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Weak cosmic censorship and the rotating quantum BTZ black hole

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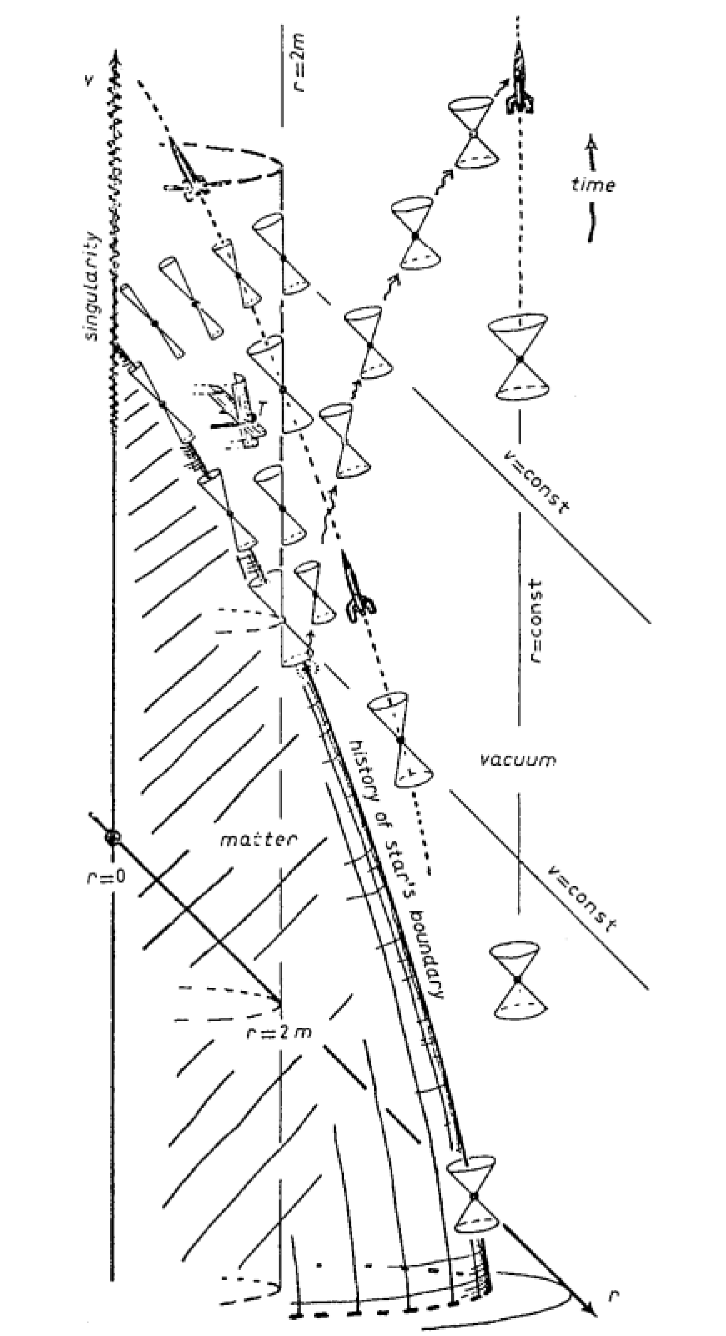
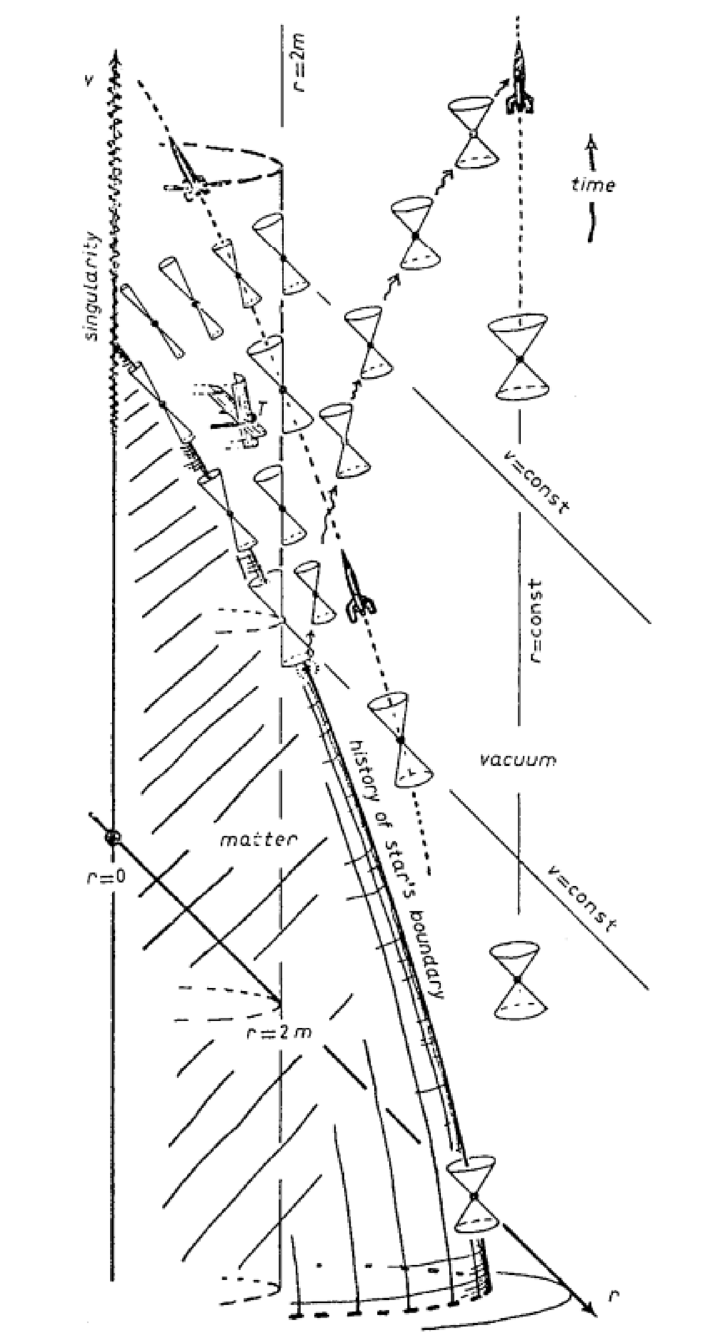


Penrose, Riv. Nuovo Cim. **1** (1969)



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Weak Cosmic Censorship Conjecture

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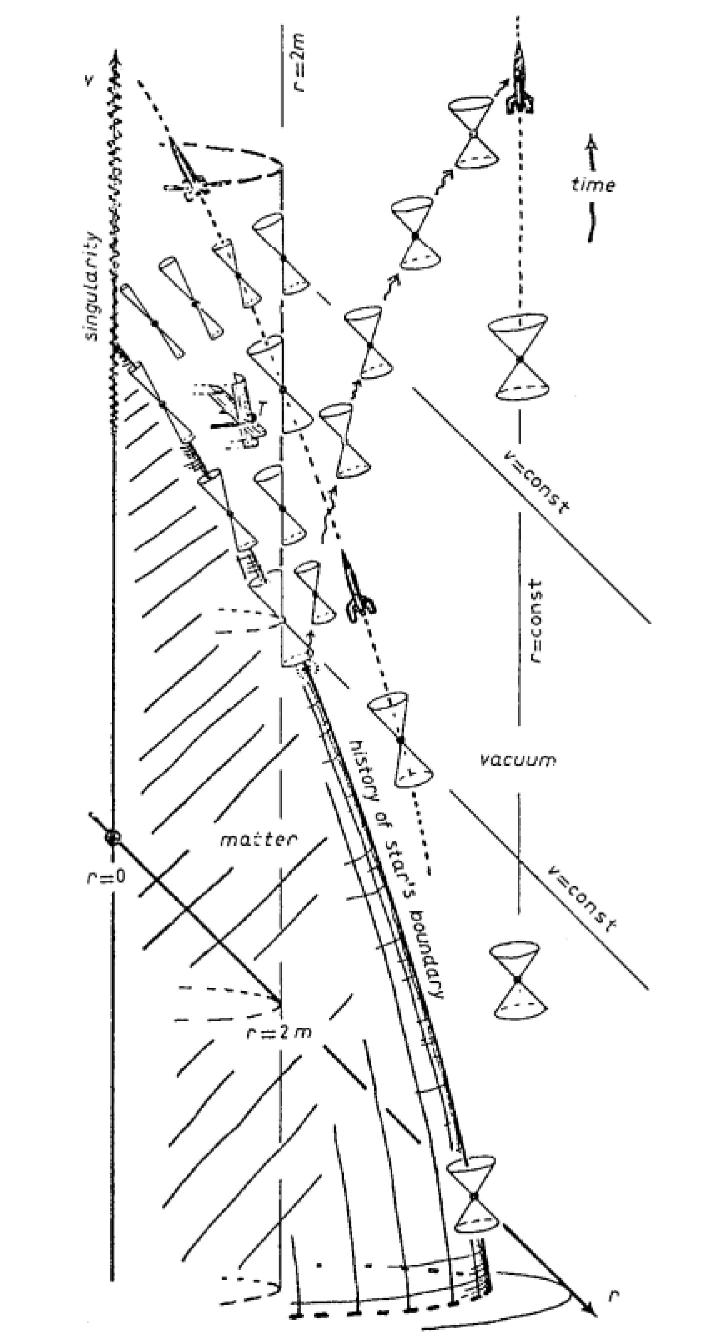

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Weak Cosmic Censorship Conjecture

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Assuming physically reasonable matter source and genericity of initial conditions

A regular initial configuration cannot develop a spacetime singularity under time evolution with the classical equations of motion unless it is cloaked behind an event horizon

Wald’s test

Wald, Annals Phys. **82** (1974)

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Throwing **test** particles at an extremal Kerr black hole

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Can one spin up the black hole past extremality?

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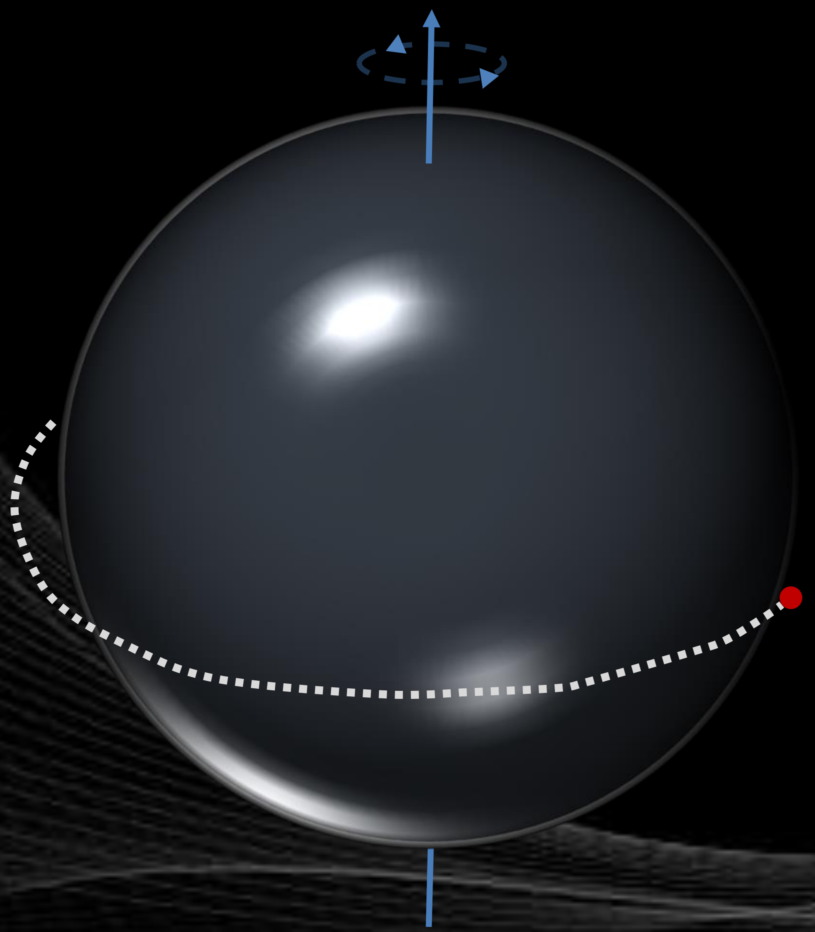

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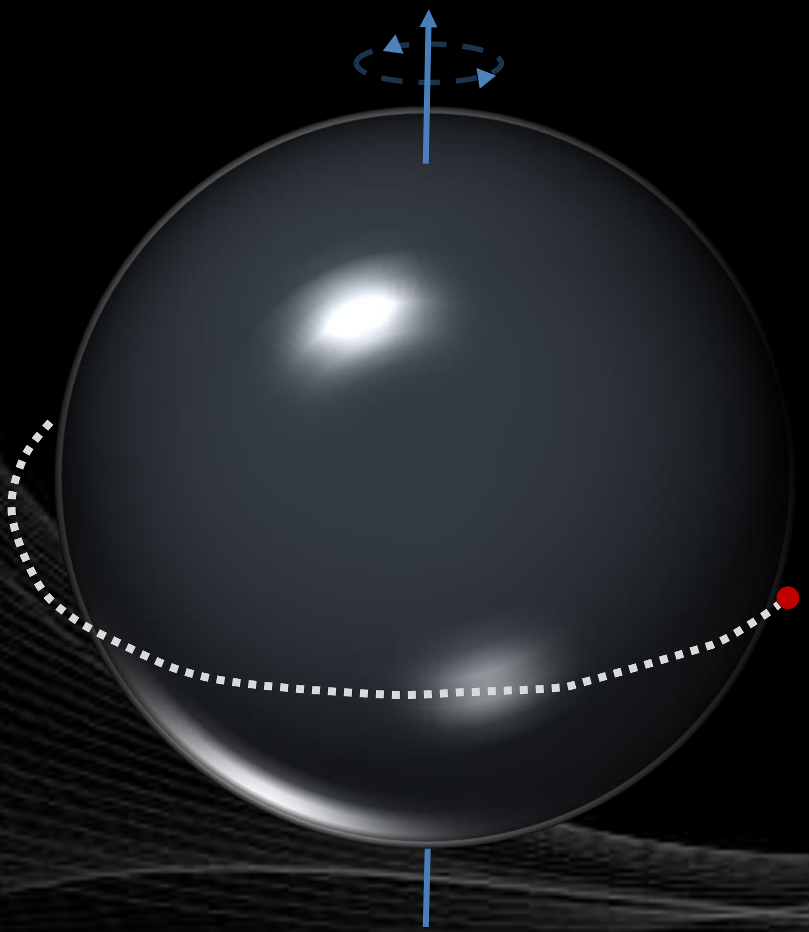

Particles with dangerously high angular momenta are **not captured** by the black hole

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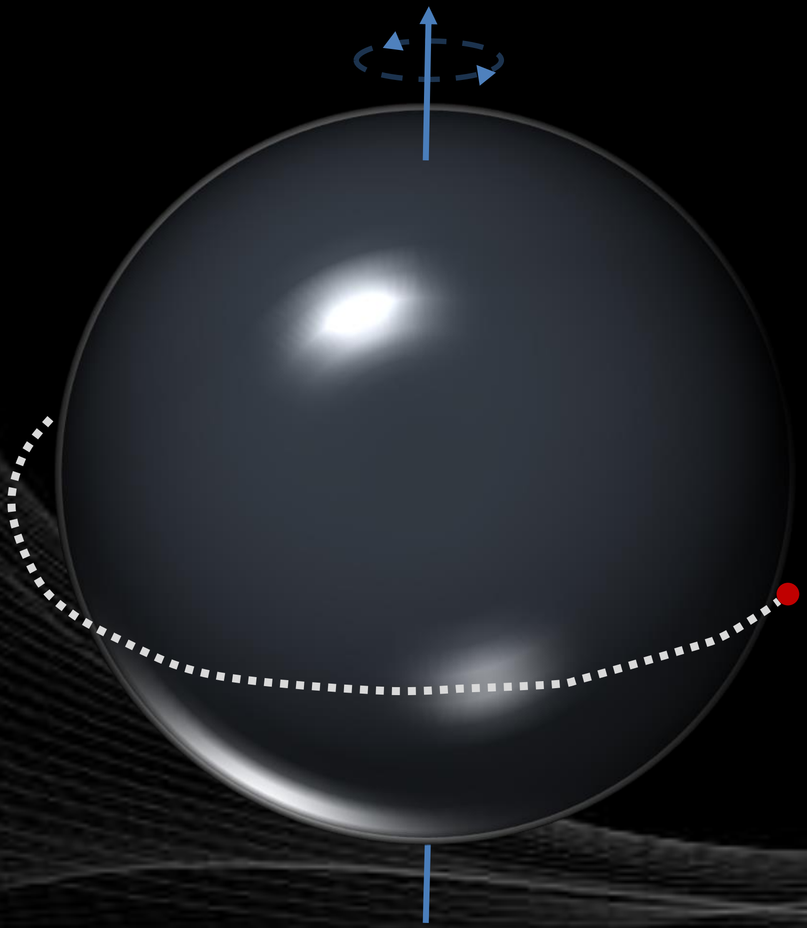

Can one spin up the black hole past extremality?

Particles with dangerously high angular momenta are **not captured** by the black hole

No naked singularity is formed

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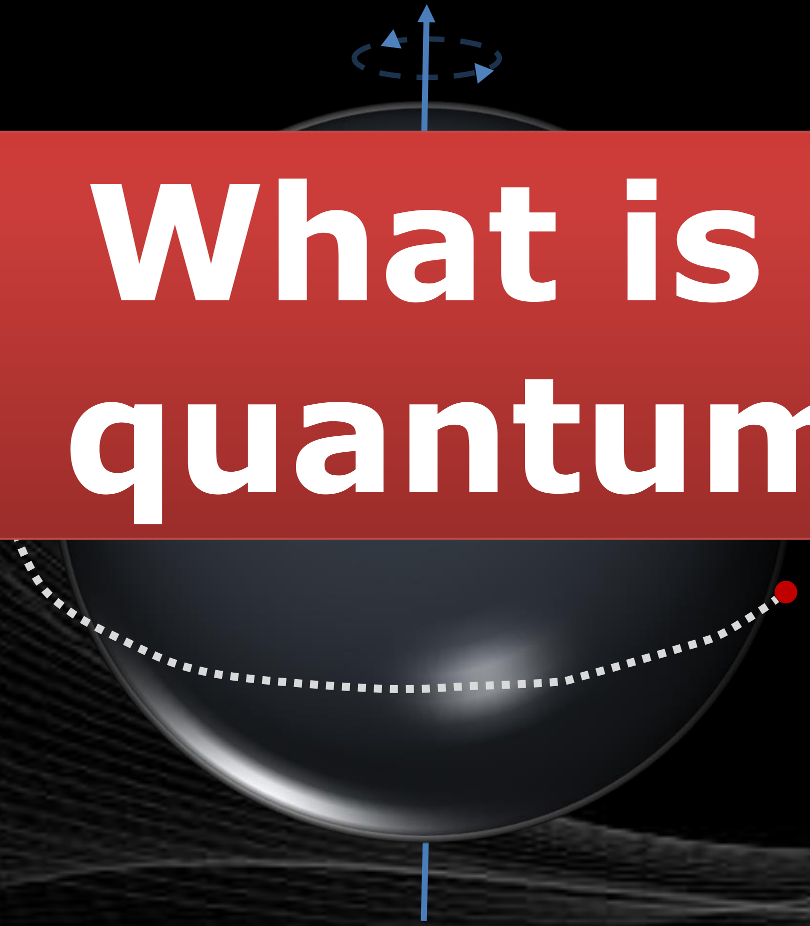

What is the impact of quantum corrections?

hole

No naked singularity is formed

quantum black holes

Semiclassical gravity

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G\langle T_{\mu\nu}(g_{\alpha\beta})\rangle$$

Classical geometry of a black hole modified by quantum fields

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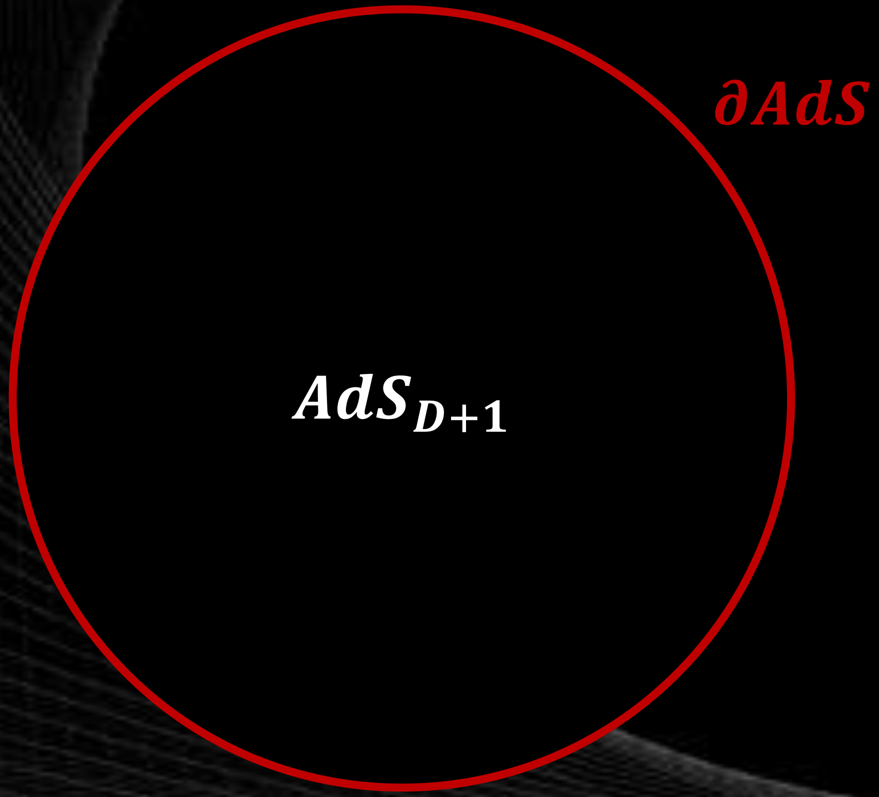
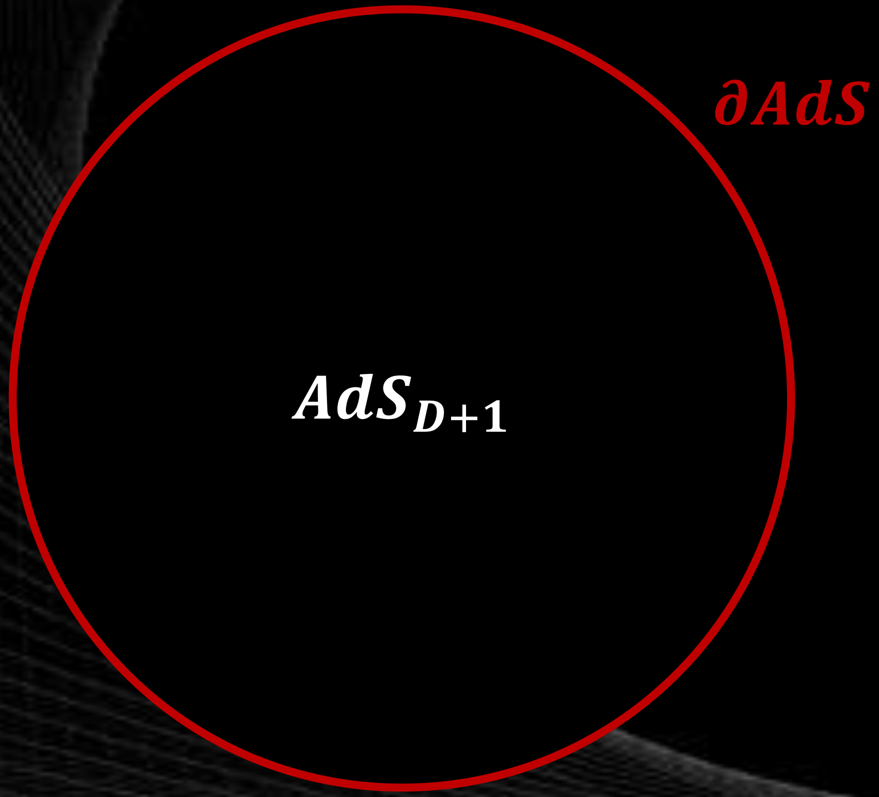
Very hard to solve in higher dimensions → Holographic formulation

Braneworld Holography



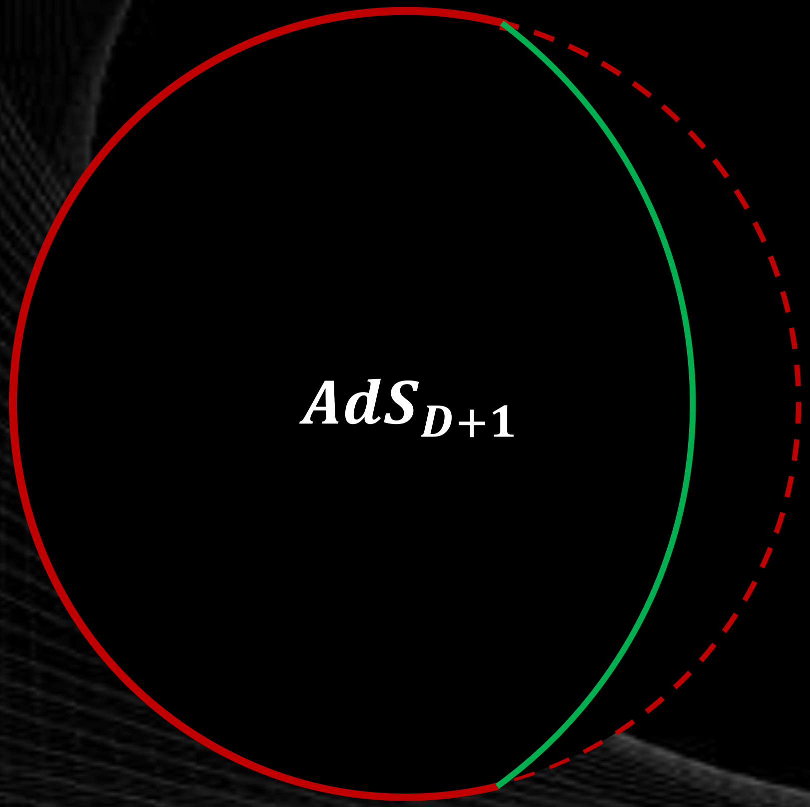
Braneworld Holography

Limitation of conventional AdS/CFT: boundary of AdS is **fixed**



Braneworld Holography

Limitation of conventional AdS/CFT: boundary of AdS is **fixed** → Introducing the brane



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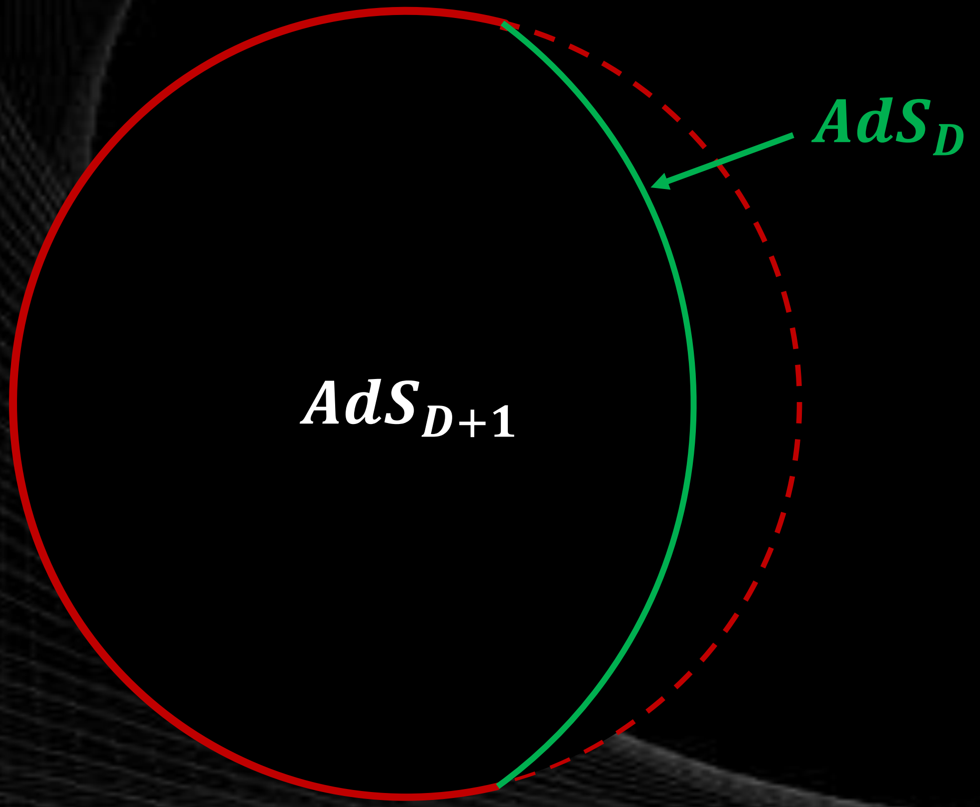

The geometry on the brane is dynamical!
Graviton modes are induced on the brane

Randall and Sundrum, PRL **83** (1999)
Karch and Randall, JHEP **05** (2001)

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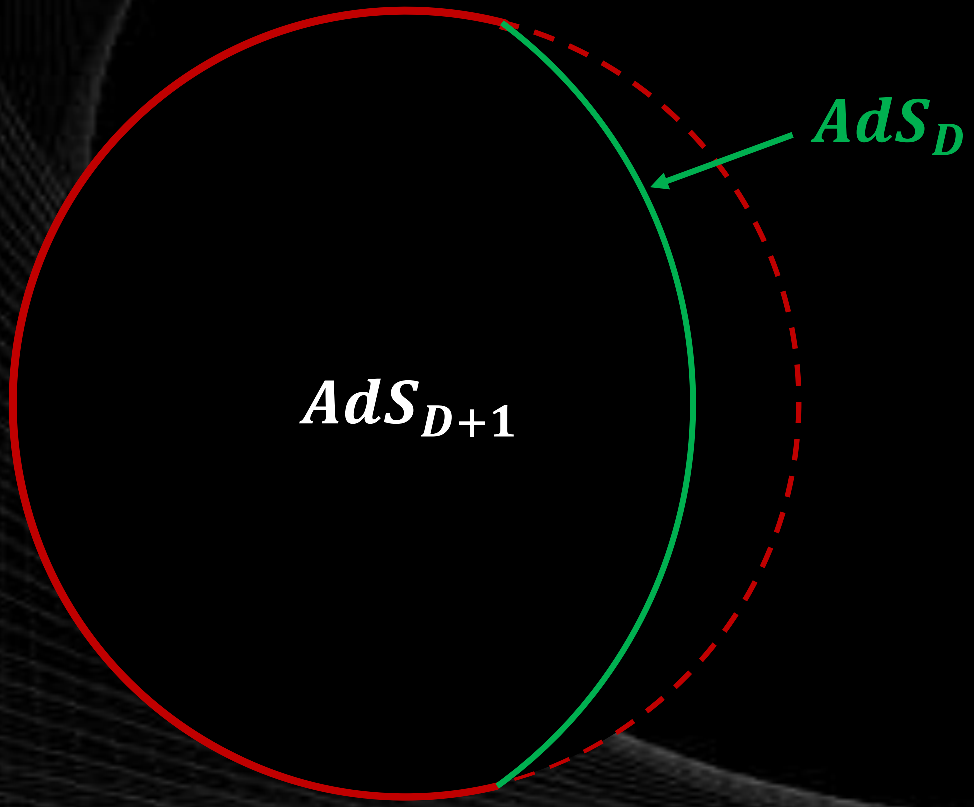

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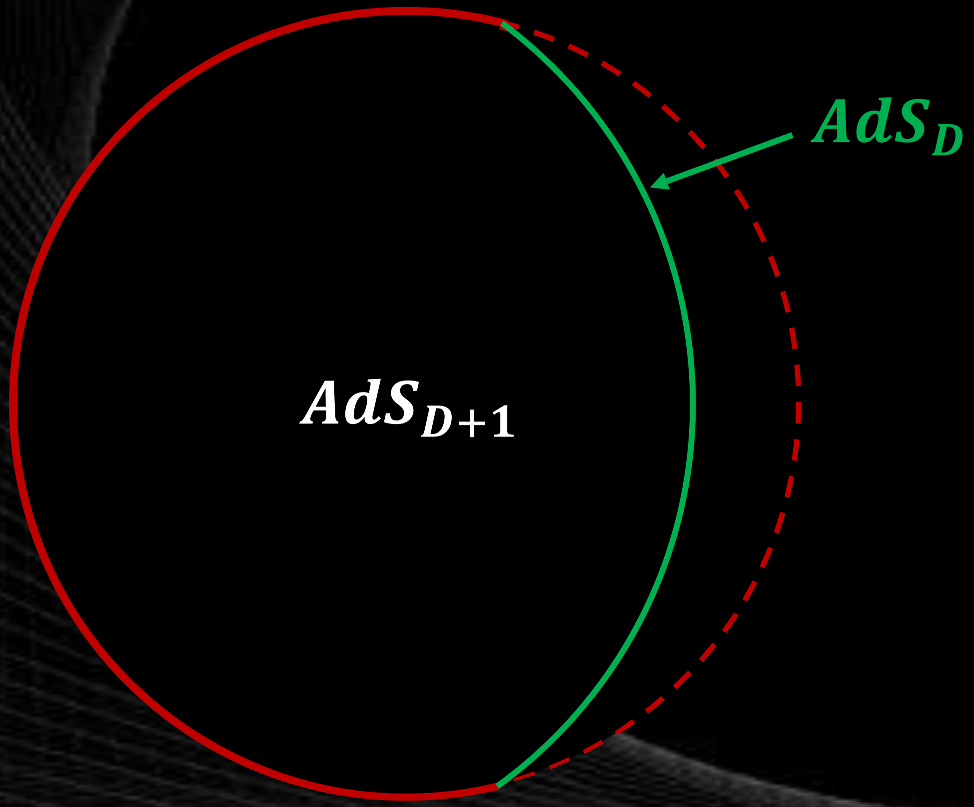

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$$\mathcal{S} = \frac{\ell_4}{8\pi G_4}\int \mathrm{d}^3x\,\sqrt{-h}\left[\frac{4}{\ell_4^2}\left(1-\frac{\ell_4}{\ell}\right)+R+\ell_4^2\left(\frac{3}{8}R^2-R_{\mu\nu}R^{\mu\nu}\right)+\ldots\right]+\mathcal{S}_{\mathrm{CFT}}$$

Braneworld Holography

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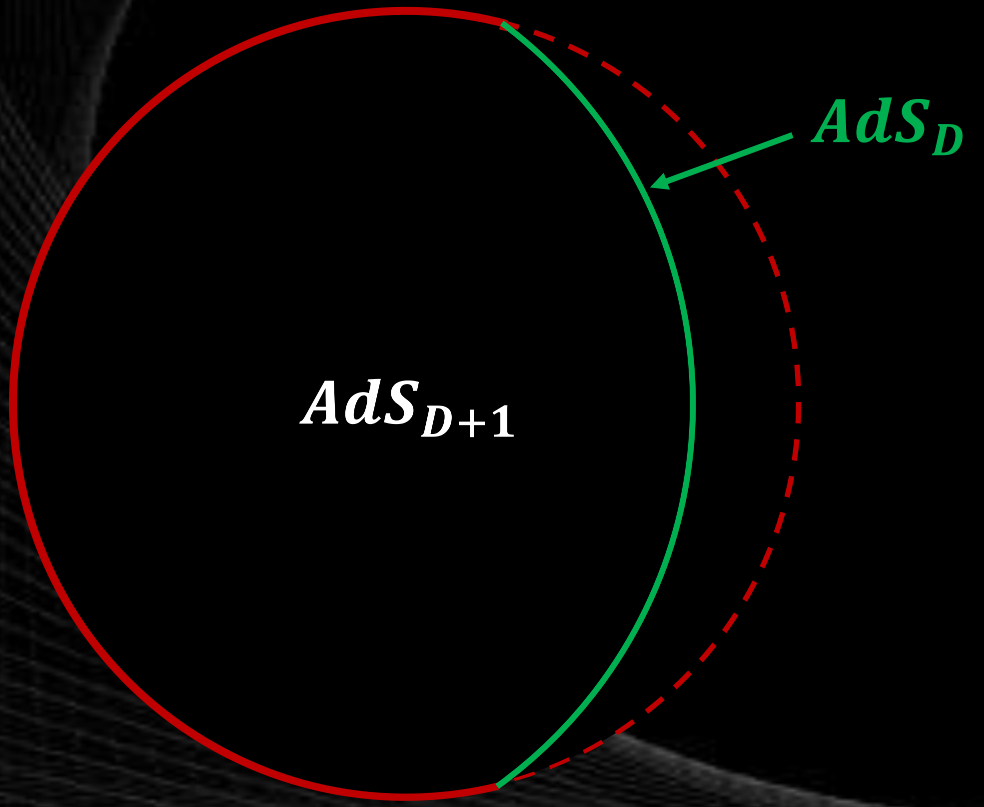

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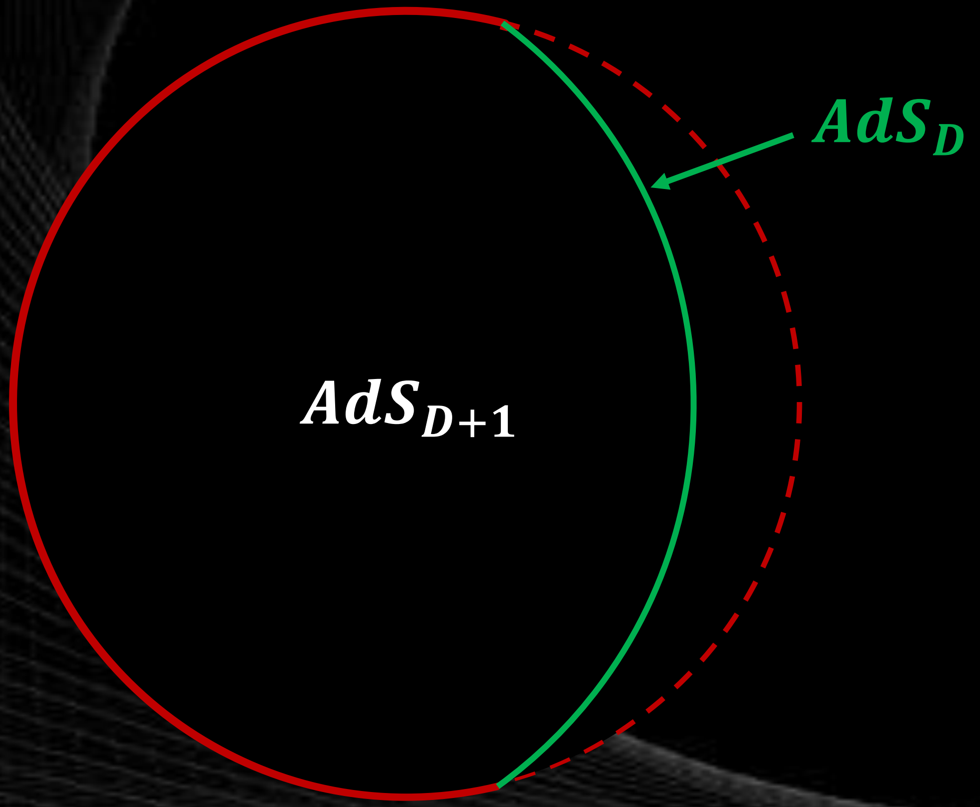
ℓ^{-1} ~ Brane tension ~ Cutoff

Braneworld Holography

Limitation of conventional AdS/CFT: boundary of AdS is **fixed** → Introducing the brane



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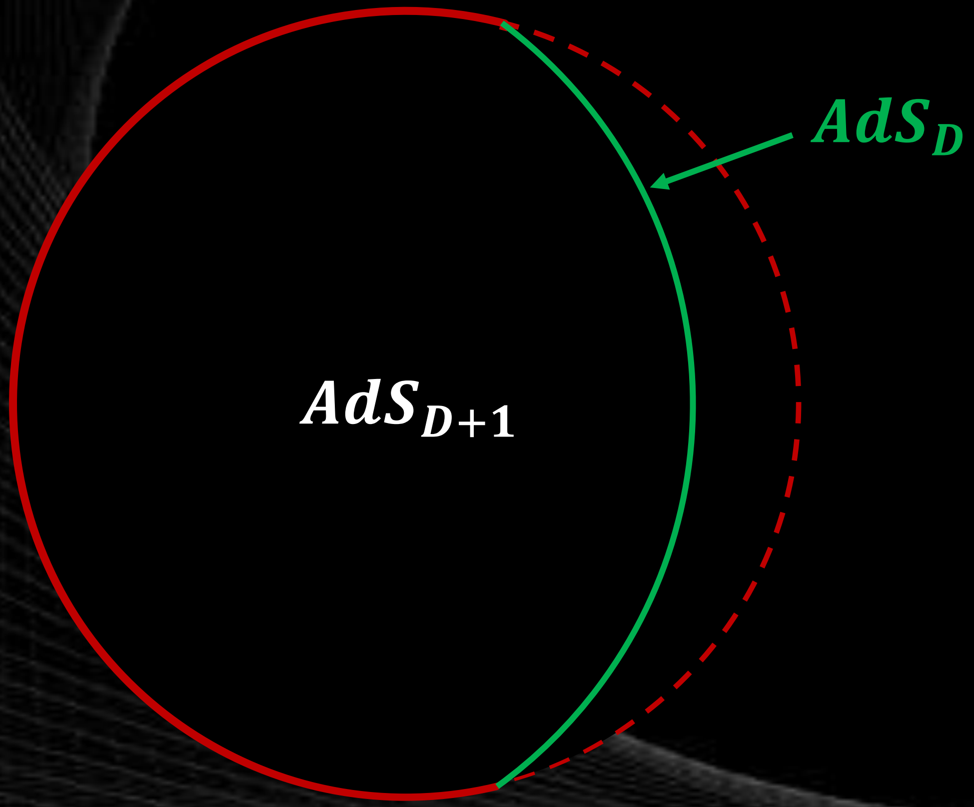

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Einstein + higher curvature terms

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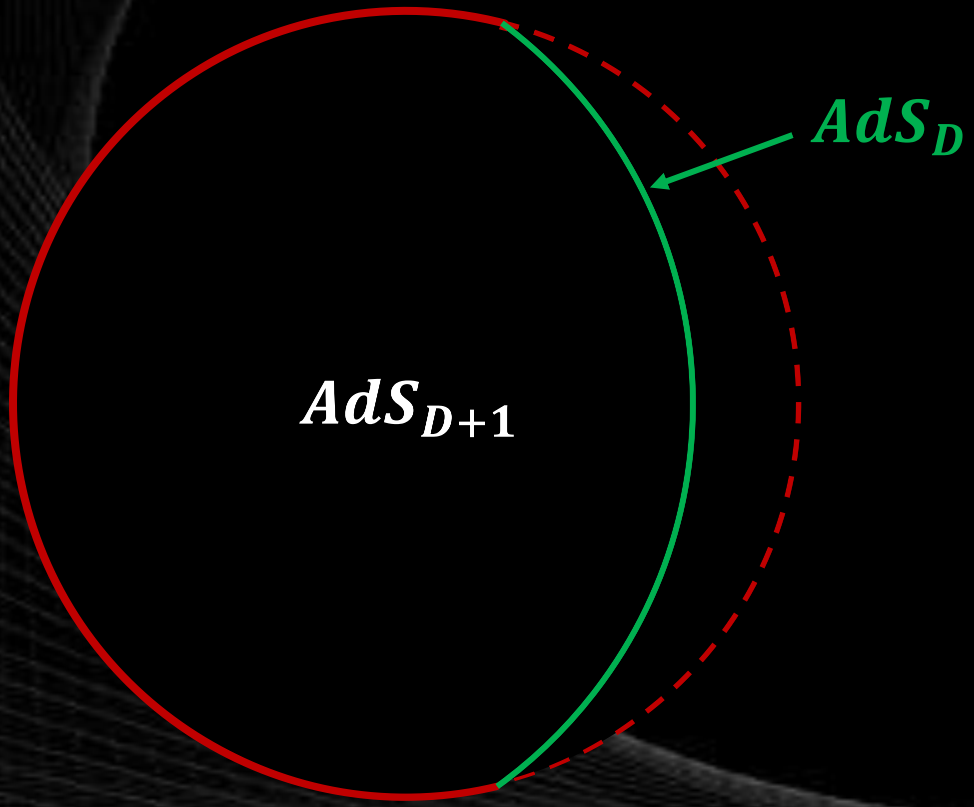

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Planar CFT with a cutoff

Braneworld Holography

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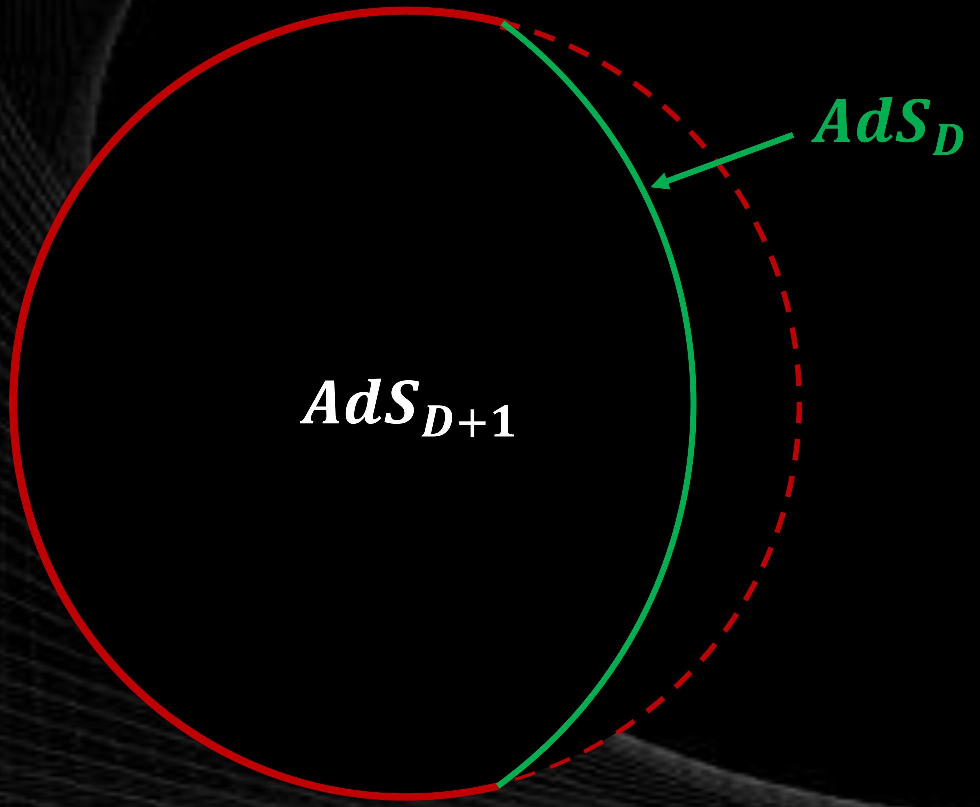

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Solutions of semiclassical brane EoM ↔ Solutions of classical bulk EoM with brane boundary conditions

Braneworld Holography

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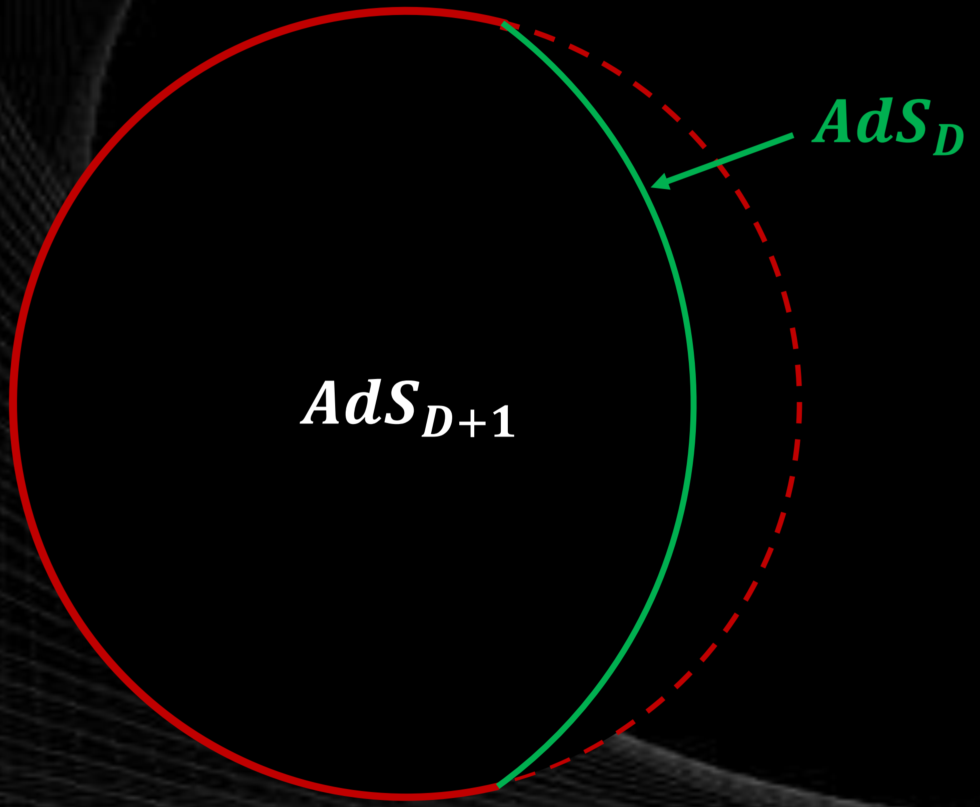

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Einstein + higher curvature terms

Planar CFT with a cutoff

The black hole solutions localized on the brane in the AdS_{D+1} braneworld which are found by solving the classical bulk equations in AdS_{D+1} with the brane boundary conditions, correspond to quantum-corrected black holes in D dimensions, rather than classical ones.

Emparn, Fabbri, Kaloper, JHEP **08** (2002)
Emparan, Horowitz, Myers, JHEP **01** (2000) 007
Emparan, Horowitz, Myers, JHEP **01** (2000) 021

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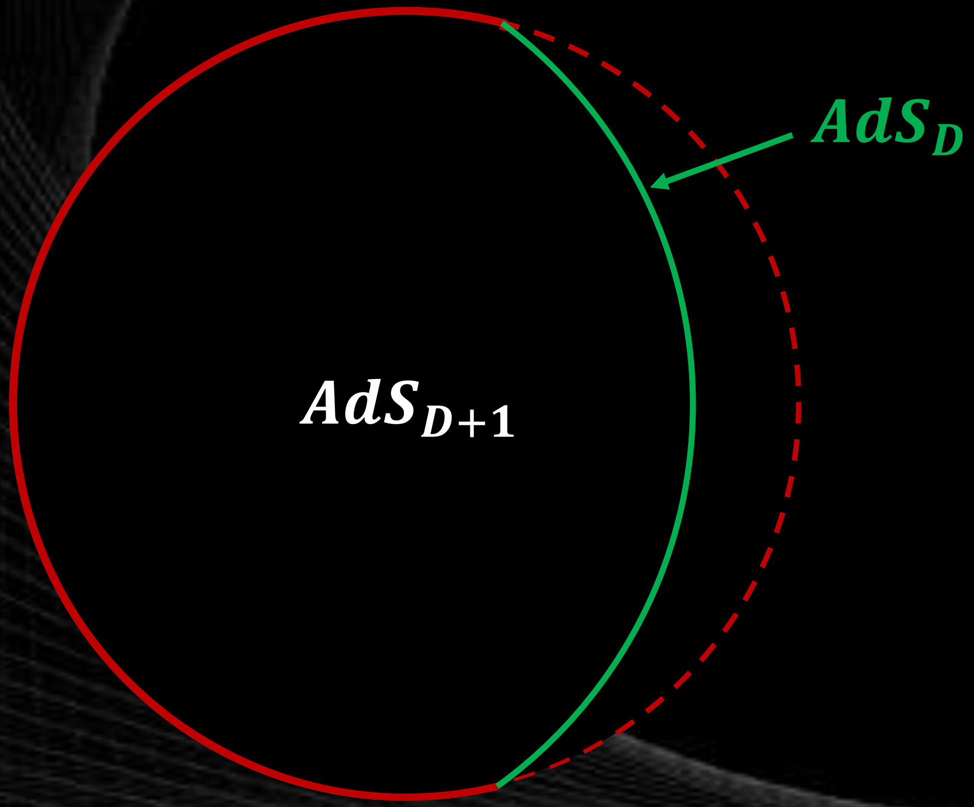

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Planar CFT with a cutoff

Consequences

Braneworld Holography

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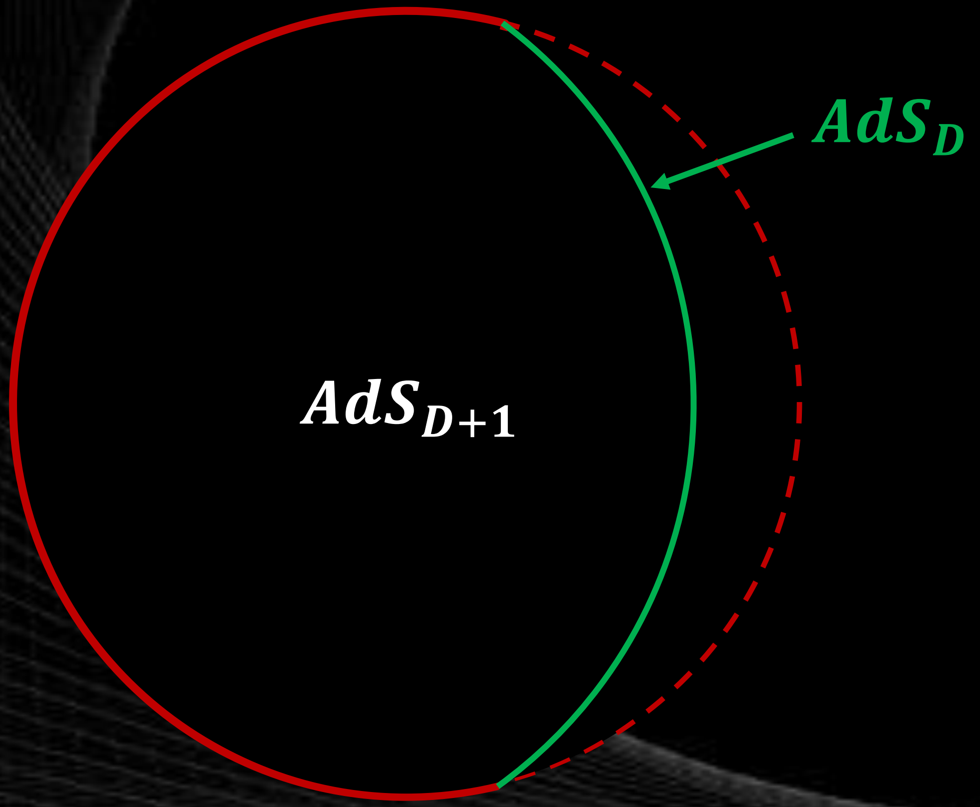

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Planar CFT with a cutoff

Consequences

- Existence of new BH solutions

Braneworld Holography

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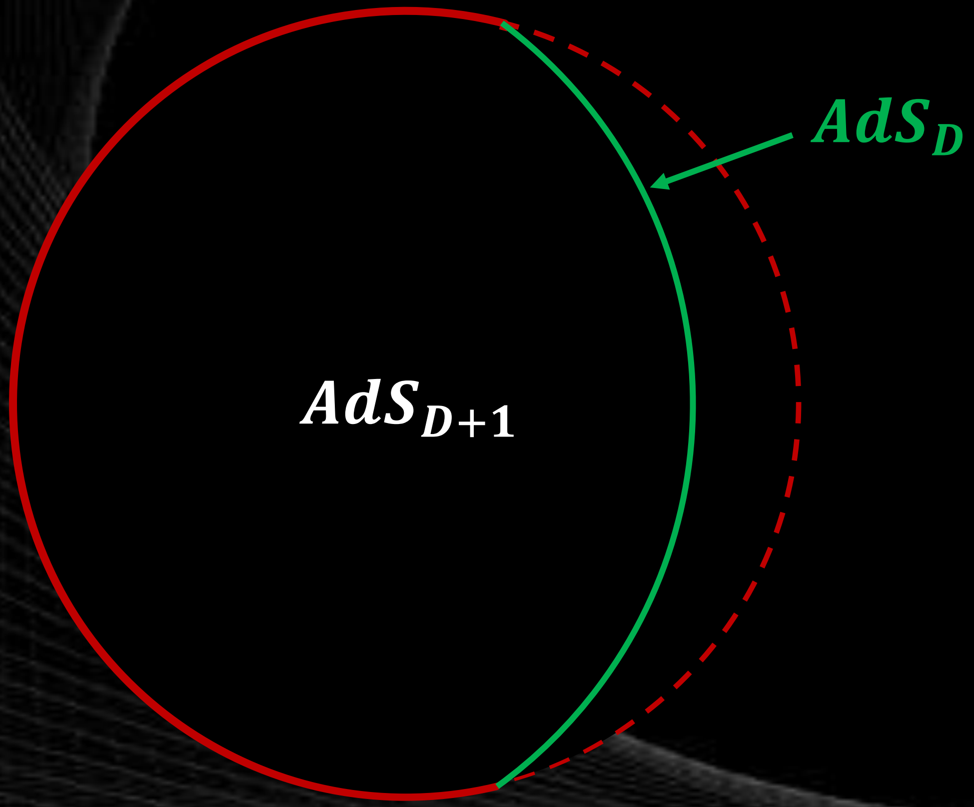

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Braneworld Holography

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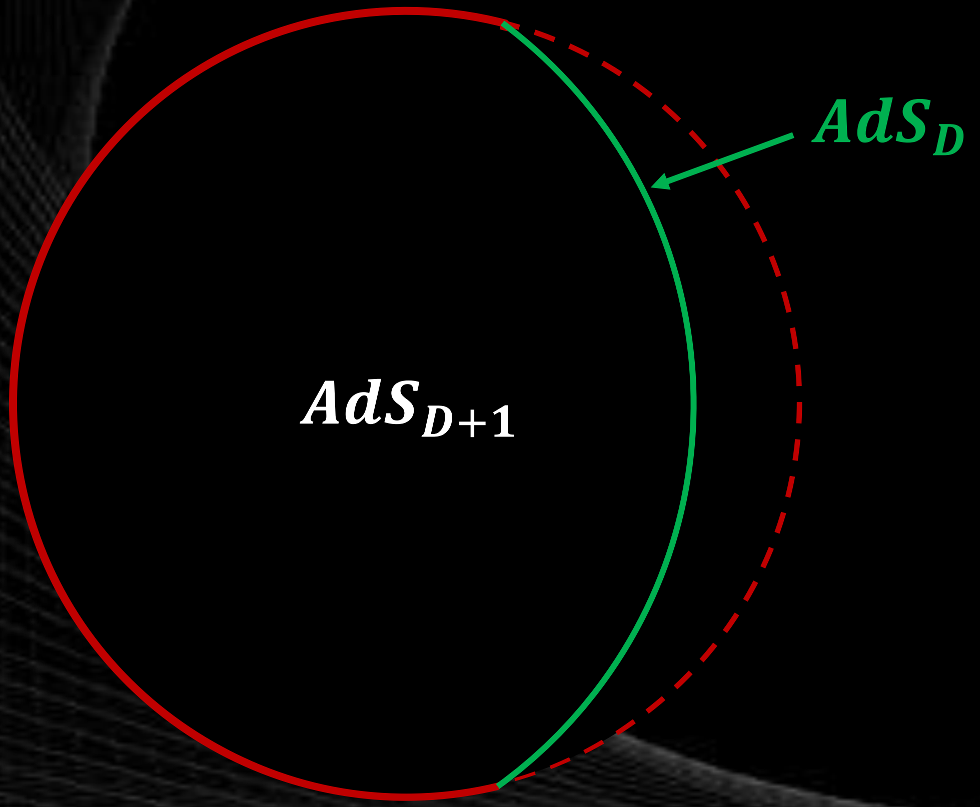

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Braneworld Holography

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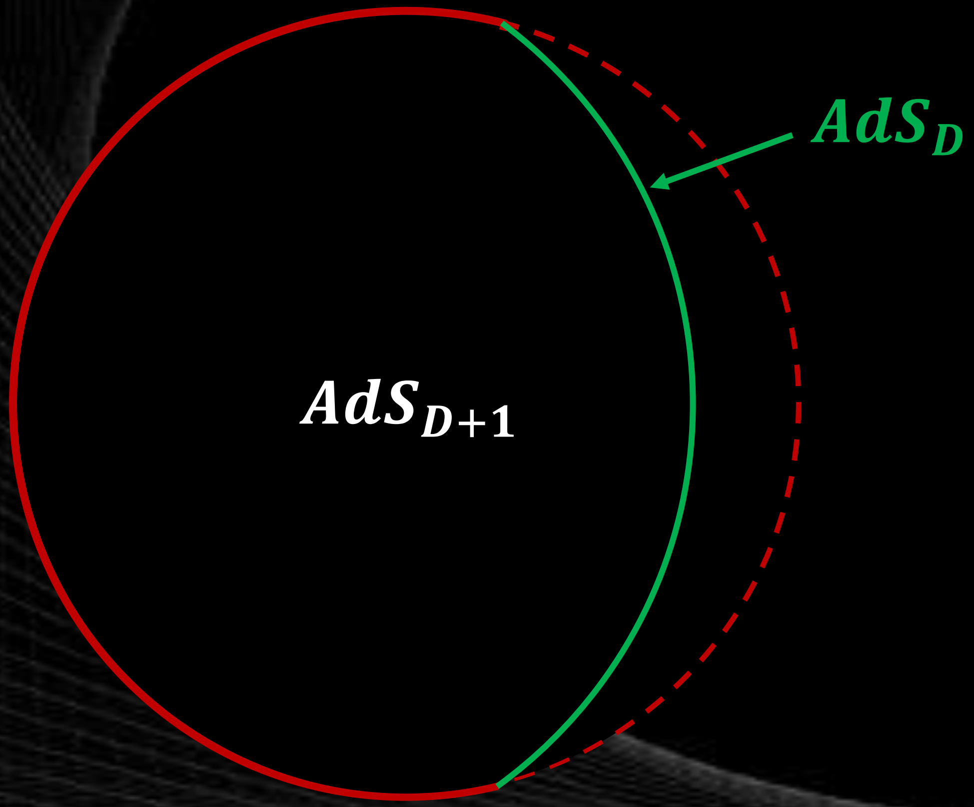

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Braneworld Holography

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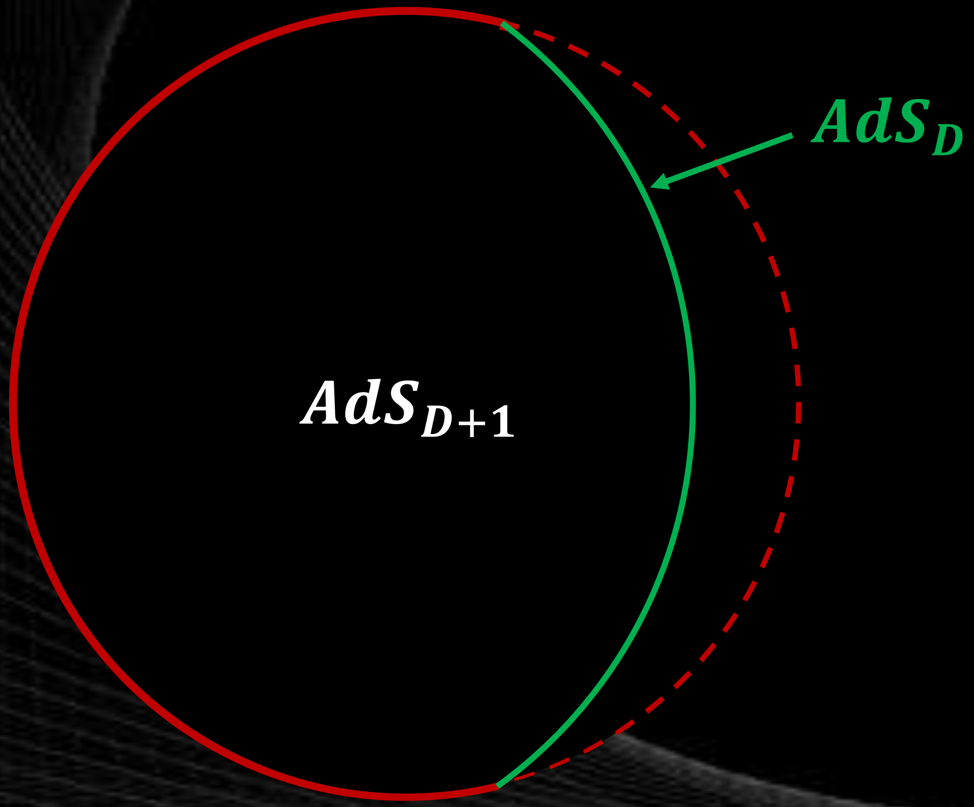

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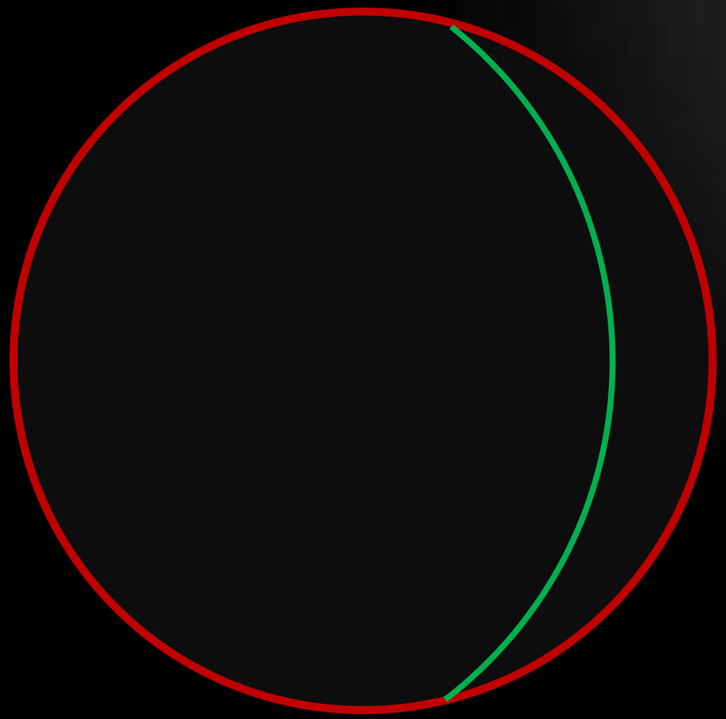
quantum BTZ

Black holes on branes



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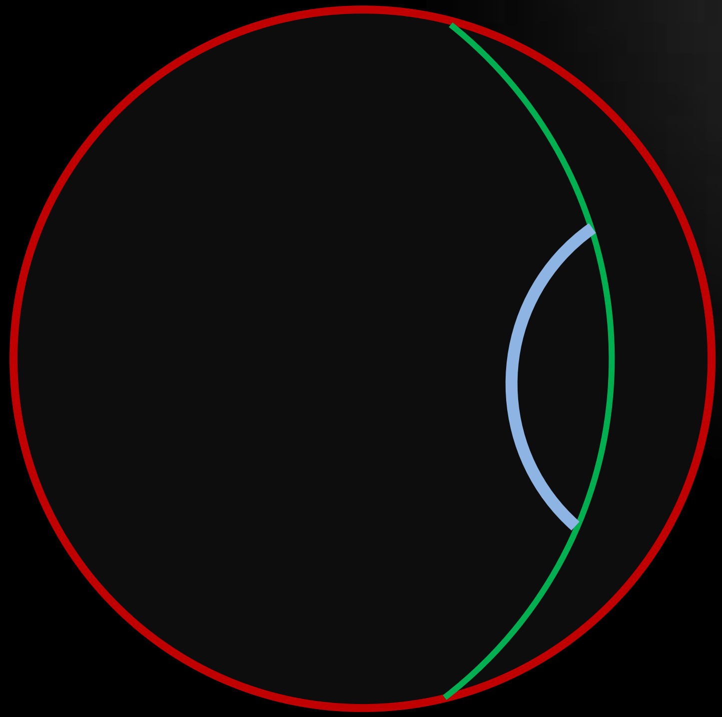

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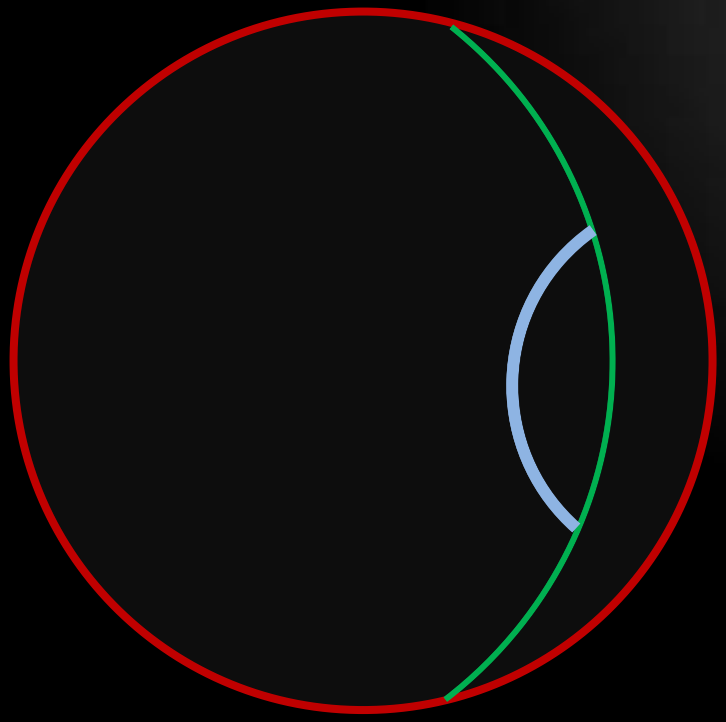

A bulk black hole whose horizon intersects the brane is accelerating towards the boundary!

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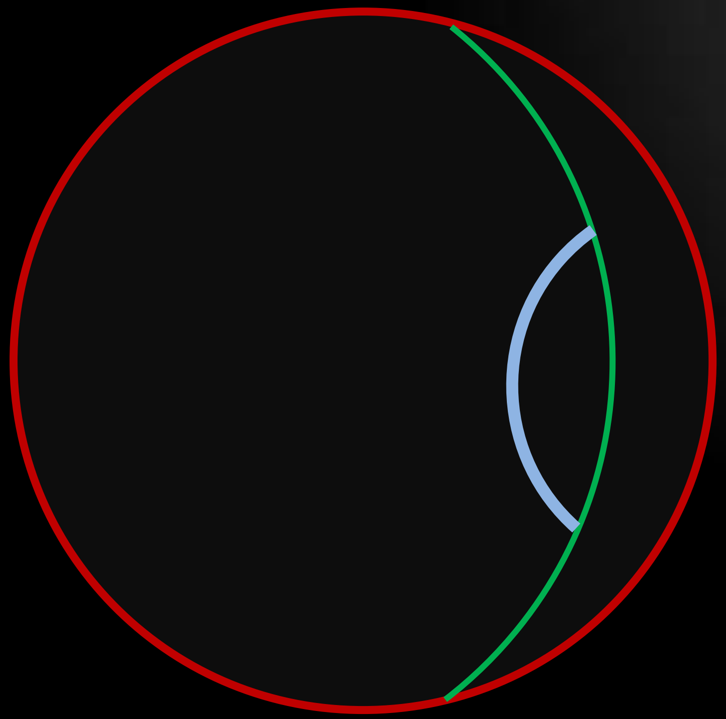

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4D **classical** bulk solution: rotating AdS C-metric sliced with a brane

Plebanski & Demianski, Annals Phys. **98** (1976)

Rotating qBTZ metric

Emparan, Frassino, Way, JHEP **11** (2020)

$$ds^2 = g_{tt}\,dt^2 + g_{\phi\phi}\,d\phi^2 + 2g_{t\phi}\,dt\,d\phi + g_{rr}\,dr^2$$

Rotating qBTZ metric

Emparan, Frassino, Way, JHEP **11** (2020)

$$\mathrm{d}s^2 = g_{tt}\,\mathrm{d}t^2 + g_{\phi\phi}\,\mathrm{d}\phi^2 + 2g_{t\phi}\,\mathrm{d}t\,\mathrm{d}\phi + g_{rr}\,\mathrm{d}r^2$$

$$g_{tt} = -\frac{8\sqrt{1-\tilde{a}^2}\,\nu\,\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^3\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}} + \frac{16\tilde{a}^2-4\left(\tilde{a}^2+1\right)\kappa x_1^2}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2} - \frac{r^2}{\ell_3^2}$$

$$g_{\phi\phi} = r^2 - \frac{8\tilde{a}^2\sqrt{1-\tilde{a}^2}\nu\ell_3^3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^3\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}$$

$$g_{t\phi} = -\frac{4\tilde{a}\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(3-\tilde{a}^2-\kappa x_1^2\right)^2}\left(1+\frac{2\sqrt{1-\tilde{a}^2}\nu\ell_3}{\left(3-\tilde{a}^2-\kappa x_1^2\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}\right)$$

$$g^{rr} = \frac{r^2}{\ell_3^2} - \frac{8\left(1-\tilde{a}^2\right)^{3/2}\nu\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}{r^2\left(3-\tilde{a}^2-\kappa x_1^2\right)^3} + \frac{16\tilde{a}^2\ell_3^2\left(\tilde{a}^2-\kappa x_1^2+1\right)^2}{r^2\left(\tilde{a}^2+\kappa x_1^2-3\right)^4} + \frac{4\left[\left(\tilde{a}^2+1\right)\kappa x_1^2-4\tilde{a}^2\right]}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}$$

Rotating qBTZ metric

Emparan, Frassino, Way, JHEP **11** (2020)

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$$g_{t\phi} = -\frac{4\tilde{a}\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(3-\tilde{a}^2-\kappa x_1^2\right)^2}\left(1+\frac{2\sqrt{1-\tilde{a}^2}\nu\ell_3}{\left(3-\tilde{a}^2-\kappa x_1^2\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}\right)$$

$$g^{rr} = \frac{r^2}{\ell_3^2} - \frac{8\left(1-\tilde{a}^2\right)^{3/2}\nu\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}{r^2\left(3-\tilde{a}^2-\kappa x_1^2\right)^3} + \frac{16\tilde{a}^2\ell_3^2\left(\tilde{a}^2-\kappa x_1^2+1\right)^2}{r^2\left(\tilde{a}^2+\kappa x_1^2-3\right)^4} + \frac{4\left[\left(\tilde{a}^2+1\right)\kappa x_1^2-4\tilde{a}^2\right]}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}$$

Relevant parameters

$\tilde{a},\, x_1,\, \kappa,\, \ell_3,\, \nu$

$\kappa = 0, \pm 1$

Rotating qBTZ metric

Emparan, Frassino, Way, JHEP **11** (2020)

$$ds^2 = g_{tt}\,\mathrm{d}t^2 + g_{\phi\phi}\,\mathrm{d}\phi^2 + 2g_{t\phi}\,\mathrm{d}t\,\mathrm{d}\phi + g_{rr}\,\mathrm{d}r^2$$

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Emparan, Frassino, Way, JHEP **11** (2020)

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$$M = \frac{1}{2\mathcal{G}_3}\,\frac{-\kappa x_1^2+\tilde{a}^2(4-\kappa x_1^2)}{(3-\kappa x_1^2-\tilde{a}^2)^2}$$

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$$\mathcal{G}_3 \propto G_4/\ell$$

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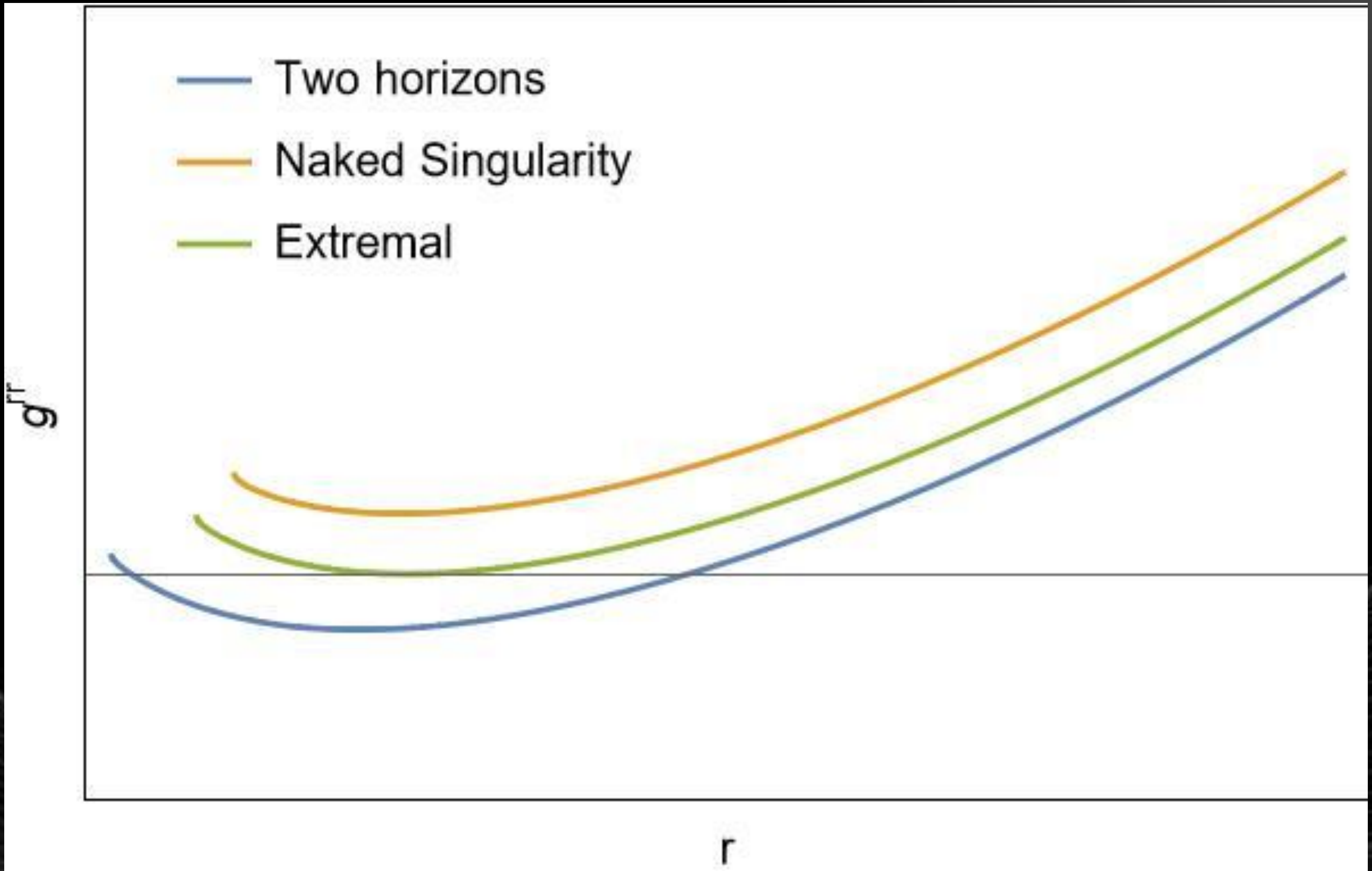
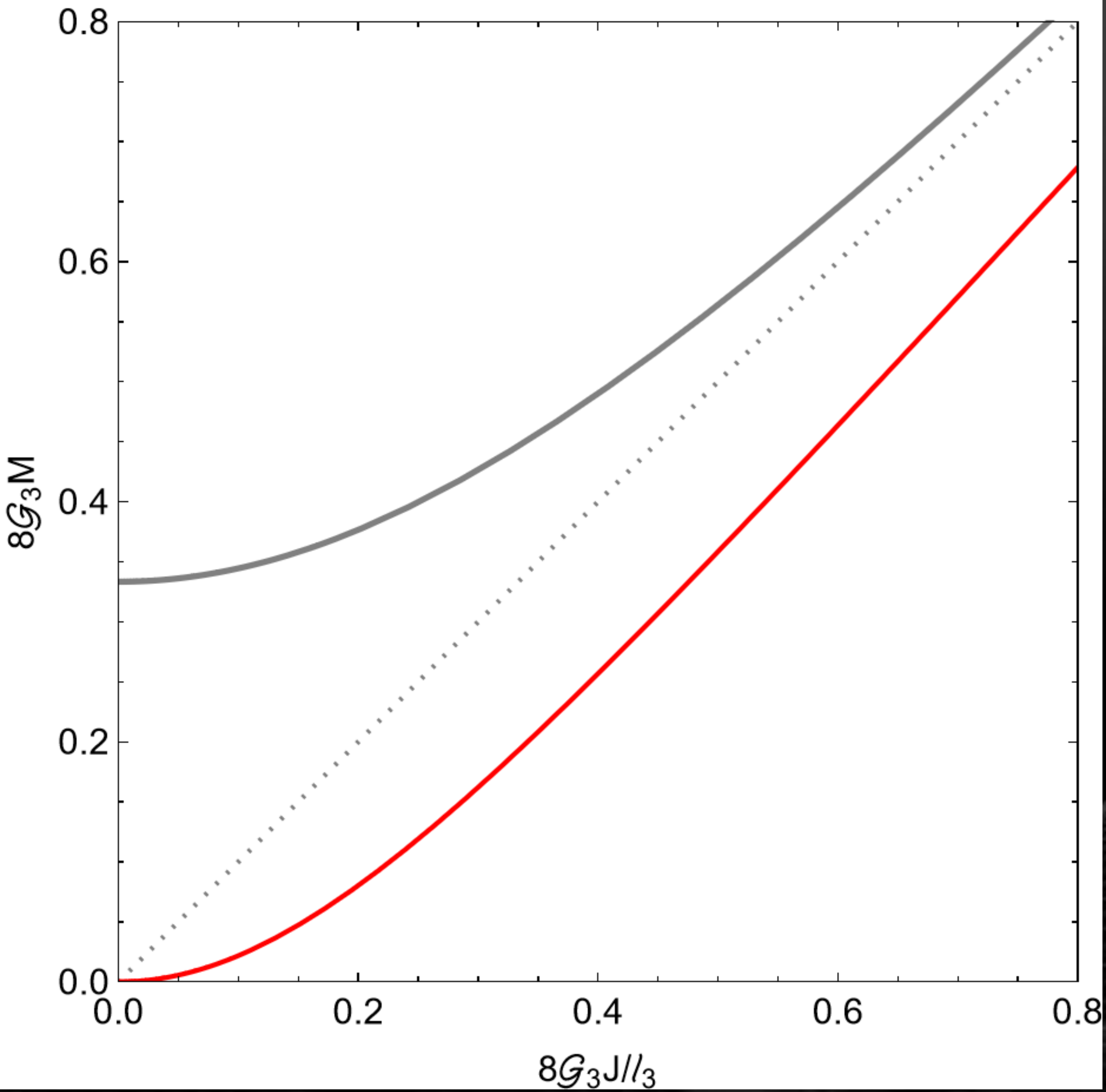
Metric structure



M vs J diagram

Emparan, Frassino, Way, JHEP **11** (2020)



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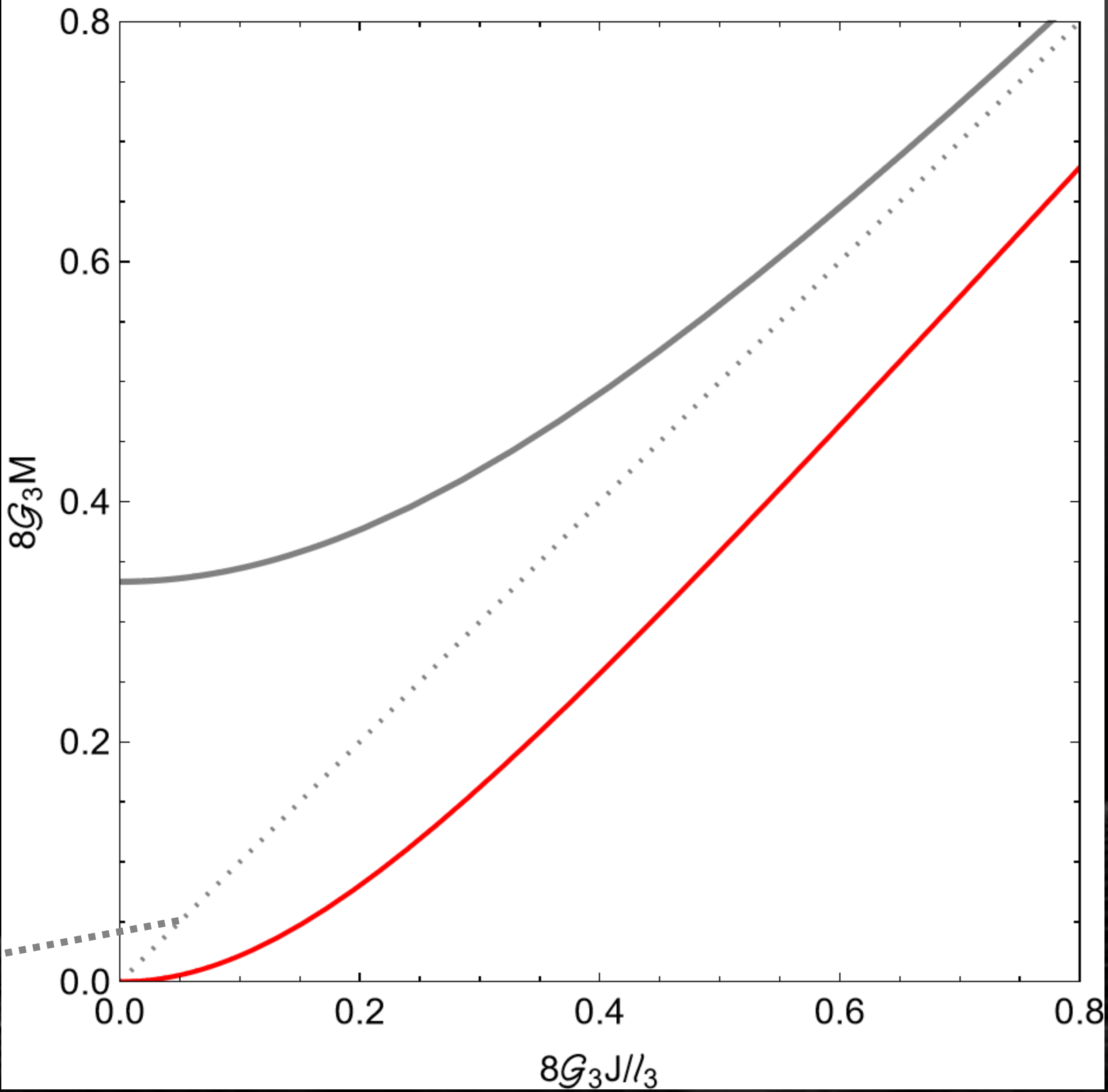
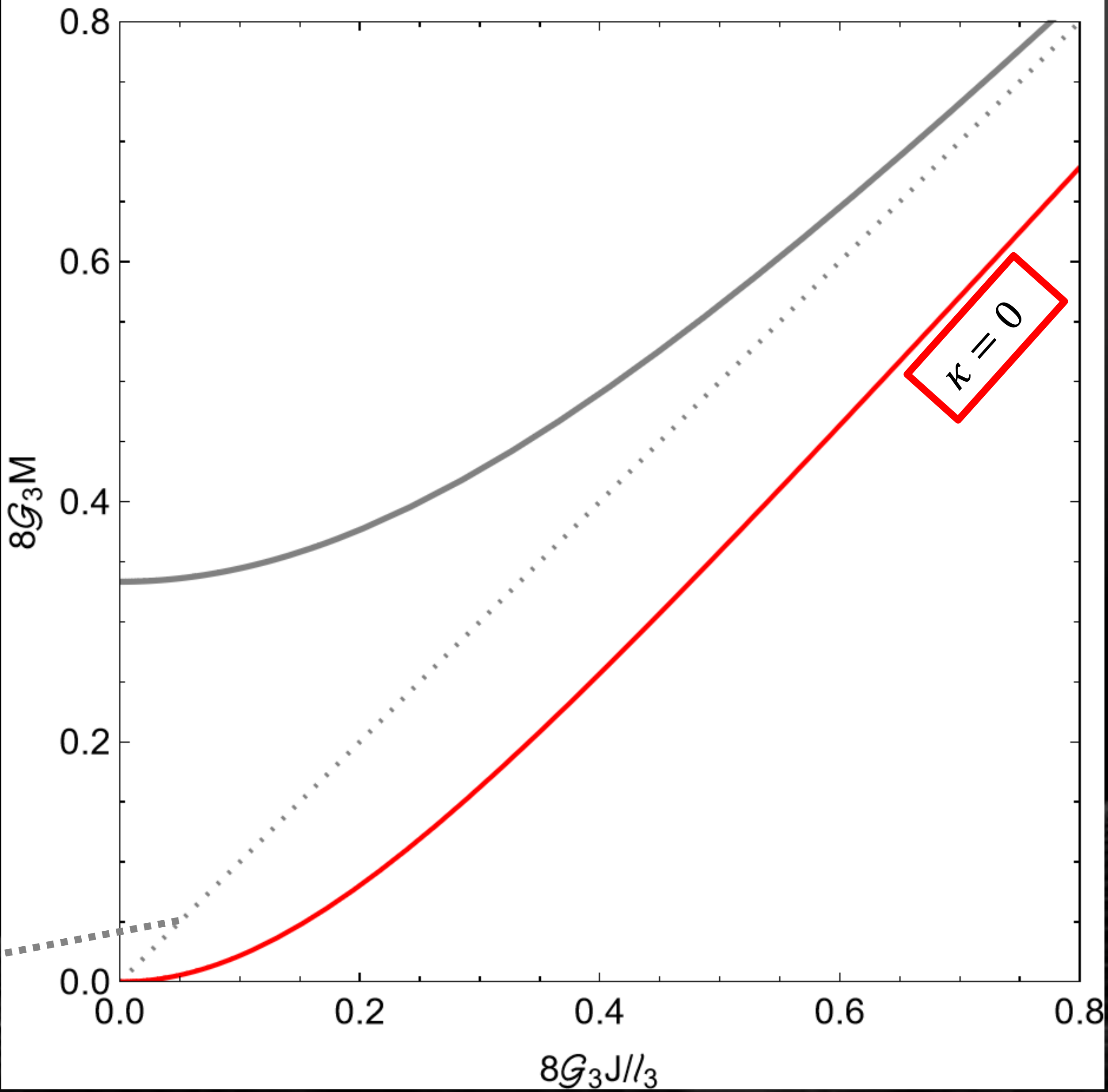


Classical BTZ extremality

M vs J diagram

Emparan, Frassino, Way, JHEP **11** (2020)



Classical BTZ extremality

M vs J diagram

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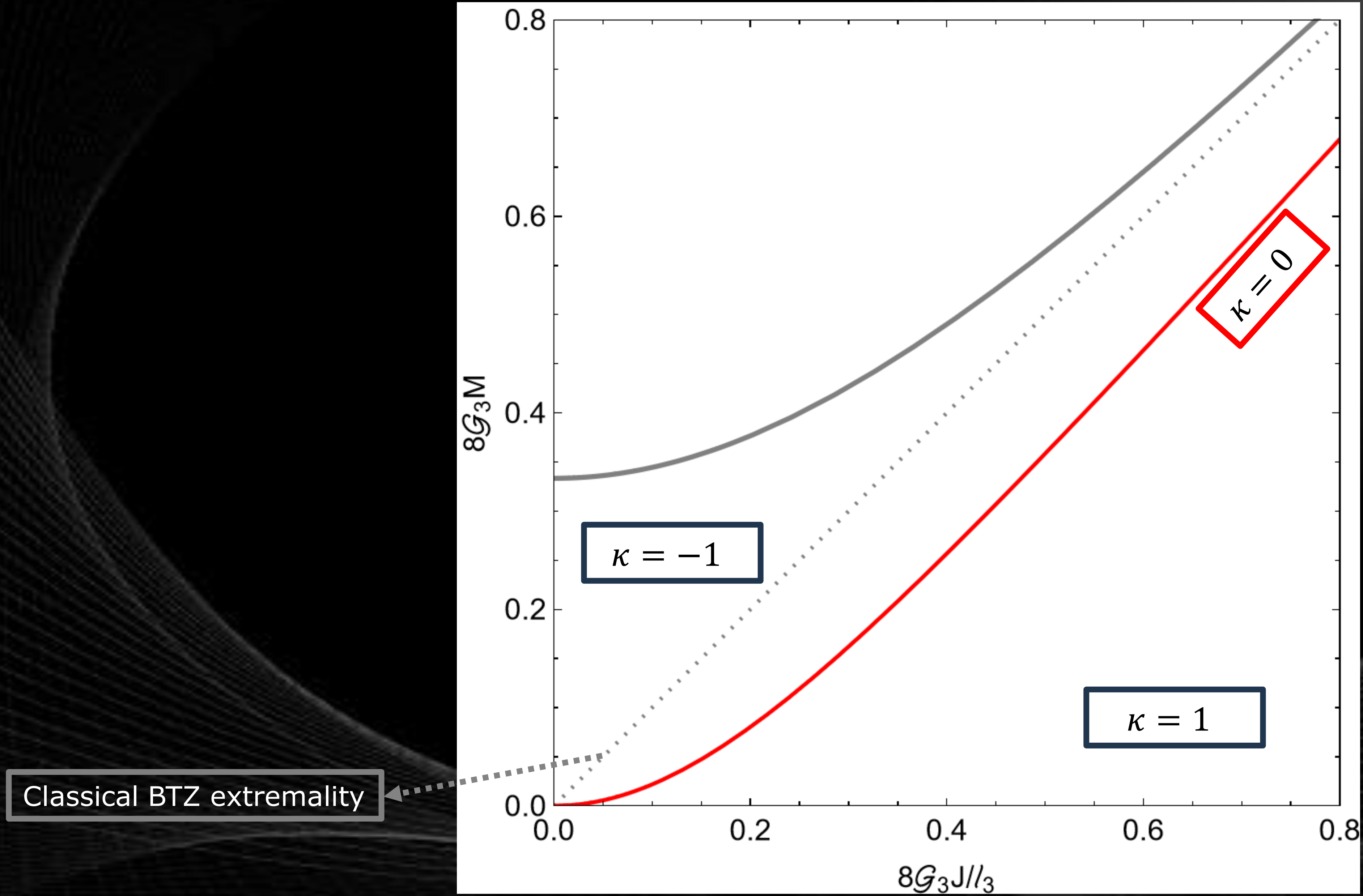

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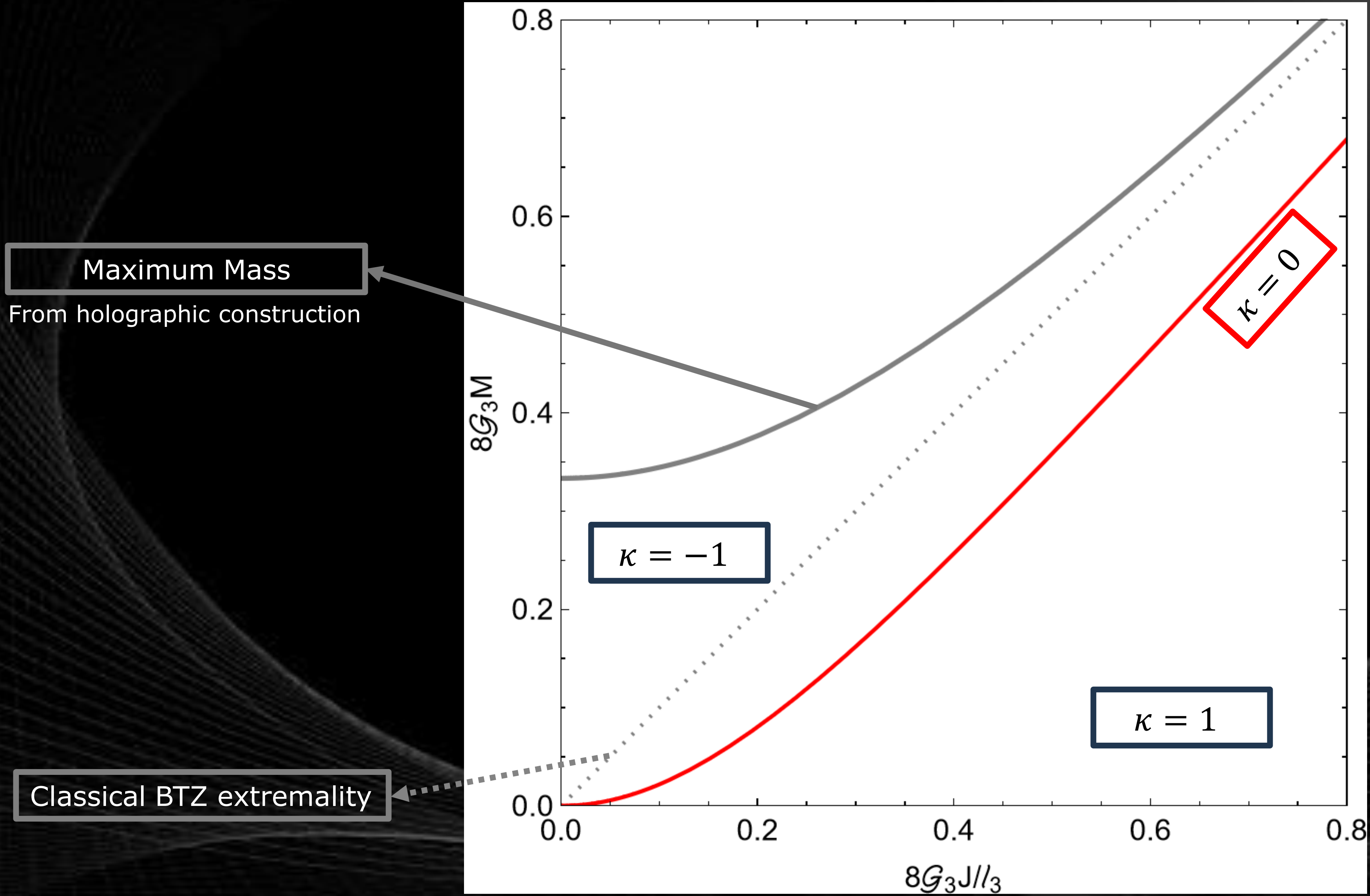

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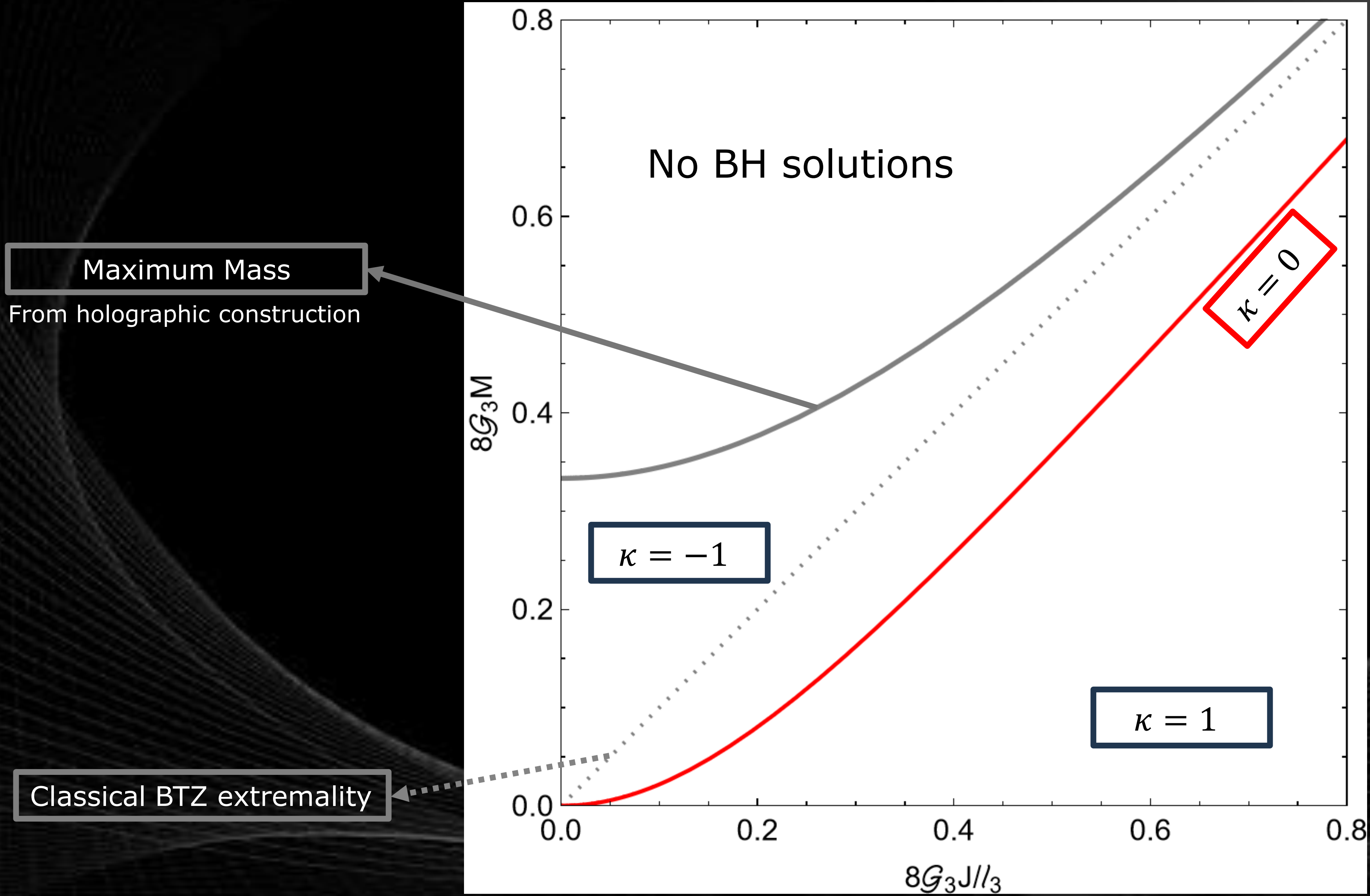

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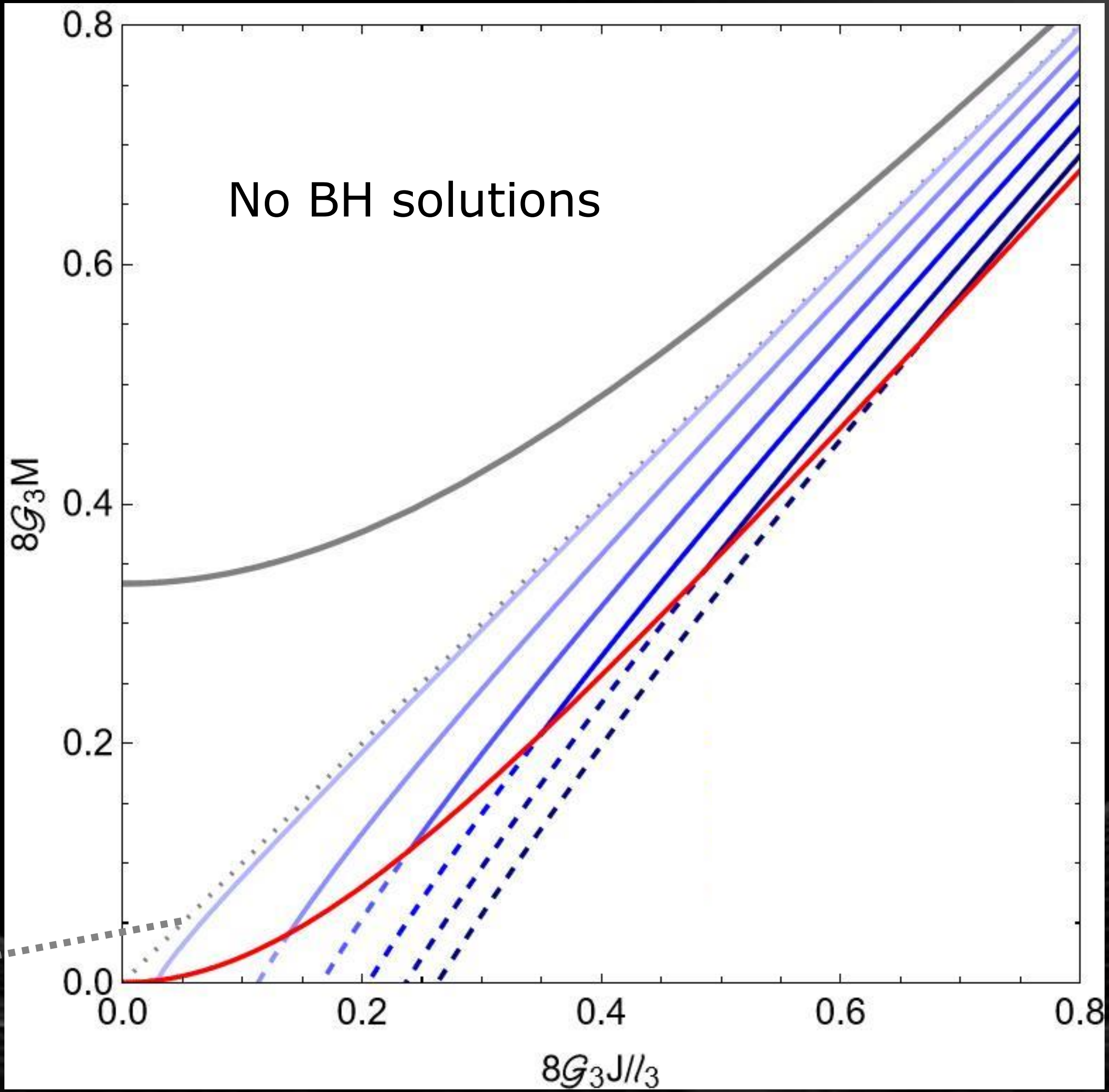

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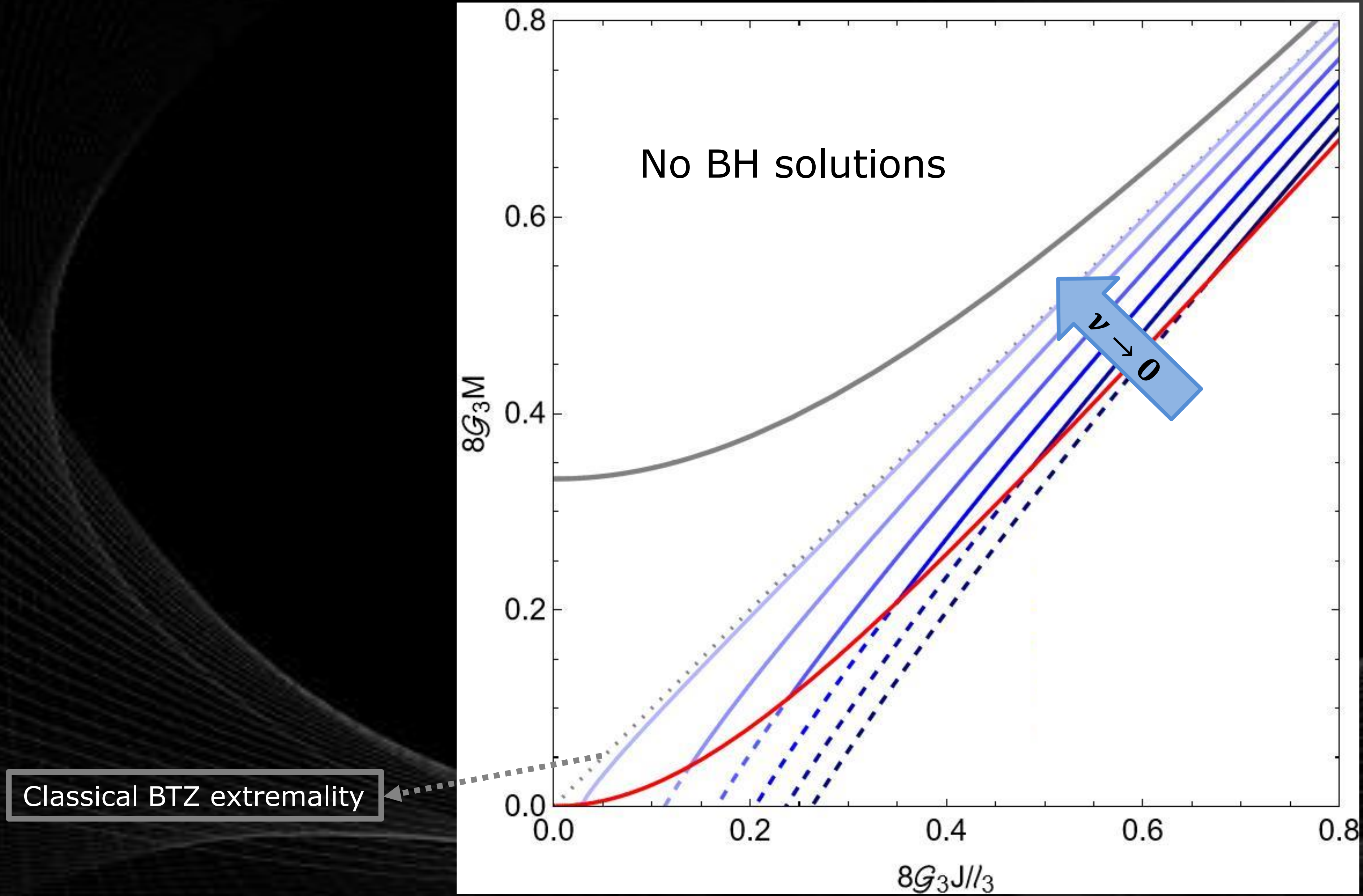

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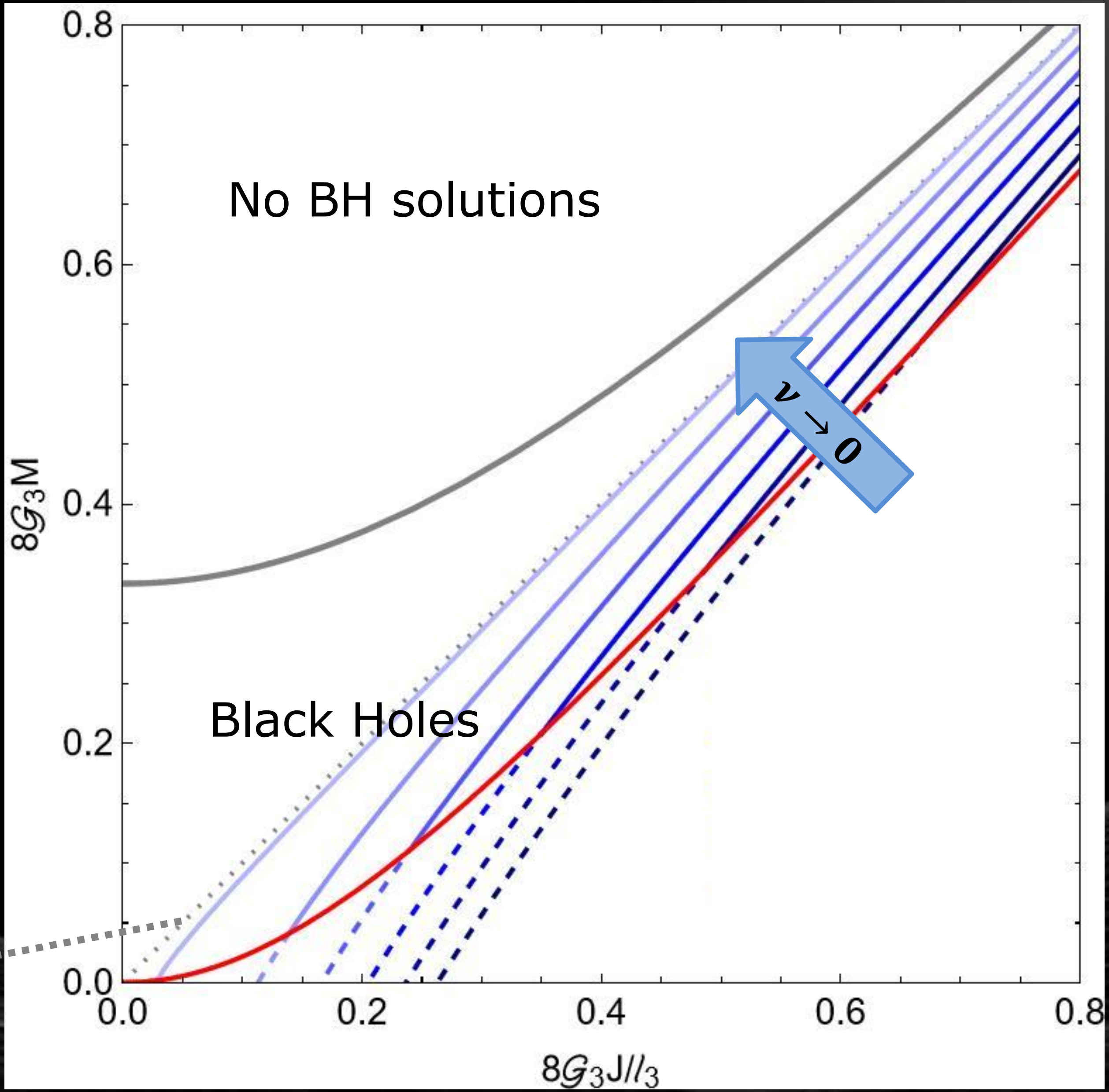

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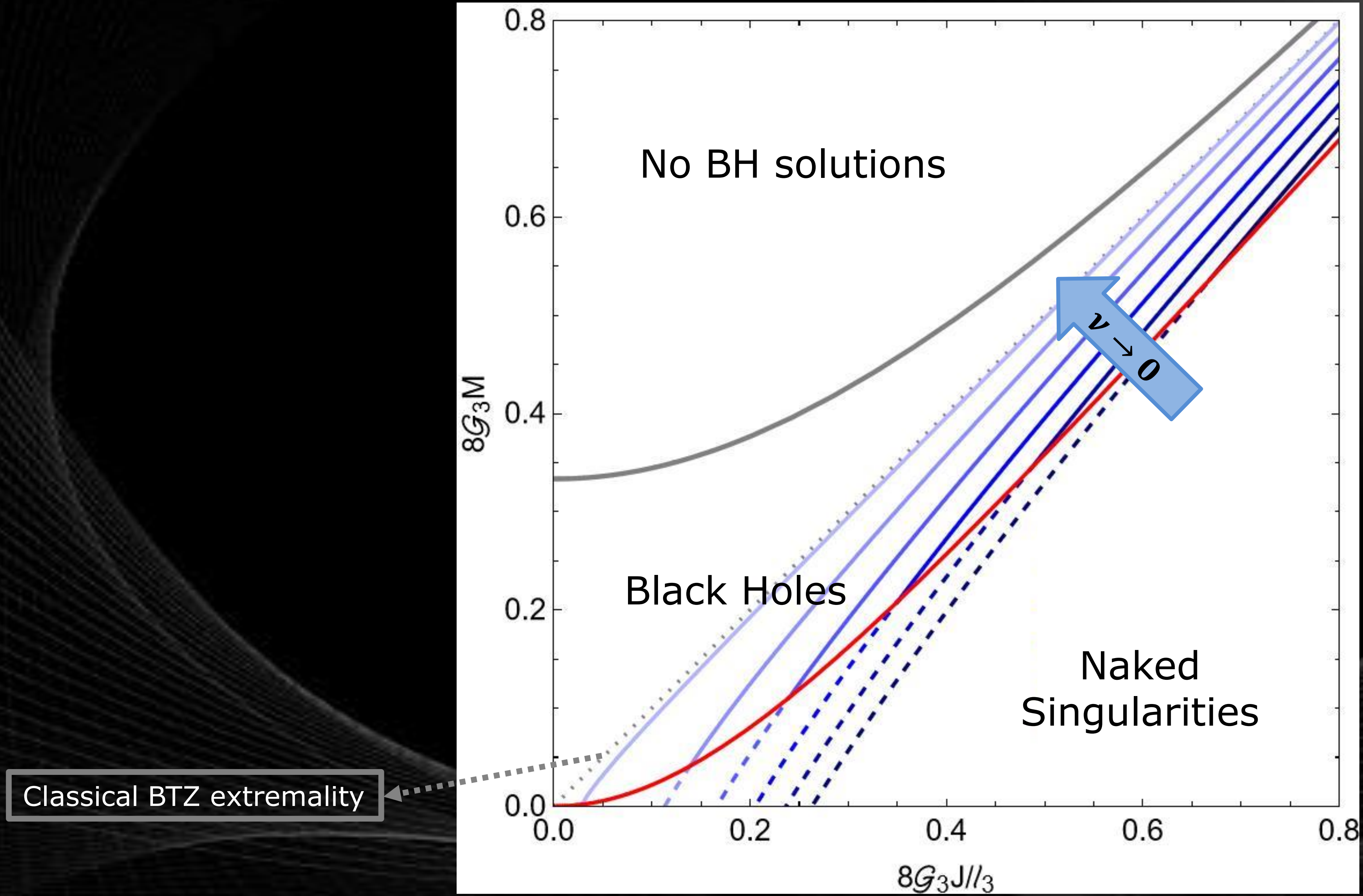

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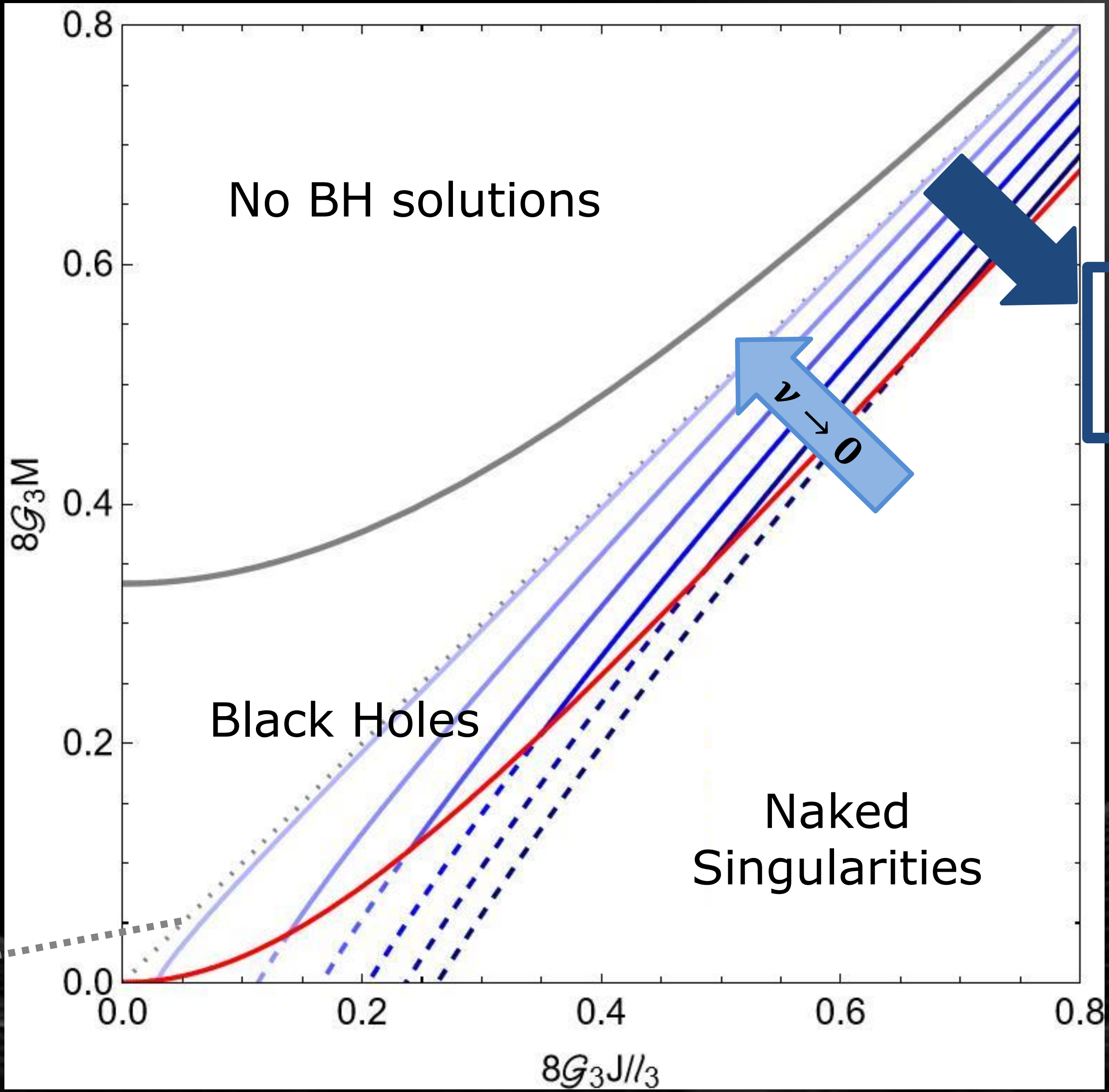

BH solution parameter space grows with ν

Classical BTZ extremality

wCCC tests w/ qBTZ

qBTZ metric: tests of wCCC

Main Idea

qBTZ metric: tests of wCCC

Main Idea

Start with the extremal configuration

qBTZ metric: tests of wCCC

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Start with the extremal configuration

↓

Particle Absorption

qBTZ metric: tests of wCCC

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Particle Absorption

↓ **(*)**

Check the sign of g^{rr} at the minimum of the final state

qBTZ metric: tests of wCCC

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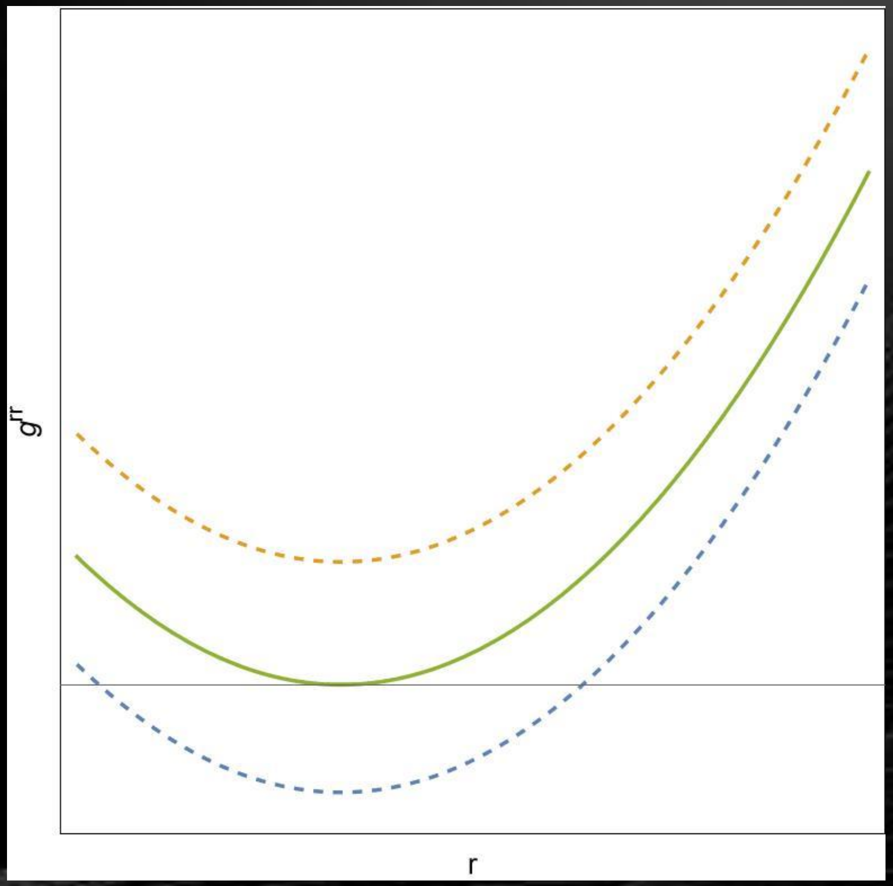

qBTZ metric: tests of wCCC

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↓

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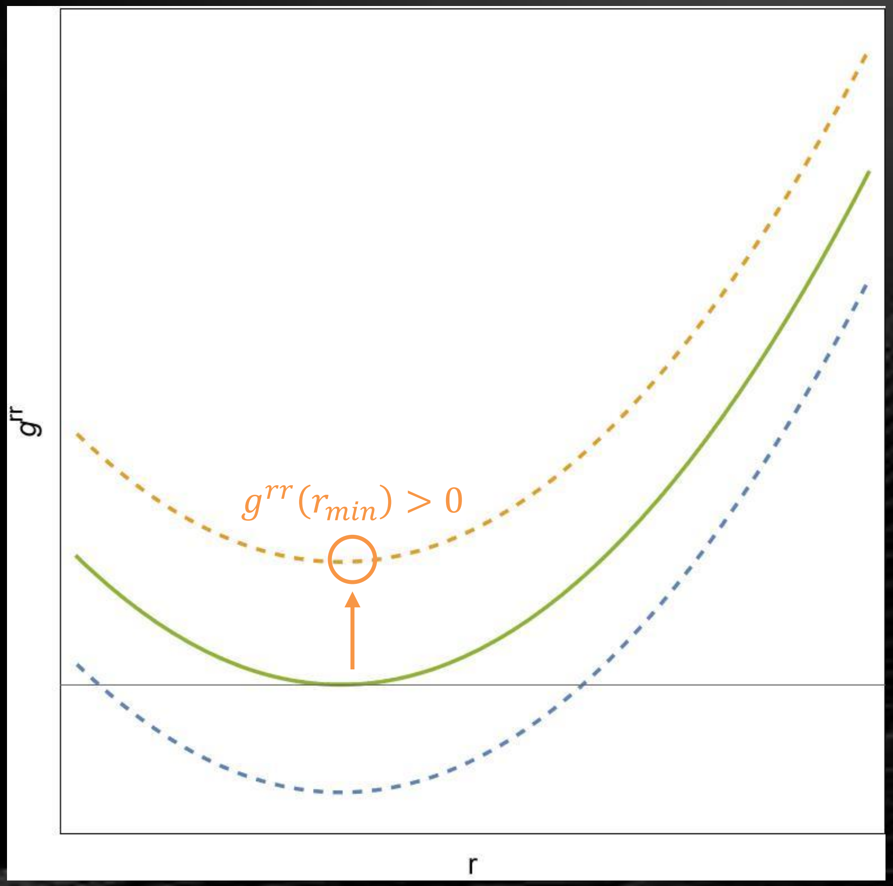

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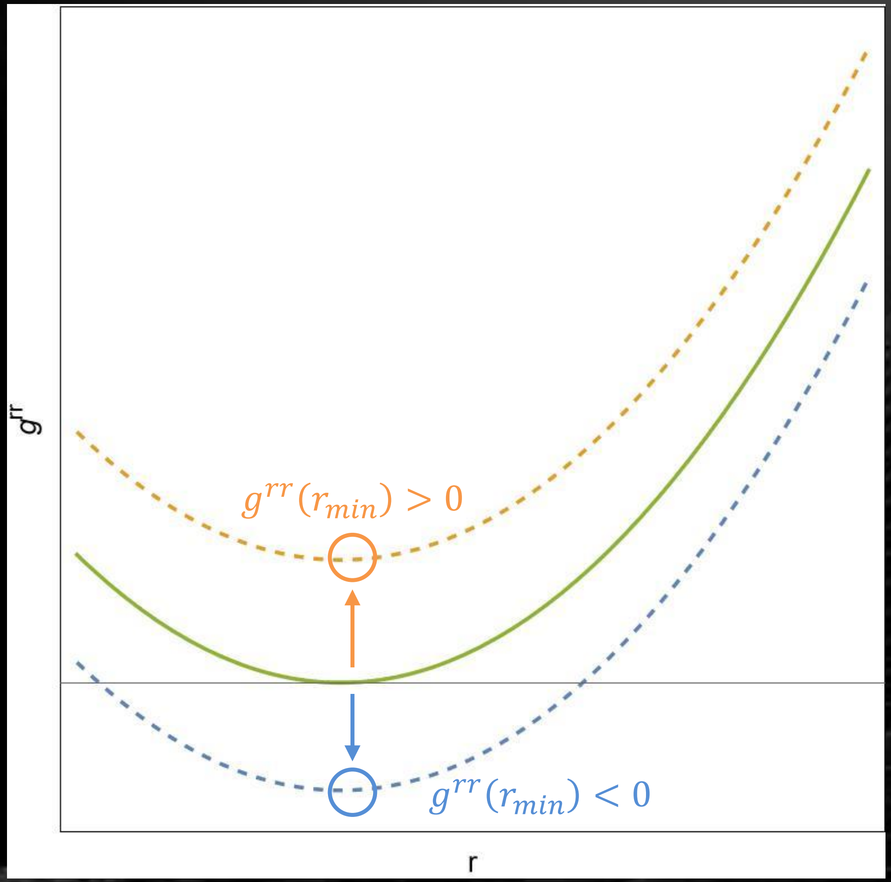


Perturbative analysis

Perturbative analysis

Linear response after absorption of a test particle

Perturbative analysis

Linear response after absorption of a test particle

Strategy

Perturbative analysis

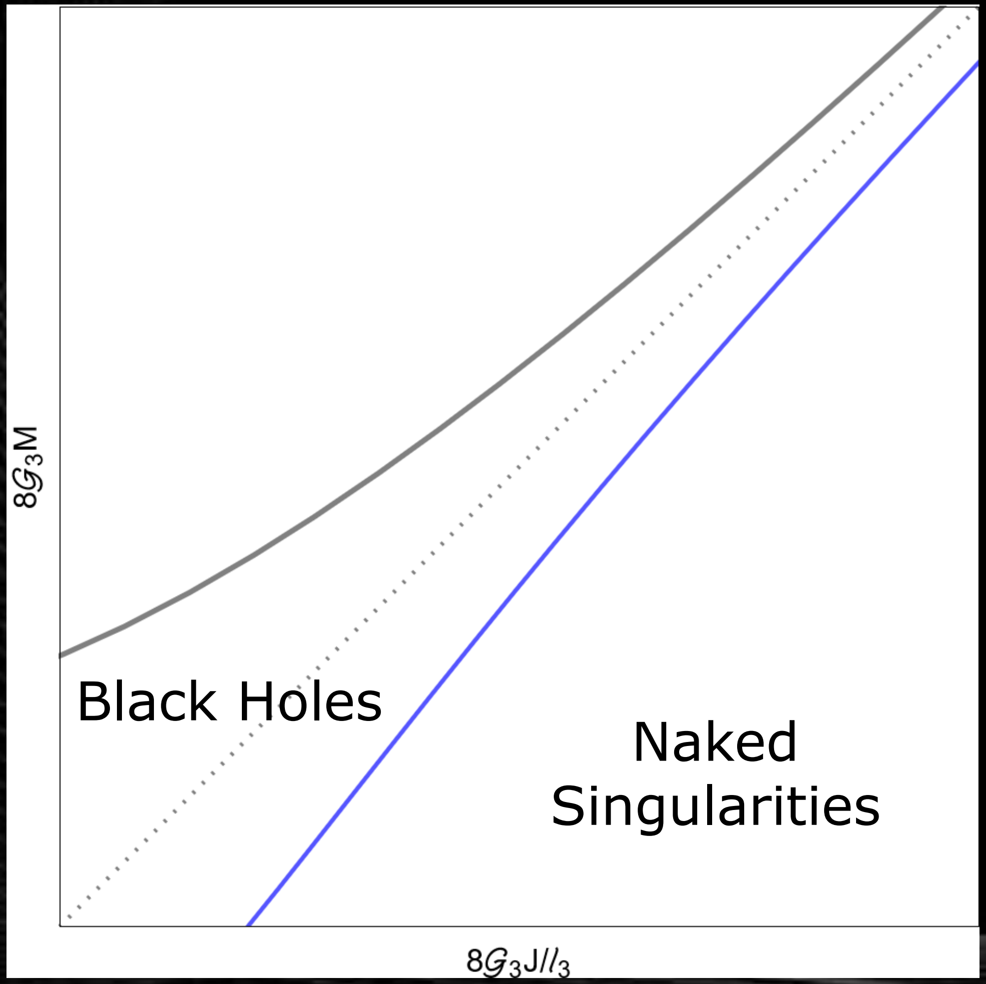
Linear response after absorption of a test particle

Strategy

- Choose an initial extremal configuration: fix ℓ_3, κ, x_1 and $\tilde{a}$

Perturbative analysis

Linear response after absorption of a test particle



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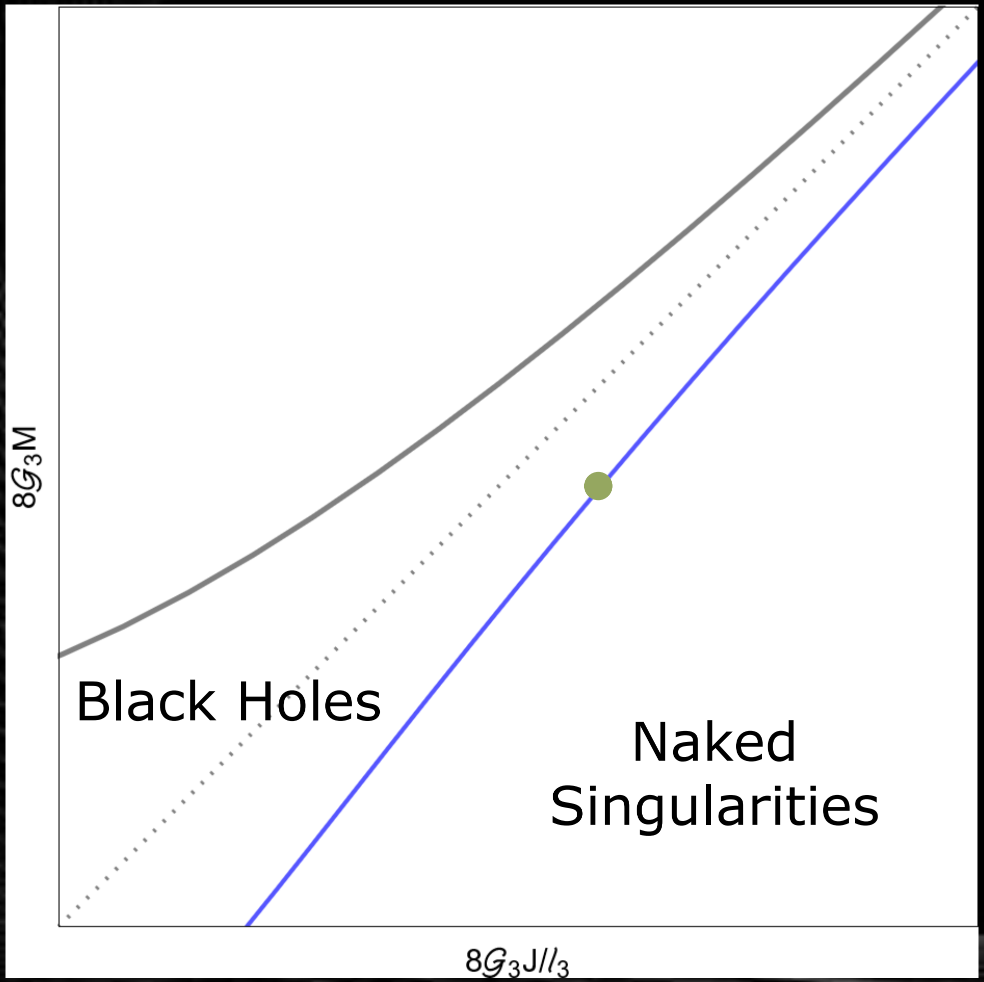
- Choose an initial extremal configuration: fix ℓ_3, κ, x_1 and $\tilde{a}$

$$\nu = \nu_{ext}(\ell_3, x_1, \tilde{a})$$

Perturbative analysis

Linear response after absorption of a test particle



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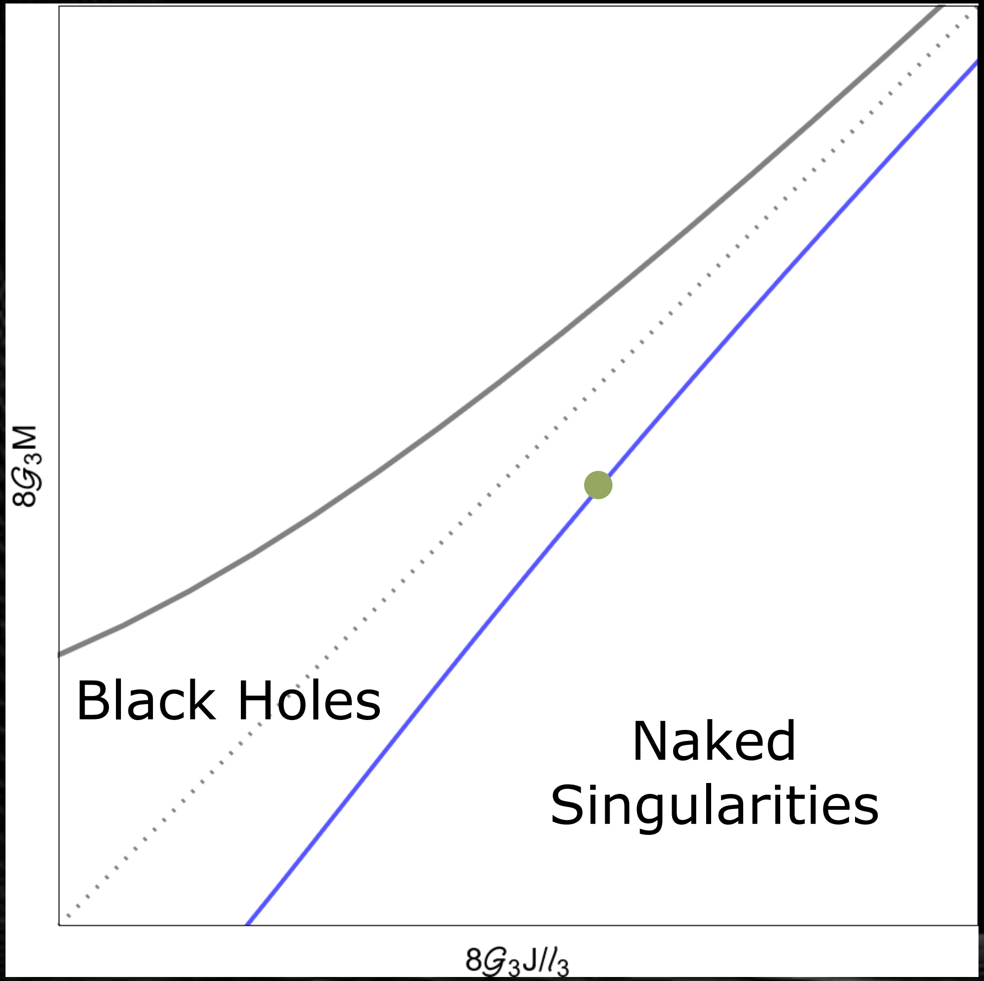

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$$M \to M + \delta M$$
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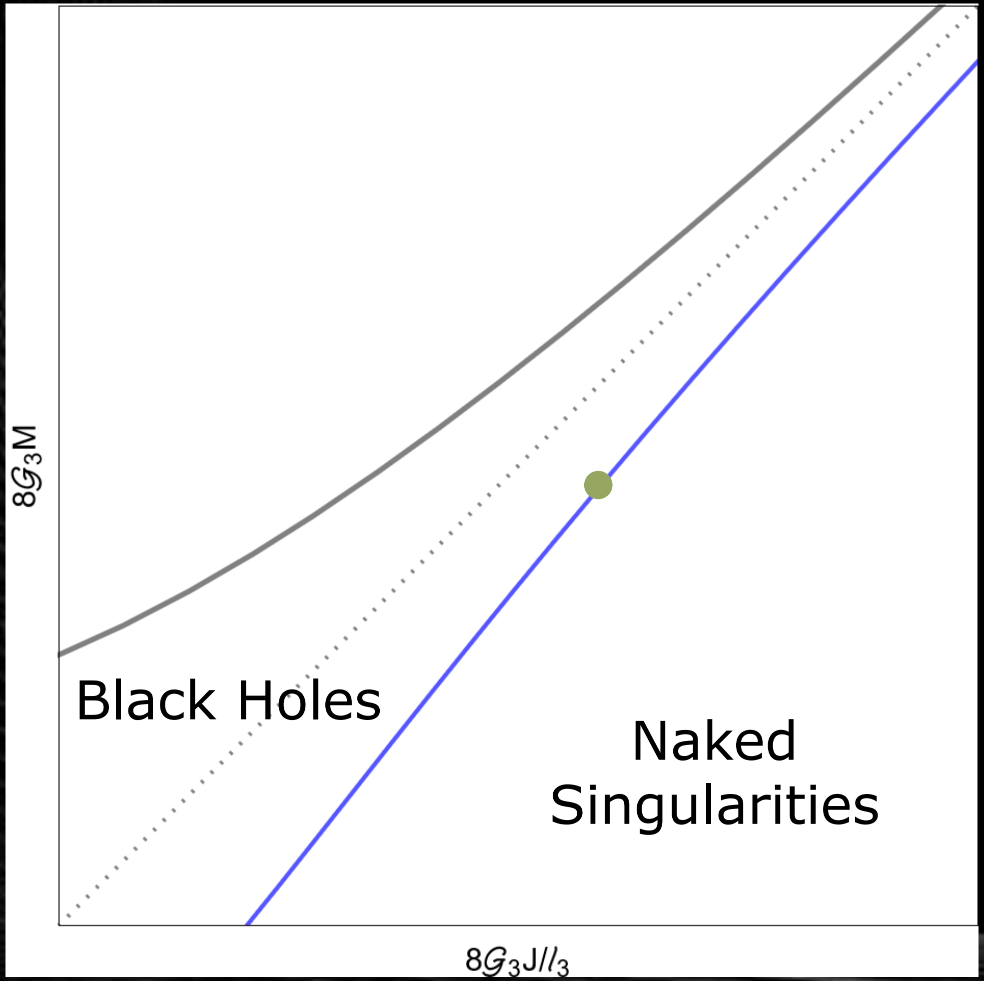
Perturbative analysis

Linear response after absorption of a test particle



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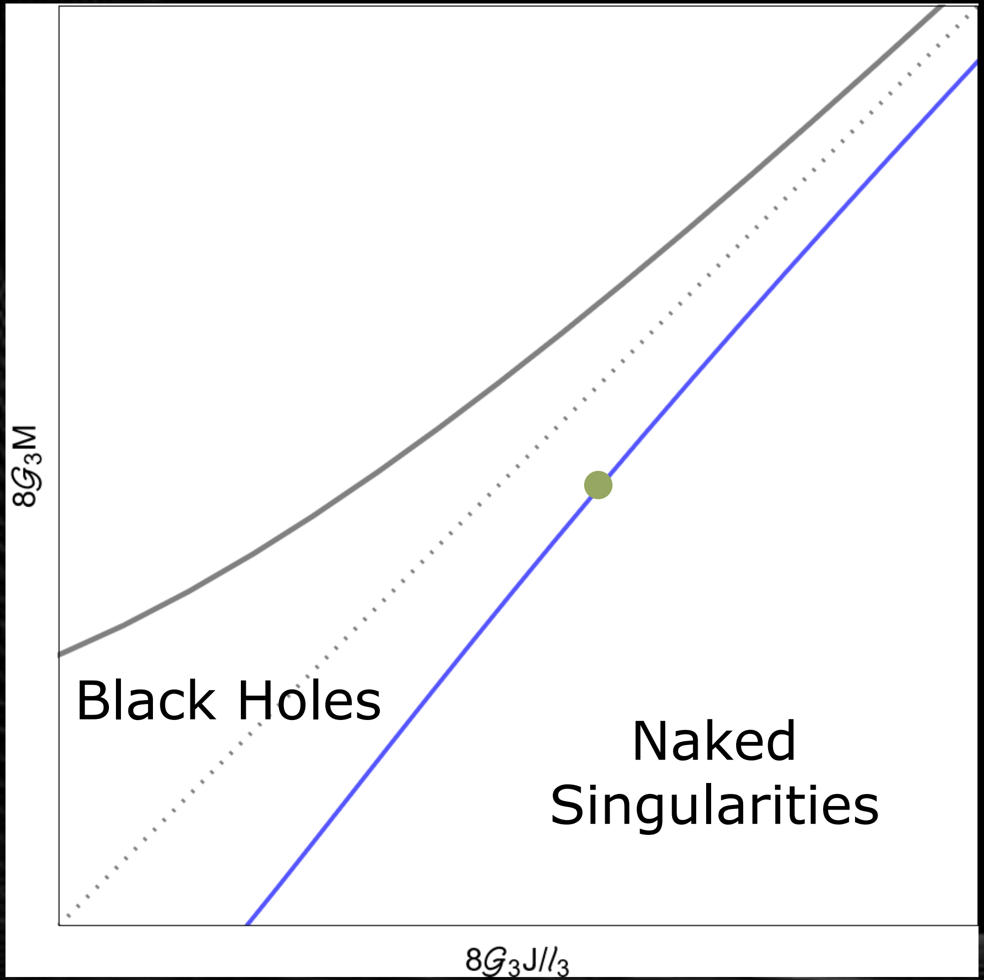
- $(\delta M, \delta J) \to (\delta \tilde{a}, \delta x_1)$

Perturbative analysis

Linear response after absorption of a test particle

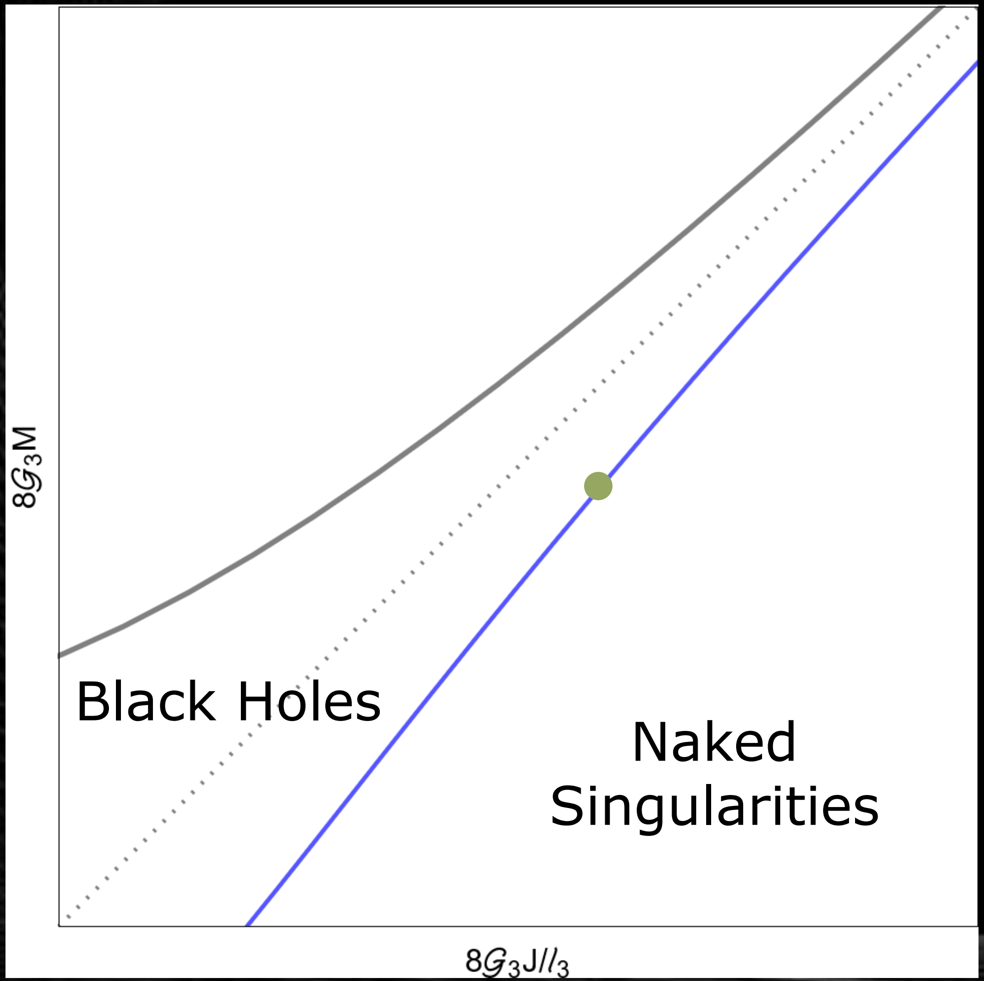


Strategy

$$g^{rr}(r_{\min} + \delta r, \tilde{a} + \delta\tilde{a}, x_1 + \delta x_1) = g^{rr}(r_{\min}, \tilde{a}, x_1) + \delta g^{rr}$$

Perturbative analysis

Linear response after absorption of a test particle



Strategy

$$g^{rr}(r_{\min} + \delta r, \tilde{a} + \delta\tilde{a}, x_1 + \delta x_1) = g^{rr}(r_{\min}, \tilde{a}, x_1) + \delta g^{rr}$$

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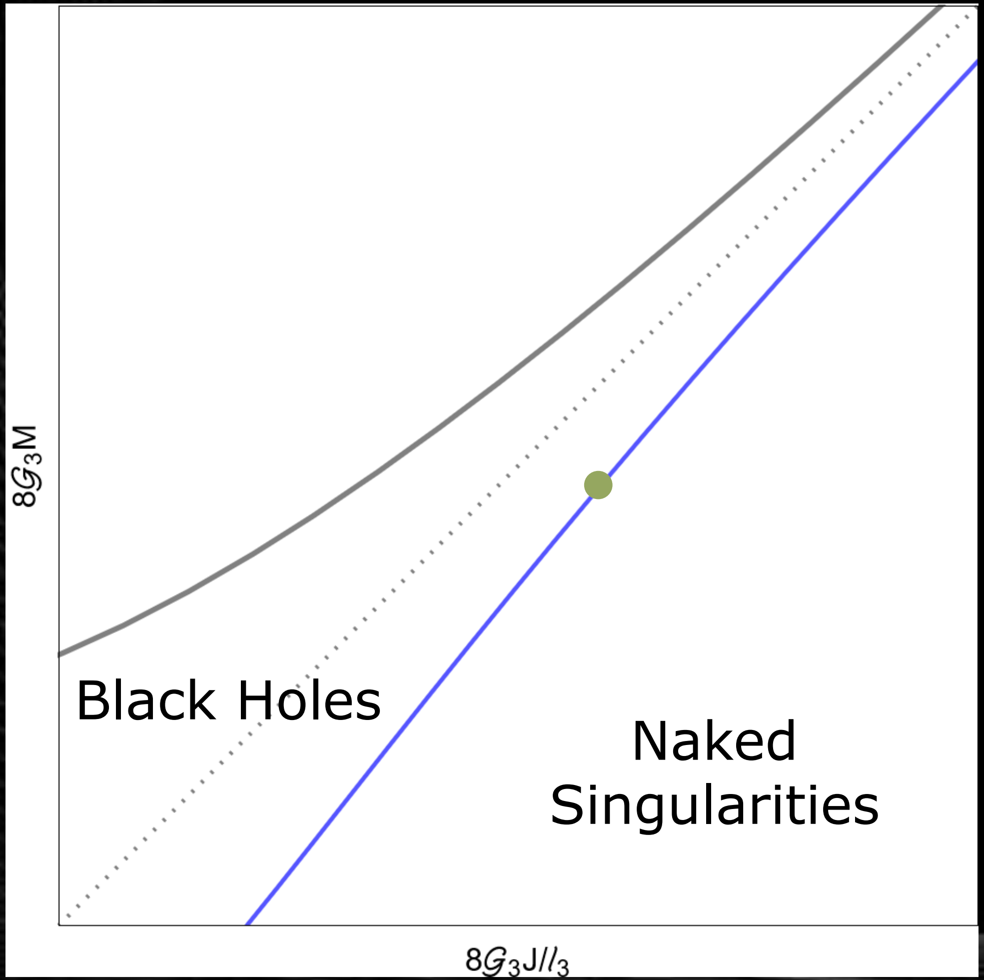

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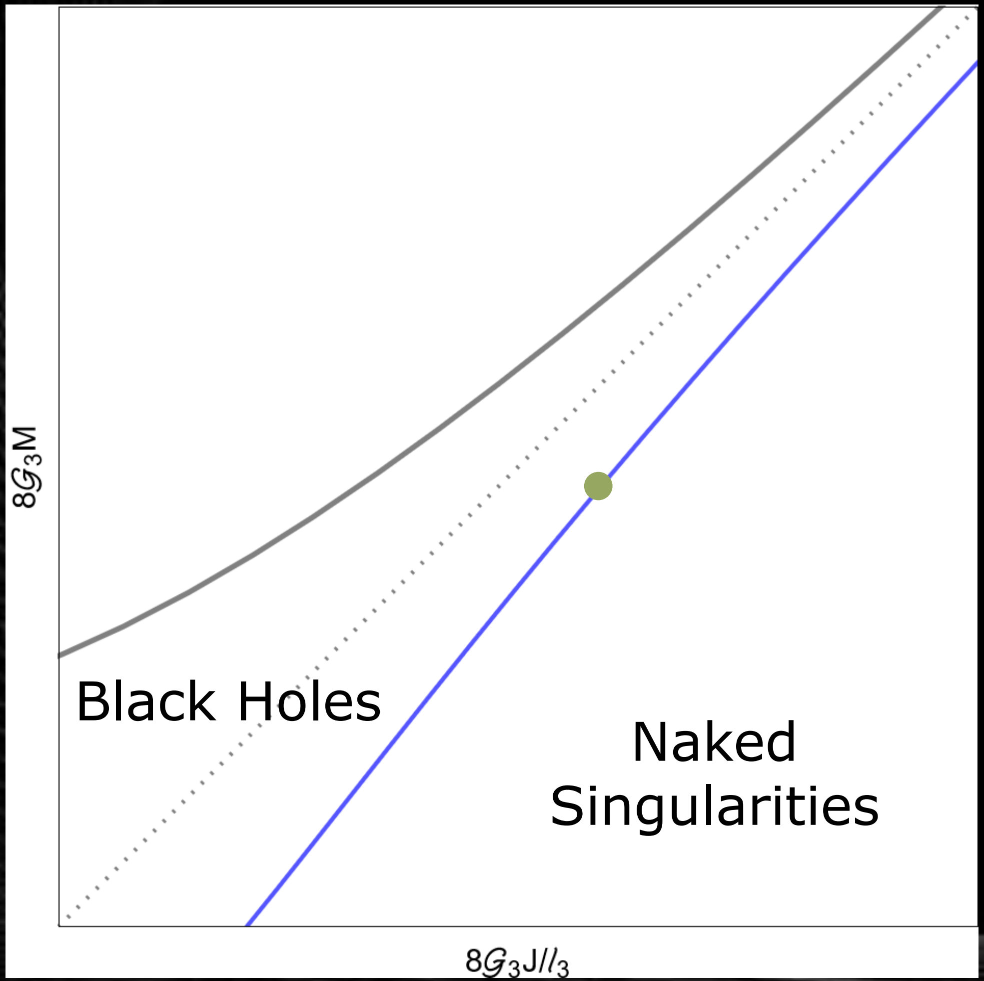

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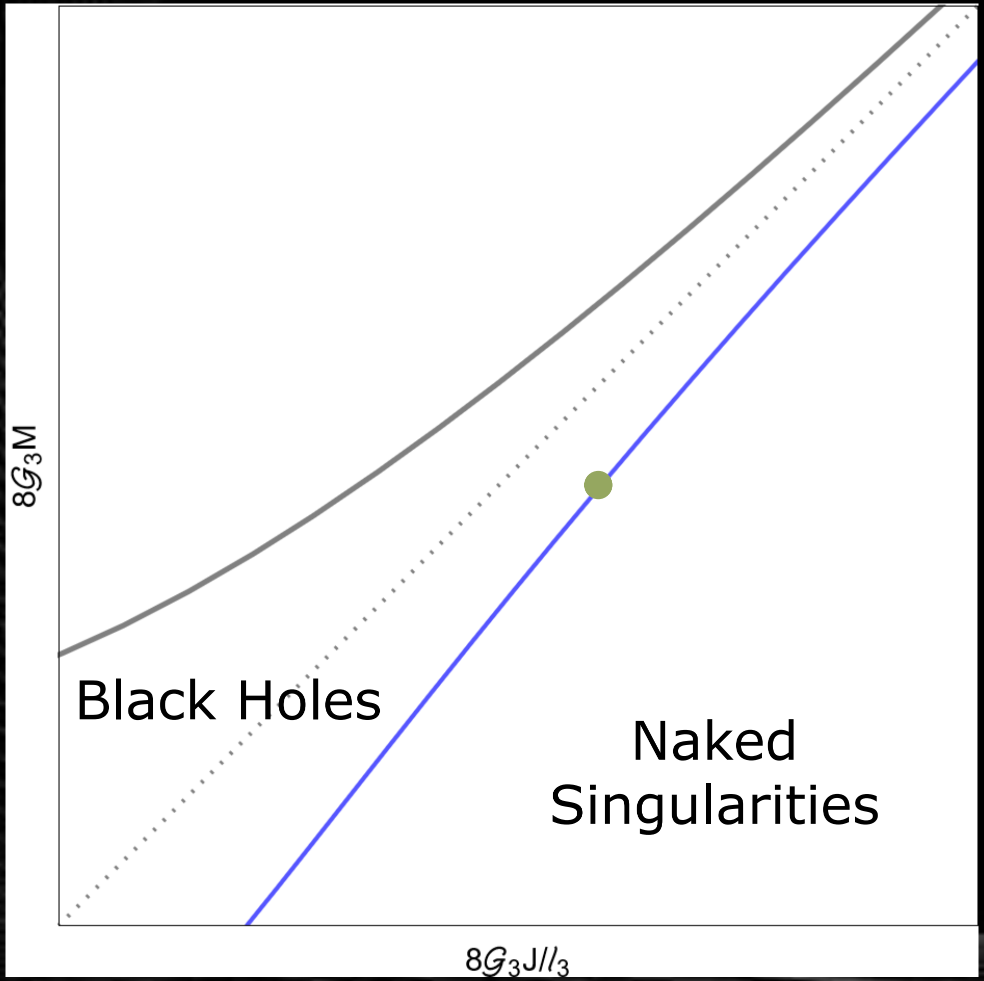

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$$\delta g^{rr} > 0$$

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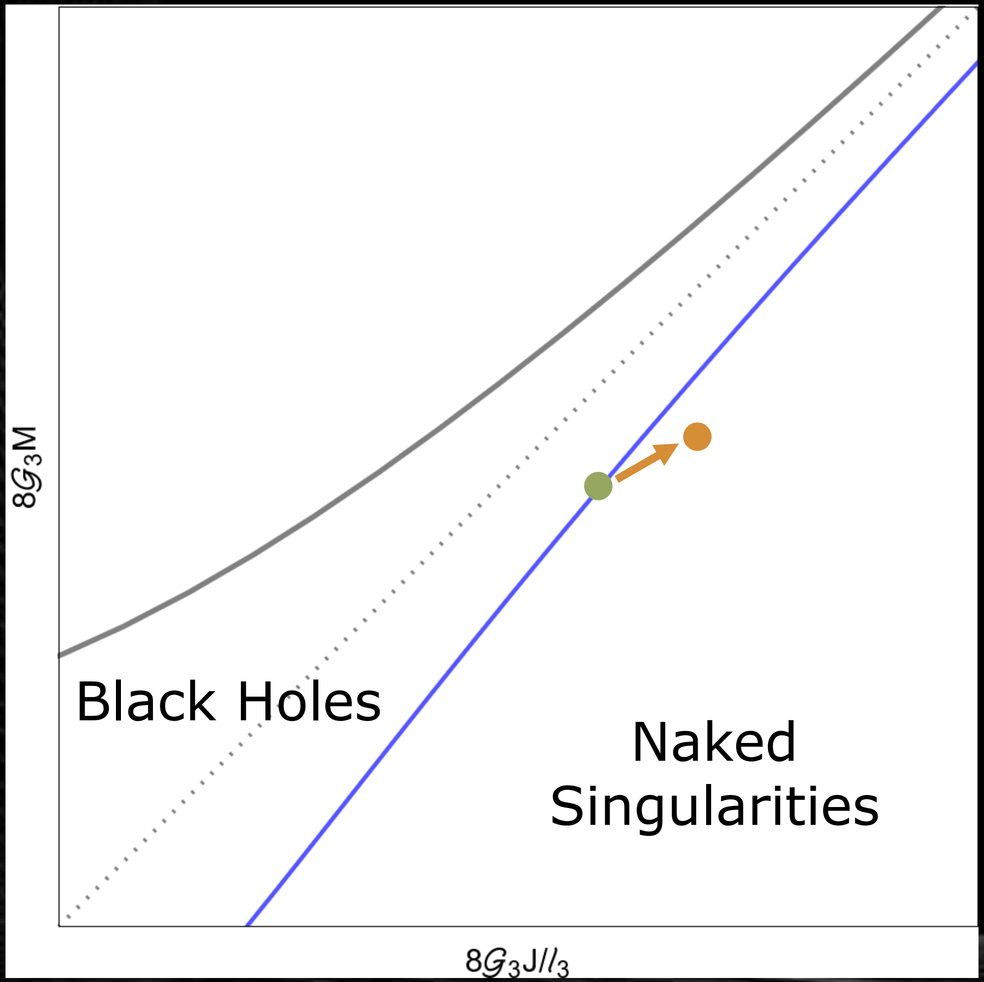

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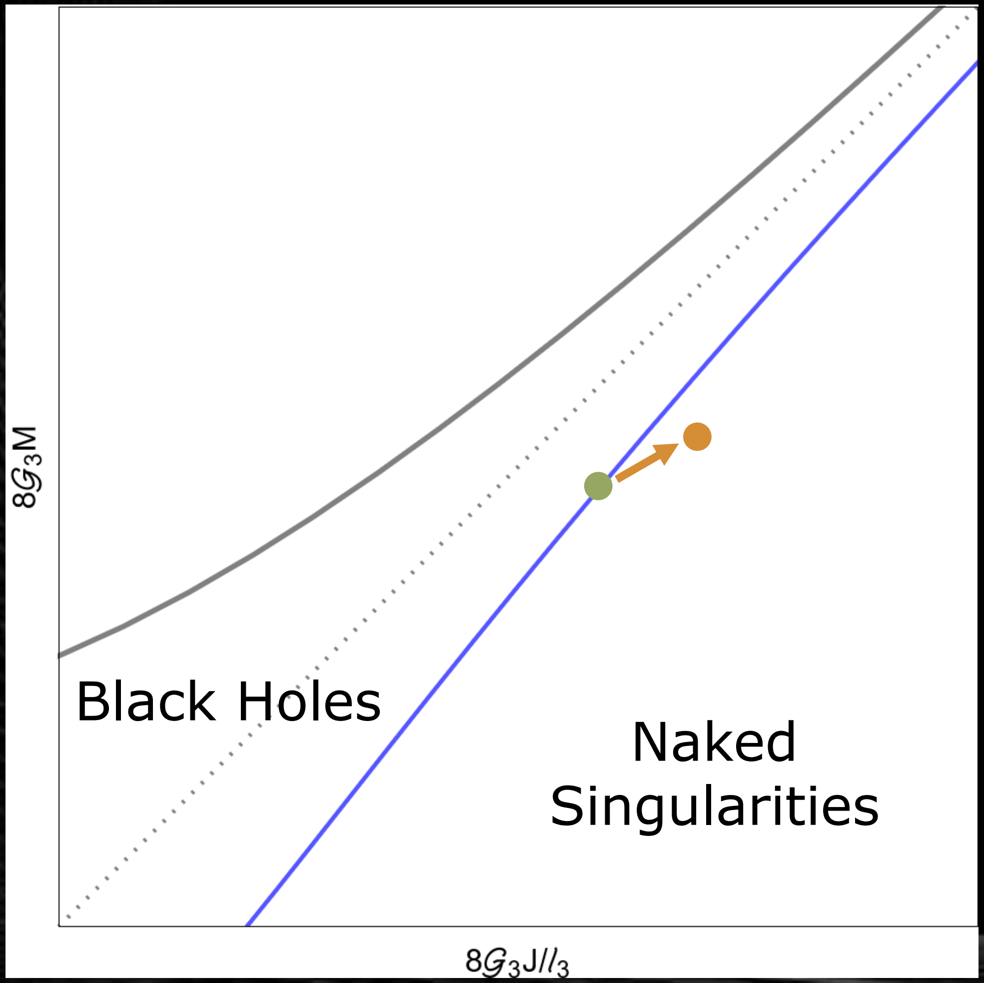
Naked singularity forms

Perturbative analysis

Linear response after absorption of a test particle



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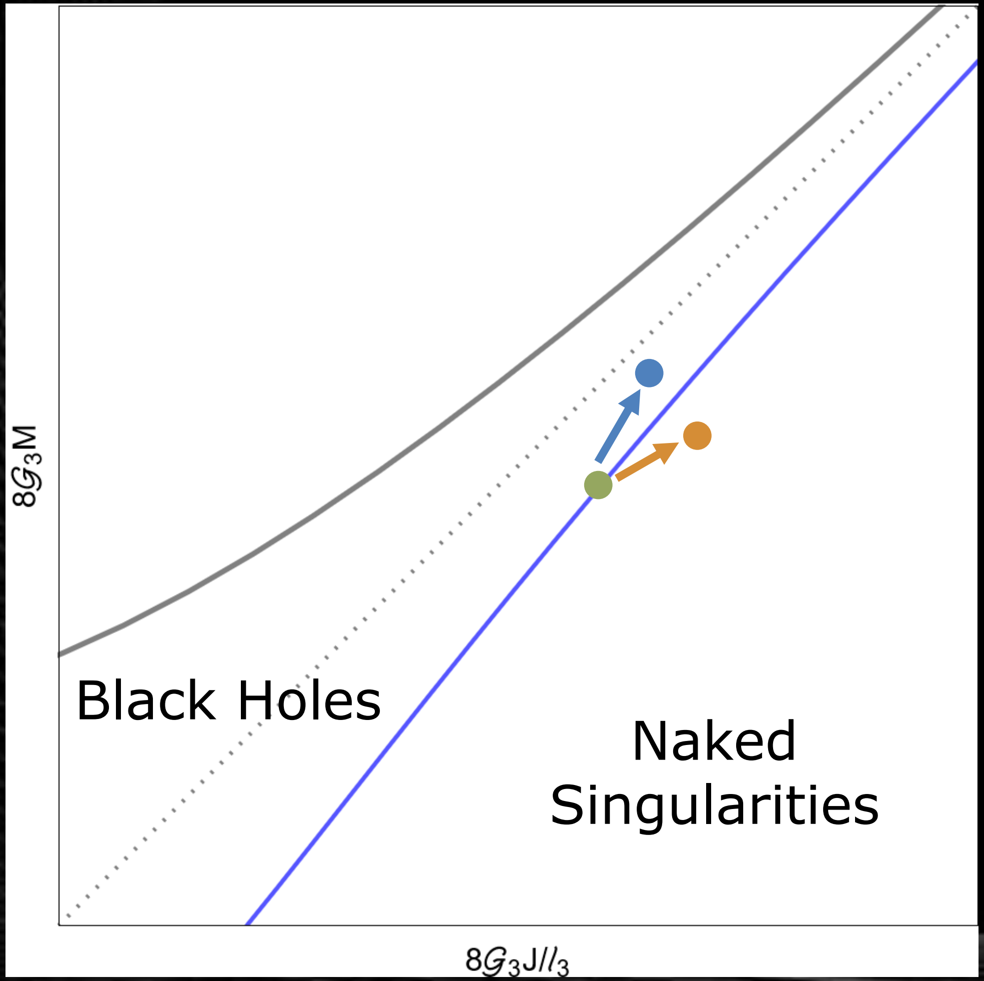
$\delta g^{rr} > 0$ Naked singularity forms

$\delta g^{rr} < 0$

Perturbative analysis

Linear response after absorption of a test particle



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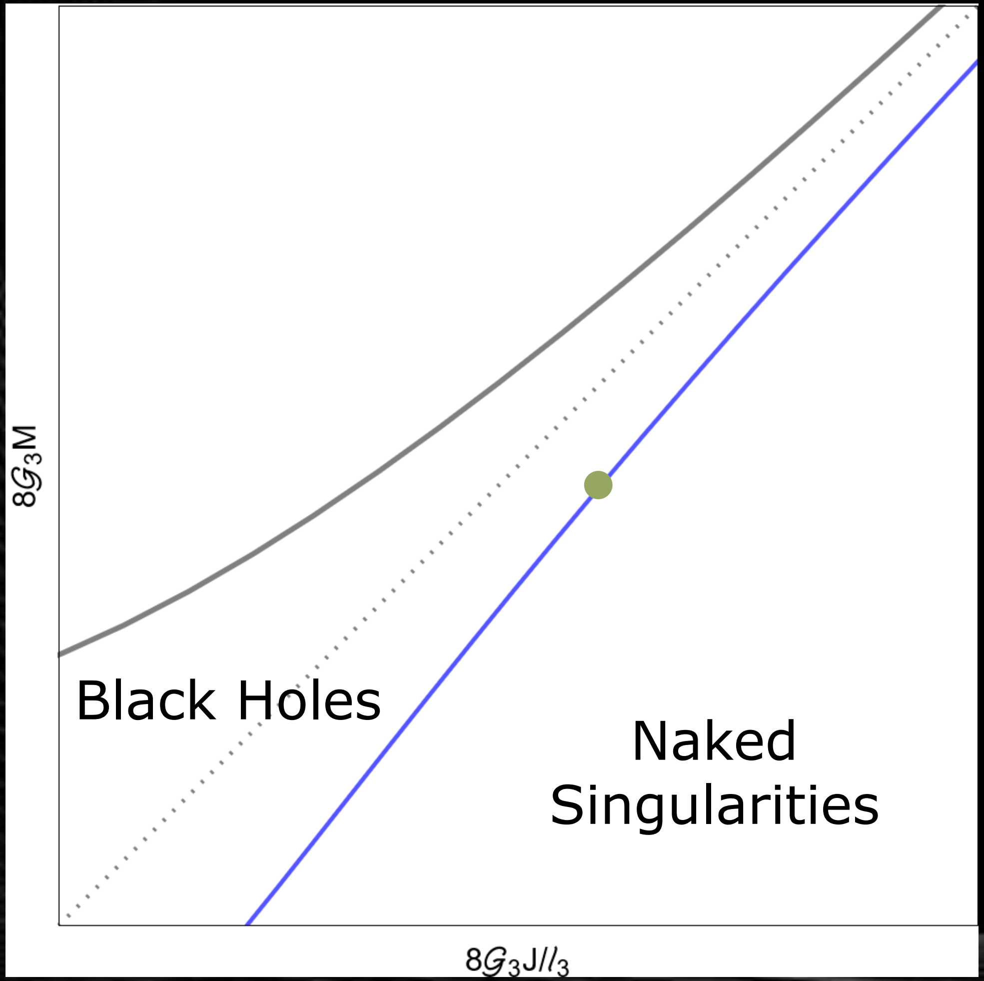
$\delta g^{rr} > 0$ Naked singularity forms

$\delta g^{rr} < 0$ Horizon forms

Perturbative analysis

Linear response after absorption of a test particle



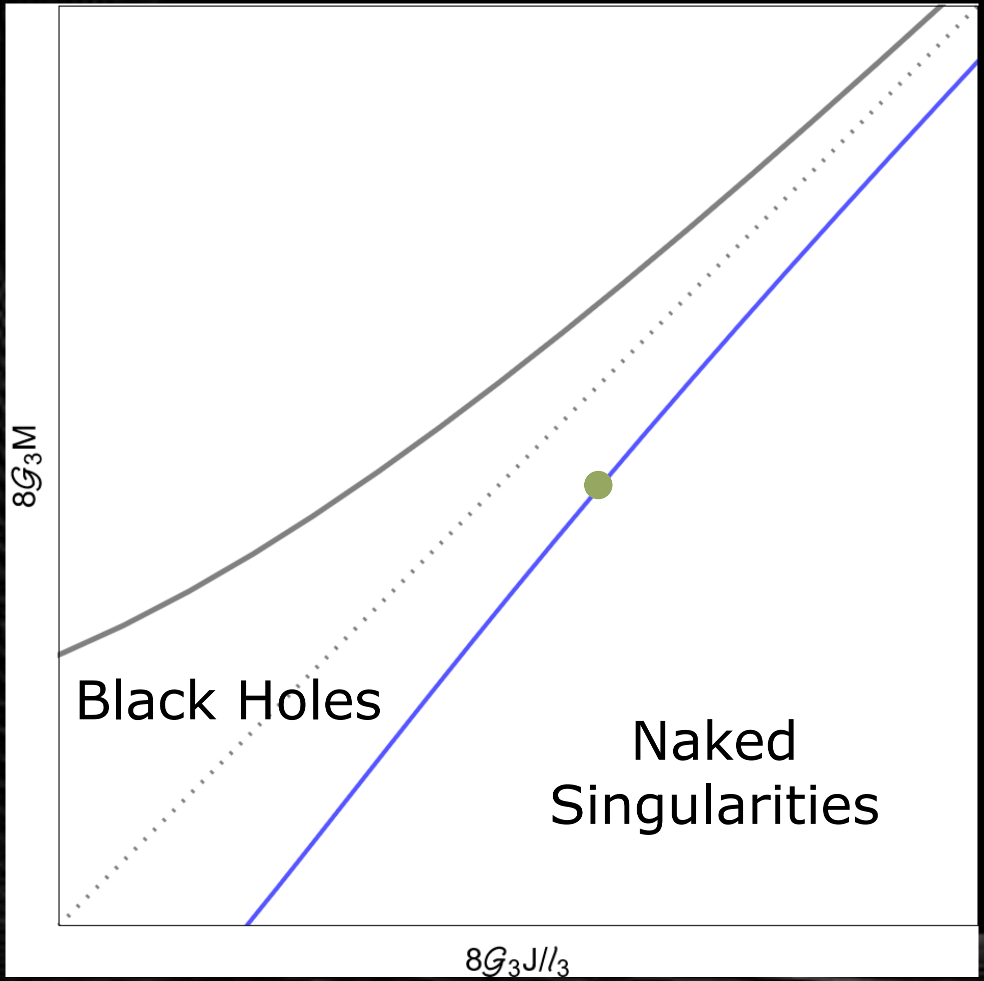
Strategy

Only particles with sufficiently low angular momentum are captured

$$\frac{\delta J}{\delta M} \leq L_{\text{max}}$$

Perturbative analysis

Linear response after absorption of a test particle



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Perturbative analysis

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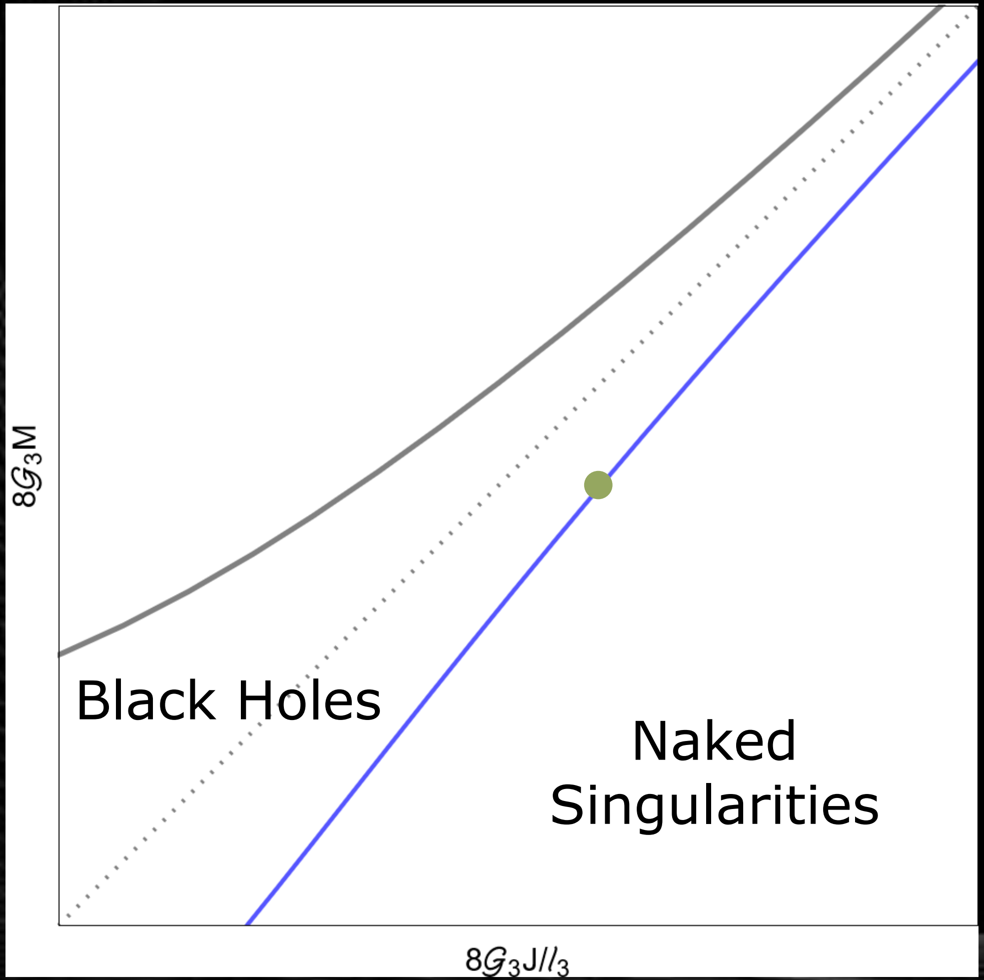

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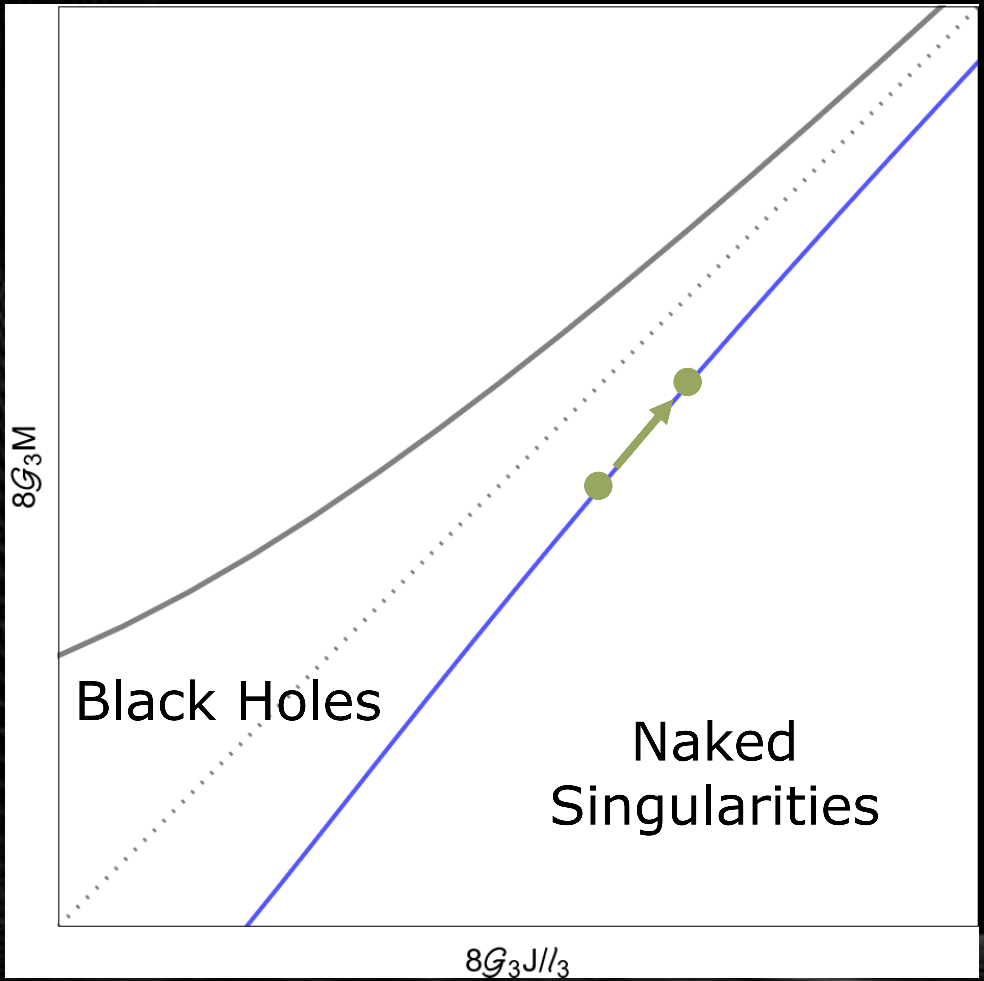

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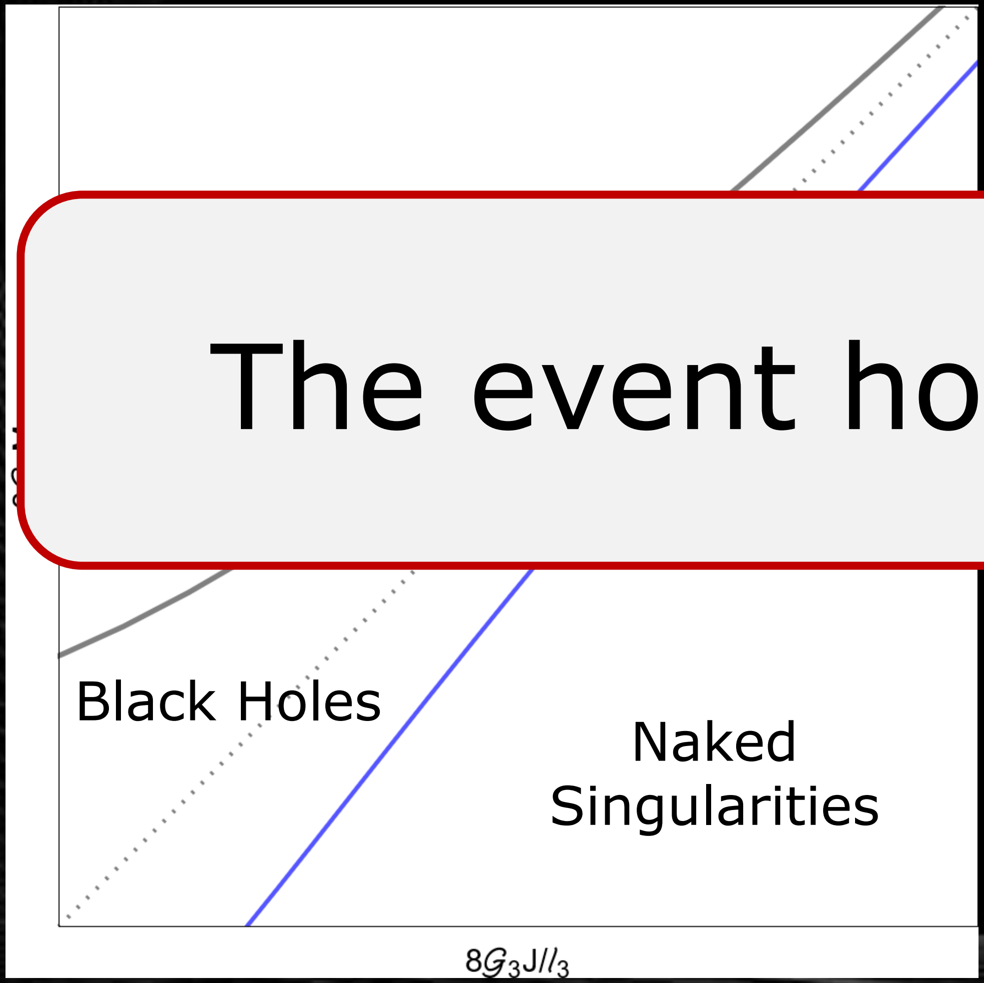

Strategy

The event horizon is not destroyed!

The worst case $\frac{\delta J}{\delta M} = L_{\max}$ → $\delta g^{rr} = 0$

Numerical assessment

Test particles with finite mass ($\delta M \not\ll M$)

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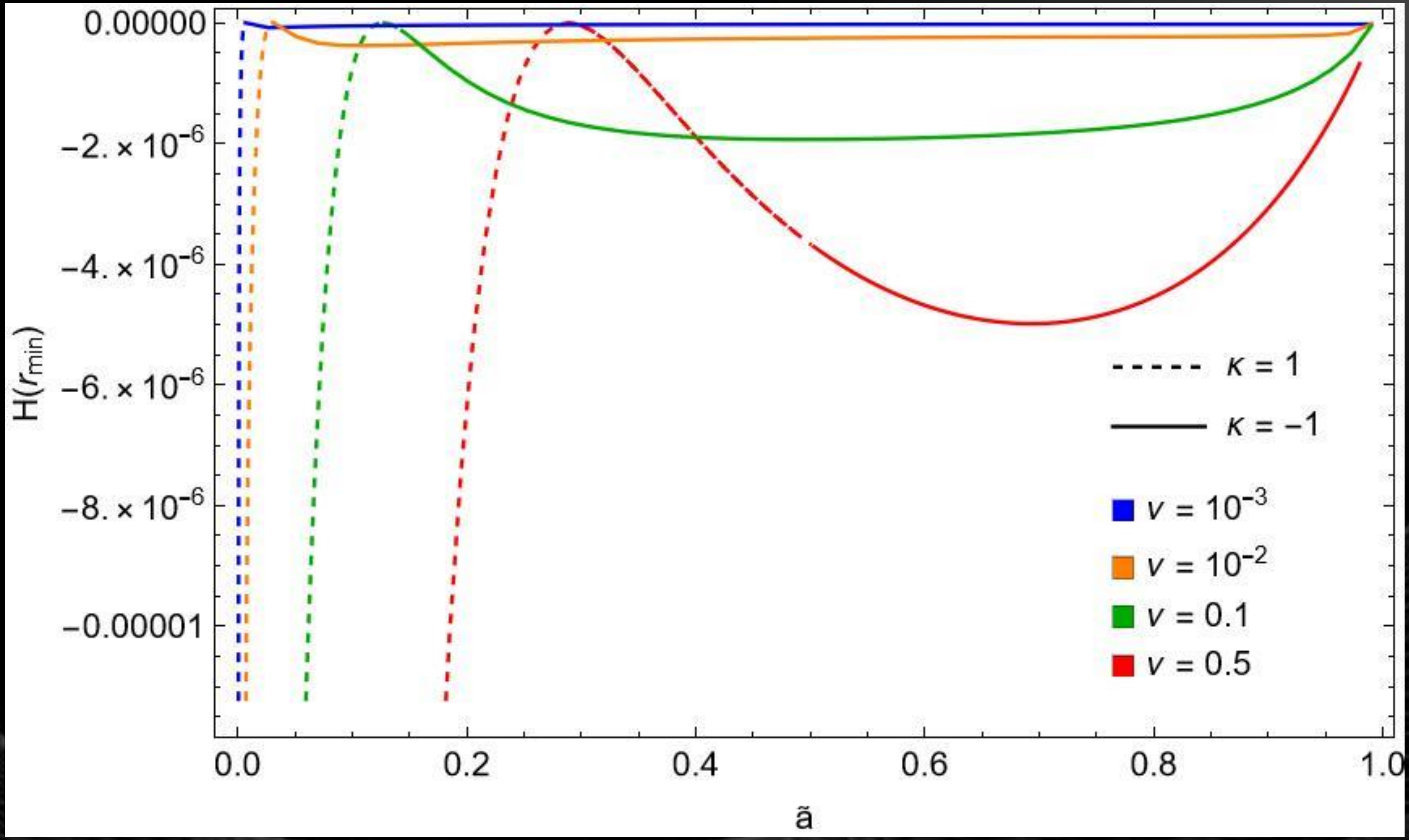
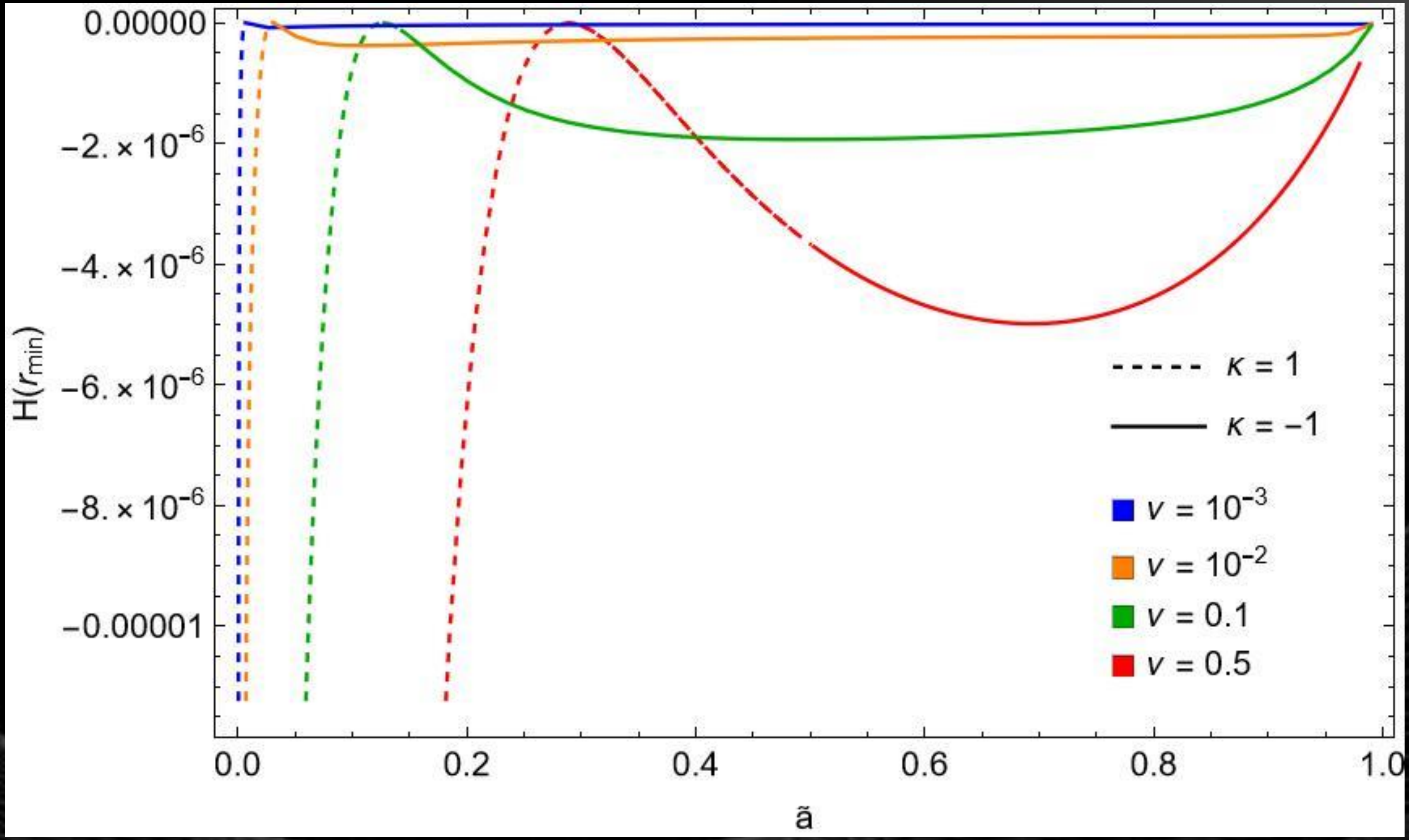


Numerical assessment

Test particles with finite mass ($\delta M \not\ll M$)



wCCC respected

Numerical assessment

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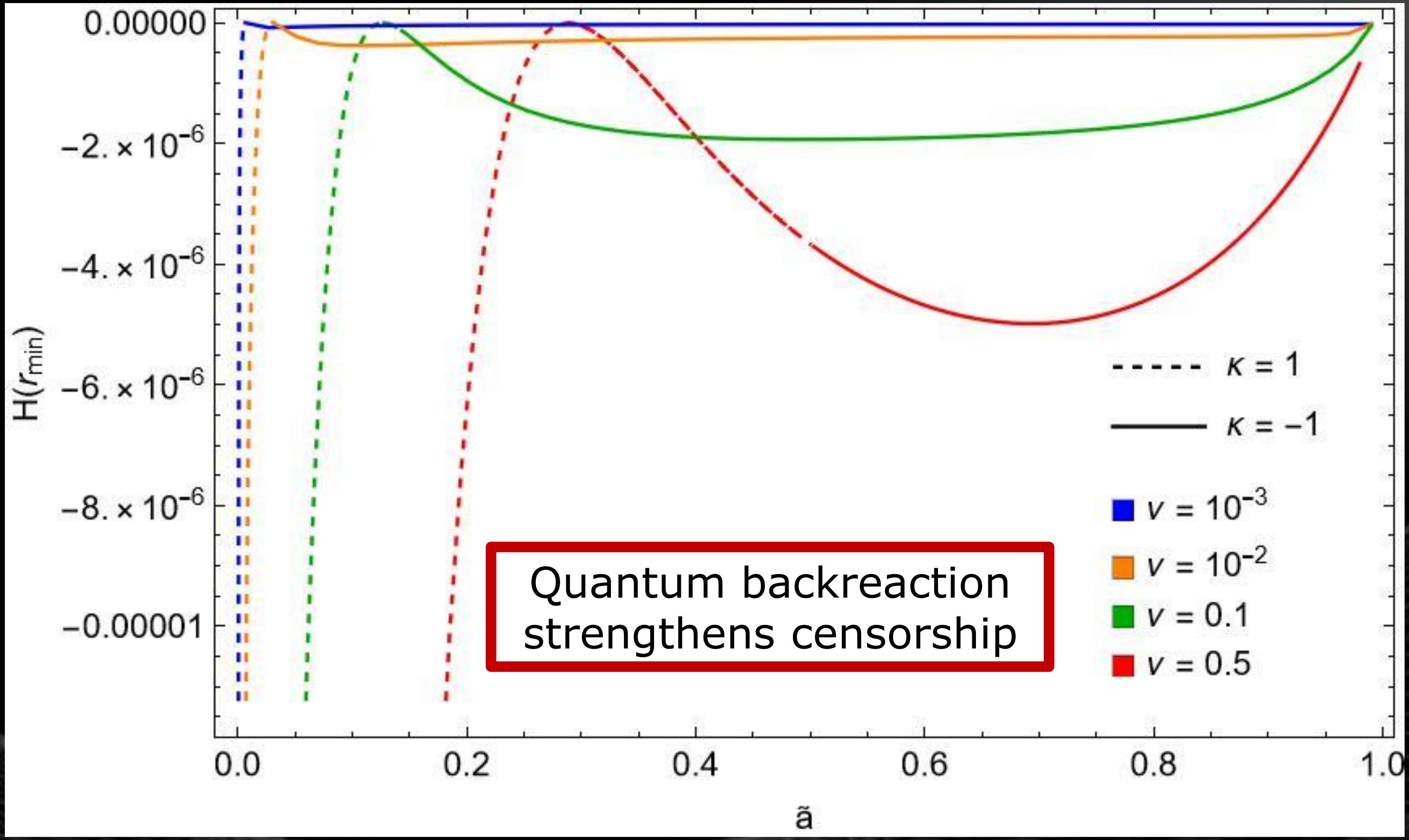

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THANKS FOR THE ATTENTION!