, the inflaton can be written to second order as

$$\phi(t, \mathbf{x}) = \phi(t) + \varphi(t, \mathbf{x}) + \varphi^{(2)}(t, \mathbf{x}) .$$

Long wavelength (LW) limit for scalar fluctuations produces:

$$\bar{\Theta} = 3H - 3H\bar{\alpha} - 3\dot{\bar{\psi}} + \frac{9}{2}H\bar{\alpha}^2 + 3\bar{\alpha}\dot{\bar{\psi}} - 6\bar{\psi}\dot{\bar{\psi}} - 3H\bar{\alpha}^{(2)} - 3\dot{\bar{\psi}}^{(2)} - \frac{1}{8}h_{ij}\dot{h}^{ij} ,$$
$$-\partial_\mu \bar{A}\partial^\mu \bar{A} = 1 - 2\bar{\alpha} + 4\bar{\alpha}^2 - 2\bar{\alpha}^{(2)} ,$$
$$\sqrt{|\bar{\gamma}|} = a^3\left(1 - 3\bar{\psi} + \frac{3}{2}\bar{\psi}^2 - \frac{1}{16}h^{ij}h_{ij} - 3\bar{\psi}^{(2)}\right) .$$

Gauge invariant construction

Neglecting tensor perturbations

$$H^2_{eff} = \left(\frac{1}{a_{eff}}\frac{\partial a_{eff}}{\partial A_0}\right)^2 = H^2\left[1 + \frac{2}{H}\langle\bar{\psi}\dot{\bar{\psi}}\rangle - \frac{2}{H}\langle\dot{\bar{\psi}}^{(2)}\rangle\right] .$$

scalar perturbations of the metric

Second order solution in U(1)-axion inflation

Going up to second order in the field and scalar metric perturbations***

- The uniform curvature gauge (UCG) ⇒ $\psi = E = 0$
- The Einstein tensor terms remain the usual ones, while the energy-momentum tensor is modified by the presence of the gauge fields.
- We consider the weak backreaction regime and neglect the background contribution of the gauge fields ⇒ we will treat expressions like $\mathbf{E}^2$, $\mathbf{B}^2$, $\mathbf{E} \cdot \mathbf{B}$ as second order perturbations.
- Comoving observer: homogeneous inflaton field for which ⇒ $\varphi = \varphi^{(2)} = 0$

To finally obtain

$$H_{\text{eff}}^2 = H^2 \left[1 + \frac{1}{3H^2} \left(\frac{1}{M_{Pl}^2} \frac{\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle}{2} - \frac{g^2}{\xi} \langle \mathbf{E} \cdot \mathbf{B} \rangle \right) \right]$$

Second order solution in U(1)-axion inflation

How to obtain the energy density and helicity contributions?

1) de Sitter background
2) the analytical solution of Eq.(12) is given in terms of Whittaker W − functions

$$A_\pm(\tau,k) = \frac{1}{\sqrt{2k}} e^{\pm\pi\xi/2} W_{\pm i\xi,\frac{1}{2}}(-2ik\tau)\,.$$

3) perform a proper renormalization procedure by subtracting to the bare results of the energy density and of the helicity integral their adiabatic counterparts as described in Animali et al. 2022.

$$\begin{aligned}\frac{\langle \mathbf{E}^2+\mathbf{B}^2\rangle}{2} &= \frac{2H^4}{960\pi^2} + \frac{H^4\xi^2\left(-1185\xi^4+(330+4\sqrt{15})\xi^2+435+4\sqrt{15}\right)}{960\pi^2\left(1+\xi^2\right)} \\ &- \frac{3H^4\xi^2\left(5\xi^2-1\right)\log\left(15/4\right)}{64\pi^2} + \frac{H^4\xi\left(30\xi^2-11\right)\sinh(2\pi\xi)}{64\pi^3} \\ &- \frac{3H^4\xi^2\left(5\xi^2-1\right)\left(\psi(-1-i\xi)+\psi(-1+i\xi)\right)}{32\pi^2} \\ &+ \frac{3iH^4\xi^2\left(5\xi^2-1\right)\left(\psi^{(1)}(1-i\xi)-\psi^{(1)}(1+i\xi)\right)\sinh(2\pi\xi)}{64\pi^3}\,,\end{aligned}$$

$$\begin{aligned}\langle \mathbf{E}\cdot\mathbf{B}\rangle &= \frac{H^4\xi\left(705\xi^2-330-\sqrt{15}\right)}{240\pi^2} + \frac{3H^4\xi\left(5\xi^2-1\right)\log\left(15/4\right)}{32\pi^2} \\ &+ \frac{3H^4\xi\left(5\xi^2-1\right)\left(\psi(1-i\xi)+\psi(1+i\xi)\right)}{16\pi^2} \\ &+ \frac{3iH^4\xi\left(5\xi^2-1\right)\left(-\psi^{(1)}(1-i\xi)+\psi^{(1)}(1+i\xi)\right)\sinh(2\pi\xi)}{32\pi^3} \\ &+ \frac{H^4\left(11-30\xi^2\right)\sinh(2\pi\xi)}{32\pi^3}\,.\end{aligned}$$

Numerical analysis

System of equations (reduced):

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0\,, \\ H_{\rm eff}^2 = H^2\left[1 + \frac{1}{3H^2}\left(\frac{1}{M_{Pl}^2}\frac{\langle \mathbf{E}^2 + \mathbf{B}^2\rangle}{2} - \frac{g^2}{\xi}\langle \mathbf{E}\cdot\mathbf{B}\rangle\right)\right] \end{cases}$$

$$\xi' + \left(\frac{H'}{H} + 3\frac{H}{H_{\rm eff}}\right)\xi + \frac{g}{2}\frac{V_{,\phi}}{H\,H_{\rm eff}} = 0\,,$$

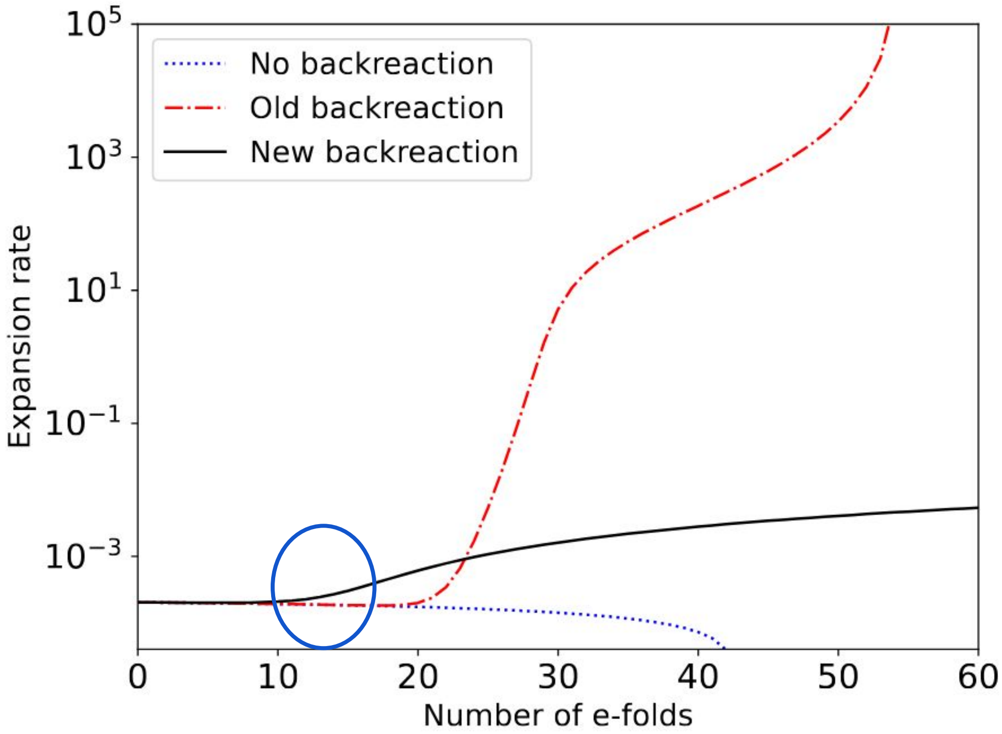
$$\phi' = \frac{2}{g}\frac{H}{H_{\rm eff}}\xi\,,$$

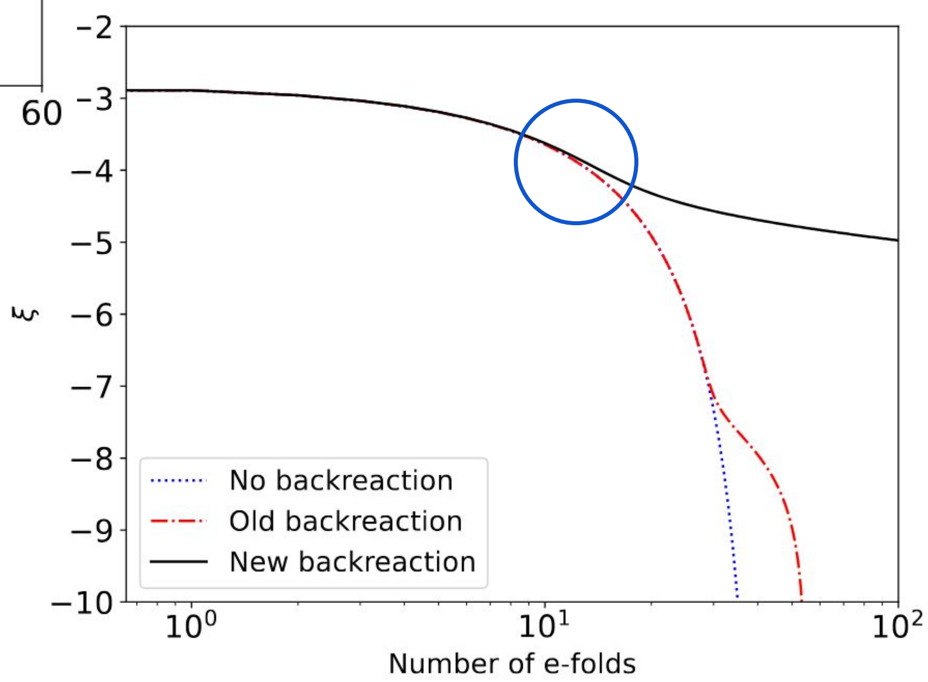
$$\mathrm{d}N_{\rm eff} \equiv H_{\rm eff}\,\mathrm{d}t\,,$$

$$\rho_{\rm V} \equiv V(\phi), \qquad \rho_{\rm k} \equiv \frac{\dot{\phi}^2}{2}, \quad \rho_{\rm EM} \equiv \frac{\langle \mathbf{E}^2 + \mathbf{B}^2\rangle}{2},$$

$$\rho_{\rm EB} \equiv -\frac{g^2}{\xi}\langle \mathbf{E}\cdot\mathbf{B}\rangle, \qquad \text{and} \qquad \rho_T = 3H_{\rm eff}^2\,. \qquad (70)$$

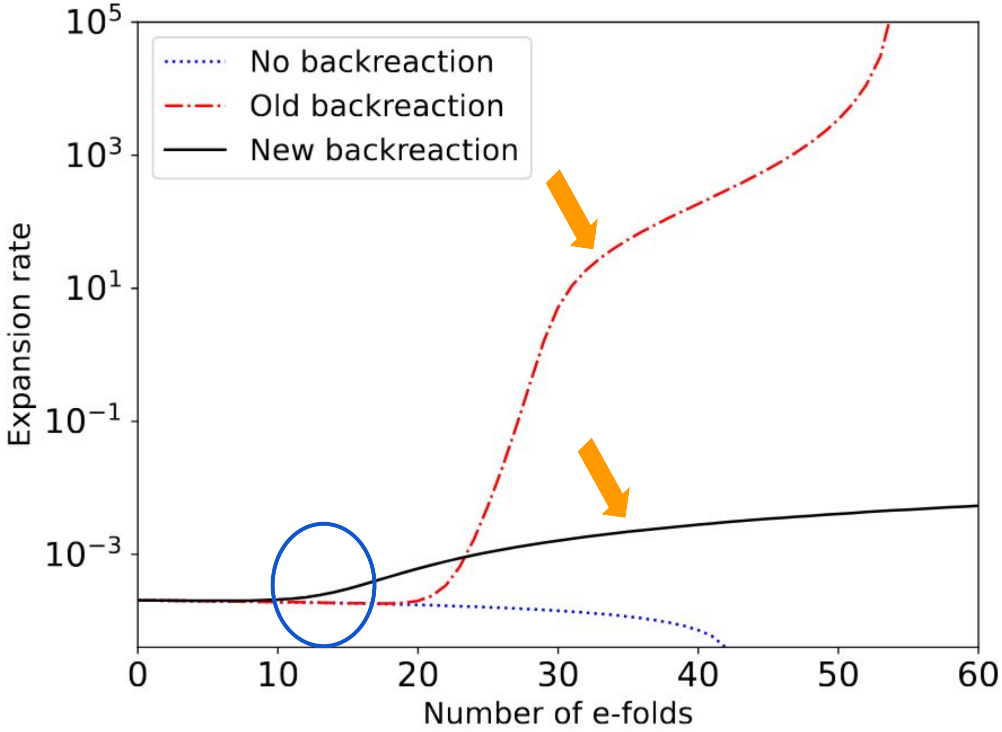
Results





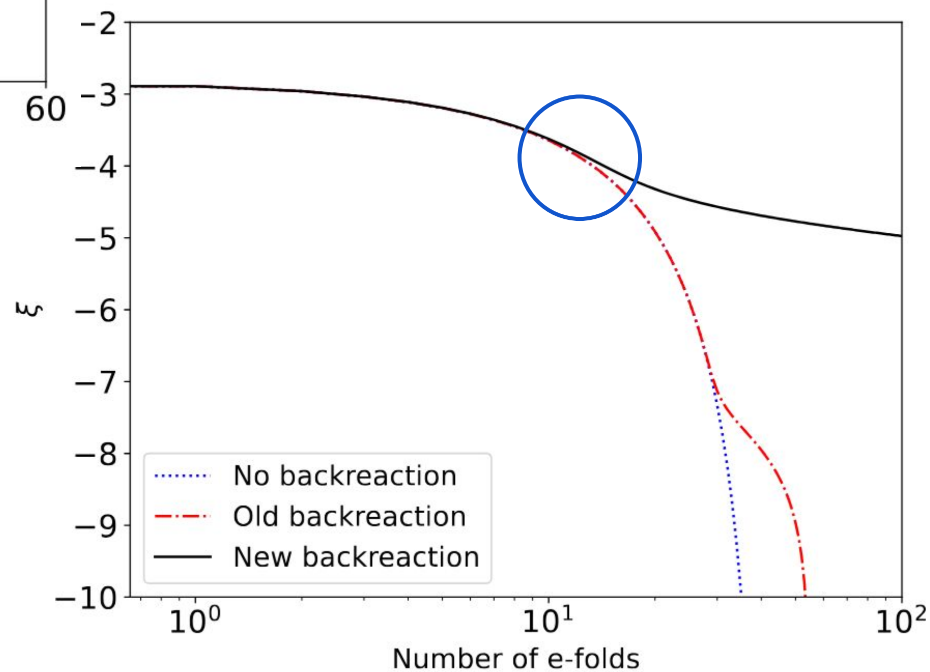
After N ~ 10 − 15 e-folds, considering the gauge fields contribution as a perturbative contribution is not valid anymore!

Results

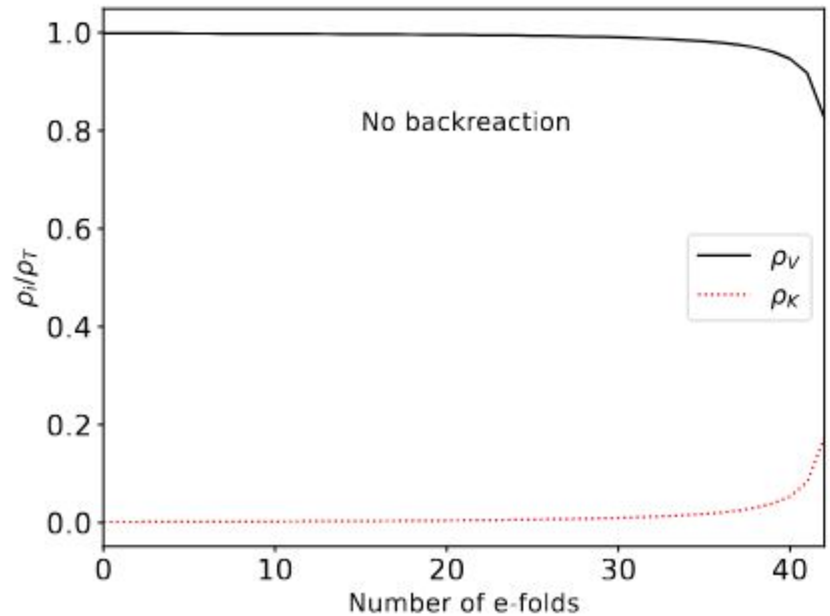


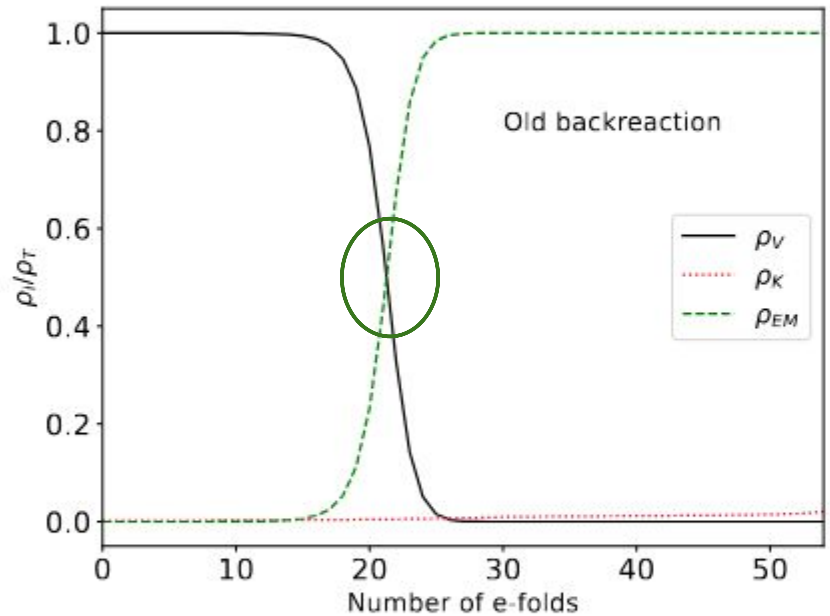
This difference happens probably because we did not consider the coupling between the scalar fluctuations of the matter/gravity sector with the gauge fields consistently.

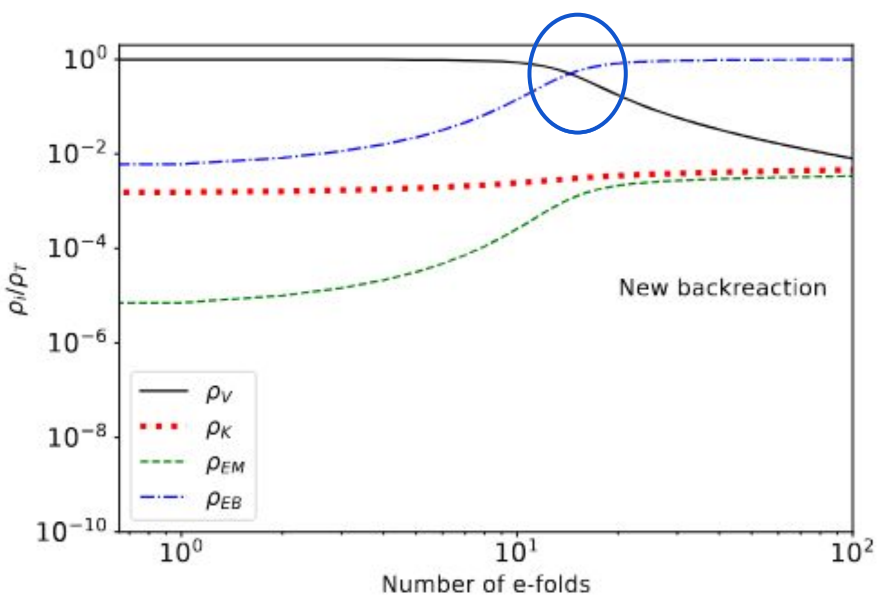
After N ~ 10 − 15 e-folds, considering the gauge fields contribution as a perturbative contribution is not valid anymore!



Results







Backreaction became non-perturbative already after N ~ 10 – 15 e-folds.

Conclusions

1. The cosmological backreaction became non-perturbative already after N ~ 10 − 15 e-folds. When this happens our assumption of considering the gauge fields contribution as a perturbative contribution is not valid anymore.

2. To go beyond this weak backreaction regime, considering all the possible contributions, one possibility could be to define an effective background for the gauge fields' contribution and write a perturbative theory on top of it, to take into consideration the coupling between the scalar fluctuations of the matter/gravity sector with the gauge fields effective fluctuations consistently.

3. The above approach is necessary to consider, in such a case, not only the backreaction of the gauge fields on the inflationary dynamics but also the backreaction that the scalar fluctuations of the matter/gravity sector have on the gauge fields' production.

4. The approach performed gives us the possibility to study the weak backreaction regime and understand when this is not valid anymore, without missing part of the effect coming from scalar induced peturbations.

GEF + axion inhomogeneities

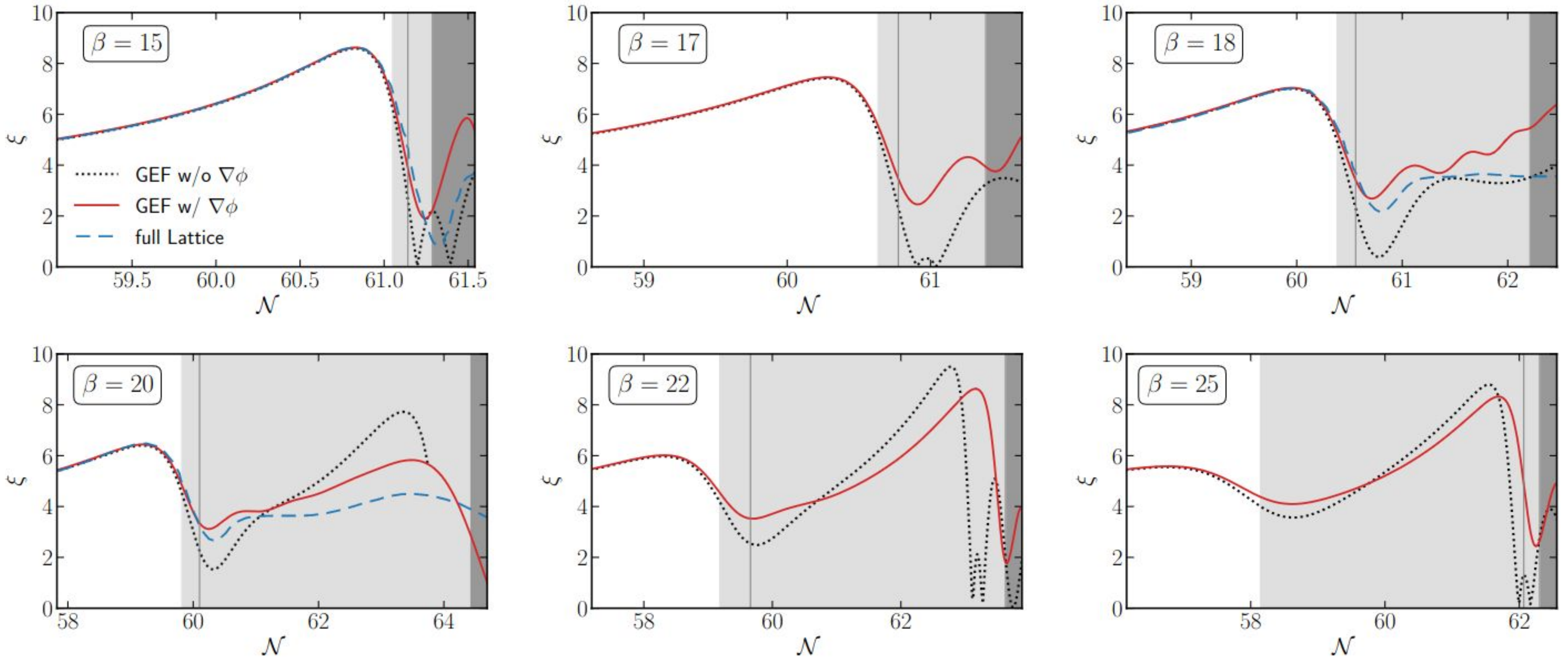


Figure 1: Evolution of ξ assuming a homogeneous axion field (dashed black) and perturbatively including axion gradients (red) for different values of β. The light (dark) gray region indicates that the gradient energy of the axion exceeds 1 % (50 %) of the kinetic energy, while the gray vertical line corresponds to 5 %. Wherever possible we compare to the result of the lattice simulation [35].

Domcke et al. JCAP 03 (2024) 019


To be continued...
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EXTRAS

U(1)-axion inflation model: iterative approach! [Domcke et al 2020 JCAP 09 (2020) 009]

Iterative approach for a chaotic inflaton potential: $V(\phi) = m^2\phi^2/2$

Coupled system

$$\frac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \quad \text{with} \quad \xi \equiv \frac{\lambda\phi'}{2f} > 0\,,$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} - \frac{1}{fH^2}\langle\vec{E}\vec{B}\rangle = 0\,,$$

$$3H^2 M_P^2 = V(\phi) + \frac{1}{2}H^2(\phi')^2 + \left\langle\frac{E^2+B^2}{2}\right\rangle,$$

1)

$$A_{-\lambda}(\tau,\vec{k}) = \frac{e^{\pi\xi/2}}{\sqrt{2k}} W_{-i\xi,1/2}(2ik\tau)$$

$$\langle\vec{E}\vec{B}\rangle_{(0)} = \frac{1}{2^{21}\pi^2}\frac{H_0^4}{\xi^4} e^{2\pi\xi}\int_0^{8\xi} x^7 e^{-x}dx\,,$$

$$\langle\rho_{EB}\rangle_{(0)} = \langle\frac{E^2+B^2}{2}\rangle_{(0)} = \frac{6!}{2^{19}\pi^2}\frac{H_0^4}{\xi^3}e^{2\pi\xi}\,,$$

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1

$$A_{-\lambda}(\tau,\vec{k}) = \frac{e^{\pi\xi/2}}{\sqrt{2k}} W_{-i\xi,1/2}(2ik\tau)$$

$$\phi''_{(j)} + (3-\epsilon_{(j)})\phi'_{(j)} + \frac{1}{H^2_{(j)}}\left(V_\phi(\phi_{(j)}) + \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle_{(j-1)}\right) = 0\,,$$

$$H^2_{(j)} = \frac{V(\phi_{(j)}) + \langle\rho_{EB}\rangle_{(j-1)}}{3 - \frac{\phi'^2_{(j)}}{2}}\,; \qquad \epsilon_{(j)} = \frac{1}{2}\phi'^2_{(j)} + \frac{2}{3H^2_{(j)}}\langle\rho_{EB}\rangle_{(j-1)}\,.$$

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U(1)-axion inflation model: iterative approach!

[Domcke et al 2020 JCAP 09 (2020) 009]

Iterative approach for a chaotic inflaton potential: $V(\phi) = m^2\phi^2/2$

Coupled system

$$A''_{k,\pm} + (1-\epsilon_{(j)})A'_{k,\pm} + \frac{k}{aH_{(j)}}\left(\frac{k}{aH_{(j)}} \mp 2\,\xi_{(j)}(N)\right)A_{k,\pm} = 0\,.$$

$$\phi''_{(j)} + (3-\epsilon_{(j)})\phi'_{(j)} + \frac{1}{H^2_{(j)}}\left(V_\phi(\phi_{(j)}) + \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle_{(j-1)}\right) = 0\,,$$

$$H^2_{(j)} = \frac{V(\phi_{(j)}) + \langle\rho_{EB}\rangle_{(j-1)}}{3 - \frac{\phi'^2_{(j)}}{2}}\,; \qquad \epsilon_{(j)} = \frac{1}{2}\phi'^2_{(j)} + \frac{2}{3H^2_{(j)}}\langle\rho_{EB}\rangle_{(j-1)}\,.$$

1

$$A_{-\lambda}(\tau,\vec{k}) = \frac{e^{\pi\xi/2}}{\sqrt{2k}}W_{-i\xi,1/2}(2ik\tau)\,.$$

2

$$\langle\vec{E}\vec{B}\rangle_{(0)} = \frac{1}{2^{21}\pi^2}\frac{H_0^4}{\xi^4}e^{2\pi\xi}\int_0^{8\xi}x^7e^{-x}dx\,,$$

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3

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$$A''_{k,\pm} + (1-\epsilon_{(j)})A'_{k,\pm} + \frac{k}{aH_{(j)}}\left(\frac{k}{aH_{(j)}} \mp 2\,\xi_{(j)}(N)\right)A_{k,\pm} = 0\,.$$

4

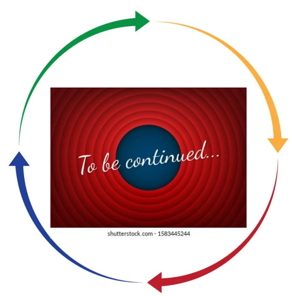
$$\langle \vec{E}\vec{B}\rangle_{(j)} = \sigma\frac{H_{(j)}}{4\pi^2 a^3}\sum_{i=1}^{M} d\ln k_i\, k_i^3\,\frac{\partial}{\partial N}|A^{\sigma}_{k_i}|^2\theta\,(N-N_i)$$

3

$$\langle \rho_{EB}\rangle_{(j)} = \frac{1}{4\pi^2 a^4}\sum_{i=1}^{M} d\ln k_i\,\left(k_i^3 a^2 H_{(j)}^2|A'^{\sigma}_{k_i}|^2 + k_i^5|A^{\sigma}_{k_i}|^2 - k_i^4\right)\theta\,(N-N_i)$$

$$\phi''_{(j)} + (3-\epsilon_{(j)})\phi'_{(j)} + \frac{1}{H_{(j)}^2}\left(V_\phi(\phi_{(j)}) + \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle_{(j-1)}\right) = 0\,,$$

$$H_{(j)}^2 = \frac{V(\phi_{(j)}) + \langle\rho_{EB}\rangle_{(j-1)}}{3-\frac{\phi'^2_{(j)}}{2}}\,; \qquad \epsilon_{(j)} = \frac{1}{2}\phi'^2_{(j)} + \frac{2}{3H_{(j)}^2}\langle\rho_{EB}\rangle_{(j-1)}\,.$$



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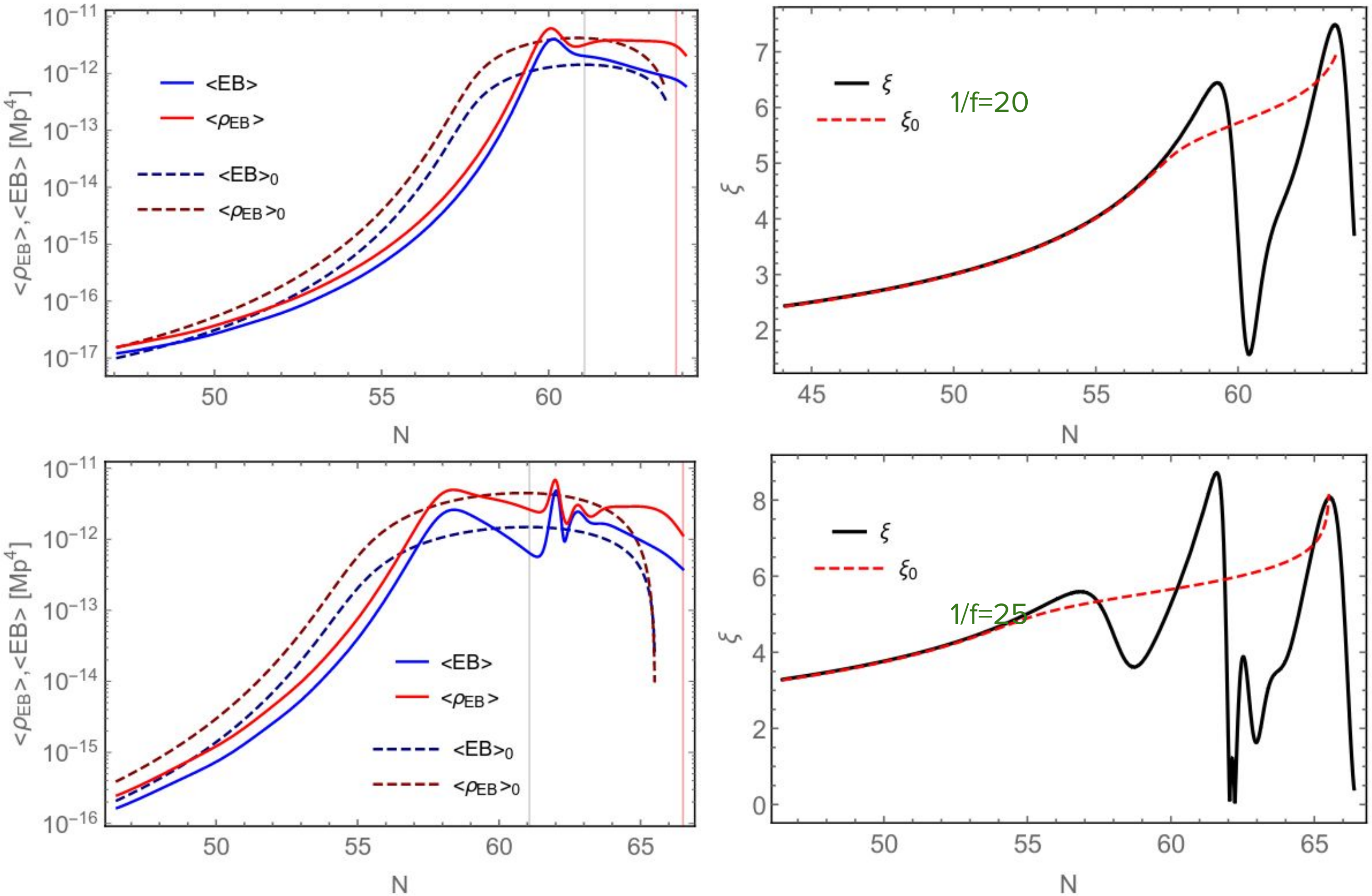

Gradient Expansion Formalism

To handle the complicated dynamics of the inflaton and gauge fields, taking into account the Schwinger effect and the backreaction of the generated gauge fields and primordial plasma on the cosmological evolution, a novel gradient expansion formalism was developed.

In the standard approach we have seen that one works with separate Fourier modes of the gauge field which evolve in a given inflationary background.

In contrast, in the gradient expansion formalism (GEF), one considers vacuum expectation values of a truncated set of bilinear functions of the electric and magnetic fields in coordinate space that include all physically relevant modes at once.

Important: the number of relevant modes constantly grows during inflation as new modes cross the horizon and undergo the quantum to classical transition.

To account for the growth of the number of relevant modes outside the horizon, *boundary terms* are added to the equations of motion for the electromagnetic bilinear functions.

Advantage: it does not rely on an iterative procedure that needs to be repeated over and over again before it converges to a self-consistent result.

Gorbar et al. Phys.Rev.D 104 (2021) 12, 123504

Gradient Expansion Formalism

Everything said translates to:

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{E}\rangle\,,$$

$$\mathcal{G}^{(n)} = -\frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle\,,$$

$$\mathcal{B}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{B}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle$$

Coupled system

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = I'(\phi)\,\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle\,,$$

$$\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\mathrm{rot}\boldsymbol{B} + I'(\phi)\,\dot{\phi}\,\boldsymbol{B} + \boldsymbol{J} = 0,$$

$$\dot{\boldsymbol{B}} + 2H\boldsymbol{B} + \frac{1}{a}\mathrm{rot}\boldsymbol{E} = 0,$$

$$\mathrm{div}\boldsymbol{E} = 0, \qquad \mathrm{div}\boldsymbol{B} = 0.$$

New Coupled system

$$\dot{\mathcal{E}}^{(n)} + [(n+4)H + 2\sigma]\,\mathcal{E}^{(n)} - 2I'(\phi)\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_b,$$

$$\dot{\mathcal{G}}^{(n)} + [(n+4)H + \sigma]\,\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - I'(\phi)\dot{\phi}\,\mathcal{B}^{(n)} = [\dot{\mathcal{G}}^{(n)}]_b,$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_b.$$

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infinite chain that needs to be truncated**

Gradient Expansion Formalism

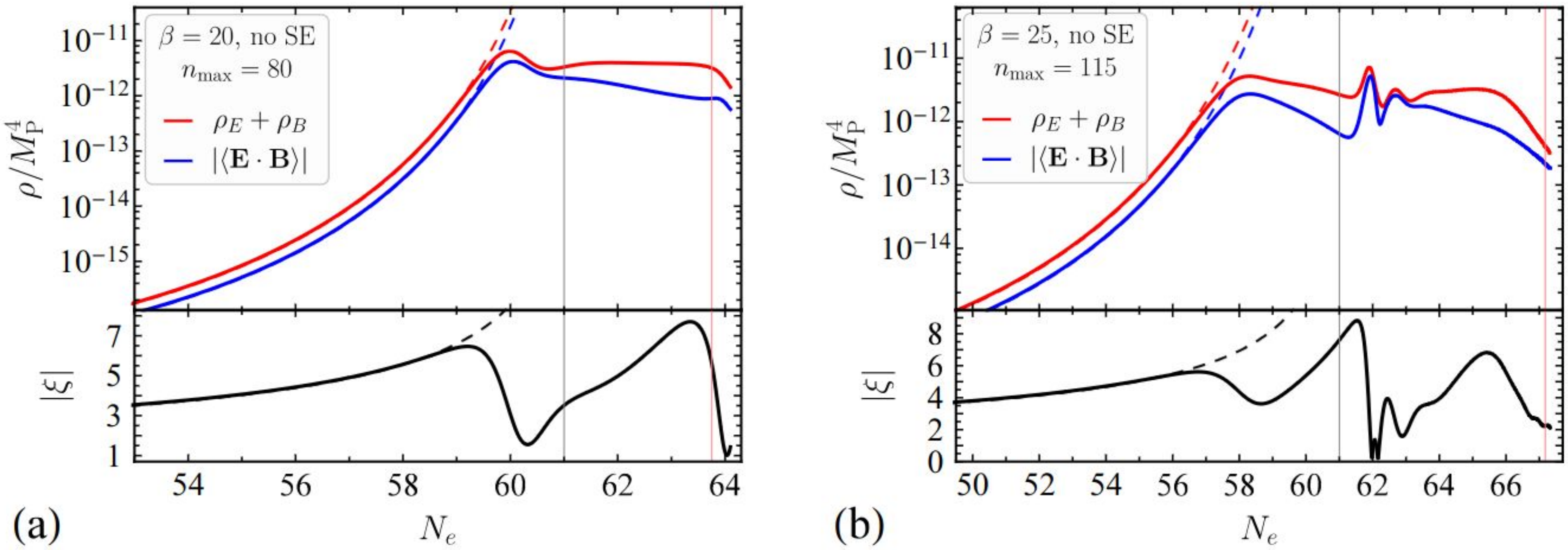

Figure 5. Top plots: the total electromagnetic energy density (red solid lines) and the scalar product $|\langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle|$ (blue solid lines) as functions of N_e generated in the axial coupling model with (a) $\beta = 20$ and (b) $\beta = 25$ in the absence of the Schwinger effect. Bottom plots show the absolute value of the parameter ξ. The corresponding dashed lines show the same dependences in the absence of backreaction. The pink vertical lines mark the end of inflation in each case while the gray vertical lines show the end of inflation in the absence of backreaction. These solutions obtained from the gradient expansion formalism are in good accordance with the results of the iterative solution of the mode equation (40), presented in Ref. [60], cf. Fig. 6 there.

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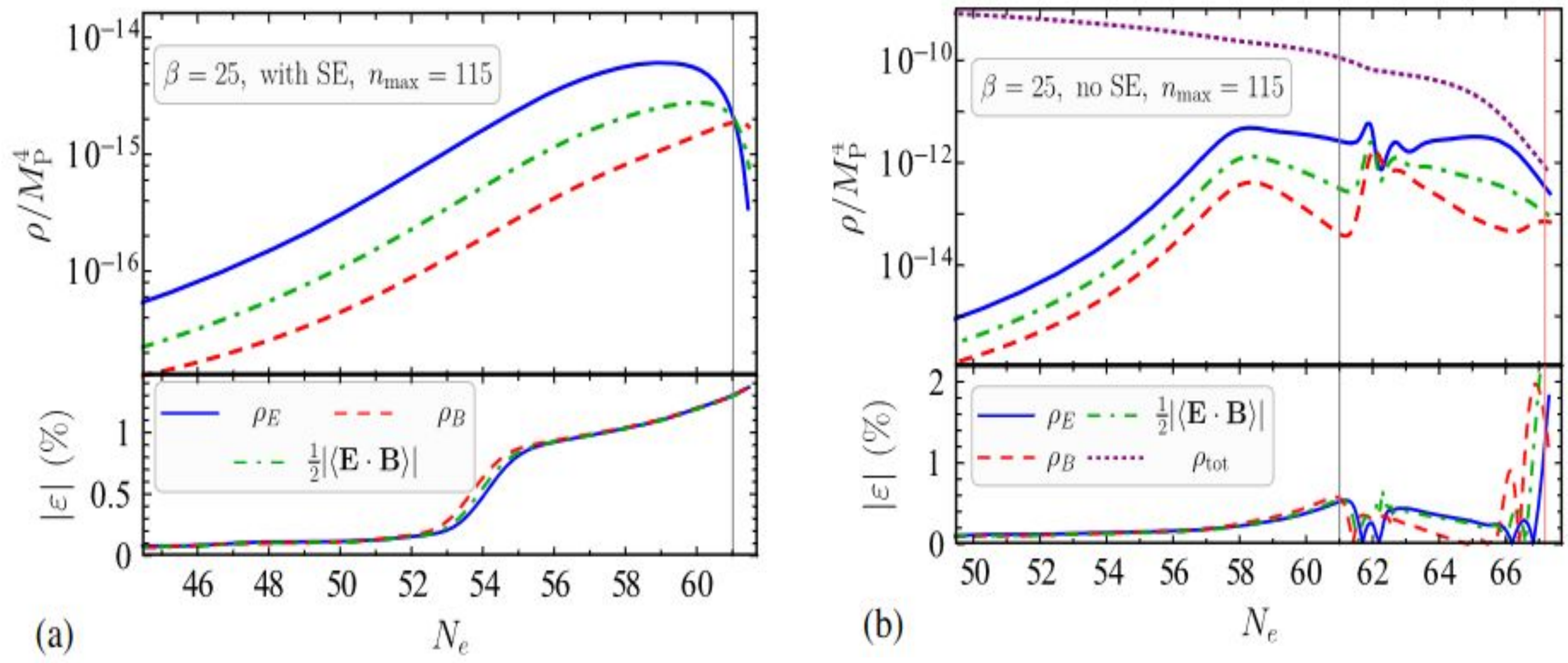
Gradient Expansion Formalism

$$\dot{\rho}_\chi + 4H\rho_\chi = \sigma\mathcal{E}^{(0)},$$



The Schwinger effect significantly suppresses magnetogenesis!

As a result, the inflation stage has the same duration as in the unperturbed case.

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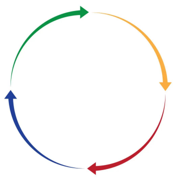
GEF + axion inhomogeneities

Domcke et al. JCAP 03 (2024) 019

Backreaction induced by the gauge fields on the axion

1. Changes in the axion velocity impact gauge field modes withi tachyonic instability window, which contribute to the friction fc
2. As a result, the friction term reacts with some time delay to the changes in the axion velocity, leading to a resonantly coupled system with distinct peaks in the axion velocity.



Previously: the axion field is taken to be homogeneous ⇒ *breaks down in the strong backreaction regime!*

rapid growth of the axion perturbations ⇒ significant departure from the standard slow-roll regime and the strong non-linearities involved

GEF + axion inhomogeneities

Domcke et al. JCAP 03 (2024) 019

Backreaction induced by the gauge fields on the axion

Using the GEF to perturbatively include axion gradients requires evolving not only the 2-point functions but also higher p-point functions:

$$\mathcal{P}_X^{(n)} = \frac{1}{a^n}\left\langle \vec{X}\cdot(\vec{\nabla}\times)^n\vec{X}\right\rangle\,, \quad \mathcal{P}_{XY}^{(n)} = -\frac{1}{a^n}\left\langle \vec{X}\cdot(\vec{\nabla}\times)^n\vec{Y}\right\rangle\,.$$

The main idea of this approach is to self-consistently determine the breakdown of perturbativity.

GEF + axion inhomogeneities

Domcke et al. JCAP 03 (2024) 019

Matter sector

$$0 = \ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi - \frac{\beta}{M_P}\left\langle \vec{E}\cdot\vec{B} \right\rangle ,$$

$$0 = \ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2\chi}{a^2} + m_\phi^2\chi - \frac{\beta}{M_P}\left(\vec{E}\cdot\vec{B} - \left\langle \vec{E}\cdot\vec{B} \right\rangle\right) ,$$

$$0 = \dot{\vec{E}} + 2H\vec{E} - \frac{1}{a}\vec{\nabla}\times\vec{B} + \frac{\beta}{M_P}\left(\dot{\phi} + \dot{\chi}\right)\vec{B} + \frac{\beta}{M_P}\frac{1}{a}\vec{\nabla}\chi\times\vec{E} ,$$

$$0 = \dot{\vec{B}} + 2H\vec{B} + \frac{1}{a}\vec{\nabla}\times\vec{E} ,$$

$$0 = \vec{\nabla}\cdot\vec{E} + \frac{\beta}{M_P}\vec{\nabla}\chi\cdot\vec{B} , \quad 0 = \vec{\nabla}\cdot\vec{B} ,$$

Gravity sector

$$H^2 = \frac{1}{3M_P^2}\left\langle \frac{1}{2}\left(\dot{\phi}^2 + \dot{\chi}^2\right) + \frac{(\partial_i\chi)^2}{2a^2} + \frac{m_\phi^2}{2}\left(\phi^2 + \chi^2\right) + \frac{1}{2}\left(|\vec{E}|^2 + |\vec{B}|^2\right) \right\rangle ,$$

$$\dot{H} = -\frac{1}{6M_P^2}\left\langle 3\left(\dot{\phi}^2 + \dot{\chi}^2\right) + \frac{(\partial_i\chi)^2}{a^2} + 2\left(|\vec{E}|^2 + |\vec{B}|^2\right) \right\rangle ,$$

GEF + axion inhomogeneities

$$\dot{\mathcal{P}}_E^{(n)} + (n+4)H\mathcal{P}_E^{(n)} - \frac{2\beta\dot{\phi}}{M_P}\mathcal{P}_{EB}^{(n)} + 2\mathcal{P}_{EB}^{(n+1)} = \left[\dot{\mathcal{P}}_E^{(n)}\right]_b ,$$
$$\dot{\mathcal{P}}_B^{(n)} + (n+4)H\mathcal{P}_B^{(n)} - 2\mathcal{P}_{EB}^{(n+1)} = \left[\dot{\mathcal{P}}_B^{(n)}\right]_b ,$$
$$\dot{\mathcal{P}}_{EB}^{(n)} + (n+4)H\mathcal{P}_{EB}^{(n)} - \mathcal{P}_E^{(n+1)} + \mathcal{P}_B^{(n+1)} - \frac{\beta\dot{\phi}}{M_P}\mathcal{P}_B^{(n)} = \left[\dot{\mathcal{P}}_{EB}^{(n)}\right]_b .$$

Matter sector

$$\dot{\mathcal{P}}_E^{(0)} + 4H\mathcal{P}_E^{(0)} + 2\mathcal{P}_{EB}^{(1)} - \frac{2\beta\dot{\phi}}{M_P}\mathcal{P}_{EB}^{(0)} - \frac{2\beta}{M_P}\mathcal{B}_{\dot{\chi};EB}^{(0)} = \left[\dot{\mathcal{P}}_E^{(0)}\right]_b ,$$
$$\dot{\mathcal{P}}_B^{(0)} + 4H\mathcal{P}_B^{(0)} - 2\mathcal{P}_{EB}^{(1)} = \left[\dot{\mathcal{P}}_B^{(0)}\right]_b , \quad (2.14)$$
$$\dot{\mathcal{P}}_{EB}^{(0)} + 4H\mathcal{P}_{EB}^{(0)} - \mathcal{P}_E^{(1)} + \mathcal{P}_B^{(1)} - \frac{\beta\dot{\phi}}{M_P}\mathcal{P}_B^{(0)} - \frac{\beta}{M_P}\mathcal{B}_{\dot{\chi};B}^{(0)} - \frac{\beta}{M_P}\left(\mathcal{B}_{\chi;EB}^{(1,0)} - \mathcal{B}_{\chi;EB}^{(0,1)}\right) = \left[\dot{\mathcal{P}}_{EB}^{(0)}\right]_b , \quad (2.15)$$

and

$$\dot{\mathcal{P}}_E^{(1)} + 5H\mathcal{P}_E^{(1)} + 2\mathcal{P}_{EB}^{(2)} - \frac{2\beta\dot{\phi}}{M_P}\mathcal{P}_{EB}^{(1)} - \frac{2\beta}{M_P}\mathcal{B}_{\dot{\chi};EB}^{(1,0)} = \left[\dot{\mathcal{P}}_E^{(1)}\right]_b , \quad (2.16)$$
$$\dot{\mathcal{P}}_B^{(1)} + 5H\mathcal{P}_B^{(1)} - 2\mathcal{P}_{EB}^{(2)} = \left[\dot{\mathcal{P}}_B^{(1)}\right]_b ,$$
$$\dot{\mathcal{P}}_{EB}^{(1)} + 5H\mathcal{P}_{EB}^{(1)} - \mathcal{P}_E^{(2)} + \mathcal{P}_B^{(2)} - \frac{\beta\dot{\phi}}{M_P}\mathcal{P}_B^{(1)} - \frac{\beta}{M_P}\mathcal{B}_{\dot{\chi};B}^{(1)} = \left[\dot{\mathcal{P}}_{EB}^{(1)}\right]_b ,$$

where we have defined

$$\mathcal{B}_{f;E}^{(n)} = \frac{1}{a^n}\left\langle f\left(\left(\vec{\nabla}\times\right)^n\vec{E}\right)\cdot\vec{E}\right\rangle , \quad \mathcal{B}_{f;B}^{(n)} = \frac{1}{a^n}\left\langle f\left(\left(\vec{\nabla}\times\right)^n\vec{B}\right)\cdot\vec{B}\right\rangle , \quad \mathcal{B}_{f;EB}^{(0)} = -\left\langle f\vec{E}\cdot\vec{B}\right\rangle ,$$
$$\mathcal{B}_{f;EB}^{(1,0)} = -\frac{1}{a}\left\langle f\left(\vec{\nabla}\times\vec{E}\right)\cdot\vec{B}\right\rangle , \quad \mathcal{B}_{f;EB}^{(0,1)} = -\frac{1}{a}\left\langle f\vec{E}\cdot\left(\vec{\nabla}\times\vec{B}\right)\right\rangle , \quad (2.19)$$

Gravity sector

$$\dot{\mathcal{P}}_\chi^{(0)} - 2\mathcal{P}_{\chi\dot{\chi}}^{(0)} = 0 ,$$
$$\dot{\mathcal{P}}_{\chi\dot{\chi}}^{(0)} + 3H\mathcal{P}_{\chi\dot{\chi}}^{(0)} + m_\phi^2\mathcal{P}_\chi^{(0)} + \frac{\beta}{M_P}\mathcal{B}_{\chi;EB}^{(0)} - \mathcal{P}_{\dot{\chi}}^{(0)} = 0 ,$$
$$\dot{\mathcal{P}}_{\dot{\chi}}^{(0)} + 6H\mathcal{P}_{\dot{\chi}}^{(0)} + 2m_\phi^2\mathcal{P}_{\chi\dot{\chi}}^{(0)} + \frac{2\beta}{M_P}\mathcal{B}_{\dot{\chi};EB}^{(0)} = 0 ,$$

Boundary terms and truncation**

Domcke et al. JCAP 03 (2024) 019

GEF + axion inhomogeneities

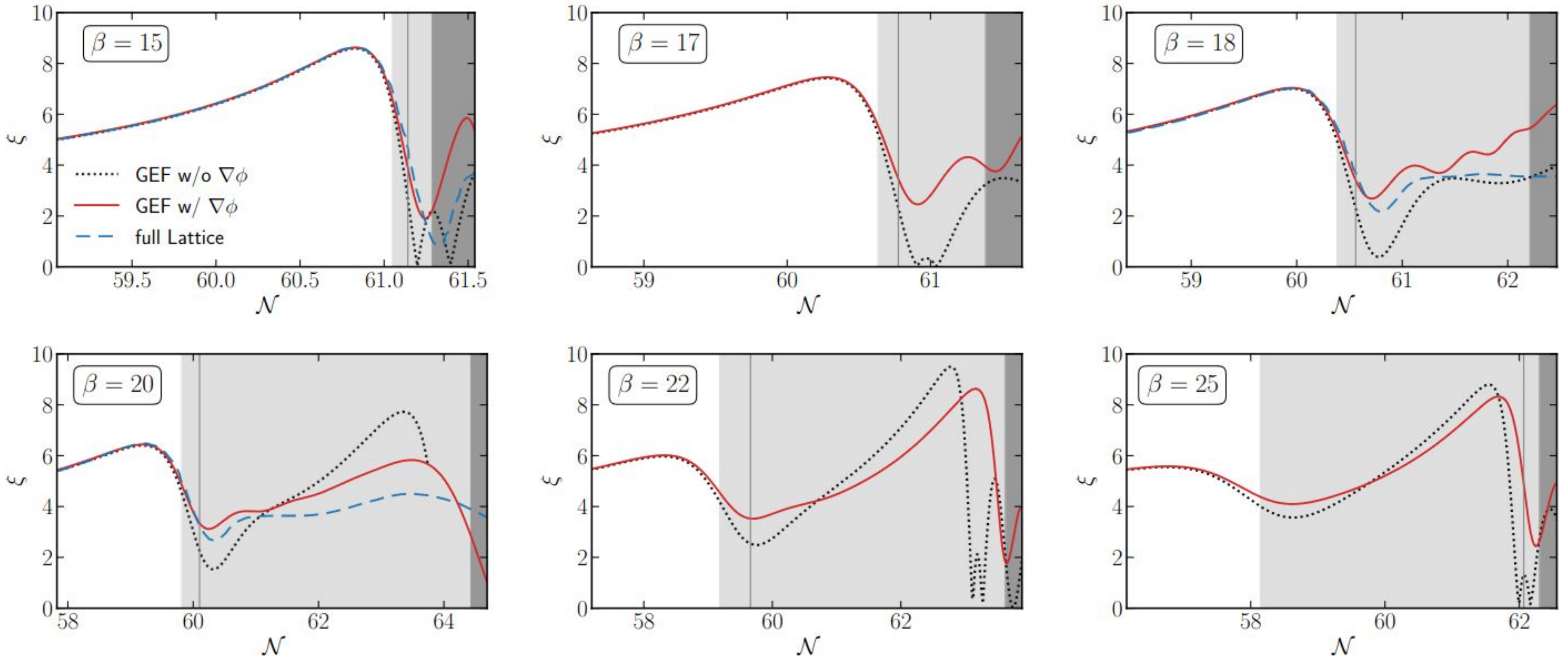


Figure 1: Evolution of ξ assuming a homogeneous axion field (dashed black) and perturbatively including axion gradients (red) for different values of β. The light (dark) gray region indicates that the gradient energy of the axion exceeds 1 % (50 %) of the kinetic energy, while the gray vertical line corresponds to 5 %. Wherever possible we compare to the result of the lattice simulation [35].

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GEF + axion inhomogeneities

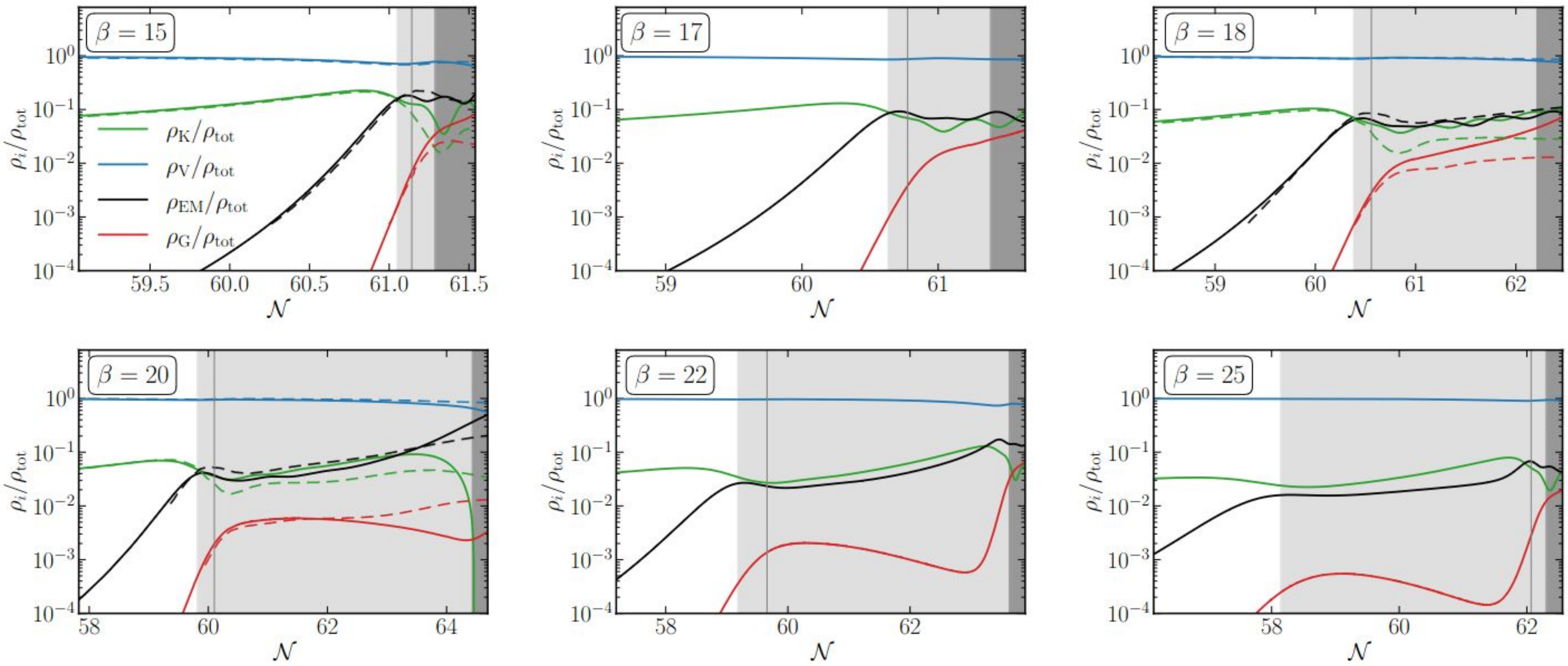

Figure 2: Evolution of the energy densities for different values of β. Gray bands as in Fig. 1. Results from lattice simulations for $\beta = 15, 18$ and 20 from Ref. [35] are shown in dashed.

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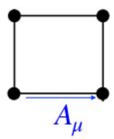
CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe



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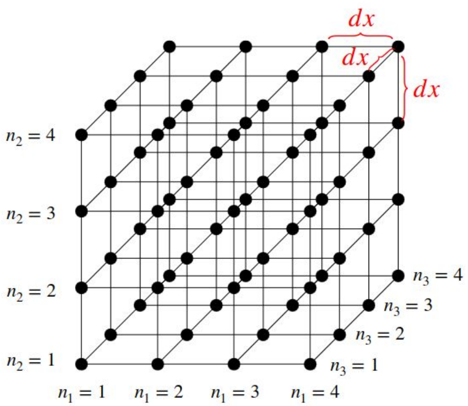

Two ways of **formulating a SF@EB in the lattice**:

- Lattice EOM approach
- Lattice action approach

Different **numerical algorithms** to solve the EOM

- Staggered leapfrog
- Position- and velocity-Verlet
- Runge-Kutta
- Yoshida methods

Figueroa et al. Phys.Rev.Lett. 131 (2023) 15, 151003

Lattice simulations ⇒ Captures the strong backreaction regime!

Lattice simulations

Figueroa et al. Phys.Rev.Lett. 131 (2023) 15, 151003

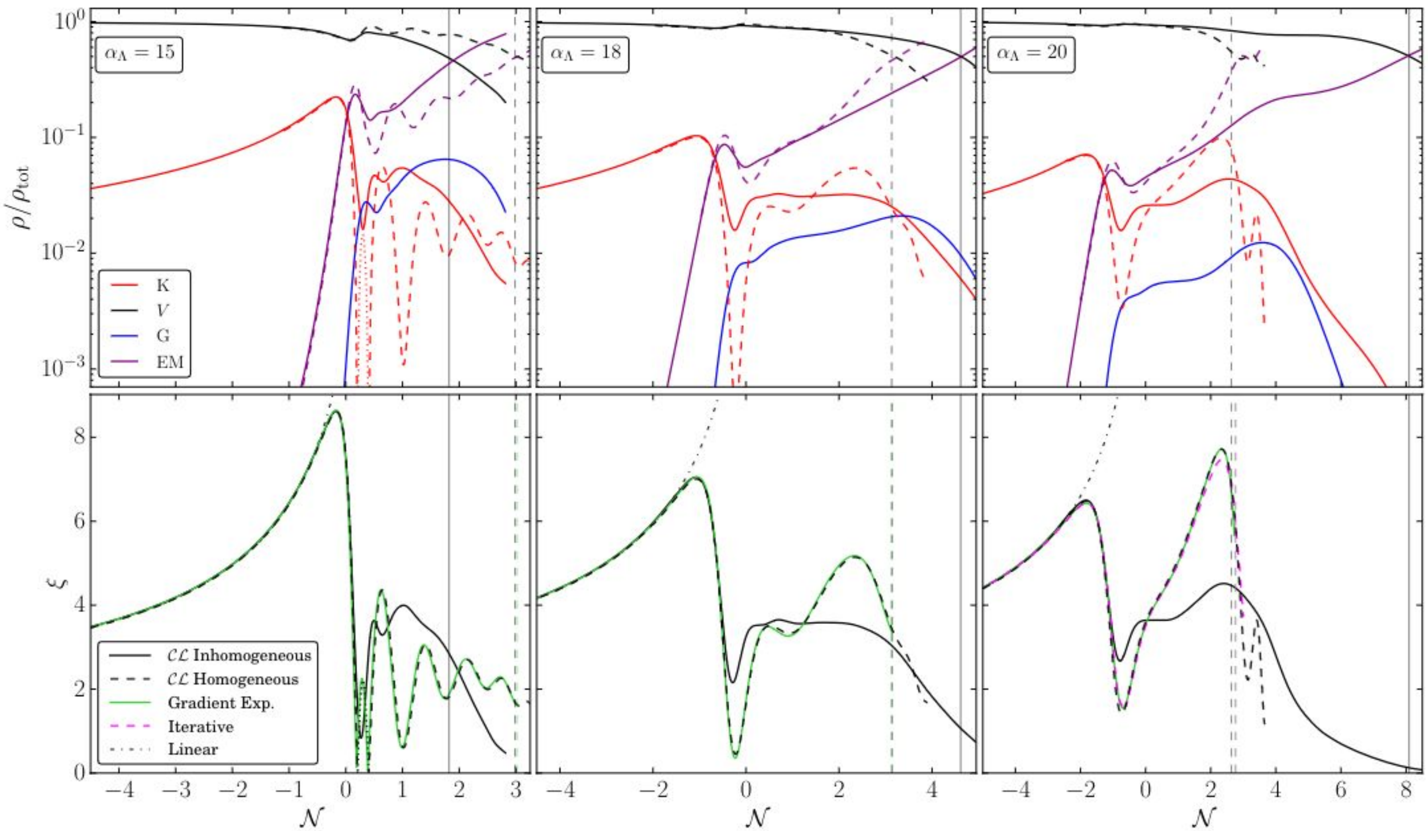

FIG. 1. *Top Row*: Evolution of the electromagnetic (purple) and inflaton potential (black), kinetic (red) and gradient (blue) energy densities, all normalized to the total energy density of the system, for $\alpha_\Lambda = 15,\ 18,\ 20$. Solid (dashed) lines correspond to lattice simulations with inhomogeneous (homogeneous) backreaction. *Bottom Row*: Evolution of ξ for the same coupling constants, corresponding to simulations with inhomogeneous (black solid) and homogeneous (black dashed) backreaction, and to gradient expansion [58, 59] (green solid) and iterative method [19] (magenta dashed). Solid and dashed vertical lines signal the end of inflation in each case. Evolution in the linear regime (black dash-dotted) is also shown for completeness.

The **effect of the inhomogeneity is highly non-trivial** and requires a dedicated study for each coupling.

In general, the excitation and **backreaction of the gauge field is no longer controlled by a homogeneous ξ** parameter, and **resonant oscillatory backreaction** features reported by previous homogeneous analyses are **quite attenuated**.

The resulting gauge field spectra during inhomogeneous backreaction become smoother than in the homogeneous case, as no spectral oscillatory features are developed.