Phase Space for the Cauchy Horizon (In)Stability of Regular Black Holes

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Content of the talk

1)Motivations

2)Cauchy horizon instability in a nutshell

3)Ori model and mass-inflation effect for the Reissner-Nordstrom solution

4.A)Phase space for regular black hole spacetimes: ← "Phase space for the Cauchy horizon (in)stability of regular black holes", A. Bonanno, A. P. , F. Saueressig, in preparation → 4.C)

- Bardeen solution
- Hayward solution

4.B)Summary for model of asymptotically safe gravitational collapse ← "Dust collapse in asymptotic safety: a path to regular black holes", A. Bonanno, D. Malafarina, A. P. , PRL 132 (2024) 3, 031401

4.C)Phase space for the regular BH obtained in 4.B)

5)Conclusions and outlooks

Motivations

- Big picture: the ones we see in nature are black holes or regular black holes?

1)First global study of the perturbed spacetime at the Cauchy horizon (CH); previous analyses in the literature focus only on a portion of the phase space related to such peturbed system.

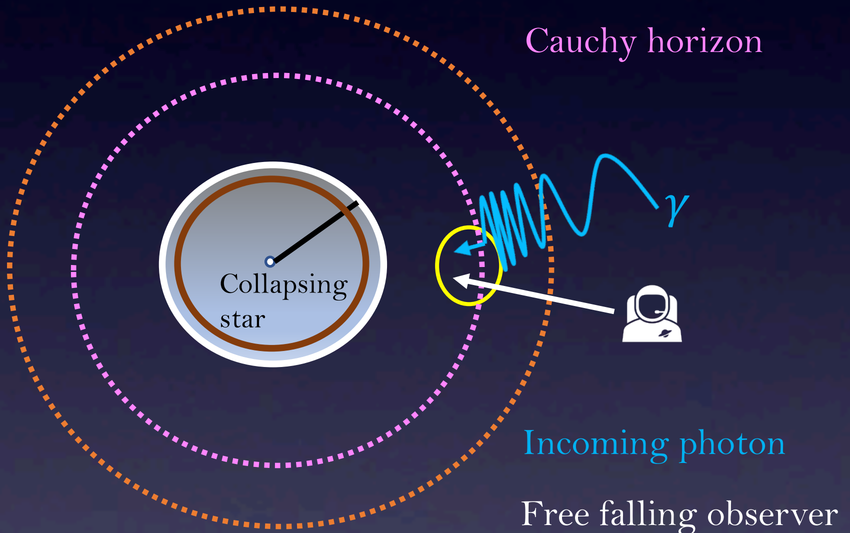
2)In general, why studying the Cauchy horizon instability? Can we cure it?

- a)It seems that regular black holes imply the presence of the Cauchy horizon
- b)It is a crucial theoretical open problem and an open problem of internal "consistency"
- c)It is related to the destinity of the cosmic censorship conjecture
- d)It is related to geodesic completeness in a (regular) black hole spacetime
- e)It can tell us something about the astrophysical viability of (regular) black holes

Interrelated problems

Cauchy horizon instability in a nutshell

- The Cauchy horizon is a surface of infinite blueshift



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

with $f(r) \equiv 1 - 2M(r)/r = 0$ having two different roots, $r_{\rm EH}$ and $r_{\rm CH}$.

p_μ for γ, $\quad p_0 = E$, $\quad p^0 = f(r)^{-1}E$

u_μ for (observer), $\quad u_0 = \tilde{E}$, $\quad u^0 = f(r)^{-1}\tilde{E}$

$$E_{obs.} = p_\mu u^\mu$$

$$E_{obs.} = p_0 u^0 + p_1 u^1 = E f(r)^{-1}\tilde{E} + g_{11} p^1 u^1 = E f(r)^{-1}\tilde{E} + f(r)^{-1} p^r u^r$$

$$E_{obs.} = f(r)^{-1}(E\tilde{E} - p^r u^r)$$

$$g_{\mu\nu}p^\mu p^\nu = 0 \qquad g_{00}(p^0)^2 + g_{11}(p^1)^2 = f(r)E^2 f(r)^{-2} - f(r)^{-1}(p^r)^2 = 0$$

$\rightarrow$ $p^r = E$

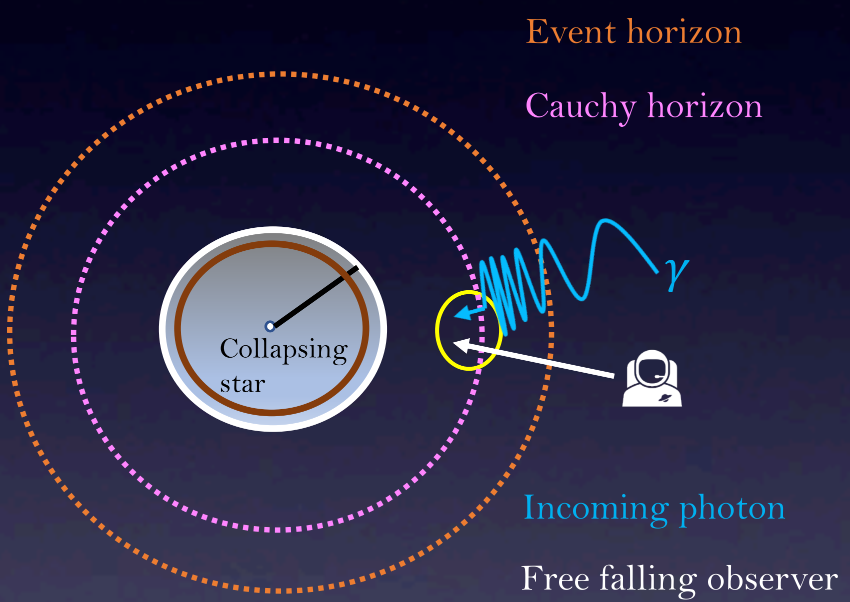
$$g_{\mu\nu}u^\mu u^\nu = 1 \qquad g_{00}(u^0)^2 + g_{11}(u^1)^2 = f(r)f(r)^{-2}\tilde{E}^{-2} - f(r)^{-1}(u^r)^2 = 1$$

$\rightarrow$ $u^r = \sqrt{\tilde{E}^2 - f(r)}$

$\rightarrow$ $E_{obs.}(r) = f(r)^{-1}\left[E\tilde{E} - E\sqrt{\tilde{E}^2 - f(r)}\right]$

Cauchy horizon instability in a nutshell

- The Cauchy horizon is a surface of infinite blueshift



$$E_{obs.}(r) = f(r)^{-1}\left[E\tilde{E} - E\sqrt{\tilde{E}^2 - f(r)}\right]$$

For $r \to r_{\mathrm{CH}}$ we have $f(r) \to 0^-$.

Since $dt/ds = u^0 = f(r)^{-1}\tilde{E}$, and $f(r) < 0$ for $r_{\mathrm{CH}} < r < r_{\mathrm{EH}}$, and since $dt/ds > 0$, then $\tilde{E} < 0$.

$\Downarrow$

Then, at the meeting point:

$$\lim_{r \to r_{\mathrm{CH}}} E_{\mathrm{obs.}}(r) \sim \lim_{f(r) \to 0^-} \frac{2E\tilde{E}}{f(r)} = +\infty$$

$\Downarrow$

A free falling radial observer approaching the Cauchy horizon will measure an infinite blueshift for a radially incoming photon. →

The system composed by (regular) black hole + incoming photon faces an ultraviolet catastrophe.

The Ori model

VOLUME 67, NUMBER 7 PHYSICAL REVIEW LETTERS 12 AUGUST 1991

Inner Structure of a Charged Black Hole: An Exact Mass-Inflation Solution

Amos Ori
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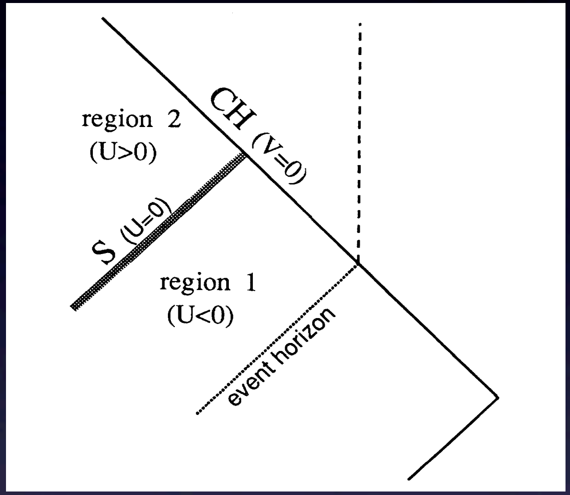
(Received 4 March 1991)

Recently, Poisson and Israel have shown how when an electrically charged black hole is perturbed its inner horizon becomes a singularity of infinite spacetime curvature—the *mass-inflation singularity*. In this paper we construct an exact mass-inflation solution of the Einstein-Maxwell equations, and use it to analyze the mass-inflation singularity. We find that this singularity is weak enough that its tidal gravitational forces do not necessarily destroy physical objects which attempt to cross it. The possible continuation of the spacetime through this weak singularity is discussed.

PACS numbers: 04.20.Jb, 97.60.Lf



I)It is an exact spherically symmetric mass-inflation solution of the Einstein(-Maxwell) equations.

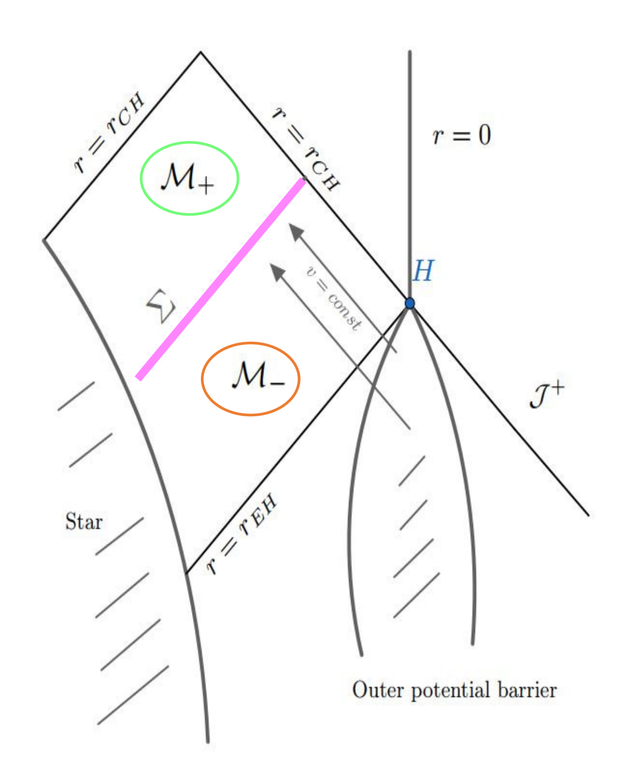
II)More "realistic" (i.e. physical) setting, if compared to the one where the perturbation is represented by a single incoming photon.

III)1° incoming flux: incoming perturbation.

IV)2° outgoing flux: portion of the originally incoming perturbation backscattered by the black hole's curvature near the Cauchy horizon (main novelty with respect to previous analyses in the literature at that time).

The Ori model

"Regular black holes with stable core",
A. Bonanno, F, Saueressig, A. Khosravi,
Phys. Rev. D 103, 124027 (2021)



V)Total flux: balance given by the two fluxes, one ingoing and one outgoing, both of positive energy, that cross each other.

VI)Total flux: *modelled as a pressureless speherical shell* Σ *composed by massless particles.*

VII)The solutions is constructed matching two patches of spacetime $\mathcal{M}_+$ (future sector) and $\mathcal{M}_-$ (past sector) through the null–like shell Σ.

VIII)*Advaced Eddington-Finkelstein coordinates* $\{v, r, \theta, \varphi\}$, with $v = t + r^*$:

$$ds^2 = -f_\pm(r, v_\pm)dv_\pm^2 + 2drdv_\pm + r^2 d\Omega^2 \text{ where } f_\pm = 1 - 2M_\pm(r, v_\pm)/r$$

IX)Equation of motion for the shell Σ:

$$f_- dv_- = 2dr$$

X)Enstein's equations (Units: $c = 1,\ G_N = 1$):

$$T_{rr} = 0, \qquad \frac{\partial M_\pm(r,v_\pm)}{\partial r} = -4\pi r^2 T_v^v, \qquad \frac{\partial M_\pm(r,v_\pm)}{\partial v} = -4\pi r^2 T_v^r$$

XI)Continuity equation across Σ:

$$[T_{\mu\nu} s^\mu s^\nu] = 0 \longrightarrow \left[\frac{1}{f_+^2}\frac{\partial M_+(r,v_+)}{\partial v_+}\right]\Bigg|_\Sigma = \left[\frac{1}{f_-^2}\frac{\partial M_-(r,v_-)}{\partial v_-}\right]\Bigg|_\Sigma$$

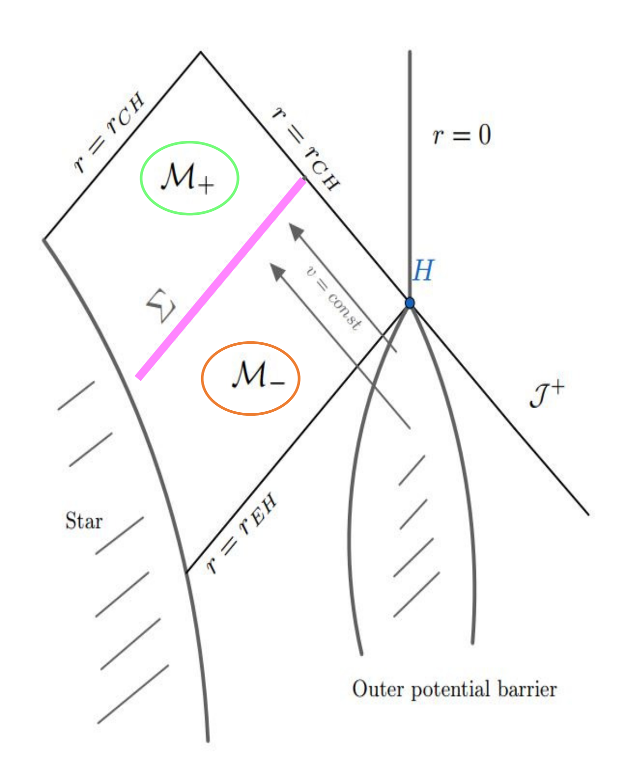
XII)Relation between v_+ and v_-:

$$f_+ dv_+ = f_+ dv_+ \quad \text{along} \quad \Sigma$$

$$\longrightarrow v_+ = v_+(v_-)$$

The Ori model dynamical system

"Regular black holes with stable core", A. Bonanno, F, Saueressig, A. Khosravi, Phys. Rev. D 103, 124027 (2021)



XIII)Thanks to XII) we can express everything in terms of $v \equiv v_-$.

XIV)Notation:

$$F(v) \equiv \left(\frac{1}{f_-}\frac{\partial M_-}{\partial v}\right)\Big|_\Sigma, \quad R(v) \equiv shell\ position, \quad \dot{y} \equiv dy/dv$$

1° Dynamical equation: $\dot{R}(v) = \frac{1}{2} f_-\Big|_\Sigma$ for $R(v)$

2° Dynamical equation: $\left(\frac{1}{f_+}\frac{\partial M_+}{\partial v}\right)\Big|_\Sigma = F(v)$ for $m_+(v)$

$m_0 \mapsto m_\pm(v) \to M_\pm(r,v) \to f_\pm(r,v)$
with m_0 unperturbed mass of the (regular) BH

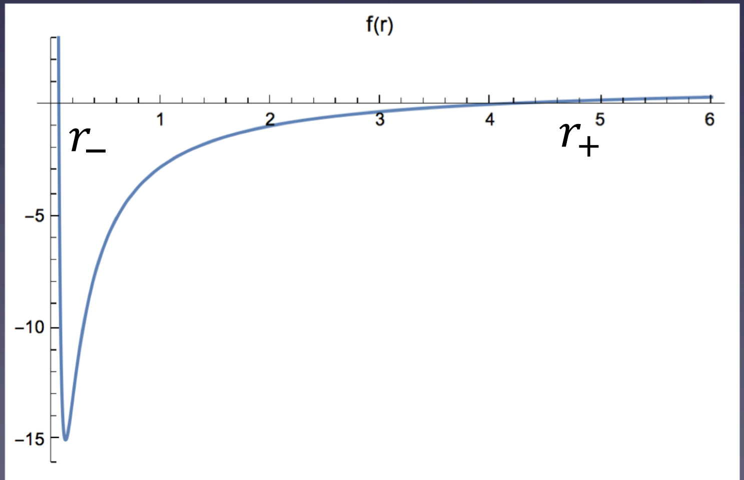
Boundary condition at the event horizon:
Price's law $m_-(v) = m_0 - \frac{\beta}{v^p}$ with $\beta > 0$, and $p \geq 11$

Reissner-Nordstrom solution: phase space for the Cauchy horizon (in)stability and mass-inflation

- Free parameters: m_0, β, p
- Degrees of freedom: $R(v),\ m_+(v)$
- Independent variable: v

- Assigning spacetime solution $M(r) \leftrightarrow f(r)$, and initial conditions $R(v_i),\ m_+(v_i)$ the system and its evoluton are fully determined

$$f(r) = \left(1 - \frac{2m_0}{r} + \frac{e^2}{r^2}\right) \qquad m_0 = 2, \qquad e = \frac{1}{2}, \qquad p = 11, \qquad \beta = 1 \longrightarrow \qquad r_- \cong 0.06, \qquad r_+ \cong 3.93$$



1) $$\mathrm{Rn}'[v] == \frac{1}{2}\left(1 - \frac{2\left(2 - v^{-p}\beta - \frac{\epsilon^2}{2\,\mathrm{Rn}[v]}\right)}{\mathrm{Rn}[v]}\right)$$

2) $$m_+'[v] == \frac{p\, v^{-1-p}\, \beta \left(1 - \frac{2\left(-\frac{\epsilon^2}{2\,\mathrm{Rn}[v]} + m_+[v]\right)}{\mathrm{Rn}[v]}\right)}{1 - \frac{2\left(2 - v^{-p}\beta - \frac{\epsilon^2}{2\,\mathrm{Rn}[v]}\right)}{\mathrm{Rn}[v]}}$$

Notation switch:

$R(v) \equiv Rn[v]$

$m_+(v) \equiv m_+[v]$

$dy/dv \equiv \dot{y} \equiv y'$

Reissner-Nordstrom solution: phase space for the Cauchy horizon (in)stability and mass-inflation

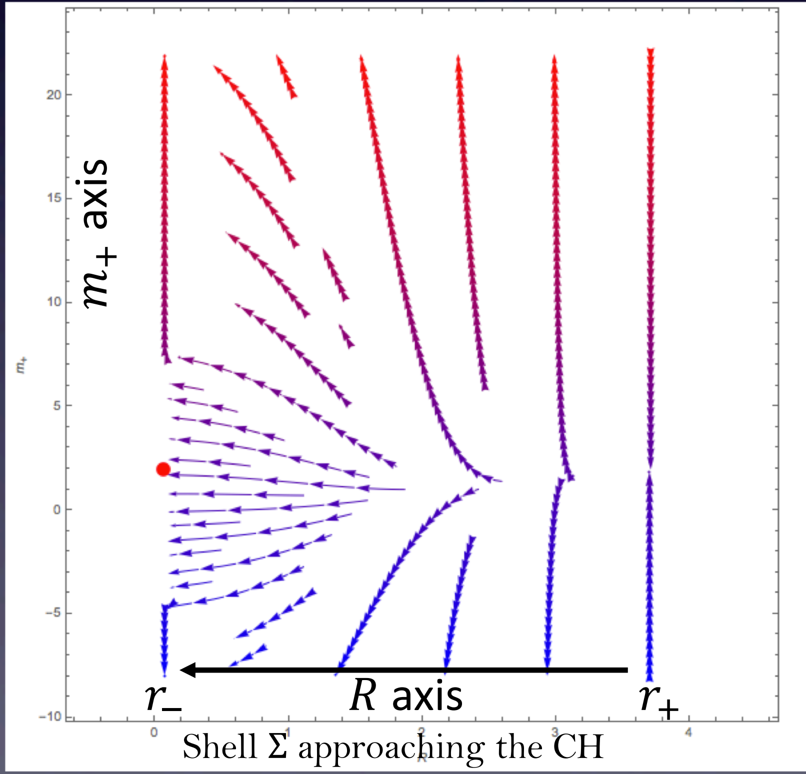
- Fixed point: $(R(v), m_+(v)) = \left(r_-, \frac{e^2+r_-^2}{2r_-}\right) \cong (0.06, 2)$

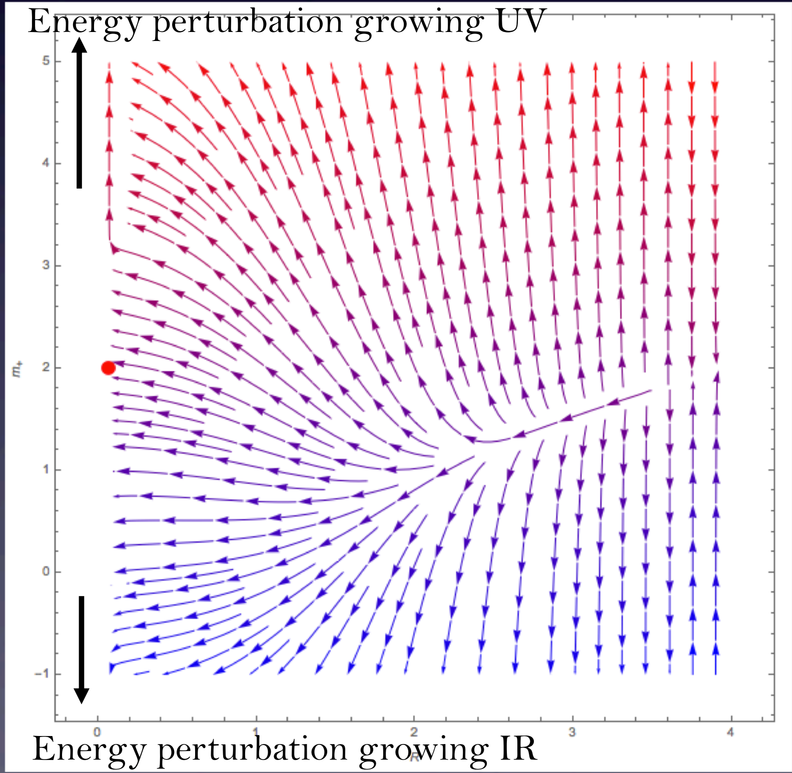
- It is $\frac{e^2+r_-^2}{2r_-} \equiv m_{+repulsor} > 0$

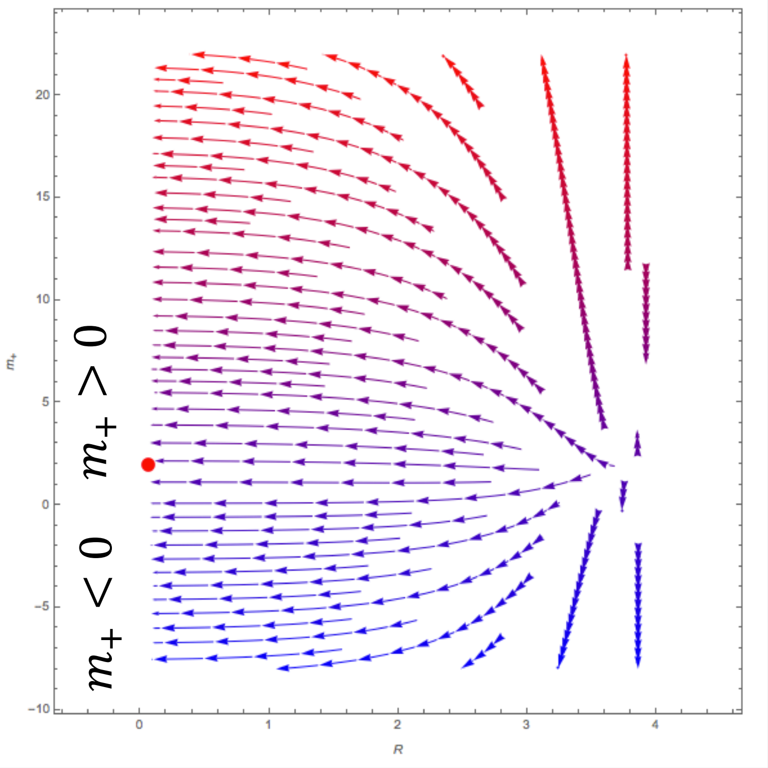
Photograh of the phase space vector field at $v = v^* = 1.2$

Zoom

Photograh at $v = v^* = 1.5$







Reissner-Nordstrom solution: phase space for the Cauchy horizon (in)stability and mass-inflation

- *Analytical solution* by *Frobenius ansatz* for $R(v)$ around its attractor r_-: $R(v) = r_- + \frac{1}{v^s}\sum_{k=0}^{\infty}\frac{a_k}{v^k}$ with $s > 0,\ a_0 \neq 0$

↓

Eq. 1) equating powers of v shows that a non-trivial solution requires $s = p$ and $p > 2$ → Eq. 2) explicitely reads

$$\frac{\frac{dm_+}{dv}}{Rn[v]^2 - 2Rn[v]\, m_+[v] + \epsilon^2} = \frac{\frac{dm_-}{dv}}{Rn[v]^2 - 2Rn[v]\, m_-[v] + \epsilon^2}$$

↓

$$\frac{\frac{dm_+}{dv}}{r_-^2 - 2r_-\, m_+[v] + \epsilon^2} \simeq \frac{p\, v^{-1-p}\, \beta}{\left[r_- + \frac{1}{v^p}\left(a_0 + \frac{a_1}{v}\right)\right]^2 - 2\left[r_- + \frac{1}{v^p}\left(a_0 + \frac{a_1}{v}\right)\right]\left(m_0 - \frac{\beta}{v^p}\right) + \epsilon^2}$$

↓

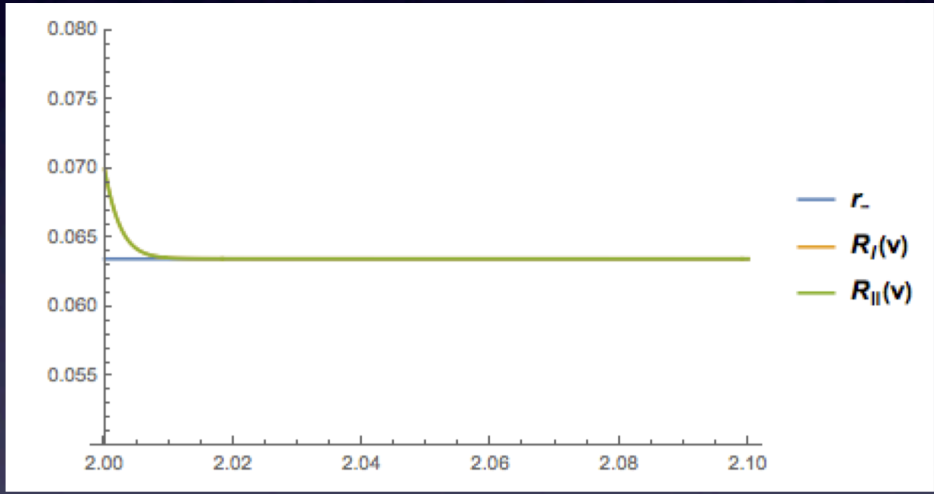
we can determine recursively all coefficients a_k

for R.-N. left-hand side analysis of (sub)leading behaviour in v is simple

↓

right-hand side analysis of (sub)leading behaviour in v takes a bit more time

Reissner-Nordstrom solution: phase space for the Cauchy horizon (in)stability and mass-inflation



$$\frac{dm_+}{r_-^2 - 2r_- m_+[v] + \epsilon^2}\frac{1}{dv} \simeq -\frac{r_- k_-}{2}\left(1-\frac{p+1}{k_-}\frac{1}{v}\right)$$

$$\frac{dm_+}{r_-^2 - 2r_- m_+ + \epsilon^2} \simeq \left[-\frac{r_- k_-}{2}\left(1-\frac{p+1}{k_-}\frac{1}{v}\right)\right] dv$$

"Surface gravity" at r_-:

$$k_- \equiv -\frac{1}{2}\frac{\partial f(r)}{\partial r}\Big|_{r_-} > 0$$

For $v \to \infty$ the shell Σ impacts the Cauchy horizon r_- and triggers the instability

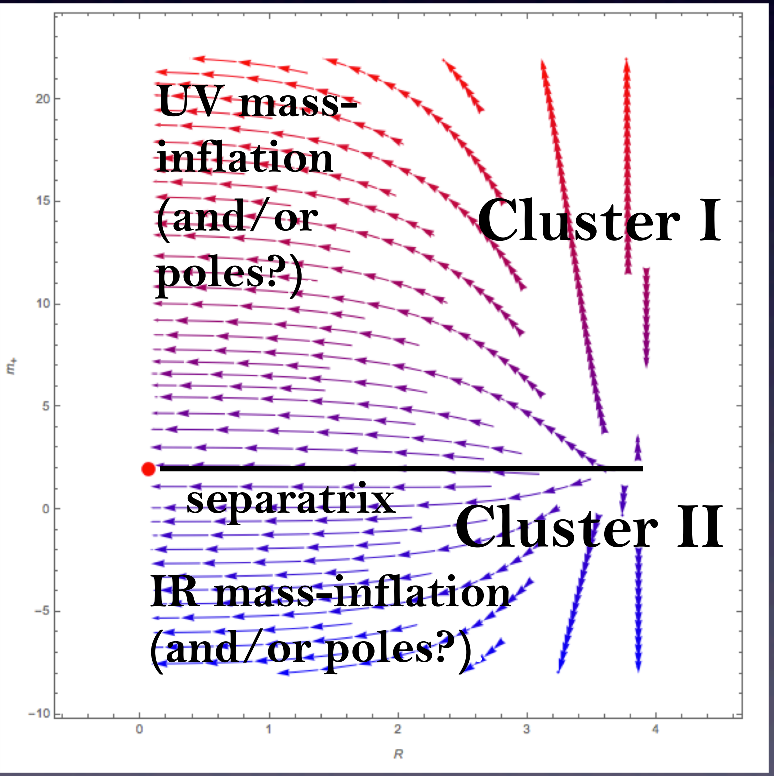
$m_+(v) \simeq C(e^{k_- v} v^{-2p})$ *exponential divergence* therefore "mass-inflation"

$M[R(v), m_+(v)]$ and $K[R(v), m_+(v)]$ will also show an exponential divergence

A curvature singularity builds up at the CH

Reissner-Nordstrom solution: phase space for the Cauchy horizon (in)stability and mass-inflation?

Photograh at $v = v^* = 1.5$

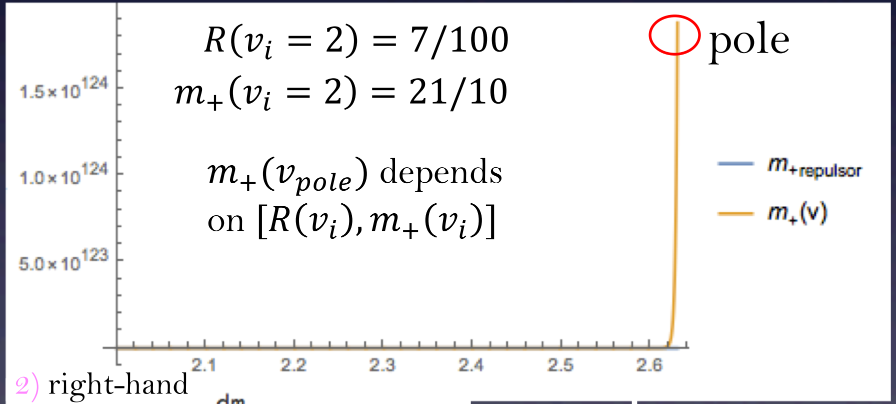


Equation for the separatrix:
$m_+ = m_{+repulsor} (> 0)$

local

One repulsive fixed point and two *clusters of initial conditions* $[R(v_i), m_+(v_i)]$: Cluster I and Cluster II, both lead to (IR and UV) mass-inflation (*)

Actually if *numerical integration* of the full Eqs. 1) and 2) is performed:



2) right-hand side: $\frac{dm_-}{dv}$

$\frac{Rn[v]^2 - 2Rn[v]\, m_-[v] + \epsilon^2}{}$

pole $[r_-, m_+(v_{pole})]$ is due to a "moving singularity" in this fraction, reached when

$m_-(v) = \frac{e^2 + R(v)^2}{2R(v)} > 0$

for $v = v_{pole}$ the curve has ∞ inclination

$m_+(v)$ reaches an accetable pole $m_+(v_{pole})$ (accetable since $R(v)$ always reaches r_- before the instant v_{pole}; if viceversa the dynamical description would just breakdowns)

(**) global

Two clusters of initial conditions: Cluster I and Cluster II, both lead to (IR and UV) poles?

Relation between (*) and (**) is still under investigation…

Simplifications, assumptions, observations and subtleties

Thanks to F. Di Filippo for a useful WhatsApp discussion (mediated by L. Buoninfante) on some of these remarks

- NB: no Hawking evaporation in the energy flux balance, and no QFT on curved backgrounds in general, is taken into account in this analysis. Here perturbations are fully classical.

Going to regular BHs:

- Ori model assumes metric has the same functional form inside and outside the shell. Since regular black holes are not sourced by vacuum GR, there is no Birkhoff theorem and this assumption becomes non-trivial.

- Ori model eqs. are obtained assuming a pressureless dust shell, and in GR this choice can always be done since there is a clear distinction between gravity and matter. Since regular black holes are sourced by an $T_{\mu\nu}^{eff}$, becomes non-trivial to justify that this choice is allowed.

- Cosmological coupling could affect, on the long time, what in the following is called ''attractor'' for the perturbed regular black hole, bringing the perturbed regular black hole out of the attractor.

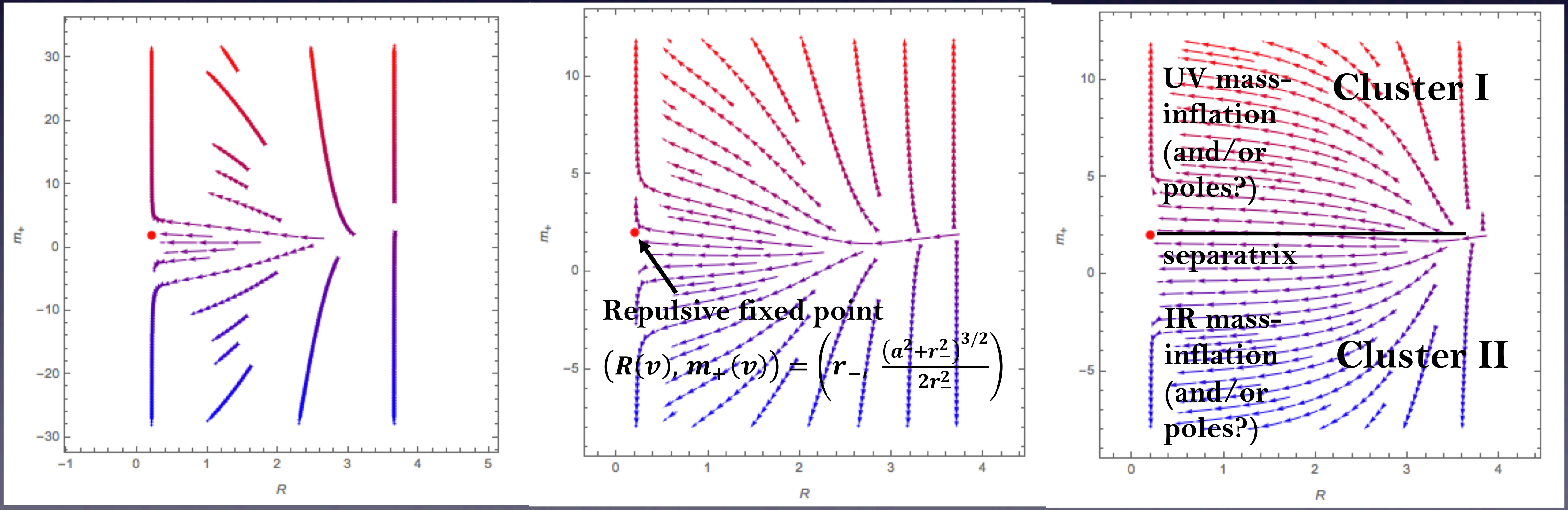
Bardeen solution:
phase space for the Cauchy horizon (in)stability

Exact photocopy of Reissner-Nordstrom phase space.

quantum charge

$$f(r) = \left[1 - \frac{2m_0 r^2}{(r^2+a^2)^{3/2}}\right] \qquad m_0 = 2, \qquad a = \frac{1}{2}, \qquad p = 11, \qquad \beta = 1$$

Photograh of the phase space vector field at $v = v^* = 1.3$ | Zoom | Photograh at $v = v^* = 1.5$



Hayward solution:
phase space for the Cauchy horizon (in)stability

$$f(r) = \left(1 - \frac{2m_0 r^2}{r^3 + 2m_0 l^2}\right) \qquad m_0 = 2, \qquad l = \frac{1}{2}, \qquad p = 11, \qquad \beta = 1 \longrightarrow \qquad r_- \cong 0.54, \qquad r_+ \cong 3.93$$

1) $$Rn'[v] == \frac{1}{2} - \frac{(2 - v^{-p}\beta)\, Rn[v]^2}{2\, l^2\, (2 - v^{-p}\beta) + Rn[v]^3}$$

2) $$m_+'[v] == \frac{p\, v^{-1-p}\, \beta\, \left(Rn[v]^3 + 2\, l^2\, m_+[v]\right)\left(Rn[v]^3 - 2\left(-l^2 + Rn[v]^2\right) m_+[v]\right)}{\left(2\, l^2\, (2 - v^{-p}\beta) + Rn[v]^3\right)\left(Rn[v]^3 - 2\, (2 - v^{-p}\beta)\left(-l^2 + Rn[v]^2\right)\right)}$$

- Fixed point: $(R(v), m_+(v)) = \left(r_-, \frac{r_-^3}{2(r_-^2 - l^2)}\right) \cong (0.54, 2)$ - It is $\frac{r_-^3}{2(r_-^2 - l^2)} \equiv m_{+_{repulsor}} > 0$

- Fixed point: $(R(v), m_+(v)) = \left(r_-, -\frac{r_-^3}{2l^2}\right) \cong (0.54, -0.31)$ - It is $-\frac{r_-^3}{2l^2} \equiv m_{+_{attractor}} < 0$

Hayward solution: $m_+(v)$ Frobenius solution around the attractor

1)*Analytical solution* by *Frobenius ansatz* for $R(v)$ around its attractor r_-: $R(v) = r_- + \frac{1}{v^s}\sum_{k=0}^{\infty}\frac{a_k}{v^k}$ with $s > 0,\ a_0 \neq 0$

↓

2)Plugging this ansatz in Eq. 1) and equating powers of v shows that a non-trivial solution requires $s = p$ and $p > 2$

↓

3)We determine recursively coefficients a_k up to $k = 2$ ⟶ analytical solution for $R(v)$ at order $k = 2$:

$$a_0 = \beta \frac{r_-^2}{(4 m_0 - 3 r_-)\, m_0}$$

$$a_1 = \beta p \frac{4 r_-^3}{(3 r_- - 4 m_0)^2}$$

$$a_2 = \beta p\, (p+1) \frac{16 r_-^4 m_0}{(4 m_0 - 3 r_-)^3}$$

↓

4)Plugging this result into $F(v)$, the right-hand side of Eq. 2), carrying on a careful analysis of powers of v in the numerator and in the denominator shows:

$$\left(\frac{1}{f_+}\frac{\partial M_+}{\partial v}\right)\Bigg|_{\Sigma} = F(v) \quad \text{with} \quad F[v] = \frac{R[v]^6\, m_-[v]}{\left(R[v]^3 + 2\, l^2\, m_-[v]\right)\left(R[v]^3 - 2\, m_-[v]\left(R[v]^2 - l^2\right)\right)}$$

$$F(v) \simeq -\frac{r_-\, k_-}{2}\left(1 - \frac{p+1}{k_-}\,\frac{1}{v}\right)$$

1)*Analytical solution* by *Frobenius ansatz* for $m_+(v)$ around its attractor: $m_+(v) = m_{+attractor} + \frac{1}{v^p}\sum_{k=0}^{\infty}\frac{b_k}{v^k}$

↓

2)Plugging this ansatz in the left-hand side of Eq. 2), carrying on a careful analysis of powers of v in the numerator and in the denominator, and comparing with right-hand side of Eq. 2), we can determine b_0 and recursively $b_1, b_2, \ldots$

↓

Hayward solution: $m_+(v)$ Frobenius solution around the attractor

↓

$$b_0 = -\frac{r_-^3}{2\,l^2}\, 3\beta\, \frac{1}{4\, m_0^2}\, \frac{4\, r_-\, m_0}{(4\, m_0 - 3\, r_-)}$$

$$b_1 = -\frac{r_-^3}{2\,l^2}\, 3\beta\, p\, \frac{1}{4\, m_0^2}\, \frac{32\, r_-^2\, m_0^2}{(3\, r_- - 4\, m_0)^2}$$

$$b_2 = -\frac{r_-^3}{2\,l^2}\, 3\beta\, p\,(1+p)\, \frac{1}{4\, m_0^2}\, \frac{4\times 2\times 32\, r_-^3\, m_0^3}{(4\, m_0 - 3\, r_-)^3}$$

In "Regular black holes with stable core", A. Bonanno, F, Saueressig, A. Khosravi, Phys. Rev. D 103, 124027 (2021) *they stop at* b_0

Hayward solution: $M_+[R(v), m_+(v)]$ and $K_+[R(v), m_+(v)]$ Frobenius solution around the attractor

1)Keep enough subleading k in the solutions for $R(v)$ and $m_+(v)$ (*the same order k for both otherwise you commit an inconsistency!*) to be safe

↓

2)Replace the solutions in $M[R(v), m_+(v)] \mapsto M_+(v)$ and carefully determine the leading term of the expression

↓

$$M_+[R(v),\, m_+(v)] \simeq \frac{2\, r_-^3\, m_0^2}{3\, l^2\, p\, \beta}\, k_-^2\, v^{p+1}$$

$$K_+[R(v),\, m_+(v)] \simeq \frac{4}{9}\, \frac{1}{l^4} \left(\frac{4\, m_0^2\, k_-^2}{p\, \beta}\, v^{p+1}\right)^6$$

Correct expressions: leading term depends on a_1 *and* b_1

$$M_+(r_-, m_+(v)) \simeq \frac{2\, r_-^3\, \kappa_-\, m_0^2}{3 l^2 \beta}\, v^p$$

$$K \simeq \frac{4}{9 l^4} \left(\frac{m_0^2\, \kappa_-\, v^p}{\beta}\right)^6$$

Wrong expressions: they use zero order r_- and first subleading b_0

Hayward solution: $M_+[R(v), m_+(v)]$ and $K_+[R(v), m_+(v)]$

Frobenius solution around the attractor

$$m_+(v) \simeq -\frac{r_-^3}{2l^2} + \frac{b_0}{v^p} + \frac{b_1}{v^{p+1}}$$

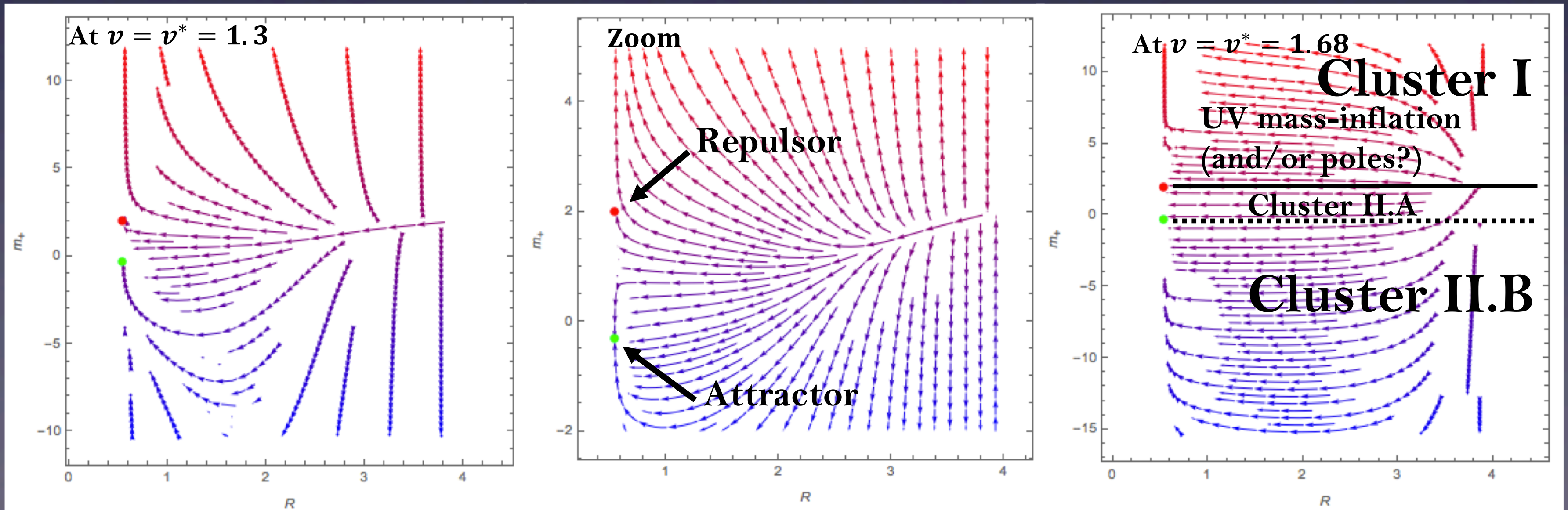
$$M_+[R(v), m_+(v)] \simeq \frac{2 r_-^3 m_0^2}{3 l^2 p \beta} k_-^2 v^{p+1}$$

$$K_+[R(v), m_+(v)] \simeq \frac{4}{9}\frac{1}{l^4}\left(\frac{4 m_0^2 k_-^2}{p \beta} v^{p+1}\right)^6$$

Cluster II.A and II.B

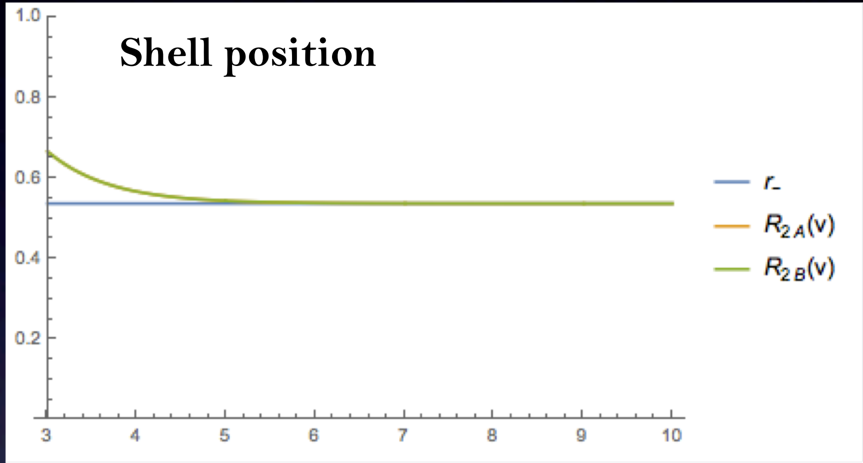
- For $v \to \infty$ *there is no mass-inflation.*
- Divergence of the Misner-Sharp and Kretschmann is *power-law* and not exponential.
- Exponent is $p+1$ with p from Price's law.
- Parameter p appears also in the numerical pre-factor.
- *Singularity at the CH builds up but is quenched* ⟶ could be integrable, then geodesics completeness at the CH could be preserved.

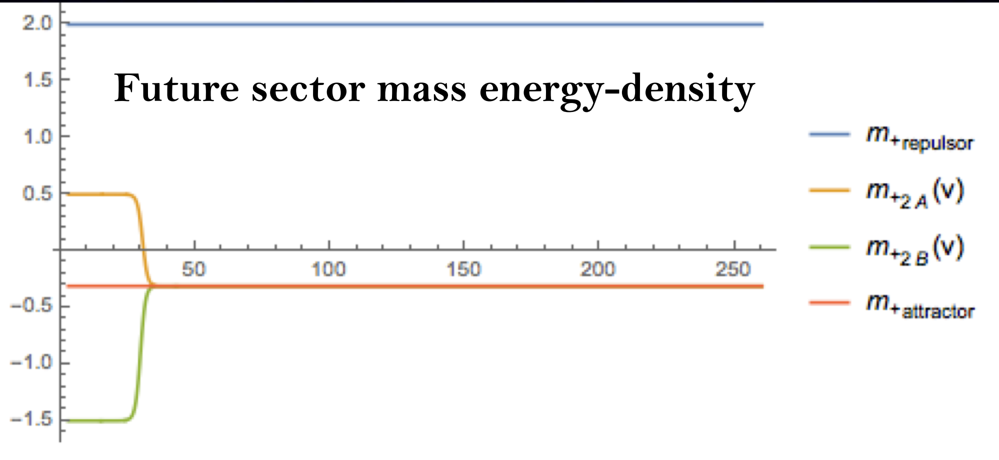
Hayward solution: phase space for the Cauchy horizon (in)stability



More articulated phase space

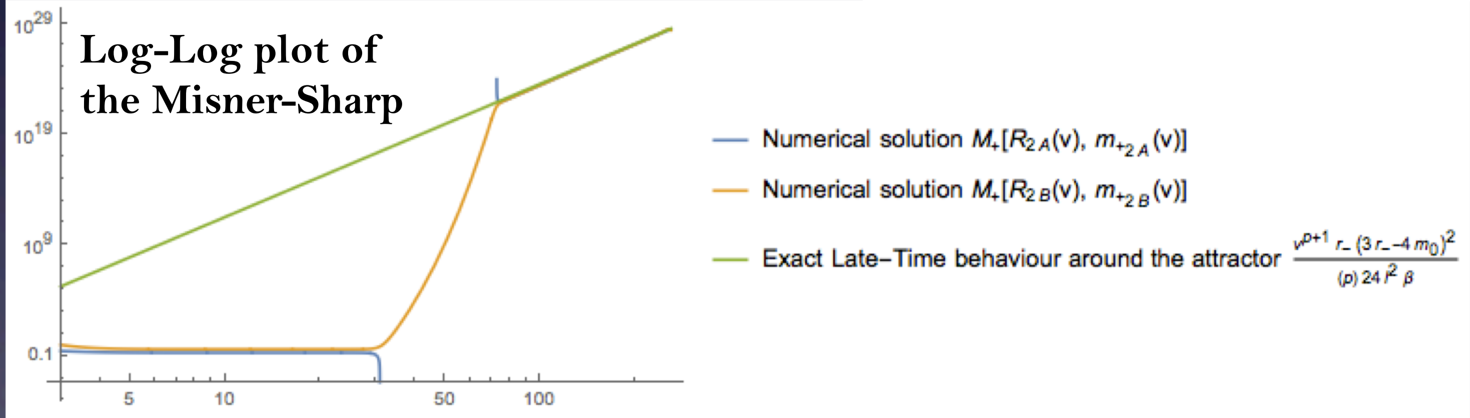
Hayward: Frobenius solution "VS" numerical solution for Cluster II

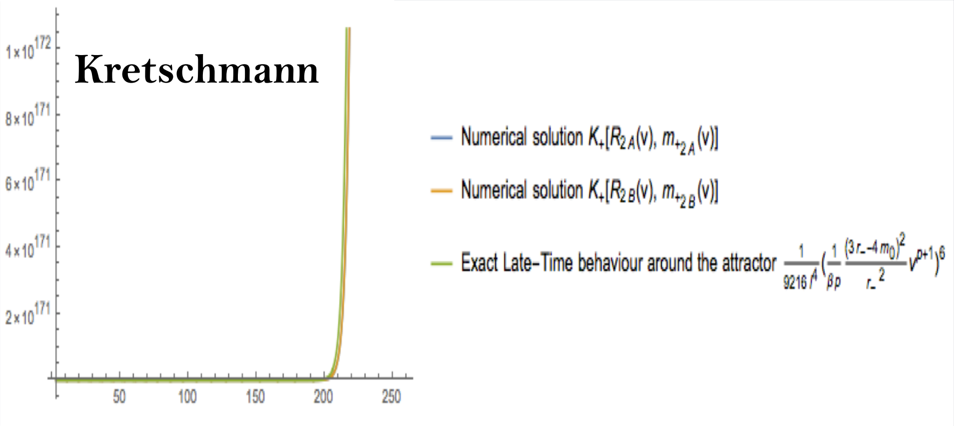


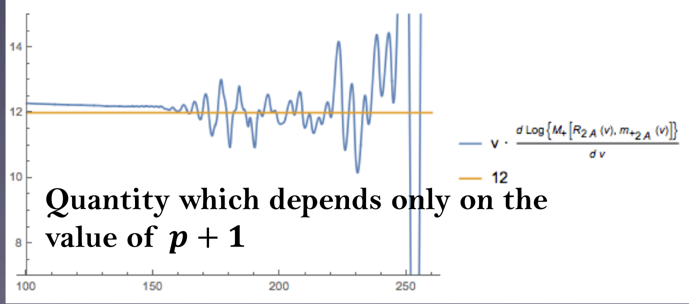


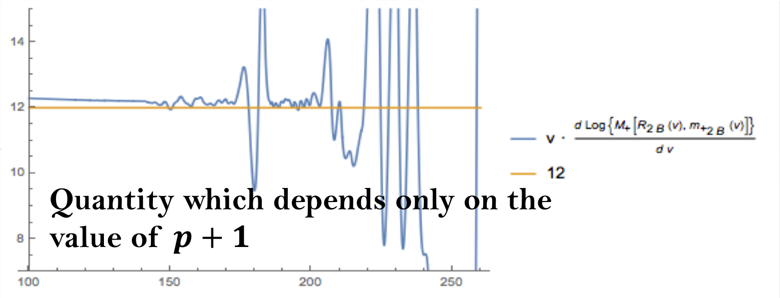
Trajectory II.A
$R(v_i = 3) = 2/3$
$m_+(v_i = 3) = 1/2$

Trajectory II.B
$R(v_i = 3) = 2/3$
$m_+(v_i = 3) = -3/2$





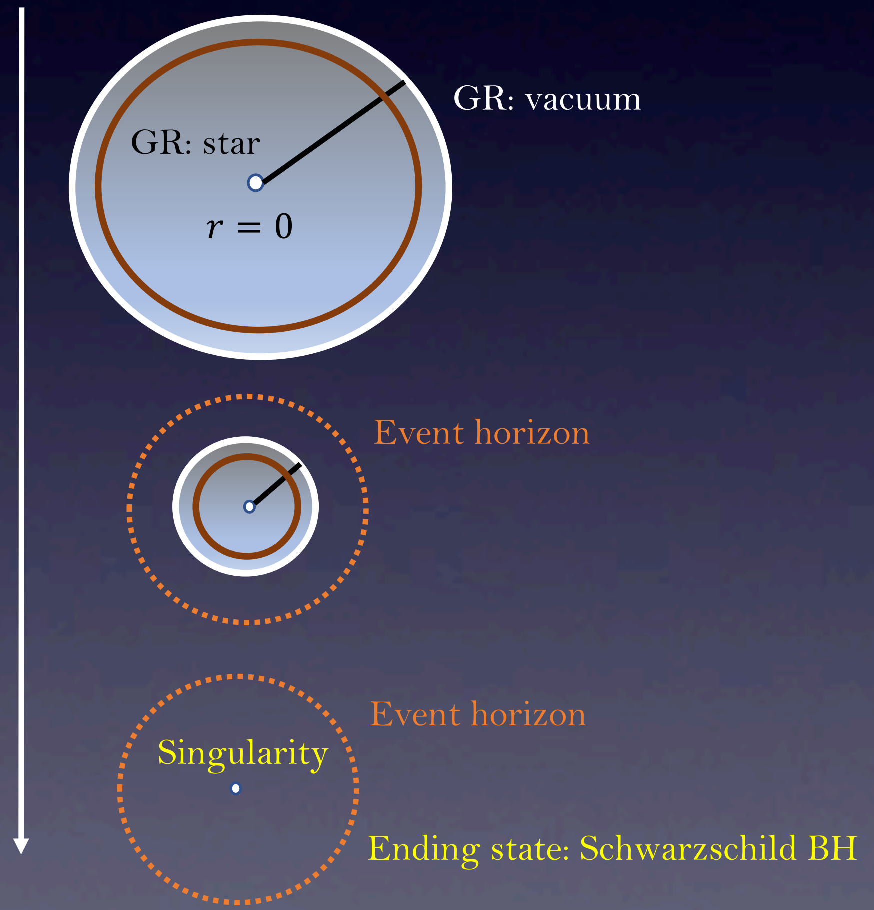




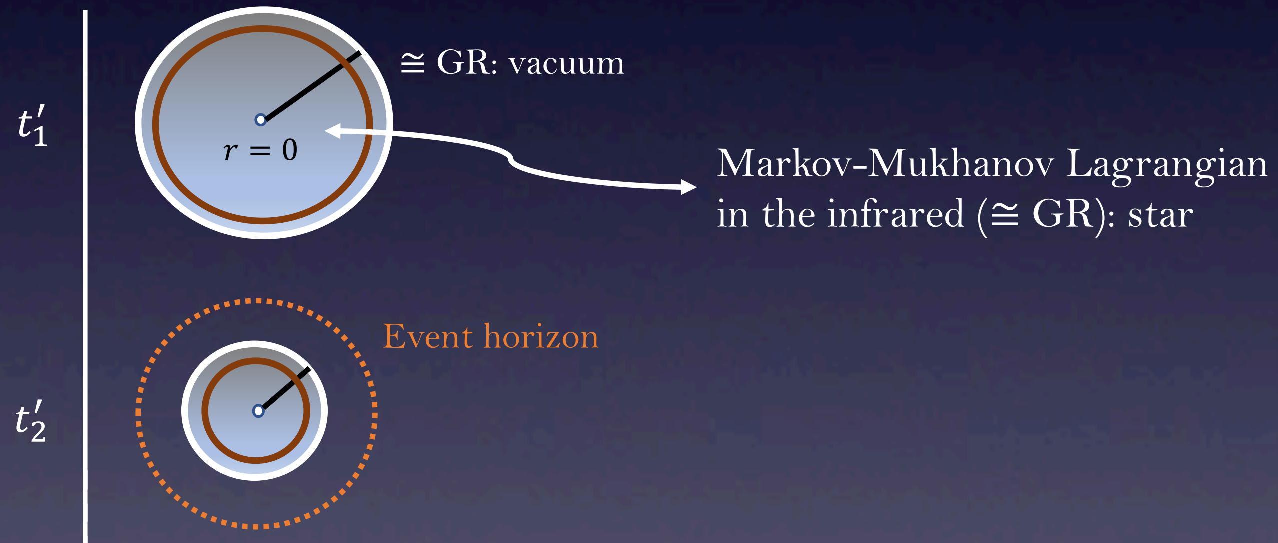
Extremely good agreement

Model for asymptotically safe gravitational collapse

Oppenheimer-Snyder collapse in General Relativity:
gravitational collapse → Schwarzschild BH



Our model of collapse implementing the idea of an *asymptotically safe gravitational interaction* (by means of a modified classical theory of gravity):
gravitational collapse → A new *regular* BH



Then the process enters in the *semiclassical regime*, after an energy density threshold is reached:
- running of the Newtonian coupling becomes significant
- gravitational potential turns repulsive (N.B. but the star keeps contracting)
- an hypothesis of the singularity theorem is violated

$$\mathcal{M} = \mathcal{M}_{star} \cup \mathcal{M}_{exterior}$$

$$ds^2_{star} = -dt^2 + a(t)^2 dr^2 + a(t)^2 r^2 d\Omega^2 \qquad \{t, r, \theta, \varphi\} \qquad 0 \le r \le r_{\rm b}$$

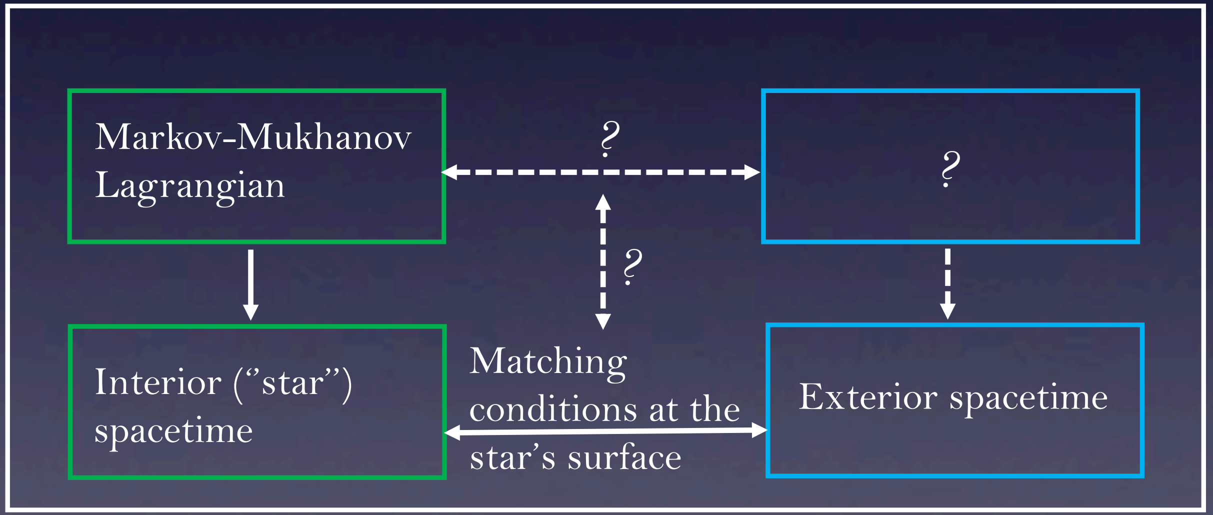
$$\frac{da}{dt} = -\sqrt{\frac{\log(1+3m_0\xi/a^3)}{3\xi}}\, a^2 \qquad a(t) \sim e^{-\frac{t^2}{4\xi}}, \quad t \to \infty$$

$$\updownarrow$$

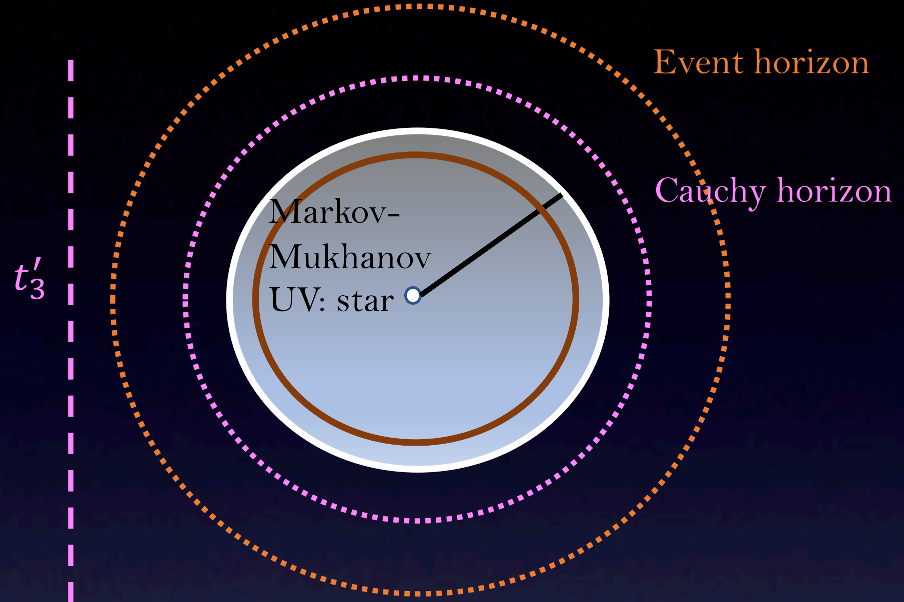
$$ds^2_{exterior} = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2 d\Omega^2 \qquad \{T, R, \theta, \varphi\}$$

$$R_b \le R < +\infty$$

$$f(R) \equiv \left[1 - \frac{2R^2}{6\xi} \log\left(1 + \frac{6M_0}{R^3}\xi\right)\right] \qquad R \ge R_b(T) = r_b a(t) > 0$$

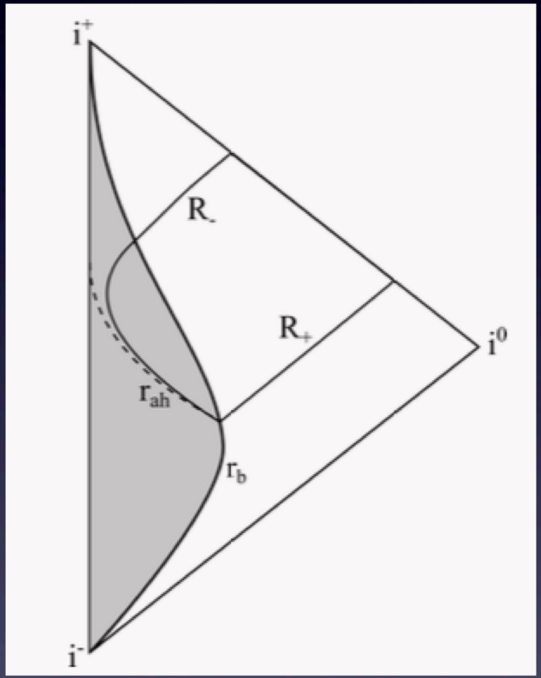


Ending state is that there is no ending state: an ongoing *“eternal collapse”* in the core



?Which effective theory (i.e. a classical modified theory of gravity that implements the leading quantum correction to vacuum GR) for the strong-gravity vacuum (i.e. outside the collapsing star)?

Solution from asymptotically safe gravitational collapse: phase space for the Cauchy horizon (in)stability



Penrose diagram courtesy of D. Malafarina.

We assume that perturbation arrives in this spacetime at $t > t_{CH} \equiv$ instant in which the CH forms

$$f(r) = \left[1 - \frac{r^2}{3\xi} Log\left(1 + \frac{6\xi m_0}{r^3}\right)\right] \qquad m_0 = 2, \quad \xi = 1, \quad p = 11, \quad \beta = 1$$

$\longrightarrow$ $r_- \cong 1.21, \quad r_+ \cong 3.54$

1) $$Rn'[v] == \frac{1}{2} - \frac{Log\left[\left(1 + \frac{6\,(2 - v^{-p}\beta)\,\xi}{Rn[v]^3}\right)^2\right] Rn[v]^2}{12\,\xi}$$

2) $$m_+'[v] == \frac{p\, v^{-1-p}\, \beta \left(1 - \frac{Log\left[\left(1 + \frac{6\,\xi\, m_+[v]}{Rn[v]^3}\right)^2\right] Rn[v]^2}{6\,\xi}\right) \left(Rn[v]^3 + 6\,\xi\, m_+[v]\right)}{\left(1 - \frac{Log\left[\left(1 + \frac{6\,(2 - v^{-p}\beta)\,\xi}{Rn[v]^3}\right)^2\right] Rn[v]^2}{6\,\xi}\right) \left(6\,(2 - v^{-p}\beta)\,\xi + Rn[v]^3\right)}$$

- Fixed point: $(R(v), m_+(v)) = \left(r_-, \frac{r_-^3}{6\xi}\left(e^{\frac{3\xi}{r_-^2}} - 1\right)\right) \cong (1.21, 2)$ - It is $\frac{r_-^3}{6\xi}\left(e^{\frac{3\xi}{r_-^2}} - 1\right) \equiv m_{+_{repulsor}} > 0$
- Fixed point: $(R(v), m_+(v)) = \left(r_-, \; -\frac{r_-^3}{6\xi}\right) \cong (1.21, -0.29)$ - It is $-\frac{r_-^3}{6\xi} \equiv m_{+_{attractor}} < 0$

Spacetime from asymptotically safe gravitational collapse: $m_+(v)$ Frobenius solution around the attractor

$$a_0 = \beta \frac{r_-^3}{r_-^3 + 6\xi m_0 - 3 r_-^2 m_0}$$

$$a_1 = \beta p \frac{r_-^4 (r_-^3 + 6\xi m_0)}{(r_-^3 + 6\xi m_0 - 3 r_-^2 m_0)^2}$$

$$a_2 = \beta p (1+p) \frac{r_-^5 (r_-^3 + 6\xi m_0)^2}{(r_-^3 + 6\xi m_0 - 3 r_-^2 m_0)^3}$$

⟷ 1) $R(v) = r_- + \frac{1}{v^p}\sum_{k=0}^{\infty} \frac{a_k}{v^k}$

↓

2) Frobenius ansatz $m_+(v) = -\frac{r_-^3}{6\xi} + \frac{1}{v^p}\sum_{k=0}^{\infty}\frac{b_k}{v^k}$ fails because the singular point of the differential eq. 2) is inside a logarithm. Nevertheless attractor is there.

↓

3) With some intuition we can still obtain exact expression for Misner-Sharp attractor

$$M_+[R(v), m_+(v)] \simeq \frac{r_-^3}{6\xi} \operatorname{Log}\left[\frac{\alpha}{v^{p+1}}\right]$$

$$K_+[R(v), m_+(v)] \simeq \frac{9}{\xi^2}\left(\frac{v^{p+1}}{p\gamma}\right)^4$$

$p\gamma = \alpha$

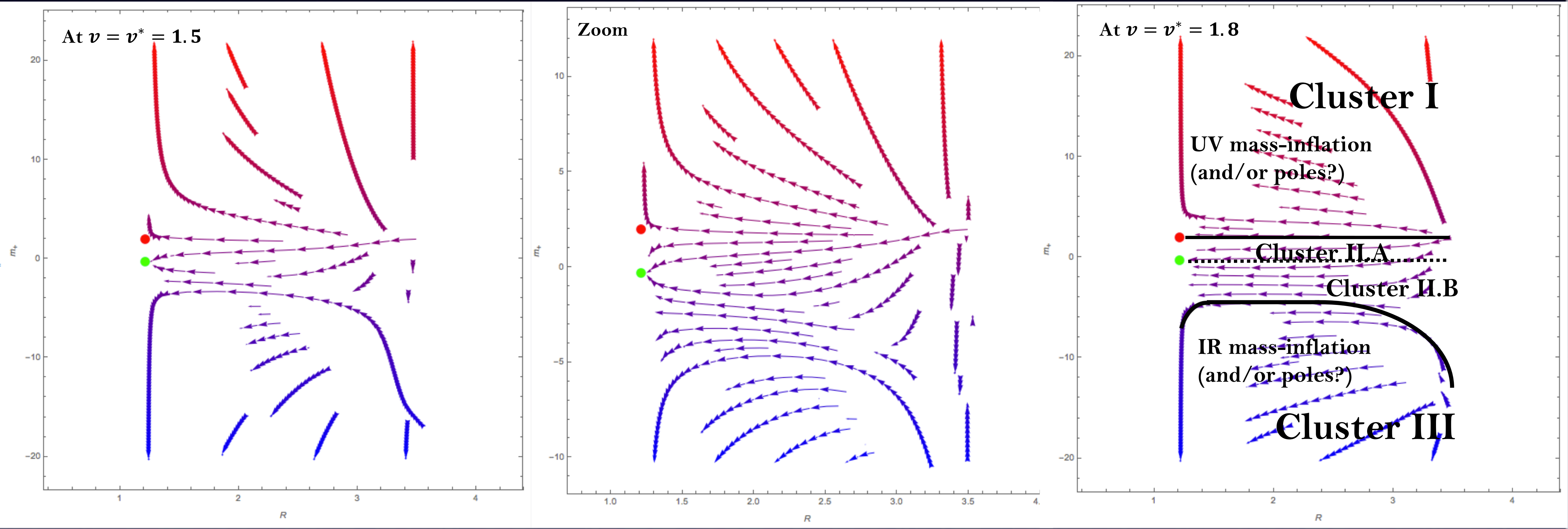
- For $v \to \infty$ *there is no mass-inflation.*
- Divergence of the Misner-Sharp and Kretschmann is, respectively, *logarithmic* and of *power-law* type, not exponential.
- Exponent is $p+1$ with p from Price's law.
- *Singularity at the CH builds up but is quenched* ⟶ could be integrable, then geodesics completeness at the CH could be preserved.

Cluster II.A and II.B

Coefficient α is assumed to depend on p in a multiplicative way and is obtained by a best-fit of points from the numerical solution once attractor regime is reached

Solution from asymptotically safe (AS) gravitational collapse: phase space for the Cauchy horizon (in)stability

Two fixed points, *two separatrices*, three Clusters of initial conditions: Cluster I, Cluster II, Cluster III

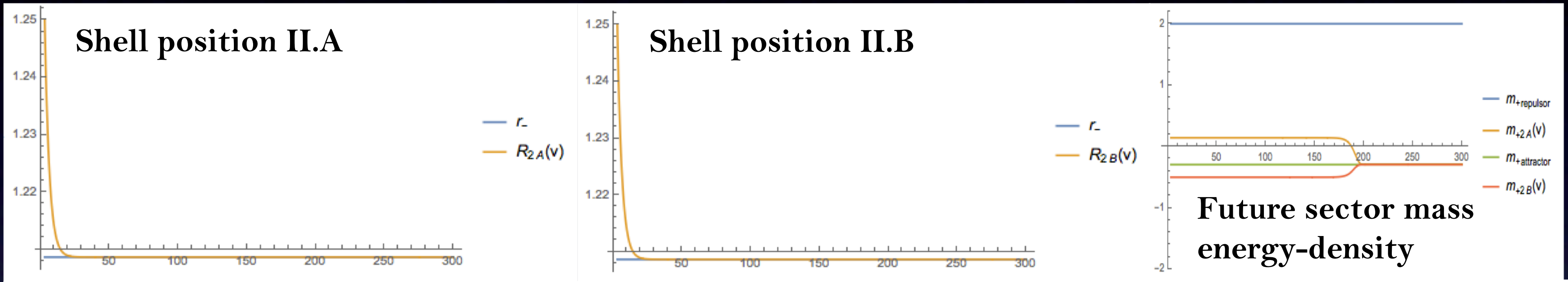


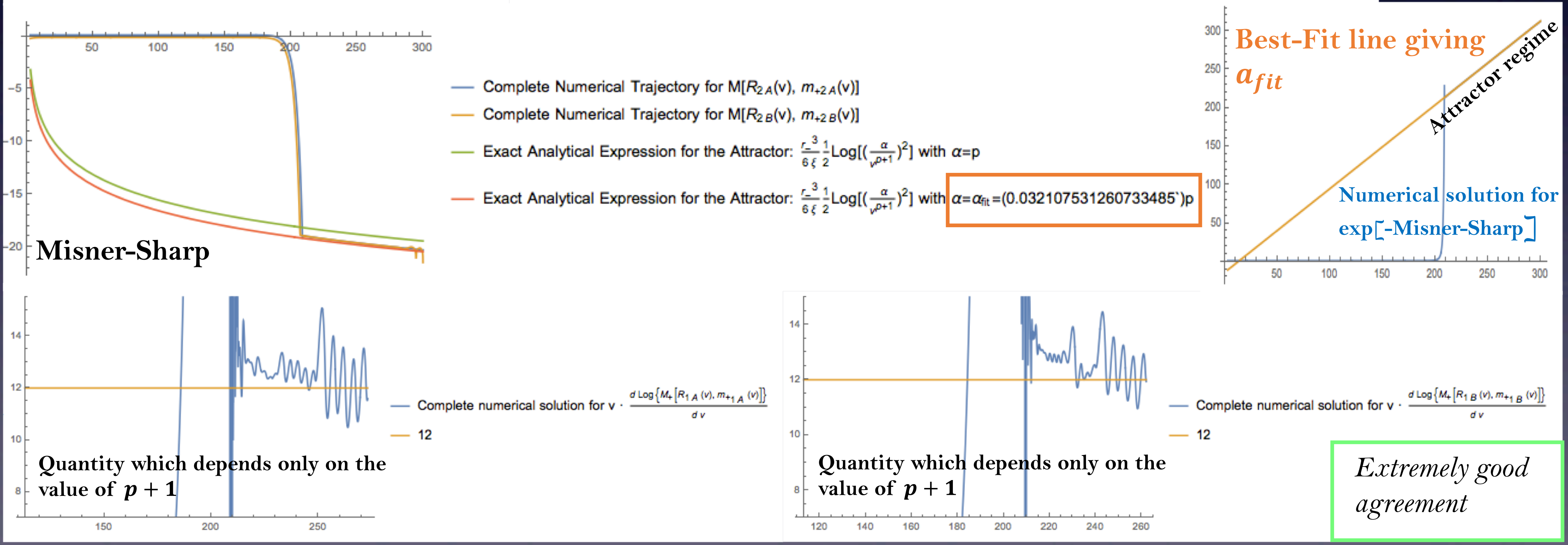
A different phase space structure

AS: Frobenius solution "VS" numerical solution for Cluster II

Trajectory II.A
$R(v_i = 3) = 5/4$
$m_+(v_i = 3) = 1/7$

Trajectory II.B
$R(v_i = 3) = 5/4$
$m_+(v_i = 3) = -1/2$





The three universality classes and their possible ending states

- Bardeen regular BH has the same phase space of the Reissner-Nordstrom BH.

- We also have studied the Bonanno-Reuter regular BH: it has the same phase space of the Hayward regular BH.

↓

What determines the phase space structure is the *functional form* of left-hand side of eq. 2) (*with respect to* m_+)

	Linear in m_+:	**Quadratic in m_+:**	**Logarithmic in m_+:**
Solutions	Reissner-Nordstrom, Bardeen	Hayward, Bonanno-Reuter	Solution from AS gravitational collapse
Clusters of initial conditions and their related ending states	Cluster I: UV mass-inflation Cluster II: IR mass-inflation	Cluster I: UV mass-inflation Cluster II: Misner-Sharp and Kretschmann scalar are power-law divergent	Cluster I: UV mass-inflation Cluster II: Misner-Sharp is logarithmically divergent Kretschmann scalar is power-law divergent Cluster III: IR mass-inflation

On the astrophysical viability of regular black hole solutions

A simple argument to show that from the study of the Cauchy horizon (in)stability maybe we could learn something about the degree of astrophysical viability of regular black hole solutions:

1)We can fix $R(v_i) \approx (r_- + r_+)/2$ without loss of generality.

↓

2)In order to talk about perturbations, it should be $\frac{|m_+(v_i)|}{m_0} < 1$, otherwise we are dealing with a competitor astrophysical object, not with a perturbation.

↓

3)Notice that $m_{+_{attractor}}$ is always one order of magnitude smaller than the mass of the black hole m_0.

↓

4)From 2) we have that a phisically motivated initial condition for the perturbation has to belong to a close and limited interval. We can take $m_{+_{attractor}}$ as center of this interval, without making an ad hoc choice because of 3):

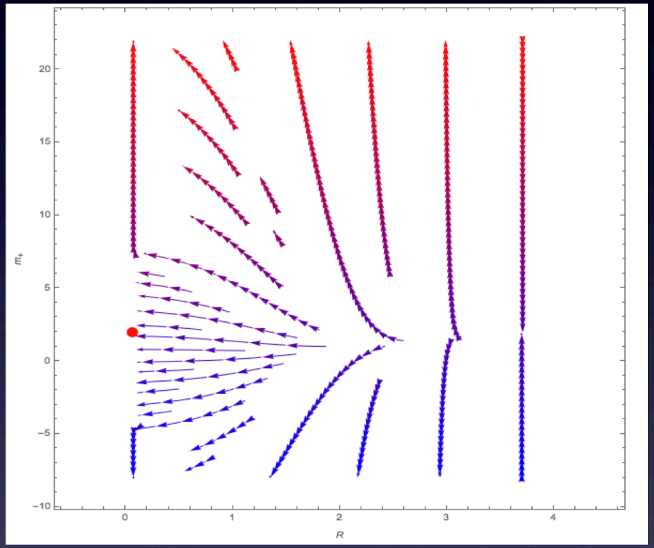
$$m_+(v_i) \in [m_{+_{attractor}} - \delta m_+,\ m_{+_{attractor}} + \delta m_+]$$

↓

5)From this study, a-posteriori, we learn that the only phase space showing a compact cluster (bounded both from above and from below) of initial conditions is the one of the solution from asymptotically safe gravitational collapse.

On the astrophysical viability of regular black hole solutions

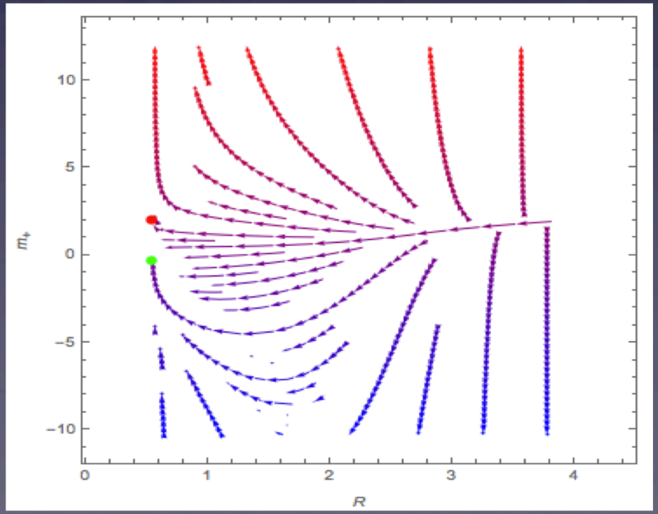
If we buy this argument:



(Regular) BHs belonging to Linear Class:
Reissner-Nordstrom, Bardeen

↓

unphysical



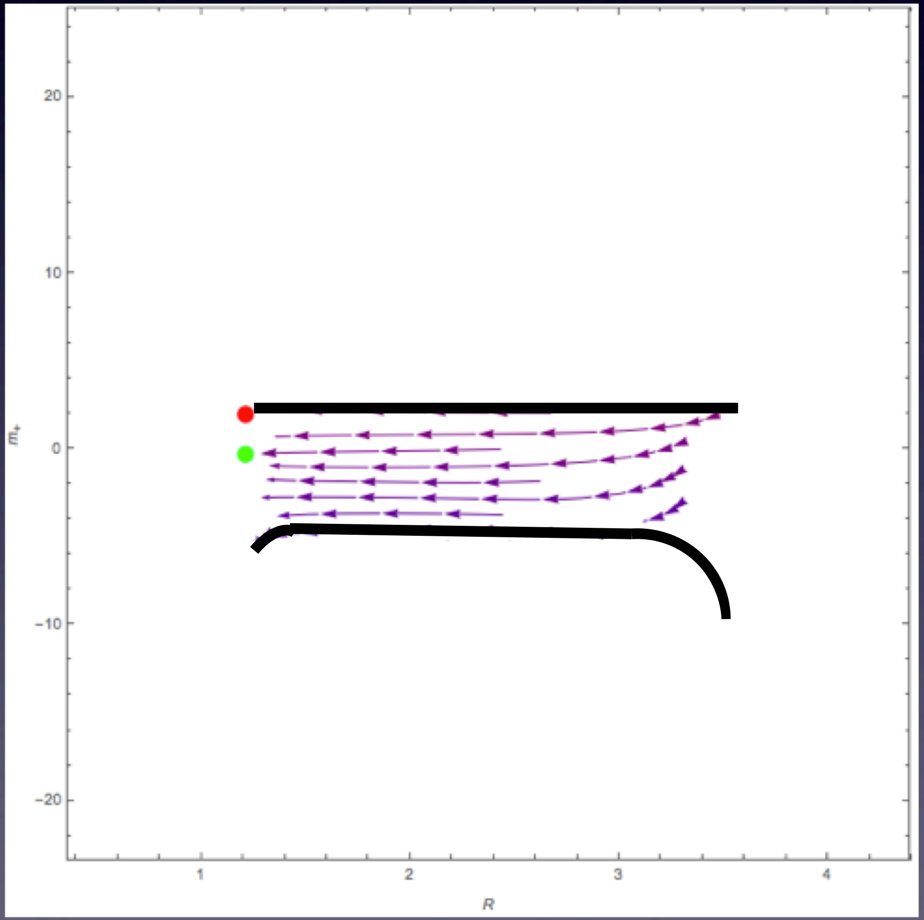
Regular BHs belonging to Quadratic Class:
Hayward, Bonanno-Reuter

↓

unphysical

On the astrophysical viability of regular black hole solutions

If we buy this argument:



Regular BH belonging to Logarithmic Class: solution from asymptotically safe gravitational collaps

↓

phase space of CH (in)stability suggests astrophysical viability

Conclusions and outlooks

Results

- Global study of the phase space related to the CH instability of (regular) BHs

- Three universality classes for the possible phase spaces

- For certain universality classes, for certain clusters of initial conditions mass-inflation instability is avoided in favour of quenched divergences

Conclusion

- Quenched divergences could allow for geodesic completeness at the CH. Then perturbed regular black holes could preserve the absence of strong curvature singularities (and actually remain regular)

Possible flaws

- Application of Ori model to regular BHs could imply non-trivial assumptions

Possible outlooks

- Including into the balance the ingoing Hawing flux bringing negative energy

- Possible geodesic completeness at the CH calls for proposals for the type of *physics beyond the CH* itself

Thank you for your attention!