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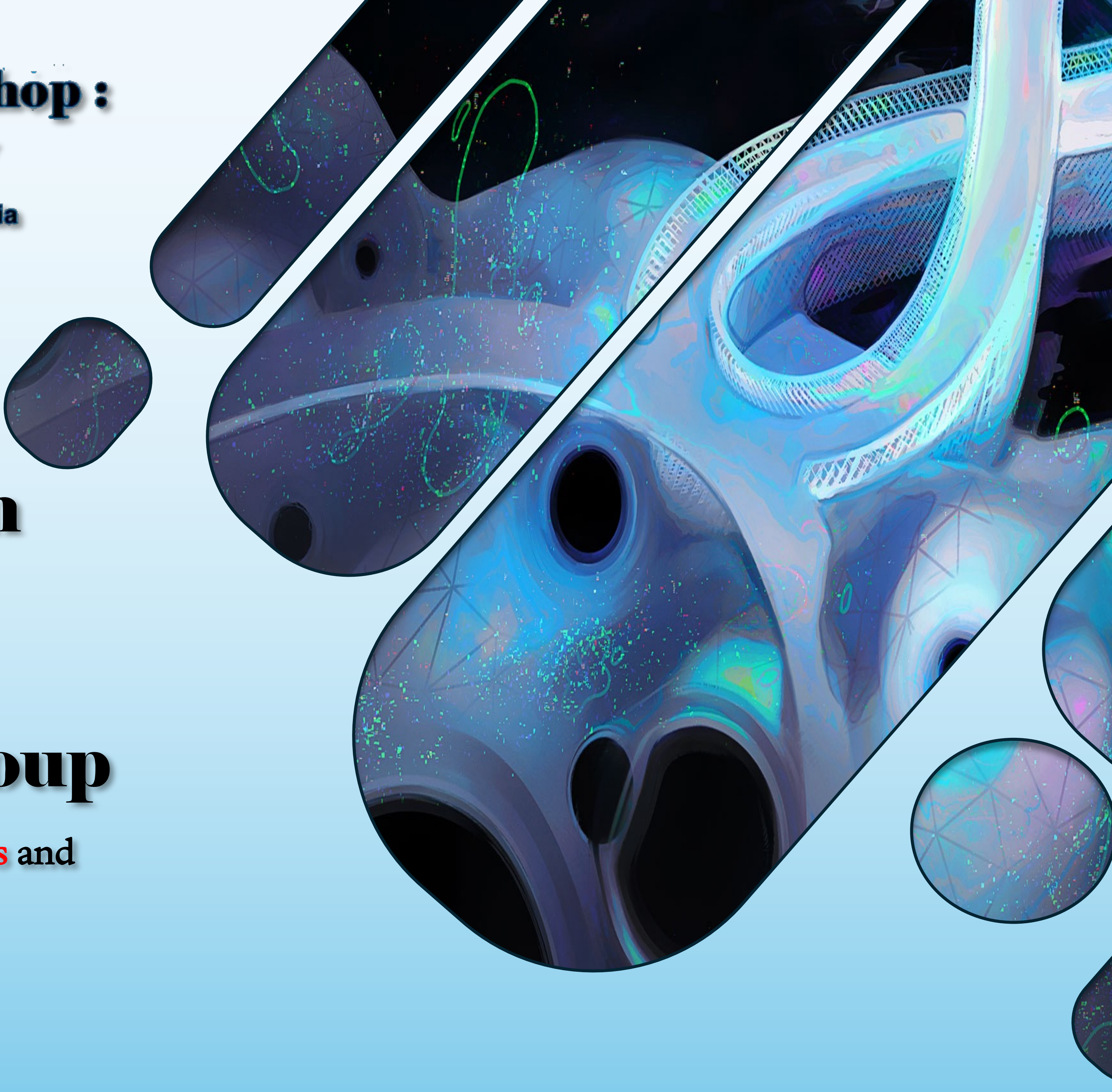
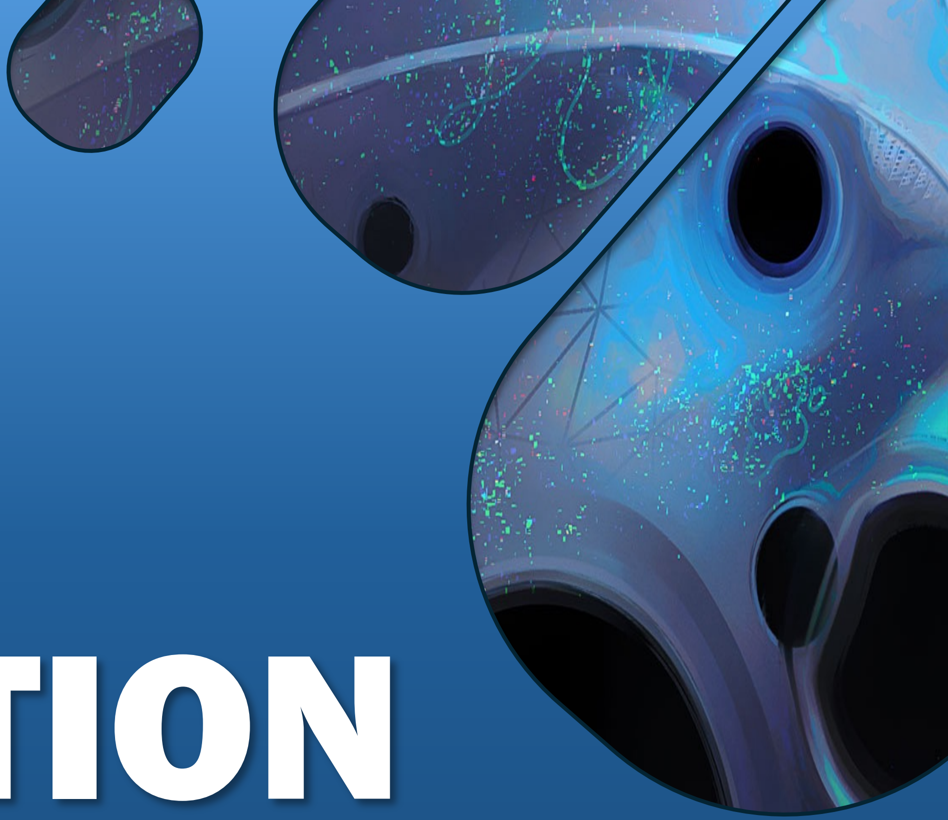
Effective Quantum Spacetime from Functional Renormalization Group

Based on A. Bonanno, M. Cadoni, M. Pitzalis and A. P. Sanna, *in preparation.*

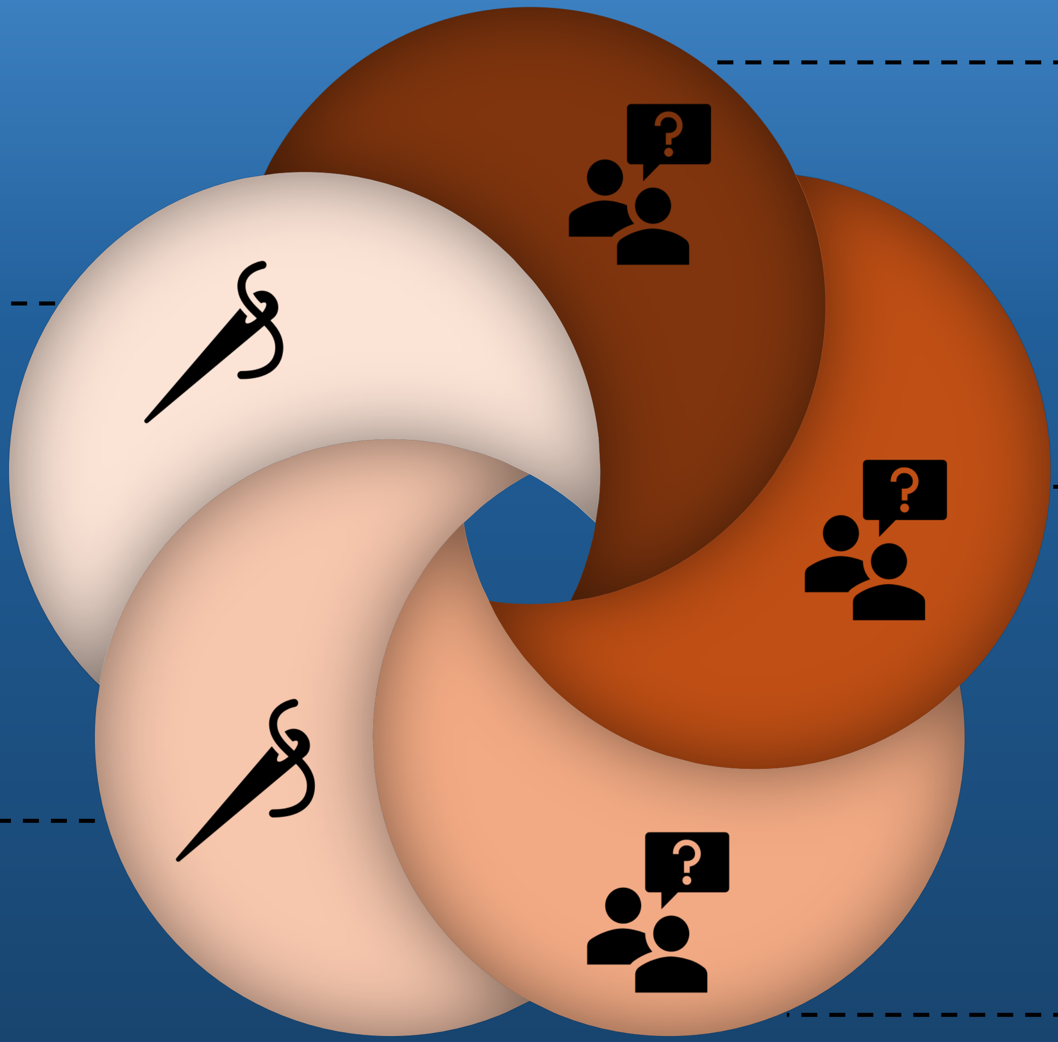




INTRODUCTION

Modern General Relativity has achieved the status of a precision science.

The upcoming statistical data on gravitational wave events will enable precise predictions in the strong field regime.



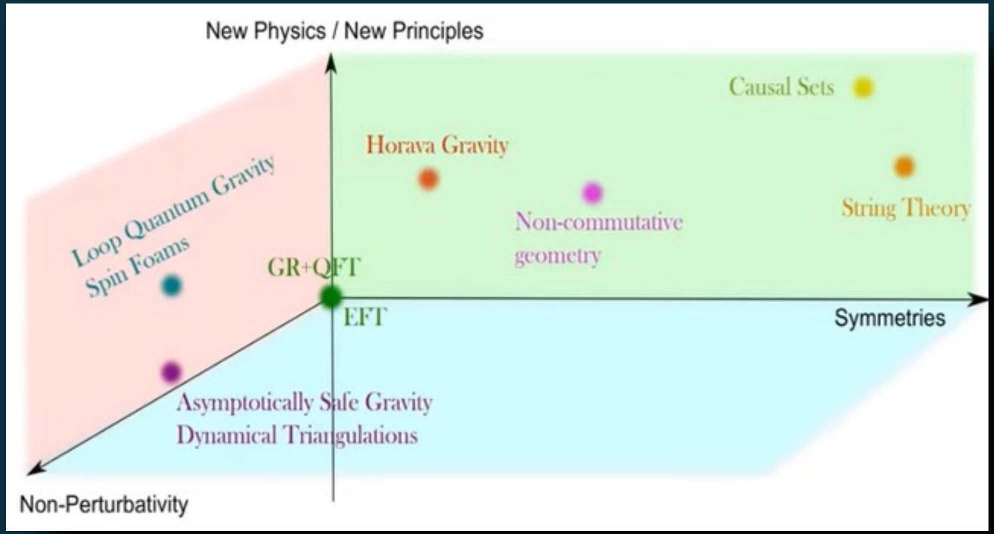
The physics of black hole interiors remains poorly understood.

Several other problems in black hole physics (information paradox).

We still lack a well-established theory of quantum gravity.

ASYMPTOTIC SAFE SCENARIO

Weinberg's conjecture: a fundamental quantum theory of gravity could perhaps be constructed nonperturbatively by taking the continuum limit as a nonGaussian fixed point (Weinberg (1979,1997)).



(A. Platania, Quantum Gravity and All of That, 2024)

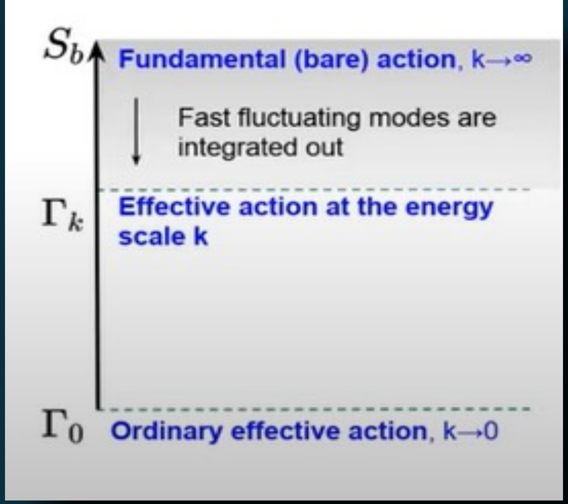
Possibility further developed by Reuter (Reuter (1998)) within the functional renormalization group approach (Wilson-type effective average action Γ_k)

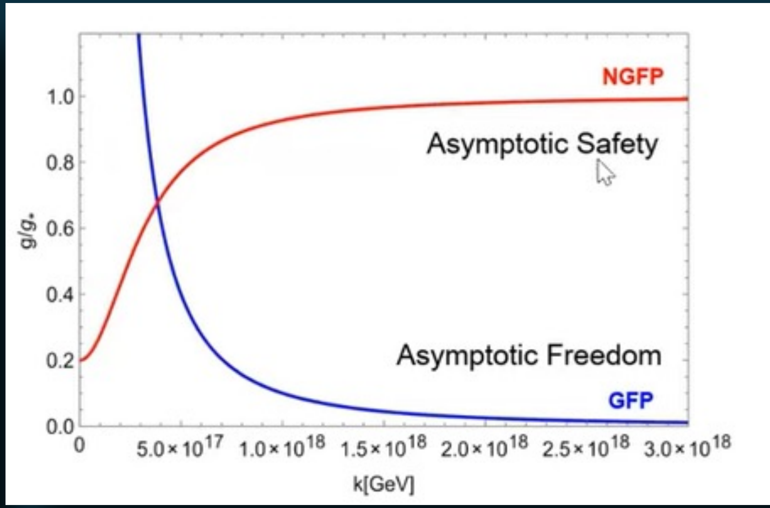
- Integrate out all fluctuation modes which have momenta larger than a certain coarse-graining momentum scale k.

- The renormalized dynamics in encoded in Γ_k whose dependence from the cutoff is governed by:

$$k\partial_k\Gamma_k = \frac{1}{2}STr\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k\mathcal{R}_k\right\},$$

the exact functional renormalization group equation (Reuter (1998), Bagnus & Bellivier (2001), Morris (1998)).





(A. Platania, Quantum Gravity and All of That, 2024)

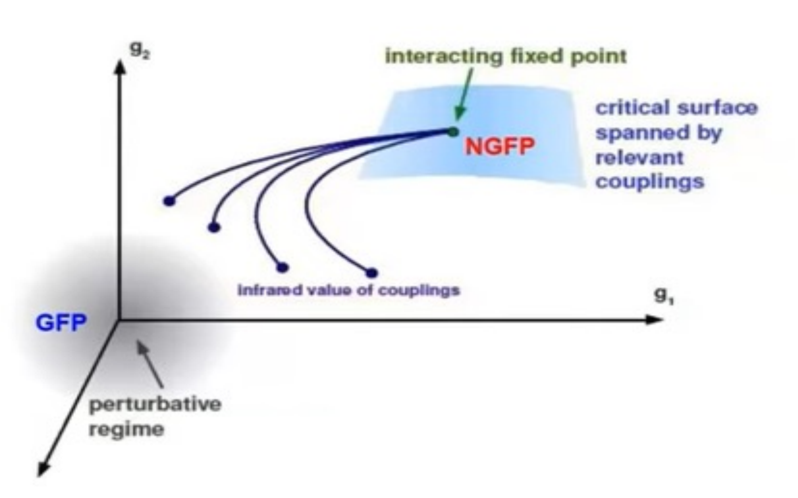
Extracting physical information:

- Truncate the theory space by projecting the renormalization group flow into a finite dimensional subspace where the coupling constants are the coordinates (Reuter (1998), Reuter & Saueressig (2002)).
- Suppose that the relevant physical information in encoded in the running of coupling constants (Dittrich & Reuter (1985)).

$$V = -\frac{e^2}{r}$$

$$V_k = -\frac{e_k^2}{r}$$

$$V_1(r) \sim -\frac{e^2 r_0^{-1}}{4\,\pi\, r}\left(1 + b\log\left(\frac{r_0}{r}\right)\right)$$



Applying the same approach to gravity → effective quantum geometries (classical solutions are replaced with effective geometries featuring a running of coupling constants, Bonanno & Reuter (2000)).

$$f(r) = 1 - \frac{2G_0 M}{r}$$

$$f(r) = 1 - \frac{2G_k M}{r}, G_k = \frac{G_0}{1 + \frac{G_0}{g^*}k^2}$$

<u>Crucial problem: finding ***k*** in a way that makes the approach consistent.</u>

- Further investigations including the running of the cosmological constant Λ have encountered various difficulties (Koch & Saueressig (2014)).
- This problem of deforming Schwarzschild solution including quantum corrections consistently has only been partially addressed in subsequent studies (Platania (2019)).

▼

Brief summary of what we did:

- We seek an approximate solution of the quantum-corrected Schwarzschild - de Sitter metric in the ultraviolet and infrared regimes by proposing a cutoff k that makes the solution consistent.
- We interpolate the two approximate solutions by a numerical procedure known as shooting such that we obtain the quantum correct metric function in the entire domain

GENERALITIES

We truncate the theory space by considering Einstein-Hilbert action (leading contribution to quantum geometry):

$$\Gamma_k = \frac{1}{16\,\pi\,G_k}\int d^4x\sqrt{-g}\,[R\ - 2\Lambda_k]$$

We introduce quantum corrections in the Schwarzschild-de Sitter solution promoting the bare constants to running coupling constants

$$f(r) = 1 - \frac{R_s}{r} - \frac{\Lambda_0 r^2}{3},\ R_s = 2\ G_0\ M$$

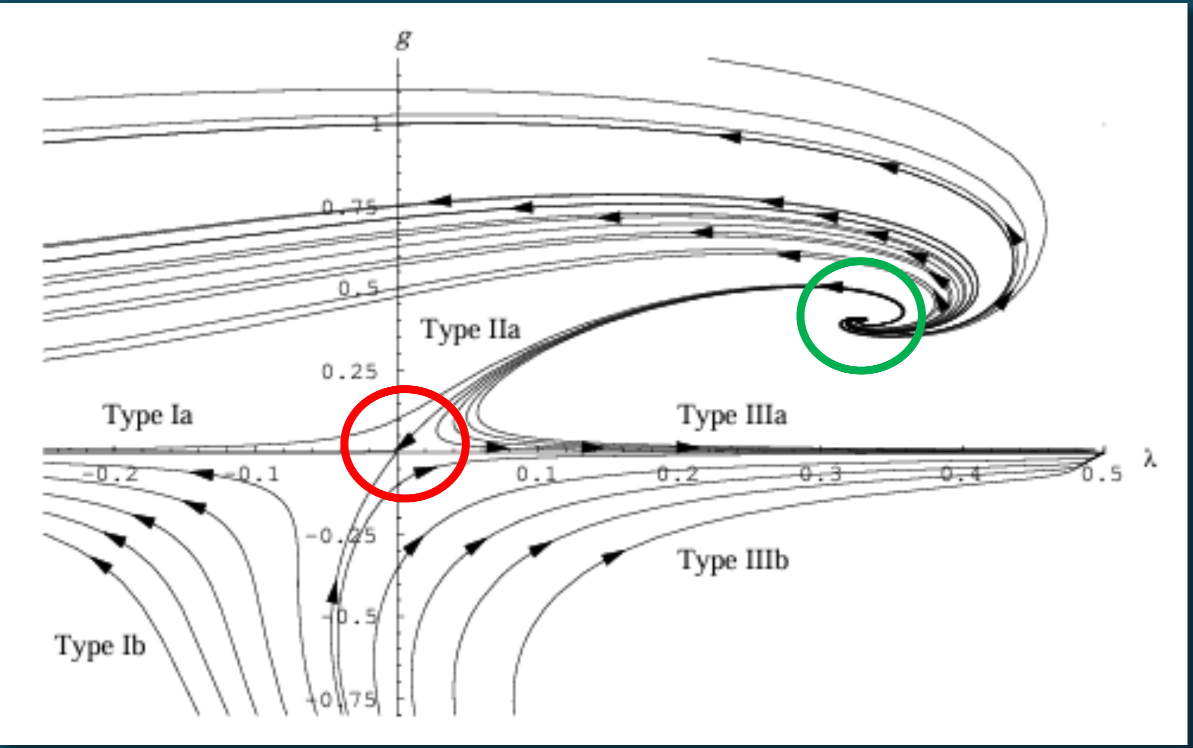
$$G_0 \to G_k, \Lambda_0 \to \Lambda_k$$

$$f_d = 1 - \frac{2G_kM}{r} - \frac{\Lambda_k r^2}{3}$$

- We are assuming the validity of the Einstein-Hilbert truncation at every momentum scale k (we expect that near the nongaussian fixed point also higher curvature terms could play a role).
- The most general way to account for the running of coupling constants is to consider it at the level of the action. A second possibility is to include them in the field equations. Including them in the solution is the simplest way.
- We are assuming that the form of the metric always remains the same at every momentum scale k.
- We are implicitly assuming the existence of an effective description of quantum gravity effects in terms of a smooth geometry.

GENERALITIES

Projecting the flow onto the subspace spanned by the Einstein-Hilbert truncation we obtain for the dimensionless coupling constants $g(k) = G_k k^2$ and $\lambda(k) = \Lambda_k k^{-2}$

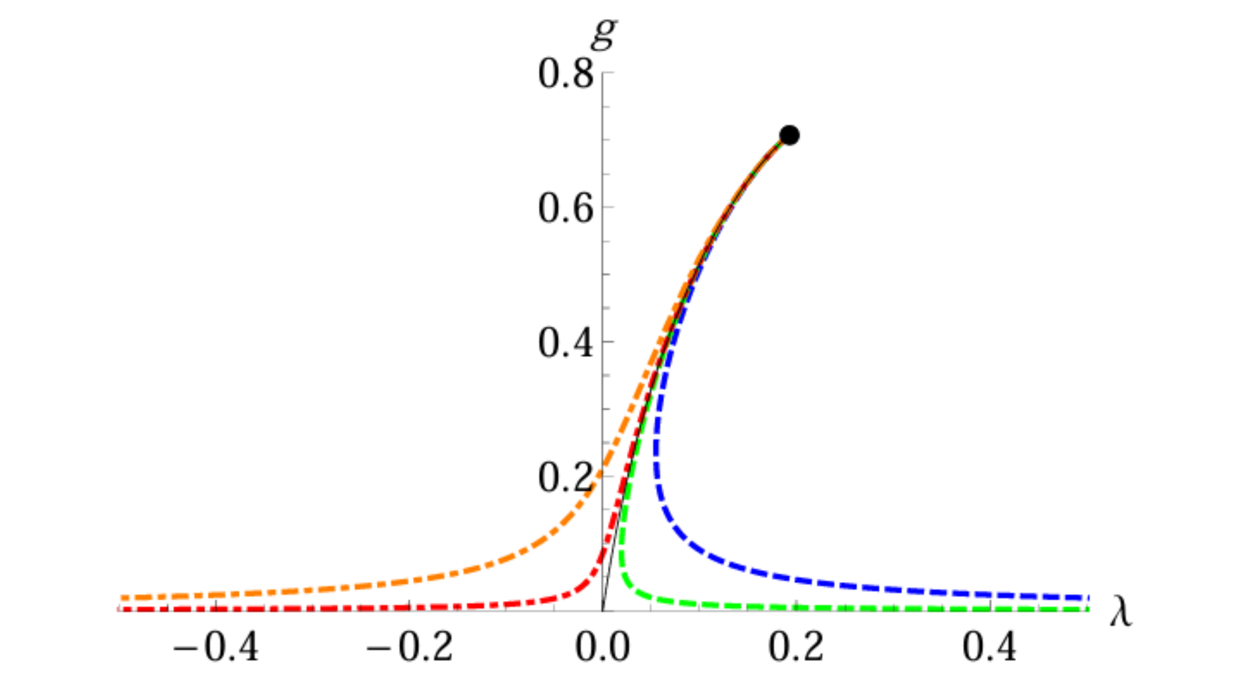


(Koch & Saueressig, 2014)

- Gaussian fixed point located at the origin (infrared regime), $g^* = 0, \lambda^* = 0$
- NonGaussian fixed point located at λ^*=0.193, $g^* = 0.707$ acting as an ultraviolet attractor (and giving the Ultraviolet completion of the theory).

GENERALITIES

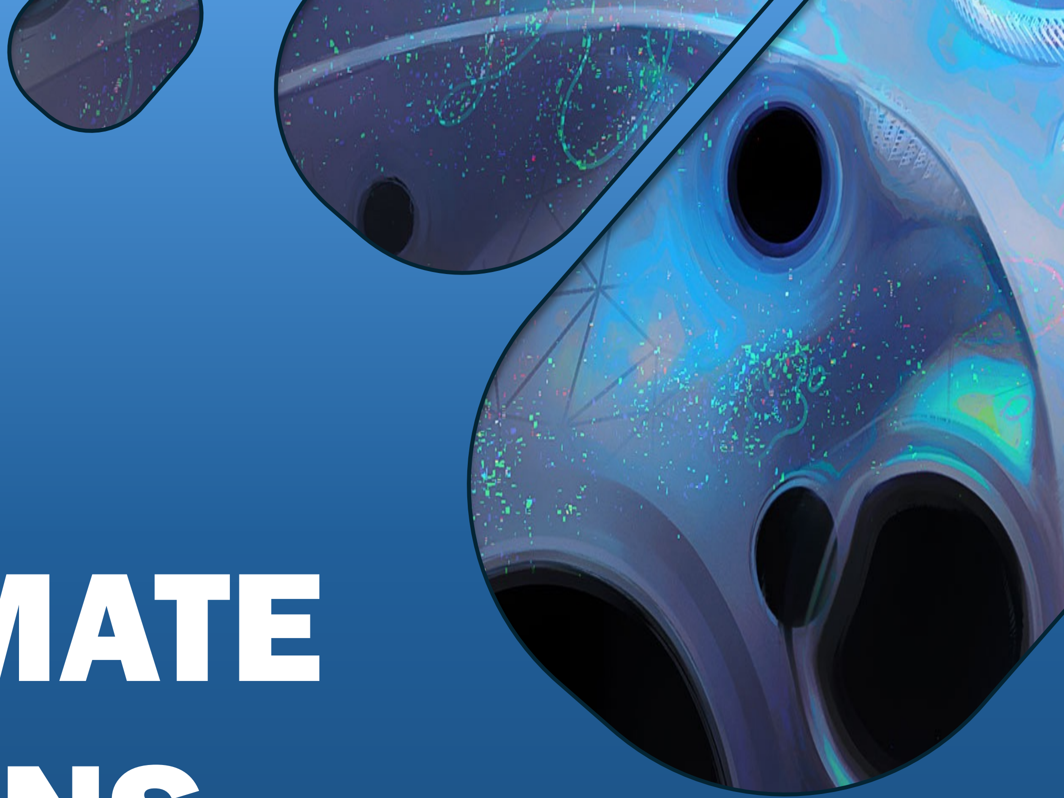
Approximated analytic form of the Renormalization Group trajectories



(Koch & Saueressig, 2014)

$$g(k) = \frac{G_0 k^2}{1 + \frac{G_0}{g^*} k^2}, \qquad \lambda(k) = \frac{g^*\lambda^*}{g(k)}\left[\left(5 + \frac{\Lambda_0 G_0}{g^*\lambda^*}\right)\left(1 - \frac{g(k)}{g^*}\right)^{\frac{3}{2}} - 5 + \frac{3\,g(k)}{2\,g^*}\left(5 - \frac{g(k)}{g^*}\right)\right]$$

The observed (infrared) present value of Λ_0 is quite small and can be set to zero whenever one is considering black holes.



APPROXIMATE SOLUTIONS

APPROXIMATE SOLUTIONS

To get a well-defined spacetime geometry we need to exploit the dependence from the cutoff in the dressed metric function.

k must respect the symmetries of the classical solution.

k must be invariant under coordinate transformations.

$$k(r) = \frac{\xi}{\mathcal{L}(r)}$$

$$\mathcal{L}(r) = \int ds = \int \frac{dr}{f_d(r)}$$

$$\mathcal{L}'(r) = \frac{1}{\sqrt{f_d(r)}}$$

We solve this differential equation in an approximate way.

$k(r)$ depends on the form of the dressed metric function which in turns depends-on the form of $g(k)$ and $\lambda(k)$ which is a backreaction between the dressed metric function and the cutoff (Until now completely neglected).

APPROXIMATE SOLUTIONS – ULTRAVIOLET REGIME

Ultraviolet approximate solutions can be found from a series expansion (Frobenius method)

$$k(r) = \frac{\beta}{r^2} + \frac{\gamma}{r} + \delta + \mathcal{O}(r) \quad \rightarrow \quad k(r) = \frac{\xi}{\mathcal{L}(r)} \quad \rightarrow \quad \mathcal{L}(r)' = \frac{1}{\sqrt{f_d(r)}}$$

$$f_d(r) = 1 - \frac{\sigma}{3} - \omega r - \Lambda_{eff} r^2 + \mathcal{O}(r^3)$$

$$R \sim \frac{\sigma}{r^2} \qquad R \sim \frac{\omega}{r}$$

$$\sigma = \beta\lambda^*$$

$$\omega = \frac{4\mathcal{M}(\sigma-3)}{\sigma(\sigma-6)}, \mathcal{M} = Mg^*\lambda^*$$

$$\Lambda_{eff} = 9\mathcal{M}^2 \frac{(3-\sigma)(24-5\sigma)}{\sigma^2(\sigma-6)^2(9-2\sigma)} - \frac{7\mathcal{M}_p^2}{2}\frac{3-\sigma}{9-2\sigma}$$

$$\mathcal{M} = Mg^*\lambda^*, \mathcal{M}_p^2 = m_p^2 g^*\lambda^*$$

- The flow of G_k generates a curvature singularity (linear term) «milder than the usual Schwarzschild one $\left(\sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}} = 1/r^3\right)$», while the flow of Λ generates a new conical singularity term (extensive origin).
- The functional form of the metric is rather similar to that derived by Mannheim and Kazanas in the context of Weyl gravity (Mannheim & Kazanas (1998)).

APPROXIMATE SOLUTIONS – UV REGIME

We can try to remove the singularity coming from the linear term (ωr) by translating the radial coordinate $r \to r + \ell$:

$$ds_d^2 = -f_d(r)\, dt^2 + \frac{dr^2}{f_d(r)} + (r+\ell)^2 d\Omega^2,\ f_d(r) = 1 - \mathcal{C} - \frac{\Lambda_{eff}\, r^2}{3}$$

$$\ell = -\frac{3\omega}{2\Lambda_{eff}},\ \mathcal{C} = \frac{\sigma}{3} - \frac{3\omega^2}{4\Lambda_{eff}}$$

The form of the UV geometry depends crucially on the sign of Λ_{eff}:

$$\Lambda_{eff} = 0 \to M_t^2 = \frac{1}{g^*\lambda^*}\frac{7}{18}\frac{(\sigma-6)^2\sigma^2}{24-5\sigma} m_p^2$$

We avoid the conical singularity by cutting the spacetime to a finite geodesic length, therefore:

$M < M_t$

- $\Lambda_{eff} < 0 \to AdS_2 \times S^2$ (appearance of an AdS_2 phase of quantum gravity (widely used for addressing black hole information paradox and microscopic origin of black hole entropy, Cadoni et al (2023), Bonanno & Reuter (2007))

$M > M_t$

- $\Lambda_{eff} > 0 \to dS_2 \times S^2$

Phase transition at Planck scale geometrically as $AdS_2 \times S^2 \to dS_2 \times S^2$ (first predicted by Polyakov (1993))

APPROXIMATE SOLUTIONS – IR REGIME

The same differential equation is solved in the infrared regime, i.e. where $\frac{r}{R_s} \gg 1$.

$$f_d^{(IR)} = 1 - \frac{R_s}{r} - \frac{\zeta^4 G_0 \lambda_*}{8 g_* r^2} + \frac{\zeta^2 G_0 R_s}{g_* r^3} + \frac{d\zeta^4 G_0 \lambda_* + R_s \zeta^4 G_0 \lambda_* ln\left(\sqrt{\frac{r}{R_s}}\right)}{2\, g_* r^3} + \mathcal{O}\left[\frac{ln^2\left(\sqrt{\frac{r}{R_s}}\right)}{r^4}\right]$$

- Long range infrared terms induced by quantum corrections.
- Long range gravitational contribution of vacuum polarization of conformal field theory degrees of freedom ($\rho_v = -\frac{\zeta^4}{32\,\pi} \frac{\lambda_*}{g_*} \frac{1}{r^4}$, typical vacuum energy density expected for conformal fields in 4D)
- Other mass polarization effects $\rho_1 = \frac{\zeta^2}{4\pi g_*} \frac{R_s}{r^5}$ $\rho_2 = \frac{\lambda_* \zeta^4}{8\,\pi\, g_*}\left(\frac{R_s ln\left(\sqrt{\frac{r}{R_s}}\right)}{r^5} + \frac{d}{r^5}\right)$
- The general scaling of the $1/r^3$ long range quantum corrections behave as $\sim G_0^2 M$ implying for naturalness arguments that $d \sim R_s$ (we cannot exclude a "superplanckian" coming from an infrared term that in the effective average action goes to zero in the limit $k \to 0$)

NUMERICAL SOLUTIONS

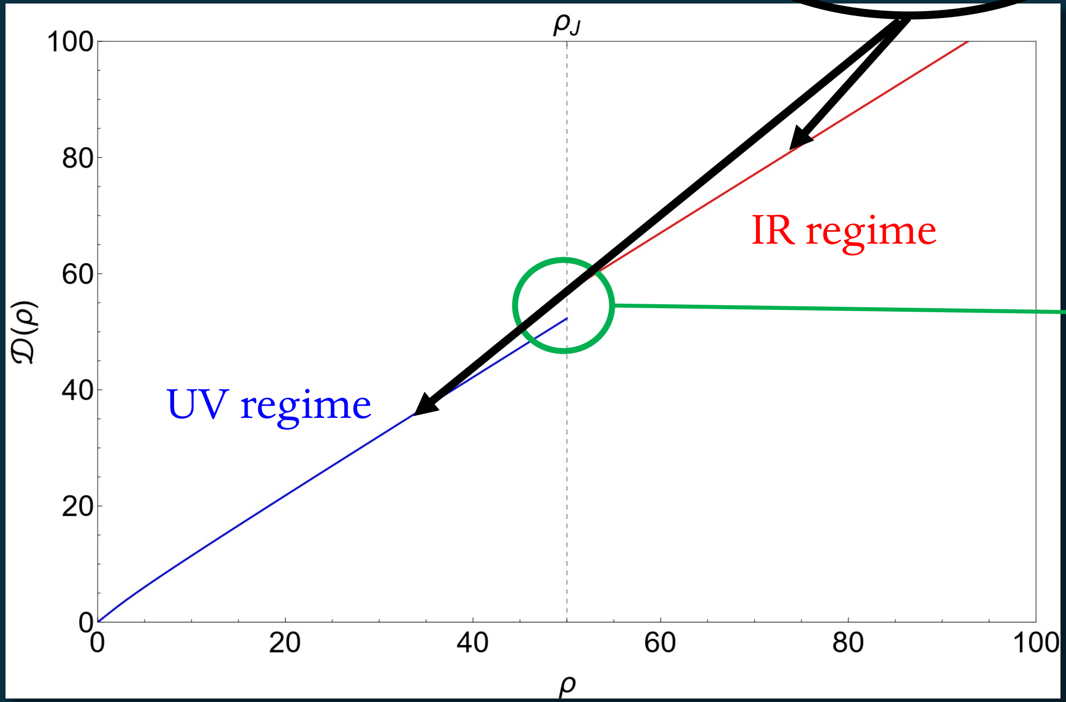
NUMERICAL SOLUTIONS

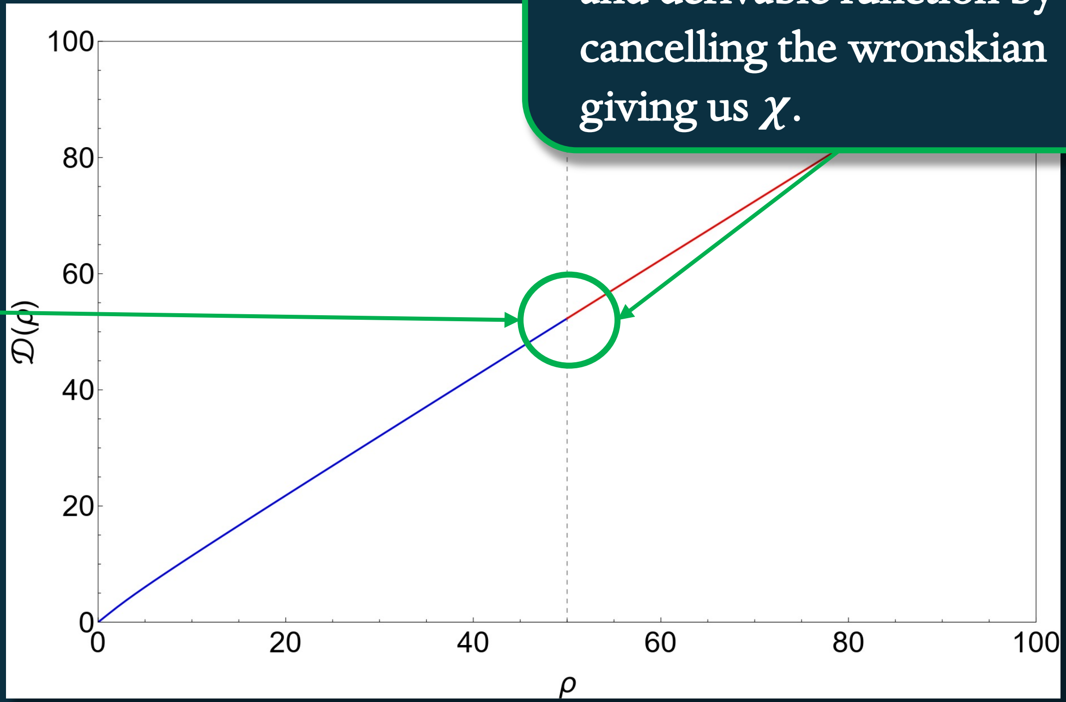
To determine the form of the metric function in the whole domain interpolating between the two asymptotic solutions we employ a numerical method called shooting. By defining some adimensional quantities:

$$\rho = \frac{r}{R_s}, \quad \mathcal{G} = \frac{G_0}{R_s^2} = \frac{m_p^2}{4\,M^2}, \quad \mu = g_*\lambda_* M R_s = 2 g_*\lambda_* \frac{M^2}{m_p^2}, \quad \mu = \frac{g_*\lambda_*}{2}\frac{1}{\mathcal{G}}$$

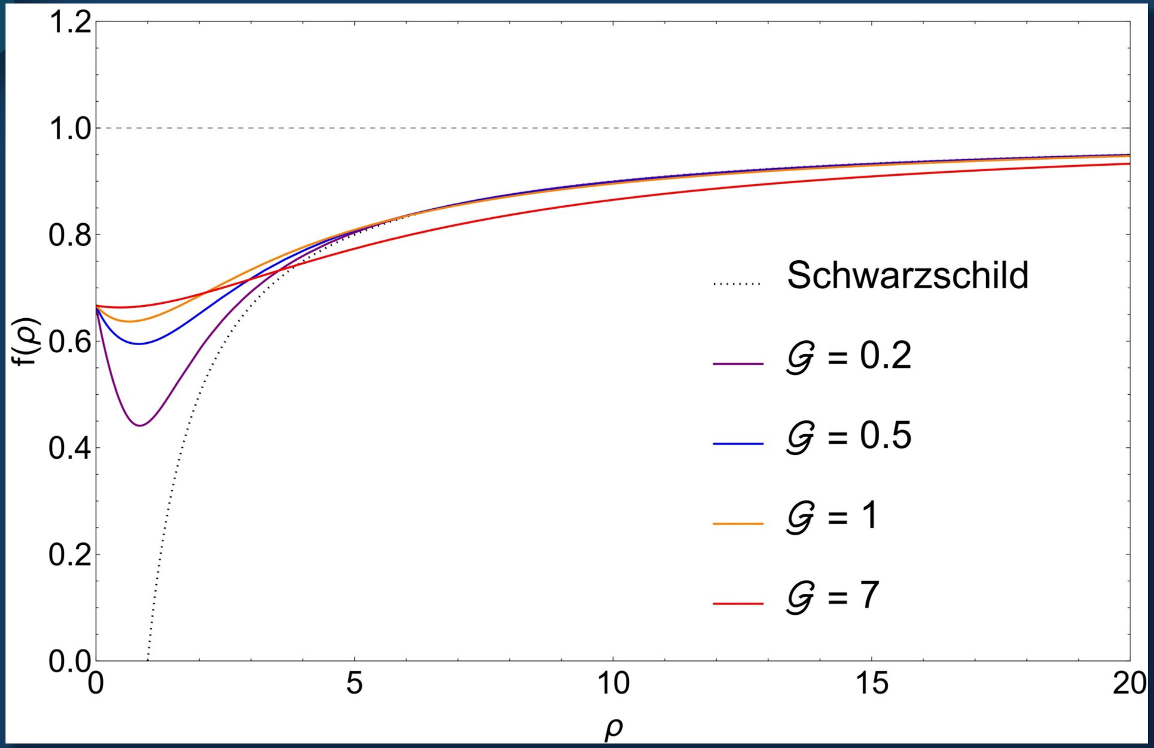
$$\mathcal{D}(\rho) = \frac{\mathcal{L}(\rho)}{R_s} \qquad \frac{d\mathcal{D}(\rho)}{d\rho} = \frac{1}{\sqrt{f_d(\rho)}} \qquad d = \chi R_s/2$$

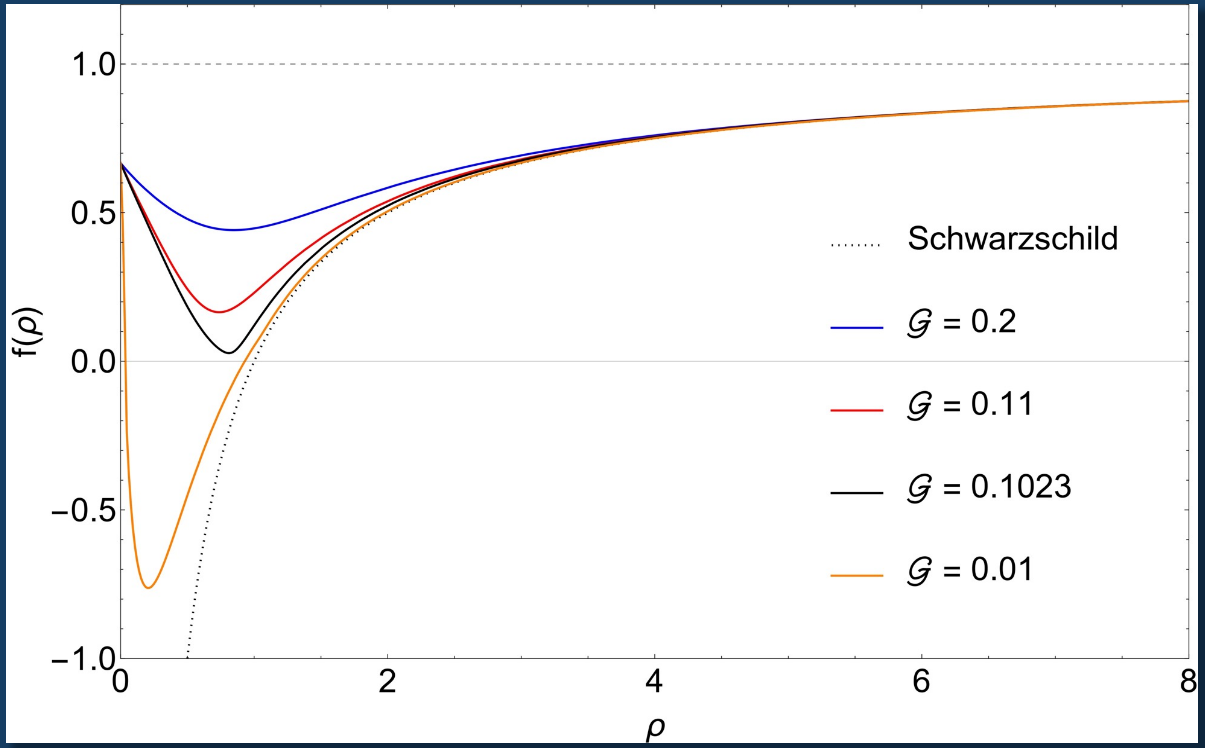
In the junction point we ensure to have a continuous and derivable function by cancelling the wronskian giving us χ.





NUMERICAL SOLUTIONS





Anti de Sitter – de Sitter transition (lowering $\mathcal{G}$ → «raising object's mass» (naked singularity))

Formation of horizons above critical mass $M_c^2 = \frac{1}{4\mathcal{G}_c} m_p^2$

FINAL REMARKS

The running of the coupling constants obtained by truncating the effective action to the Einstein-Hilbert action, although including the 'backreaction' of the metric, does not eliminate the curvature singularity, makes the Schwarzschild one milder and generates a conical singularity.

The non-linearity of the coupling constant flow equations gives rise to a phase transition from a regime with a negative effective cosmological constant, linked to the Planck mass and corresponding to the vacuum energy domain, to a regime with a positive cosmological constant in which matter excitations, controlled by the mass of the black hole, dominate. This is correlated with a change in the topology of spacetime in the vicinity of the fixed point, i.e. $AdS_2 \times S^2 \rightarrow dS_2 \times S^2$.

We shows the appearance of an AdS_2 phase of quantum gravity near the nonGaussian fixed point.

The effective quantum spacetimes we obtained from the running of the coupling constants show, above a critical mass value very close to the Planck mass, the formation of event horizons.

THANKS FOR THE ATTENTION